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Title: Designing Universal Causal Deep Learning Models: The Case of Infinite-Dimensional Dynamical Systems from Stochastic Analysis

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Causal operators (CO), such as various solution operators to stochastic differential equations, play a central role in contemporary stochastic analysis; however, there is still no canonical framework for designing Deep Learning (DL) models capable of approximating COs. This paper proposes a "geometry-aware" solution to this open problem by introducing a DL model-design framework that takes suitable infinite-dimensional linear metric spaces as inputs and returns a universal sequential DL model adapted to these linear geometries.

We call these models Causal Neural Operators (CNOs). Our main result states that the models produced by our framework can uniformly approximate on compact sets and across arbitrarily finite-time horizons Hölder or smooth trace class operators, which causally map sequences between given linear metric spaces.

Our analysis uncovers new quantitative relationships on the latent state-space dimension of CNOs which even have new implications for (classical) finite-dimensional Recurrent Neural Networks (RNNs). We find that a linear increase of the CNO's (or RNN's) latent parameter space's dimension and of its width, and a logarithmic increase of its depth imply an exponential increase in the number of time steps for which its approximation remains valid. A direct consequence of our analysis shows that RNNs can approximate causal functions using exponentially fewer parameters than ReLU networks.