



Optical model and compound nucleus model

S. Hilaire CEA, DAM, DIF

ICTP-IAEA Workshop on Simulation of Nuclear Reaction Data with the TALYS Code - TRIESTE - October 2023

- Introduction

- General features about nuclear reactions

- Time scales and associated models
- Types of data needed
- Data format = f (users)

- Nuclear Models

- Basic structure properties
- Optical model
- Pre-equilibrium model
- Compound Nucleus model

- Model ingredients

- Level densities
- Gamma-ray strengths
- Fission transmission coefficients

- Fission reactions

- Generalities about fission
- Fission neutrons and gammas
- Fission yields
- Fission cross sections

- Prospects



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THURSDAY

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INTRODUCTION

The references today





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Nuclear data needed for

Understanding basic reaction mechanism between particles and nuclei

Good accuracy if possible \Rightarrow good understanding or room for improvements



Nuclear data needed for

Understanding basic reaction mechanism between particles and nuclei Astrophysical applications (Age of the Galaxy, element abundances ...)

Good accuracy if possible \Rightarrow good understanding or room for improvements Predictive power important \Rightarrow sound physics (first principles)



Nuclear data needed for

Understanding basic reaction mechanism between particles and nuclei Astrophysical applications (Age of the Galaxy, element abundances ...) Existing or future nuclear reactor simulations

Good (Excellent) accuracy required \Rightarrow reproduction of data, safety Predictive power less important \Rightarrow Reproductive power



Nuclear data needed for

Understanding basic reaction mechanism between particles and nuclei Astrophysical applications (Age of the Galaxy, element abundances ...) Existing or future nuclear reactor simulations Medical applications, oil well logging, waste transmutation, fusion, ...

Good accuracy required \Rightarrow reproduction of data Predictive power less important \Rightarrow Reproductive power



Nuclear data needed for

Understanding basic reaction mechanism between particles and nuclei Astrophysical applications (Age of the Galaxy, element abundances ...) Existing or future nuclear reactor simulations Medical applications, oil well logging, waste transmutation, fusion, ...

Good accuracy required \Rightarrow reproduction of data Predictive power less important \Rightarrow Reproductive power

But

Finite number of experimental data (price, safety or counting rates) Complete measurements restricted to low energies (< 1 MeV) and scarce nuclei

Predictive & Robust Nuclear models (codes) are essential

Optical model and compound nucleus model



Predicted nuclei



 \Rightarrow nearly 9200 nuclei predicted up to Z=130

 \Rightarrow 256 stable nuclei + few tens of quasi-stable (half life many years)



Predicted nuclei, known masses



 \Rightarrow nearly 9200 nuclei predicted up to Z=130

- \Rightarrow 256 stable nuclei + few tens of quasi-stable (half life many years)
- \Rightarrow less than 2600 known masses up to Z=110



Predicted nuclei, known masses, known capture



 \Rightarrow nearly 9200 nuclei predicted up to Z=130

- \Rightarrow 256 stable nuclei + few tens of quasi-stable (half life many years)
- \Rightarrow 2550 known masses up to Z=110
- \Rightarrow less than 150 known capture cross sections

Do application (except astrophysics) care about exotic nuclei ?



Cross section stongly vary !





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GENERAL FEATURES ABOUT NUCLEAR REACTIONS

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Typical spectrum shape

⁵⁶Fe(p,xp) @ 62 MeV Double differential cross sections



3 regions :

- Red : « evaporation » peak always present and low outgoing energy
- Green : « flat » intermediate region
 width increases with incident energy
- Blue : « discrete » peaks
 outgoing energy close to incident energy









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Models sequence and required ingredients





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Models sequence and required ingredients









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Cross sections :

total, reaction, elastic (shape & compound), non-elastic, inelastic (discrete levels & total)

BRC AIEA stand

20

+ LANL

LOUGHEED

MÉOT (1985)

15

10

10

(n,2n)

5

Energie Neutron (MeV)

10

1.5

0.6

0.4

20

total particle (residual) production

all exclusive reactions (n,2n), (n,p),, (n,nd2a)

all exclusive isomer production

15

10

5

2

1.4

2

1

10 -3

10 -2

Sections efficaces (barn)

all exclusive discrete and continuum γ-ray production

(n,tot)

10 -1

10 -2

(n,n'

10 -1

(n,2n)

1

10

a) (b)







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total particle (residual) production

Cross sections :

Types of data needed

all exclusive reactions (n,2n), (n,p),, (n,nd2a)

all exclusive isomer production

all exclusive discrete and continuum γ-ray production

total, reaction, elastic (shape & compound), non-elastic, inelastic (discrete levels & total)

Spectra :

elastic and inelastic angular distribution or energy spectra all exclusive double-differential spectra

total particle production spectra

compound and pre-equilibrium spectra per reaction stage.





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²³⁹Pu neutron multiplicity

Types of data needed

Cross sections :

total, reaction, elastic (shape & compound), non-elastic, inelastic (discrete levels & total)

total particle (residual) production

all exclusive reactions (n,2n), (n,p),, (n,nd2a)

all exclusive isomer production

all exclusive discrete and continuum γ-ray production

Spectra :

elastic and inelastic angular distribution or energy spectra all exclusive double-differential spectra total particle production spectra compound and pre-equilibrium spectra per reaction stage.

Fission observables :

cross sections (total, per chance) fission fragment mass and isotopic yields fission neutrons (multiplicities, spectra)



Optical model and compound nucleus model

Types of data needed **Cross sections :**

total, reaction, elastic (shape & compound), non-elastic, inelastic (discrete levels & total)

total particle (residual) production

all exclusive reactions (n,2n), (n,p),, (n,nd2a)

all exclusive isomer production

all exclusive discrete and continuum y-ray production

Spectra :

elastic and inelastic angular distribution or energy spectra all exclusive double-differential spectra total particle production spectra compound and pre-equilibrium spectra per reaction stage.

Fission observables :

cross sections (total, per chance) fission fragment mass and isotopic yields fission neutrons (multiplicities, spectra)

Miscellaneous :

recoil cross sections and ddx particle multiplicities astrophysical reaction rates covariances informations





Fig. 8 Example of the difference between exact and approximative recoil treatment in TALYS.



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- Trivial for basic nuclear science : x,y,(z) file
- Complicated (even crazy) for data production issues : ENDF file


Data format : ENDF file



NUCLEAR MODELS



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Basic structure properties : what is needed ?

Nuclear Masses :

 \Rightarrow basic information to determine reaction threshold

Excited levels :

- \Rightarrow Angular distributions (depend on spin and parities)
- ⇒ Decay properties (branching ratios)
- \Rightarrow Excitation energies (reaction thresholds)

Target levels' deformations :

- \Rightarrow Required to select appropriate optical model
- \Rightarrow Required to select appropriate coupling scheme

Many different theoretical approaches if experimental data are missing Recommended databases (RIPL !)



Basic structure properties : mass models



- Typical deviations for the best mass formulas:
- $rms(M) = 500-700 \text{ keV on } Z \ge 8 \text{ experimental masses}$



Accuracy

Reliability



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Basic structure properties : mass models' predictive power

Comparison between several mass models adjusted with 2003 exp and tested with 2012 exp masses



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Basic structure properties : Nuclear structure & level scheme

 \Rightarrow General level sequence for a spherical even-even nucleus



3 coupled phonons

2 coupled phonons

1 phonon

other levels





Basic structure properties : Nuclear structure & level scheme



Basic structure properties : Nuclear structure & level scheme



106Pd₆₀-1



 $^{106}_{46}\mathrm{Pd}_{60}$ -1

 \Rightarrow energy, spin, parity, decay modes, ... :

informations compiled in ENSDF database

 \Rightarrow Level scheme = deformation evidence

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 $^{106}_{46} Pd_{60} - 14$



















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Optical model and competing measure measure

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Basic structure properties : RIPL inputs









Basic structure properties : RIPL inputs





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Basic structure properties : RIPL inputs





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The optical model : why such a name ?



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⇒ diffraction patterns observed
experimentally
⇒ optical model

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The optical model : basics



Direct interaction of a projectile with a target nucleus considered as a whole Quantum model \rightarrow Schrödinger equation



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The optical model : output ?





The optical model yields :

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The optical model : various approaches



Phenomenological

20 adjusted parameters Weak predictive power Very precise (\approx 1%) Important work



Optical model and compound nucleus model

(Semi-)microscopic

No adjustable parameters Usable without exp. data Less precise (\approx 5-10 %) Quasi-automated



The optical model : phenomenological model



- \approx 20 adjusted parameters
- Very precise (1%)
- Weak predictive power
- Requires data
- Local models
 - => Global models



The optical model : phenomenological model



 $U(r,E) = V(E,r) + i W(E,r) + \{V_{so}(E,r) + i W_{so}(E,r)\}$ I.s

Central term

Spin orbit term

The optical model : phenomenological model





Let's neglect SO

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U(r,E) = V(E,r) + i W(E,r)

Central term



$$U(r,E) = \begin{bmatrix} V_V(E) f(r, \mathbf{R}_V, a_V) + V_S(E) g(r, \mathbf{R}_S, a_S) \end{bmatrix}$$

+ i
$$\begin{bmatrix} W_V(E) f(r, \mathbf{R}_V, a_V) + W_S(E) g(r, \mathbf{R}_S, a_S) \end{bmatrix}$$












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- No adjustable parameters
- Based on nuclear structure properties
 - \Rightarrow usable for any nucleus
- Less precise than the phenomenological approach













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Unique description of elastic scattering

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Unique description of elastic scattering (n,n)





Unique description of elastic scattering (n,n), (p,p)







Unique description of elastic scattering (n,n), (p,p) and (p,n)



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Enables to give predictions for very exotic nuclei for which there exist no experimental data



Enables to give predictions for very exotic nuclei for which there exist no experimental data



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Enables to give predictions for very exotic nuclei for which there exist no experimental data





The optical model : direct reaction description

- Spherical OMP

- \Rightarrow Shape elastic only
- \Rightarrow Absorbed flux distributed with CN model
- \Rightarrow Generally bad for inelastic scattering off collective levels

- Coupled chanel approach

- \Rightarrow Both elastic and few « well chosen » inelastic levels
- \Rightarrow Absorbed flux distributed with CN model
- \Rightarrow Good description of collective levels if the coupling model is appropriate
 - DWBA approximation for spherical or weakly deformed nuclei
 - CC approach for well deformed nuclei (various options)

 \Rightarrow The choice of the « well chosen » inelastic levels relies on the experimental level scheme or on theoretical predictions of the nuclear deformation and excited levels

Nuclear structure & level scheme : spherical nucleus





Nuclear structure & level scheme : deformed nucleus



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The optical model : DWBA vs CC



The optical model : coupling scheme saturation







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The optical model : coupling scheme saturation

Optical model and compound nucleus model



All data must be simultaneously described at best



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All data must be simultaneously described at best







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and the whole reaction process matters

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All data must be simultaneously described at best



and the whole reaction process matters

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The optical model : impact of deformation on cross sections



Compound nucleus component

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The optical model : impact of deformation on cross sections



Compound nucleus component Direct component Sum

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The optical model : impact of structure description



- \Rightarrow Nearly spherical nucleus
- \Rightarrow Structure with 5DCH (GCM) probably not perfect

 \Rightarrow QRPA option could be more appropriate

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The optical model : impact of structure description







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The optical model : deformation and outgoing spectrum



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The optical model : deformation and outgoing spectrum



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The optical model : deformation and outgoing spectrum



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The optical model : RIPL inputs





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The pre-equilibrium model : history

Semi-classical approaches

- called « exciton model »
- « simple » to implement
- initially only able to describe angle integrated spectra (1966 & 1970)
- extended to ddx spectra in 1976
- link with Compound Nucleus established in 1987
- systematical underestimation of ddx spectra at backward angles
- complemented by Kalbach systematics (1988) to improve ddx description
- link with OMP imaginary performed in 2004

Quantum mechanical approaches

- distinction between MSC and MSD processes

MSC = bound p-h excitations, symetrical angular distributions

MSD = unbound configuration, smooth forward peaked ang. dis.

- MSD dominates pre-equ xs above 20 MeV
- 3 approaches : FKK (1980)
 - TUL (1982)
 - NWY (1986)
- ddx spectra described as well as with Kalbach systematics

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 $\lambda_{a, b}$ (E) = Transition rate from an initial state a towards a state b for a given energy E.



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Evolution equation

dP(n,E,t) dt

- Disparition

Apparition



 $\lambda_{a,b}$ (E) = Transition rate from an initial state a towards a state b for a given energy E.

Evolution equation

$$\frac{dP(n,E,t)}{dt} = P(n-2, E, t) \lambda_{n-2, n}(E) + P(n+2, E, t) \lambda_{n+2, n}(E)$$

- **Disparition**



 $\lambda_{a,b}$ (E) = Transition rate from an initial state a towards a state b for a given energy E.

$\frac{dP(n,E,t)}{dt} = P(n-2, E, t) \lambda_{n-2, n}(E) + P(n+2, E, t) \lambda_{n+2, n}(E)$ $- P(n, E, t) \left[\lambda_{n, n+2}(E) + \lambda_{n, n-2}(E) + \lambda_{n, emiss}(E)\right]$



 $\lambda_{a, b}$ (E) = Transition rate from an initial state a towards a state b for a given energy E.

$$\frac{dP(n,E,t)}{dt} = P(n-2, E, t) \lambda_{n-2, n}(E) + P(n+2, E, t) \lambda_{n+2, n}(E)$$
$$-P(n, E, t) \left[\lambda_{n, n+2}(E) + \lambda_{n, n-2}(E) + \lambda_{n, emiss}(E)\right]$$
$$Emission cross section in channel c$$
$$d\sigma_{c}(E, \varepsilon_{c}) = \sigma_{R} \int_{0}^{\infty} \sum_{n, \Delta n=2} P(n, E, t) \lambda_{n, c}(E) dt d\varepsilon_{c}$$

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Initialisation

 $P(n, E, 0) = \delta_{n,n_0}$ with $n_0=3$ for nucleon induced reactions

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 $P(n, E, 0) = \delta_{n,n_0}$ with $n_0=3$ for nucleon induced reactions

Transition rates

$$\lambda_{n, n-2}(E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \quad \omega(p,h,E) \text{ with } p+h=n-2$$

$$\lambda_{n, n+2}(E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \quad \omega(p,h,E) \text{ with } p+h=n+2$$

$$\lambda_{n, n+2}(E) = \frac{2s_c+1}{\hbar} \langle M^2 \rangle \quad \omega(p,h,E) \text{ with } p+h=n+2$$

$$\lambda_{n, n+2}(E) = \frac{2s_c+1}{\pi^2 \hbar^3} \mu_c \, \varepsilon_c \, \sigma_{c,inv}(\varepsilon_c) \frac{\omega(p-p_b,h,E-\varepsilon_c-B_c)}{\omega(p,h,E)}$$
Original formulation

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Initialisation

 $P(n, E, 0) = \delta_{n, n_0}$ with $n_0=3$ for nucleon induced reactions

Transition rates

$$\begin{split} \lambda_{n, n-2} (E) &= \frac{2\pi}{\hbar} \langle M^2 \rangle \ \omega(p,h,E) \text{ with } p+h=n-2 \\ \lambda_{n, n+2} (E) &= \frac{2\pi}{\hbar} \langle M^2 \rangle \ \omega(p,h,E) \text{ with } p+h=n+2 \\ \lambda_{n, c} (E) &= \frac{2s_c+1}{\pi^2 \hbar^3} \ \mu_c \ \varepsilon_c \ \sigma_{c,inv} (\varepsilon_c) \frac{\omega(p-p_b,h,E-\varepsilon_c-B_c)}{\omega(p,h,E)} \ Q_c(n) \ F_c \\ Corrections for proton-neutron distinguishability \\ & & \\ &$$

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Initialisation

 $P(n, E, 0) = \delta_{n,n_0}$ with $n_0=3$ for nucleon induced reactions

Transition rates

$$\lambda_{n, n-2} (E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \quad \omega(p,h,E) \text{ with } p+h=n-2$$

$$\lambda_{n, n+2} (E) = \frac{2\pi}{\hbar} \langle M^2 \rangle \overset{\text{Corrections for proton-neutron}}{\omega(p,h,E)} \overset{\text{K}}{\underset{m, c}{\text{M}^2}} \sum_{\substack{n, n+2 \\ m \in \mathbb{C}}}^{k} \langle M^2 \rangle \overset{\text{Corrections for proton-neutron}}{\omega(p,h,E)} \overset{\text{K}}{\underset{m}{\text{M}^2}} \sum_{\substack{n, n+2 \\ m \in \mathbb{C}}}^{k} \langle M^2 \rangle \overset{\text{Corrections for proton-neutron}}{\omega(p,h,E)} Q_c(n) F_c$$

State densities

 $\omega(p,h,E)$ = number of ways of distributing p particles and h holes among all accessible single particle levels with the available excitation energy E

The pre-equilibrium model : state densities



State densities in ESM

- Ericson 1960 : no Pauli principle
- Griffin 1966 : no distinction between particles and holes
- Williams 1971 : distinction between particles and holes as well as between neutrons and protons but infinite number of accessible states for both particle and holes

$$\omega_{p_{\pi}h_{\pi}p_{\nu}h_{\nu}}(U) = g_{\pi}^{p_{\pi}+h_{\pi}}g_{\nu}^{p_{\nu}+h_{\nu}}\frac{(U-B)^{M-1}}{p_{\pi}!p_{\nu}!h_{\pi}!h_{\nu}!(M-1)!},$$

where M is the total number of particles and holes of both kinds and

$$B = \frac{1}{4} \left(\frac{p_{\pi}^2 + h_{\pi}^2 + p_{\pi} - h_{\pi}}{g_{\pi}} + \frac{p_{\nu}^2 + h_{\nu}^2 + p_{\nu} - h_{\nu}}{g_{\nu}} \right) - \frac{1}{2} \left(\frac{h_{\pi}}{g_{\pi}} + \frac{h_{\nu}}{g_{\nu}} \right)$$

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State densities in ESM

- Ericson 1960 : no Pauli principle
- Griffin 1966 : no distinction between particles and holes
- Williams 1971 : distinction between particles and holes as well as between neutrons and protons but infinite number of accessible states for both particle and holes
- Běták and Doběs 1976 : account for finite number of holes' states
- Obložinský 1986 : account for finite number of particles' states (MSC)
- Anzaldo-Meneses 1995 : first order corrections for increasing number of p-h
- Hilaire and Koning 1998 : generalized expression in ESM
The pre-equilibrium model : master equation





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The pre-equilibrium model : RIPL inputs





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After direct and pre-equilibrium emission

$$\sigma_{\text{reaction}} = \sigma_{\text{dir}} + \sigma_{\text{pre-equ}} + \sigma_{\text{NC}}$$



After direct and pre-equilibrium emission

$$\sigma_{\text{reaction}} = \sigma_{\text{dir}} + \sigma_{\text{pre-equ}} + \sigma_{\text{NC}}$$

$$\sum_{\substack{z_0 \\ E^*_0 \\ J_0}}^{N_0}$$



After direct and pre-equilibrium emission

$$\sigma_{reaction} = \sigma_{dir} + \sigma_{pre-equ} + \sigma_{NC}$$

$$N_0 = N_0 - dN_D$$

$$Z_0 = Z_0 - dZ_D$$

$$E^*_0 = E^*_0 - dE^*_D$$

$$J_0 = J_0 - dJ_D$$



After direct and pre-equilibrium emission





After direct and pre-equilibrium emission





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After direct and pre-equilibrium emission





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After direct and pre-equilibrium emission





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Compound nucleus hypothesys

- Continuum of excited levels
- Independence between incoming channel a and outgoing channel b

$$\sigma_{ab} = \sigma_a^{(CN)} P_b$$



Compound nucleus hypothesys

- Continuum of excited levels
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Compound nucleus hypothesys

- Continuum of excited levels
- Independence between incoming channel a and outgoing channel b





Compound nucleus hypothesys

- Continuum of excited levels
- Independence between incoming channel a and outgoing channel b



⇒ Hauser- Feshbach formula

$$\sigma_{ab} = \frac{p}{k_a^2} \qquad \frac{T_a T_b}{S T_c}$$

The compound nucleus model : qualitative feature





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The compound nucleus model : complete channel definition

Channel Definition

a + A
$$\rightarrow$$
 (CN)* \rightarrow b+B
Incident channel a = $(\vec{l}_a, \vec{j}_a = \vec{l}_a + \vec{s}_a, \vec{J}_A, \pi_A, E_A, E_a)$

Conservation equations

- Total energy : $E_a + E_A = E_{CN} = E_b + E_B$
- Total momentum : $\vec{p}_a + \vec{p}_A = \vec{p}_{CN} = \vec{p}_b + \vec{p}_B$
- Total angular momentum : $\vec{l}_a + \vec{s}_a + \vec{J}_A = \vec{J}_{CN} = \vec{l}_b + \vec{s}_b + \vec{J}_B$
- Total parity : π_{A} (-1)^I_a = $\pi_{CN} = \pi_{B}$ (-1)^I_b



In realistic calculations, all possible quantum number combinations have to be considered





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$$\sigma_{ab} = \frac{\pi}{k_a^2} \sum_{J=|I_A - s_a|}^{I_A + s_a + l_a^{max}} \sum_{\pi = \pm} \frac{(2J+1)}{(2I_A + 1)(2s_a + 1)}$$



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$$\sigma_{ab} = \frac{\pi}{k_a^2} \sum_{J=|I_A - s_a|}^{I_A + s_a + l_a^{max}} \sum_{\pi = \pm} \frac{(2J+I)}{(2I_A + 1)(2s_a + 1)}$$

$$\int_{J=|I_A - s_a|}^{J+I_A} \sum_{\substack{\pi = \pm}}^{j_a + s_a} \frac{J+I_B}{\sum_{j_a = |J-I_A|}^{J+I_B}} \sum_{\substack{j_b + s_b\\j_b = |J-I_B|}}^{J+I_B} \sum_{\substack{l_b = |j_b - s_b}}^{J_b + s_b}$$



In realistic calculations, all possible quantum number combinations have to be considered



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In realistic calculations, all possible quantum number combinations have to be considered



The compound nucleus model : quantum numbers rule



 $9/2^+$ is too weak for low energies du to too low transmission coefficients

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The compound nucleus model : width fluctuation



Breit-Wigner resonance integrated and averaged over an energy width corresponding to the incident beam dispersion

The compound nucleus model : width fluctuation



Breit-Wigner resonance integrated and averaged over an energy width corresponding to the incident beam dispersion

$$\langle \sigma_{ab} \rangle = \frac{\pi}{k_a^2} \frac{2\pi}{D} \langle \Gamma_a \Gamma_b \rangle$$

The compound nucleus model : width fluctuation



Breit-Wigner resonance integrated and averaged over an energy width corresponding to the incident beam dispersion



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The compound nucleus model : main methods for WFCF



The compound nucleus model : main methods for WFCF



• Tepel method

Simplified iterative method

The compound nucleus model : main methods for WFCF



• Tepel method

Simplified iterative method

- Moldauer method
 - Simple integral
The compound nucleus model : main methods for WFCF



• Tepel method

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- GOE triple integral
 - « exact » result

The compound nucleus model : main methods for WFCF



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Elastic enhancement with respect to the other channels

The compound nucleus model : main methods for WFCF



• Tepel method

Simplified iterative method

Moldauer method

Simple integral

- GOE triple integral
 - « exact » result

Elastic enhancement with respect to the other channels Inelastic enhancement sometimes in very particular situations ?

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The compound nucleus model : GOE triple integral



$$\begin{split} W_{a,l_{a},j_{a},b,l_{b},j_{b}} &= \int_{0}^{+\infty} d\lambda_{1} \int_{0}^{+\infty} d\lambda_{2} \int_{0}^{1} d\lambda \frac{\lambda(1-\lambda)|\lambda_{1}-\lambda_{2}|}{\sqrt{\lambda_{1}(1+\lambda_{1})\lambda_{2}(1+\lambda_{2})}(\lambda+\lambda_{1})^{2}(\lambda+\lambda_{2})^{2}} \\ &\prod_{c} \frac{(1-\lambda T_{c,l_{c},j_{c}})}{\sqrt{(1+\lambda_{1}T_{c,l_{c},j_{c}})(1+\lambda_{2}T_{c,l_{c},j_{c}})}} \quad \left\{ \delta_{ab}(1-T_{a,l_{a},j_{a}}) \right\} \\ &\left[\frac{\lambda_{1}}{1+\lambda_{1}T_{a,l_{a},j_{a}}} + \frac{\lambda_{2}}{1+\lambda_{2}T_{a,l_{a},j_{a}}} + \frac{2\lambda}{1-\lambda T_{a,l_{a},j_{a}}} \right]^{2} + (1+\delta_{ab}) \\ &\left[\frac{\lambda_{1}(1+\lambda_{1})}{(1+\lambda_{1}T_{a,l_{a},j_{a}})(1+\lambda_{1}T_{b,l_{b},j_{b}})} + \frac{\lambda_{2}(1+\lambda_{2})}{(1+\lambda_{2}T_{a,l_{a},j_{a}})(1+\lambda_{2}T_{b,l_{b},j_{b}})} \right] \end{split}$$

$$+ \frac{2\lambda(1-\lambda)}{(1-\lambda T^J_{a,l_a,j_a})(1-\lambda T_{b,l_b,j_b})} \bigg] \bigg\}$$

Optical model and compound nucleus model

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$$\sigma_{ab} = \frac{\pi}{k_{a}^{2}} \sum_{J=|I_{A}-s_{a}|}^{I_{A}+s_{a}+l_{a}^{max}} \sum_{\pi=\pm} \frac{(2J+I)}{(2I_{A}+I)(2s_{a}+I)}$$

$$\sum_{j_{a}=|J-I_{A}|}^{J+I_{A}} \sum_{l_{a}=|j_{a}-s_{a}|}^{j_{a}+s_{a}} \sum_{j_{b}=|J-I_{B}|}^{J+I_{B}} \sum_{l_{b}=|j_{b}-s_{b}|}^{j_{b}+s_{b}}$$

$$\delta_{\pi}(a) \delta_{\pi}(b) \frac{T_{a,l_{a},j_{a}}^{J\pi} T_{b,l_{b},j_{b}}^{J\pi}}{\sum_{c} T_{c,l_{c},j_{c}}^{J\pi}} W_{a,l_{a},j_{a},b,l_{b},j_{b}}^{J\pi}$$

$$W_{a,l_{a},j_{a},b,l_{b},j_{b}} = \int_{0}^{+\infty} d\lambda_{1} \int_{0}^{+\infty} d\lambda_{2} \int_{0}^{1} d\lambda \frac{\lambda(1-\lambda)|\lambda_{1}-\lambda_{2}|}{\sqrt{\lambda_{1}(1+\lambda_{1})\lambda_{2}(1+\lambda_{2})}(\lambda+\lambda_{1})^{2}(\lambda+\lambda_{2})^{2}}$$
$$\prod_{c} \frac{(1-\lambda T_{c,l_{c},j_{c}}^{J})}{\sqrt{(1+\lambda_{1} T_{c,l_{c},j_{c}}^{J})(1+\lambda_{2} T_{c,l_{c},j_{c}}^{J})}} \begin{cases} \delta_{ab}(1-T_{a,l_{a},j_{a}}^{J}) \end{cases}$$

Using OMP transmission coefficients is OK for a diagonal S matrix (no coupled channels) but not correct if off-diagonal S matrix elements are significant

()

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$$\left[\frac{\lambda_1(1+\lambda_1)}{(1+\lambda_1 T_{a,l_a,j_a}^J)(1+\lambda_1 T_{b,l_b,j_b})} + \frac{\lambda_2(1+\lambda_2)}{(1+\lambda_2 T_{a,l_a,j_a}^J)(1+\lambda_2 T_{b,l_b,j_b})}\right]$$

$$+ \frac{2\lambda(1-\lambda)}{(1-\lambda T^J_{a,l_a,j_a})(1-\lambda T_{b,l_b,j_b})} \bigg] \bigg\}$$

Optical model and compound nucleus model

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- 1. One defines $P \equiv \mathbb{I} SS^{\dagger}$ More details in Kawano T., Phys. Rev. C 94, 014612 (2016)
- 2. One diagonalize P and computes the U matrix such that $UPU^{\dagger} = D_p$
- 3. Then $A = USU^T$ is diagonal

4. Using $\tilde{T}_{\alpha} \equiv 1 - |A_{\alpha\alpha}|^2$ one gets the analogous of the usual $T_{\alpha} \equiv 1 - |S_{\alpha\alpha}|^2$

5. Finally

$$\sigma_{ab} = \sum_{\alpha} |U_{\alpha a}|^2 |U_{\alpha b}|^2 \tilde{\sigma}_{\alpha \alpha} + \sum_{\alpha \neq \beta} U^*_{\alpha a} U^*_{\beta b} (U_{\alpha a} U_{\beta b} + U_{\beta a} U_{\alpha b}) \tilde{\sigma}_{\alpha \beta}$$
$$+ \sum_{\alpha \neq \beta} U^*_{\alpha a} U^*_{\alpha b} U_{\beta a} U_{\beta b} \langle A_{\alpha \alpha} A_{\beta \beta} \rangle$$

where
$$\tilde{\sigma}_{\alpha\beta} = \frac{\tilde{T}_{\alpha}\tilde{T}_{\beta}}{\sum_{\gamma}\tilde{T}_{\gamma}}\tilde{W}_{\alpha\beta}$$
 and $\tilde{W}_{\alpha\beta}$ are calculed with

Optical model and compound nucleus model

The compound nucleus model : GOE triple integral



$$\begin{split} W_{a,l_{a},j_{a},b,l_{b},j_{b}} &= \int_{0}^{+\infty} d\lambda_{1} \int_{0}^{+\infty} d\lambda_{2} \int_{0}^{1} d\lambda \frac{\lambda(1-\lambda)|\lambda_{1}-\lambda_{2}|}{\sqrt{\lambda_{1}(1+\lambda_{1})\lambda_{2}(1+\lambda_{2})}(\lambda+\lambda_{1})^{2}(\lambda+\lambda_{2})^{2}} \\ &\prod_{c} \frac{(1-\lambda T_{c,l_{c},j_{c}}^{J})}{\sqrt{(1+\lambda_{1} T_{c,l_{c},j_{c}}^{J})(1+\lambda_{2} T_{c,l_{c},j_{c}}^{J})}} \quad \left\{ \delta_{ab}(1-T_{a,l_{a},j_{a}}^{J}) \right. \\ &\left[\frac{\lambda_{1}}{1+\lambda_{1} T_{a,l_{a},j_{a}}^{J}} + \frac{\lambda_{2}}{1+\lambda_{2} T_{a,l_{a},j_{a}}^{J}} + \frac{2\lambda}{1-\lambda T_{a,l_{a},j_{a}}^{J}} \right]^{2} + (1+\delta_{ab}) \\ &\left[\frac{\lambda_{1}(1+\lambda_{1})}{1-\lambda_{1}(1+\lambda_{1})} + \frac{\lambda_{2}(1+\lambda_{2})}{1-\lambda_{1}(1+\lambda_{2})} \right]^{2} + (1+\delta_{ab}) \right] \end{split}$$

$$\left[\frac{(1+\lambda_1 T_{a,l_a,j_a}^J)(1+\lambda_1 T_{b,l_b,j_b})}{(1+\lambda_1 T_{b,l_b,j_b})} + \frac{(1+\lambda_2 T_{a,l_a,j_a}^J)(1+\lambda_2 T_{b,l_b,j_b})}{(1+\lambda_2 T_{b,l_b,j_b})}\right]$$

$$+ \frac{2\lambda(1-\lambda)}{(1-\lambda T^J_{a,l_a,j_a})(1-\lambda T_{b,l_b,j_b})} \bigg] \bigg\}$$

Optical model and compound nucleus model

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\Rightarrow Optical model re-optimisation required !

Optical model and compound nucleus model

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Capture cross section shape modified

Y













Ceal Optical model and compound nucleus model

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Ceal Optical model and compound nucleus model

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Ceal Optical model and compound nucleus model

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Neutron emission





100



100







⇒ Little impact compared to investment

The compound nucleus model : summary



 $n + {}^{238}U$



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The compound nucleus model : summary



 $n + {}^{238}U$





CC2 Optical model and compound nucleus model

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$$\sigma_{NC} = \sum_{b} \sigma_{ab}$$
 where $b = \gamma$, n, p, d, t, ..., fission

$$\sigma_{ab} = \frac{\pi}{k_a^2} \sum_{J,\pi} \sum_{\alpha,\beta} \frac{(2J+1)}{(2s+1)(2I+1)} T_{lj}^{J\pi} \left(\alpha \right) \frac{\langle T_b^{J\pi}(\beta) \rangle}{\sum_{\delta} \langle T_d^{J\pi}(\delta) \rangle} W_{\alpha\beta}$$
with $J = I_{\alpha} + s_{\alpha} + I_A = j_{\alpha} + I_A$ and $\pi = (-1)^{l_{\alpha}} \pi_A$

and $\langle T_{b}(\beta) \rangle$ = transmission coefficient for outgoing channel β associated with the outgoing particle **b**





Possible decays

The compound nucleus model : various decay channels



Possible decays

Emission to a discrete level with energy E_d

 $\langle T_{b}(\beta) \rangle = T_{lj}^{J\pi}(\beta)$ given by the O.M.P.

The compound nucleus model : various decay channels



Possible decays

Emission to a discrete level with energy E_d

 $\langle T_{b}(\beta) \rangle = T_{lj}^{J\pi}(\beta)$ given by the O.M.P.

Emission in the level continuum

$$\langle T_{b}(\beta) \rangle = \int_{E}^{E + \Delta E} T_{lj}(\beta) \rho(E, J, \pi) dE$$

 $\rho(E,J,\pi)$ density of residual nucleus' levels (J, π) with excitation energy E

The compound nucleus model : various decay channels



Possible decays

Emission to a discrete level with energy E_d

 $\langle T_{b}(\beta) \rangle = T_{lj}^{J\pi}(\beta)$ given by the O.M.P.

Emission in the level continuum

$$\langle T_{b}(\beta) \rangle = \int_{E}^{E + \Delta E} T_{lj}(\beta) \rho(E,J,\pi) dE$$

 $\rho(E,J,\pi)$ density of residual nucleus' levels (J, π) with excitation energy E

- Emission of photons, fission
 - **Specific treatment**