Earthquake Physics and Fault Geometry

Scaling relations,
Chapter 3.

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Scaling Relations in Seismic Hazard

Uses:

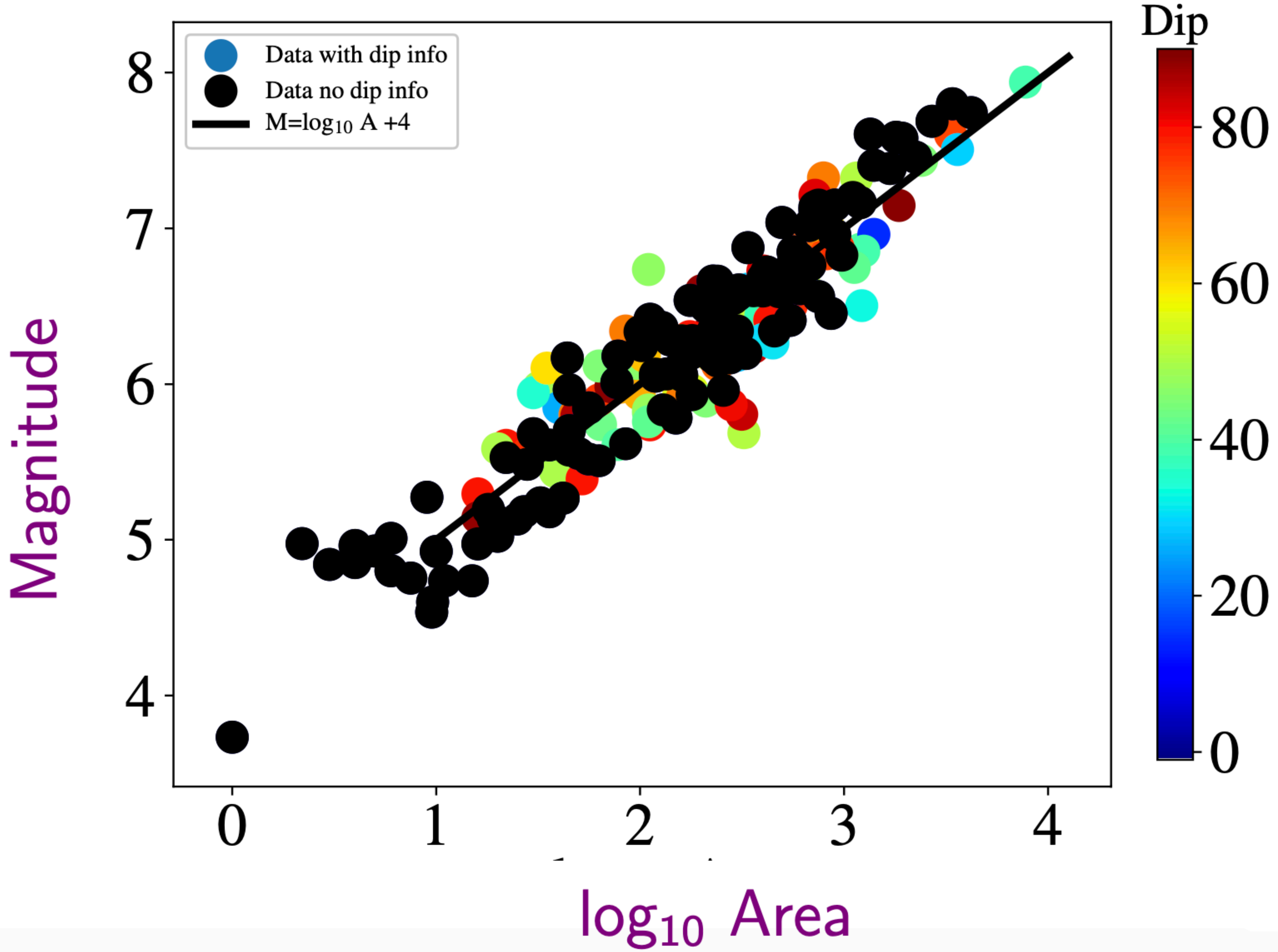
- Used to estimate sizes of events
- Used to estimate rates of events

Considerations in fault-based hazard:

- Applicable to complex multifault ruptures
- Extrapolation to data poor regions
- Good fits to largest events most important
- Robustness important
- Uncertainties in the depth extent of ruptures
- Span epistemic uncertainties
- Continuity in methods valued

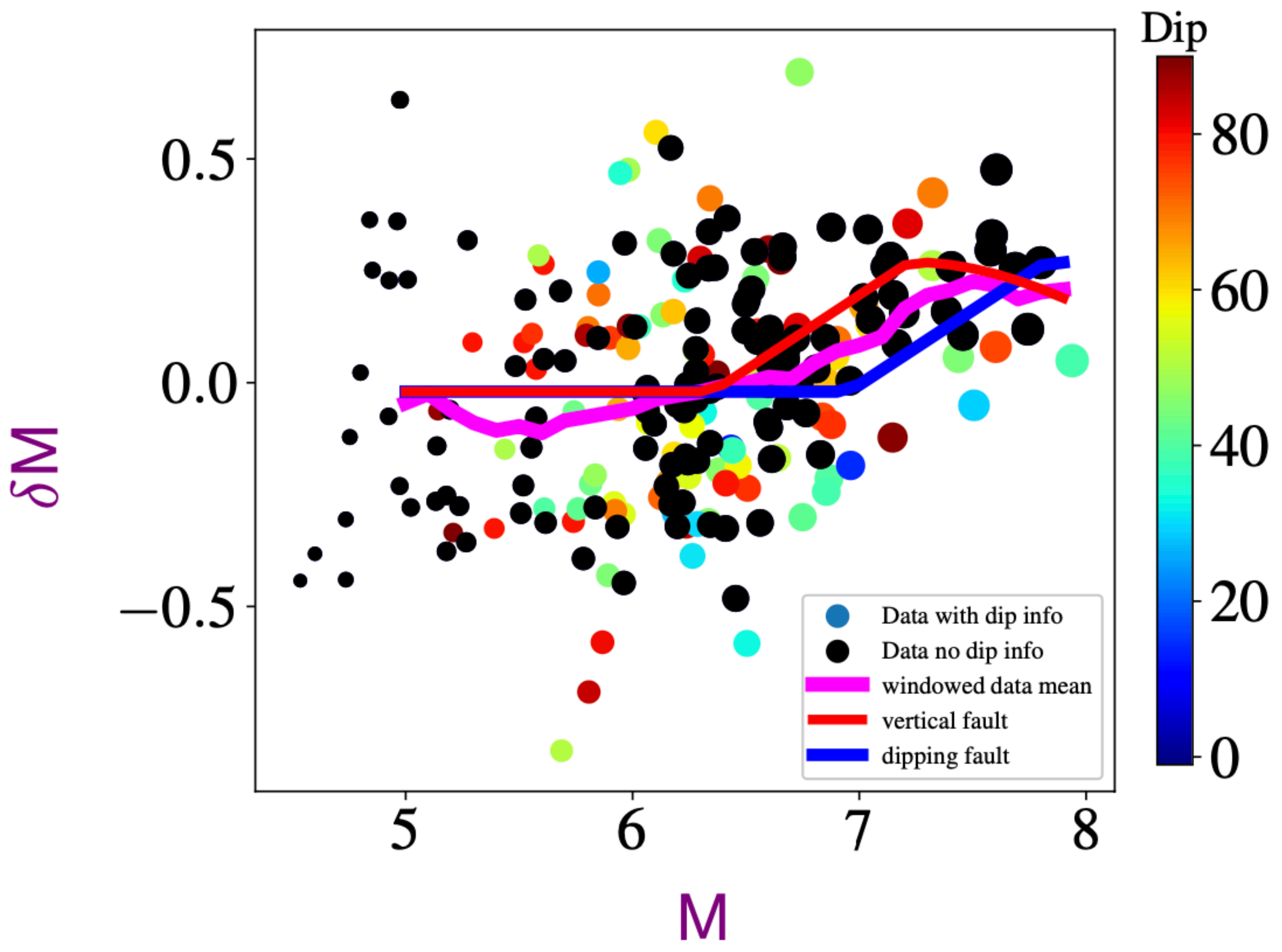
Traditional way of fitting data

Linear log-log fit [Wells and Coppersmith, 1994]



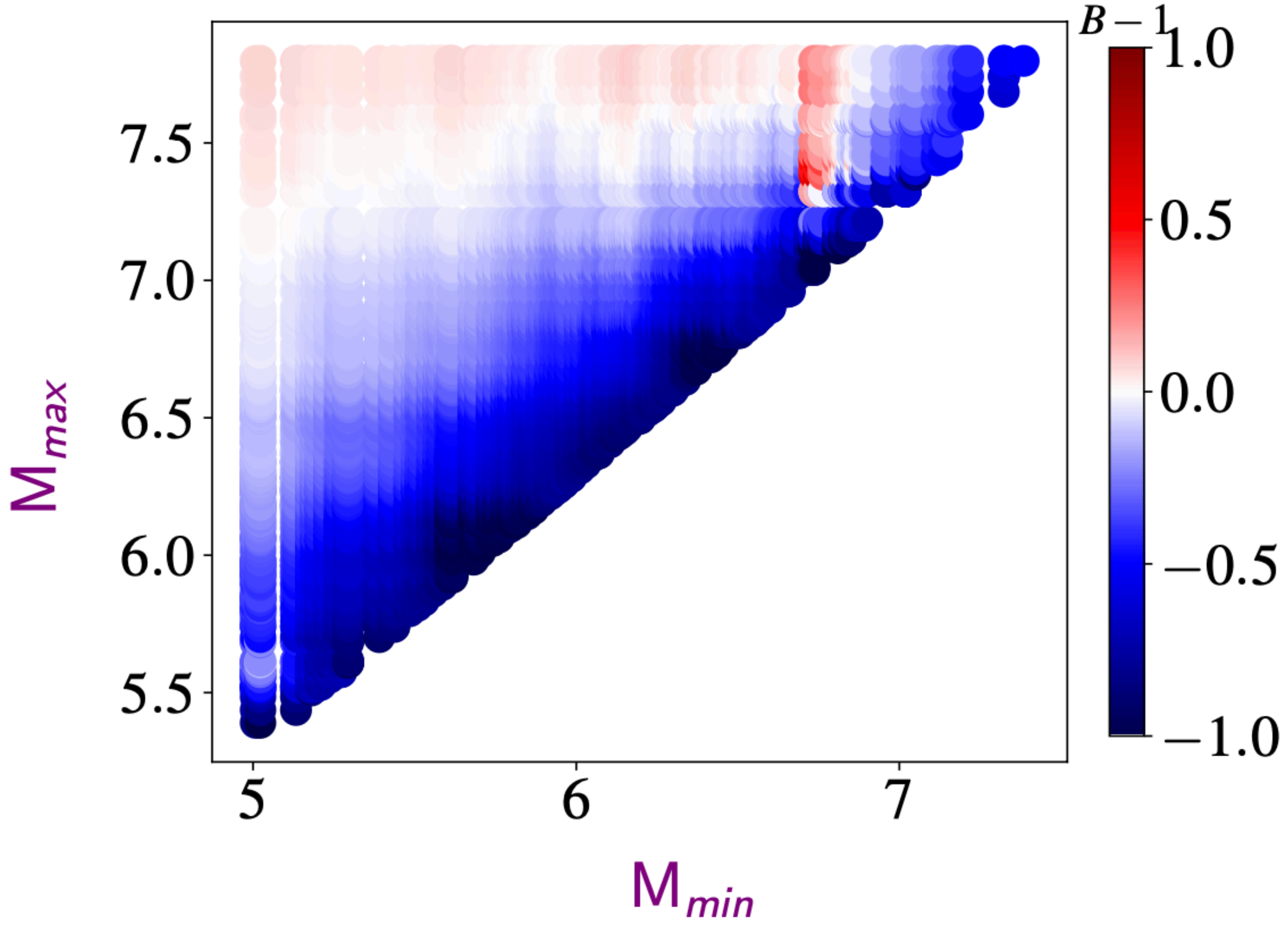
Magnitude Residual deviation from unity slope useful

Illustrates problem with fitting slope to curved data



- Magnitude residual: $\delta \mathbf{M} \equiv \mathbf{M} - \log_{10} A - 4.0$
- Problem of fitting slope when curvature in data
- Free surface and finite seismogenic width effects

Best fit slope depends on magnitude limits

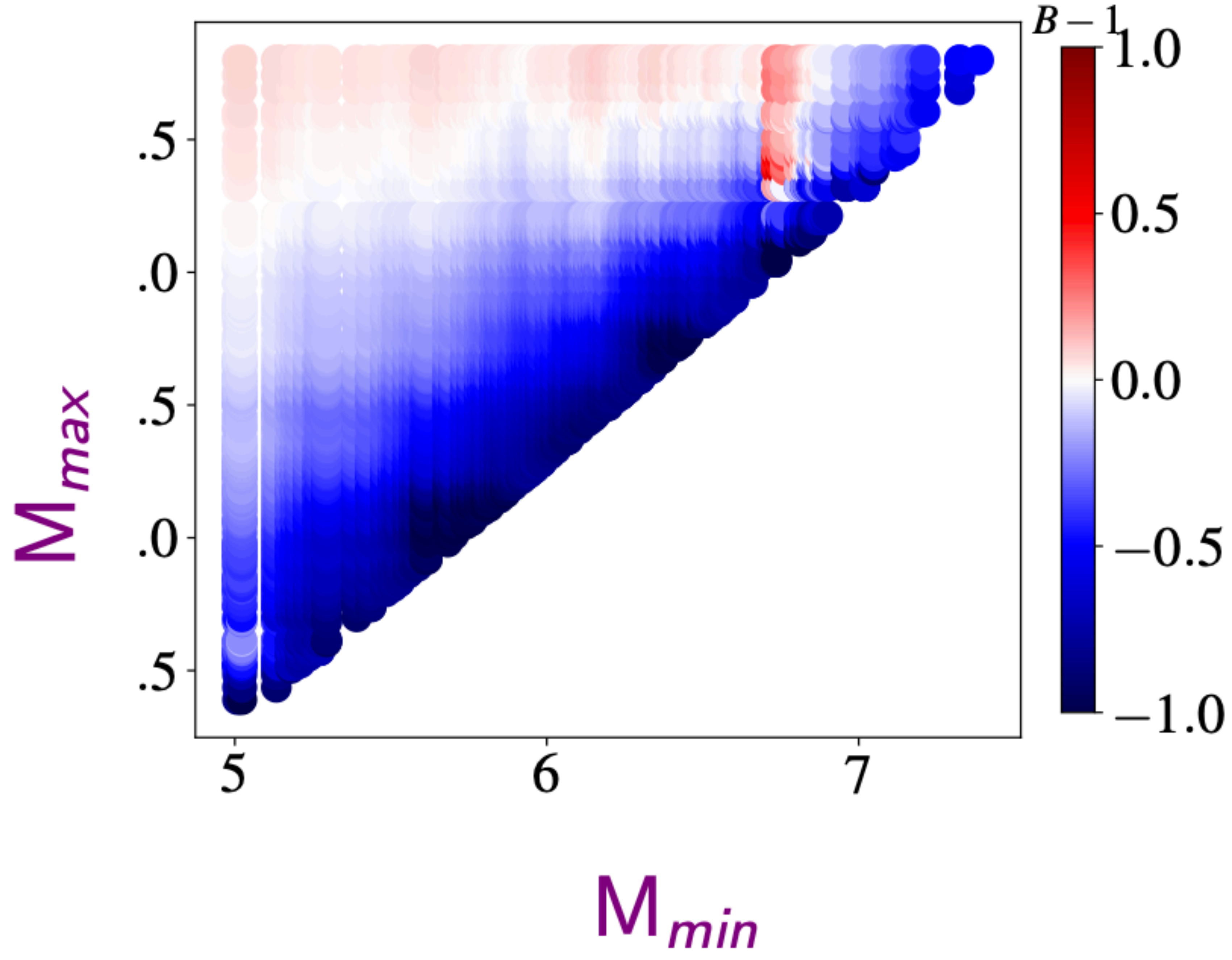


- Best fit slope depends on range of data fit over
- Note low slopes when M_{min} large
- What slope to pick?

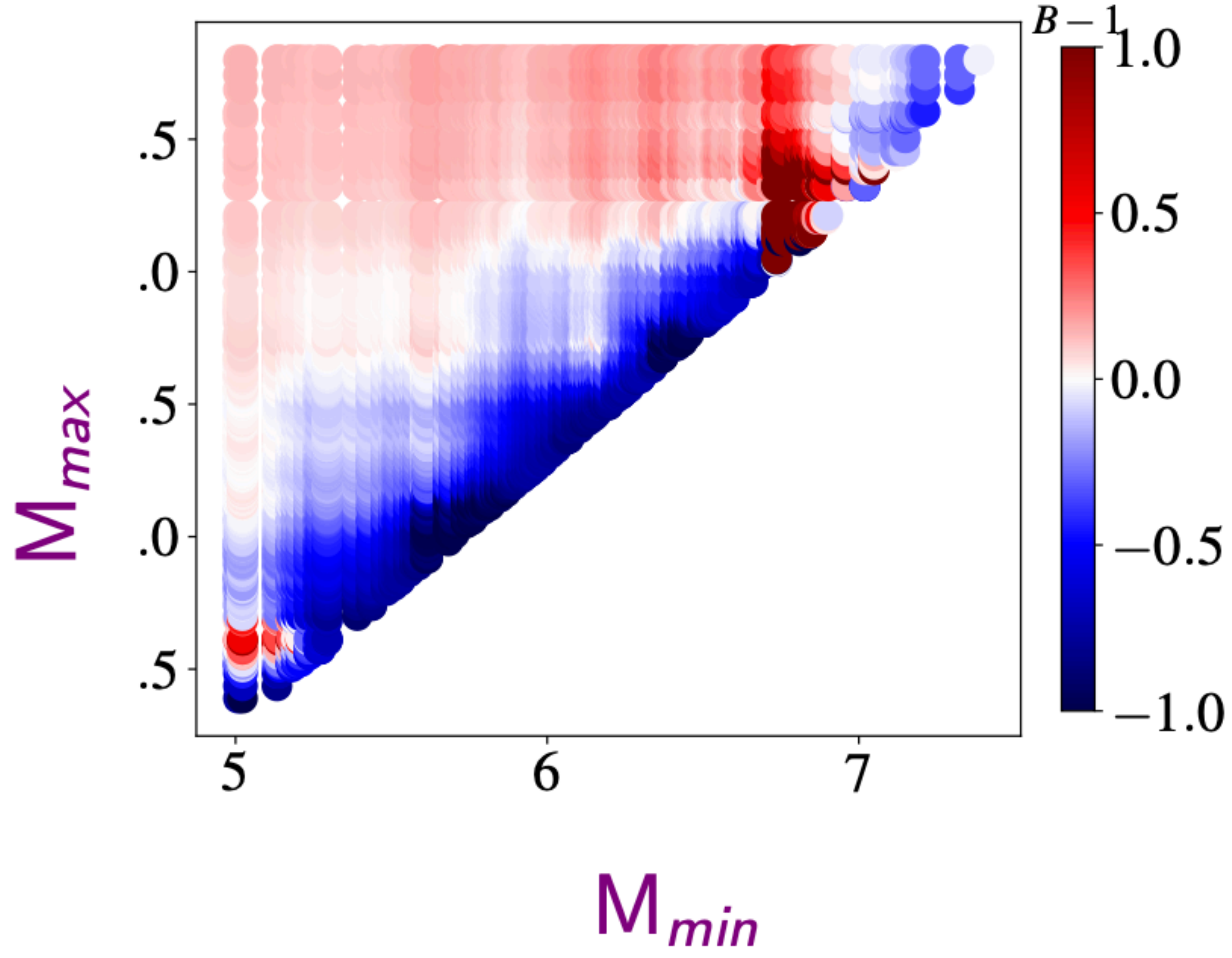
More problems with non-specific exponents

Uncertainty structure impacts best fitting slope

Equal **M** and $\log_{10}$ A Uncertainty



M half of $\log_{10}$ A Uncertainty



- Best fitting slope depends not only on M_{max} and M_{min}, but uncertainty structure as well.
- How would one pick a slope?

Use constrainted functional forms with good asymptotics

Small event scaling:

$$\mathbf{M} = \log_{10} A + C \tag{1}$$

Unconstrained scaling (reject):

$$\mathbf{M} = B \log_{10} A + C \tag{2}$$

Hanks and Bakun scaling (reject):

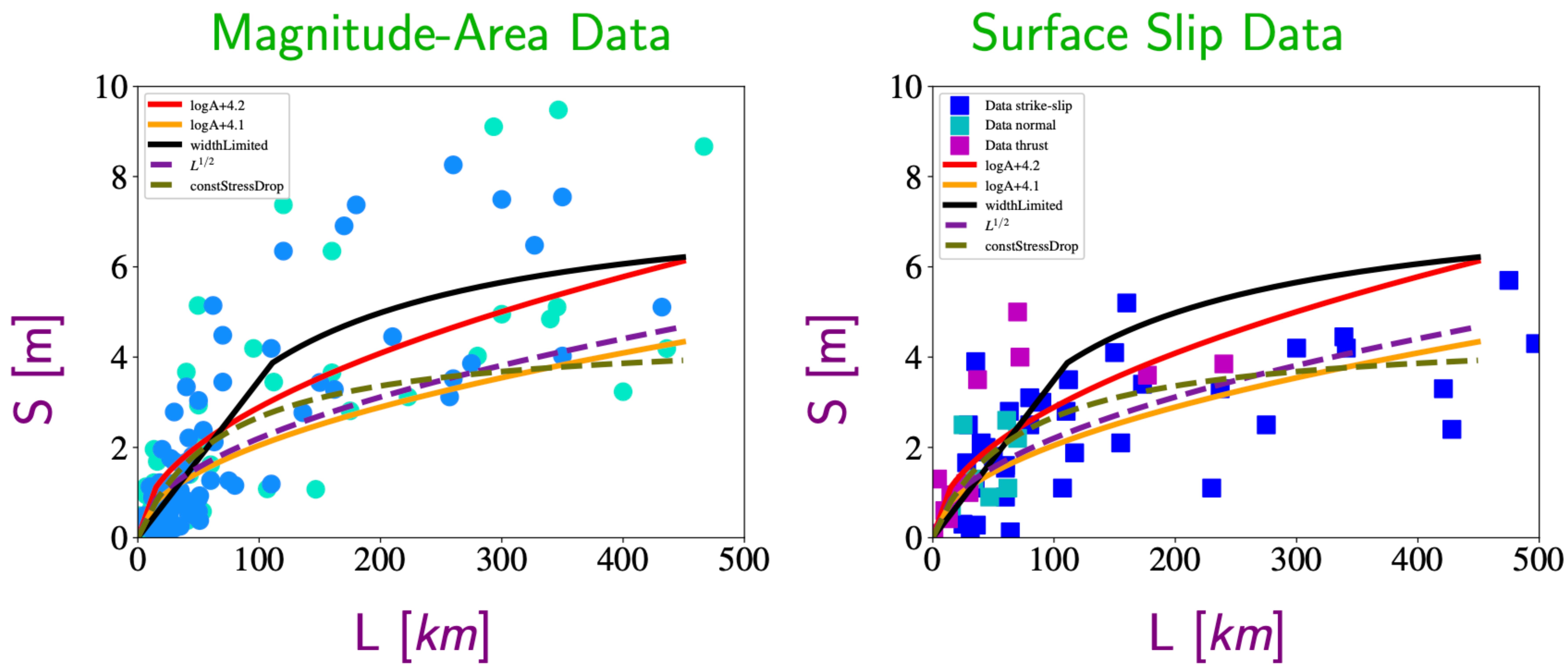
$$\mathbf{M} = \begin{cases} \log A + 3.98 & A \leq 537 \ km^2; \\ \frac{4}{3} \log A + 3.07 & A > 537 \ km^2. \end{cases} \tag{3}$$

Free surface and finite width [Shaw, 2009]:

$$\mathbf{M} = log_{10} A + \frac{2}{3} \log_{10} \frac{\max(1, \sqrt{\frac{A}{W^2}})}{(1 + \max(1, \frac{A}{W^2 \beta}))/2} + C \quad . \tag{4}$$

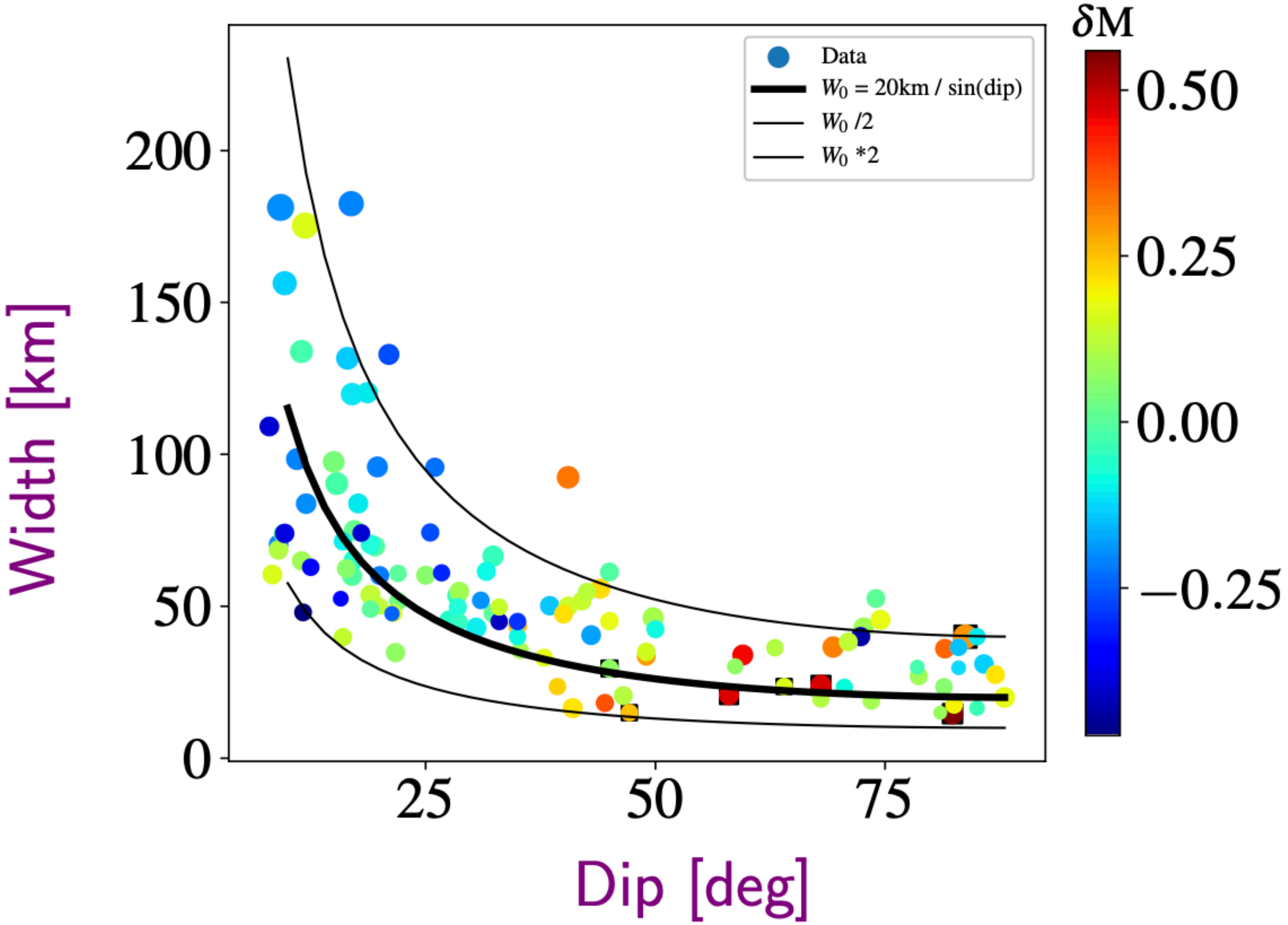
- Reject non-specific exponent B from before
- Reject Hanks-Bakun due to L scaling large slips large M

Slip Scaling



- Implied slip from magnitude-area higher than surface slip
- Surface slip may underestimate deeper slip
- But alternatively ruptures may penetrate deeper as well
- Use both to span epistemic uncertainty
- Not substantial mechanism difference for crustal events

Seismogenic Depth



- Width = Seismogenic Depth/ sin(dip)
- Factor of 2 variability about mean
- Transient deepening of aftershock contours → systematic difference between preevent widths and data estimates

Scaling comments

- Classic scaling relations of $\mathbf{M} = \log_{10} A + C$ are found to be a good approximation, with classical values of $C = 4.15$ from [Wyss, 1979] being a good reference value for plate boundary events.
- Corrections for free surface and finite seismogenic depth present an alternative option.
- Traditional fits of non-specific exponents [Wells and Coppersmith, 1994] problematic due to dependence on magnitude limits of data range fit over.
- Linear scaling of $S \sim L$ out to the largest events does not fit well and gives too large slips for the largest events.
- Subduction events have lower C value (~ -0.2).
- Stable continent interior have higher C value (~ 0.1).

Robust Statistics: going beyond ordinary least squares fits

Some Robust Statistics Concepts:

- Robustness (not sensitive to outliers)
- Median vs mean and outliers
- AIC (Akaike Information Criteria): penalizes fits that use more parameters
- Orthogonal regression (take into account uncertainties in X as well as Y axis

Be careful with Least Squares fits!

- Assumes normally distributed errors
- Sensitive to outliers (non-robust)
- Particularly sensitive to outliers at high and low ends of data
- Systematic errors can distort fits

  For example, if constant location error uncertainty,

  small magnitude area estimates biased high

  exponent fit for magnitude versus area will have lower slope

AIC (Akaike Information Criteria)

Penalizes fits that use more parameters

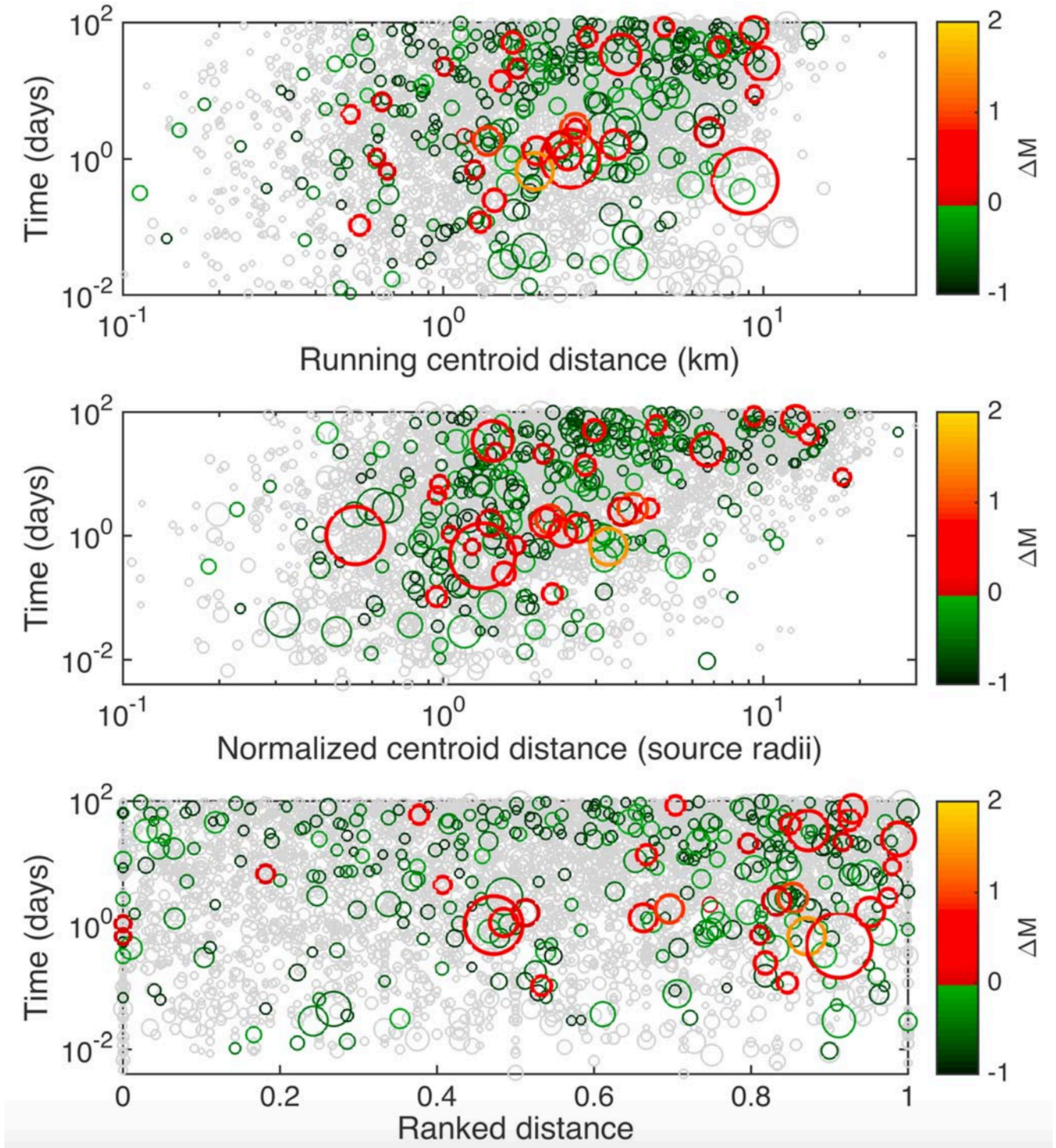
$AIC = -2\mathcal{L} + 2k$

$\mathcal{L}$ Ln likelihood

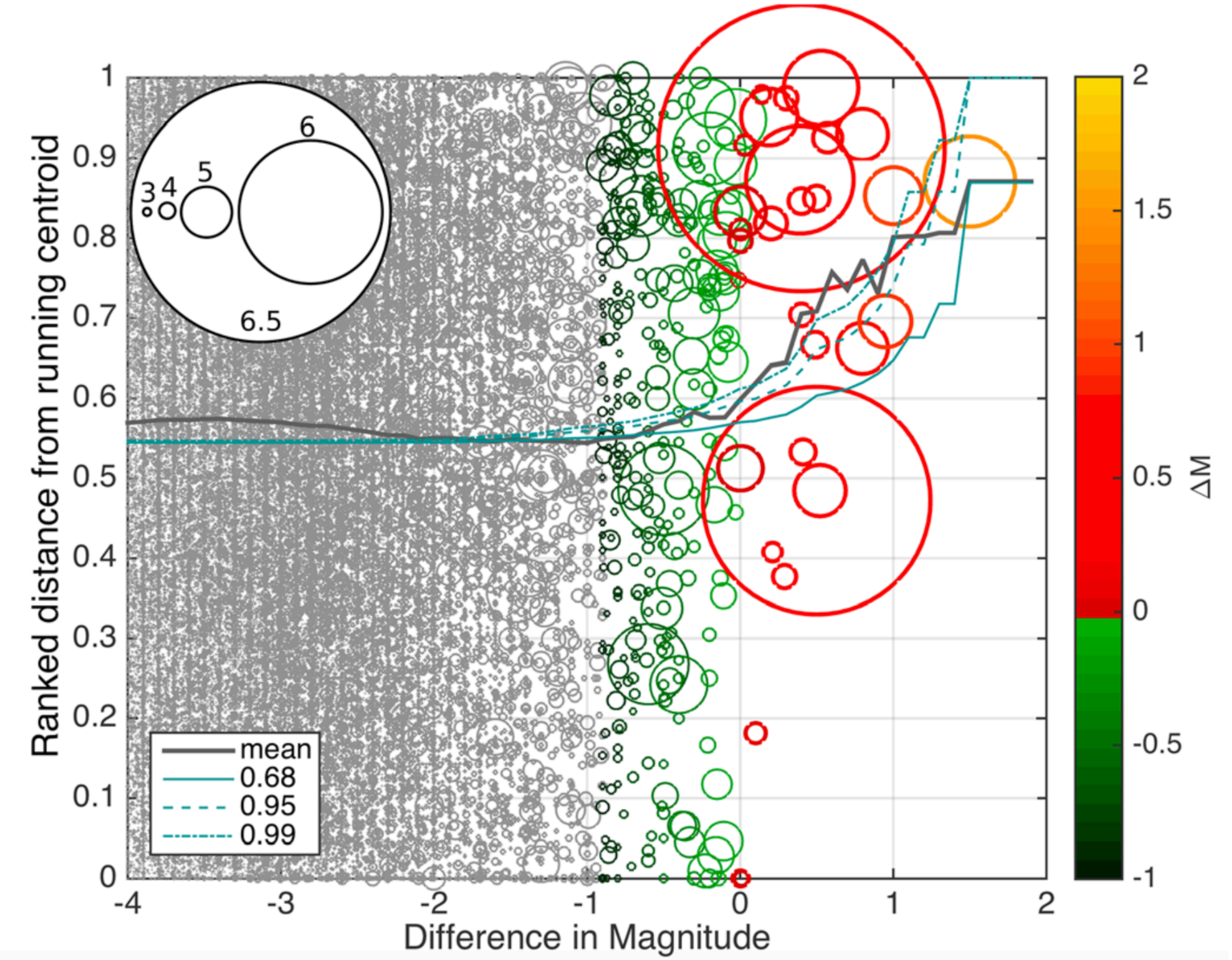
k number of parameters

Allows you to compare relative fits of different functional forms

Scaling and Robust statistics: Bigger aftershocks happen farther away



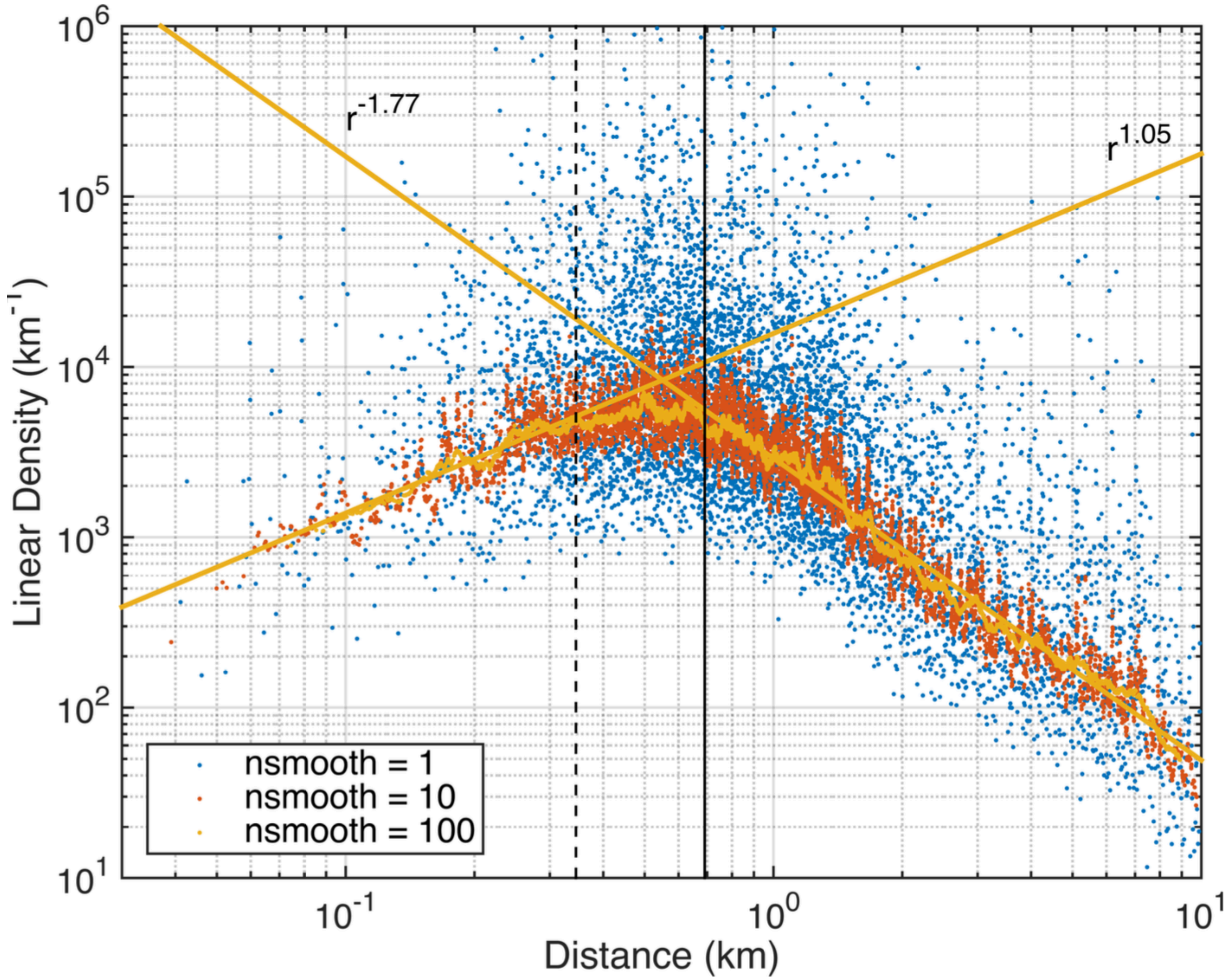
Distance from centroid of aftershocks

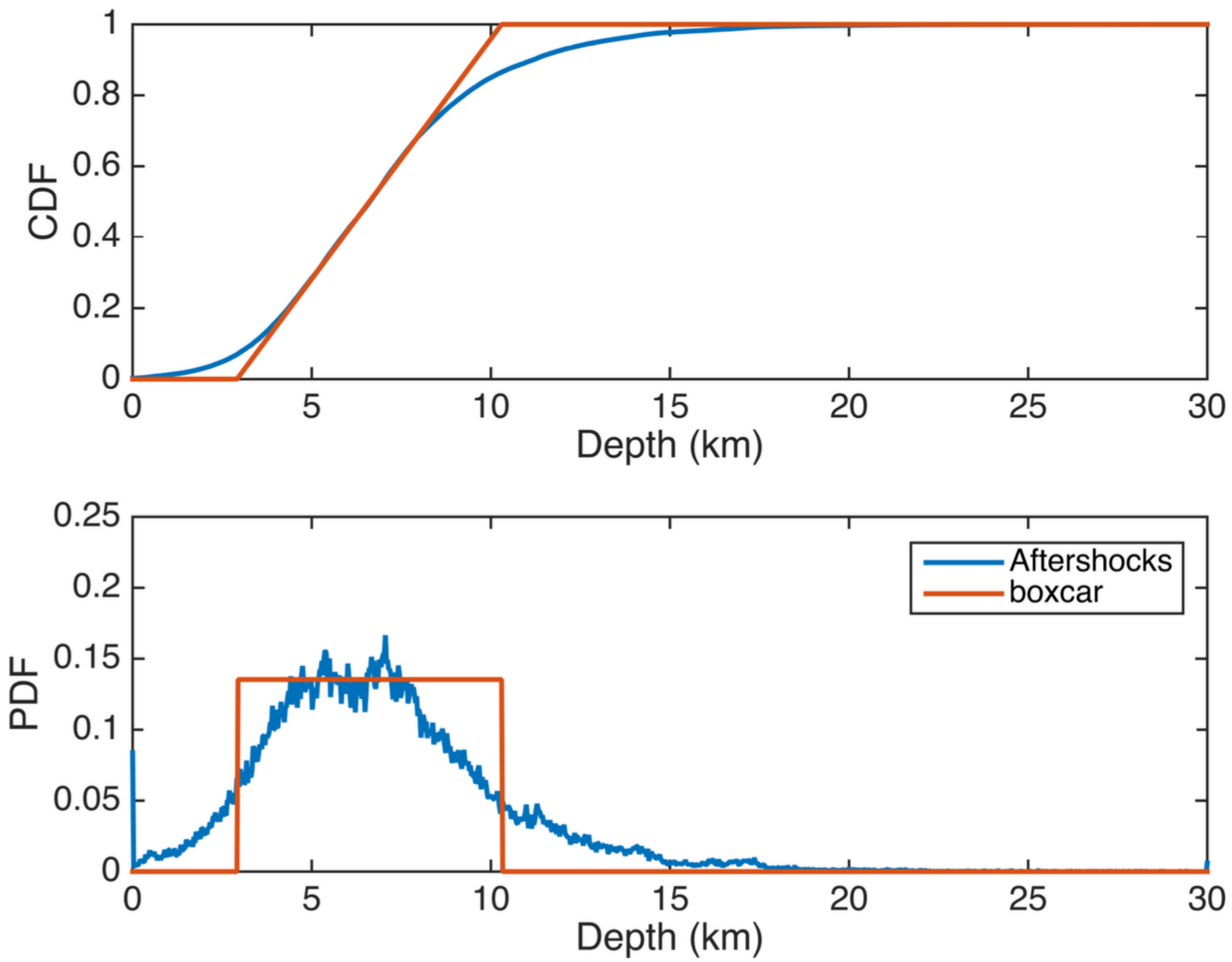


[van der Elst and Shaw, 2015]

Exponent in aftershock falloff impacted by finite seismogenic thickness

Can't just use power law fits naively



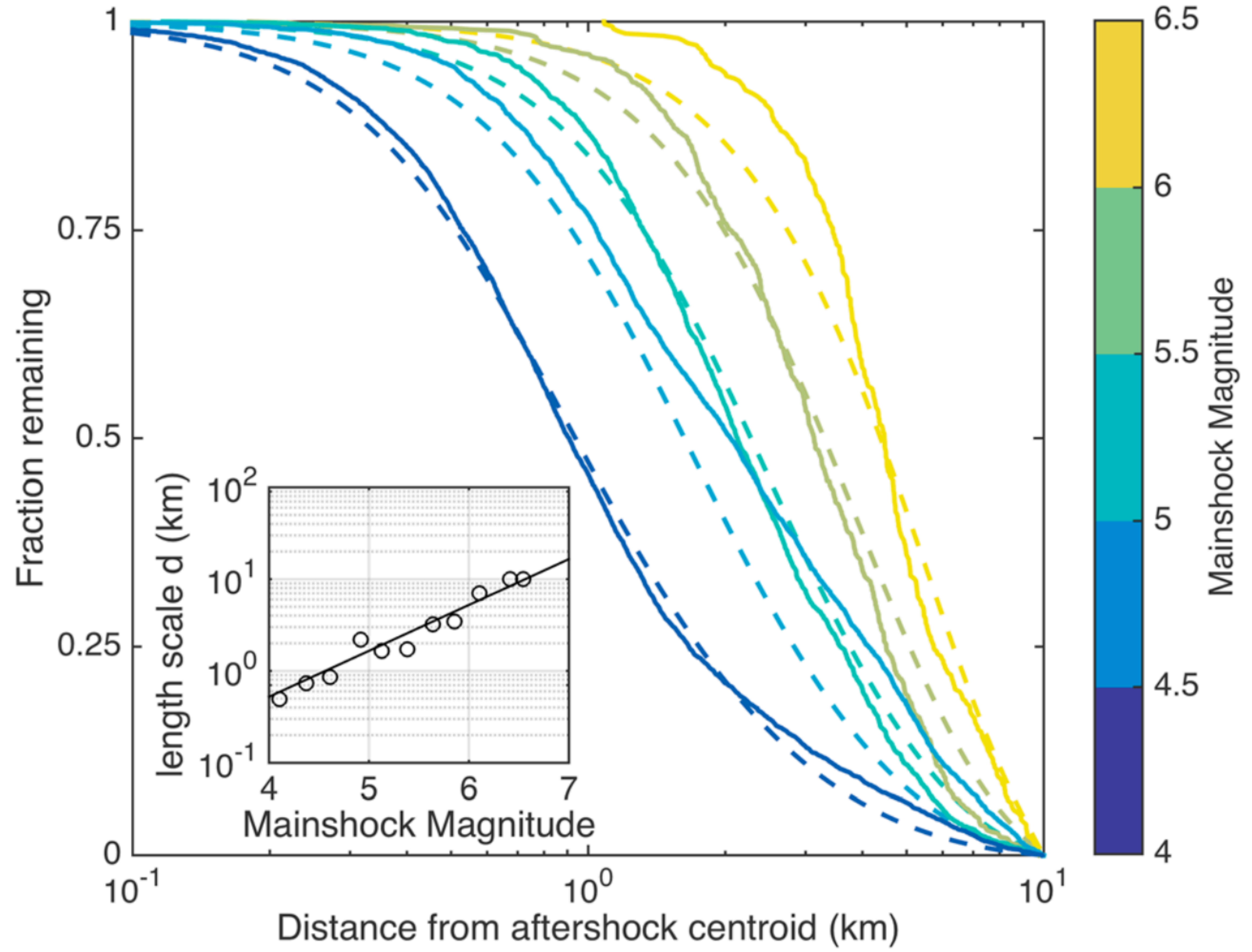


Finite thickness impacts falloff

Linear density falloff of aftershocks M4 main shock

Appears "fractal" if don't account for seismogenic depth

Falloff consistent with static stress drop



Density with distance:

$$p(r) \propto N(r)\left(r^2 + d^2\right)^{-\gamma/2}$$

$$d(M_{ms}) = 10^{\lambda(M_{ms} - M_{ref})}$$

Maximum likelihood estimate:

$\hat{\lambda} = 0.53 \quad \hat{\gamma} = 2.93$

Assuming:

$\lambda = 0.5 \quad \gamma = 3$

→ $\Delta\sigma = 3.0$ MPa.

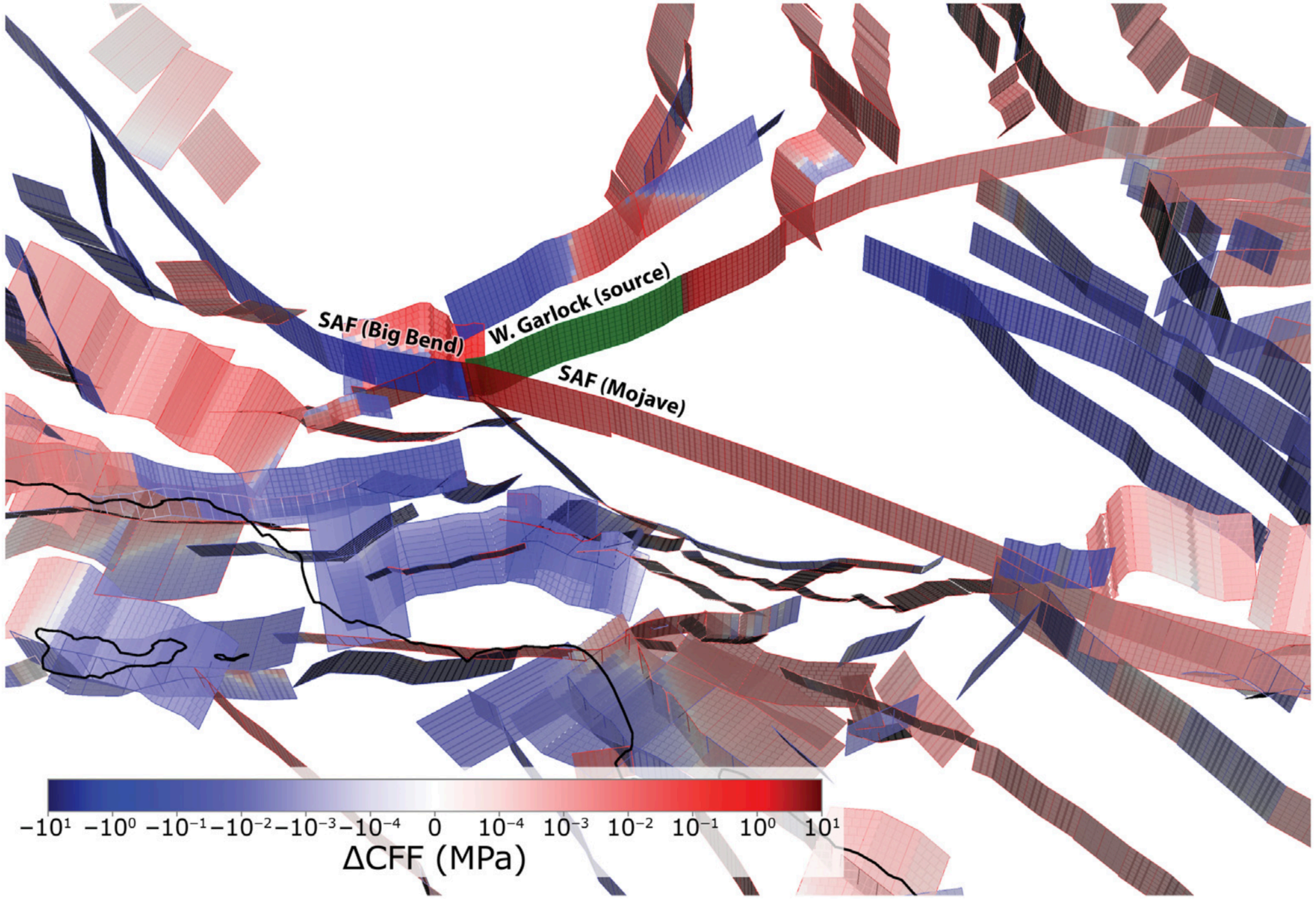
Density of aftershocks from centroid falls as r $^{-3}$

With inner scale length corresponding to 3Mpa

After accounting for finite seismogenic thickness N(r)

Rupture plausibility filter

Using simulator as guide to developing statistical rules for hazard rupture set



Develop robust measures of coulomb interactions to describe potential ruptures

Some statistical coulomb measures

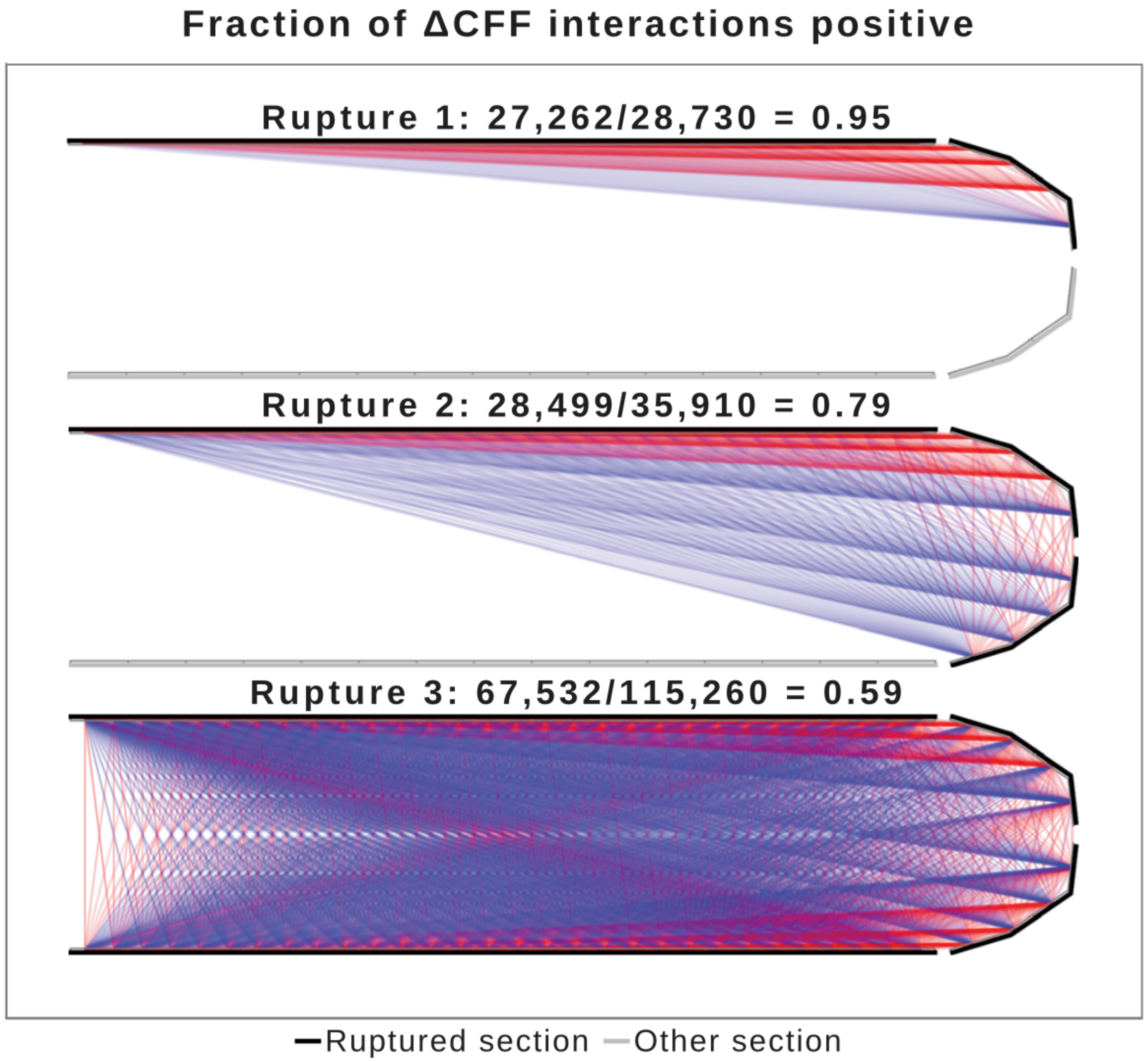
Problem with mean measure is $1/r^3$ stress interaction dominated by small r

Proposed Plausibility Filters

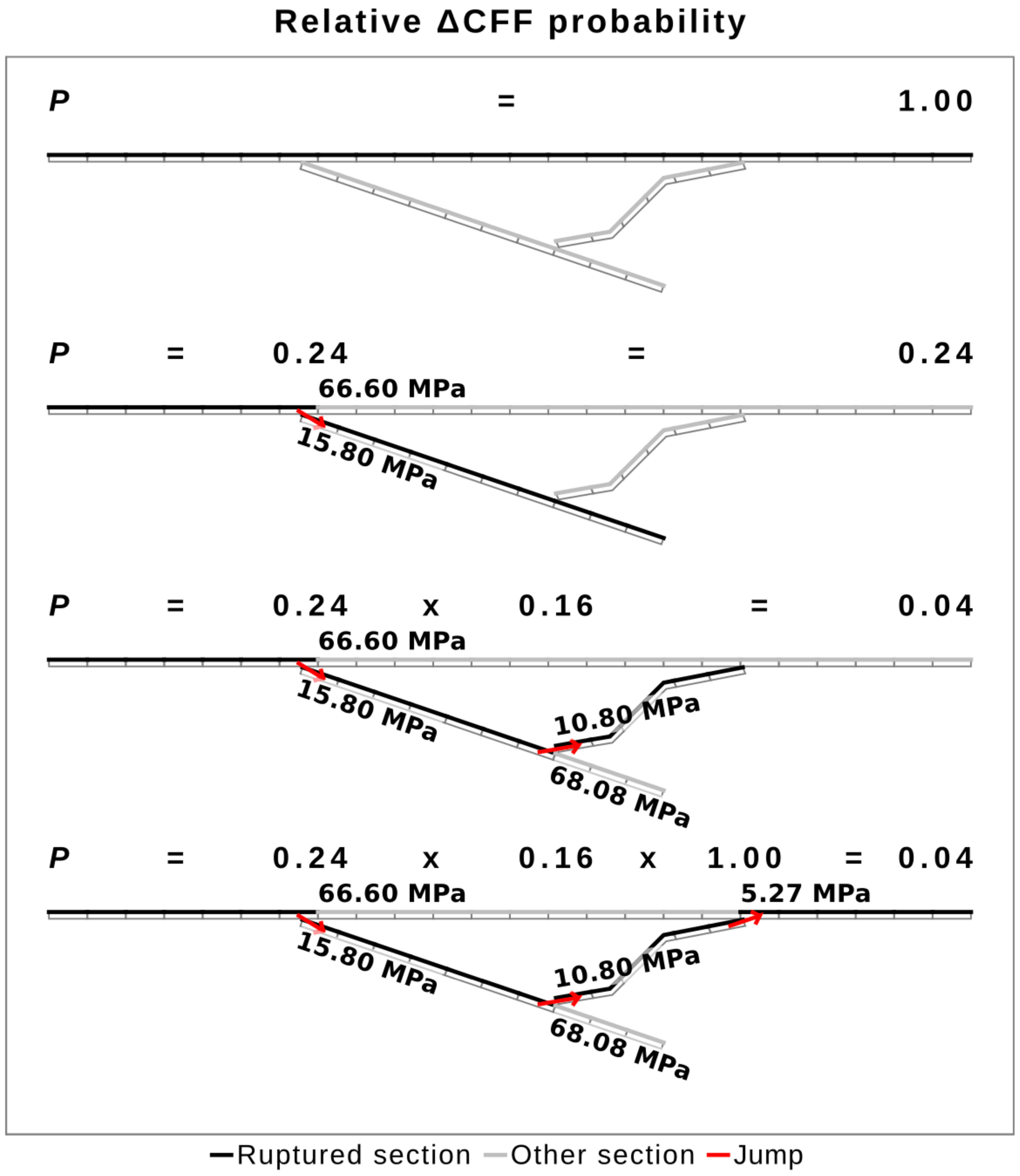
Name	Threshold	Ruptures Excluded	RSQSim Rupture Failures
Maximum jump distance	15 km	N/A	485 (0.22%)
Minimum number of subsections per cluster	2	348,858	33,214 (15.04%)
Cumulative rake change	360°	9,122	36 (0.02%)
Cumulative Coulomb favorability ratio	$R \geq 0.5$	827,224	5109 (2.31%)
Cumulative relative Coulomb probability	$P \geq 0.01$	377,211	3402 (1.54%)
Fraction of Coulomb interactions positive	0.75	539,533	3228 (1.46%)
Cumulative relative slip-rate probability	$P \geq 0.05$	513,977	73 (0.03%)
Cumulative jump distance	$P \geq 0.001$ (~21 km)	39,496	226 (0.10%)
No indirect connections	N/A	43,462	385 (0.17%)

Use more non-dimensional and fractional measures

Fraction positive Coulomb interactions



Relative Coulomb probability



Resulting rupture rules very compatible with simulator set