Workshop on Mechanics of the Earthquake Cycle

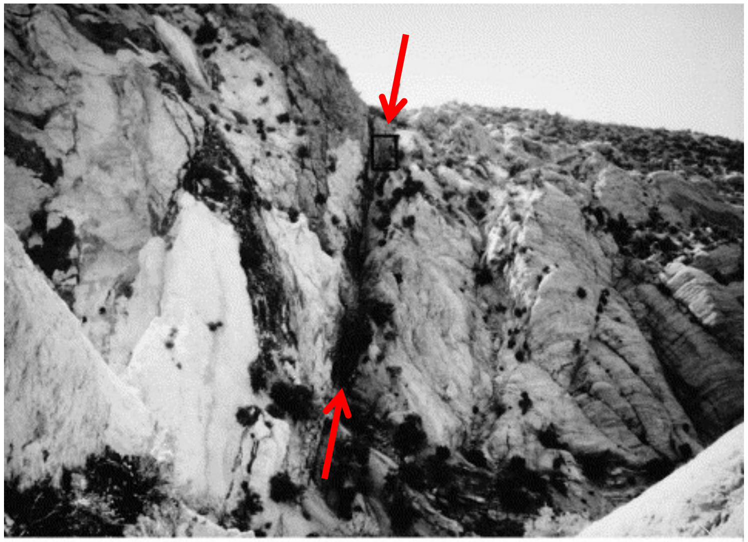
ICTP, Trieste, 16-27 October, 2023

Wednesday October 18, Lecture 1:

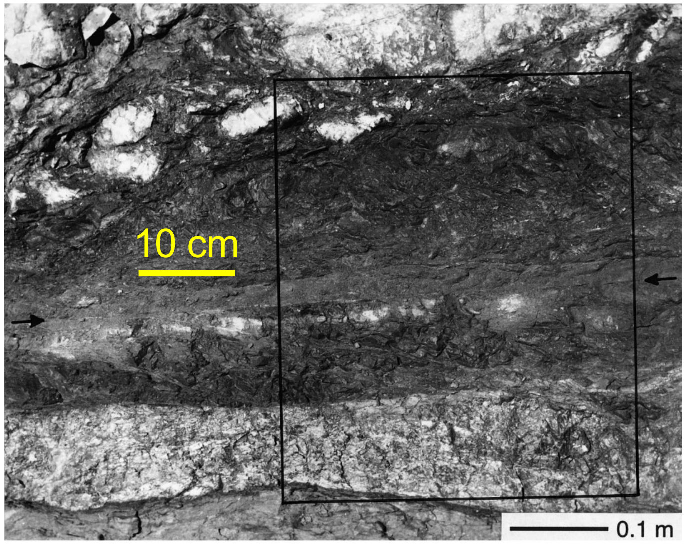
Rock friction

Allan Rubin, Princeton University

Surface expression of San Andreas fault



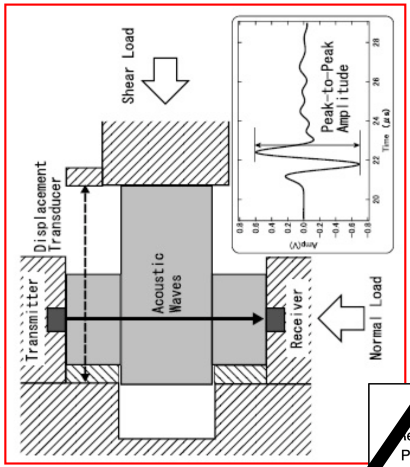
(Inactive) Punchbowl fault

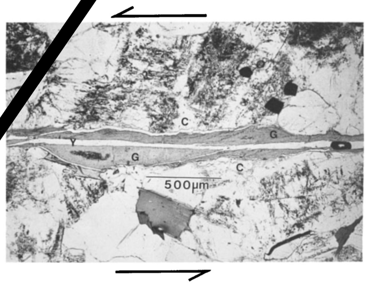


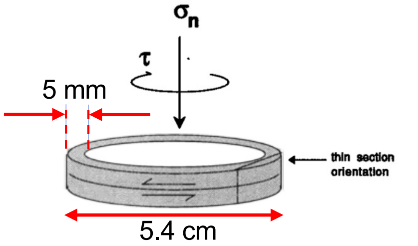
Punchbowl fault zoom-in

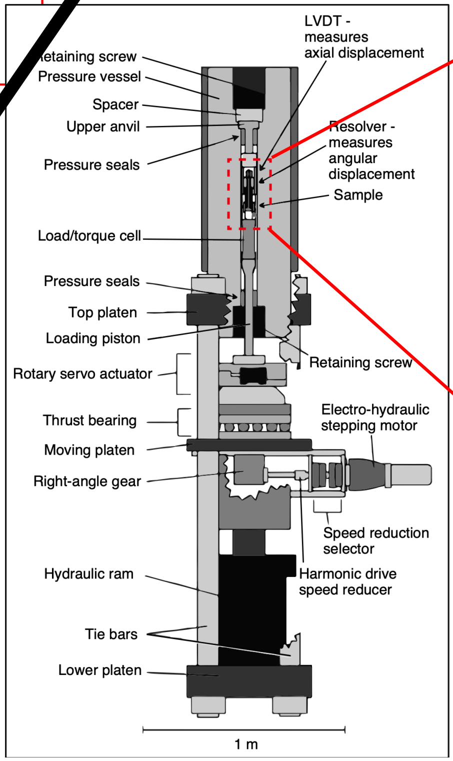
If you want to model fault slip, you need to couple a constitutive equation for fault friction with an equation for deformation of the surrounding rock (elastic; elastic-plastic; elasto-visco-plastic; etc.).

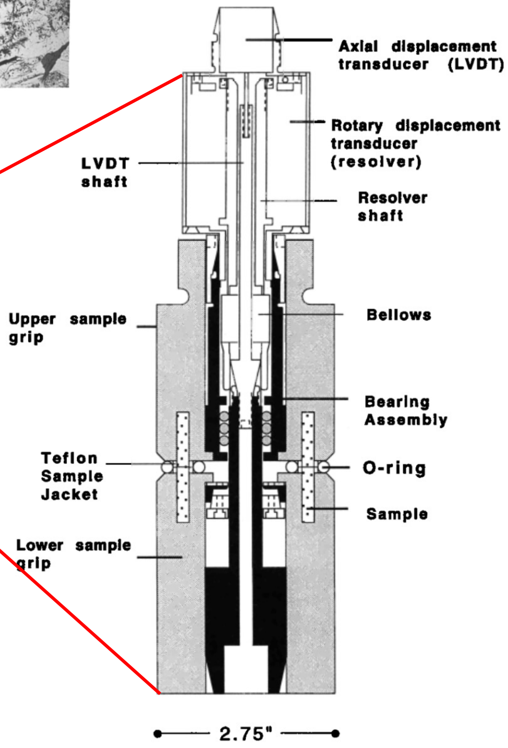
Chris Marone's lab,
Penn State Univ.









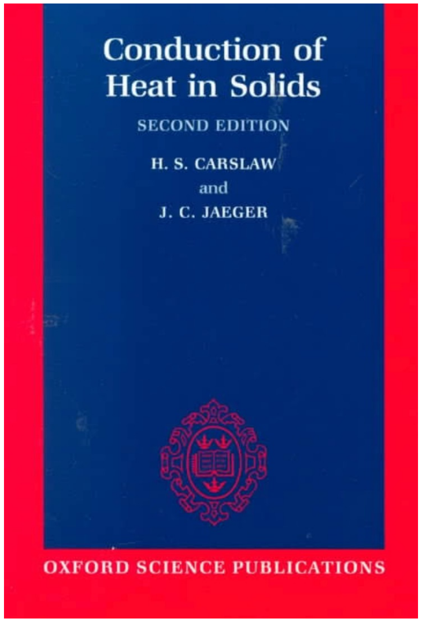


Terry Tullis' lab,
Brown Univ.

"There are only two things you need to know about friction –

It is always 0.6, and it will always make a monkey out of you."

John Conrad Jaeger
(as quoted by Mark Zoback,
Reservoir Mechanics, 2010)


Conduction of Heat in Solids
SECOND EDITION
H. S. CARSLAW
and
J. C. JAEGER
OXFORD SCIENCE PUBLICATIONS

The current state-of-the-art: "Rate- and State-dependent friction". The state-dependence gets you from "static" to "dynamic" friction in a physically plausible way.

A reliable constitutive equation for fault friction is difficult to obtain, because:

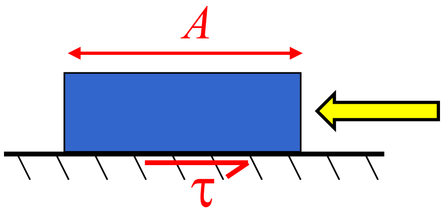
(1) Rocks, and most materials that have been investigated, are opaque.
Hard to observe “state”.

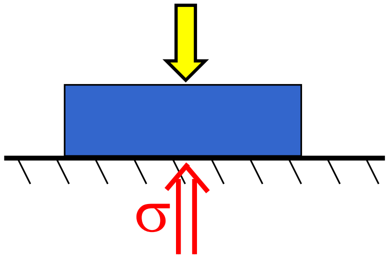
(2) Processes such as earthquake nucleation depend upon the *time variation* of friction, not its base value. At fault slip speeds less than ~1 cm/s, the “rate-dependence” and the “state-dependence” of friction amount to only a few percent of the base value. Whether or not a fault can nucleate earthquakes depends upon the small difference between these small, opposing influences.

(3) A lot of work on granular flow (numerical and experimental) has been done in the physics and engineering communities. These tend to focus more on steady-state friction, not the friction transients important to seismologists. (In rock friction experiments, "gouge” and initially bare surfaces show similar phenomenology).

Existing friction “laws” are largely empirical, making extrapolation to the Earth
a fraught endeavor.

SOME NOTATION:





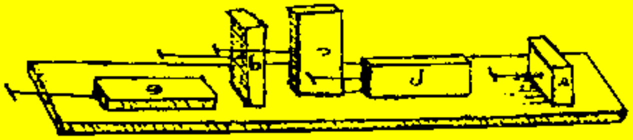
Shear stress (force/area A) is τ; contact shear stress and area are τ_c and A_c.

Normal stress is σ; contact normal stress is σ_c.

Friction, $\dfrac{\text{shear stress}}{\text{normal stress}}$ during sliding, is μ.

(But in rate-state friction, the surface is always sliding, provided $\tau \neq 0$, so $\mu = \tau/\sigma$, always.)

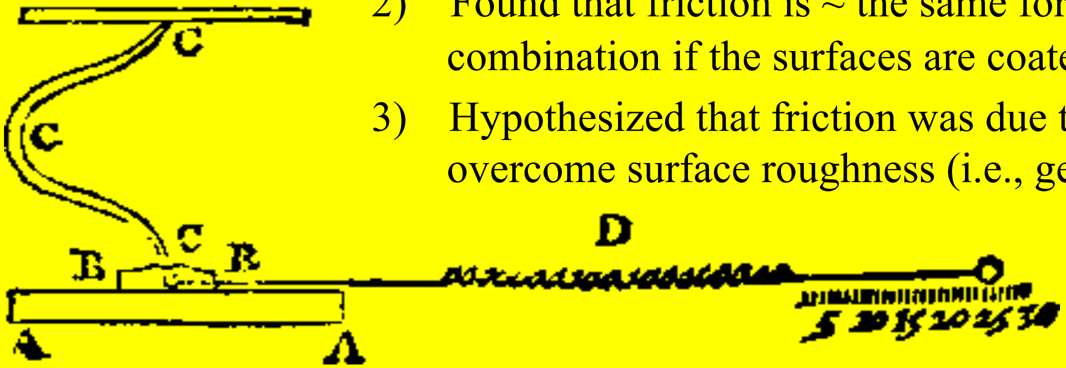
Leonardo Da Vinci (1452-1519):



1) For a given weight, the friction force (the shear force required for sliding) is independent of the surface area.
2) Friction force is proportional to weight.
   (nowadays, we would say shear stress [friction force per area] is proportional to normal stress [weight per area])
3) Found a “universal” coefficient of friction of ~0.25.

Guillaume Amontons (1663-1705):

1) Rediscovered Da Vinci’s (unpublished) laws.
2) Found that friction is ~ the same for iron, lead, copper and wood in any combination if the surfaces are coated w/pork fat (coefficient of friction ~1/3).
3) Hypothesized that friction was due to the work required to overcome surface roughness (i.e., geometric in origin).



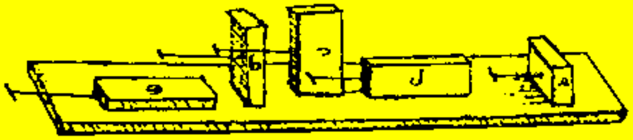
https://www.tribology-abc.com/abc/history.htm

John Theophilus Desagulier (1683-1744): Noted that Amontons' idea implied that smoother surfaces would have reduced friction, but experimentally found "flat surfaces of metals or other bodies may be so far polished as to increase friction." Suggested that chemical adhesion was responsible, and measured adhesion of freshly-cut iron balls.

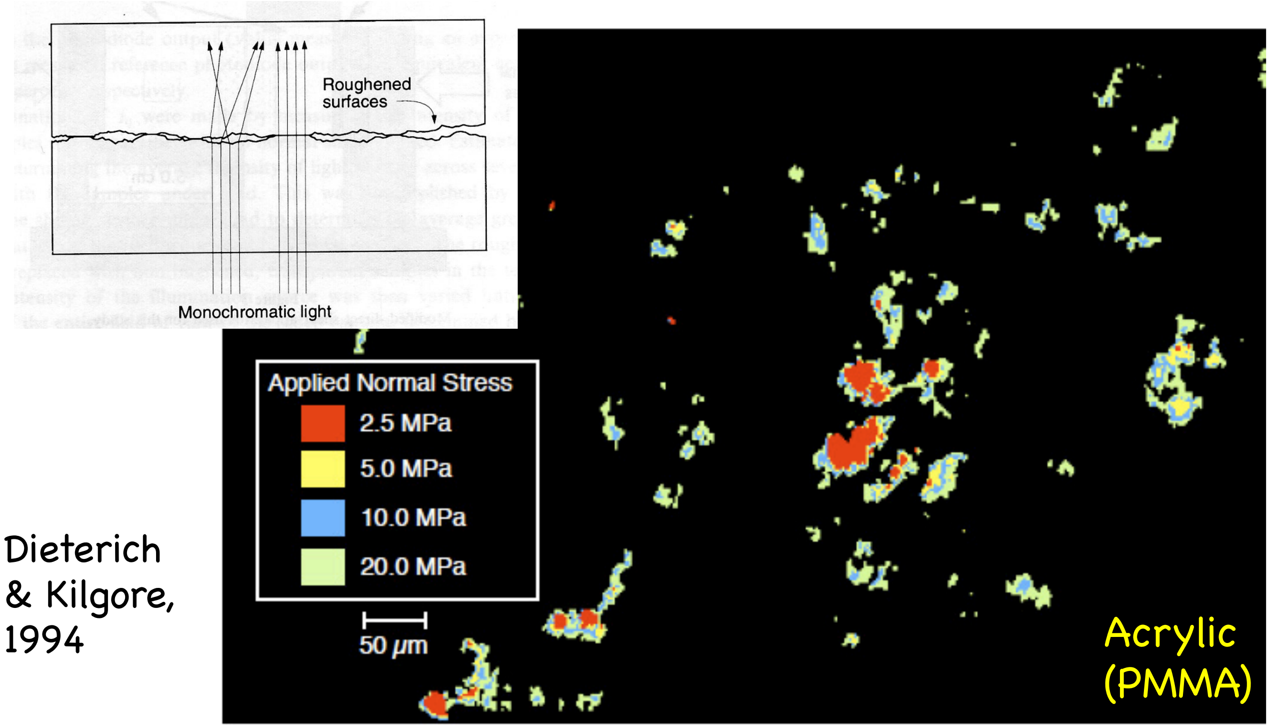
Leonhard Euler (1707-1783): Because motion of blocks down an incline begins rapidly (as slope is slowly increased), introduced the idea that "static friction" is larger than "kinetic friction".

Charles August Coulomb (1736-1806):

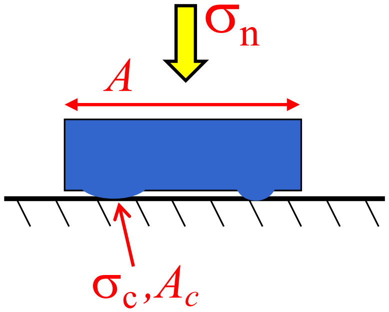
1) Kinetic friction is largely independent of sliding speed (now we would say there is a weak [logarithmic] dependence on sliding speed).
2) Static friction increases with time of rest (now we would say it increases logarithmically with time of rest).
3) Since Desagulier's adhesion idea seemed to violate area-independence, went back to Amontons' idea that friction was the force necessary to overcome surface roughness.



Rate- and state-dependent friction: Frictional resistance depends upon slip *rate* and the "*state*" of the fault surface:

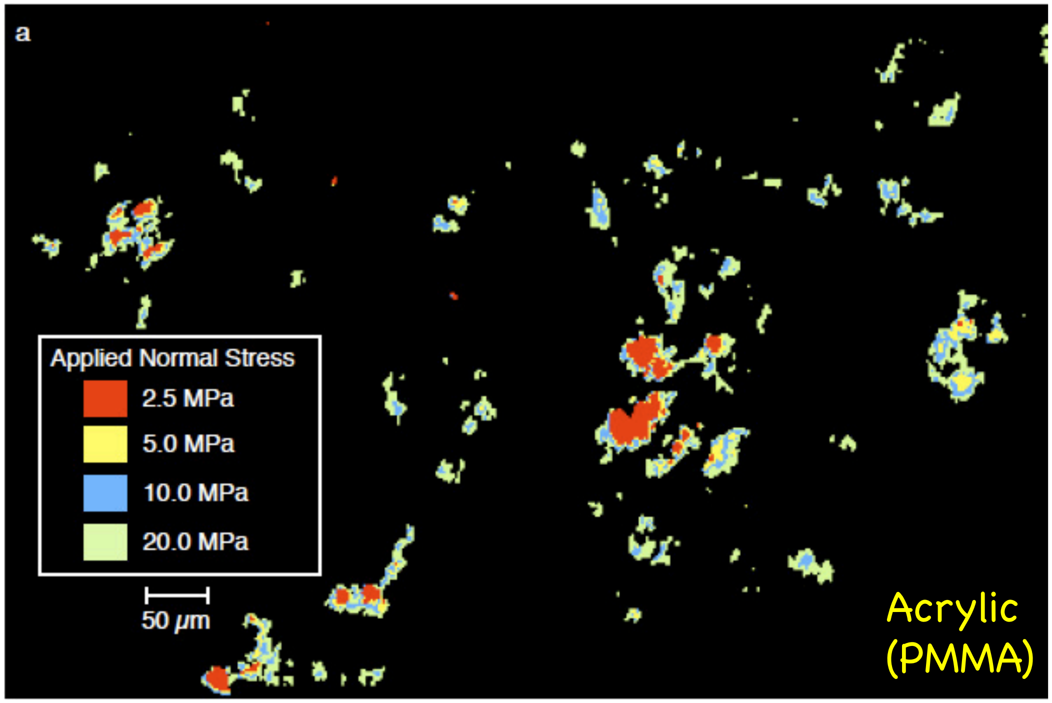


Dieterich & Kilgore, 1994

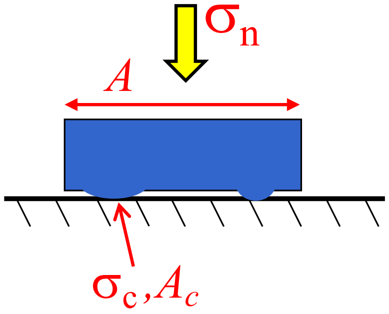


Consider an interface with nominal area A subjected to a normal stress σ_n.
If contacting asperities flow rapidly at a contact normal stress σ_c,
then from force balance $A\sigma_n = A_c\sigma_c$, and contact area $A_c/A = \sigma_n/\sigma_c$
is proportional to the applied normal stress σ_n (Bowden & Tabor, 1950).

Rate- and state-dependent friction: Frictional resistance depends upon slip *rate* and the "*state*" of the fault surface:

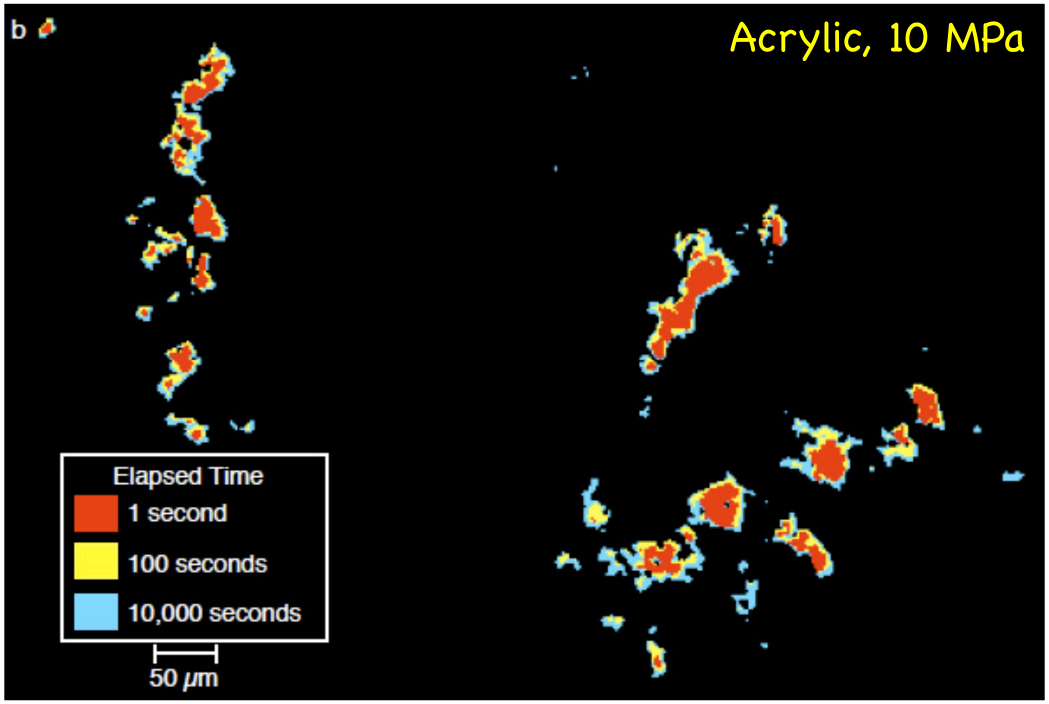


Dieterich
& Kilgore,
1994



The "rate" part of rate-state friction is reasonably well understood (we think). To slide at a faster rate requires breaking bonds at a faster rate, which requires a larger applied stress.

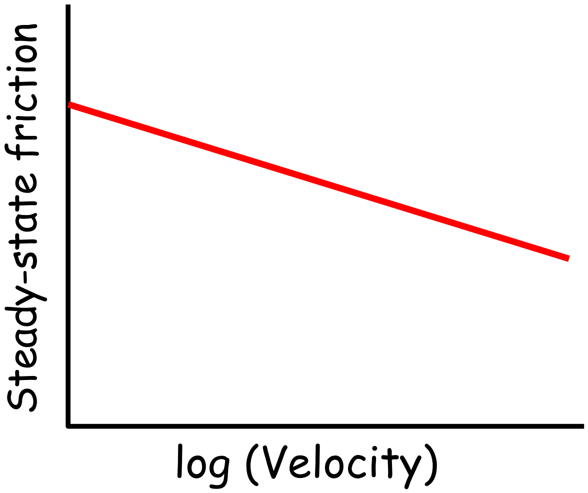
State is some measure of true contact area, and the “intrinsic strength” of those contacts (strength at a reference slip speed). Asperity size increases with age (and decreasing slip speed; if D is an asperity dimension, age $T \sim D/V$):



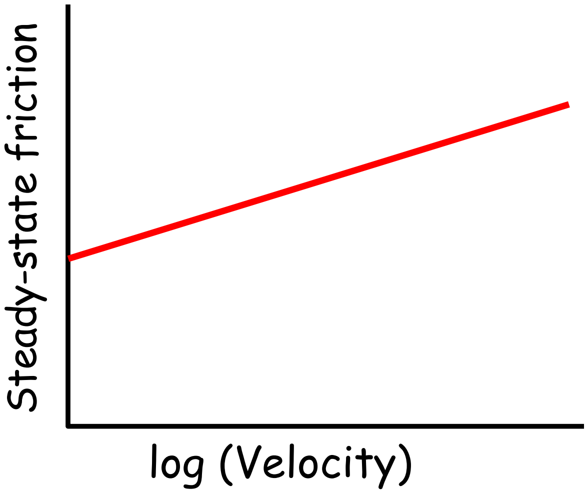
Dieterich
& Kilgore,
1994

Rate-state friction a competition between the “direct rate effect” (faster = stronger) and the “state evolution effect” (faster = weaker). Earthquakes can nucleate only if the “state evolution effect” wins out.

Over at least a few orders of magnitude in sliding speed,
steady-state friction varies approximately linearly with log(slip speed)

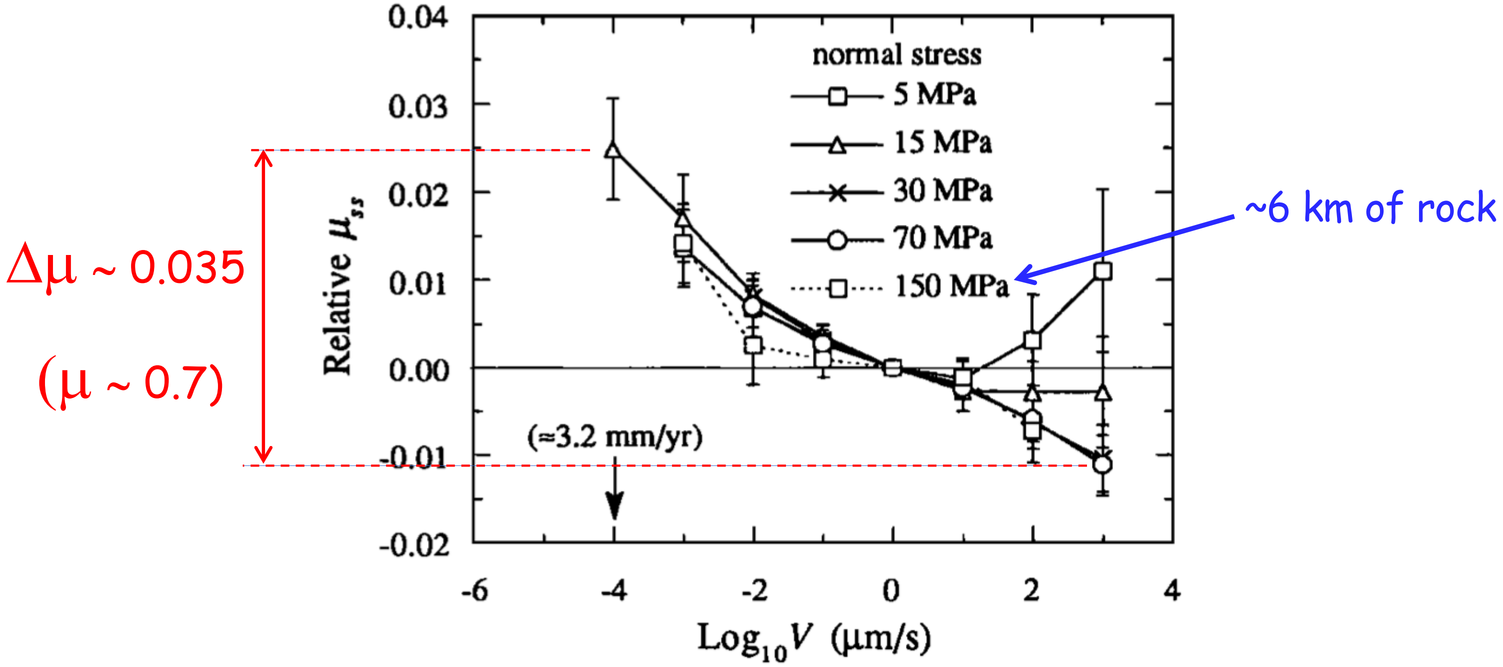


Steady-state velocity weakening
(reduction in state wins out):
Instability possible.



Steady-state velocity strengthening
(increased slip speed wins out):
Inherently stable.

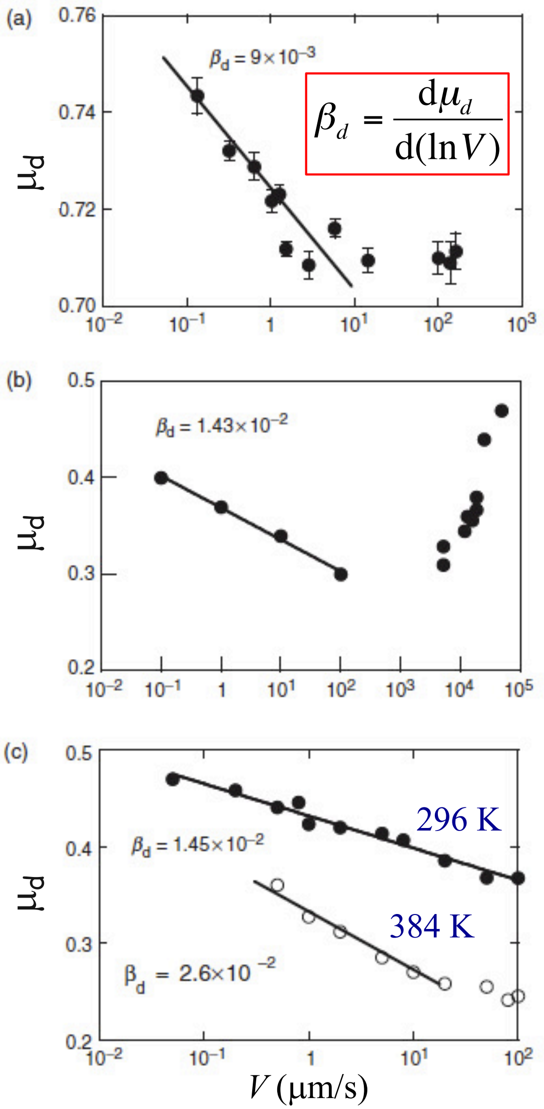
Kilgore & Dieterich, 1993



Granite surfaces at room T and humidity.
Steady-state velocity-weakening (~5% over 7 orders of magnitude in V).
State-weakening wins out over rate-strengthening.
Earthquake nucleation possible (except at low normal stress and large slip speeds?).

This behavior is shared by glass, metals, paper, acrylics, etc.

Even the magnitude of the steady-state velocity dependence is shared.



Granite

Paper

PMMA

$$\boxed{\beta_d = \frac{d\mu_d}{d(\ln V)}}$$

~0.01

Baumberger & Caroli, Advances in Physics, 2006

How the surface evolves when NOT at steady state matters!

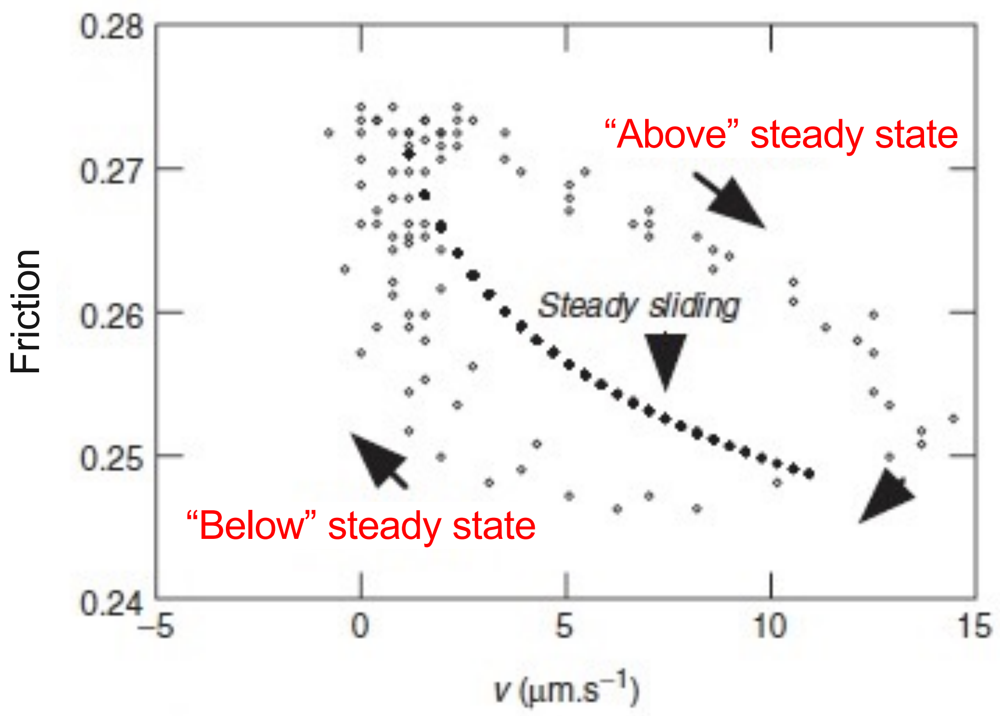
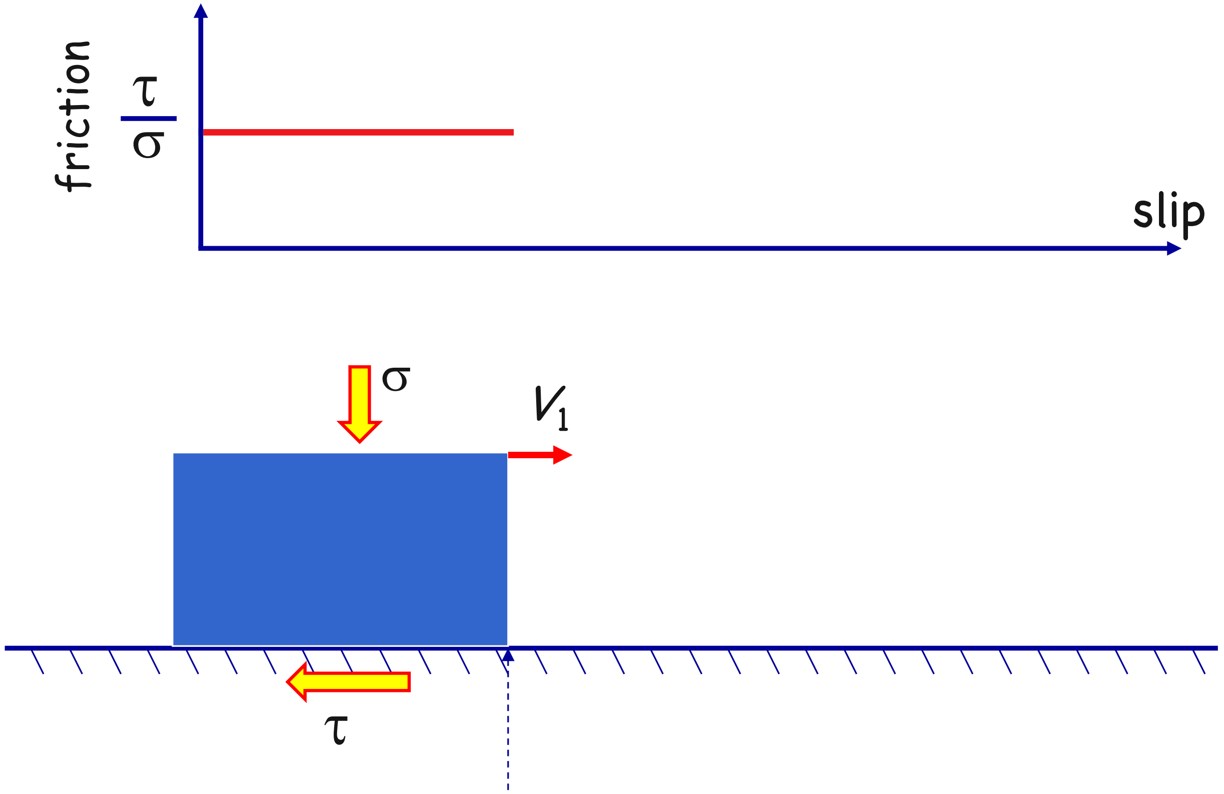
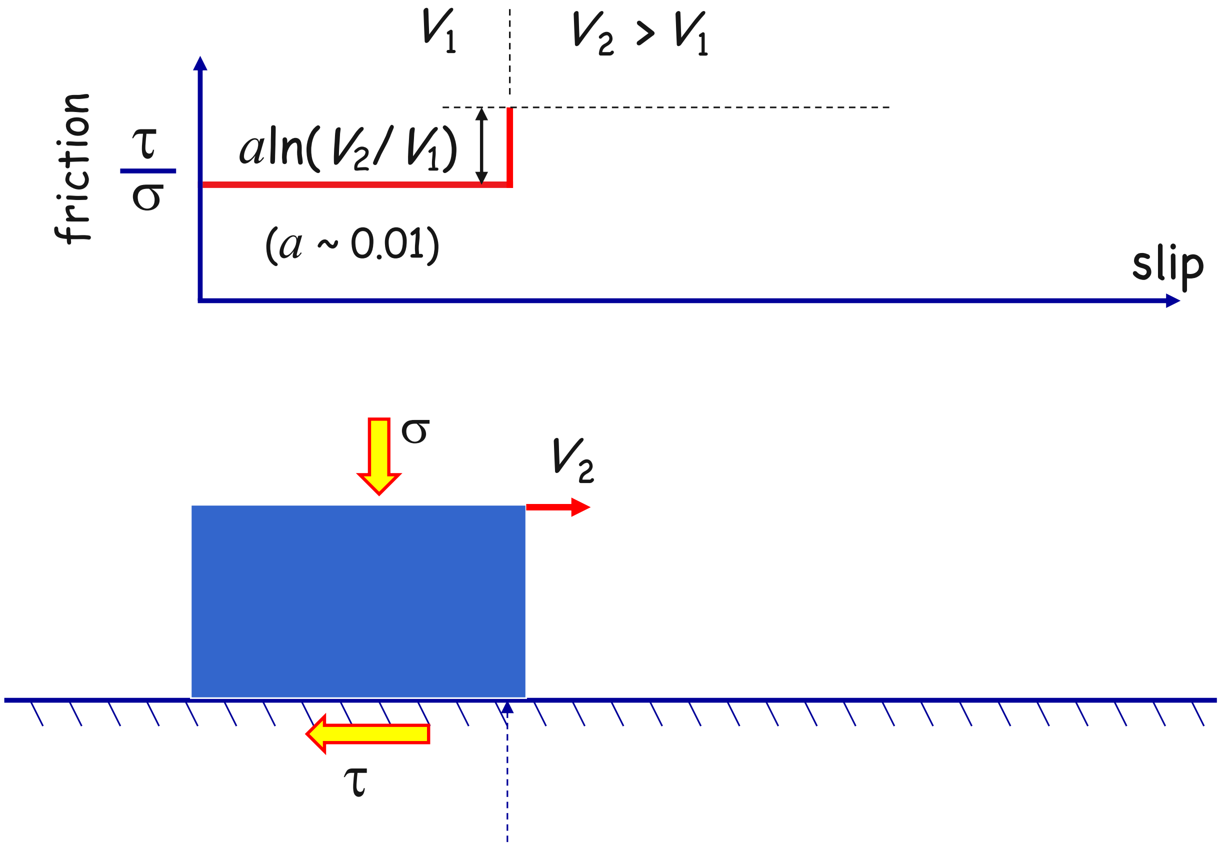


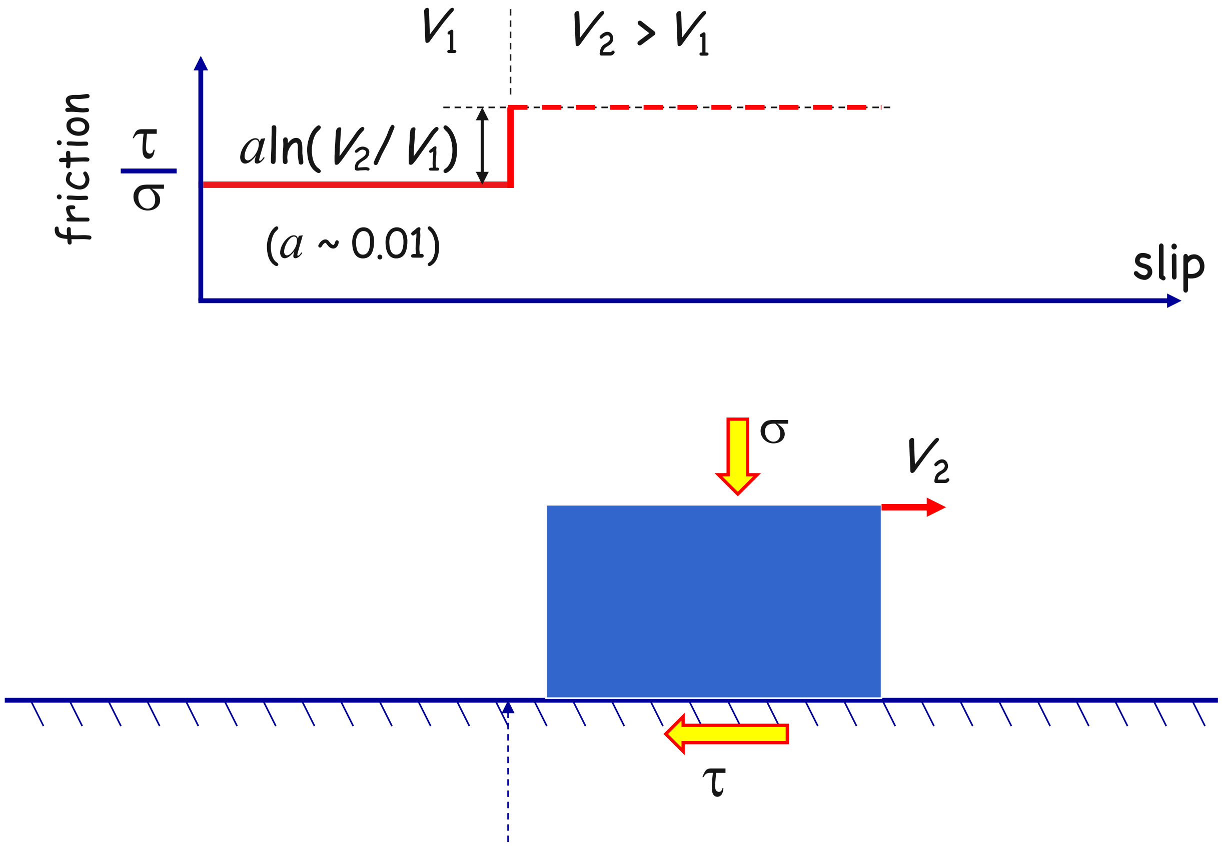
Figure 3. Hysteretic friction force response of a paper/paper system in non-steady sliding (open circles) and for steady sliding at various velocities V (filled circles).

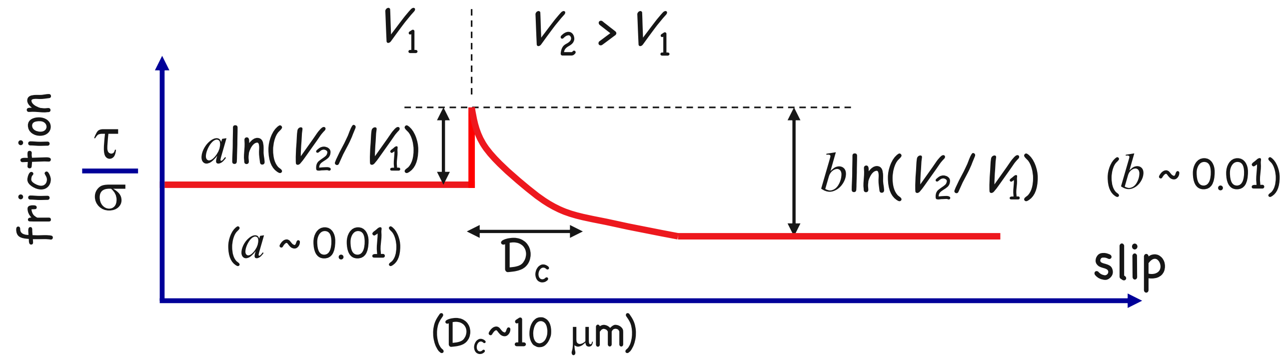
Baumberger & Caroli, Advances in Physics, 2006

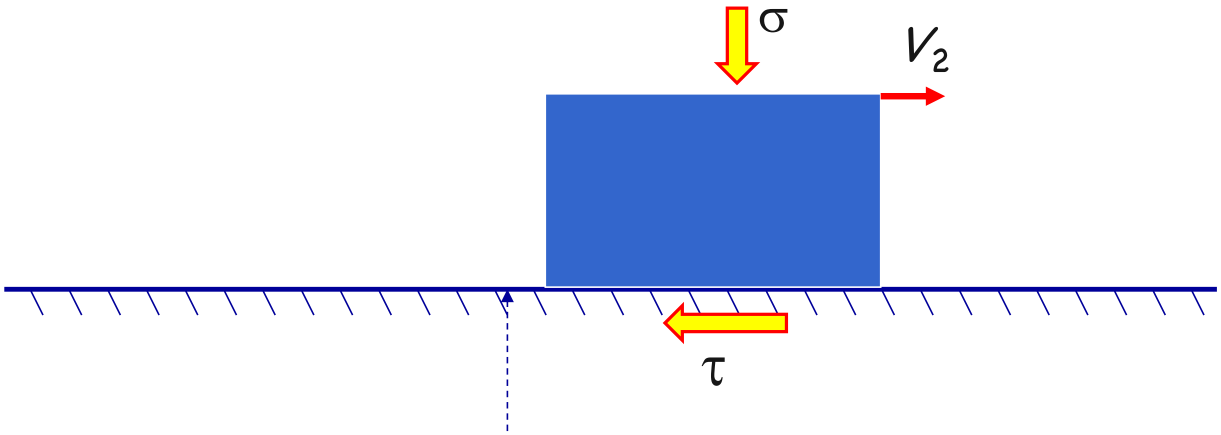
How the surface evolves when NOT at steady state matters!



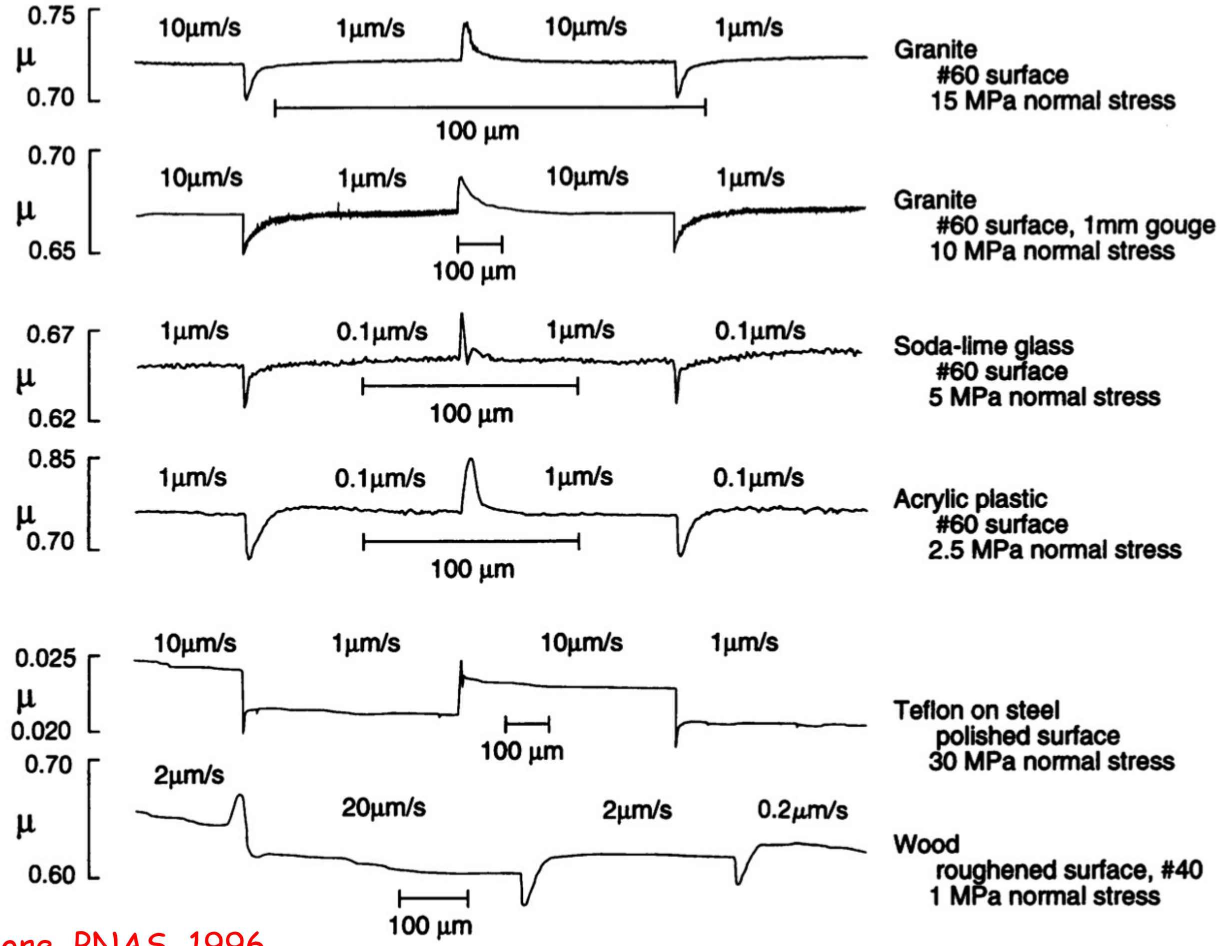

V_1
$V_2 > V_1$
friction
$\frac{\tau}{\sigma}$
$a\ln(V_2/V_1)$
$(a \sim 0.01)$
slip
σ
V_2
τ


V_1
$V_2 > V_1$
friction
$\frac{\tau}{\sigma}$
$a\ln(V_2/V_1)$
$(a \sim 0.01)$
slip
σ
V_2
τ

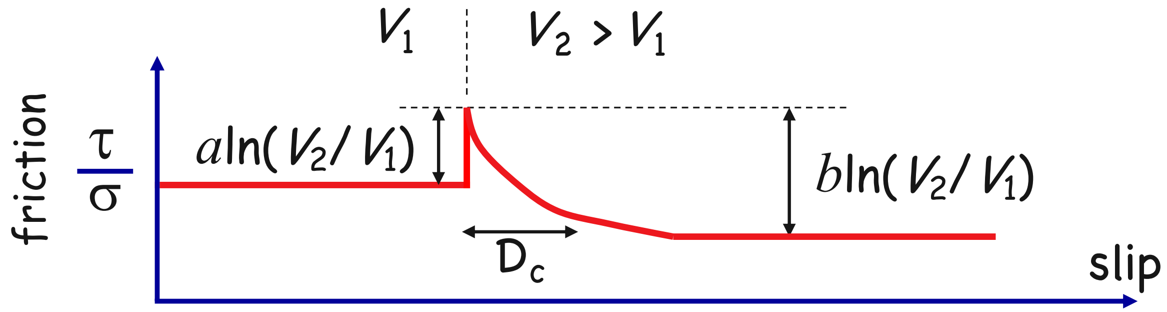

V_1
$V_2 > V_1$
friction
$\frac{\tau}{\sigma}$
$a\ln(V_2/V_1)$
$b\ln(V_2/V_1)$
$(b \sim 0.01)$
$(a \sim 0.01)$
D_c
slip
$(D_c \sim 10\ \mu m)$


σ
V_2
τ

Most surfaces seem to share this phenomenology …



Dieterich and Kilgore, PNAS, 1996



$b > a$: Steady-state velocity weakening. Earthquake nucleation possible.
$b < a$: Steady-state velocity strengthening. Earthquakes can't nucleate.
$a = b$: Velocity neutral. (Rock is pretty close to velocity-neutral.)

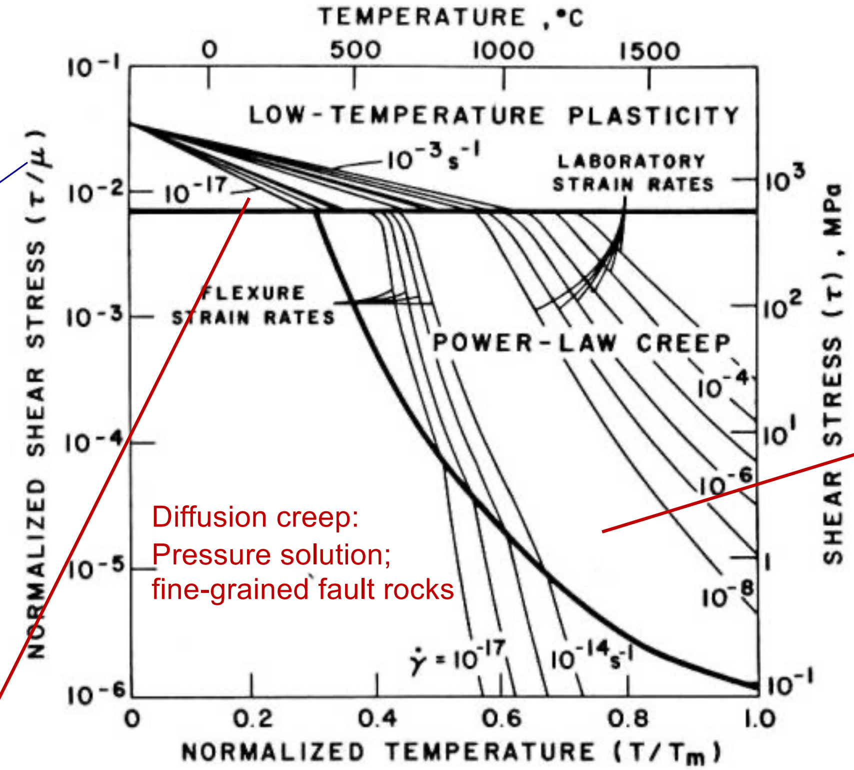
$$\underbrace{\mu}_{\text{friction}} = \frac{\tau}{\sigma} = \underbrace{\boxed{\mu^*}}_{\sim 0.7} + \underbrace{a}_{\sim 0.01}\ln\frac{V}{V^*} + \underbrace{b}_{\sim 0.01}\ln\frac{\theta}{\theta^*}$$

Simplest rate-state
friction equation
(starred values represent
an arbitrary reference
steady state)

Taking a stab at the physics

"Deformation map" for olivine (1 mm grain size)

(this μ is elastic shear modulus)



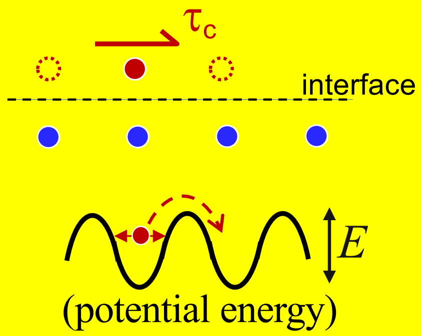
Diffusion creep:
Pressure solution;
fine-grained fault rocks

Dislocation creep:
Most of the upper mantle
and lithosphere

"Peierls" creep:

$$\dot{\varepsilon}_P = A_P \tau^{\sim 2} \exp\left(-\frac{E_P}{RT}\left(1-\left(\frac{\tau}{\tau_P}\right)^{\sim 1}\right)^{\sim 2}\right) \quad \Rightarrow \quad \ln\dot{\varepsilon} \approx \ln A_P + 2\ln\tau - \frac{E_P}{RT}\left(1-\boxed{2\frac{\tau}{\tau_P}}+(\ldots)^2\right)$$

What is *a*?



Assume an Arrhenius process: $V = V_0 \exp\left(-\frac{E - \tau_c \Lambda}{k_B T}\right)$,

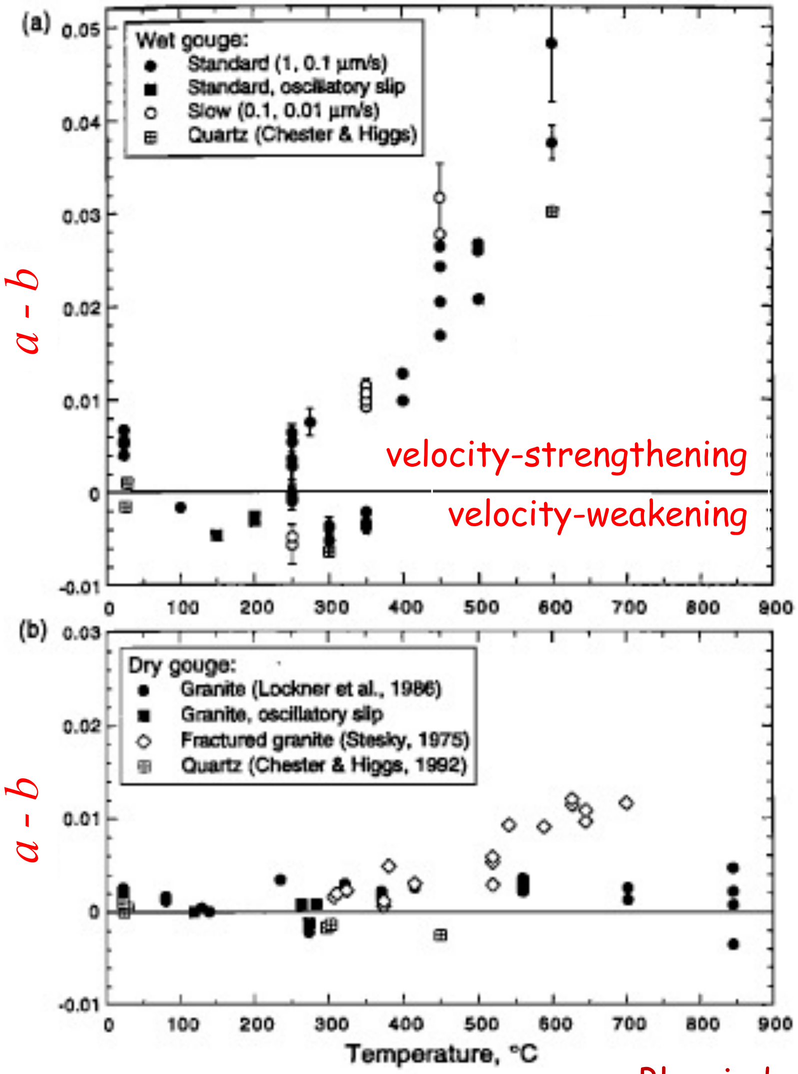
where k_B is Boltzman's constant, E an activation energy, τ_c the contact shear stress, and Λ an activation volume (or a slip distance times a slipping area?). We want to turn this into an expression for friction (τ/σ) as a function of V.

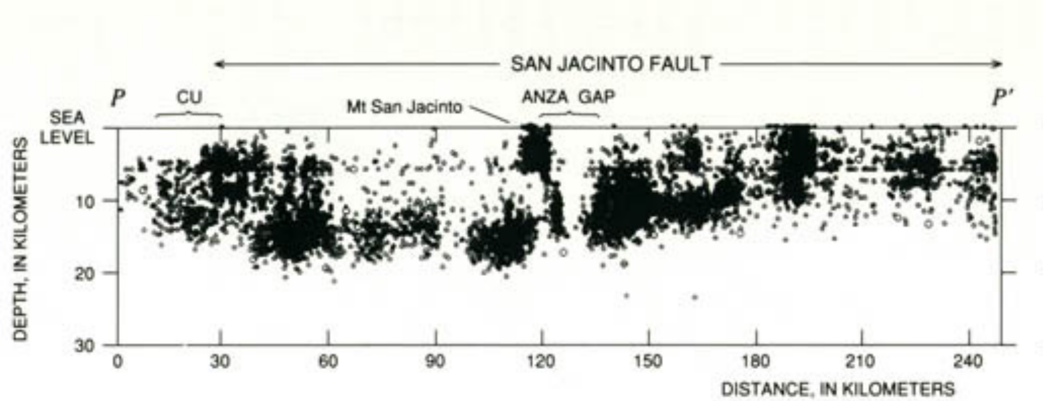
From global force balance $\frac{\tau_c}{\sigma_c} = \frac{\tau}{\sigma}$, so $\tau_c = \sigma_c \frac{\tau}{\sigma}$, where σ_c is contact normal stress.

Substituting for τ_c, taking the logarithm, and rearranging, $\frac{\tau}{\sigma} = \frac{E}{\sigma_c \Lambda} + \frac{k_B T}{\sigma_c \Lambda} \ln \frac{V}{V_0}$.

Comparing to $\frac{\tau}{\sigma} = \mu* + b \ln \frac{\theta}{\theta_0} + a \ln \frac{V}{V_0}$ yields $a = \frac{k_B T}{\sigma_c \Lambda}$.

For $T = 300$ K, $\sigma_c = 1$ GPa, $k_B = 1.4\text{x}10^{-23}$, and $\Lambda = 1\text{nm}^3$, $a \sim 0.005$.





(For the dependence of a upon temperature to explain the limiting depth of seismicity, any T-dependence of b must be secondary, or of the opposite sign.)

Blanpied et al., 1995

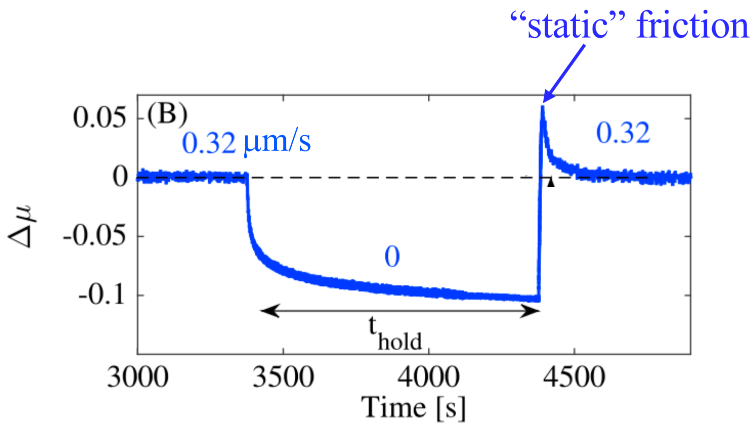
What is *b*?

$$\mu = \mu^* + a \ln \frac{V}{V^*} + b \ln \frac{\theta}{\theta^*}$$ (Friction equation)

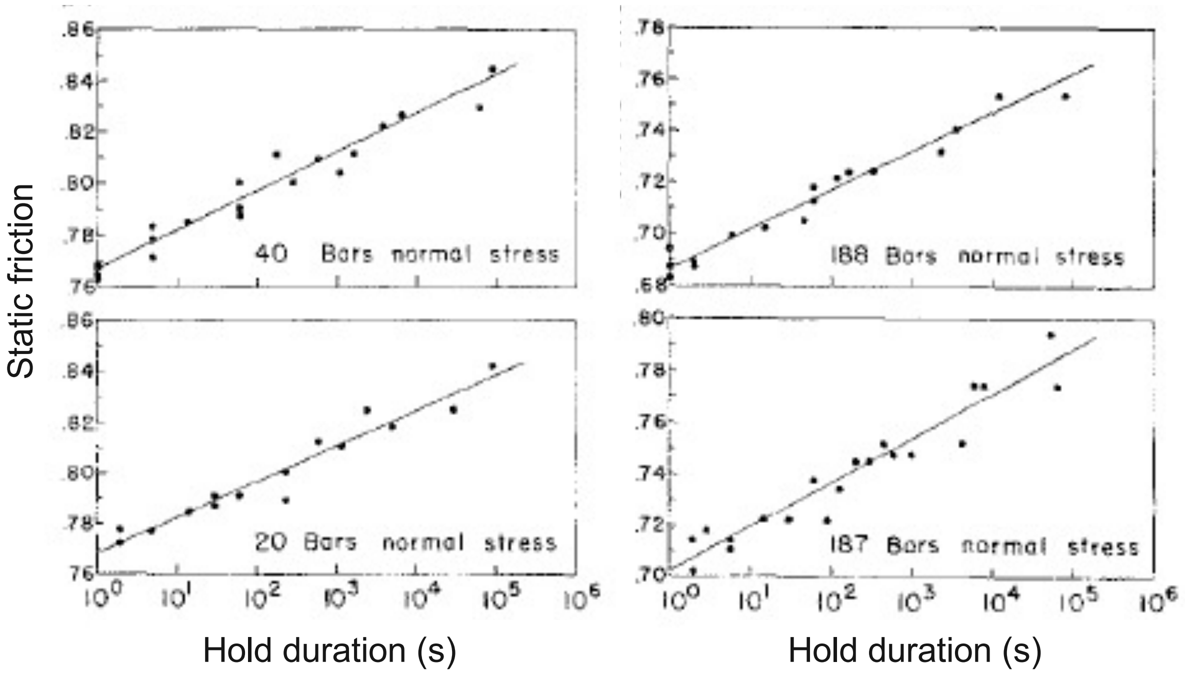
What is “state”?

Some combination of the true contact area (or the deviation from $[\sigma_n / \sigma_c]A$), and the quality of bonding across those contacts.

In (near-)stationary contact, surfaces strengthen with time (Coulomb – 1770's).
Approximately as log time (known since the 1950's). Because "state" is increasing.

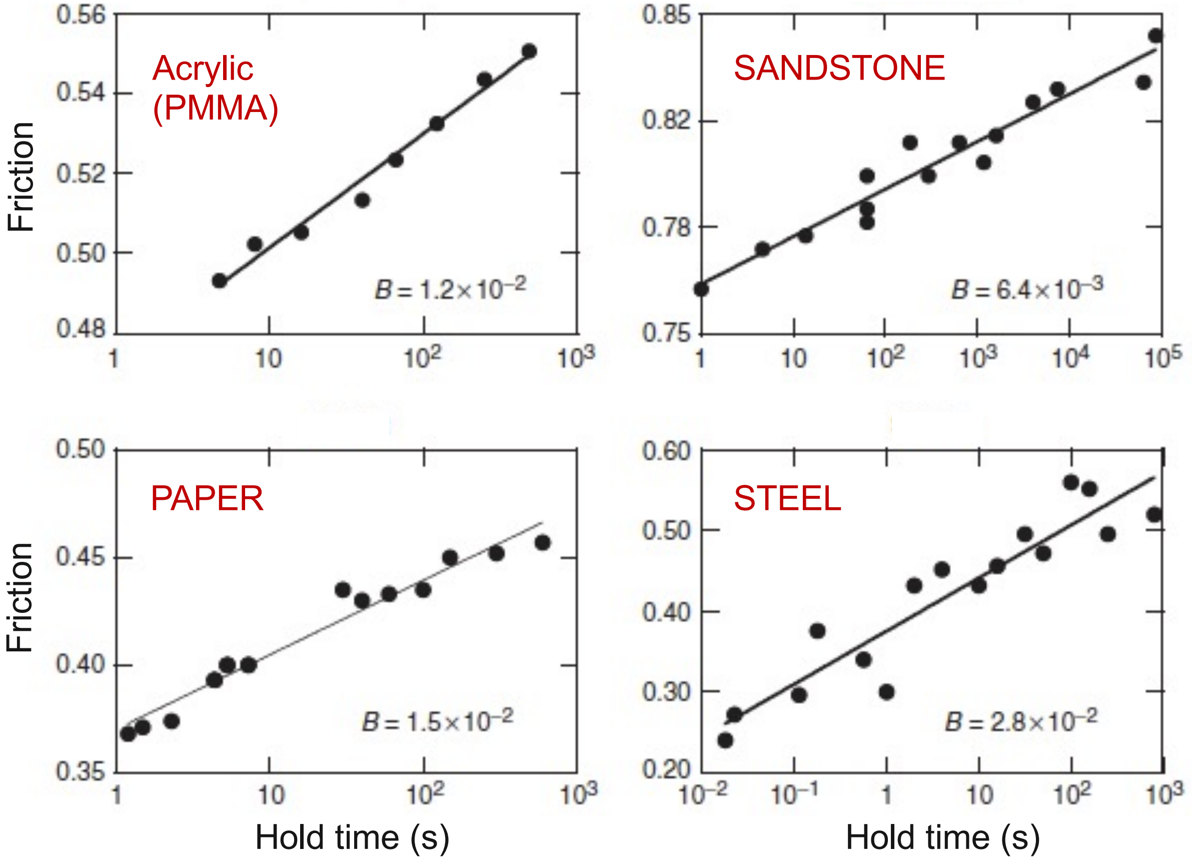


"Slide-hold-slide" experiments
("hold" refers to the load point,
not the sliding surface).

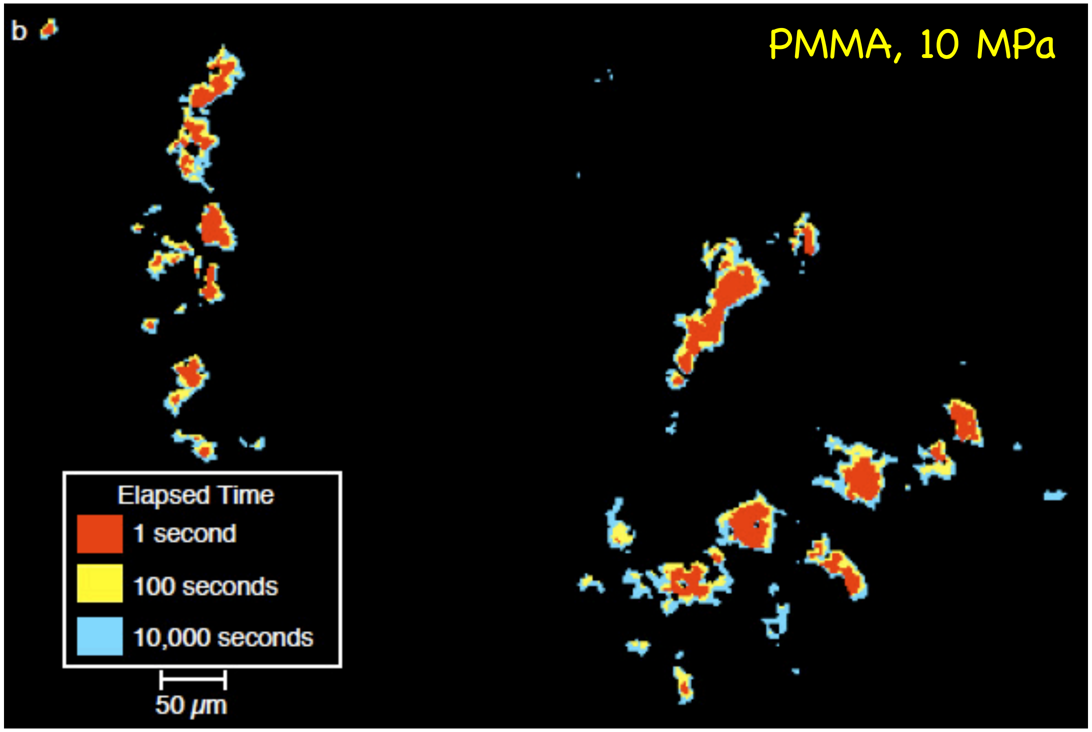


Dieterich, 1972

T. Baumberger and C. Caroli



Contact area also grows approximately as log time: Dieterich & Kilgore, 1994



PMMA shear modulus: 1.7 GPa
Contact stress for 4% contact area at 10 MPa: 250 MPa
This is 15% of the shear modulus, or roughly the material strength.
Contacts are flowing plastically.

In transparent materials, contact area seems to track state…

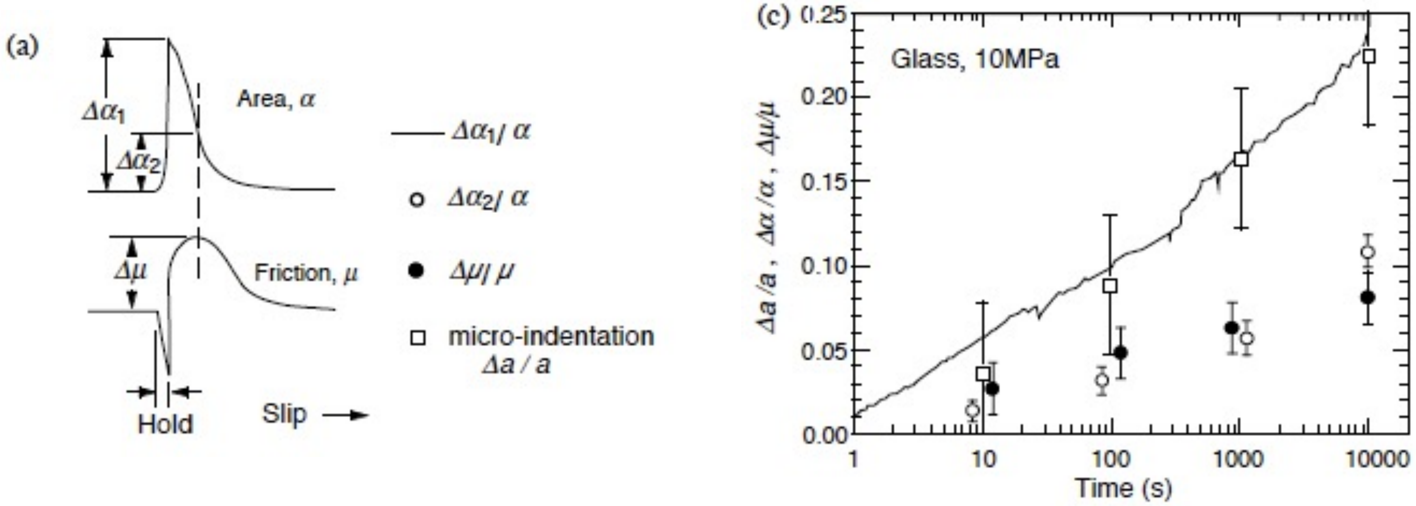


Figure 7

Normalized increases of micro-indentation area, contact area and peak friction versus logarithm of time for acrylic and glass (Figures 7b and 7c, respectively). Contact area and friction in Figure 7a are plotted against slip which has been corrected to remove the effects of finite apparatus stiffness. The contact area and friction data are normalized by α and μ, area and friction respectively, at the start of the hold. The data for increase of indentation area with time are normalized by the 1-second indentation area a. Time-dependent increases of contact area measured during a hold $\Delta\alpha_1/\alpha$ (solid curve) agree with the micro-indentation area increases. Contact area measured at peak friction, $\Delta\alpha_2/\alpha$, agrees with normalized change of peak friction, $\Delta\mu/\mu$ confirming a direct dependence of friction on contact area.

Dieterich and Kilgore, 1994

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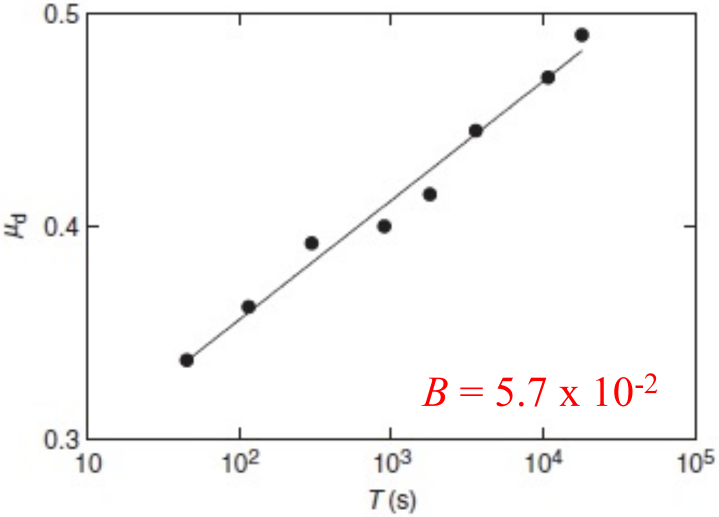
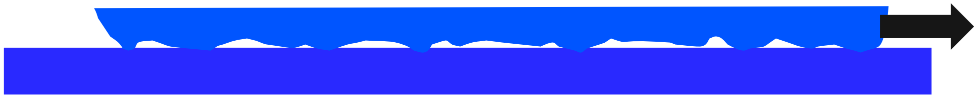
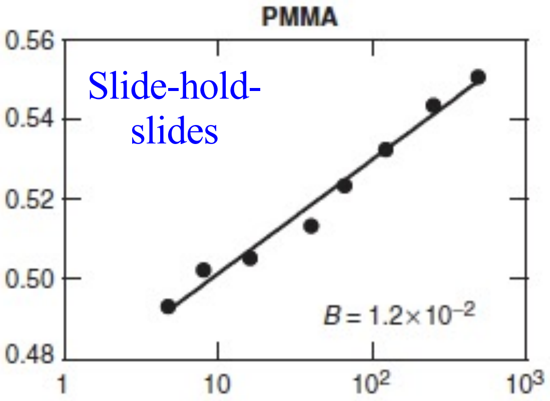


Figure 21. Logarithmic growth of the *steady sliding* friction coefficient μ_d at velocity $V = 50\,\mu\text{m/s}$ for a rough PMMA/flat glass MCIs *versus* time T elapsed since the interface was created. (Adapted from [61].) (Reprinted with permission from [61]). [61] Copyright 2002





But - It may not all be contact area!

Terry Tullis, extended abstract,
USGS “Red-Book” volume, 1993.

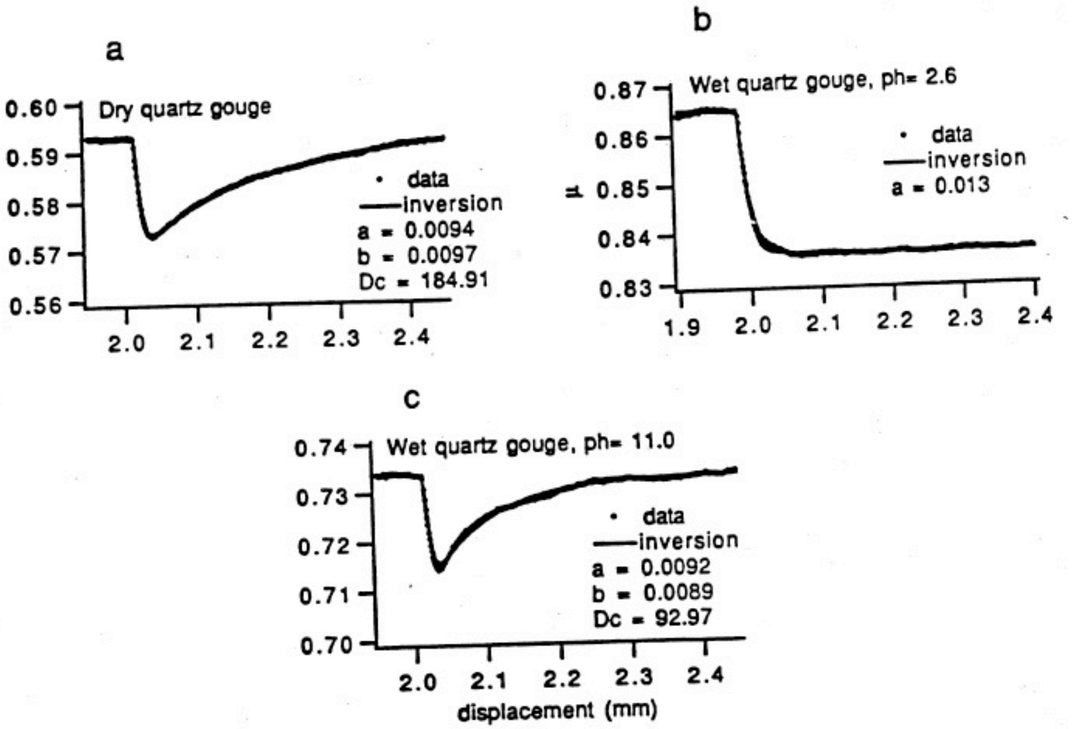


Figure 1. Comparison of frictional response to an decrease in loading velocity from 10.0 to 1.0 μm/sec for three layers of fine quartz gouge 1 mm thick, each run under different chemical environment. Although the fit is so good as to be virtually invisible, each trace is composed of a data set from an experiment (dots) and a numerical simulation using parameters determined by iterative least-squares inversion. The data are from very early in the runs (note the displacement scale) and include a substantial background trend that has been removed as part of the inversion. (A) Control experiment - room dry. This is a standard type response to a velocity step, showing a direct effect and evolution. In this case, the steady-state dependence on velocity is nearly neutral. Note the rather large value of D_c. (B) Nitric acid solution having a pH of 2.6. This is the point of zero charge, where the tendency of water to become bound to silicate surfaces will be eliminated or reduced due to saturation of the surface with protons. Thus the contaminant layer, which we believe may be responsible for the evolution effect, should not exist. In fact, the response is well fit with a model having no evolution effect. (C) Buffered solution having pH = 11.0. The magnitude of the evolution effect is not increased relative to the dry run, but D_c is a factor of two smaller.

But - It may not all be contact area!

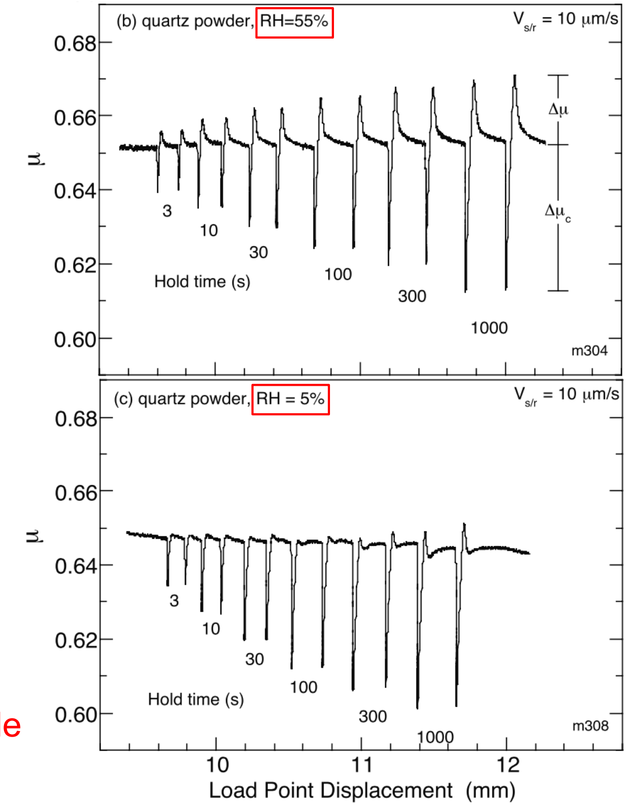


Figure 3. SHS tests for quartz powder. (a) Typical 30 s hold shows frictional creep when load point stops and stress peak when loading resumes. (b) Typical set of SHS tests performed at 55% RH illustrate frictional creep ($Δμ_c$) and frictional healing (Δμ). (c) Under dry conditions, with the same displacement history shown for panel b, healing is negligible.

Slide-Hold-Slide tests of Frye & Marone, 2002

Recognized earlier by Dieterich and Conrad (1984) (on westerly granite, quartz, and quartzite), and ascribed by them to the importance of water (H) in facilitating plastic flow of silicates.

But - It may not all be contact area!

Slide-Hold-Slide tests of Frye & Marone, 2002

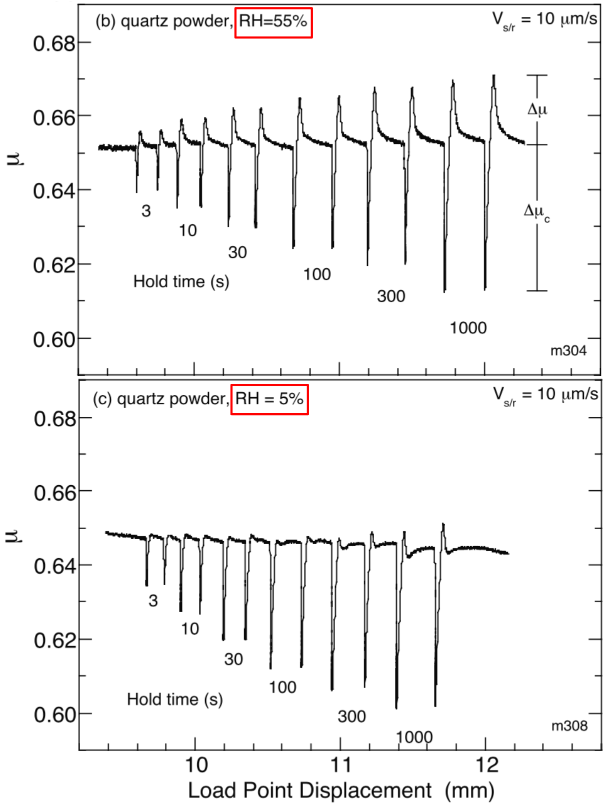


Figure 3. SHS tests for quartz powder. (a) Typical 30 s hold shows frictional creep when load point stops and stress peak when loading resumes. (b) Typical set of SHS tests performed at 55% RH illustrate frictional creep ($\Delta\mu_c$) and frictional healing ($\Delta\mu$). (c) Under dry conditions, with the same displacement history shown for panel b, healing is negligible.

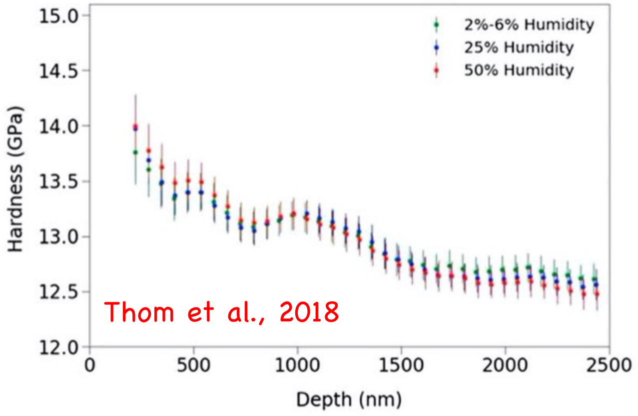


Figure 1. Nanoindentation hardness at several RH conditions.

(hardness = load/contact area)

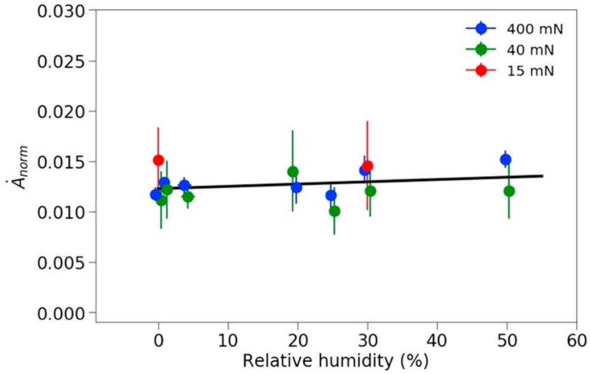


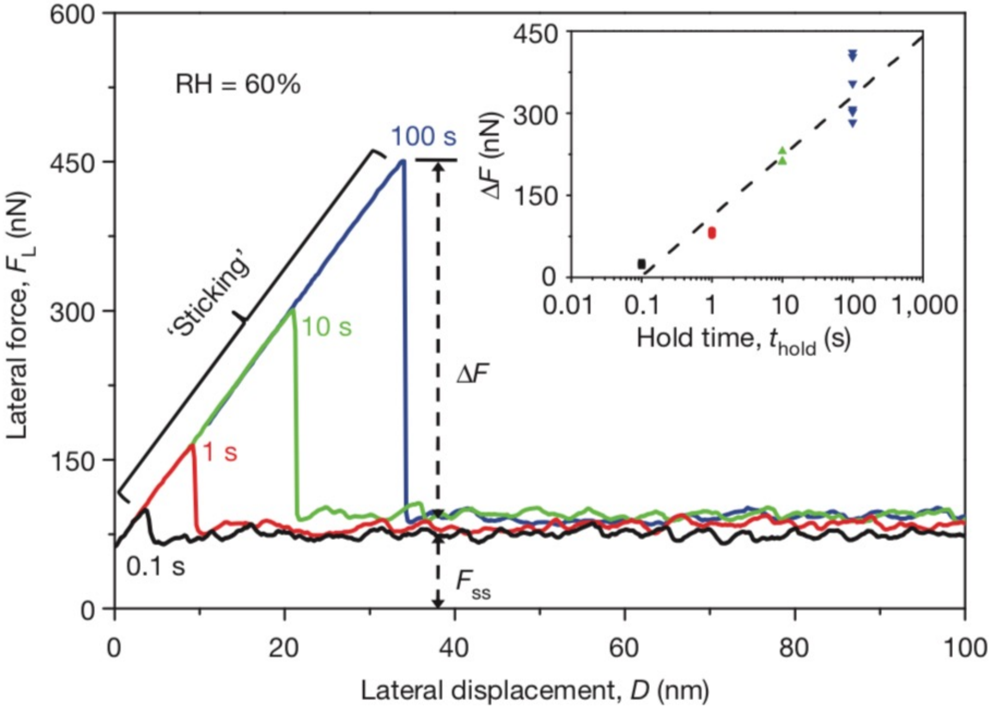
Figure 4. Rate of change of the normalized projected contact area, $\dot{A}_{norm}$, as a function of relative humidity. Each data point represents an average of 3 to 12 tests at a given load (represented by the color) and humidity, and error bars represent one standard deviation from the mean. A best fit line is shown in black, demonstrating no dependence of the rate of normalized contact area growth on humidity. If aging were caused by contact area growth,

But more recent AFM experiments (Thom et al., 2018) indicate that plastic flow rates in quartz are insensitive to humidity.

Frictional ageing from interfacial bonding and the origins of rate and state friction

Qunyang Li[1]†, Terry E. Tullis[2], David Goldsby[2] & Robert W. Carpick[1]

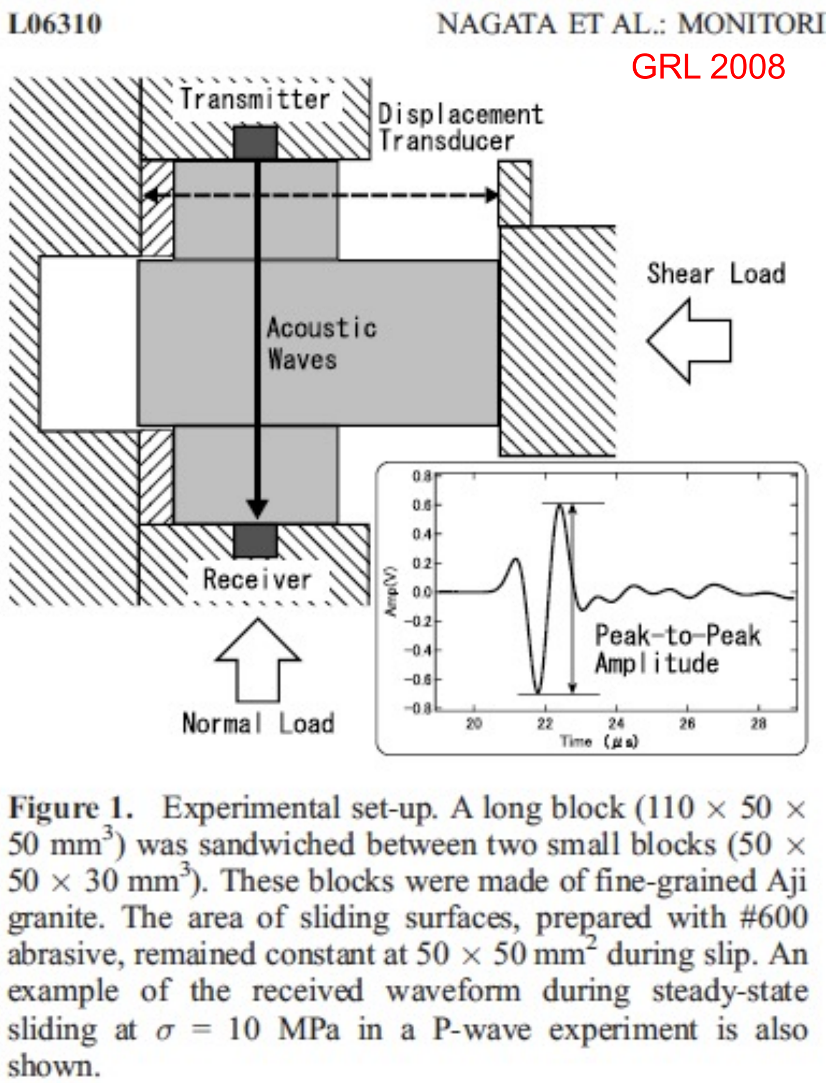
Nature, 2011



Atomic Force Microscope (AFM) tip on silica glass; force too low to generate ductile flow.

Number of Si-O-Si bonds grows as log time (MD calculations of Liu and Szlufarska,2012):

$$\text{Si-OH} + \text{Si-OH} = \text{Si-O-Si} + H_2O$$



L06310 NAGATA ET AL.: MONITORI

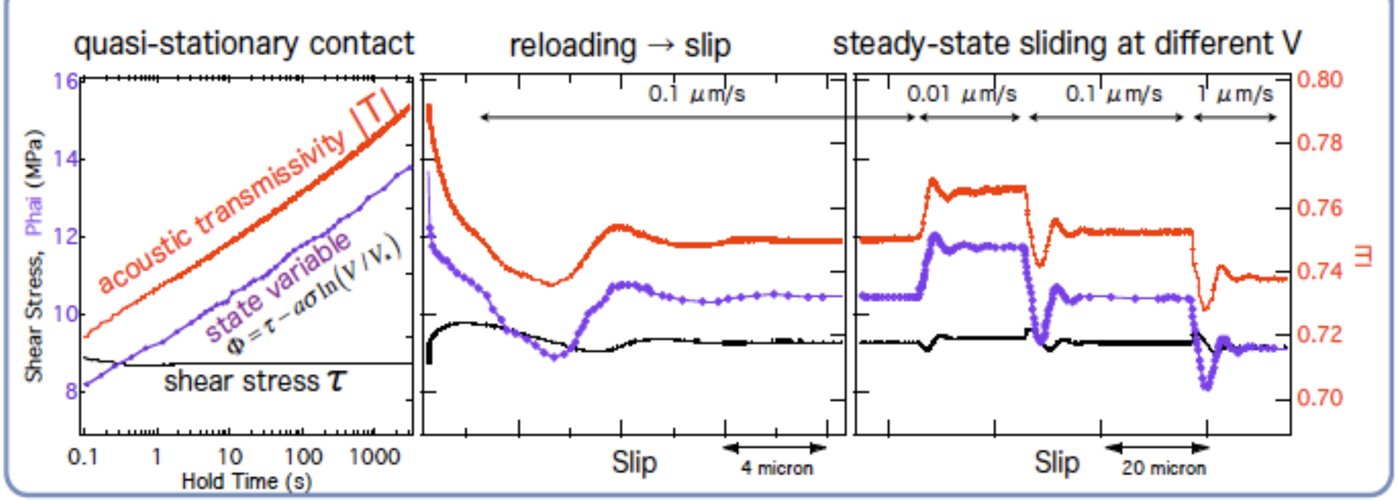
GRL 2008

Figure 1. Experimental set-up. A long block (110 × 50 × 50 mm^3) was sandwiched between two small blocks (50 × 50 × 30 mm^3). These blocks were made of fine-grained Aji granite. The area of sliding surfaces, prepared with #600 abrasive, remained constant at 50 × 50 mm^2 during slip. An example of the received waveform during steady-state sliding at $\sigma = 10$ MPa in a P-wave experiment is also shown.

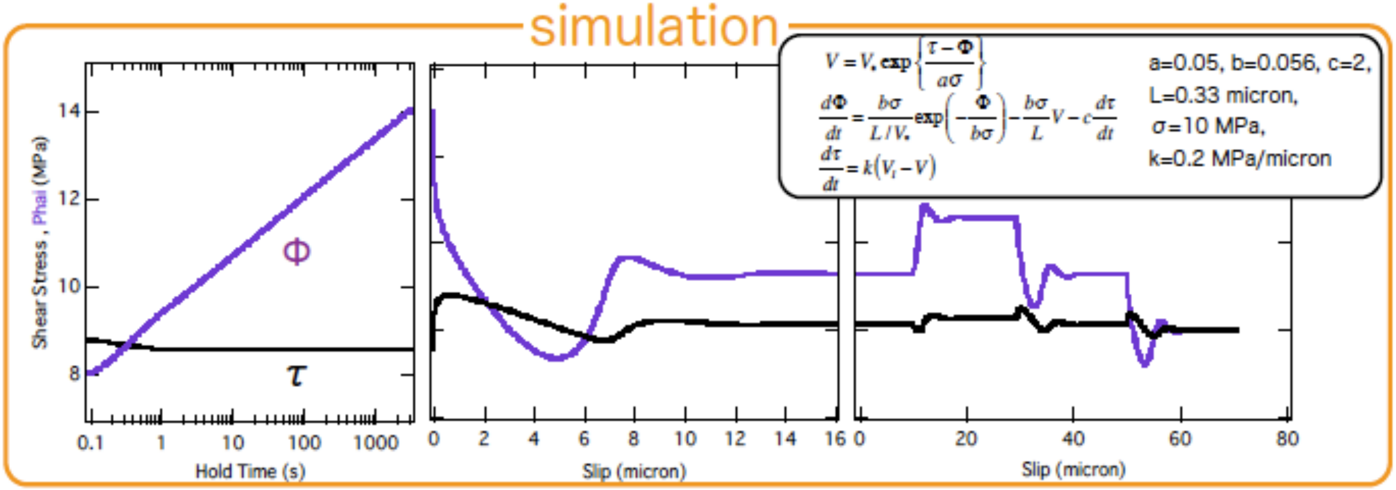
For wavelengths much longer than the asperity size, transmissivity depends upon the stiffness of the interface (ratio of stress change to displacement discontinuity), which increases with contact area (may also depend upon distribution of contact sizes).

Nagata et al., 2008

observation



simulation



$$V = V_* \exp\left\{\frac{\tau - \Phi}{a\sigma}\right\}$$

$$\frac{d\Phi}{dt} = \frac{b\sigma}{L / V_*}\exp\left(-\frac{\Phi}{b\sigma}\right) - \frac{b\sigma}{L}V - c\frac{d\tau}{dt}$$

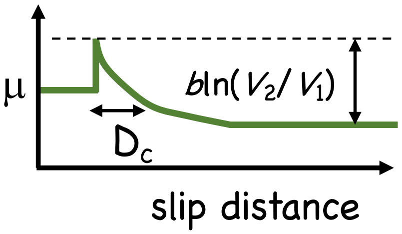
$$\frac{d\tau}{dt} = k\left(V_t - V\right)$$

a=0.05, b=0.056, c=2,
L=0.33 micron,
σ=10 MPa,
k=0.2 MPa/micron

We still need an evolution equation for “state” (θ):

$$\mu = \mu^* + a \ln \frac{V}{V^*} + b \ln \frac{\theta}{\theta^*}$$

$$\dot{\theta} = ??$$



$$\boxed{\mu = \mu^* + a\ln\frac{V}{V^*} + b\ln\frac{\theta}{\theta^*}}$$

The evolution of state: end-member candidates

"Aging law"

$$\dot{\theta} = 1 - \frac{V\theta}{D_c}$$ (*Dieterich*, 1972)

Steady state: $\frac{V\theta}{D_c} = 1$

$\dot{\theta} \sim 1$ at $V \sim 0$

θ evolves with time even in the absence of slip

"Slip law"

$$\dot{\theta} = -\frac{V\theta}{D_c}\ln\left(\frac{V\theta}{D_c}\right)$$ (*Ruina*, 1983)

Steady state: $\frac{V\theta}{D_c} = 1$

(near steady state:
$\ln[1+\varepsilon] \sim \varepsilon$)

$\dot{\theta} \sim 0$ at $V \sim 0$.

θ evolves only with slip.

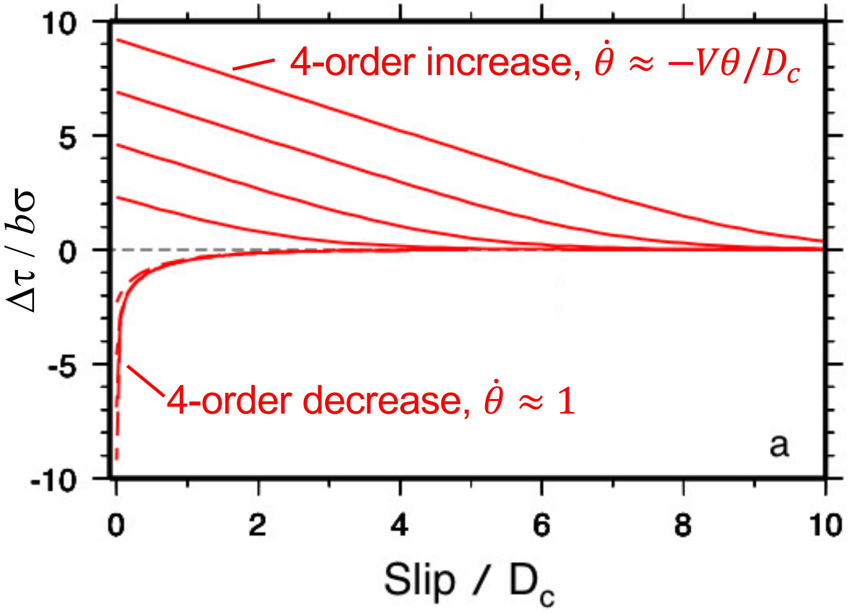
"Aging law" evolution:

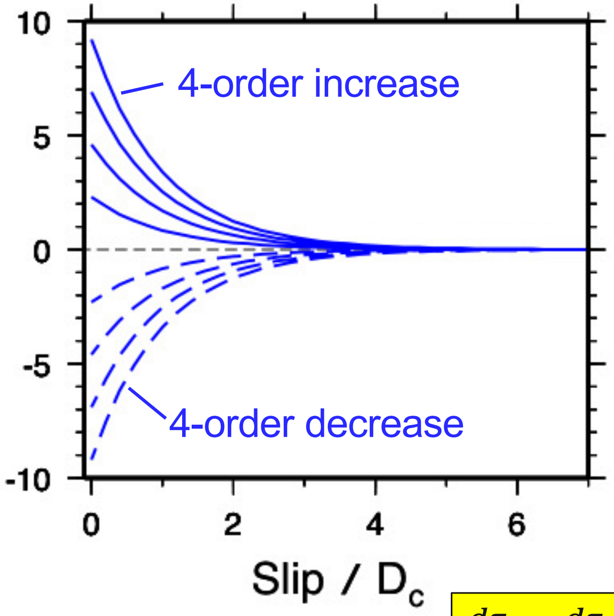
$$\dot{\theta} = 1 - \frac{V\theta}{D_c}$$

"Slip law" evolution:

$$\dot{\theta} = -\frac{V\theta}{D_c}\ln\frac{V\theta}{D_c}$$

Prior to step:
steady state,
$V\theta/D_c$=1



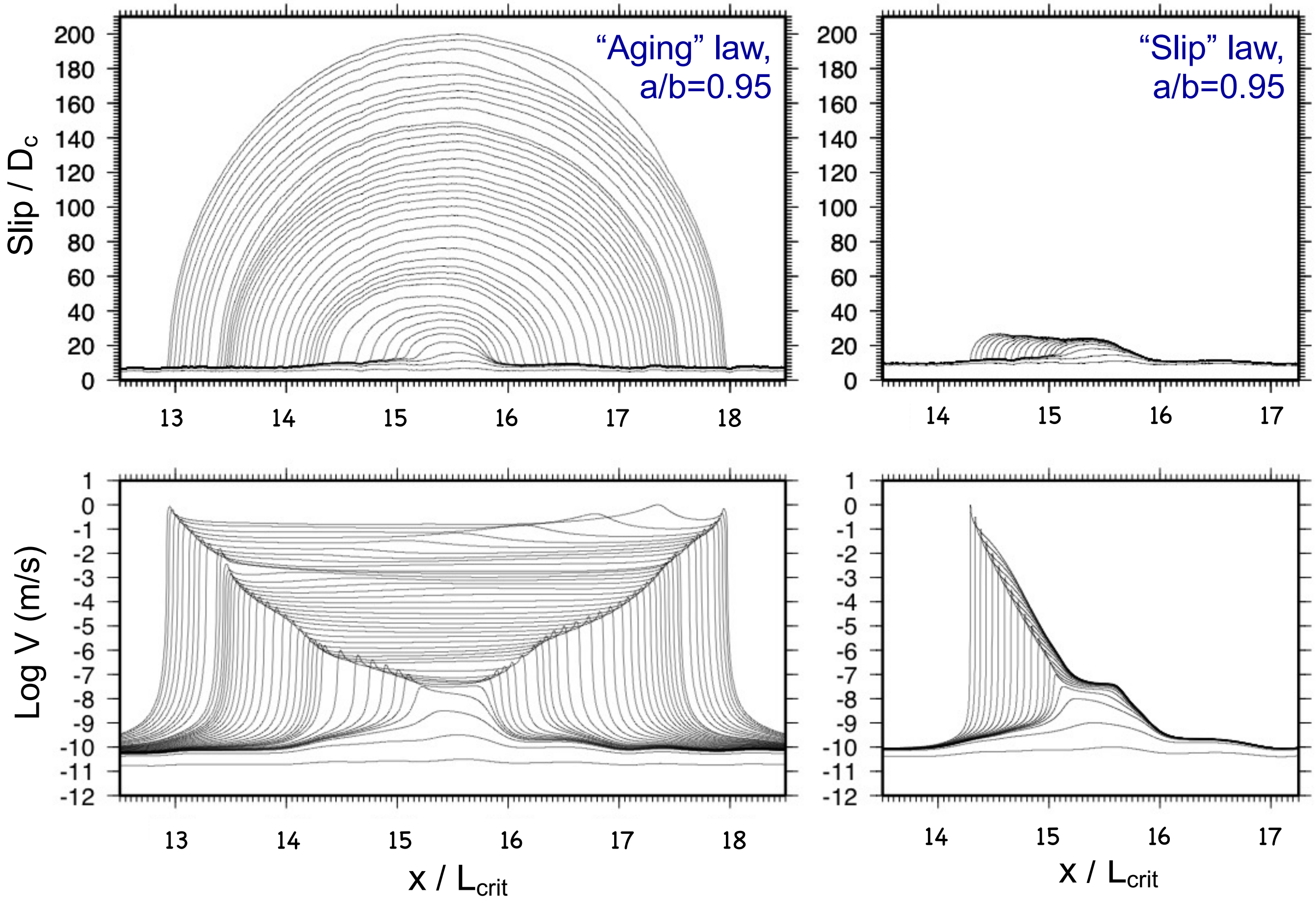


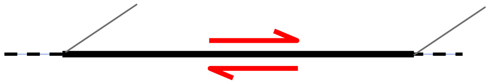
For large velocity increases from steady state, $\dot{\theta}$ ~ $-V\theta/D_c$:

$$\tau = \tau^* + a\sigma\ln\frac{V}{V^*} + b\sigma\ln\frac{\theta}{\theta^*} \quad \Rightarrow \quad \dot{\tau} = a\sigma\frac{\dot{V}}{V} + b\sigma\frac{\dot{\theta}}{\theta} = -b\sigma\frac{V}{D_c} \quad \Rightarrow \quad \frac{d\tau}{d\delta} = -\frac{b\sigma}{D_c}$$

$$\frac{d\tau}{dt} = \frac{d\tau}{d\delta}\frac{d\delta}{dt} = \frac{d\tau}{d\delta}V$$

The equation for state evolution matters!



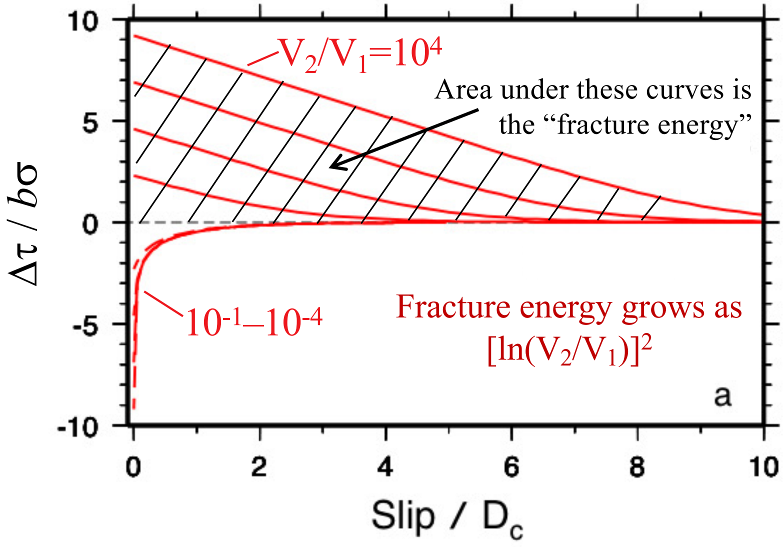


Ampuero and Rubin, 2008

$$L_{crit} = \frac{GD_c}{(b-a)\sigma}$$

Aging law evolution

$$\dot{\theta} = 1 - \frac{V\theta}{D_c}$$

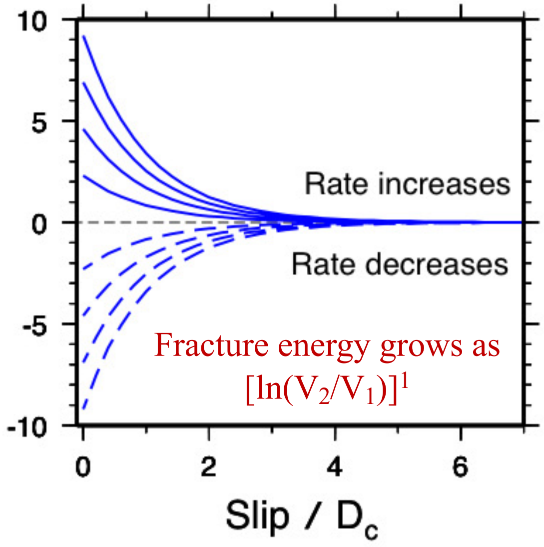


Area under these curves is the “fracture energy”

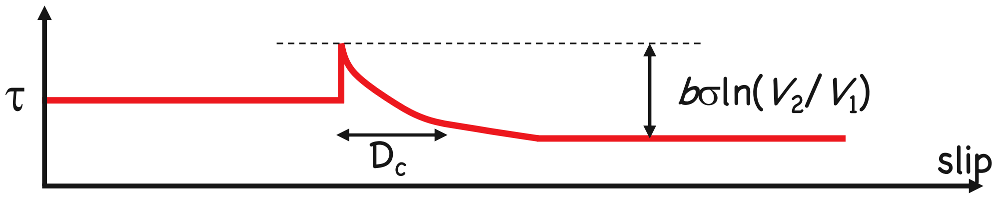
Fracture energy grows as $[\ln(V_2/V_1)]^2$

Slip law evolution

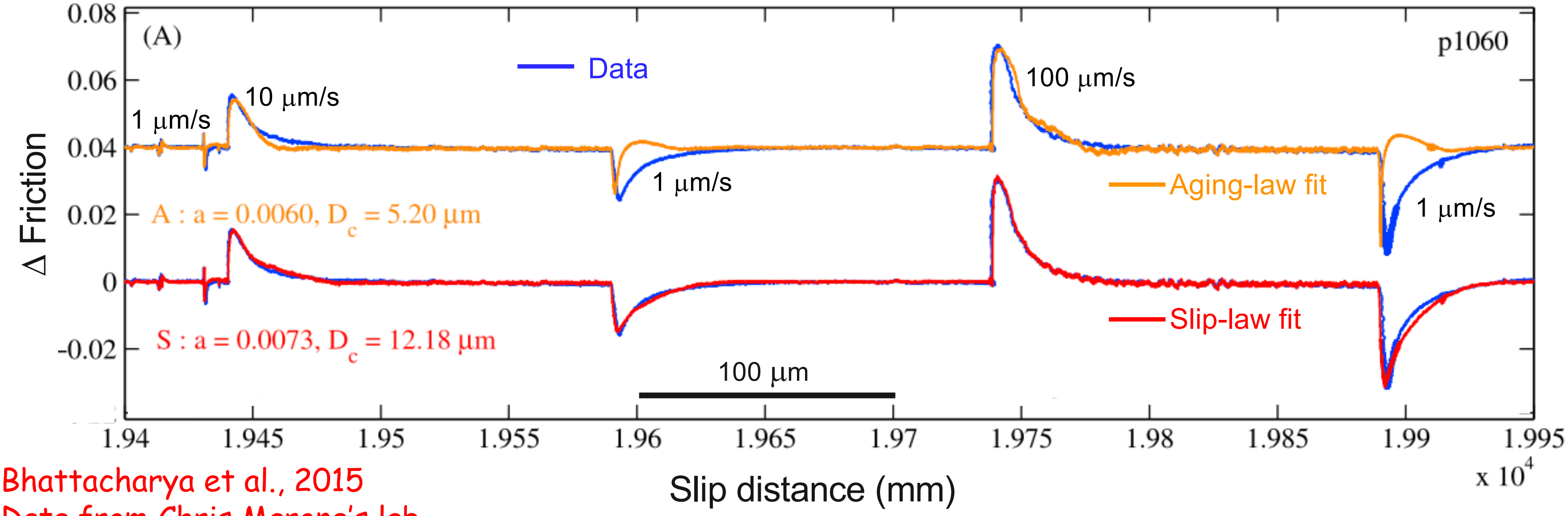
$$\dot{\theta} = -\frac{V\theta}{D_c}\ln\frac{V\theta}{D_c}$$



Fracture energy grows as $[\ln(V_2/V_1)]^1$

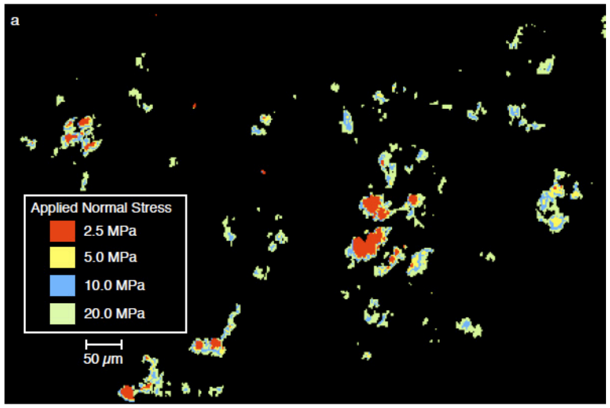


Laboratory velocity-step tests are unambiguous.
At constant sliding speed, state evolves with slip, not time:
The approach to the new value of steady-state friction occurs over a
characteristic slip distance, independent of sliding speed.



Bhattacharya et al., 2015
Data from Chris Marone's lab
(synthetic quartz gouge)

The "Slip law" was invented because it fits data like these. It has no well-accepted theoretical basis (but see Sleep [2006]).



A justification for the Slip law?
Swap out the old contacts; swap in the new…

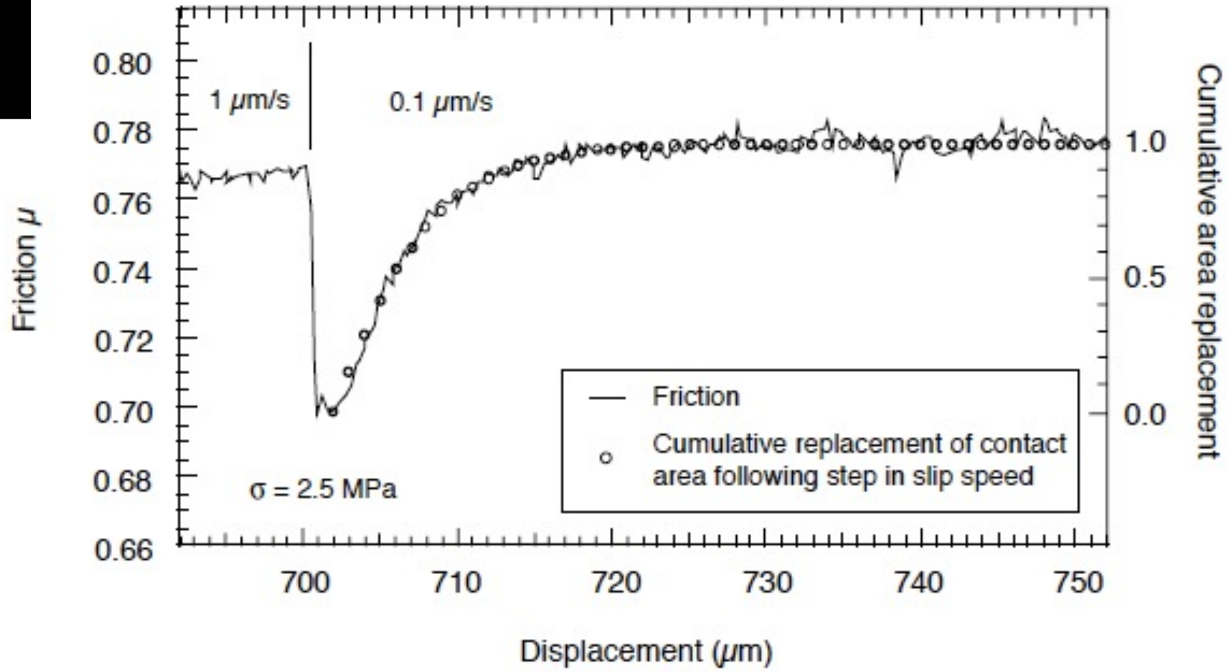


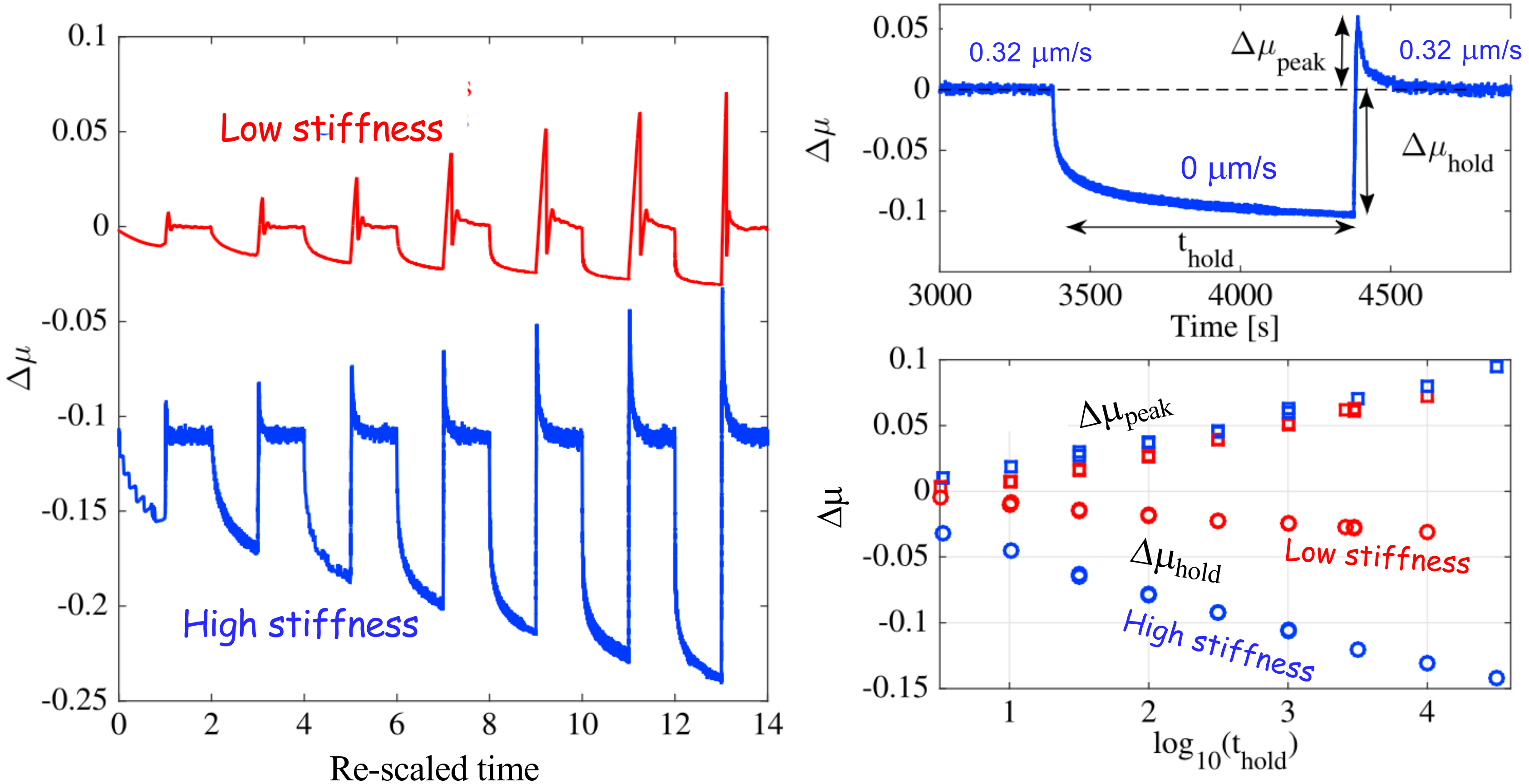
Figure 9

Displacement dependence of friction in acrylic following step in slip speed from 1 μm/s to 0.1 μm/s. Solid curve in experimental data, circles give the simulated replacement of contact area following the step in slip speed calculated from contact image data for this surface.

Dieterich and Kilgore, 1994

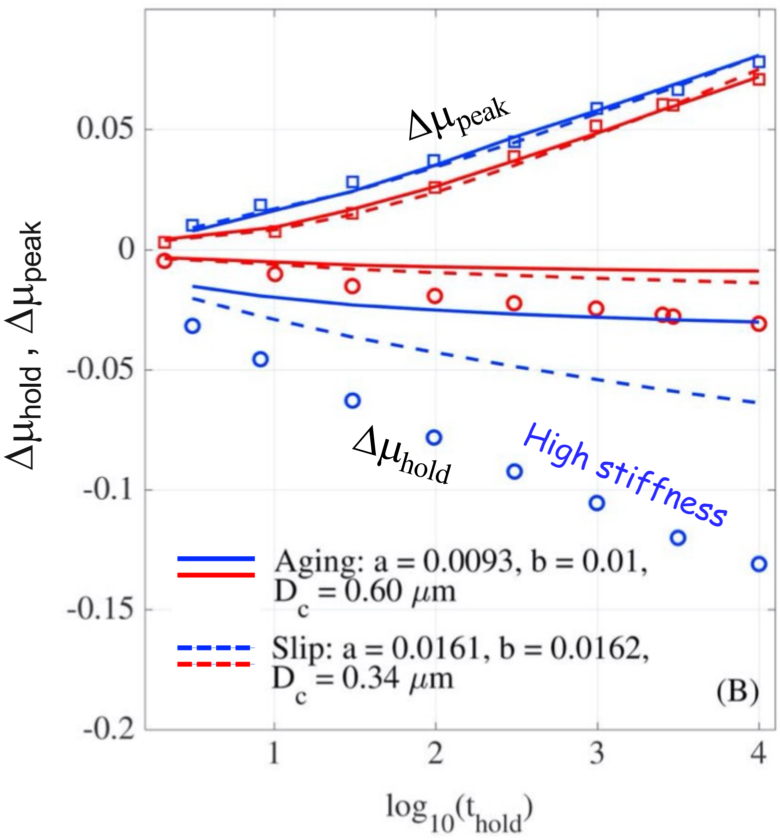
Where is the evidence for time-dependent healing?

Beeler et al., 1994: Slide-hold-slides at 2 different machine stiffnesses (2 different slip distances during the load-point hold) interpreted as showing that strengthening depends on time and not slip.

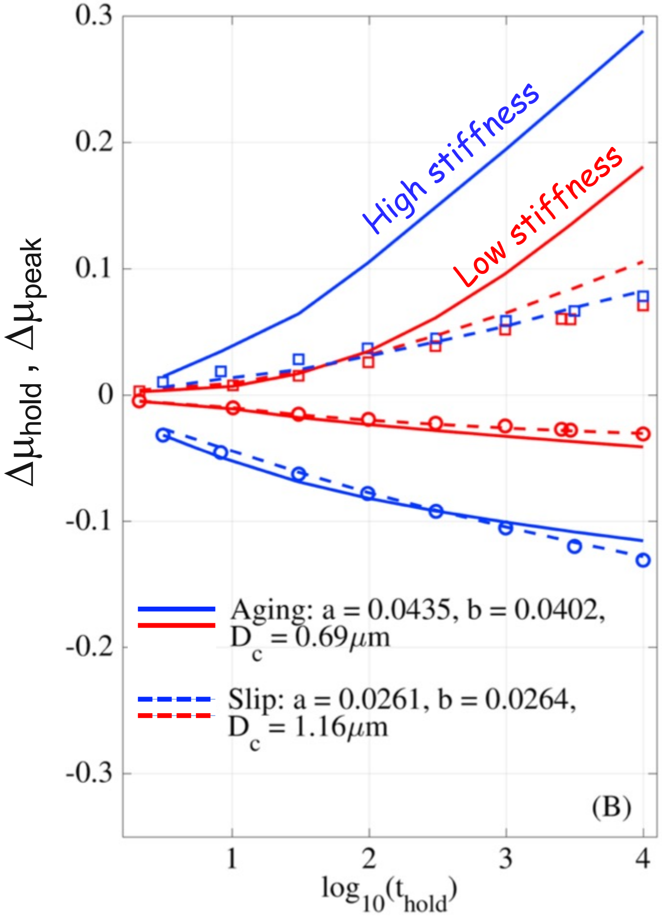


(re-interpreted by Bhattacharya et al., 2017)

Fitting $\Delta\mu_{peak}$, predicting $\Delta\mu_{hold}$



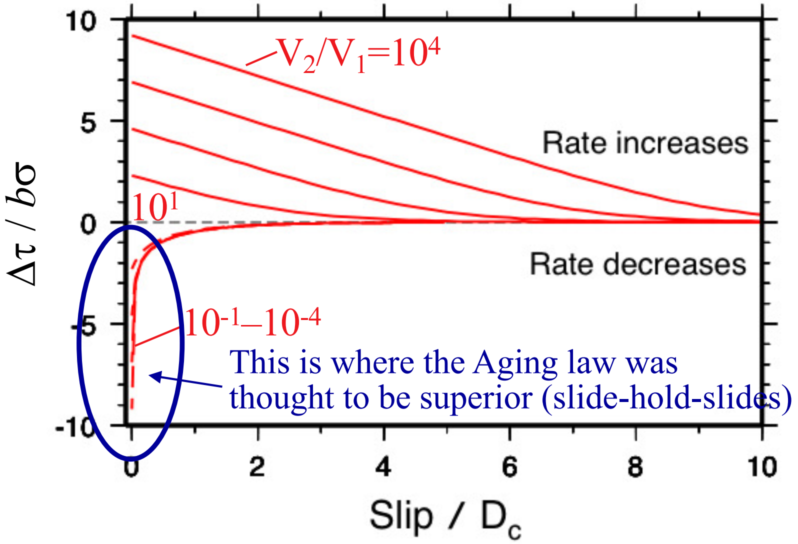
Fitting $\Delta\mu_{hold}$, predicting $\Delta\mu_{peak}$



Bhattacharya et al., 2017

Aging law evolution

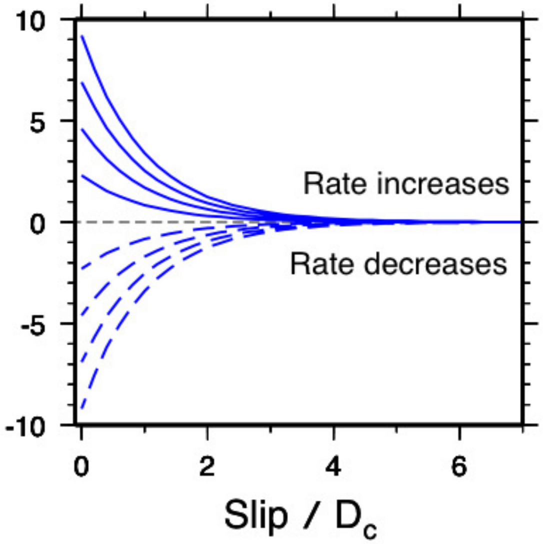
$$\dot{\theta} = 1 - \frac{V\theta}{D_c}$$



This is where the Aging law was thought to be superior (slide-hold-slides)

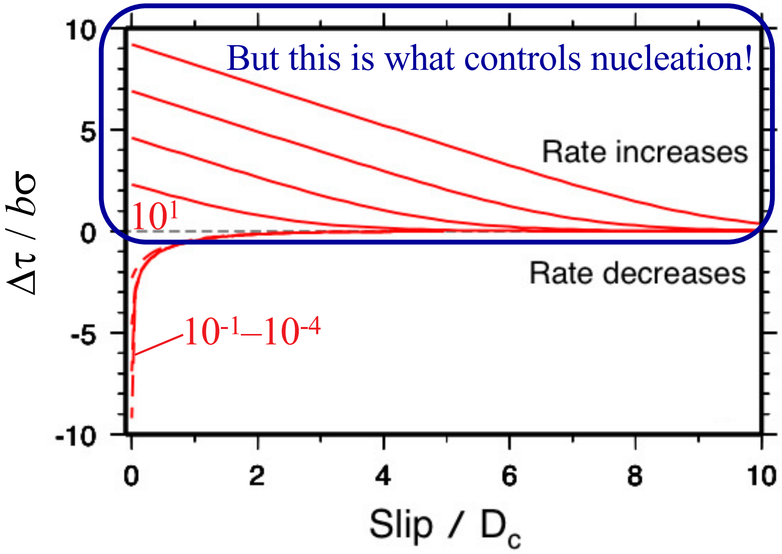
Slip law evolution

$$\dot{\theta} = -\frac{V\theta}{D_c}\ln\frac{V\theta}{D_c}$$



Aging law evolution

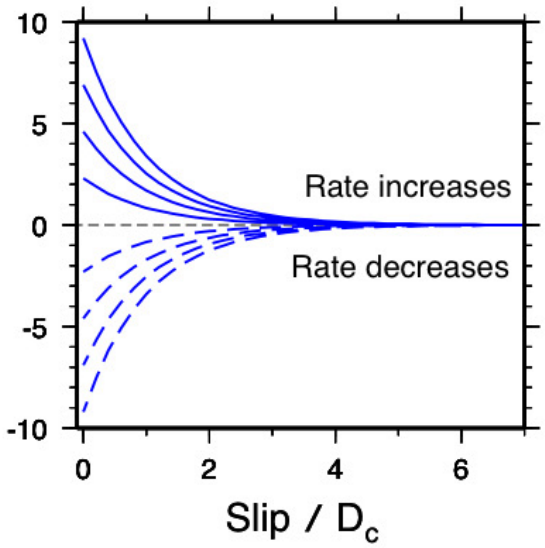
$$\dot{\theta} = 1 - \frac{V\theta}{D_c}$$



But this is what controls nucleation!

Slip law evolution

$$\dot{\theta} = -\frac{V\theta}{D_c}\ln\frac{V\theta}{D_c}$$

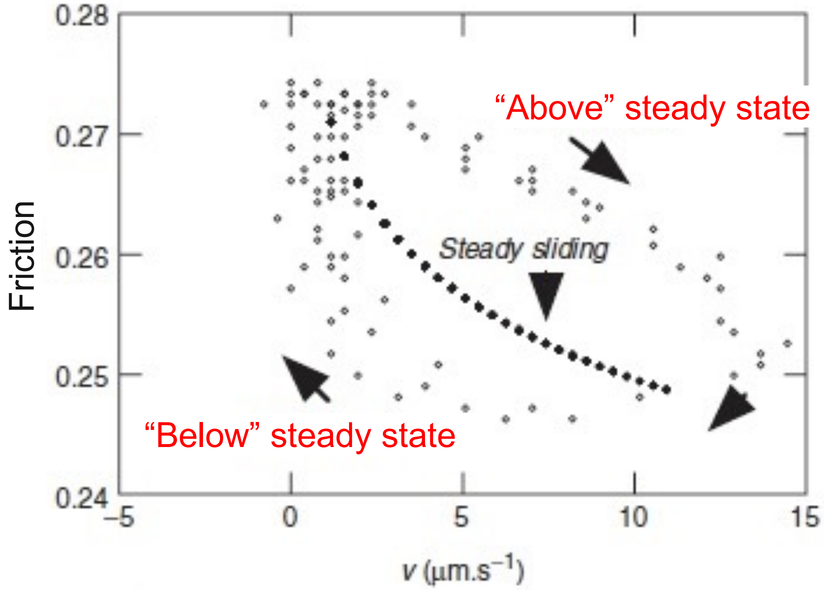


Conventional wisdom:

Velocity-step tests are described better by the "Slip" law.

Strengthening with log hold time of the peak stress upon resliding ("static friction") is described better by the "Aging" law. (perhaps)

These views aren't obviously consistent, because because large velocity-step decreases and holds probe the region of parameter space where the sliding surface is far below steady state.



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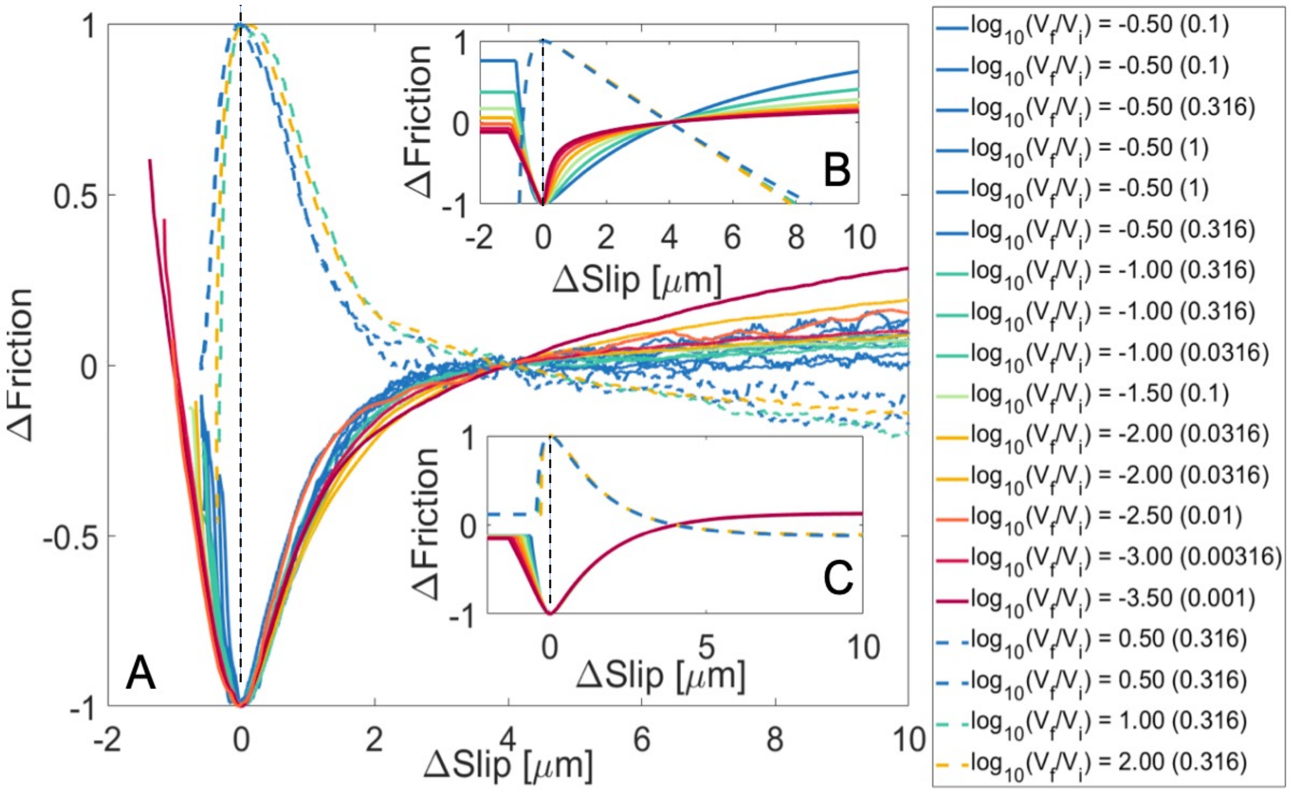
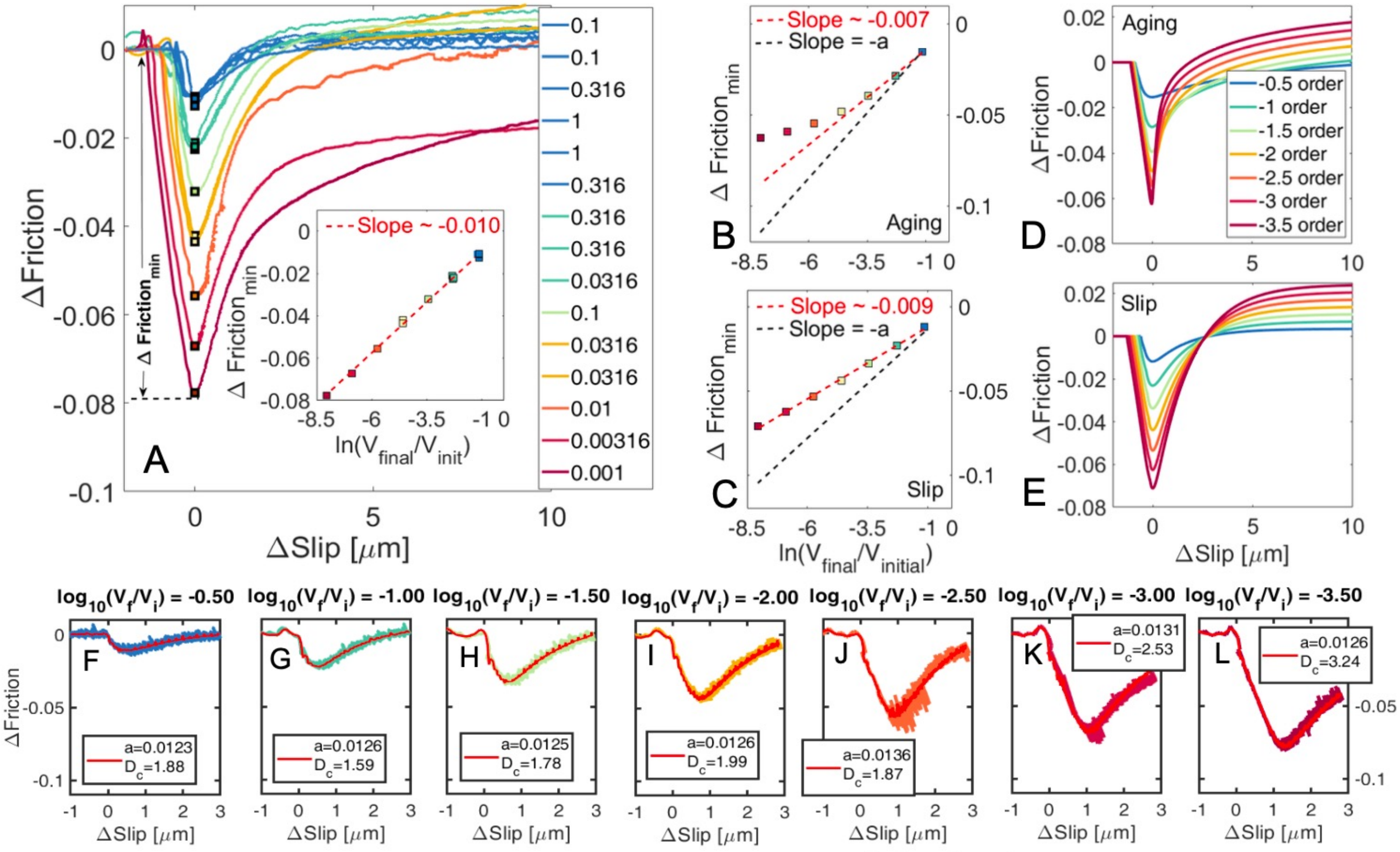
These views aren’t obviously consistent, because because large velocity-step decreases and holds probe the region of parameter space where the sliding surface is far below steady state.

BUT – it has been possible to ascribe this apparent inconsistency to different physics operating at the very low sliding speeds accessed by holds, which are much lower than those of conventional velocity-step tests.

Large (0.5 – 3.5 orders of magnitude) velocity decreases, in the (very stiff) Brown University rotary shear apparatus (3 to 0.001 μm/s).
Sliding velocities get as low as those during moderately long holds.



Bhattacharya et al., PNAS, 2022



Following the stress minima, the surface strengthens with *slip* and not *time* **!**

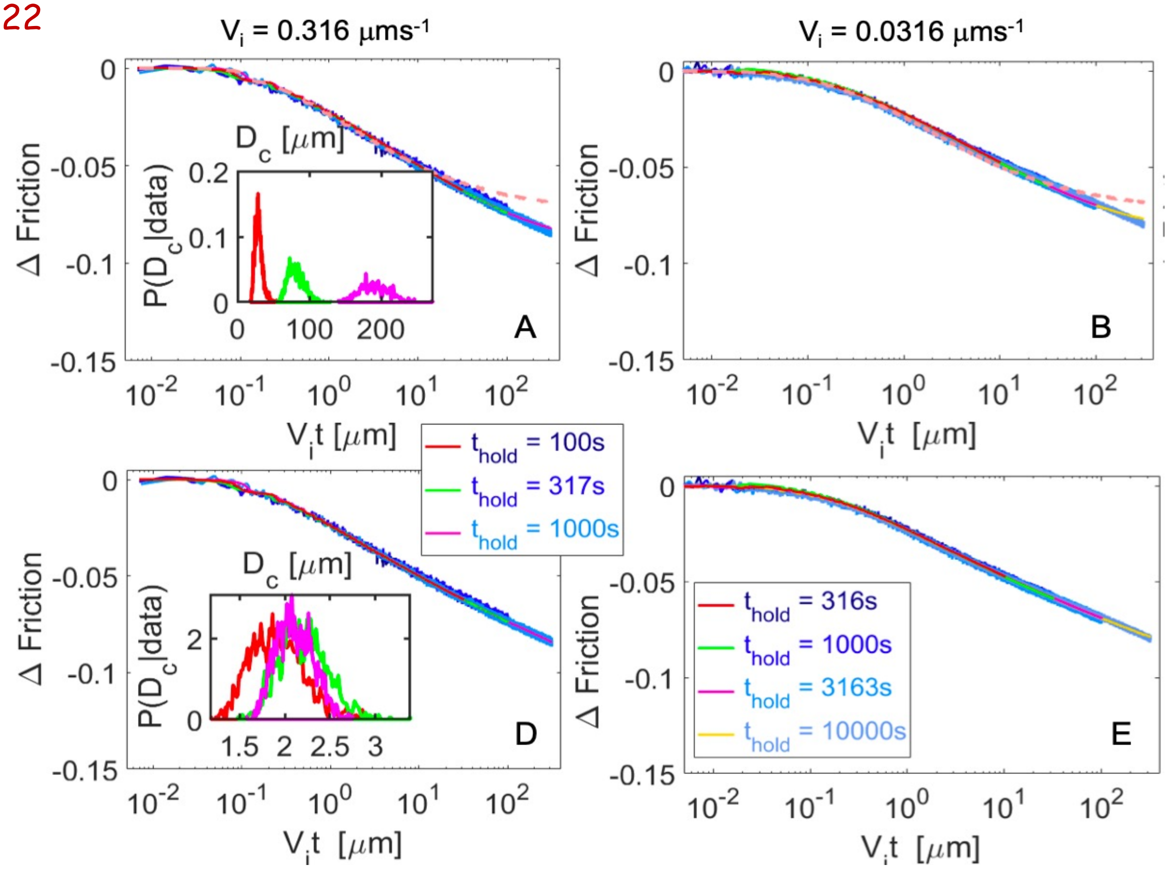
Bhattacharya et al., PNAS, 2022

Best-fitting Slip law: $a = 0.0126$; $b = 0.0157$; $D_c = 2.07$ μm.

Bhattacharya et al., PNAS, 2022

Aging-law fits to the holds:

Slip-law fits to the holds:



Bottom line: The "Slip law" describes very well the evolution of friction
during velocity steps and load-point holds, with a single set of parameter values.
The "Aging law" describes moderately well the peak stress during the reslide portion
of slide-hold-reslide tests (but it only gets the slope to within a factor of 2).
We don't understand why the Slip law is so successful.

A view from the physics community (Baumberger and Caroli, 2006):

$$\begin{aligned}\tau &= A_c(\theta,\sigma_n)\tau_c(V)\\ &= A_{t=0}\left[1+b'\ln\left(\frac{\theta}{t_0}+1\right)\right]\tau_{c(V=V_0)}\left[1+a'\ln\left(\frac{V}{V_0}\right)\right]\\ &= A_{t=0}\tau_{c(V=V_0)}+A_{t=0}a'\ln\left(\frac{V}{V_0}\right)+\tau_{c(V=V_0)}b'\ln\left(\frac{\theta}{t_0}+1\right)+a'\ln\left(\frac{V}{V_0}\right)b'\ln\left(\frac{\theta}{t_0}+1\right)\end{aligned}$$

with a', b', t_0 given by combinations of more fundamental properties.

The “standard” friction law just drops the $ab\ln^2$ term

SUMMARY:

(1) Rate-state effects amount to only a few % of a representative friction value, but are shared by a wide range of materials and are essential to models of earthquake nucleation and fault slip at low sliding speeds.

(2) The observed log-time increase of contact area has a sensible physical explanation and seems consistent with observed log-time increase in frictional strength (“static” friction) during holds. But the equation derived to capture this behavior adequately describes very little lab friction data. An empirical equation in which state evolves only with slip does much better (Sleep 2006 for a possible explanation of the “Slip” state evolution equation).

(3) We have a long way to go before a “first principles” understanding of friction even at room P-T conditions, let alone at high T and in the presence of reactive pore fluids relevant to faults in the Earth. Observations of earthquake “nucleation phases” and injection-induced seismicity offer only indirect constraints, but will be essential in aiding this understanding.

Not to mention …

Changes in state that result from changes in normal stress (Linker and Dieterich 1992 for experiments; Norm Sleep, Sylvain Barbot for theory):

$$\tau = \tau^* + a\sigma \ln \frac{V}{V^*} + b\sigma \ln \frac{\theta}{D_c / V^*}$$

$$\dot{\theta} = f\left(\frac{V\theta}{D_c}, \dot{\sigma}\right)$$

Changes in state that imply changes in porosity that entail pore fluid flow, and hence changes in effective normal stress.
Loss of state evolution at modest slip speeds (steady-state velocity-weakening to velocity-strengthening transition).

High-temperature weakening effects, that include (perhaps in order of increasing slip speed) increases in pore pressure, “flash heating” (heating and weakening of asperities), and melting (Jim Rice, Dmitry Garagash, Sylvain Barbot for theory).
And decomposition of carbonate minerals to solid and gas (CO_2).