

# Level densities and gamma-ray strengths

S. Hilaire

CEA, DAM, DIF

# Content

## - Introduction

## - General features about nuclear reactions

- Time scales and associated models
- Types of data needed
- Data format = f (users)

## - Nuclear Models

- Basic structure properties
- Optical model
- Pre-equilibrium model
- Compound Nucleus model

## - Model ingredients

- Level densities
- Gamma-ray strengths
- Fission transmission coefficients

## - Fission reactions

- Generalities about fission
- Fission neutrons and gammas
- Fission yields
- Fission cross sections

## - Prospects

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MONDAY

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TODAY

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THURSDAY

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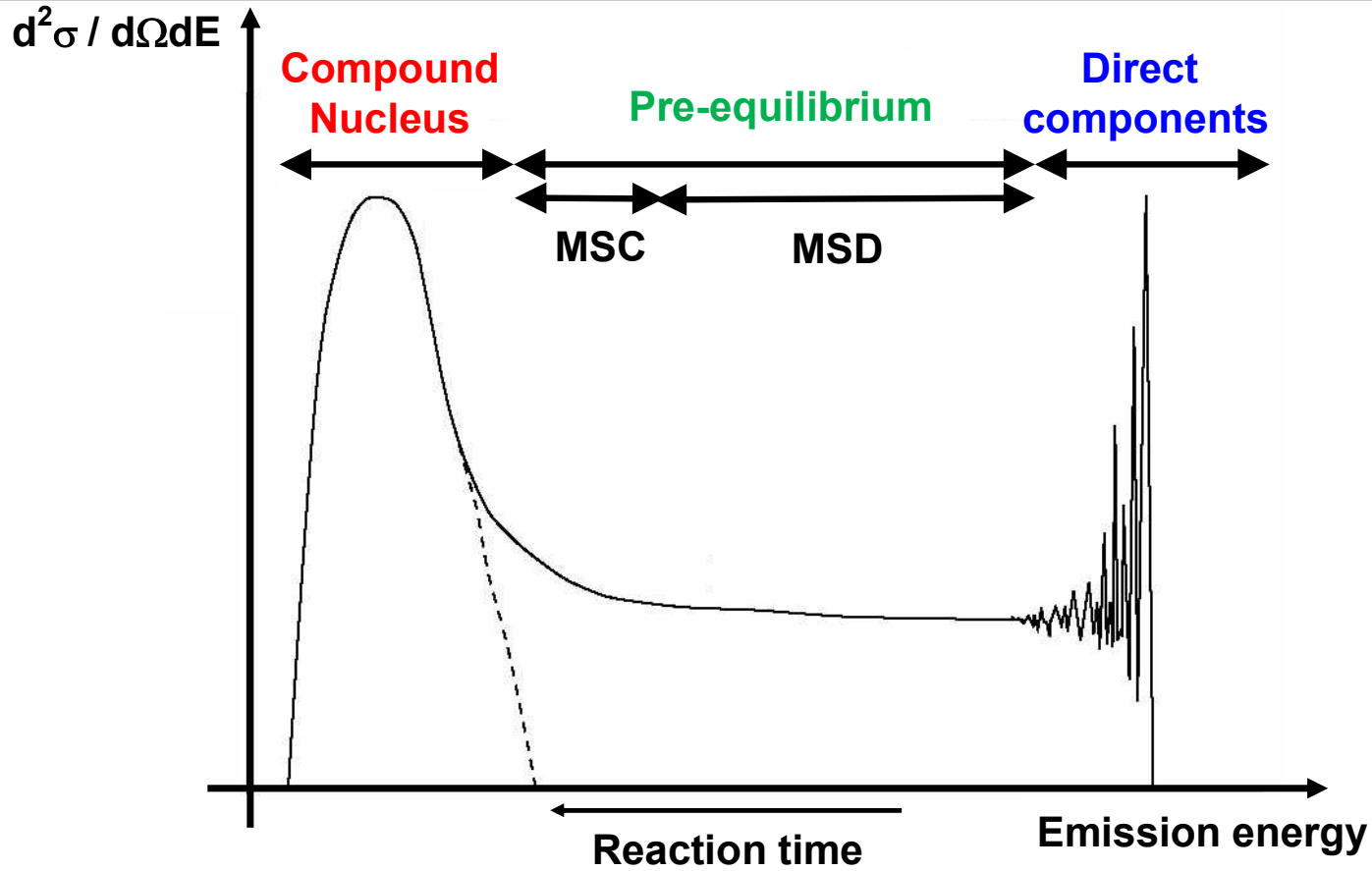
## - Prospects



# 1. FEW REMINDERS



# Time scales and associated models

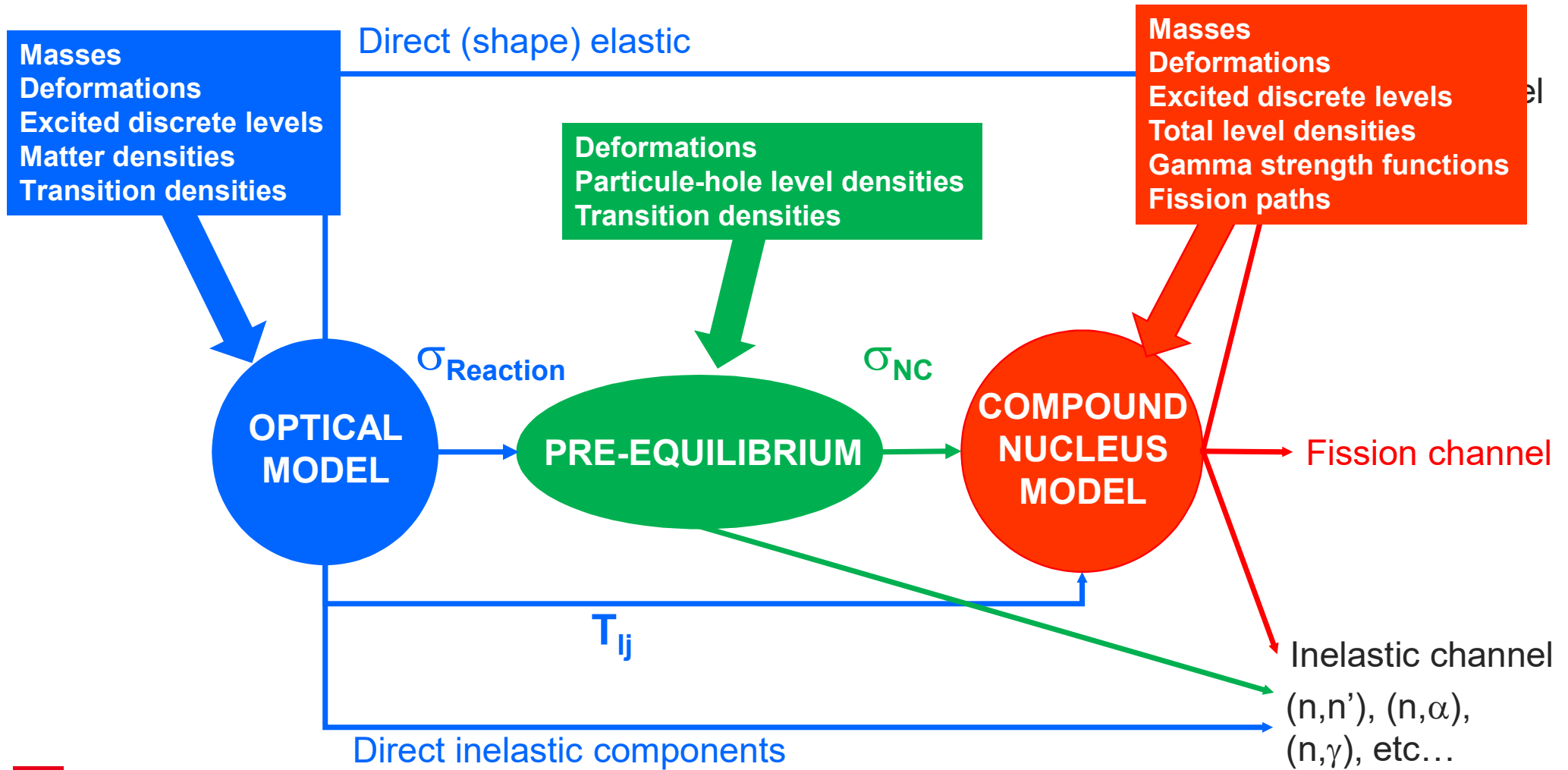


Real scale :  $10^{-15}$  s  
Human scale : year

$10^{-22}$  s  
s



# Models sequence and required ingredients





# 2. LEVEL DENSITIES



# Level densities

## - Why and where do we need them ?

- Why ?
- Where ?

## - Particle-hole level densities for pre-equilibrium

- The equidistant spacing model
- Beyond the ESM

## - Total level densities

- Qualitative features
- Quantitative analysis with analytical approaches
- Shell Model Monte Carlo approach
- HFB+BCS Statistical approach
- Combinatorial approach

## - Impacts on cross sections

- Parity non equipartition
- Non-Gaussian spin distribution
- Governing competition
- Tabulated data adjustment



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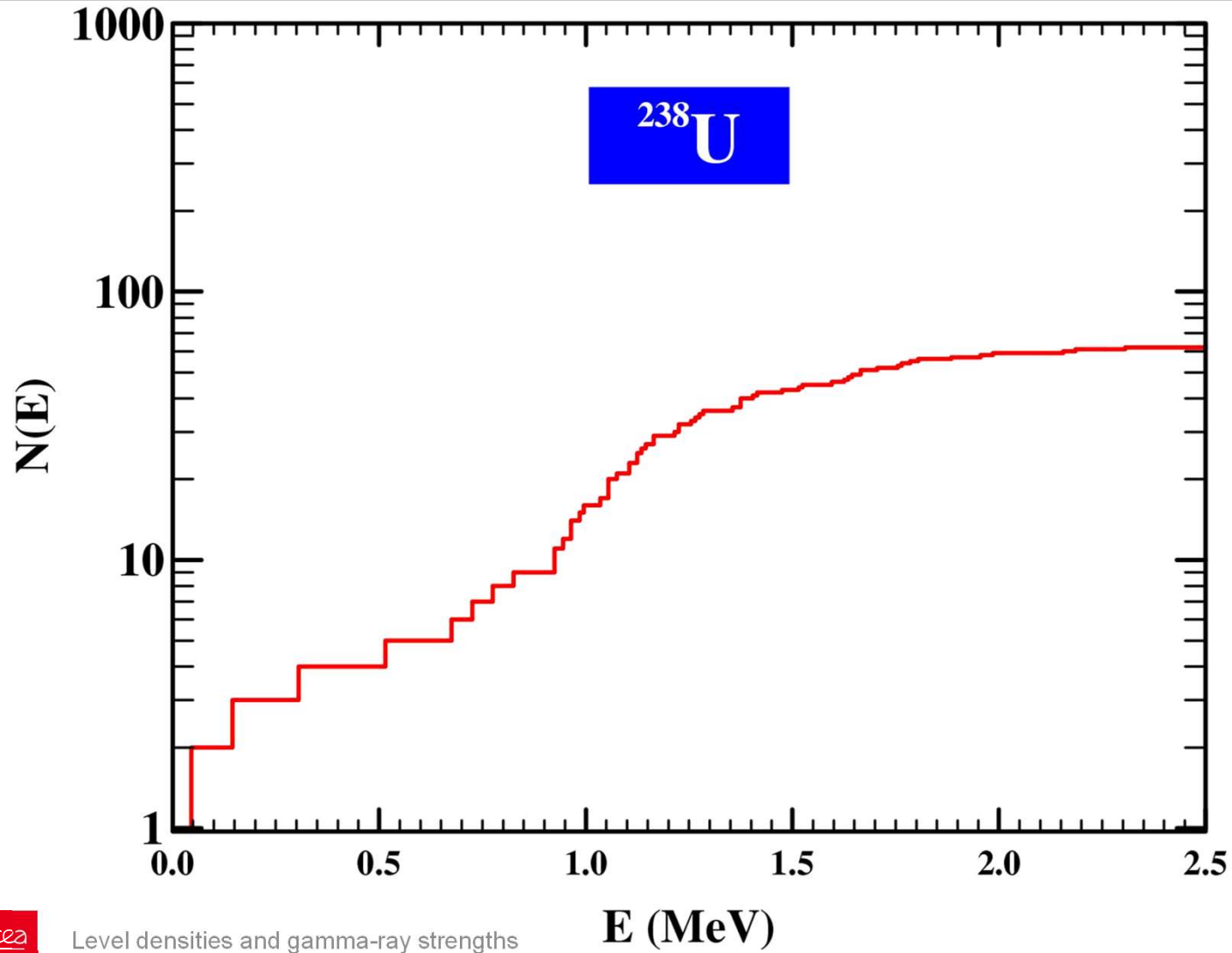
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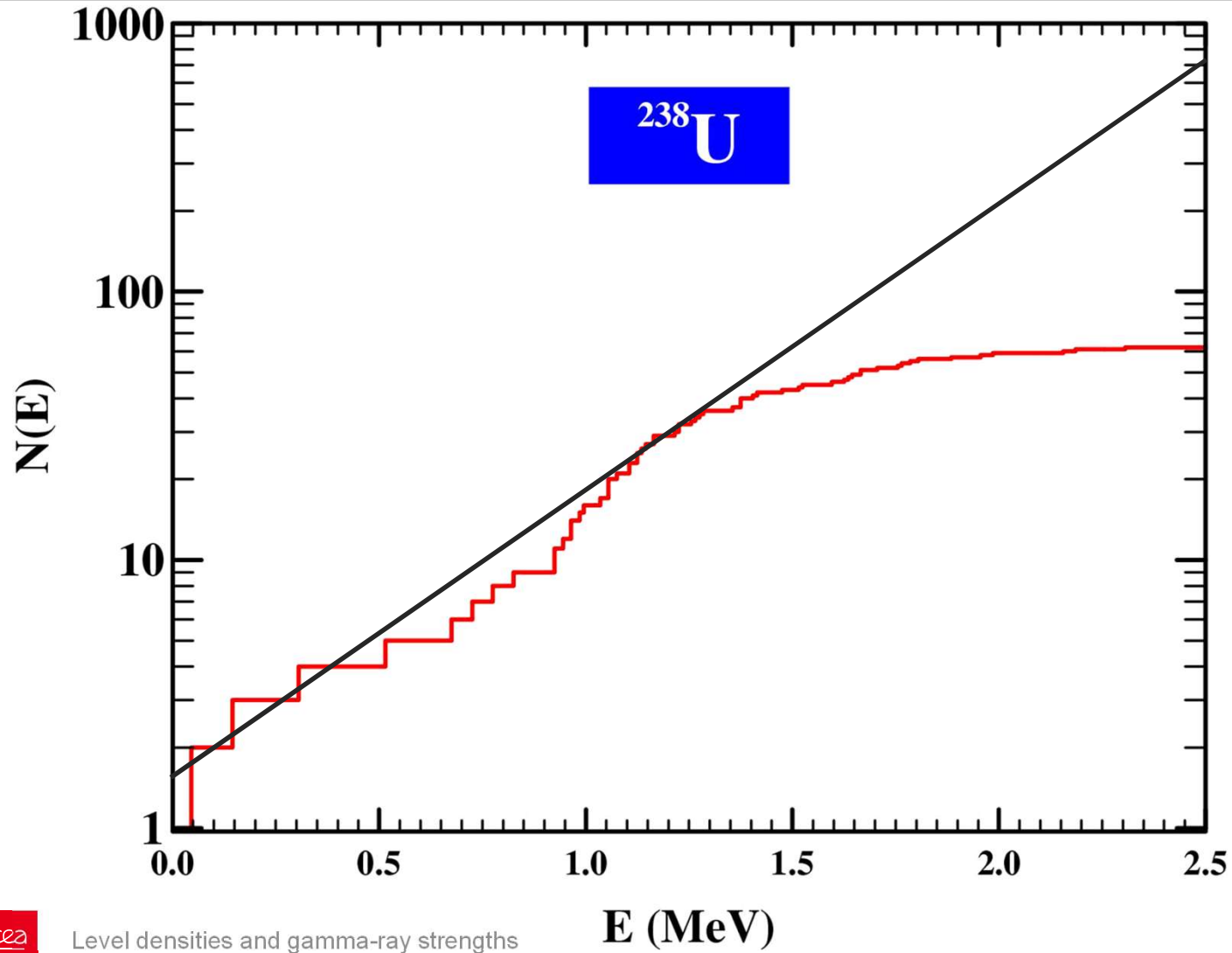


# Level densities : why ?



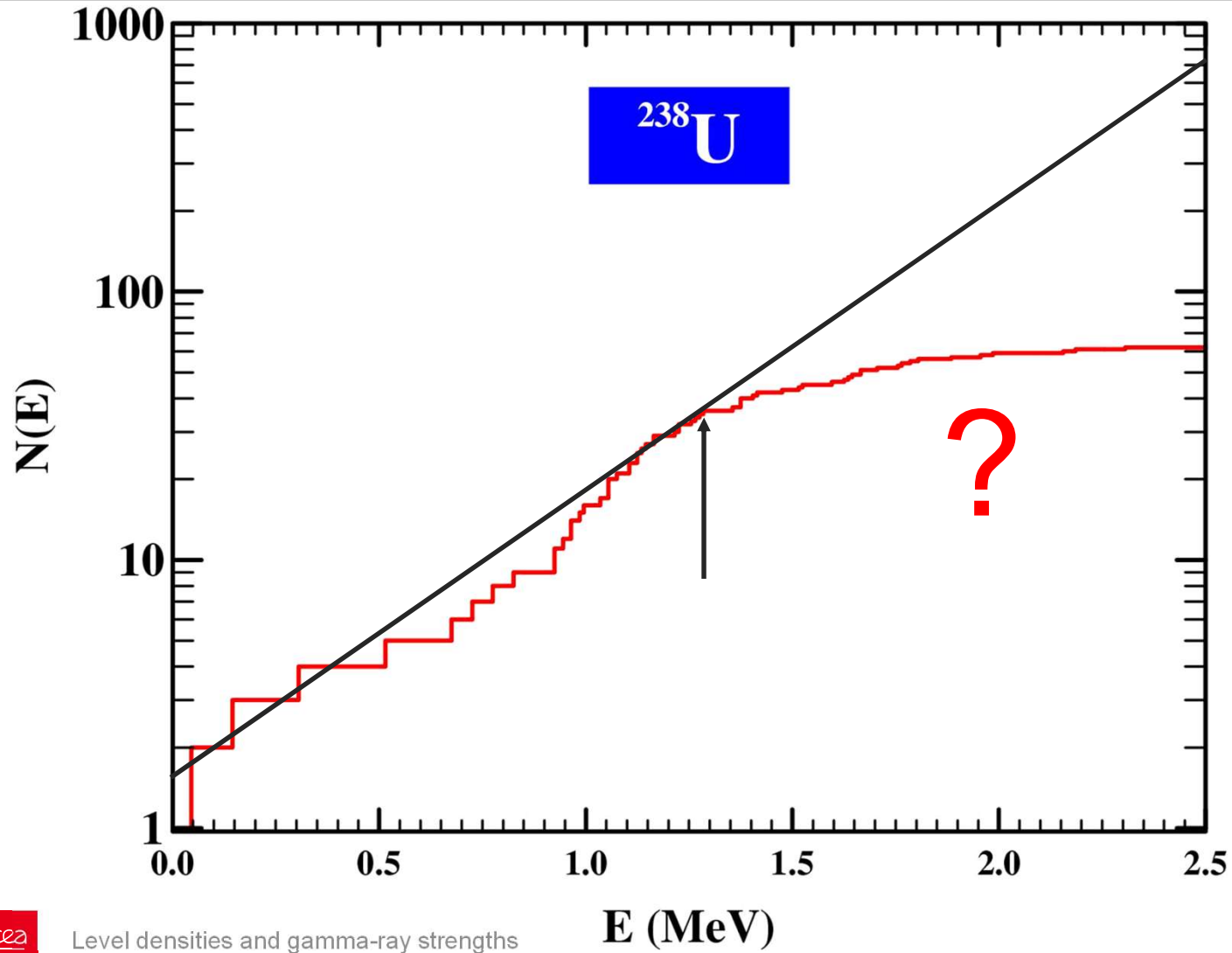


# Level densities : why ?





# Level densities : why ?





## Level densities : where do we need them ?

⇒ partial or p-h level densities for pre-equilibrium model





## The pre-equilibrium model : master equation

$P(n, E, t)$  = **Probability** to find for at **time t** the composite system with an **energy E** and an **exciton number n**.

$\lambda_{a,b}(E)$  = Transition rate from an initial state **a** towards a state **b** for a given energy **E**.

### Evolution equation

$$\frac{dP(n, E, t)}{dt} = P(n-2, E, t) \lambda_{n-2, n}(E) + P(n+2, E, t) \lambda_{n+2, n}(E) - P(n, E, t) \left[ \lambda_{n, n+2}(E) + \lambda_{n, n-2}(E) + \lambda_{n, \text{emiss}}(E) \right]$$

### Emission cross section in channel **c**

$$d\sigma_c(E, \varepsilon_c) = \sigma_R \int_0^{\infty} \sum_{n, \Delta n=2} P(n, E, t) \lambda_{n, c}(E) dt d\varepsilon_c$$



# The pre-equilibrium model : initialisation & transition rates

## Initialisation

$$P(\mathbf{n}, \mathbf{E}, \mathbf{0}) = \delta_{\mathbf{n}, \mathbf{n}_0} \text{ with } n_0=3 \text{ for nucleon induced reactions}$$

## Transition rates

$$\lambda_{\mathbf{n}, \mathbf{n}-2}(\mathbf{E}) = \frac{2\pi}{\hbar} \langle M^2 \rangle \omega(\mathbf{p}, \mathbf{h}, \mathbf{E}) \text{ with } \mathbf{p}+\mathbf{h}=\mathbf{n}-2$$

$$\lambda_{\mathbf{n}, \mathbf{n}+2}(\mathbf{E}) = \frac{2\pi}{\hbar} \langle M^2 \rangle \omega(\mathbf{p}, \mathbf{h}, \mathbf{E}) \text{ with } \mathbf{p}+\mathbf{h}=\mathbf{n}+2$$

$$\lambda_{\mathbf{n}, \mathbf{c}}(\mathbf{E}) = \frac{2s_c+1}{\pi^2 \hbar^3} \mu_c \varepsilon_c \sigma_{c,inv}(\varepsilon_c) \frac{\omega(\mathbf{p}-\mathbf{p}_b, \mathbf{h}, \mathbf{E}-\varepsilon_c-\mathbf{B}_c)}{\omega(\mathbf{p}, \mathbf{h}, \mathbf{E})} Q_c(\mathbf{n}) F_c$$

## State densities

$\omega(\mathbf{p}, \mathbf{h}, \mathbf{E})$  = number of ways of distributing  $\mathbf{p}$  particles and  $\mathbf{h}$  holes among all accessible single particle levels with the available excitation energy  $\mathbf{E}$



## Level densities : where do we need them ?

⇒ partial or p-h level densities for pre-equilibrium model

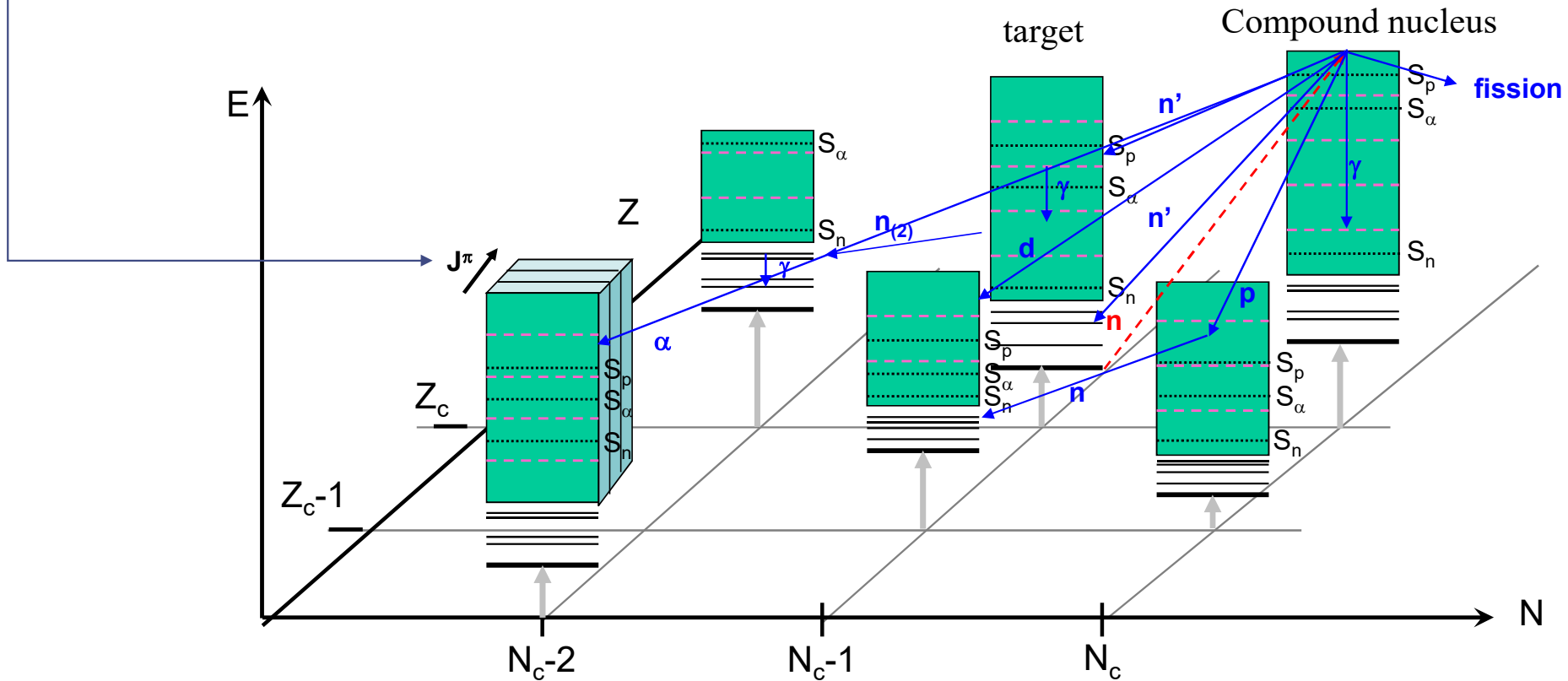
⇒ total level densities for compound-nucleus model

- Light particle emission in continuum bins
- Gamma decay
- Fission cross section



# The compound nucleus model : multiple emission

+ loop over compound nucleus spins and parities





## The compound nucleus model : compact expression

$$\sigma_{\text{NC}} = \sum_{\mathbf{b}} \sigma_{\mathbf{ab}} \quad \text{where } \mathbf{b} = \gamma, n, p, d, t, \dots, \text{ fission}$$

$$\sigma_{\mathbf{ab}} = \frac{\pi}{k_a^2} \sum_{\mathbf{J}, \pi} \sum_{\alpha, \beta} \frac{(2\mathbf{J}+1)}{(2s+1)(2\mathbf{I}+1)} T_{\mathbf{Ij}}^{\mathbf{J}\pi}(\alpha) \frac{\langle T_{\mathbf{b}}^{\mathbf{J}\pi}(\beta) \rangle}{\sum_{\delta} \langle T_{\mathbf{d}}^{\mathbf{J}\pi}(\delta) \rangle} W_{\alpha\beta}$$

with  $\mathbf{J} = \mathbf{l}_{\alpha} + \mathbf{s}_{\alpha} + \mathbf{I}_{\text{A}} = \mathbf{j}_{\alpha} + \mathbf{I}_{\text{A}}$  and  $\pi = (-1)^{\mathbf{l}_{\alpha}} \pi_{\text{A}}$

and  $\langle T_{\mathbf{b}}(\beta) \rangle =$  transmission coefficient for outgoing channel  $\beta$   
associated with the outgoing particle  $\mathbf{b}$



# The compound nucleus model : various decay channels

## Possible decays

- Emission to a discrete level with energy  $E_d$

$$\langle T_b(\beta) \rangle = T_{ij}^{J\pi}(\beta) \quad \text{given by the O.M.P.}$$

- Emission in the level continuum

$$\langle T_b(\beta) \rangle = \int_E^{E+\Delta E} T_{ij}^{J\pi}(\beta) \rho(E, J, \pi) dE$$

$\rho(E, J, \pi)$  density of residual nucleus' levels  $(J, \pi)$  with excitation energy  $E$

- Emission of photons, fission

### Specific treatment



# The compound nucleus model : various decay channels

## Possible decays

- Emission to a discrete level with energy  $E_d$

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LDs needed



- Emission in the level continuum

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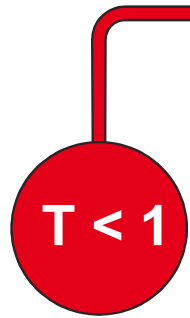
- Emission of photons, fission

**Specific treatment**



# The compound nucleus model : GOE triple integral

$$W_{a,l_a,j_a,b,l_b,j_b} = \int_0^{+\infty} d\lambda_1 \int_0^{+\infty} d\lambda_2 \int_0^1 d\lambda \frac{\lambda(1-\lambda)|\lambda_1 - \lambda_2|}{\sqrt{\lambda_1(1+\lambda_1)\lambda_2(1+\lambda_2)(\lambda+\lambda_1)^2(\lambda+\lambda_2)^2}}$$



$$\prod_c \frac{(1 - \lambda T_{c,l_c,j_c}^J)}{\sqrt{(1 + \lambda_1 T_{c,l_c,j_c}^J)(1 + \lambda_2 T_{c,l_c,j_c}^J)}} \left\{ \delta_{ab}(1 - T_{a,l_a,j_a}^J) \right.$$

$$\left[ \frac{\lambda_1}{1 + \lambda_1 T_{a,l_a,j_a}^J} + \frac{\lambda_2}{1 + \lambda_2 T_{a,l_a,j_a}^J} + \frac{2\lambda}{1 - \lambda T_{a,l_a,j_a}^J} \right]^2 + (1 + \delta_{ab})$$

$$\left[ \frac{\lambda_1(1 + \lambda_1)}{(1 + \lambda_1 T_{a,l_a,j_a}^J)(1 + \lambda_1 T_{b,l_b,j_b}^J)} + \frac{\lambda_2(1 + \lambda_2)}{(1 + \lambda_2 T_{a,l_a,j_a}^J)(1 + \lambda_2 T_{b,l_b,j_b}^J)} \right.$$

$$\left. + \frac{2\lambda(1 - \lambda)}{(1 - \lambda T_{a,l_a,j_a}^J)(1 - \lambda T_{b,l_b,j_b}^J)} \right\}$$





# Level densities

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- Why ?
- Where ?

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- Beyond the ESM

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# Level densities : particule-hole level densities

## State densities in ESM

- Ericson 1960 : no Pauli principle
- Griffin 1966 : no distinction between particles and holes
- Williams 1971 : distinction between particles and holes as well as between neutrons and protons **but** infinite number of accessible states for both particle and holes

$$\omega_{p_{\pi}h_{\pi}p_{\nu}h_{\nu}}(U) = g_{\pi}^{p_{\pi}+h_{\pi}}g_{\nu}^{p_{\nu}+h_{\nu}} \frac{(U-B)^{M-1}}{p_{\pi}!p_{\nu}!h_{\pi}!h_{\nu}!(M-1)!},$$

where  $M$  is the total number of particles and holes of both kinds and

$$B = \frac{1}{4} \left( \frac{p_{\pi}^2 + h_{\pi}^2 + p_{\pi} - h_{\pi}}{g_{\pi}} + \frac{p_{\nu}^2 + h_{\nu}^2 + p_{\nu} - h_{\nu}}{g_{\nu}} \right) - \frac{1}{2} \left( \frac{h_{\pi}}{g_{\pi}} + \frac{h_{\nu}}{g_{\nu}} \right)$$



# Level densities : particule-hole level densities

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- Ericson 1960 : no Pauli principle
- Griffin 1966 : no distinction between particles and holes
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- Běták and Doběš 1976 : account for finite number of holes' states
- Obložinský 1986 : account for finite number of particles' states (MSC)
- Anzaldo-Meneses 1995 : first order corrections for increasing number of p-h
- Hilaire and Koning 1998 : generalized expression in ESM



# Level densities : particule-hole level densities

## Refinement to the ESM

- **Fu 1984** : advanced pairing correction
- **Akkermans and Gruppelaar 1985** : ensure consistency between ph and total level densities
- **Fu 1985** : advanced spin cut-off factor
- **Kalbach 1995** : Inclusion and treatment of a gap in the ESM
- **Harangozo 1998** : Energy dependent single particle state density  $g(\epsilon)$



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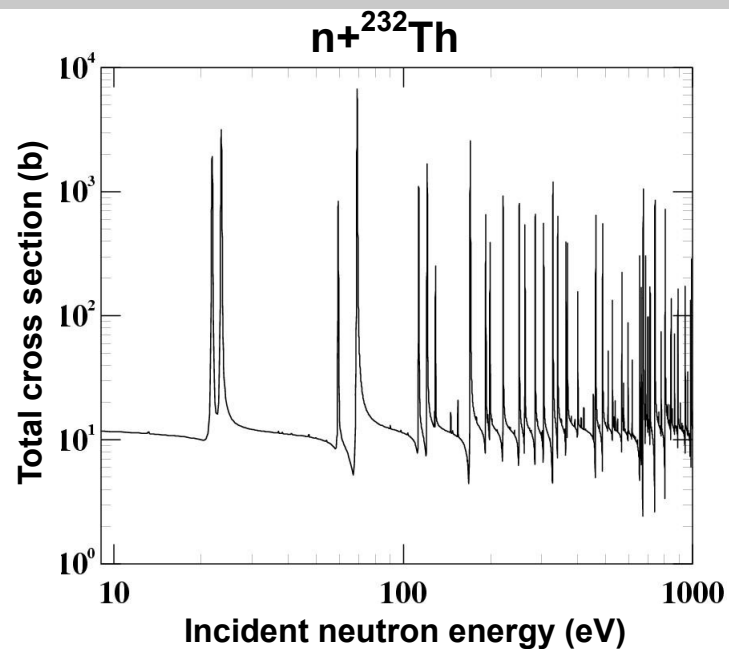
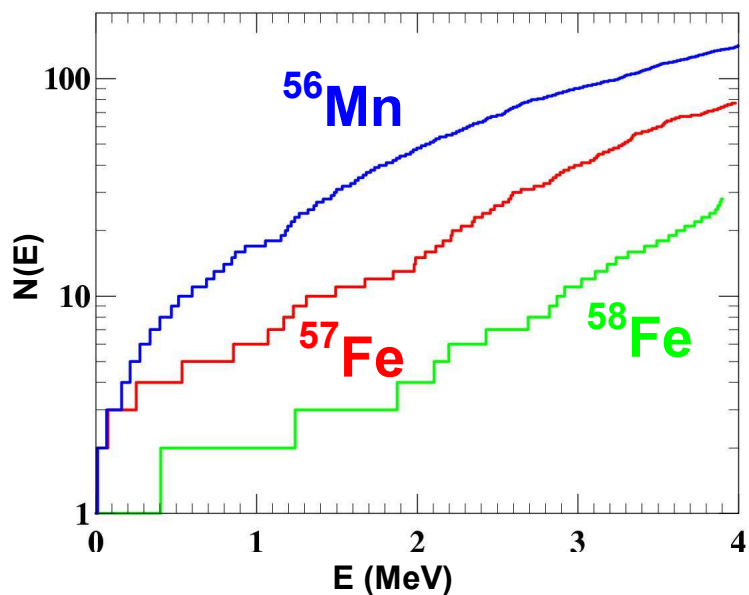
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# Level densities : qualitative aspects from experiment



- Exponential increase of the cumulated number of discrete levels  $N(E)$  with energy

$$\Rightarrow \rho(E) = \frac{dN(E)}{dE} \text{ increases exponentially}$$

$\Rightarrow$  odd-even effects

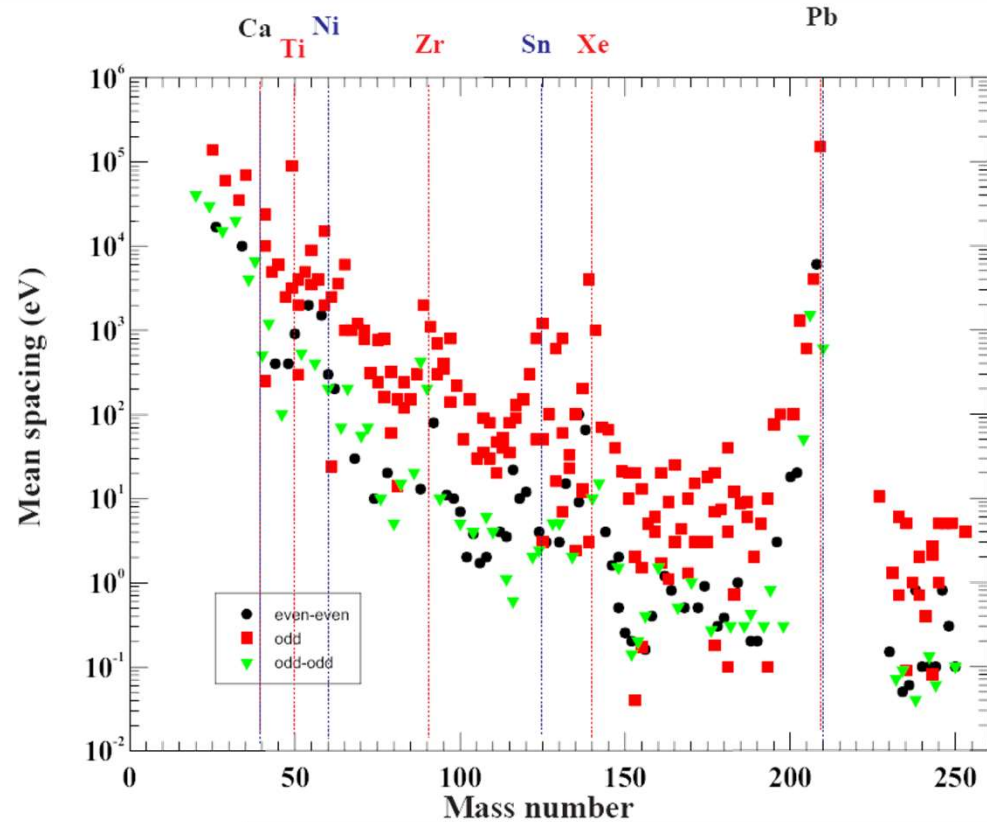
- Mean spacings of s-wave neutron resonances at  $B_n$  of the order of few eV

$$\Rightarrow \rho(B_n) \text{ of the order of } 10^4 - 10^6 \text{ levels / MeV}$$





# Level densities : qualitative aspects $D_0$ vs mass A



*Ilijin et al., NPA 543 (1992) 517.*

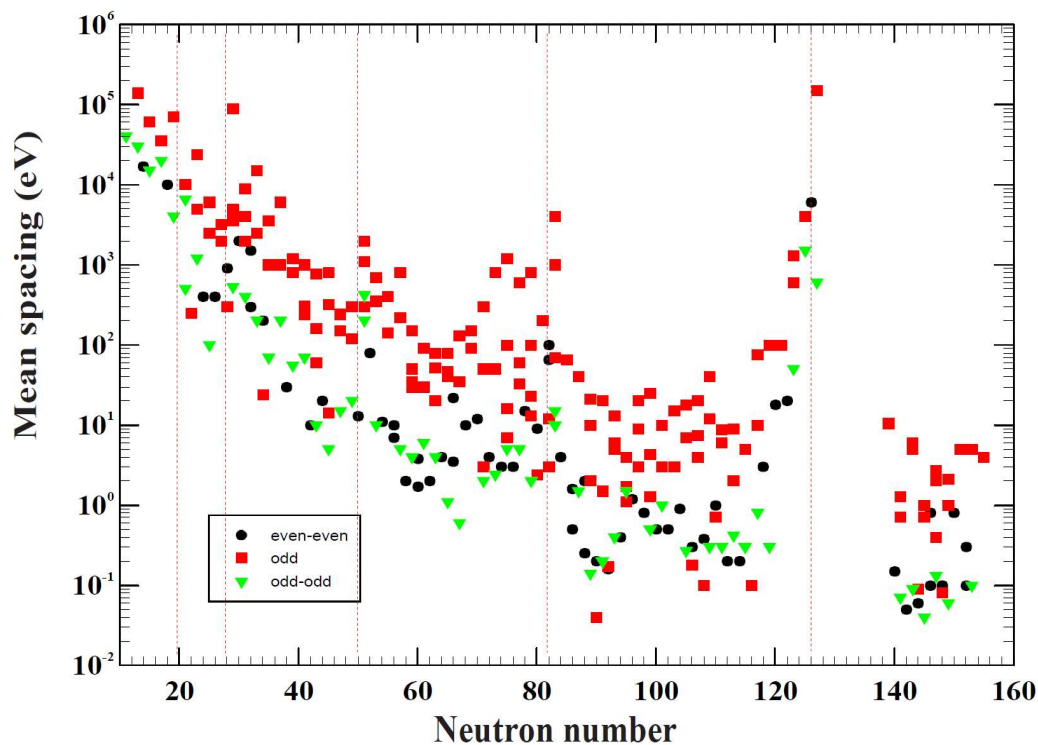
⇒ Mass dependency  
 Odd-even effects  
 Shell effects

$$\frac{1}{D_0} = \rho(B_n, 1/2, \pi_t) \text{ for an even-even target}$$

$$= \rho(B_n, I_t + 1/2, \pi_t) + \rho(B_n, I_t - 1/2, \pi_t) \text{ otherwise}$$



# Level densities : qualitative aspects $D_0$ vs neutron number $N$



*Ilijin et al., NPA 543 (1992) 517.*

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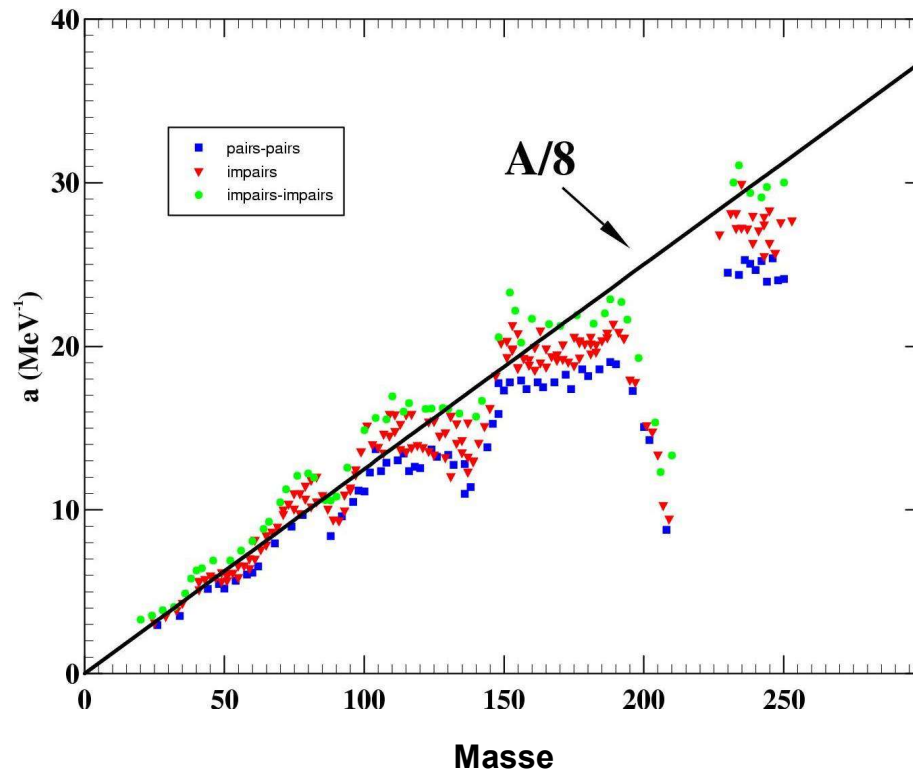
## Level densities : quantitative analysis of $D_0$

$$\rho(\mathbf{U}, \mathbf{J}, \pi) = \frac{1}{2} \frac{\sqrt{\pi}}{12} \frac{\exp(2\sqrt{aU})}{a^{1/4} U^{5/4}} \frac{2J+1}{2\sqrt{2\pi} \sigma^3} \exp\left[-\frac{(J+1/2)^2}{2\sigma^2}\right]$$
$$+ \sigma^2 = I_{\text{rig}} \sqrt{\frac{U}{a}}$$



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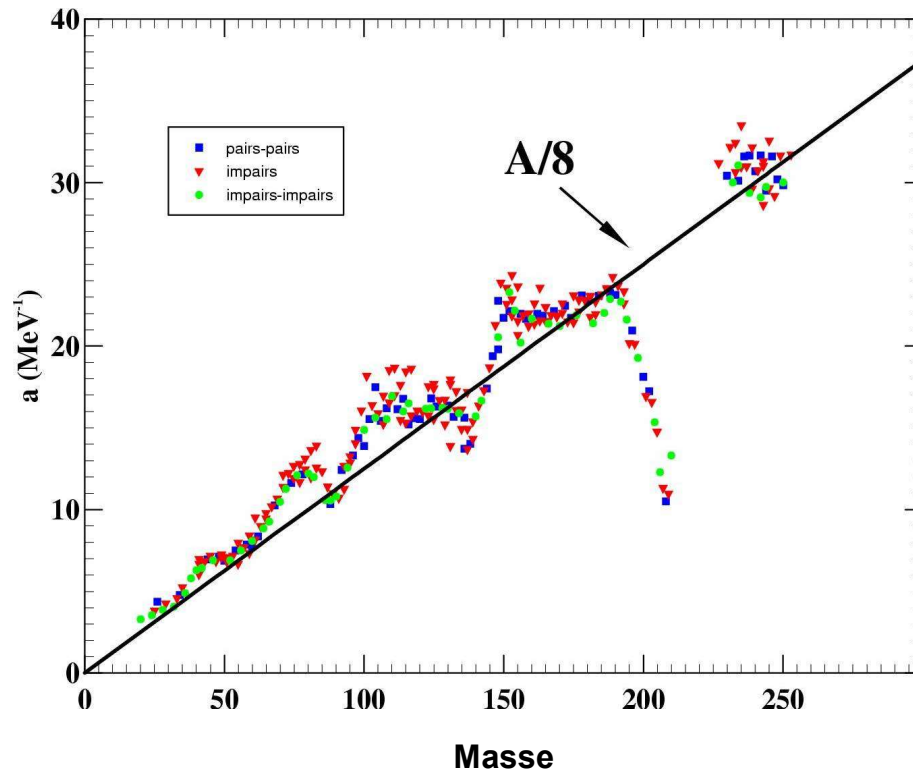


**⇒ odd-even effects**



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Odd-even effects  
accounted for

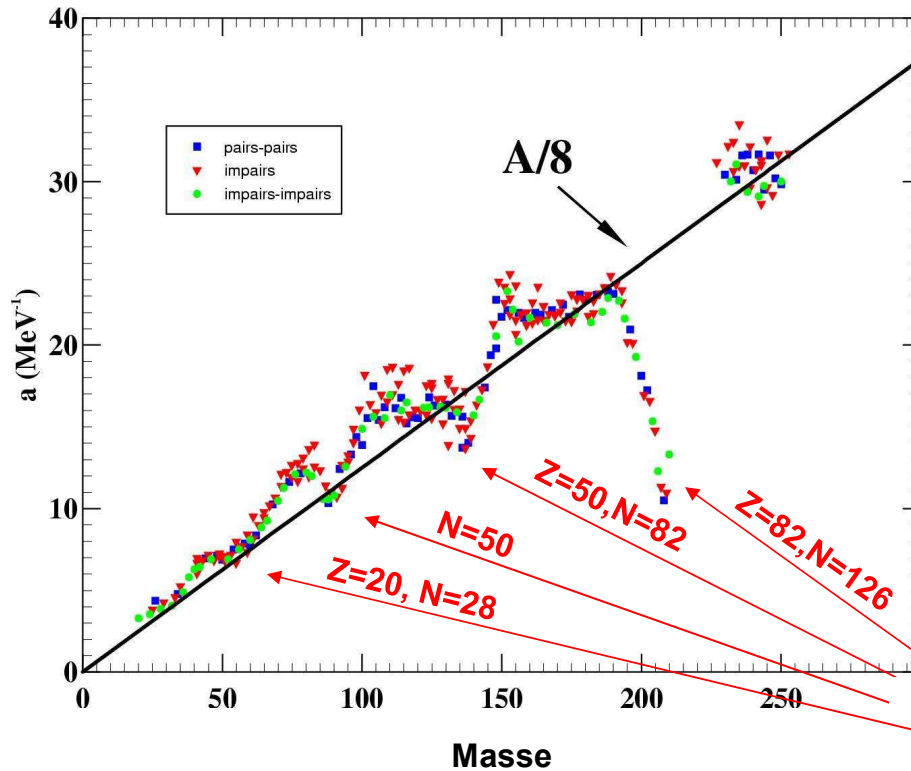
$$U \rightarrow U^* = U - \Delta$$

$$\Delta = \begin{cases} 0 & \text{odd-odd} \\ 12/\sqrt{A} & \text{odd-even} \\ 24/\sqrt{A} & \text{even-even} \end{cases}$$

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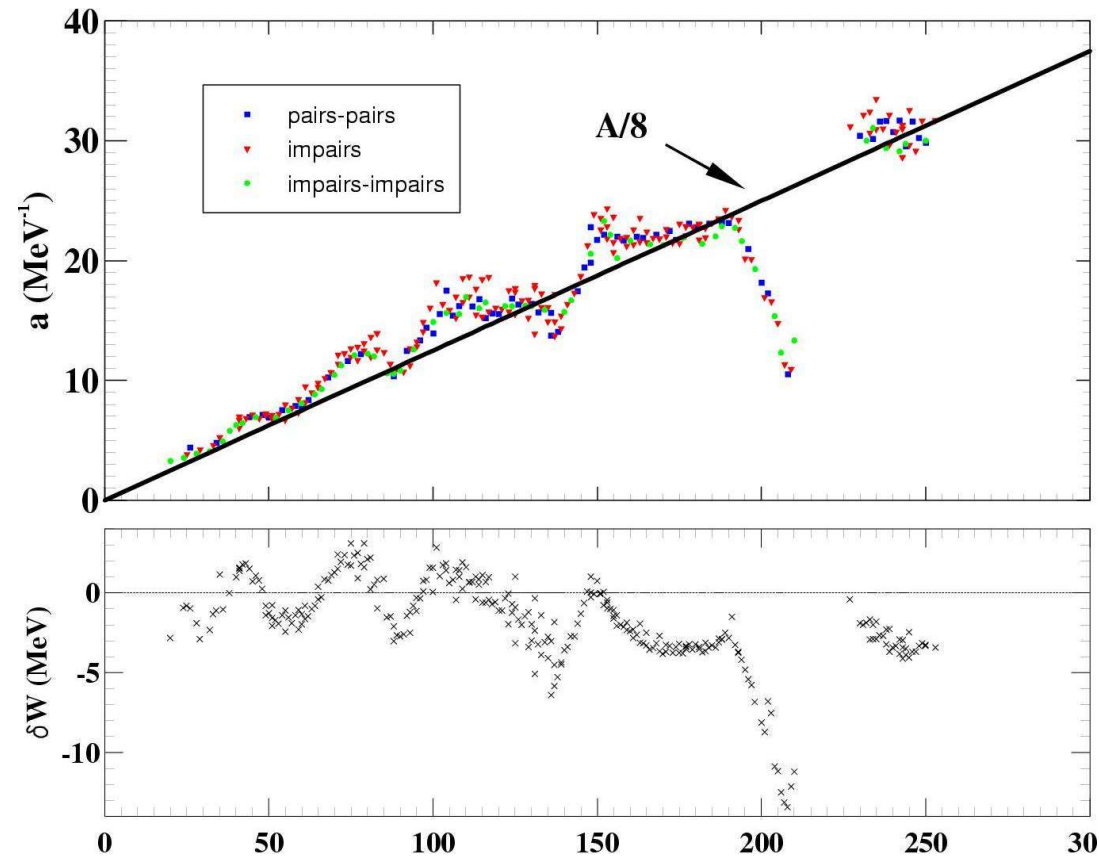
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Shell effects



# Level densities : Ignatyuk formula

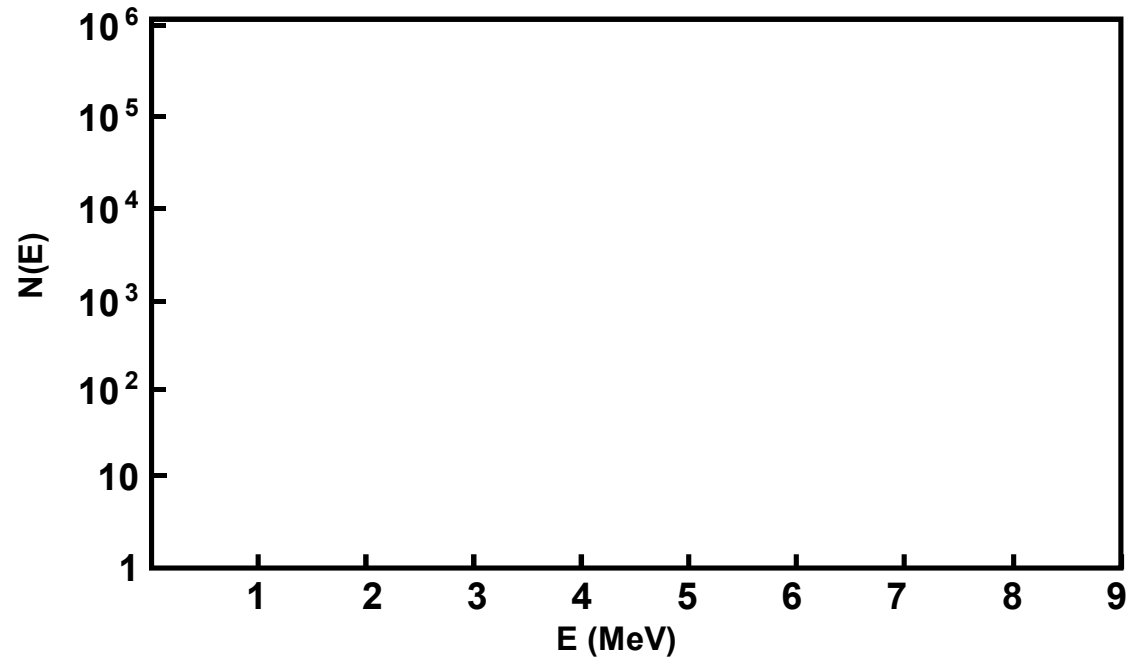


$$a(N, Z, U^*) = \tilde{a}(A) \left[ 1 + \delta W(N, Z) \frac{1 - \exp(-\gamma U^*)}{U^*} \right]$$



# Level densities : summary of analytical description

ldmodel 1 in TALYS

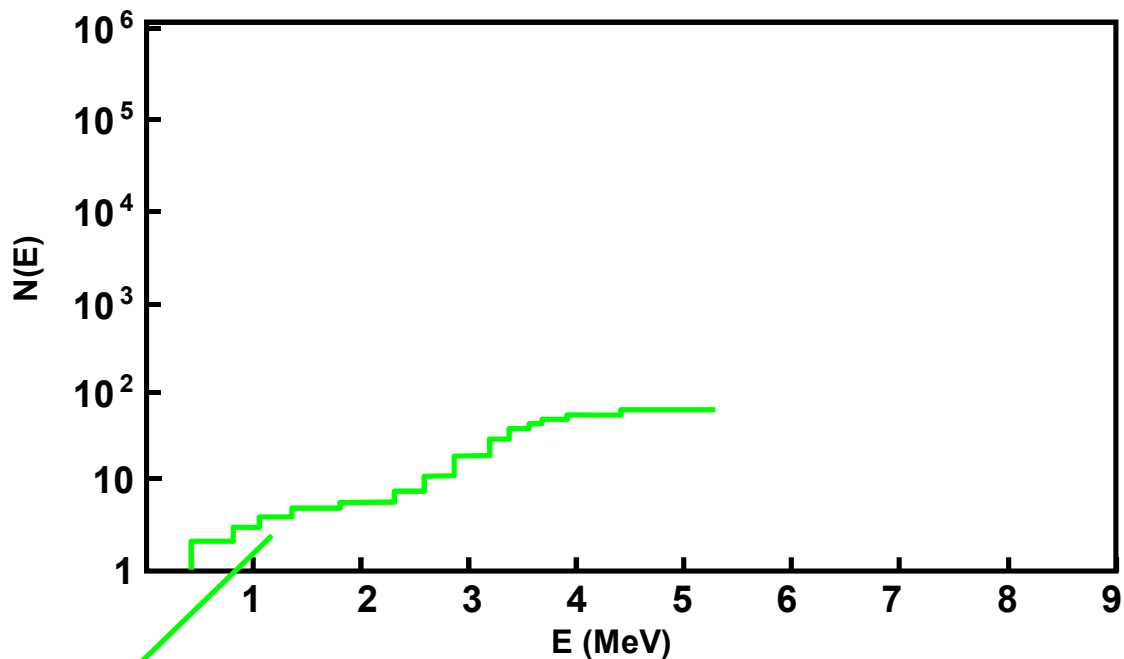






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ldmodel 1 in TALYS

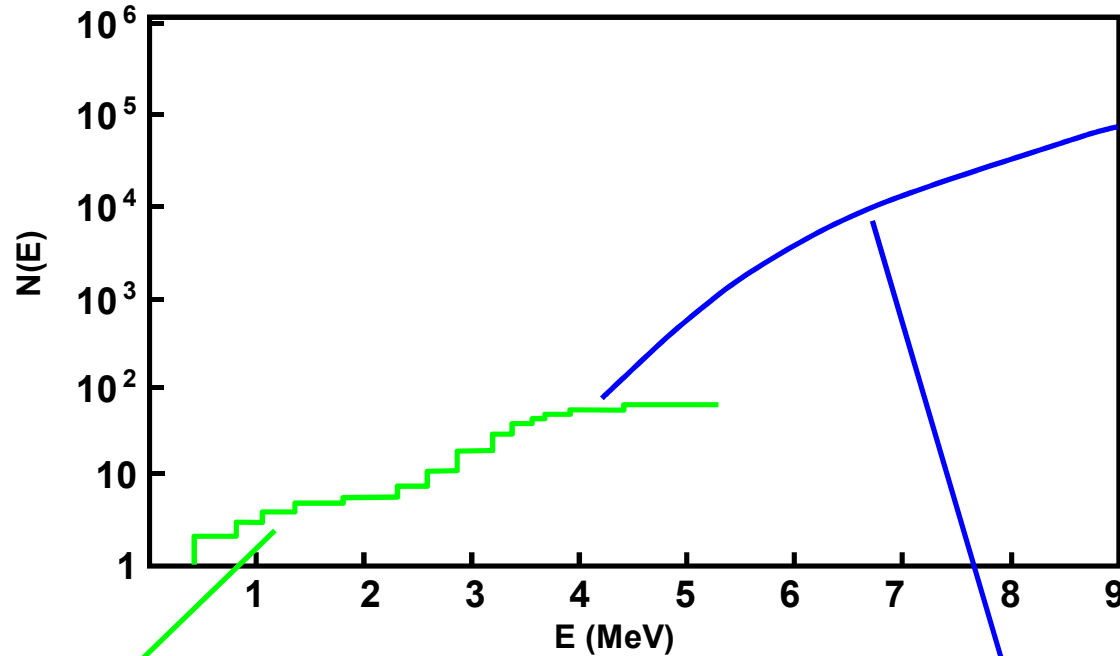


Discrete levels  
(spectroscopy)



# Level densities : summary of analytical description

ldmodel 1 in TALYS



Discrete levels  
(spectroscopy)

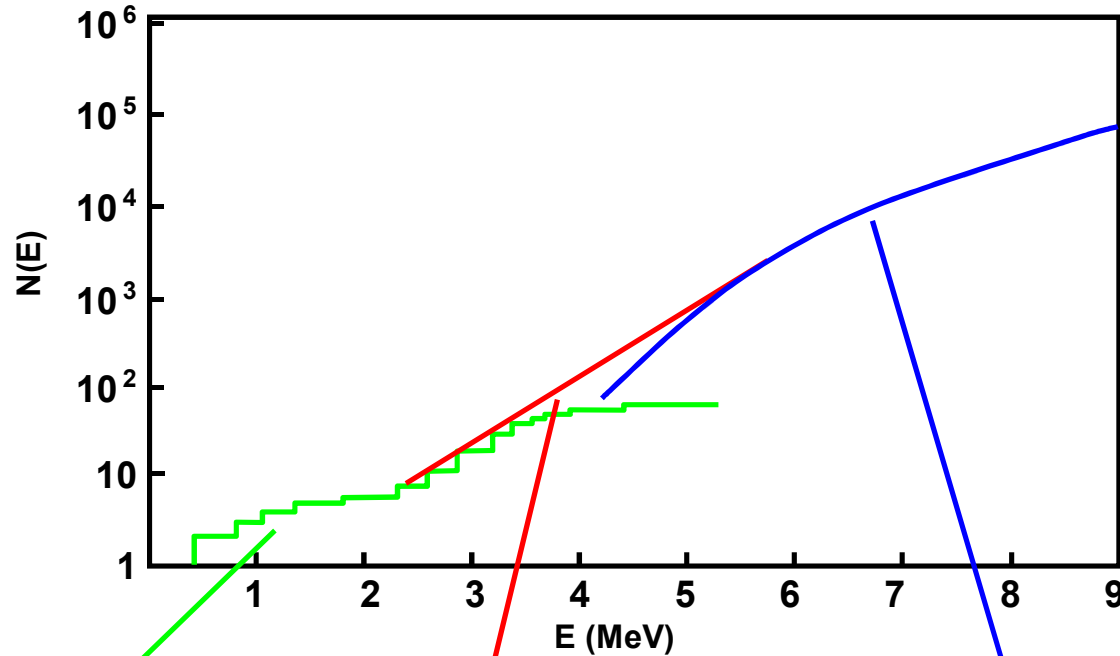
Fermi gaz (adjusted at  $B_n$ )

$$\rho(E) = \alpha \frac{\exp(2\sqrt{aU^*})}{a^{1/4} U^{*5/4}}$$

# Level densities : summary of analytical description



ldmodel 1 in TALYS



Discrete levels  
(spectroscopy)

Temperature law

$$N(E) = \exp\left(\frac{E - E_0}{T}\right)$$

Fermi gaz (adjusted at  $B_n$ )

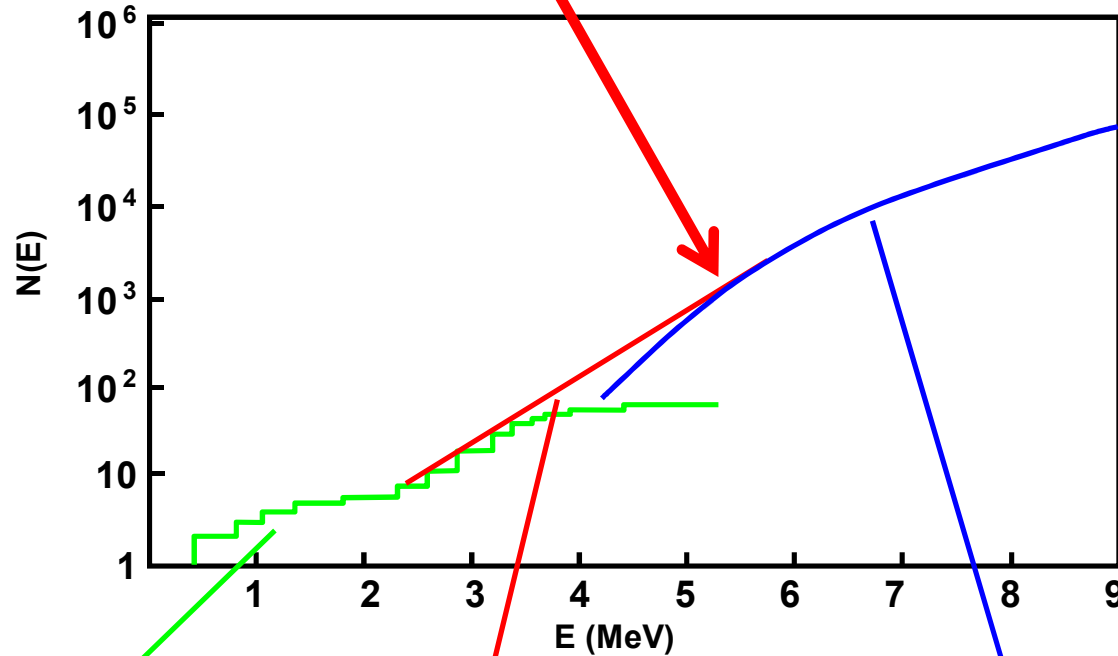
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# Level densities : summary of analytical description

Matching conditions : continuity of  $\rho$  and of its derivative (sometimes difficult)

ldmodel 1 in TALYS



Discrete levels  
(spectroscopy)

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Fermi gaz (adjusted at  $B_n$ )

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# Level densities : More sophisticated analytical expression

ldmodel 3 in TALYS

- **Superfluid model & Generalized superfluid model**

*Ignatyuk et al., PRC 47 (1993) 1504 & RIPL3 paper (IAEA)*

- ⇒ More correct treatment of pairing for low energies
- ⇒ Fermi Gas + Ignatyuk beyond critical energy
- ⇒ Explicit treatment of collective effects

$$\rho(U) = K_{\text{vib}}(U) * K_{\text{rot}}(U) * \rho_{\text{int}}(U)$$

$a_{\text{eff}} \approx A/8$

Several analytical  
or numerical options

$a \approx A/13$

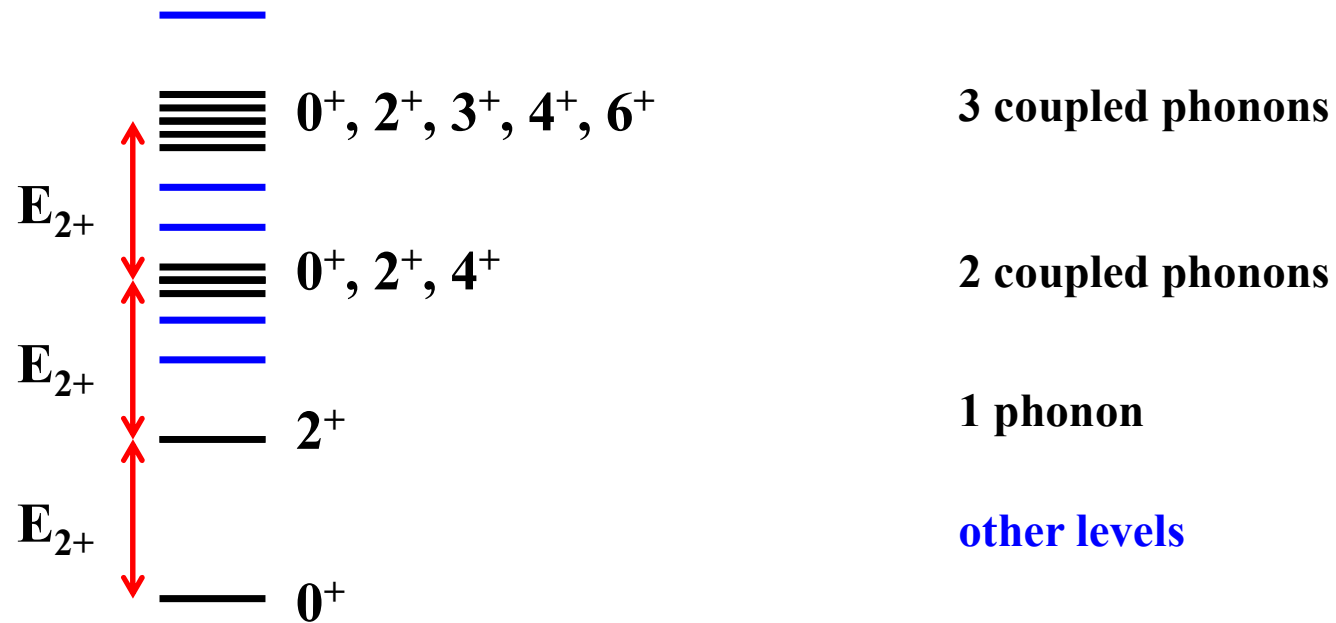
⇒ **Collective enhancement** only if  $\rho_{\text{int}}(U) \neq 0$  not correct for vibrational states

⇒ yet not the most used one in practice



# Level densities : collective levels

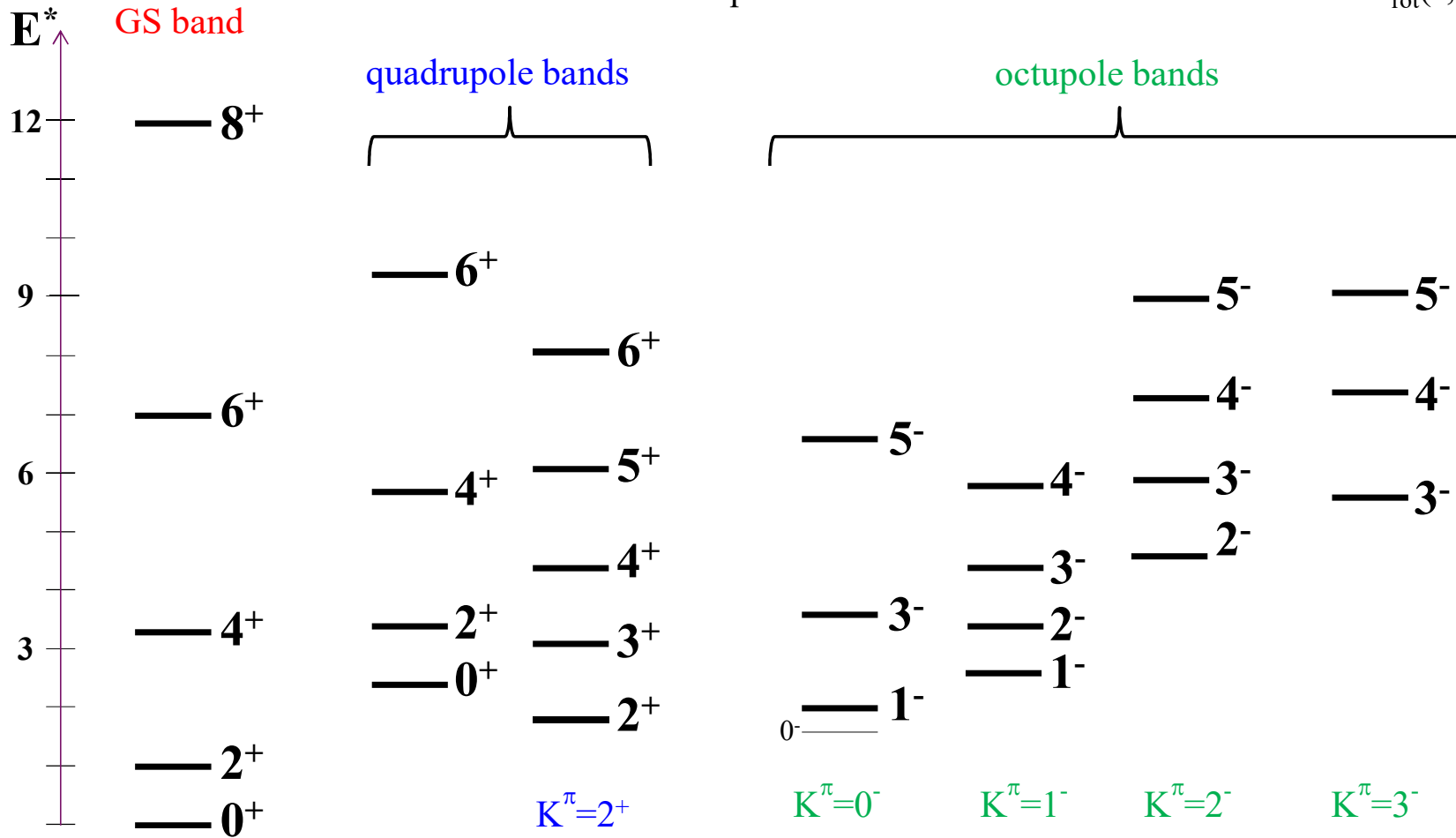
⇒ vibrational level sequence for a spherical even-even nucleus





# Level densities : collective levels

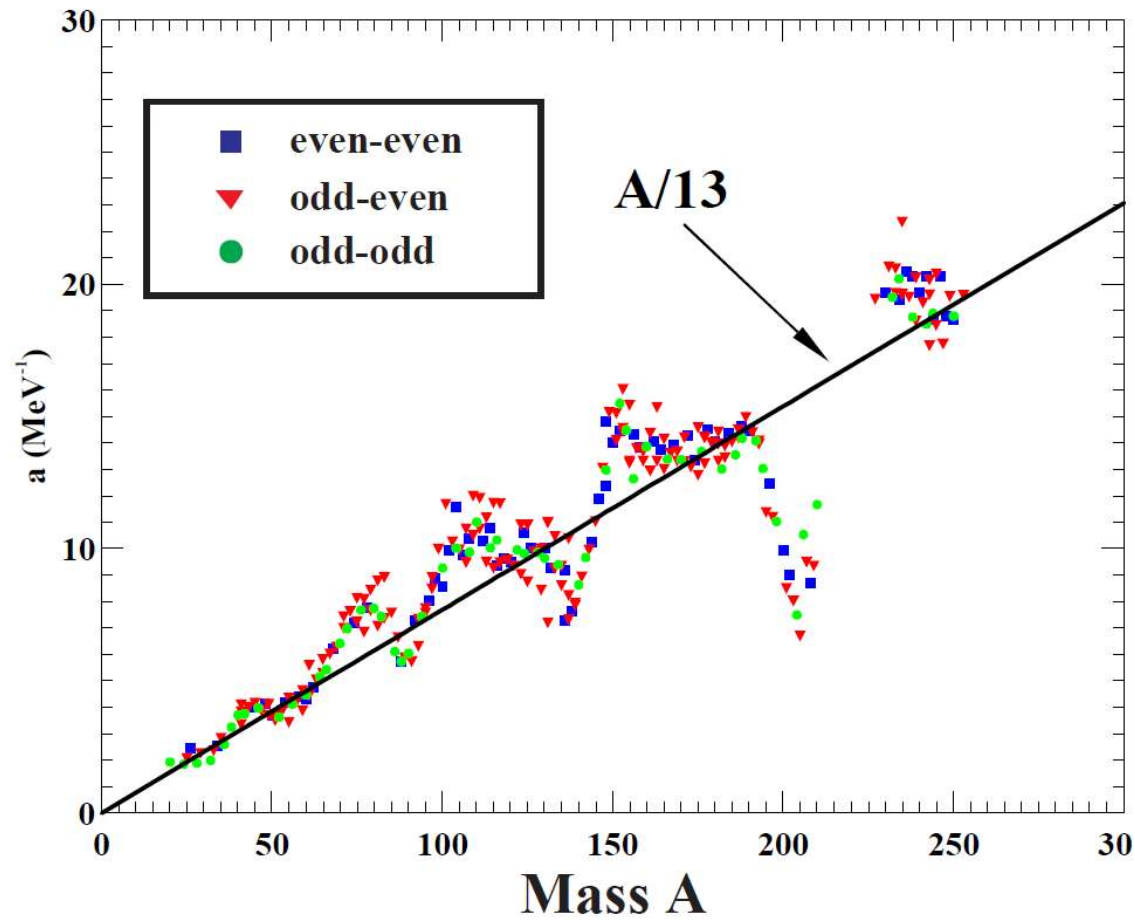
⇒ General level sequence for a deformed even-even nucleus :  $E_{\text{rot}}(J,K) = \frac{J(J+1) - K^2}{2 \mathcal{I}_{\perp}(\mathbf{U},\beta)}$





# Level densities : explicit treatment of collective effect

$$\rho(U) = K_{\text{vib}}(U) \times K_{\text{rot}}(U, \beta) \times \rho_{\text{int}}(U)$$





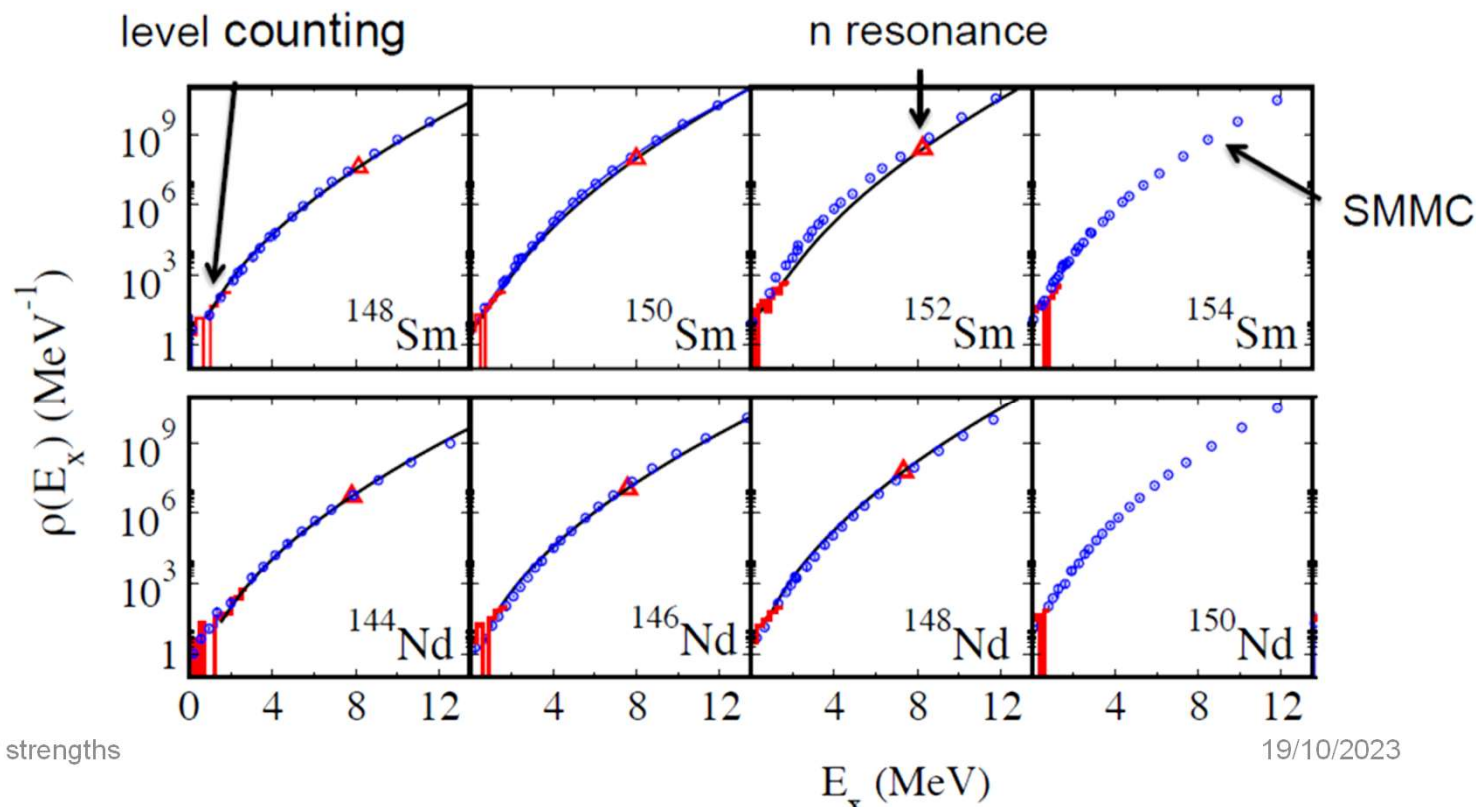


# Shell Model Monte Carlo

- **Shell Model Monte Carlo approach**

*Agrawal et al., PRC 59 (1999) 3109 + Koonin et al, Phys. Rep. 278 (1997) 1 + Alhassid et al, Phys. Rev. Lett 99 (2007) 162504.*

- ⇒ Realistic Hamiltonians but not global
- ⇒ Coherent and incoherent excitations treated on the same footing
- ⇒ Time consuming and not systematically applied

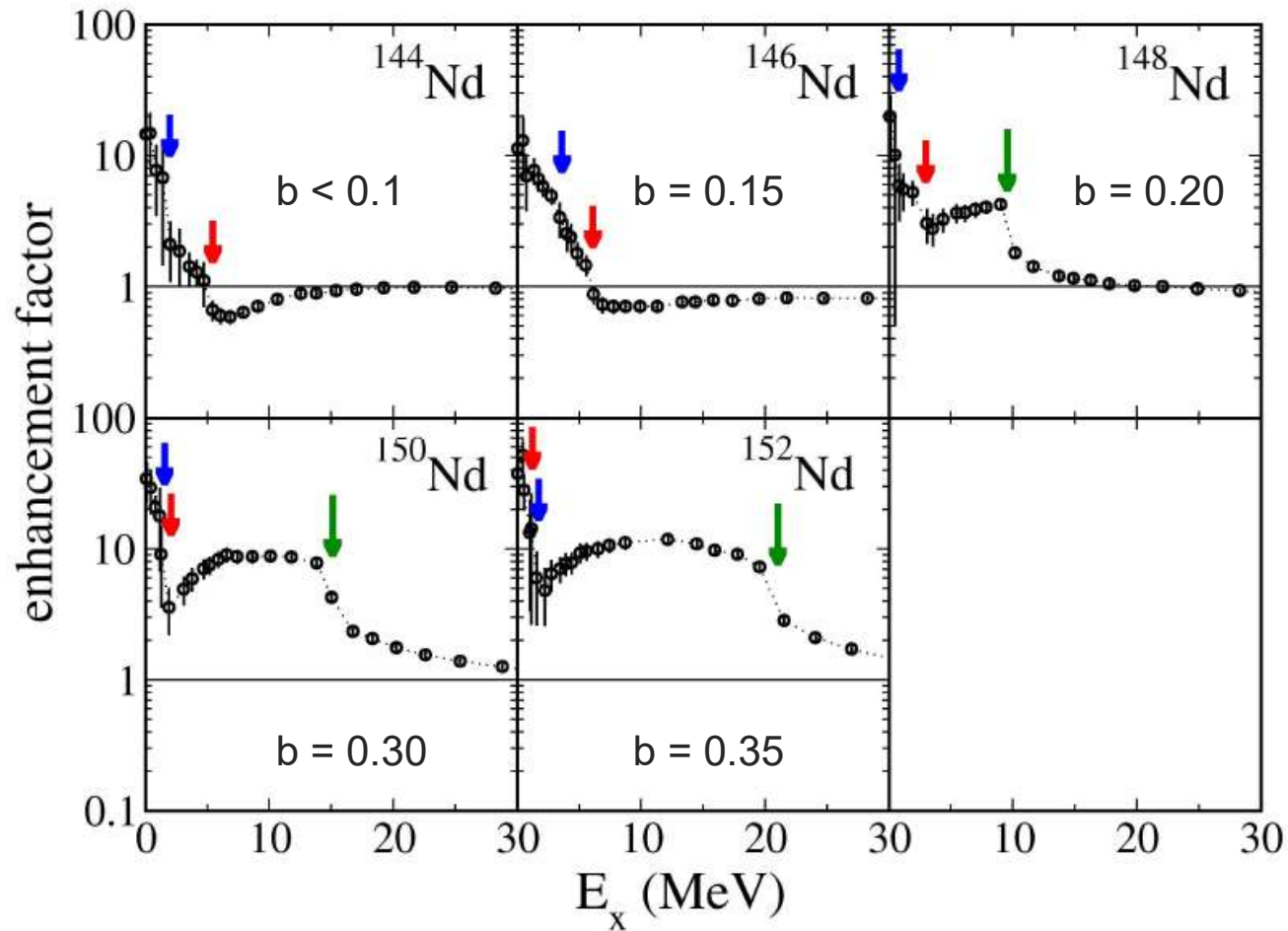


Courtesy Y. Alhassid





# Shell Model Monte Carlo (collective effects vanishing)



Courtesy Y. Alhassid

neutron pair breaking proton pair breaking shape transition



# HFBCS Statistical approach

## Mean Field + Statistical NLD formula

Idmodel 4 in TALYS

Partition function method applied to the discrete SPL scheme predicted by a MF model

$$\omega(U) = \frac{e^{S(U)}}{(2\pi)^{3/2} \sqrt{D(U)}} \quad U(T) = E(T) - E(T=0)$$
$$S(T) = 2 \sum_{q=n,p} \sum_k \ln \left[ 1 + \exp(-E_q^k/T) \right] + \frac{E_q^k/T}{1 + \exp(-E_q^k/T)}$$
$$E(T) = \sum_{q=n,p} \sum_k \varepsilon_q^k \left[ 1 - \frac{\varepsilon_q^k - \lambda_q}{E_q^k} \tanh\left(\frac{E_q^k}{2T}\right) \right] - \frac{\Delta_q^2}{G}$$
$$N_q = \sum_k \left[ 1 - \frac{\varepsilon_q^k - \lambda_q}{E_q^k} \tanh\left(\frac{E_q^k}{2T}\right) \right]$$
$$\frac{2}{G_q} = \sum_k \frac{1}{E_q^k} \tanh\left(\frac{E_q^k}{2T}\right)$$
$$\sigma^2(T) = \frac{1}{2} \sum_{q=n,p} \sum_k \omega_q^{k^2} \operatorname{sech}^2\left(\frac{E_q^k}{2T}\right)$$

Courtesy S. Goriely



## Mean Field + Statistical NLD formula

Idmodel 4 in TALYS

$$\rho_{sph}(U, J) = \frac{2J+1}{2\sqrt{2\pi}\sigma^3} e^{-\frac{J(J+1)}{2\sigma^2}} \omega(U)$$
$$\rho_{def}(U, J) = \frac{1}{2} \sum_{K=-J}^J \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\left[\frac{J(J+1)}{2\sigma_{\perp}^2} + \frac{K^2}{2} \left(\frac{1}{\sigma^2} - \frac{1}{\sigma_{\perp}^2}\right)\right]} \omega(U)$$

The inclusion of rotational bands may increase the NLD by a factor of 10-70

- Strong impact and sensitivity to the GS deformation of the nucleus !
- deformation is known to disappear with increasing excitation



$$\rho(U, J) = [1 - f_{dam}(U)] \rho_{sph}(U, J) + f_{dam}(U) \rho_{def}(U, J)$$

providing a smooth deformed ( $f_{dam}=1$ ) to spherical ( $f_{dam}=0$ ) transition, e.g

$$f_{dam}(U) = \frac{1}{1 + e^{(U - E_{def})/d_u}} \left[ 1 - \frac{1}{1 + e^{(\beta_2 - \beta^*)/d_{\beta}}} \right]$$

Courtesy S. Goriely



# HFBCS Statistical approach

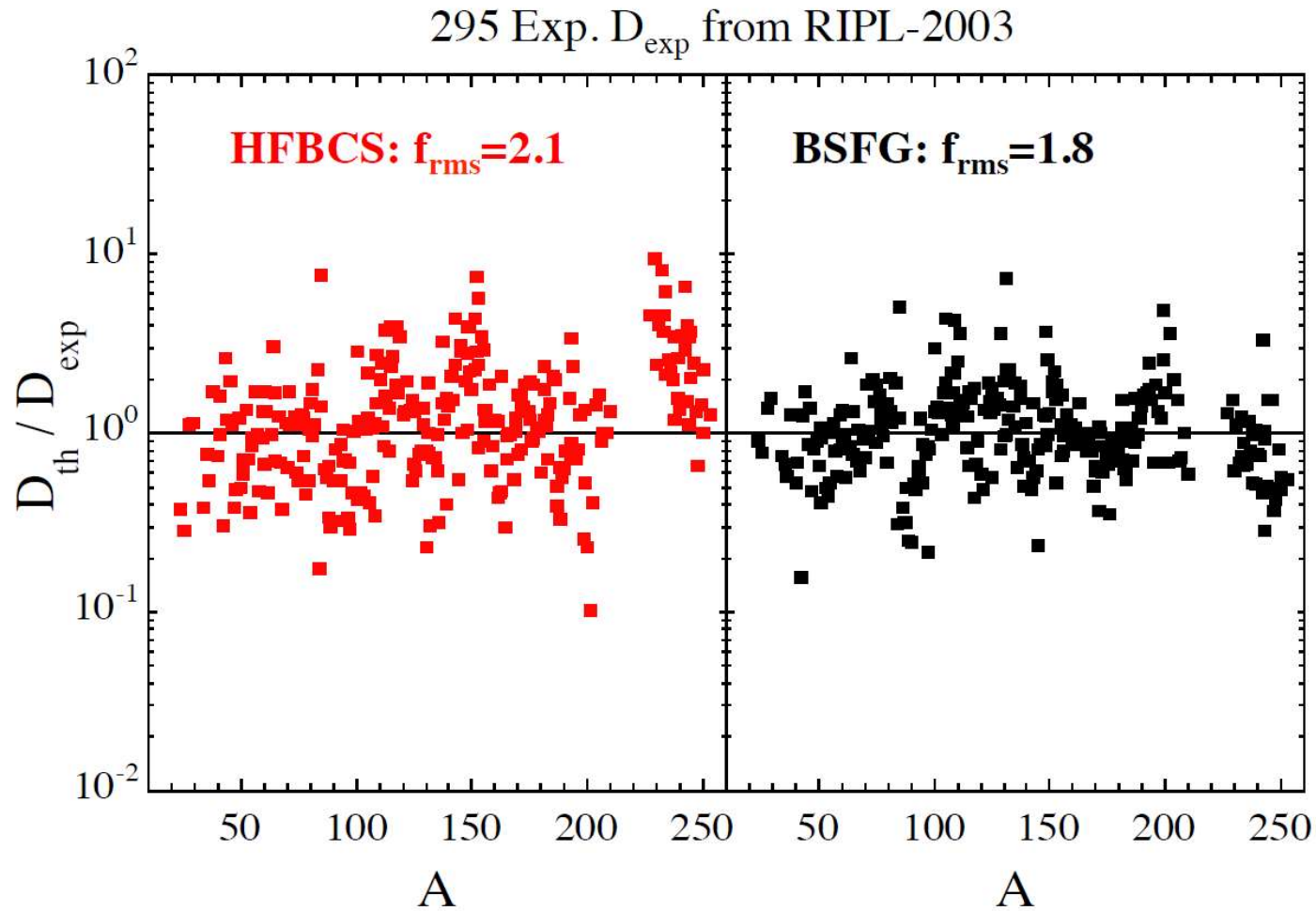
## Mean Field + Statistical NLD formula

- NLD formula within the statistical (partition function) method based on the Skyrme or Gogny HF-BCS/HFB ground-state properties
    - Single particle level scheme
    - Ground-state deformation parameters and energy
    - Pairing strength
  - Microscopic NLD formula includes
    - Shell correction inherent in the mean field s.p. level scheme
    - Pairing correction (in the constant-G approximation) with blocking effects
    - Spin-dependence with microscopic shell and pairing effects
    - Deformation effects included in
      - the single-particle level scheme
      - the collective contribution of the rotational band on top of each intrinsic state
      - disappearance of deformation effects at increasing excitation energies
- **Reliability**: Exact solution the analytical formulas tries to mimic
- **Accuracy**: Competitive with parametrized formulas in reproducing experimental data

# HFBCS Statistical approach



Comparison with experimental neutron resonance spacings **ldmodel 4 in TALYS**

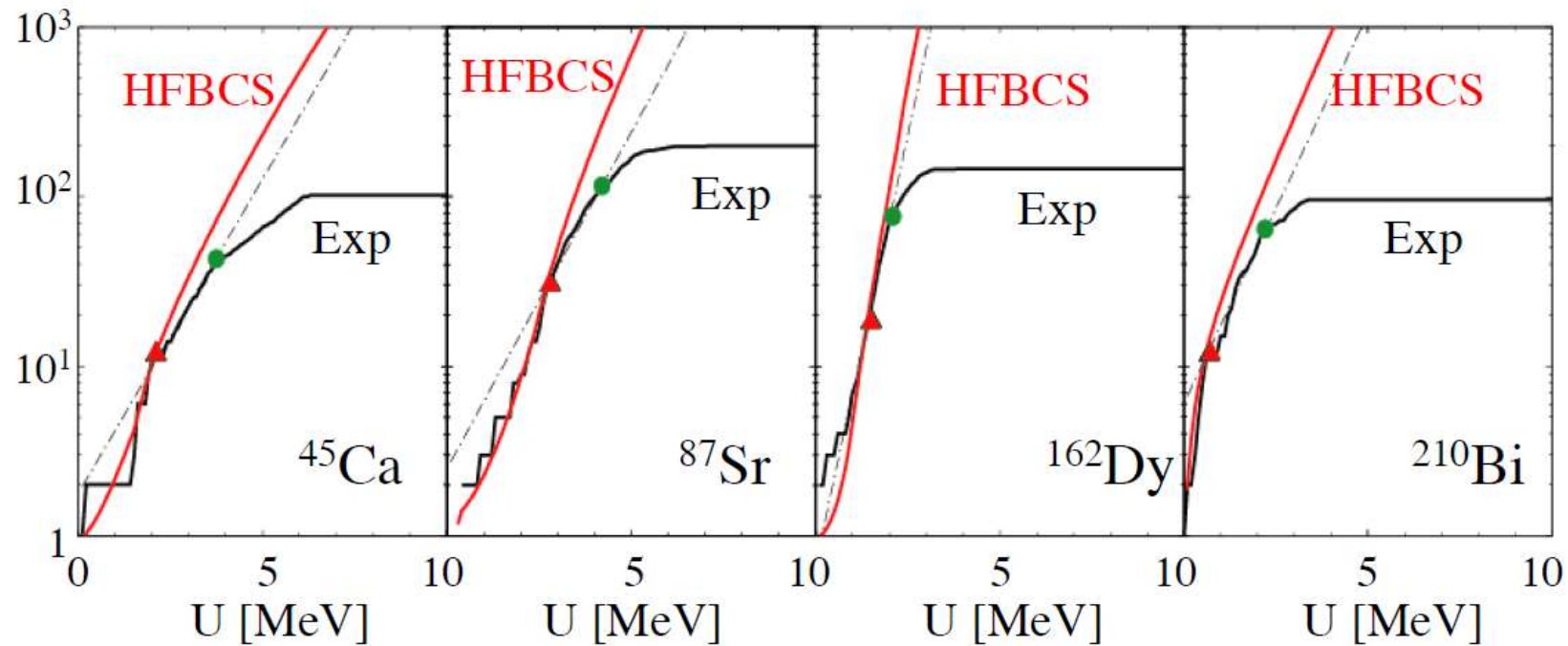


Courtesy S. Goriely

# HFBCS Statistical approach



Comparison with experimental low-lying levels Idmodel 4 in TALYS



Courtesy S. Goriely

NLD provided for all  $\sim 8000$   $8 \leq Z \leq 110$  nuclei in table format

# HFBCS Statistical approach



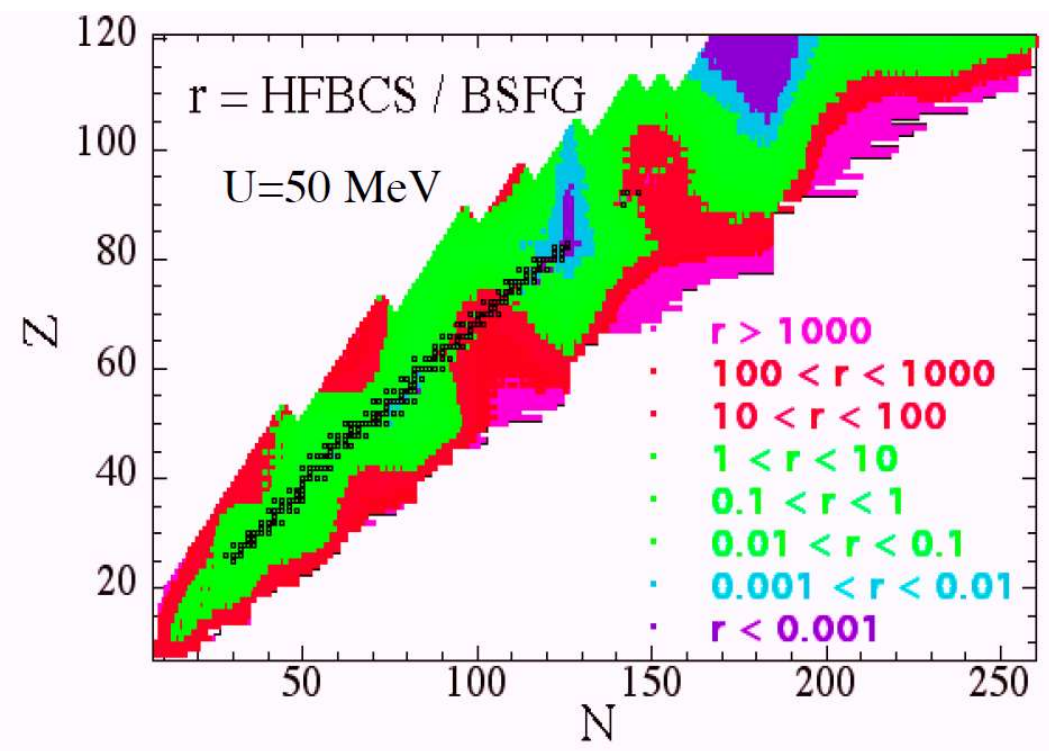
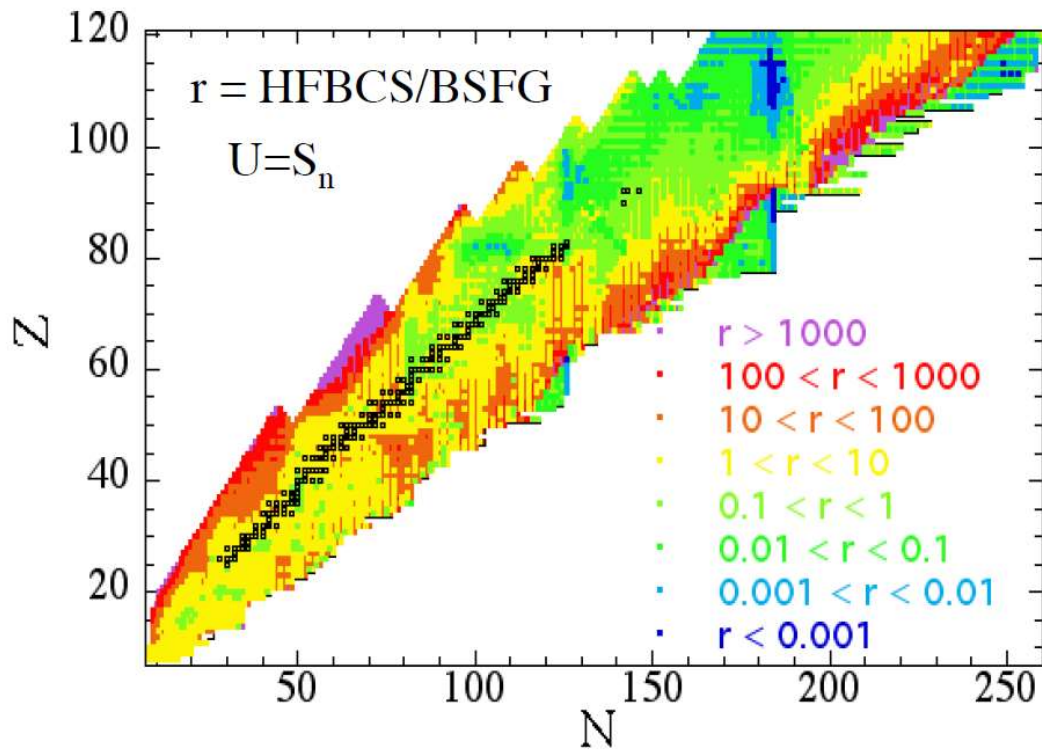
## Comparison of NLD predictions

Idmodel 4 in TALYS

HFBCS+Statistical NLD formula

vs

Analytical shell-corrected Back-Shifted Fermi Gas



Courtesy S. Goriely





## HFBCS Statistical approach : summary

### Mean Field + Statistical NLD formula

Idmodel 4 in TALYS

**Reliability:** Exact solution the analytical formulas try to mimic

**Accuracy:** Competitive with parametrized formulas in reproducing experimental data

But the MF + Statistical approach still makes fundamental approximations :

- Saddle point approximation
- Statistical distribution
- Simple vibrational / rotational enhancement
- Sensitive to the adopted potential, i.e SPL and pairing scheme
- Phenomenological deformed-to-spherical transition at increasing energies
- Partial particle-hole level densities incoherent with total NLD

Courtesy S. Goriely



# Level densities : combinatorial approach

- **Combinatorial approach**

*S. Hilaire & S. Goriely, NPA 779 (2006) 63 & PRC 78 (2008) 064307.*

⇒ Direct level counting

⇒ Total (compound nucleus) and partial (pre-equilibrium) level densities

⇒ Non statistical effects (spin and parity in particular)

⇒ **Global (tables)**



# Level densities : combinatorial approach

*See PRC 78 (2008) 064307 for details*

ldmodel 5 in TALYS

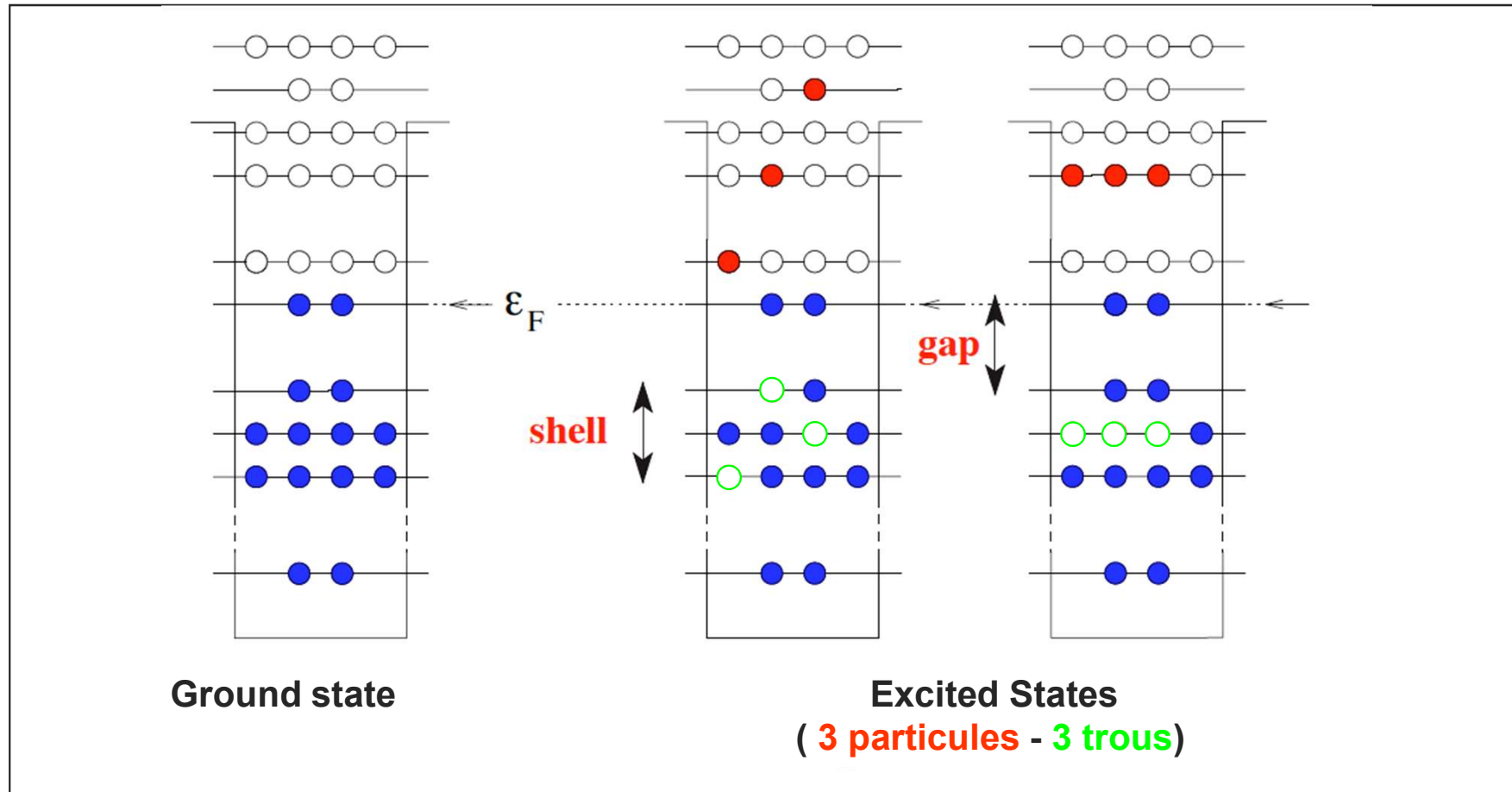
- HFB + effective nucleon-nucleon interaction  $\Rightarrow$  single particle level schemes
- Combinatorial calculation  $\Rightarrow$  intrinsic p-h and total state densities  $\omega_{\text{ph}}(U, K, \pi)$

# Level densities : combinatorial approach



See PRC 78 (2008) 064307 for details

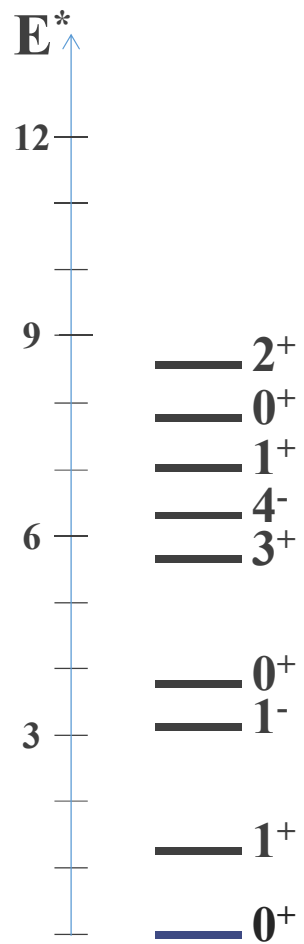
Idmodel 5 in TALYS





# Level densities : combinatorial approach

Idmodel 5 in TALYS





# Level densities : combinatorial approach

*See PRC 78 (2008) 064307 for details*

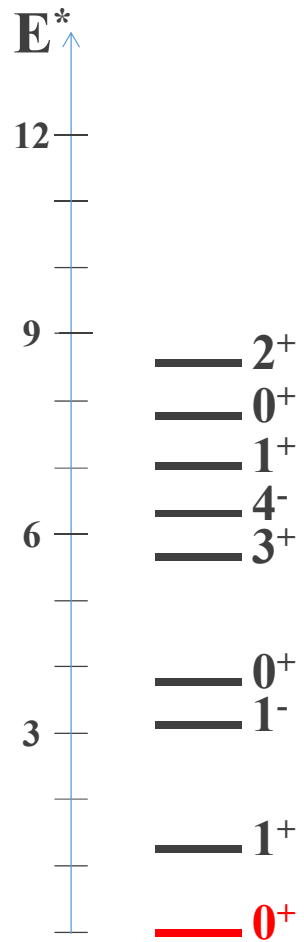
**Idmodel 5 in TALYS**

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1) folding of intrinsic states and vibrational states :  $\omega = \omega_{\text{ph}} * \omega_{\text{vib}}$



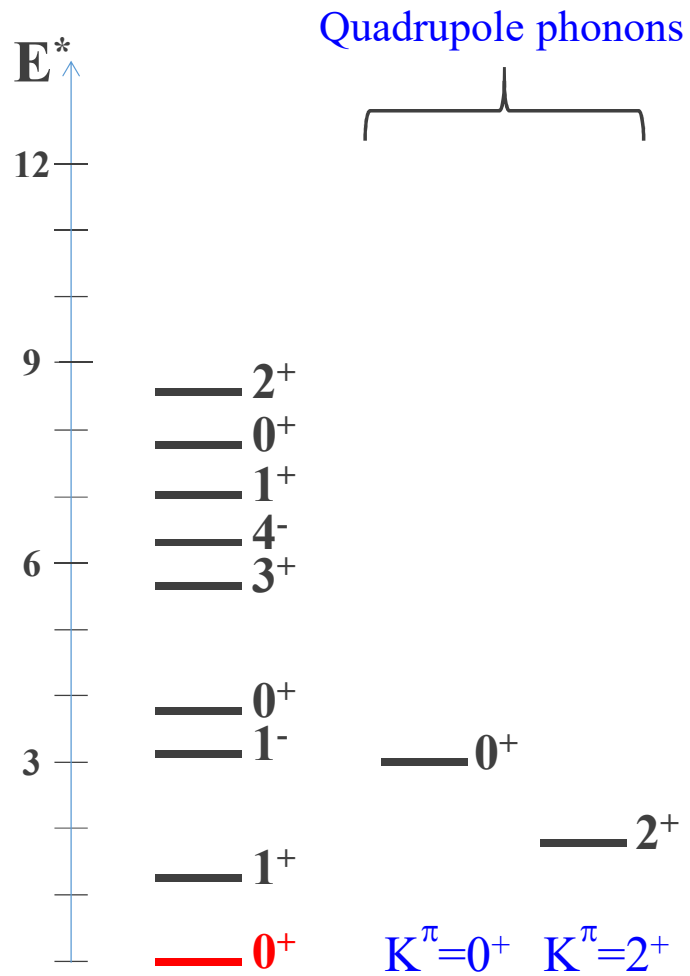
Idmodel 5 in TALYS





# Level densities : combinatorial approach

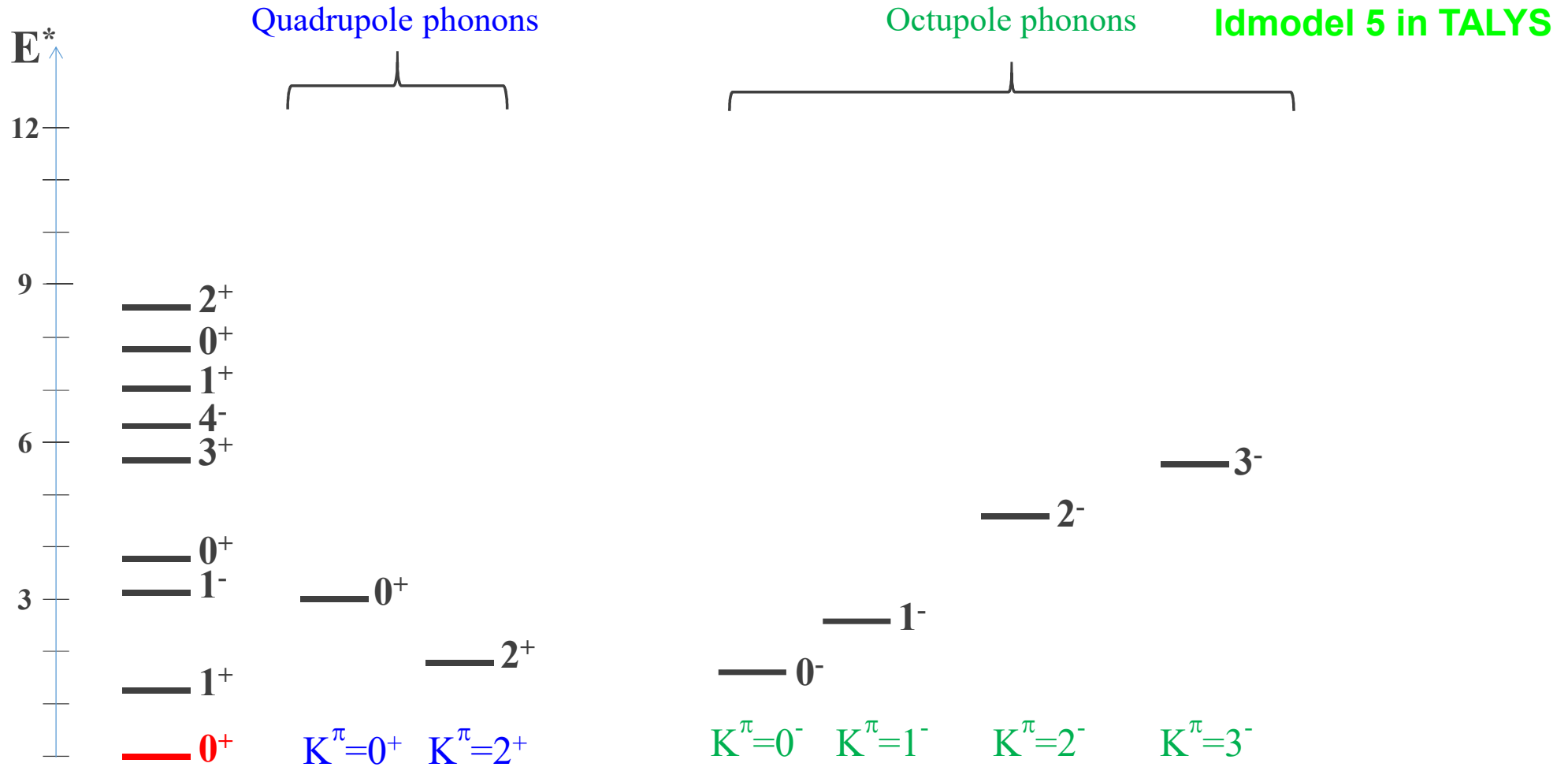
Idmodel 5 in TALYS





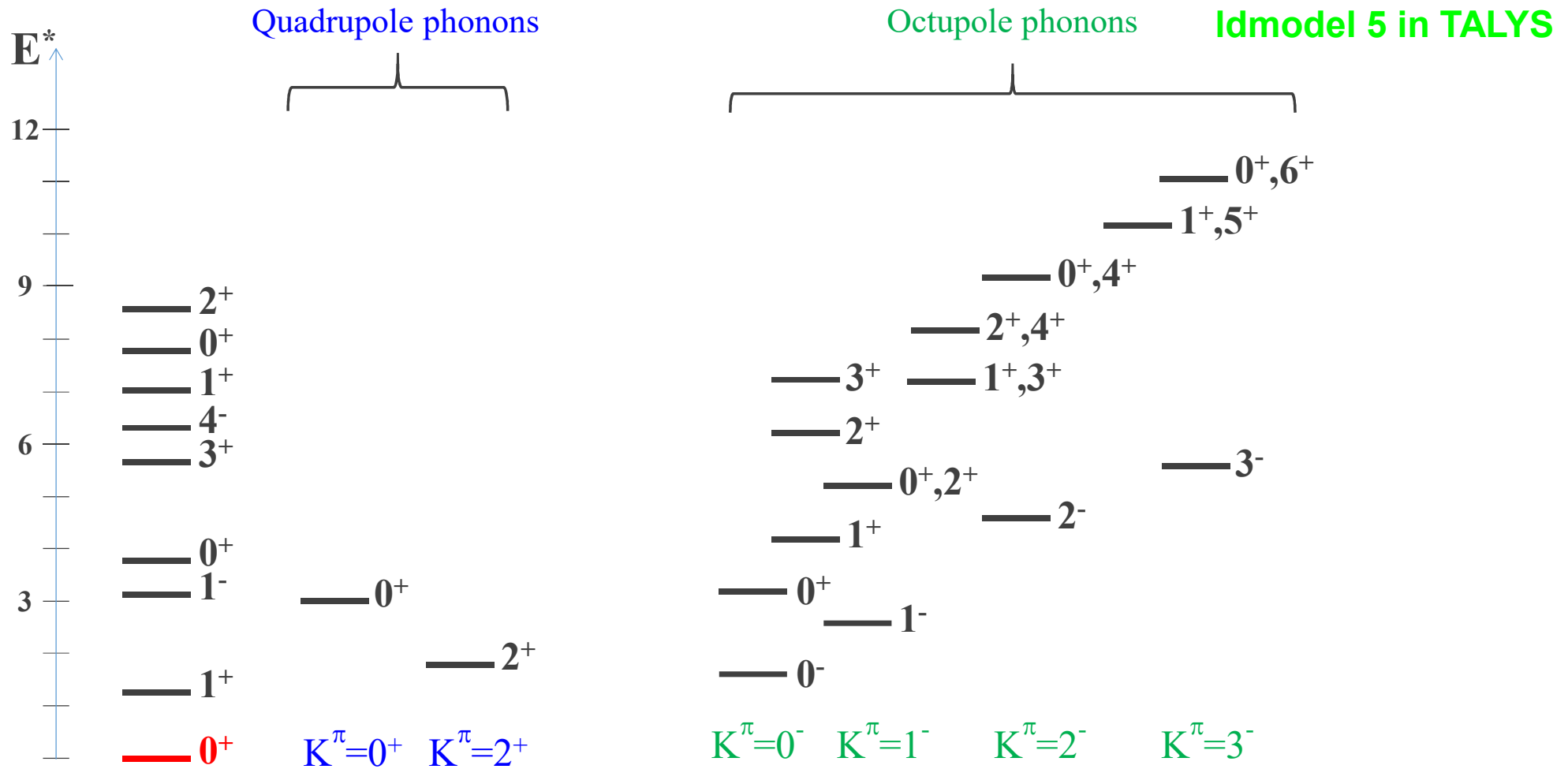


# Level densities : combinatorial approach



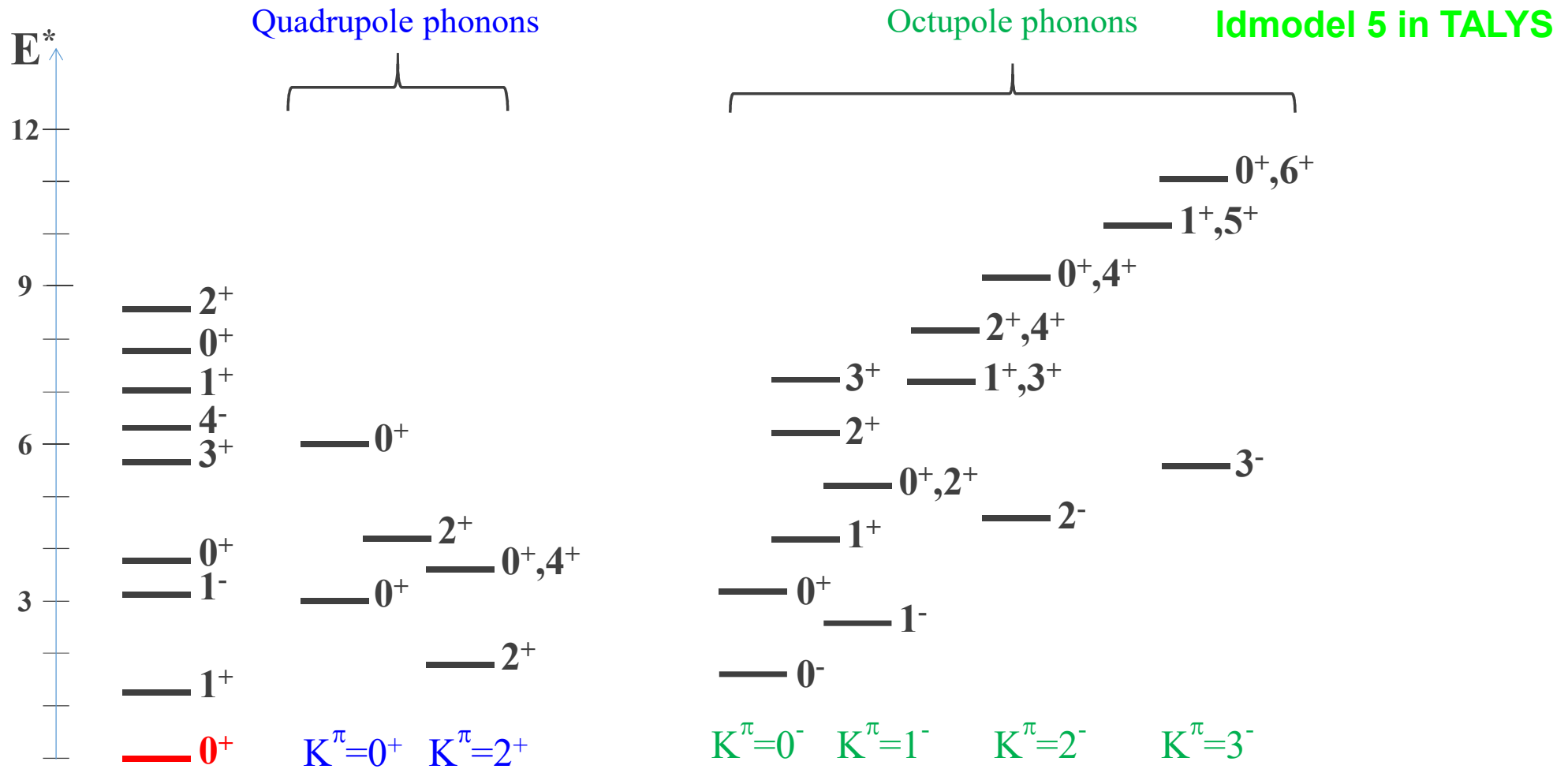


# Level densities : combinatorial approach





# Level densities : combinatorial approach

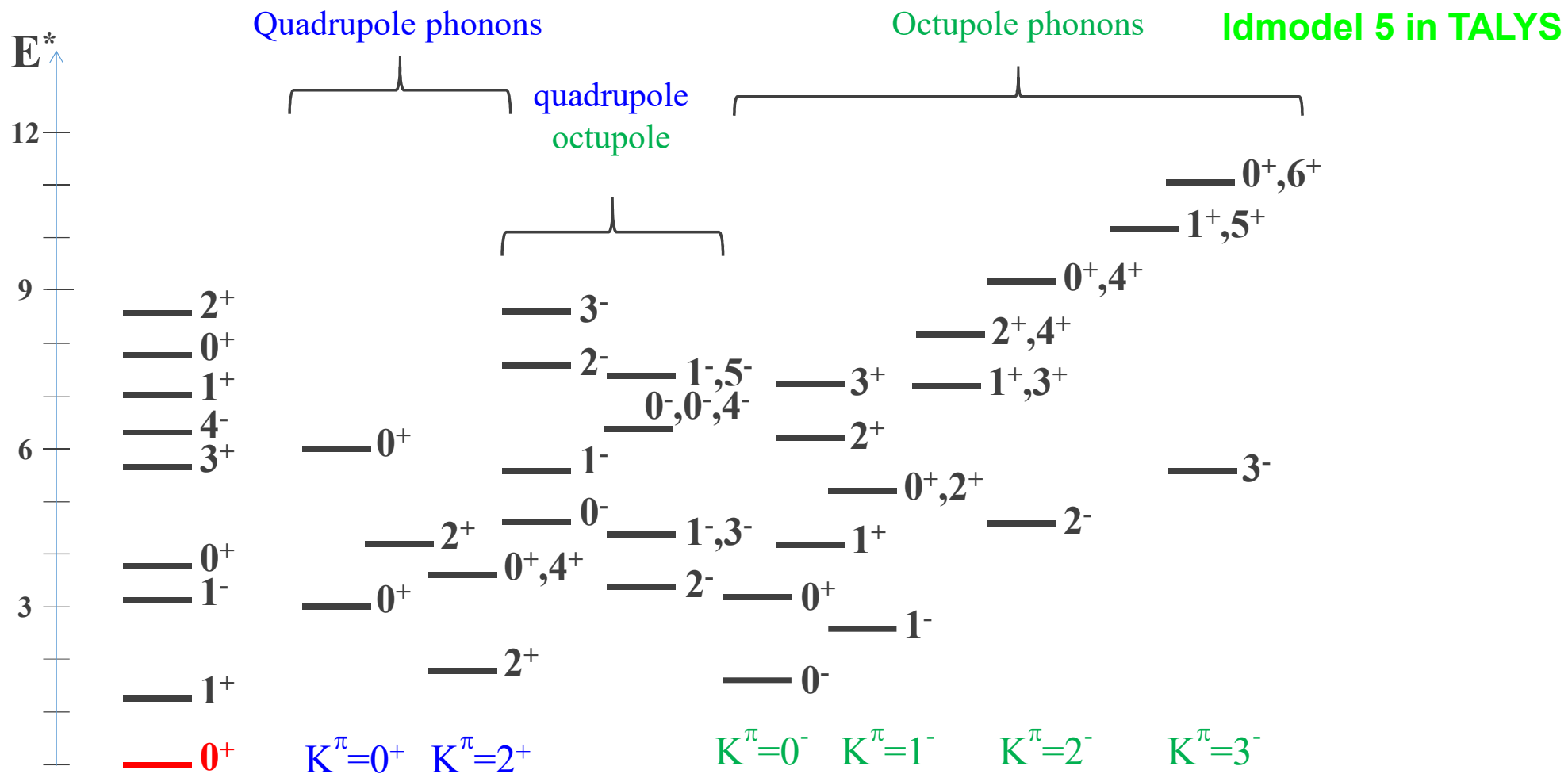








# Level densities : combinatorial approach





# Level densities : combinatorial approach

*See PRC 78 (2008) 064307 for details*

Idmodel 5 in TALYS

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  - 1) folding of intrinsic states and vibrational states :  $\omega = \omega_{\text{ph}} * \omega_{\text{vib}}$
  - 2) construction of rotational bands for deformed nuclei

$$\rho(\text{U}, \text{J}, \pi) = \sum_{\text{K}} \omega(\text{U} - \text{E}_{\text{rot}}^{\text{JK}}, \text{K}, \pi)$$

trivial relation for spherical nuclei

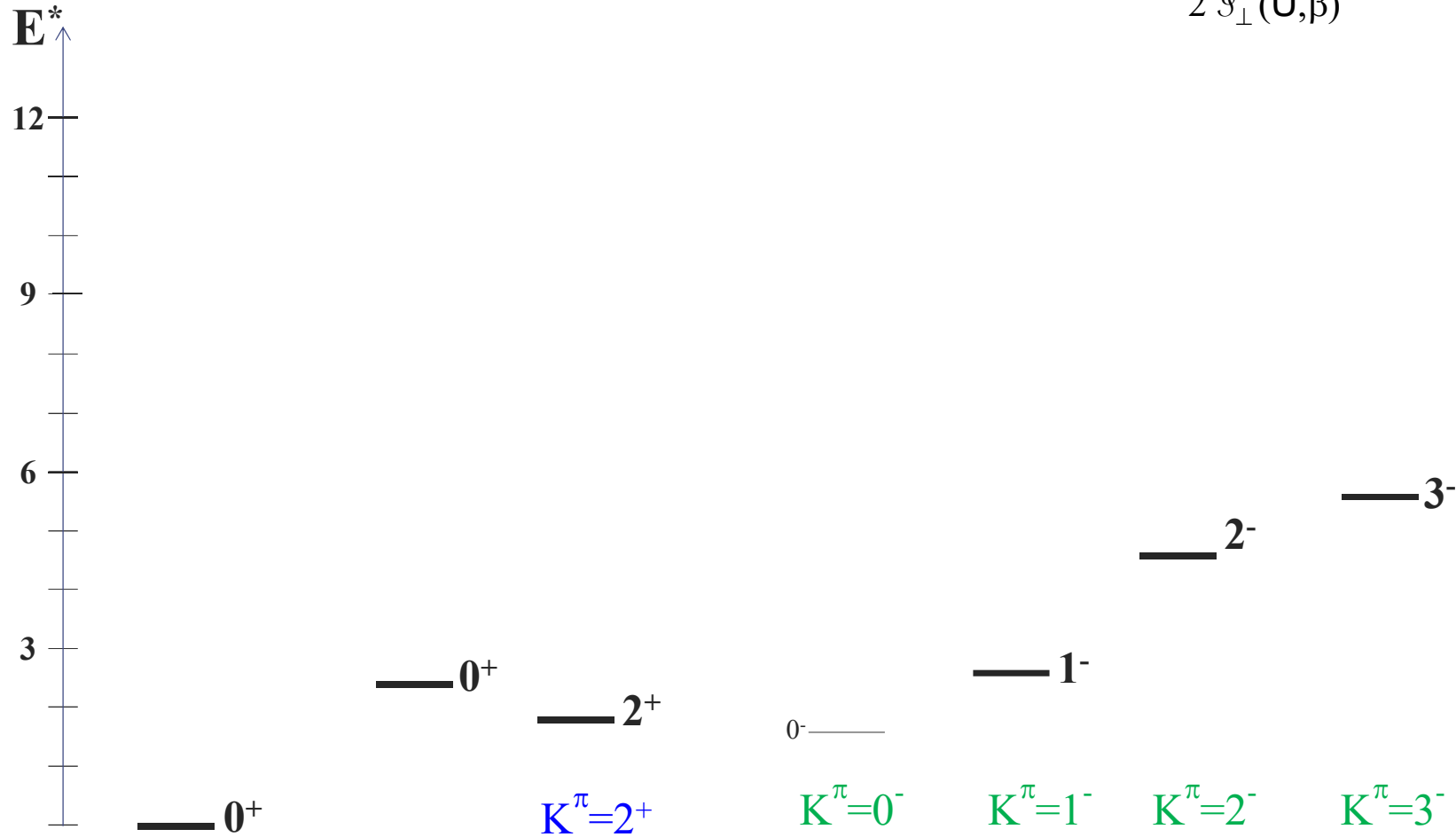
$$\rho(\text{U}, \text{J}, \pi) = \omega(\text{U}, \text{K}=\text{J}, \pi) - \omega(\text{U}, \text{K}=\text{J}+1, \pi)$$



# Level densities : combinatorial approach

⇒ General level sequence for a deformed even-even nucleus :  $E_{\text{rot}}(J,K) = \frac{J(J+1) - K^2}{2 \mathcal{I}_{\perp}(U,\beta)}$

Idmodel 5 in TALYS



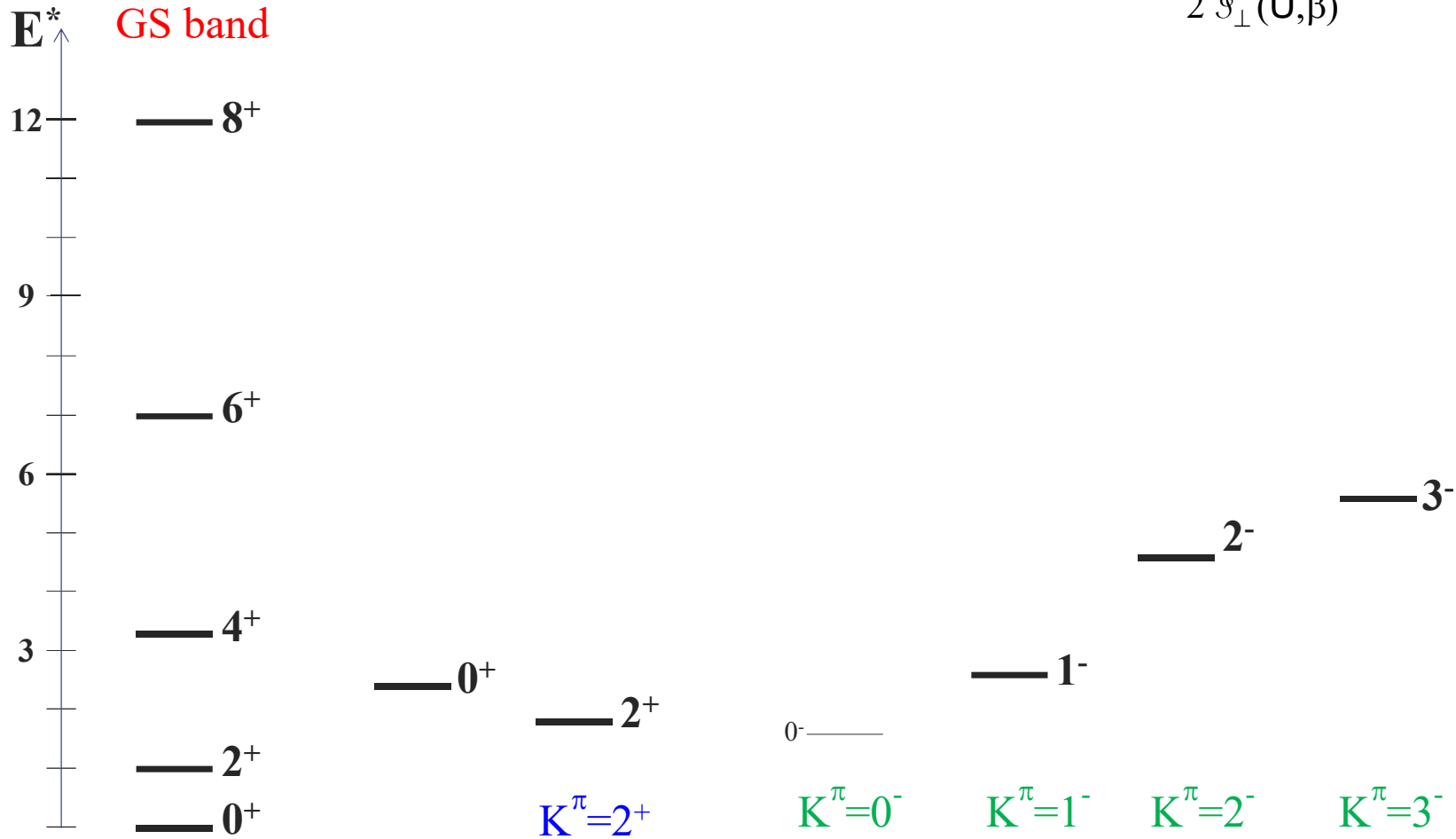




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Idmodel 5 in TALYS

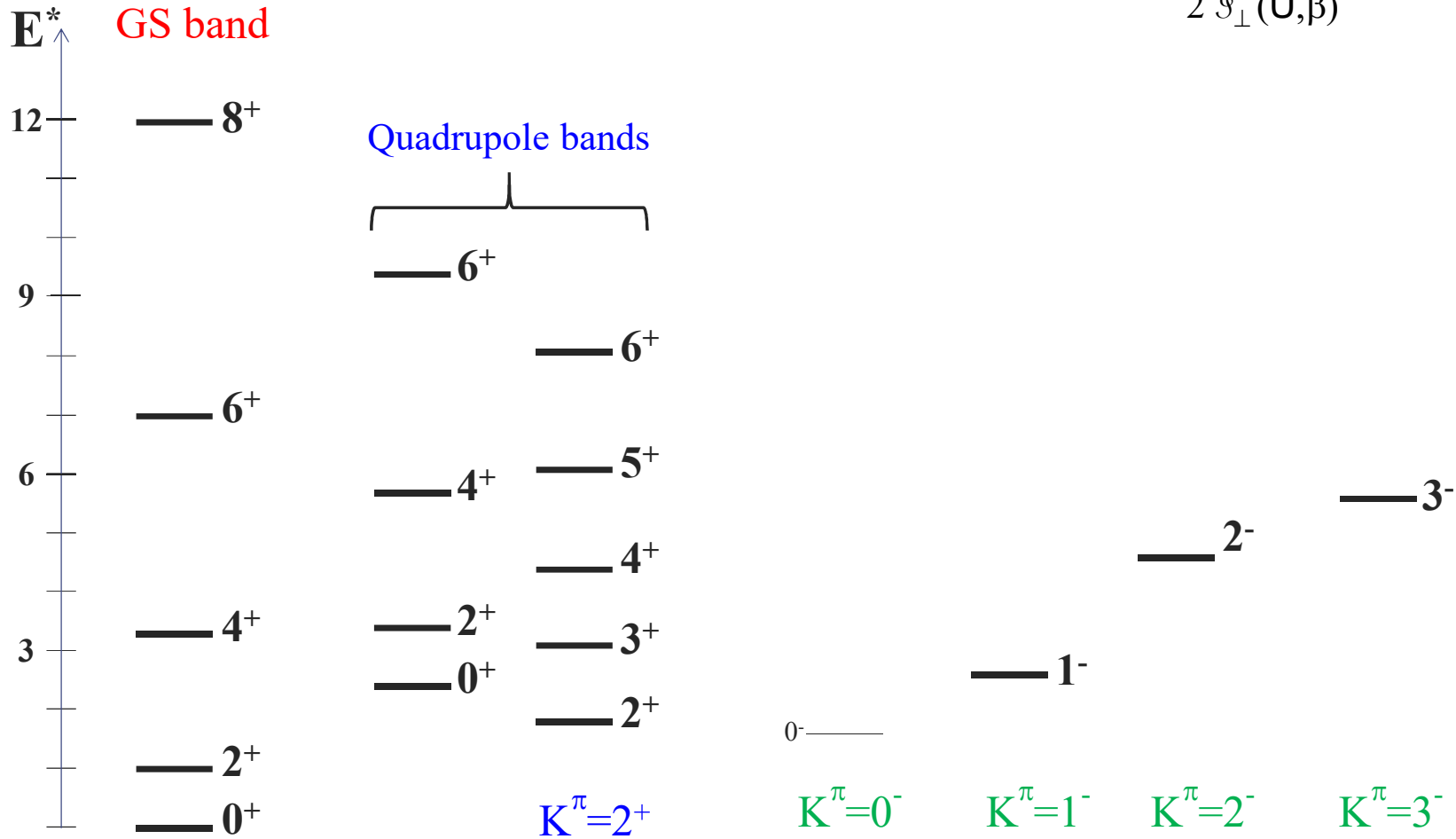




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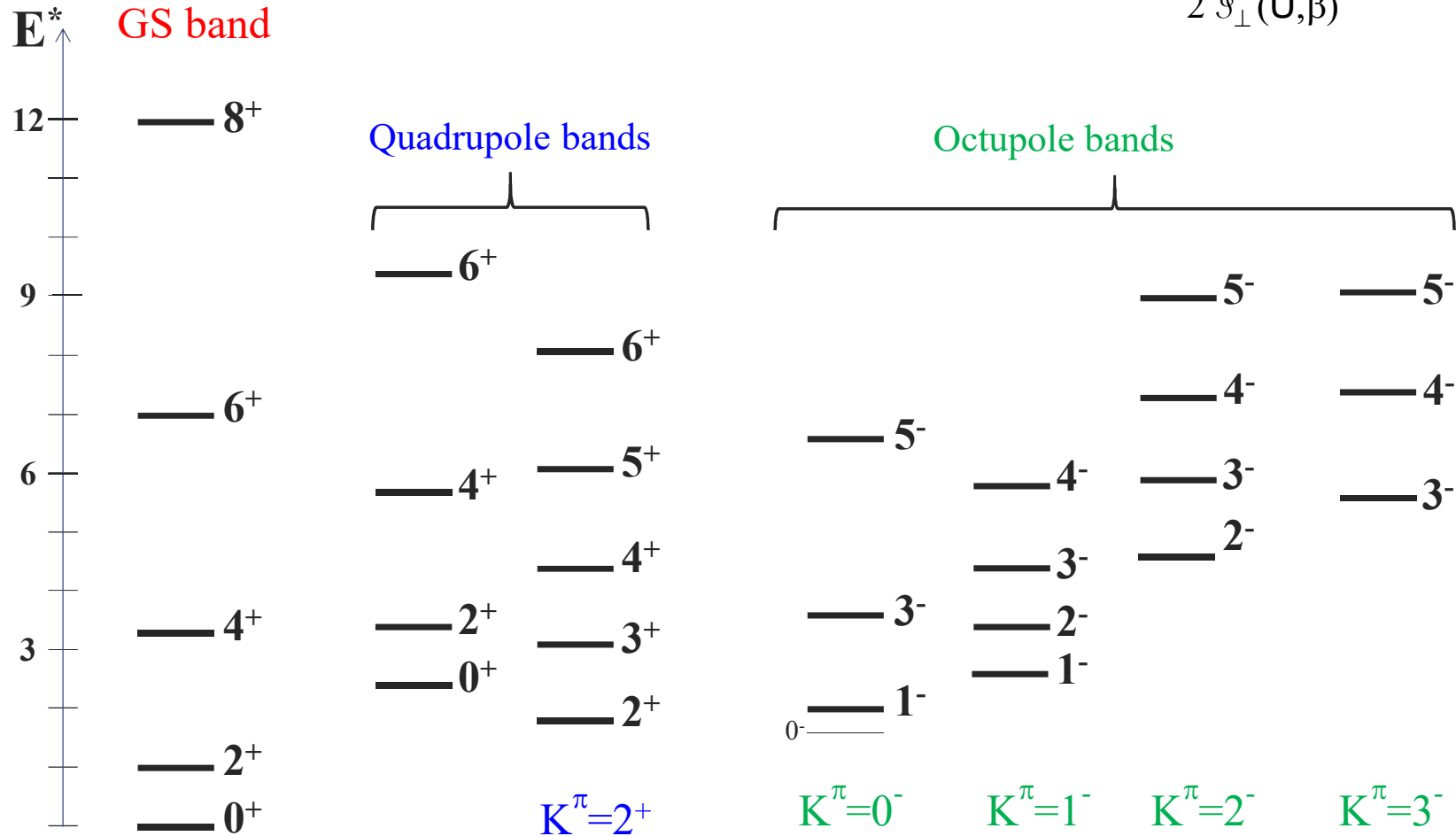




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Idmodel 5 in TALYS

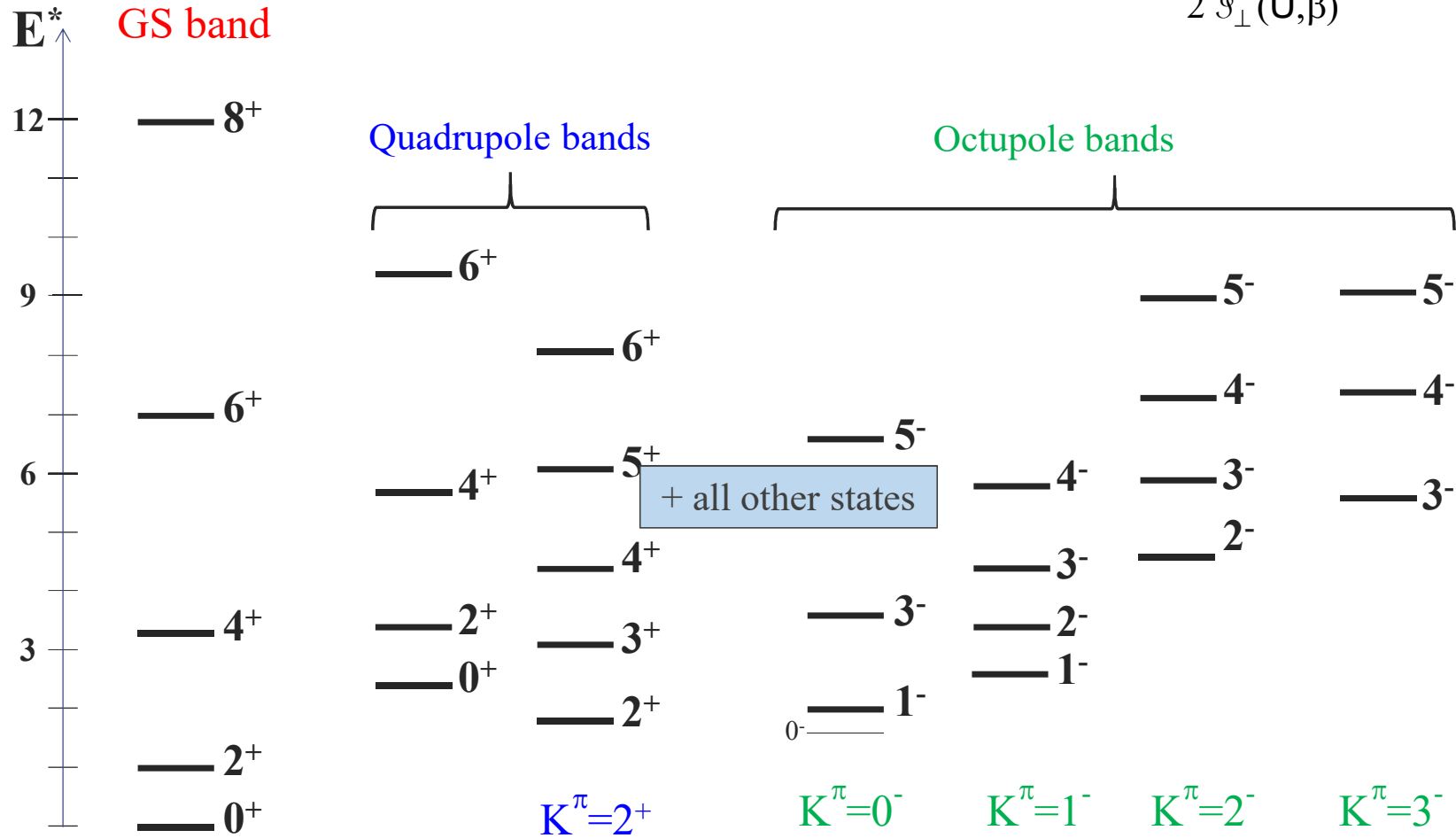




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Idmodel 5 in TALYS





# Level densities : combinatorial approach

*See PRC 78 (2008) 064307 for details*

Idmodel 5 in TALYS

- HFB + effective nucleon-nucleon interaction  $\Rightarrow$  single particle level schemes
- Combinatorial calculation  $\Rightarrow$  intrinsic p-h and total state densities  $\omega_{\text{ph}}(\text{U}, \text{K}, \pi)$
- Collective effects  $\Rightarrow$  from state to level densities  $\rho(\text{U}, \text{J}, \pi)$ 
  - 1) folding of intrinsic states and vibrational states :  $\omega = \omega_{\text{ph}} * \omega_{\text{vib}}$
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trivial relation for spherical nuclei

$$\rho(\text{U}, \text{J}, \pi) = \omega(\text{U}, \text{K}=\text{J}, \pi) - \omega(\text{U}, \text{K}=\text{J}+1, \pi)$$

- Phenomenological mixing of spherical and deformed level densities for small deformations



# Level densities : combinatorial approach + temperature

See PRC 78 (2008) 064307 and PRC 86 (2012) 064317 for details

Idmodel 6 in TALYS

- TDHFB + effective nucleon-nucleon interaction

⇒ temperature (energy) dependent single particle level schemes

- Combinatorial calculation ⇒ intrinsic p-h and total state densities  $\omega_{ph}(U, K, \pi)$

- Collective effects ⇒ from state to level densities  $\rho(U, J, \pi)$

1) folding of intrinsic states and vibrational states :  $\omega = \omega_{ph} * \omega_{vib}$

2) construction of rotational bands for deformed nuclei

$$\rho(U, J, \pi) = \sum_K \omega(U - E_{rot}^{JK}, K, \pi)$$

trivial relation for spherical nuclei

$$\rho(U, J, \pi) = \omega(U, K=J, \pi) - \omega(U, K=J+1, \pi)$$

Predicted within the same theoretical framework (coherence)

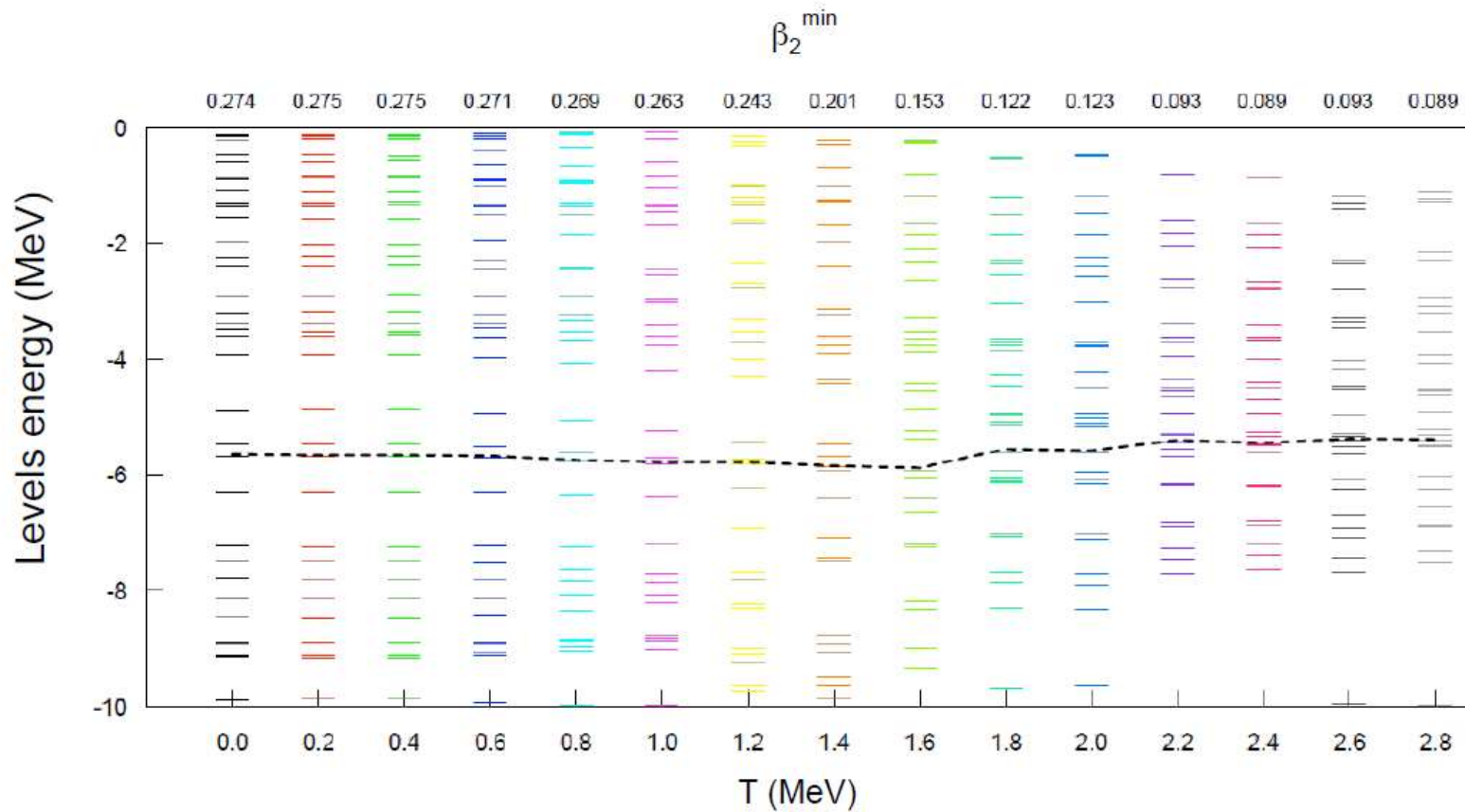
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# Level densities : combinatorial approach + temperature



Neutrons levels around Fermi energy for  $^{152}\text{Sm}$

Idmodel 6 in TALYS



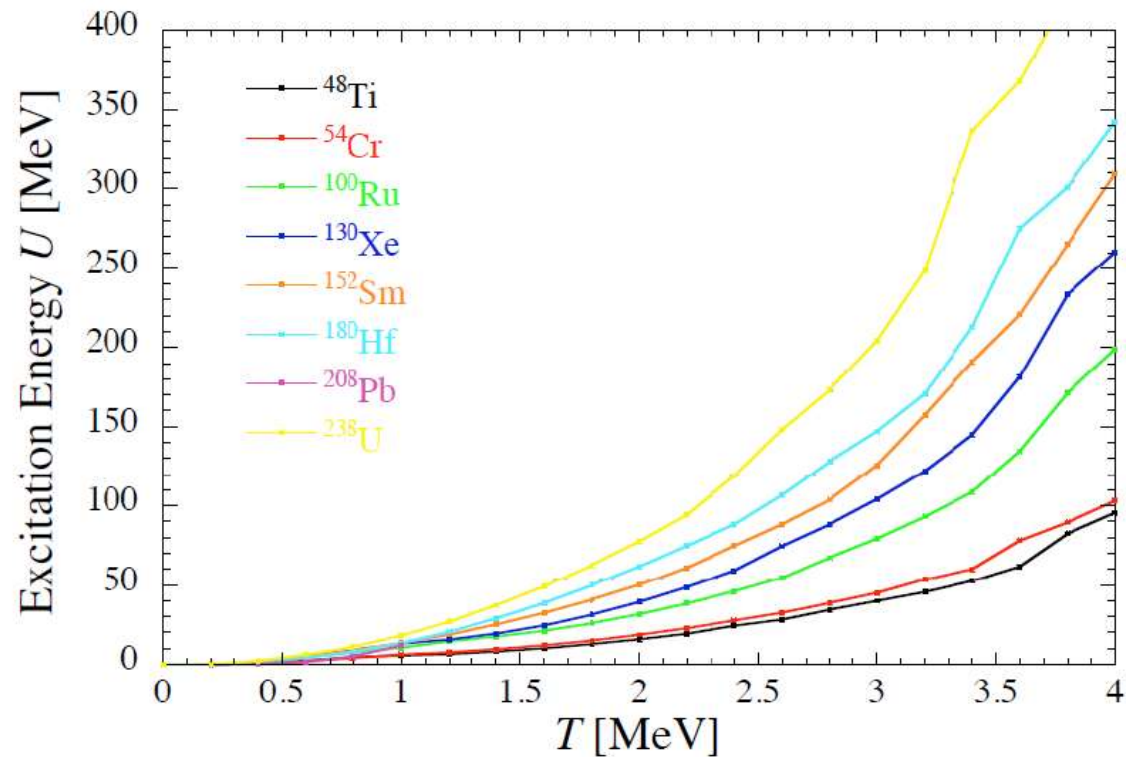


# Level densities : combinatorial approach +temperature

For each temperature, the excitation energy is determined.  
→ expected parabolic shape ( $U \propto T^2$ ) is observed.

ldmodel 6 in TALYS

Excitation energy as a function of the temperature





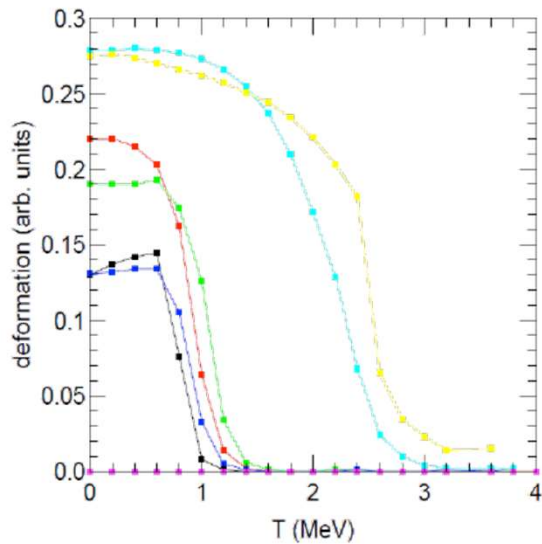


# Level densities : combinatorial approach +temperature

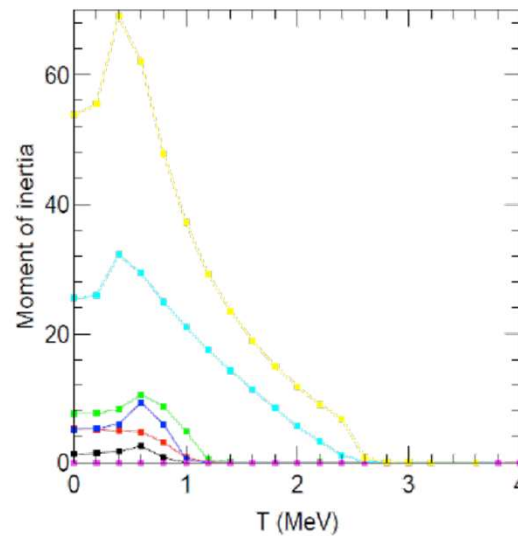
Temperature evolution of nuclear structure properties relevant for level density calculations within the combinatorial model

ldmodel 6 in TALYS

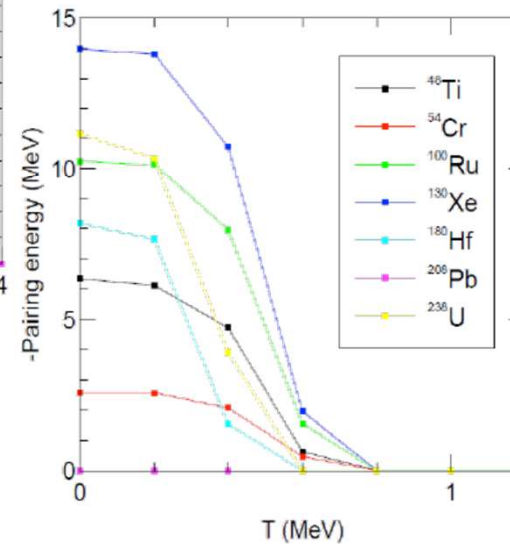
*Quadrupole deformation*



*Cranking moment of Inertia*



*Total pairing energy*

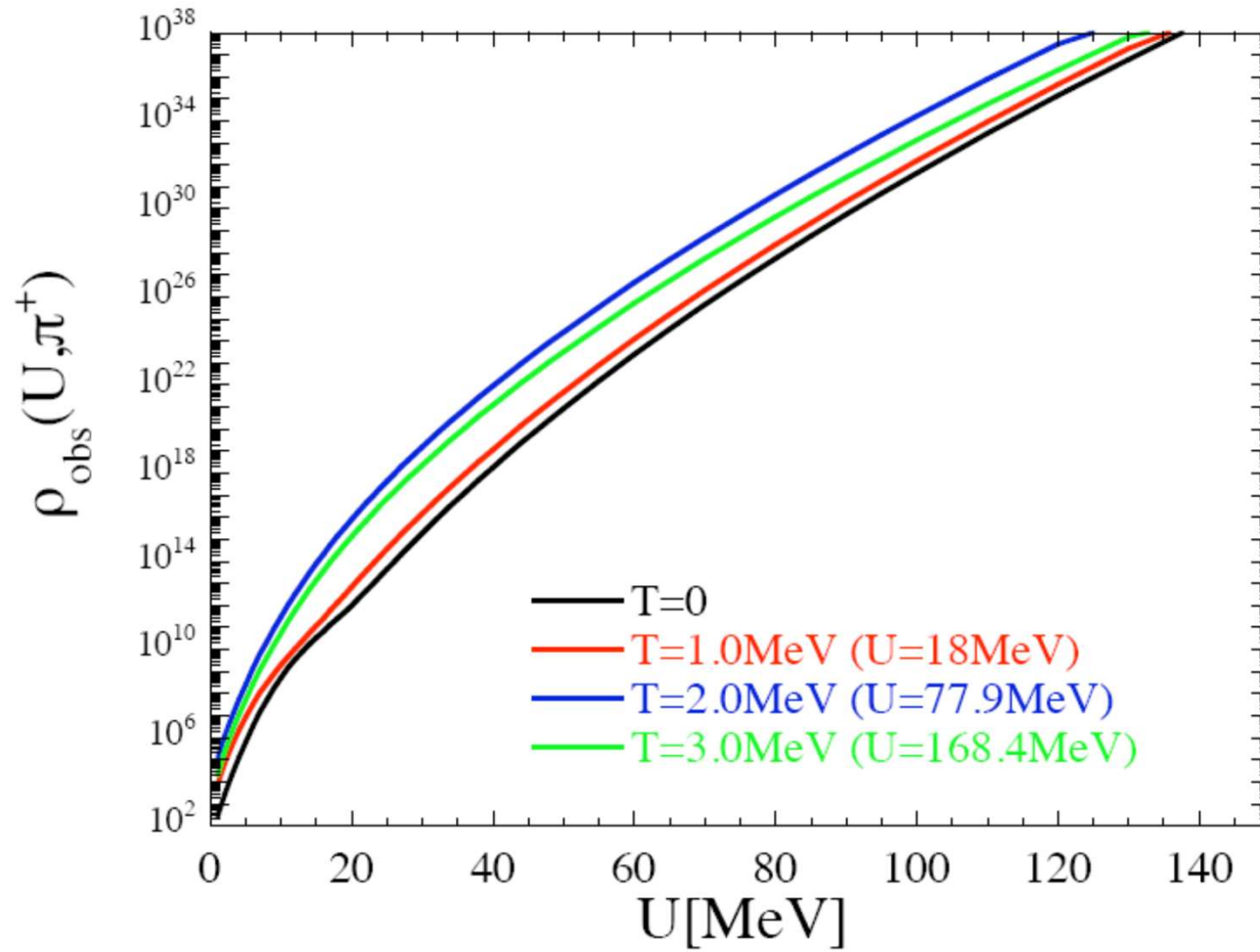


# Level densities : combinatorial approach +temperature



Level density for  $^{238}\text{U}$

Idmodel 6 in TALYS

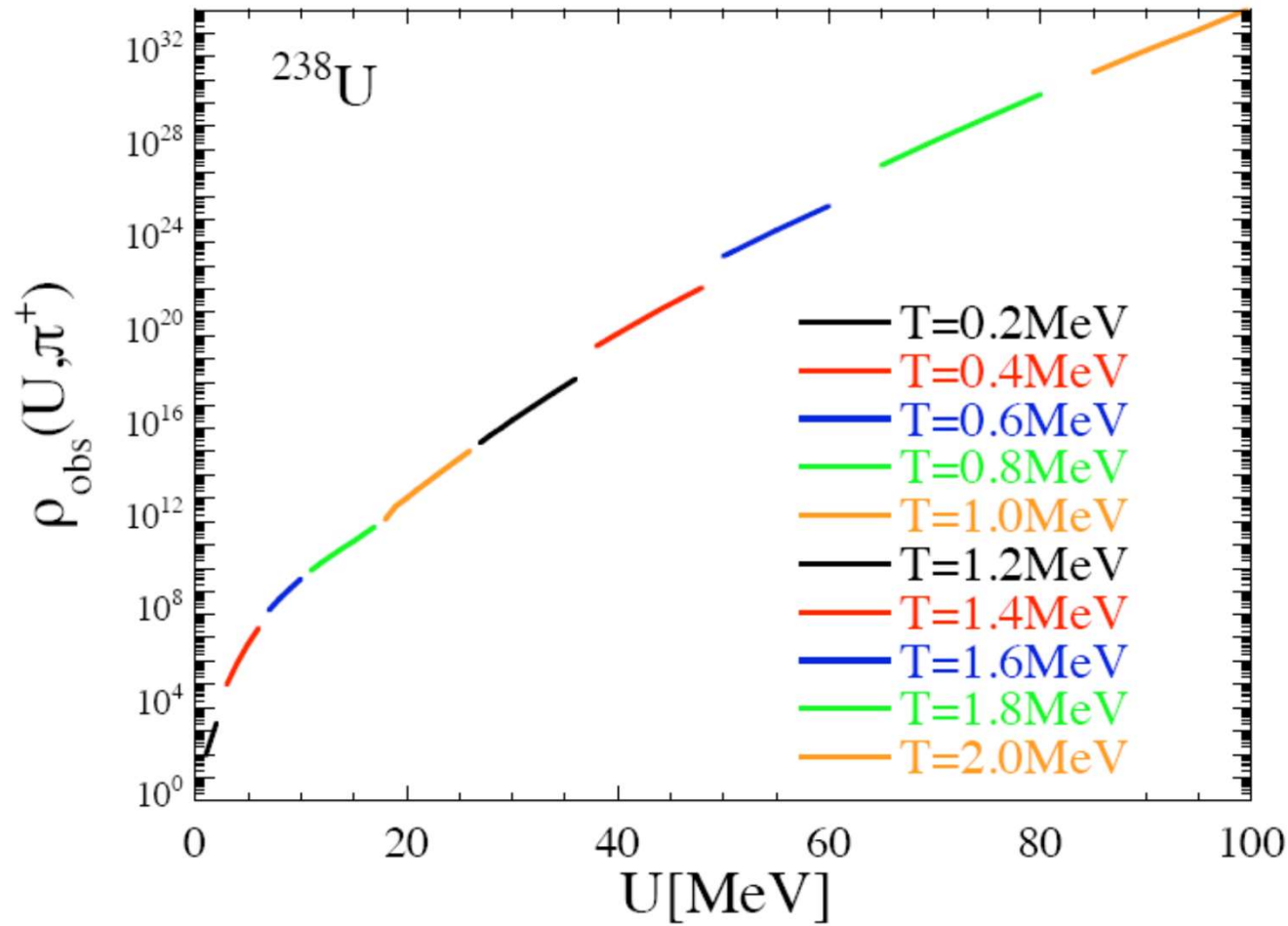


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Idmodel 6 in TALYS

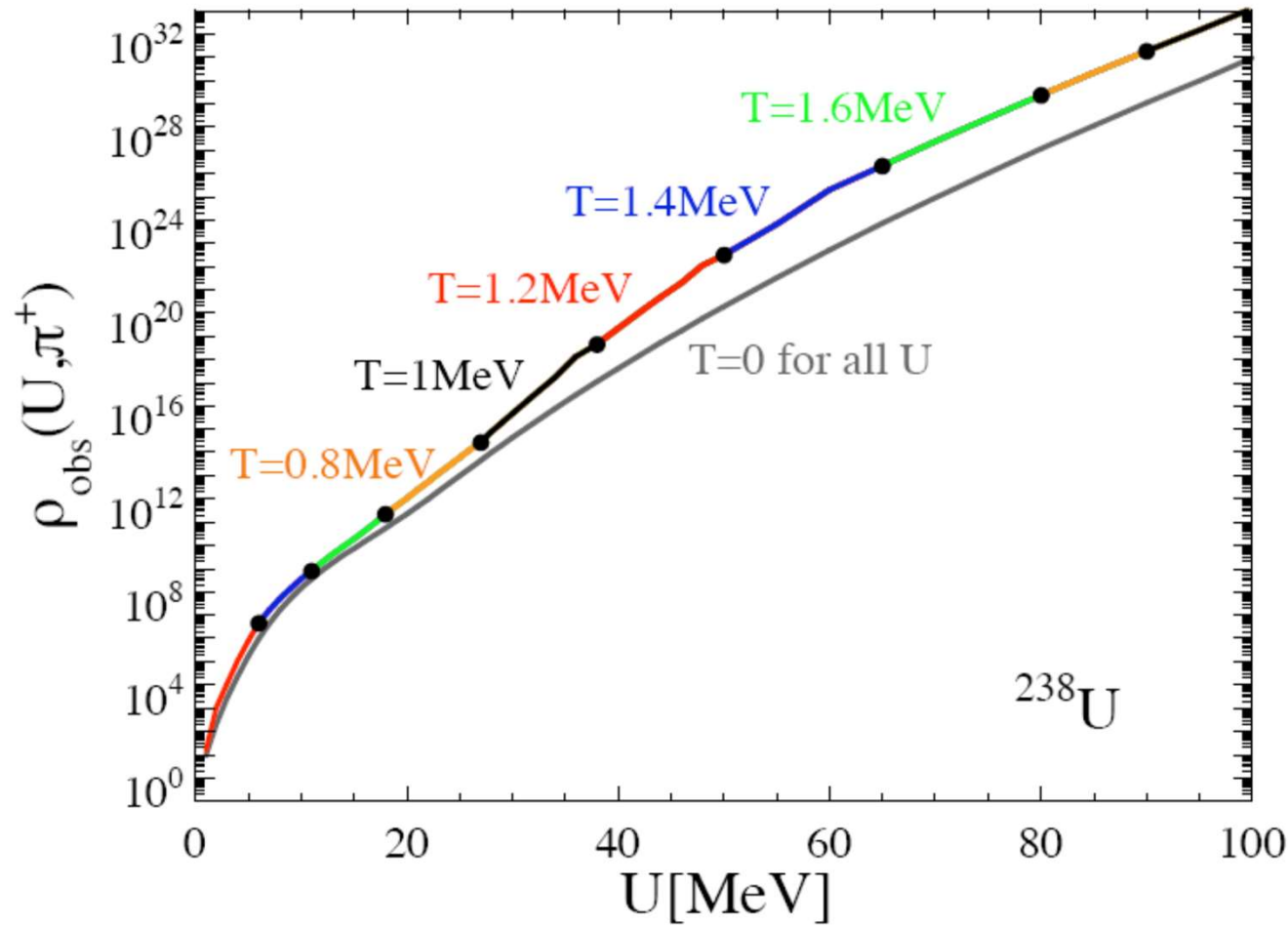


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Idmodel 6 in TALYS

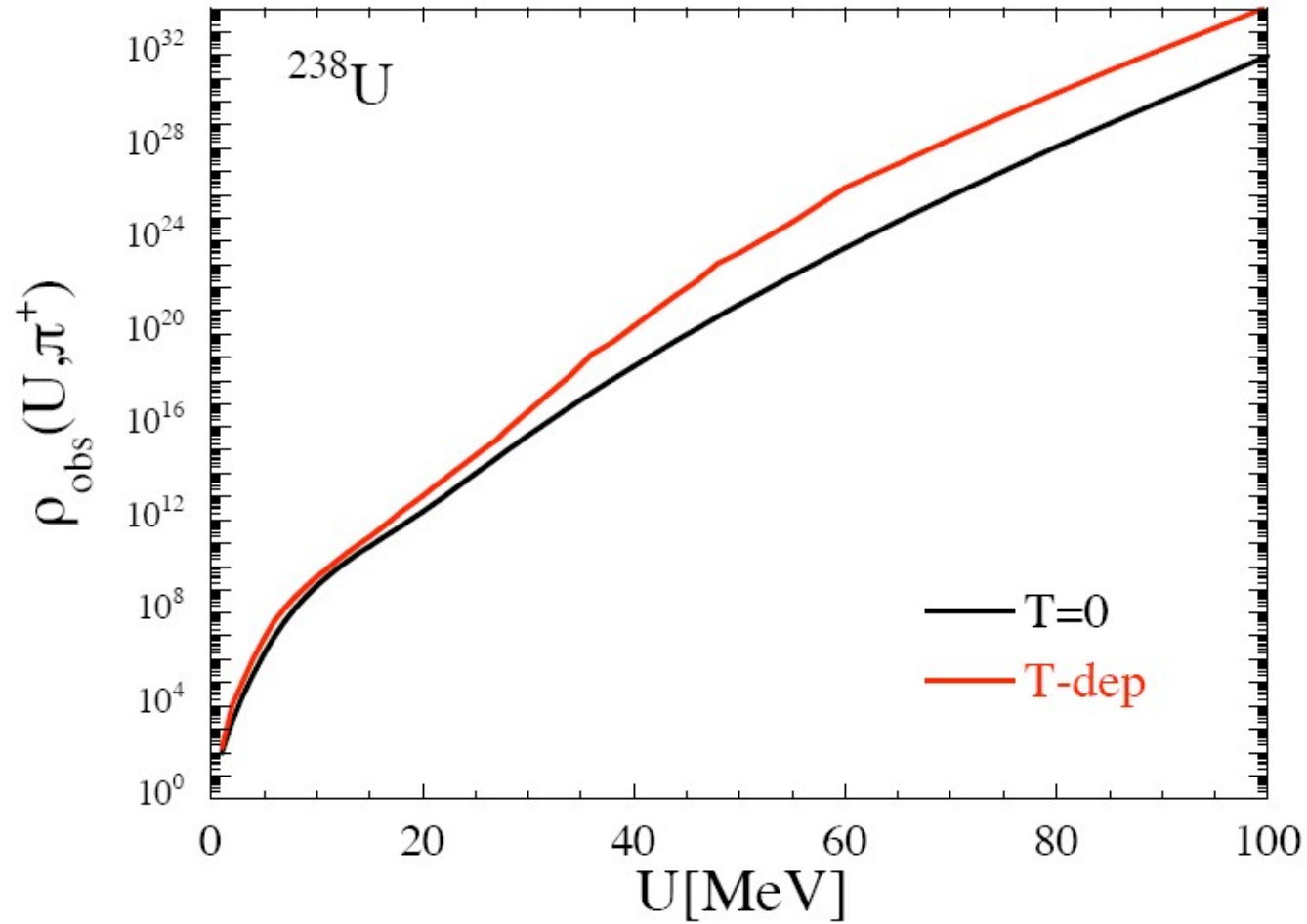


# Level densities : combinatorial approach +temperature



Level density for  $^{238}\text{U}$

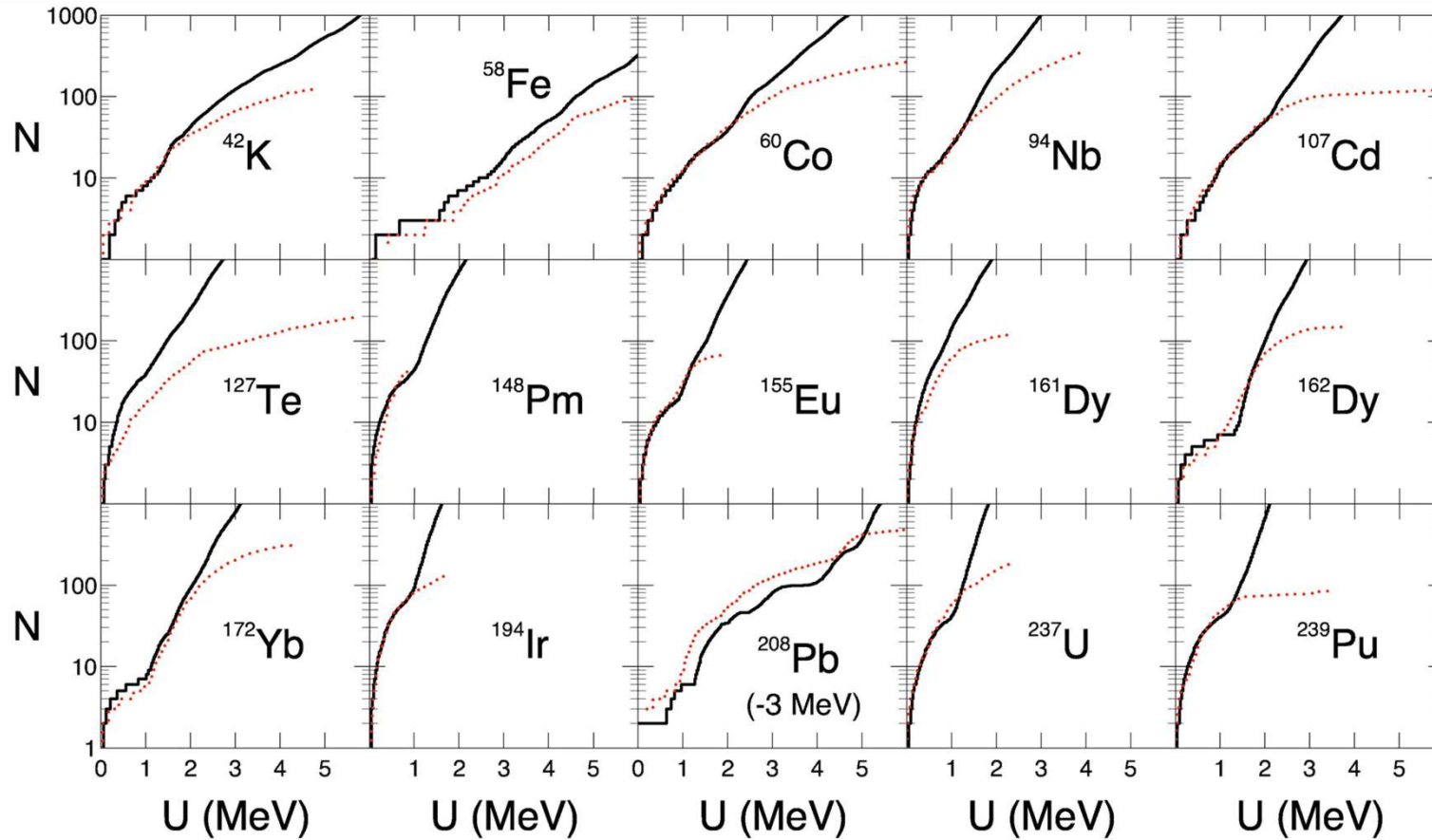
Idmodel 6 in TALYS





# Level densities : combinatorial approach +temperature

Idmodel 6 in TALYS



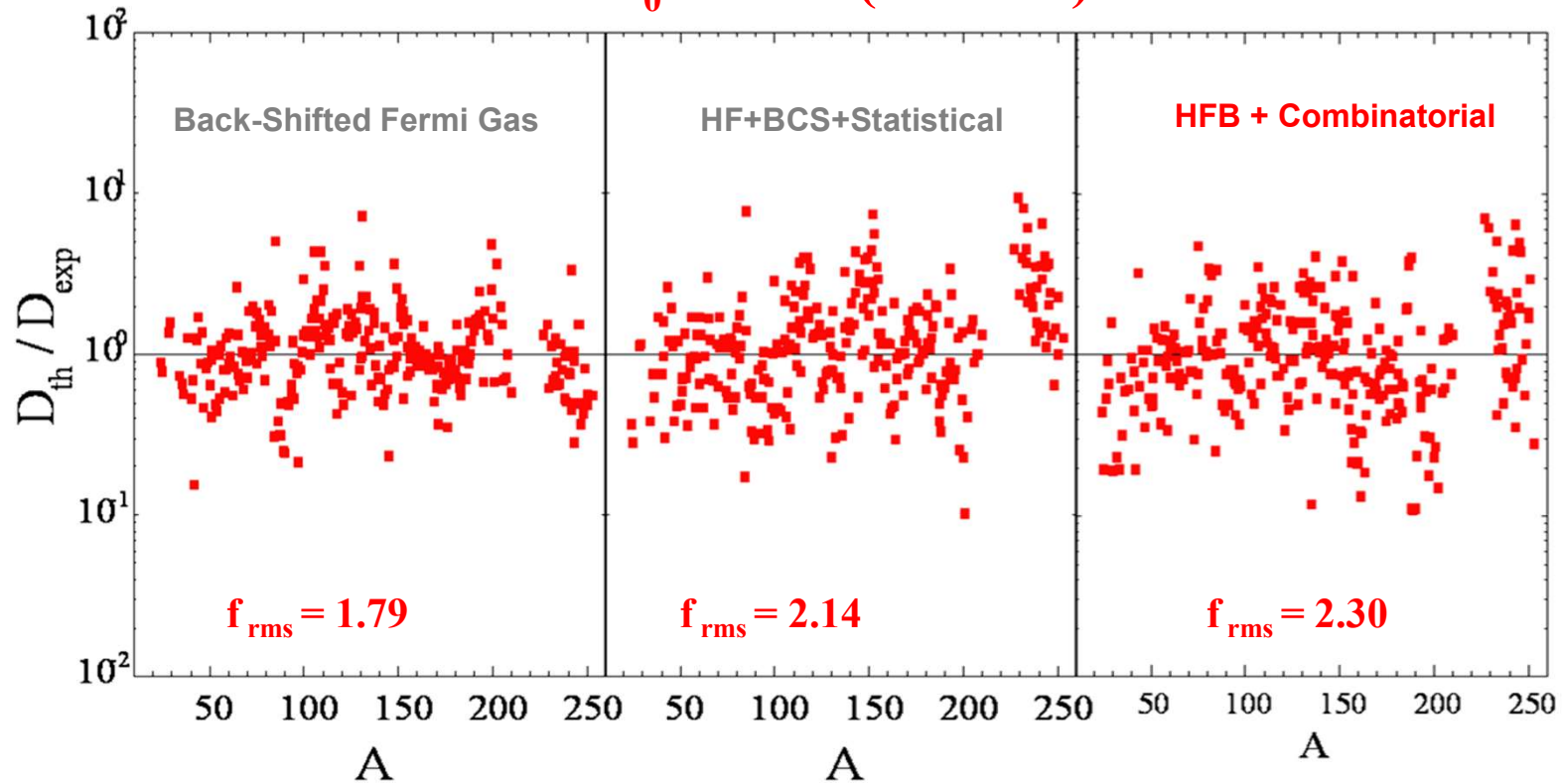
→ Structures typical of non-statistical feature



# Level densities : combinatorial approach +temperature

$D_0$  values ( s-waves)

Idmodel 6 in TALYS



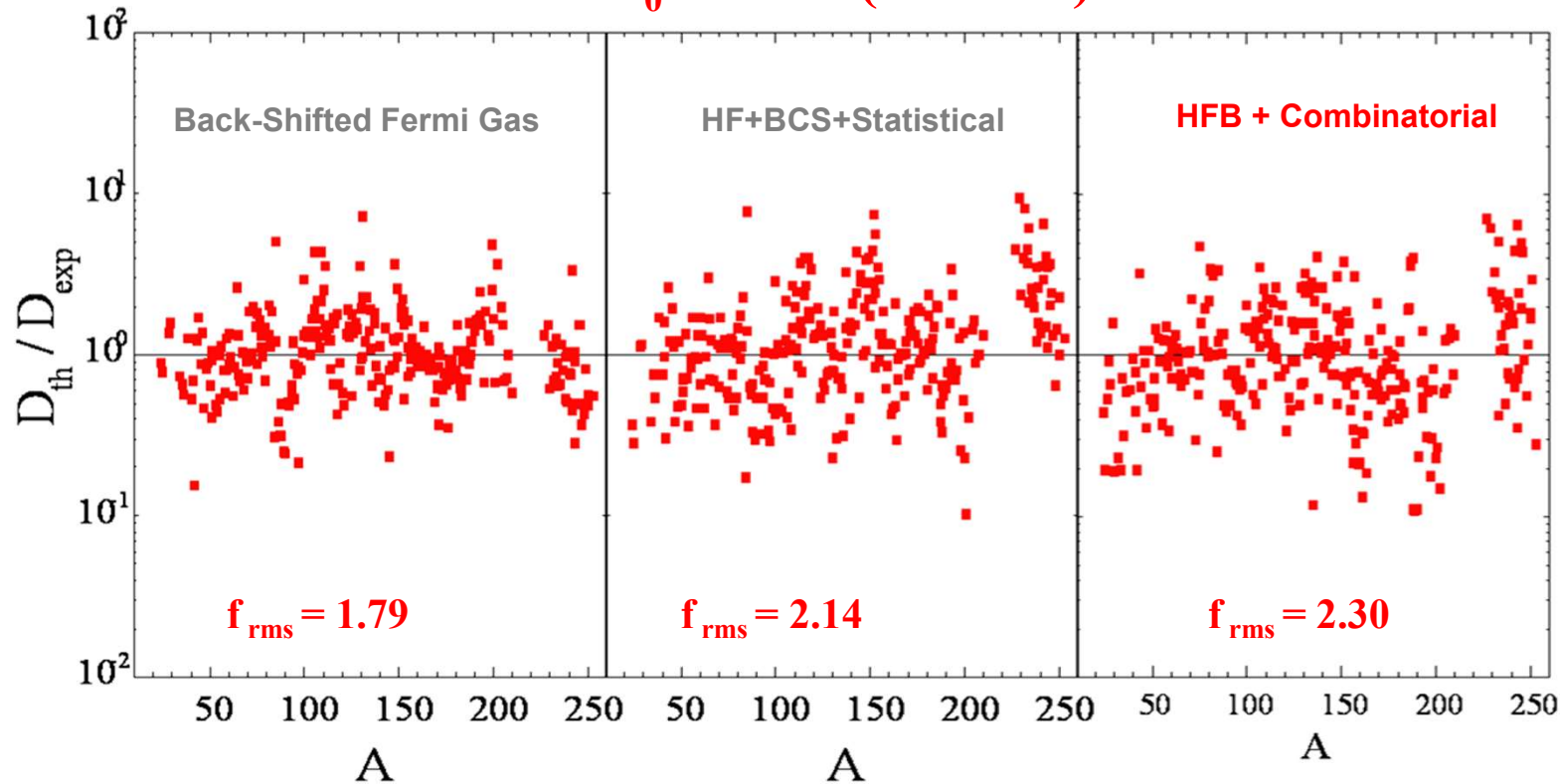
$$f_{\text{rms}} = \exp \left[ \frac{1}{N_e} \sum_{i=1}^{N_e} \ln^2 \frac{D_{\text{th}}^i}{D_{\text{exp}}^i} \right]^{1/2}$$



# Level densities : combinatorial approach +temperature

$D_0$  values ( s-waves)

Idmodel 6 in TALYS



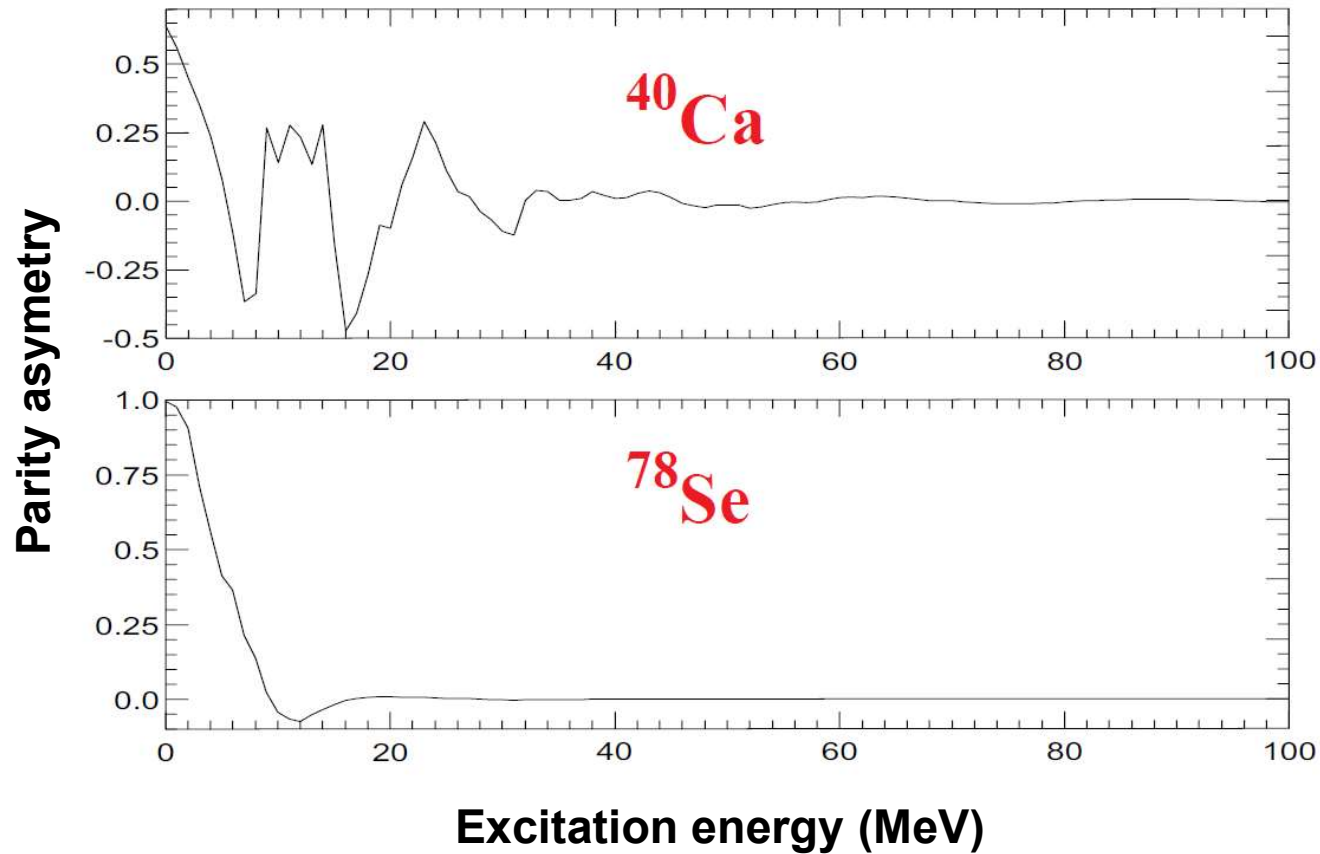
➔ Description similar to that obtained with other **global** approaches



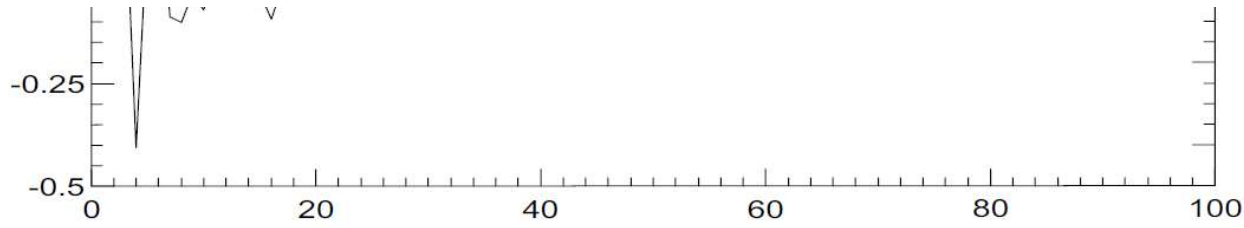
# Level densities : combinatorial approach +temperature



Idmodel 6 in TALYS



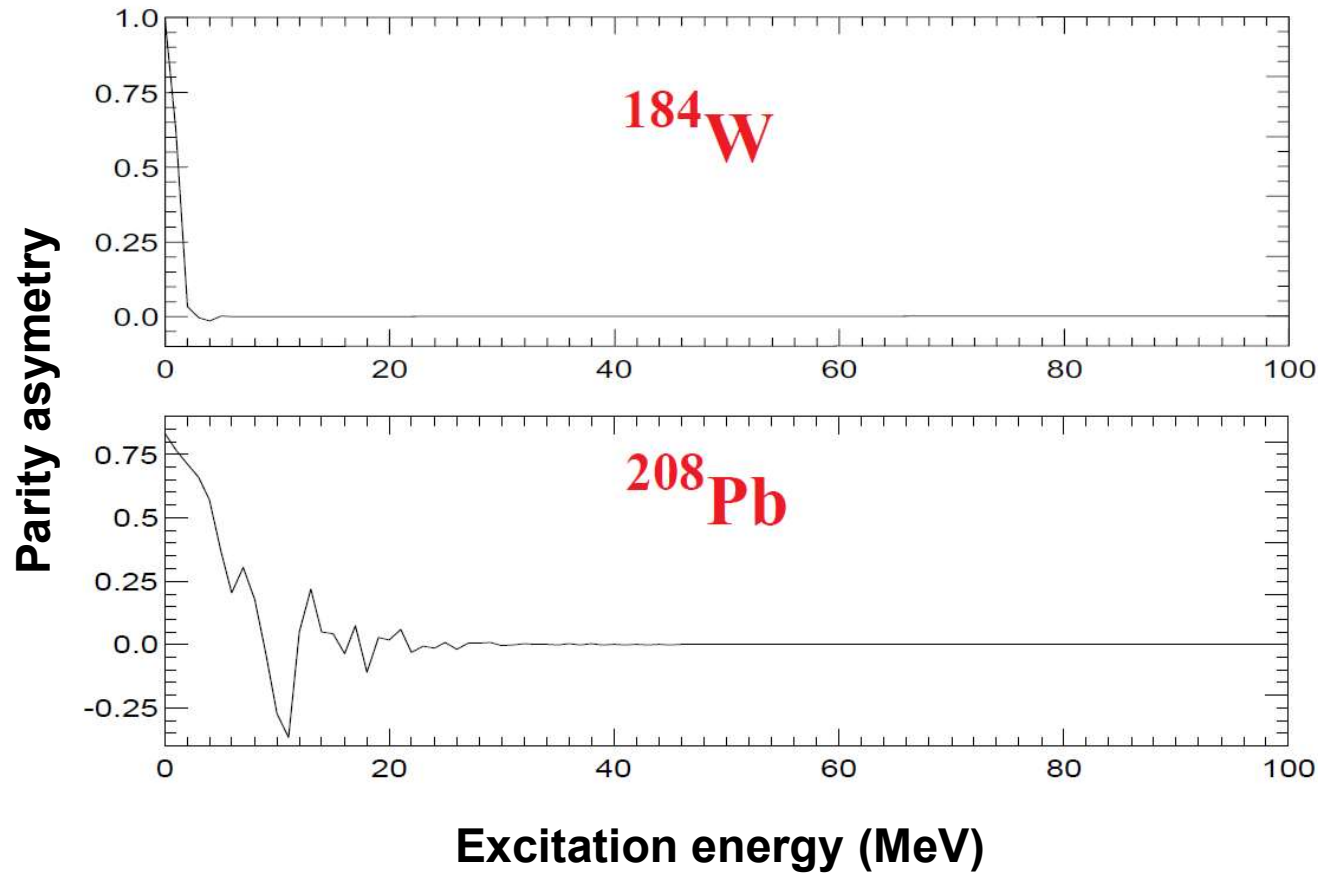
**Level d**



**rature**



**ldmodel 6 in TALYS**





# Level densities

## - Why and where do we need them ?

- Why ?
- Where ?

## - Particle-hole level densities for pre-equilibrium

- The equidistant spacing model
- Beyond the ESM

## - Total level densities

- Qualitative features
- Quantitative analysis with analytical approaches
- Shell Model Monte Carlo approach
- HFB+BCS Statistical approach
- Combinatorial approach

## - Impacts on cross sections

- Parity non equipartition
- Non-Gaussian spin distribution
- Governing competition
- Tabulated data adjustment



# Level densities

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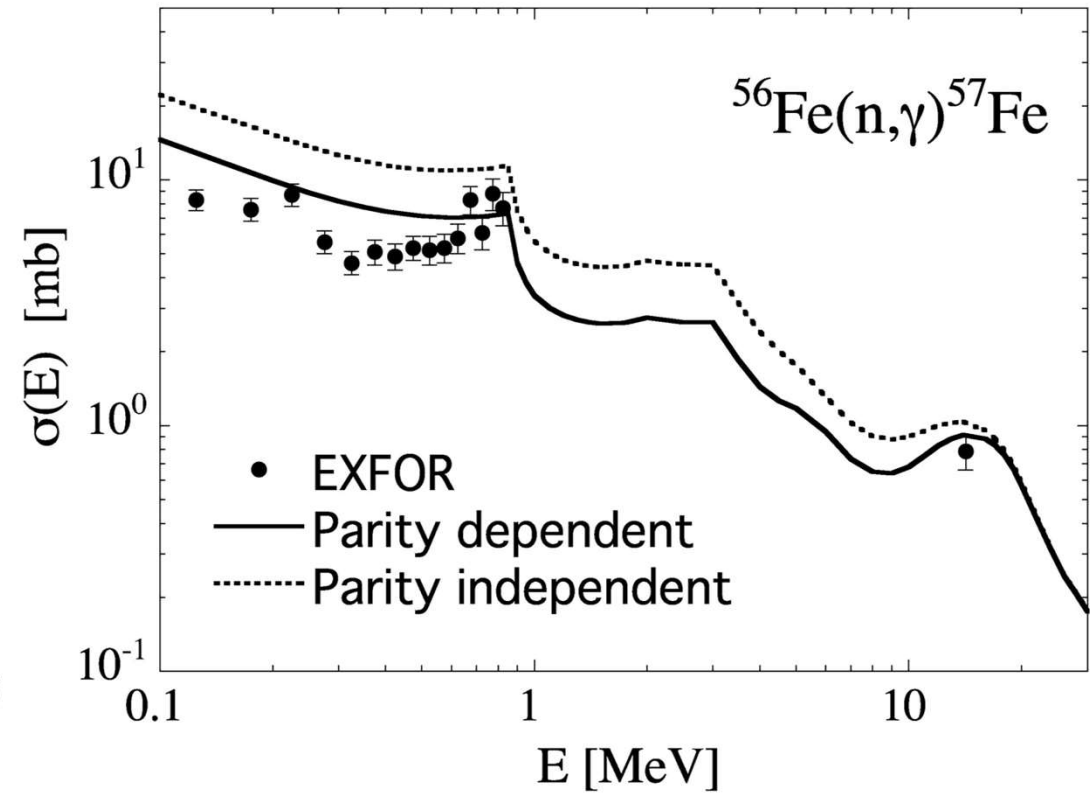
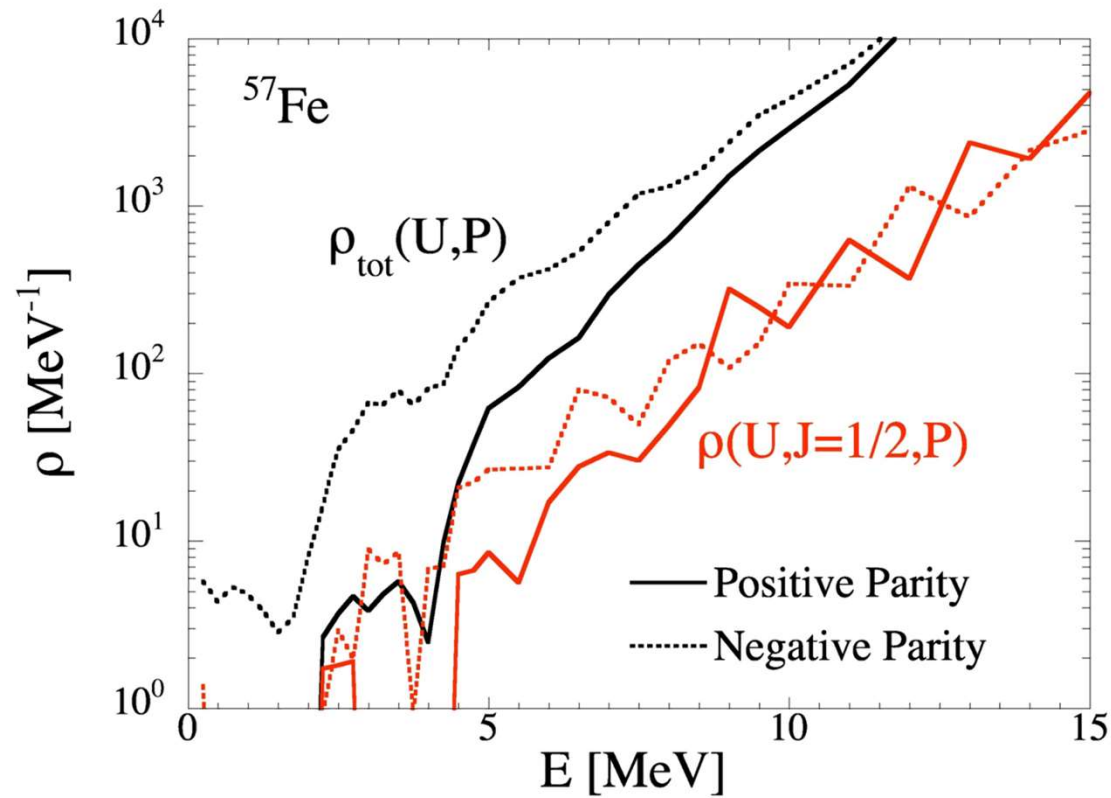
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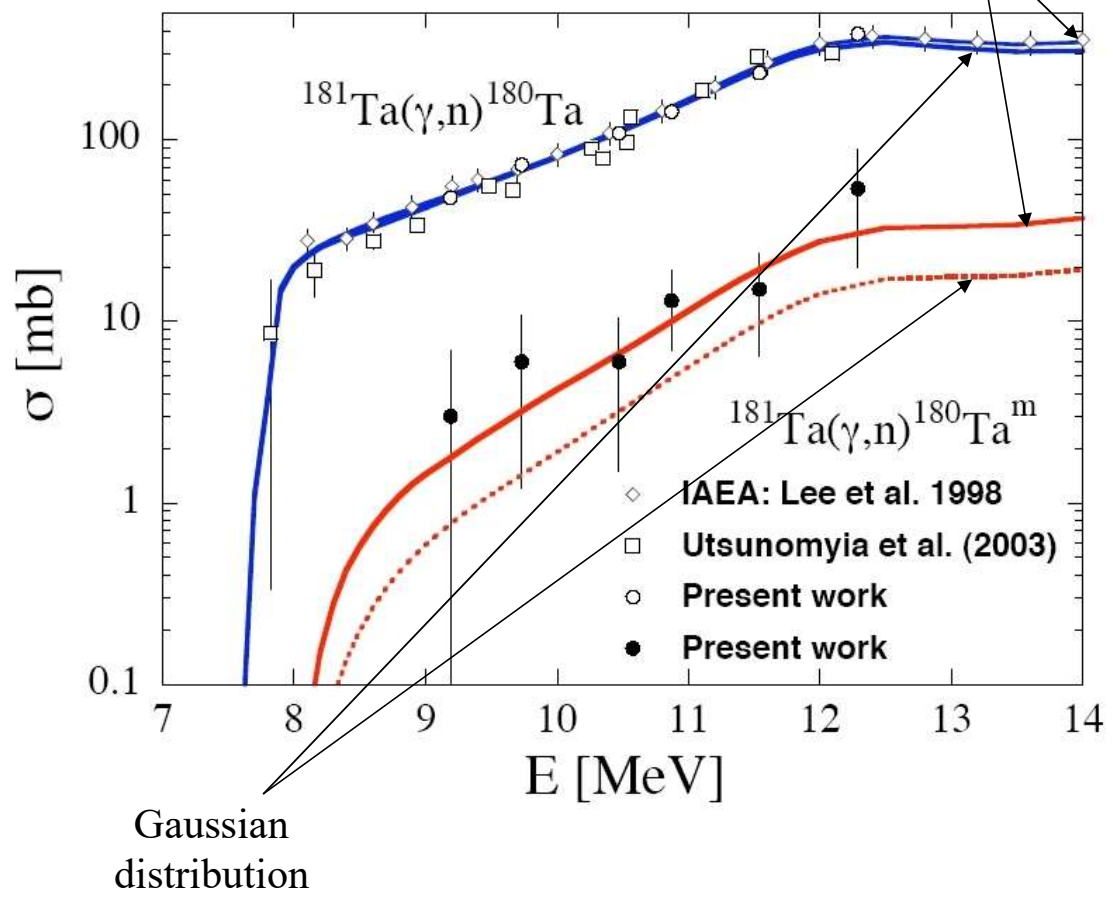
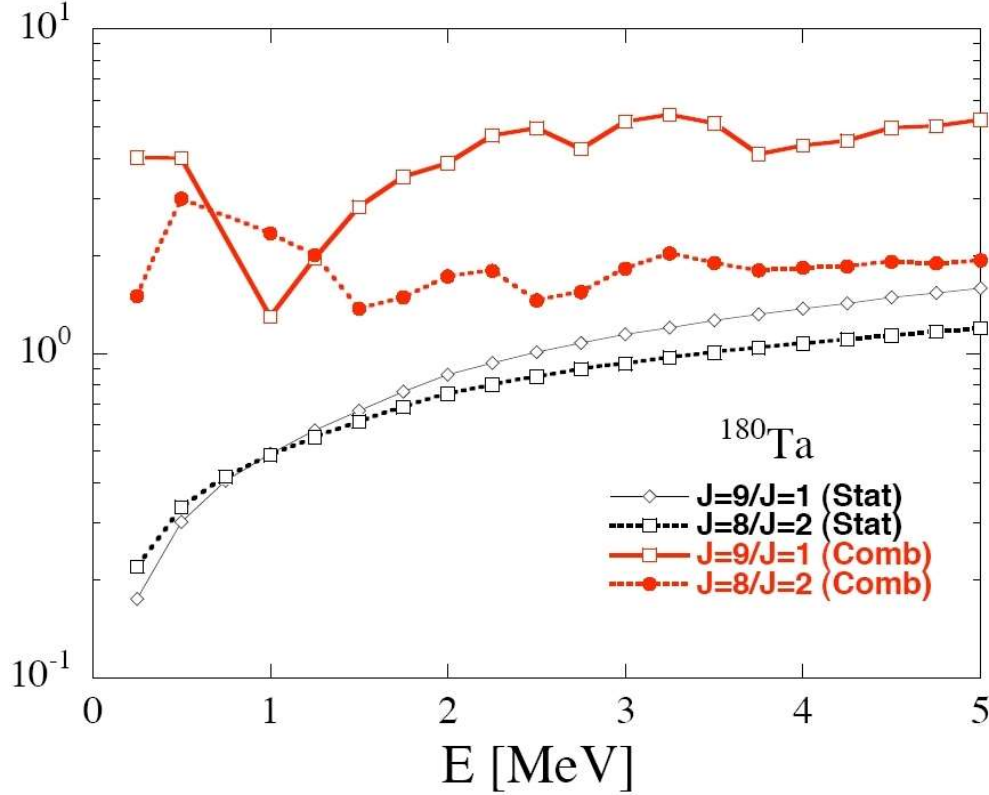
- Parity non equipartition
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# Level densities : parity non-equipartition



# Level densities : non-Gaussian spin distribution

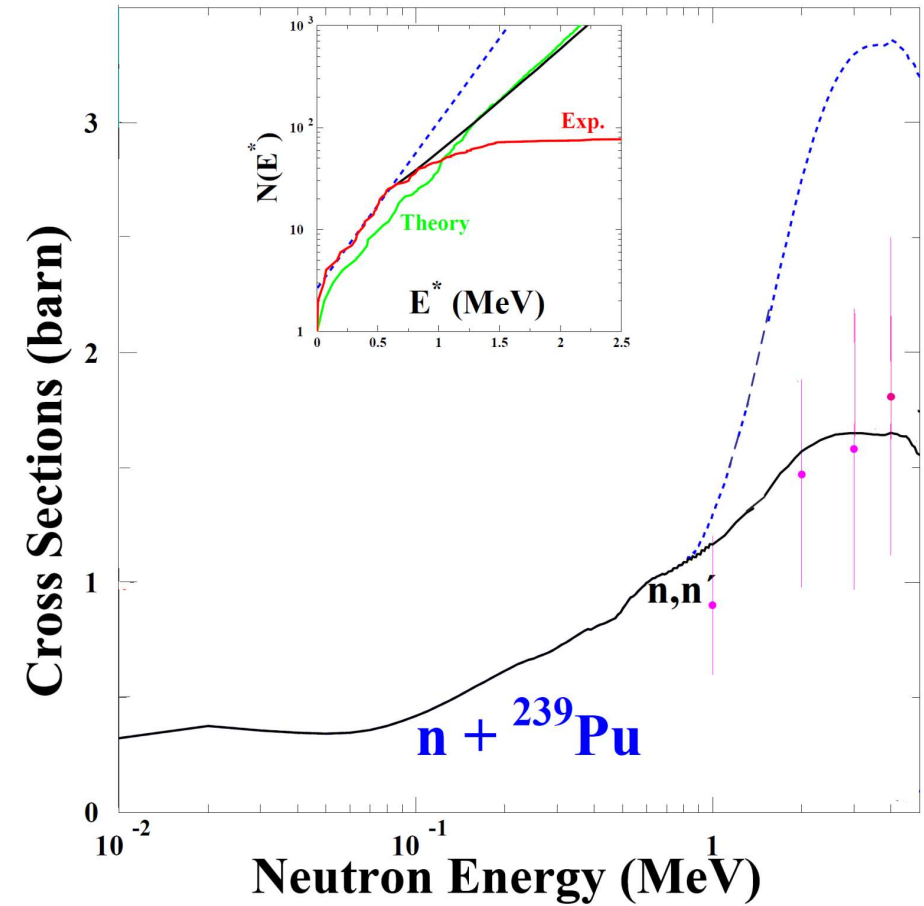
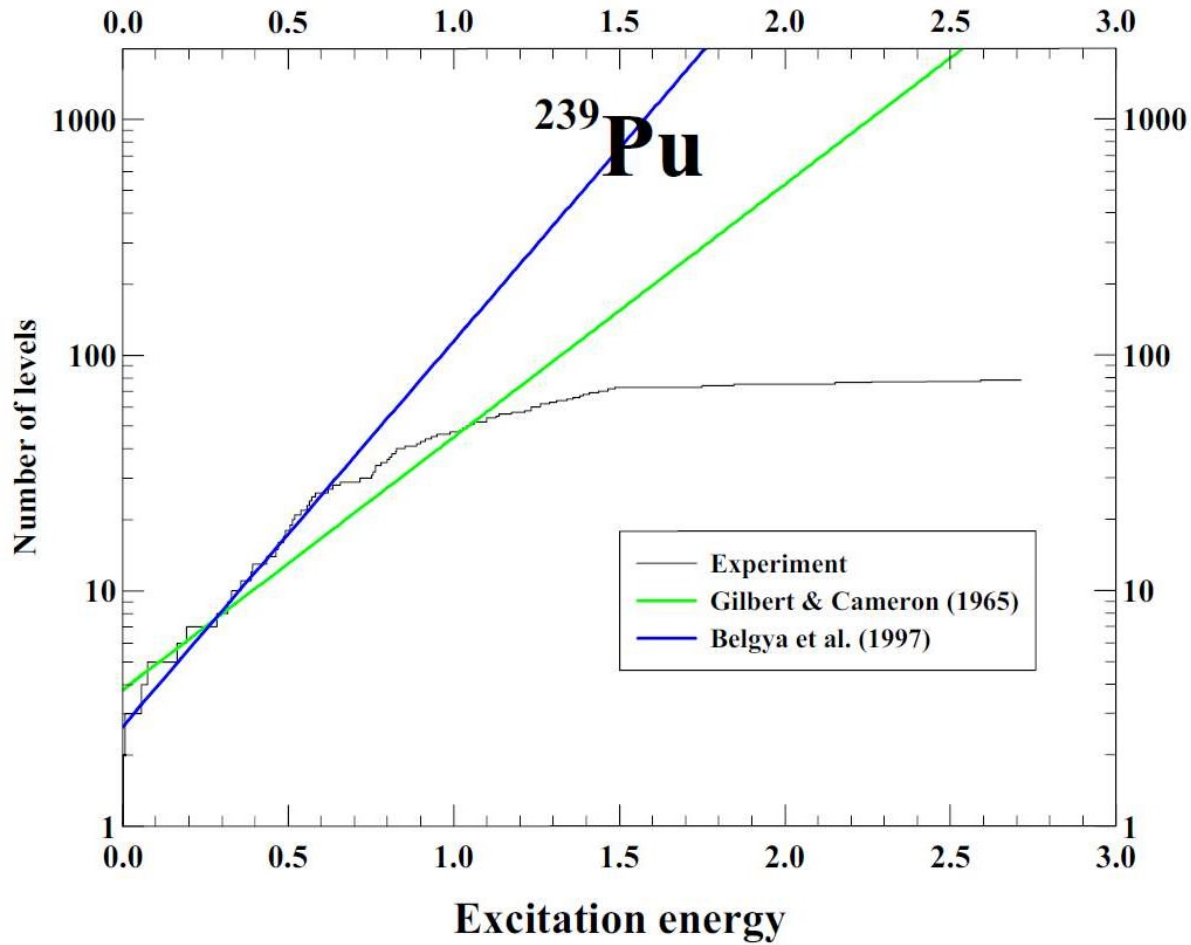


➔ Non-statistical feature imply significant deviations from the usual gaussian spin dependence which have significant impact on isomeric production

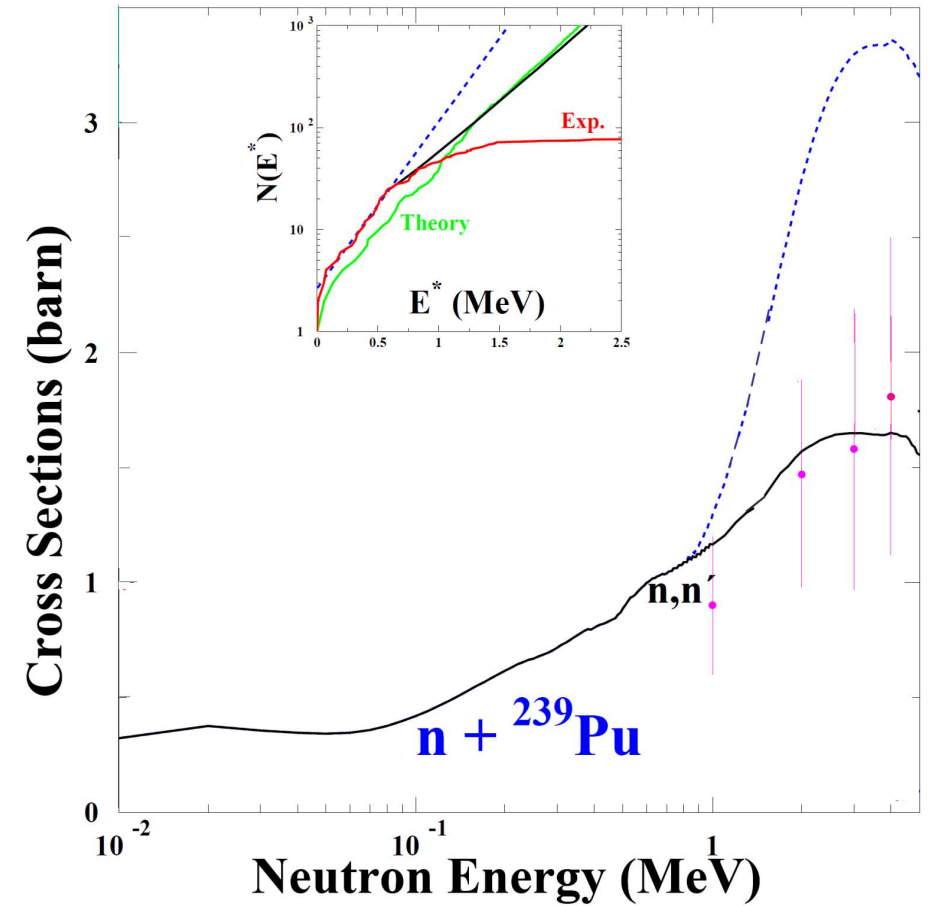
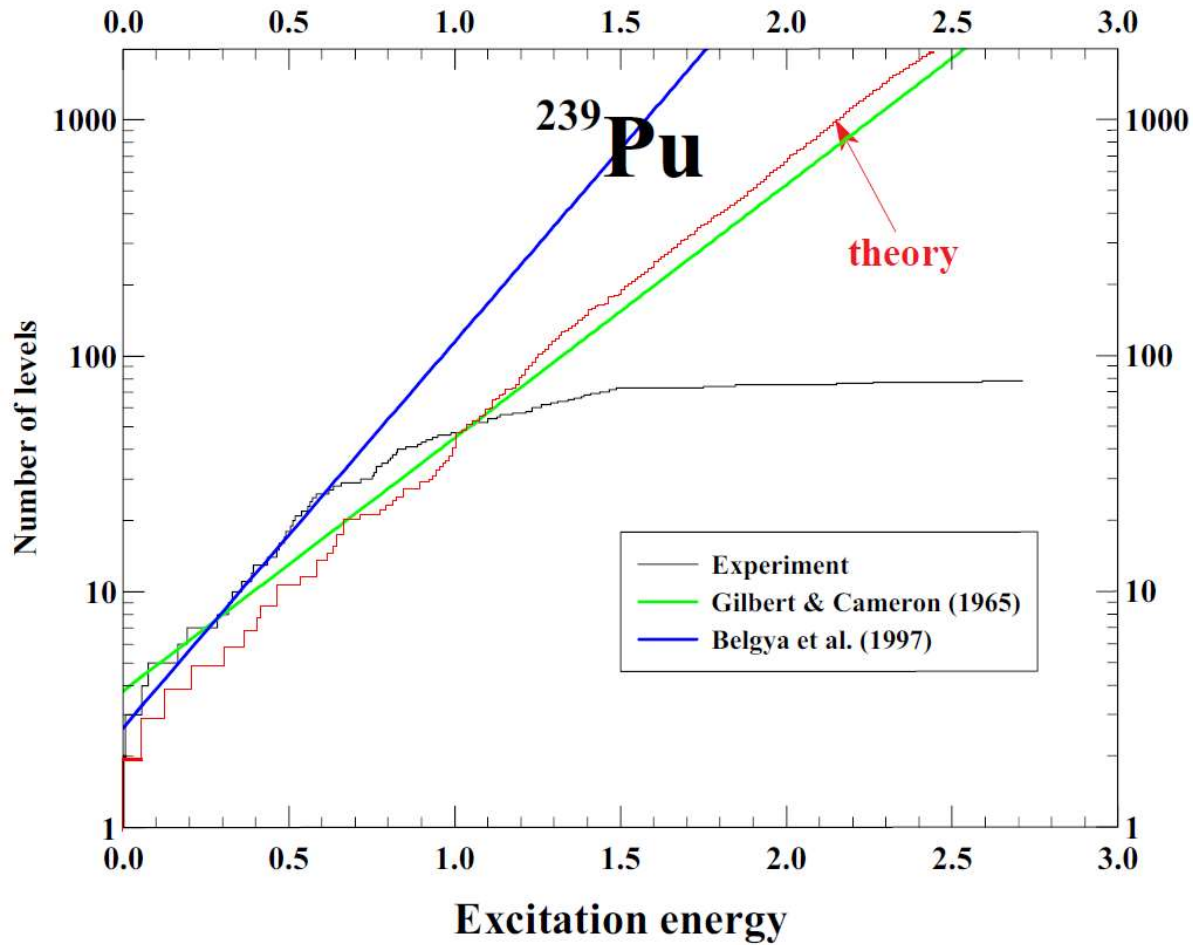




# Level densities : govern competition



# Level densities : govern competition

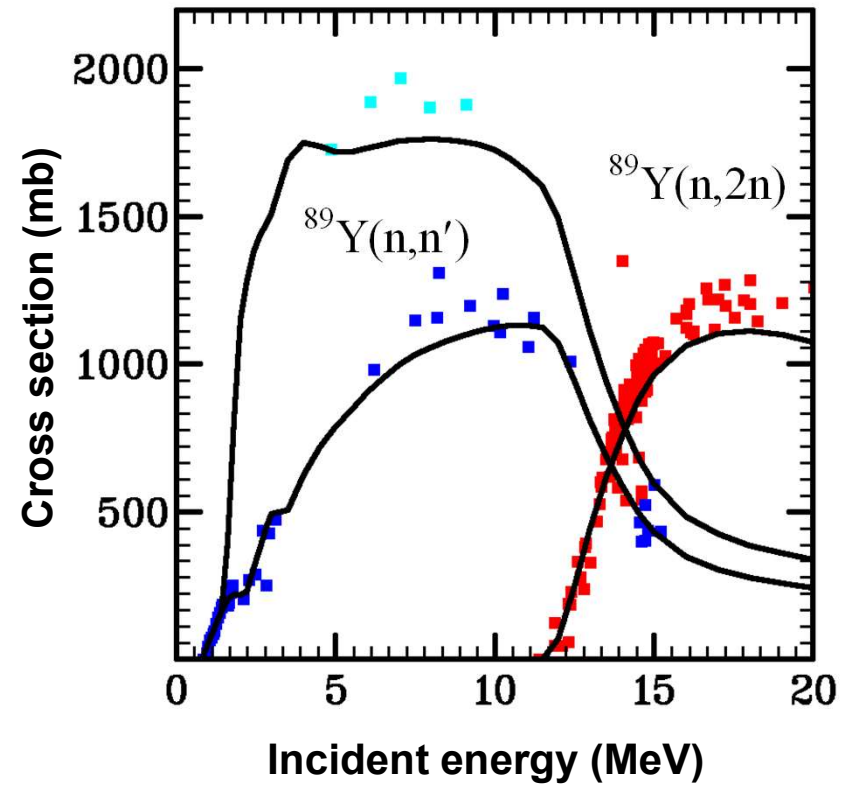
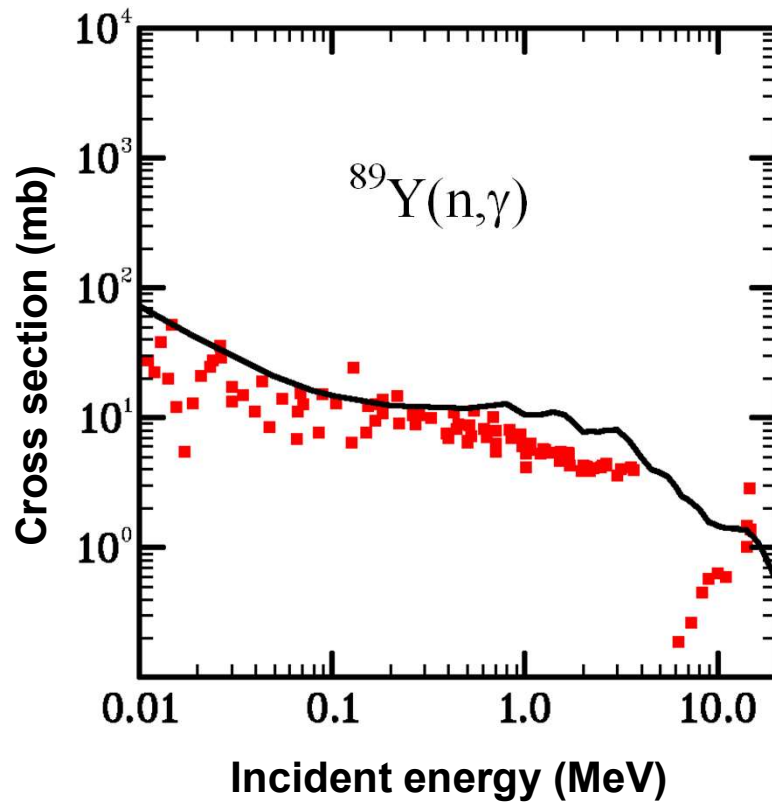




# Level densities : table adjustment



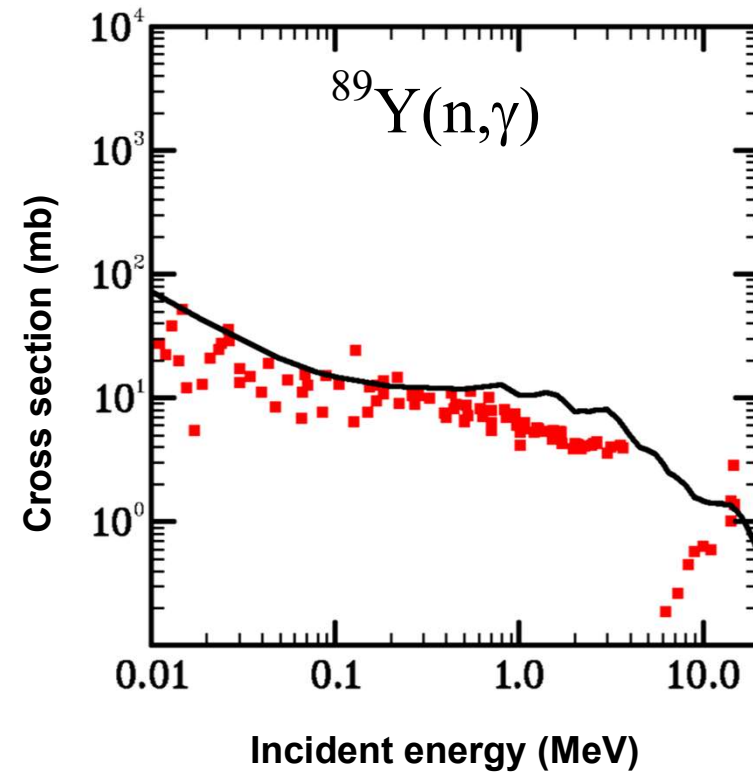
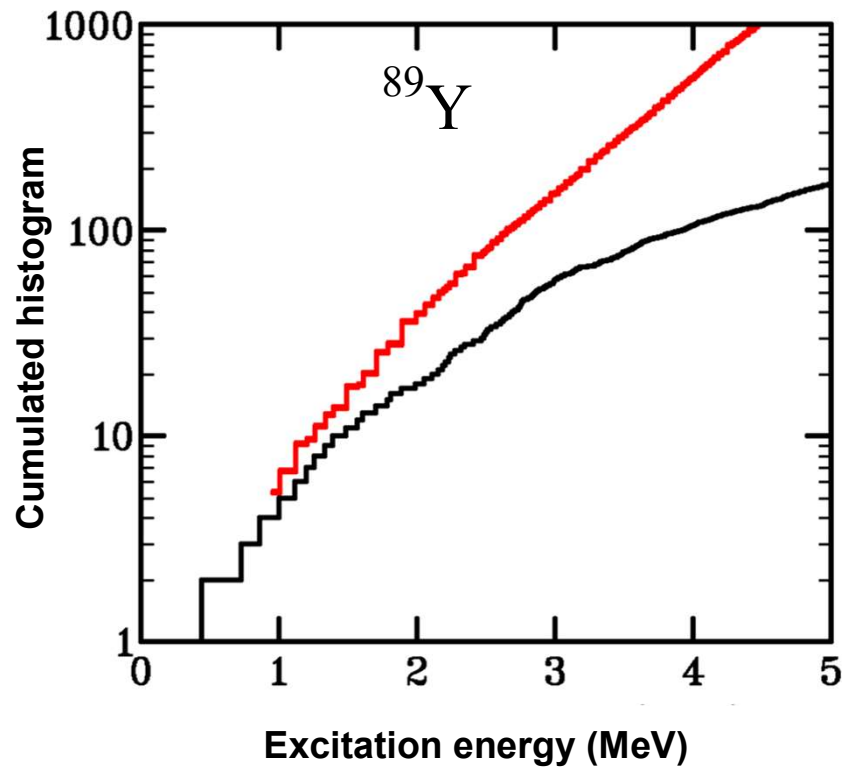
$$\rho_{\text{renorm}}(\mathbf{U}) = e^{\alpha \sqrt{(\mathbf{U} - \delta)}} \rho_{\text{global}}(\mathbf{U} - \delta)$$





# Level densities : table adjustment

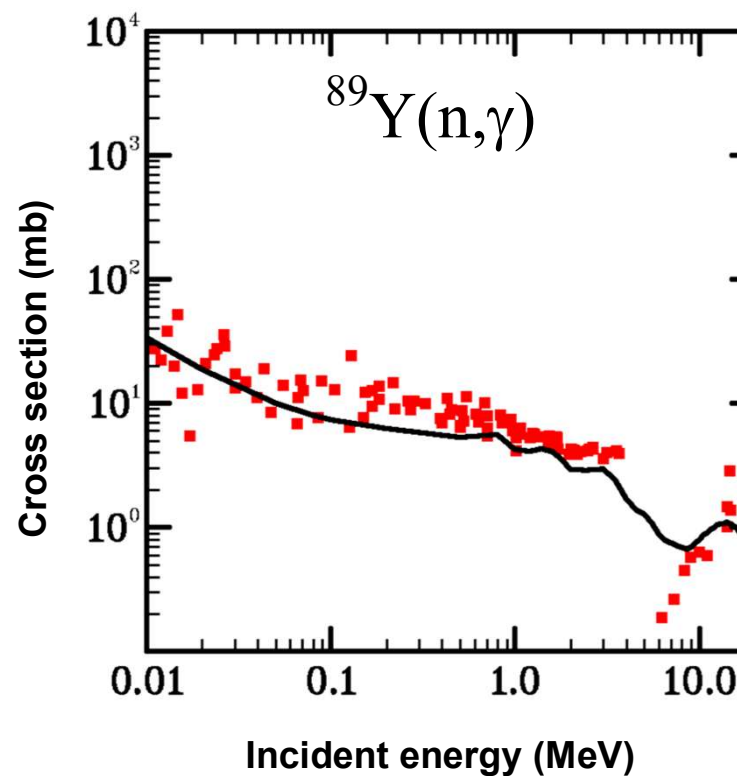
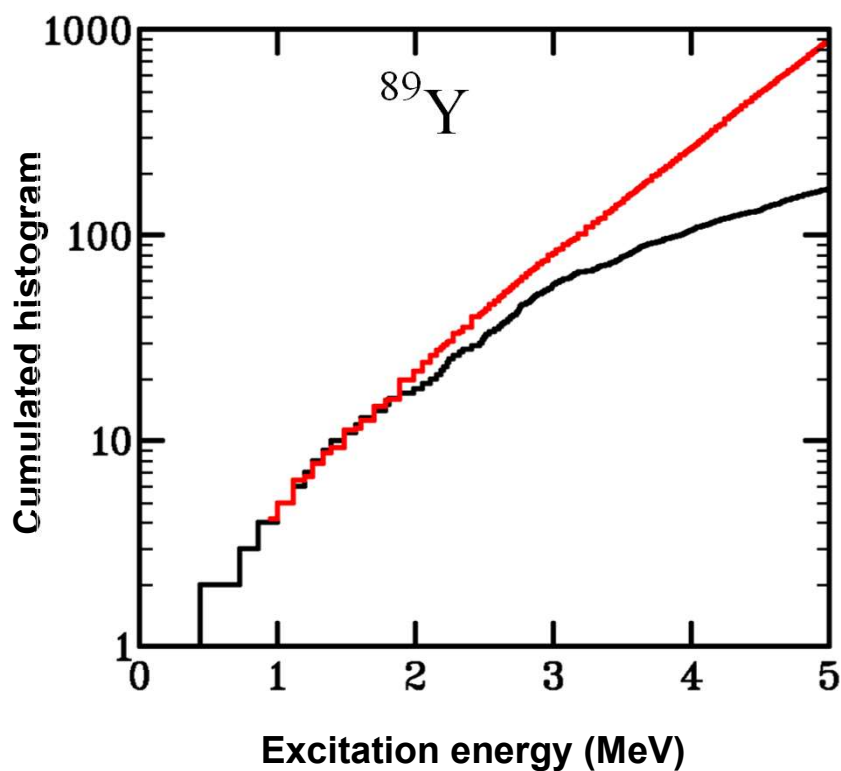
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# Level densities : table adjustment

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# Level densities : summary



International Atomic Energy Agency  
**Nuclear Data Services**  
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Databases > EXFOR | ENDF | CINDA | IBANDL | Medical | PGAA | NGAtlas | RIPL | FENDL | IRDF-2002 | IRDF

## Reference Input Parameter Library (RIPL-3)

R. Capote, M. Herman, P. Oblozinsky, P.G. Young, S. Goriely, T. Belgia, A.V. Ignatyuk, A.J. Koning, S. Hilaire, V.A. Plujko, M. Avrigeanu, O. Bersillon, M.B. Chadwick, T. Fukahori, Zhigang Ge, Yinlu Han, S. Kailas, J. Kopecky, V.M. Maslov, G. Reffo, M. Sin, E.Sh. Soukhovitskii and P. Talou

*Nuclear Data Sheets - Volume 110, Issue 12, December 2009, Pages 3107-3214*

RIPL discrete levels database should be corrected for +X... levels, new release soon.

Introduction | MASSES | LEVELS | RESONANCES | OPTICAL | **DENSITIES** | GAMMA | FISSION | CODES | Contacts

### Introduction

We describe the physics and data included in the Reference Input Parameter Library, which is devoted to input parameters needed in calculations of nuclear reactions and nuclear data evaluations. Advanced modelling codes require substantial numerical input, therefore the International Atomic Energy Agency (IAEA) has worked extensively since 1993 on a library of validated nuclear-model input parameters, referred to as the Reference Input Parameter Library (RIPL). A final RIPL coordinated research project (RIPL-3) was brought to a successful conclusion in December 2008, after 15 years of challenging work carried out through three consecutive IAEA projects. The RIPL-3 library was released in January 2009, and is available on the Web through <http://www-nds.iaea.org/RIPL-3/>. This work and the resulting database are extremely important to theoreticians involved in the development and use of nuclear reaction modelling (ALICE, EMPIRE, GNASH, UNF, TALYS) both for theoretical research and nuclear data evaluations.

The numerical data and computer codes included in RIPL-3 are arranged in seven segments: **MASSES** contains ground-state properties of nuclei for about 9000 nuclei, including three theoretical predictions of masses and the evaluated experimental masses of Audi *et al.* (2003). **DISCRETE LEVELS** contains 117 datasets (one for each element) with all known level schemes, electromagnetic and  $\gamma$ -ray decay probabilities available from ENSDF in October 2007. **NEUTRON RESONANCES** contains average resonance parameters prepared on the basis of the evaluations performed by Ignatyuk and Mughabghab. **OPTICAL MODEL** contains 495 sets of phenomenological optical model parameters defined in a wide energy range. When there are insufficient experimental data, the evaluator has to resort to either global parameterizations or microscopic approaches. Radial density distributions to be used as input for microscopic calculations are stored in the MASSES segment. **LEVEL DENSITIES** contains phenomenological parameterizations based on the modified Fermi gas and superfluid models and microscopic calculations which are based on a realistic microscopic single-particle level scheme. Partial level densities formulae are also recommended. All tabulated total level densities are consistent with both the recommended average neutron resonance parameters and discrete levels. **GAMMA** contains parameters that quantify giant resonances, experimental gamma-ray strength functions and methods for calculating gamma emission in statistical model codes. The experimental GDR parameters are represented by Lorentzian fits to the photo-absorption cross sections for 102 nuclides ranging from  $^{51}\text{V}$  to  $^{239}\text{Pu}$ . **FISSION** includes global prescriptions for fission barriers and nuclear level densities at fission saddle points based on microscopic HFB calculations constrained by experimental fission cross sections.

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Last Updated: 08/22/2013 12:00:23

# Level densities : summary

The screenshot shows the International Atomic Energy Agency (IAEA) Nuclear Data Services website. The main heading is "Reference Input Parameter Library (RIPL-3)". Below it, the authors are listed: R. Capote, M. Herman, P. Oblozinsky, P.G. Young, S. Goriely, T. Belgia, A.V. Ignatyuk, A.J. Koning, S. Hilaire, V.A. Plujko, M. Avrigeanu, O. Bersillon, M.B. Chadwick, T. Fukahori, Zhigang Ge, Yinlu Han, S. Kailas, J. Kopecky, V.M. Maslov, G. Reffo, M. Sin, E.Sh. Soukhovitskii and P. Talou. The page is dated "Nuclear Data Sheets - Volume 110, Issue 12, December 2009, Pages 3107-3214". A navigation bar includes "Introduction", "MASSES", "LEVELS", "RESONANCES", "OPTICAL", "DENSITIES", "GAMMA", "FISSION", "CODES", and "Contacts". The "DENSITIES" tab is highlighted in red. The introduction text describes the physics and data included in the RIPL-3 library, which is devoted to input parameters needed in calculations of nuclear reactions and nuclear data evaluations. It mentions that the library was released in January 2009 and is available on the web through <http://www.nds.iaea.org/RIPL-3/>. The text also lists the seven segments of the library: MASSES, DISCRETE LEVELS, NEUTRON RESONANCES, OPTICAL MODEL, LEVEL DENSITIES, GAMMA, and FISSION.

**Nuclear level densities (formulae, tables, codes)**

- spin-, parity- dependent level densities fitted to  $D_0$
- single particle level schemes
- p-h level density tables

# Level densities : summary



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# **3. GAMMA-RAY STRENGTHS**



# Gamma-ray strengths

## - Qualitative features

## - Analytical approaches

## - Microscopic approaches

- HFBCS-RPA
- HFB+QRPA
- Shell Model

## - Impacts on cross sections

- Normalizations
- Exotic nuclei
- Hot topics





# Gamma-ray strengths

## - Qualitative features

### - Analytical approaches

### - Microscopic approaches

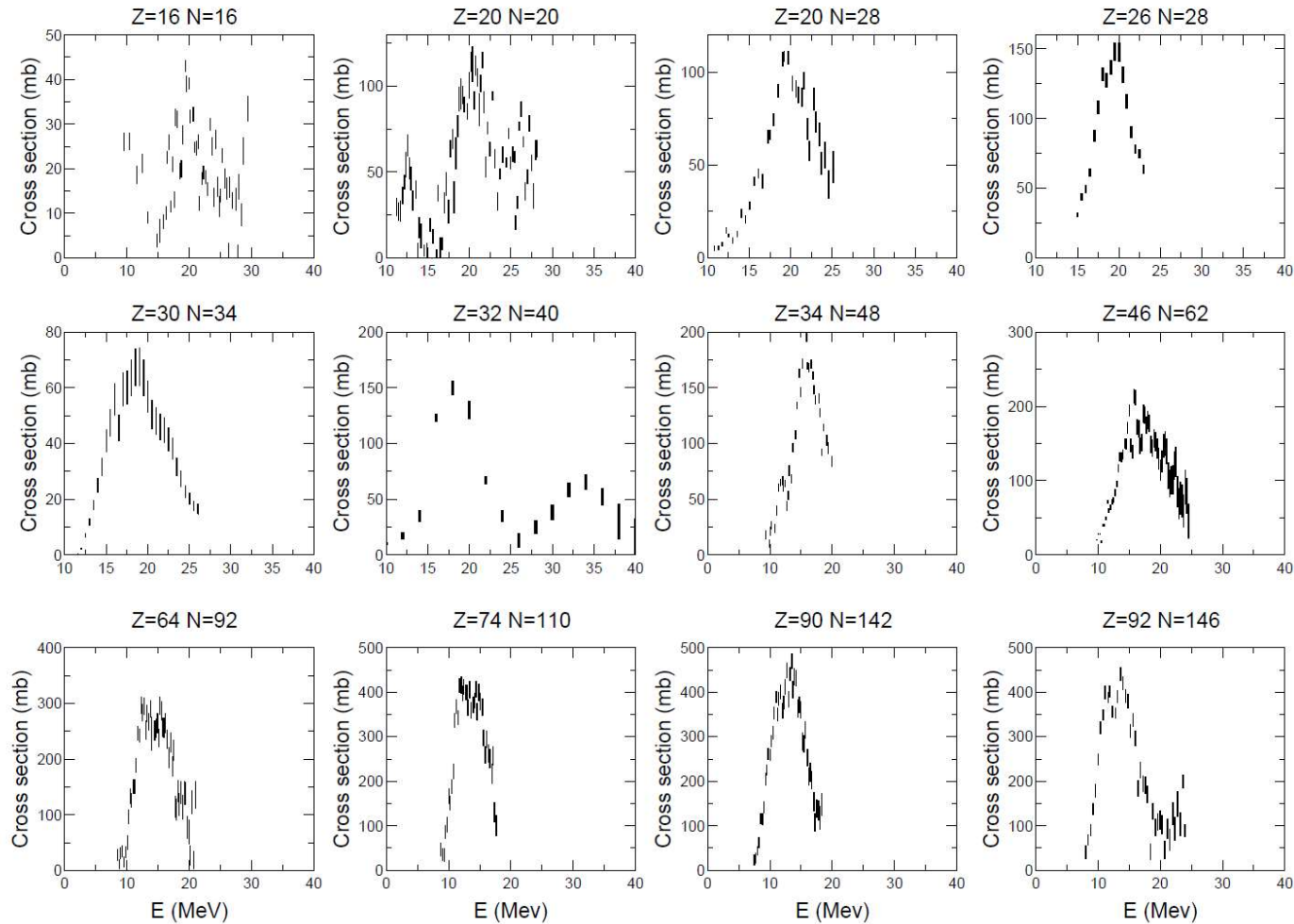
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### - Impacts on cross sections

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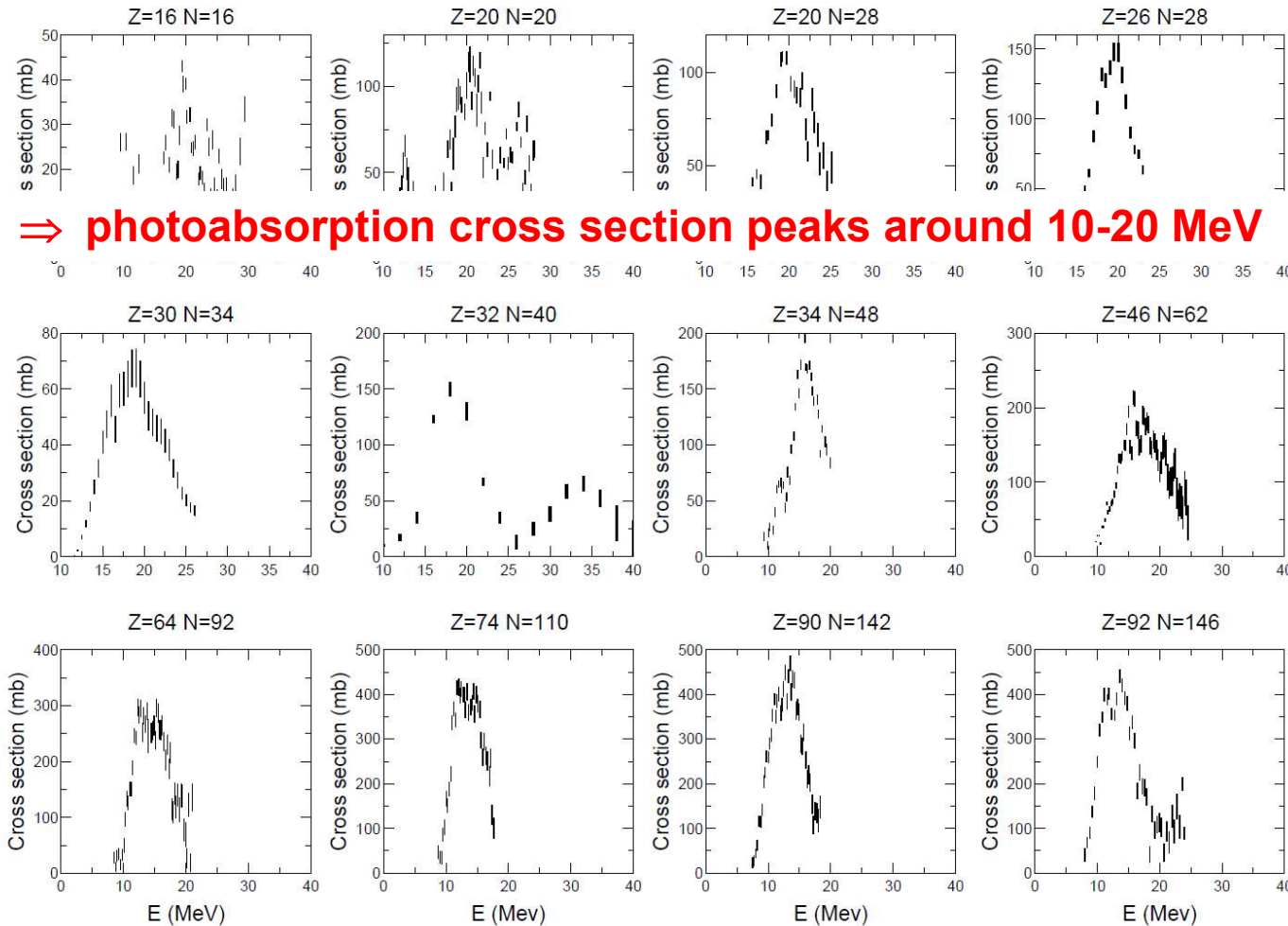


# Gamma-ray strengths : qualitative aspects from photoabsorption



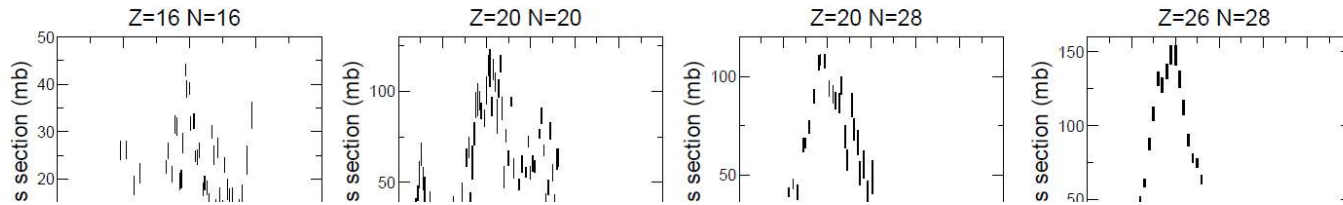


# Gamma-ray strengths : qualitative aspects from photoabsorption



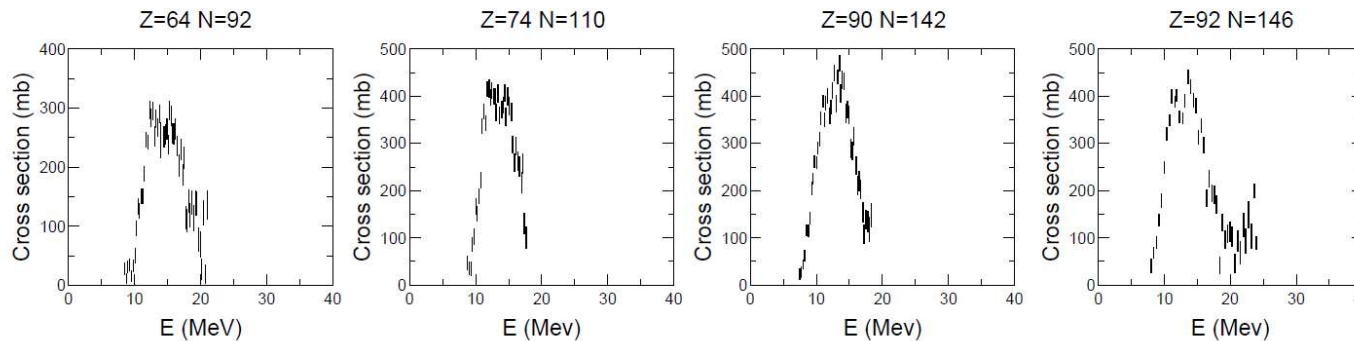
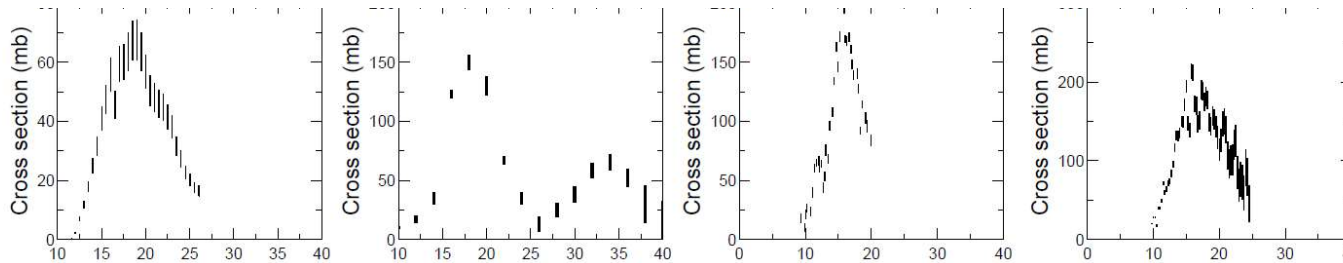


# Gamma-ray strengths : qualitative aspects from photoabsorption



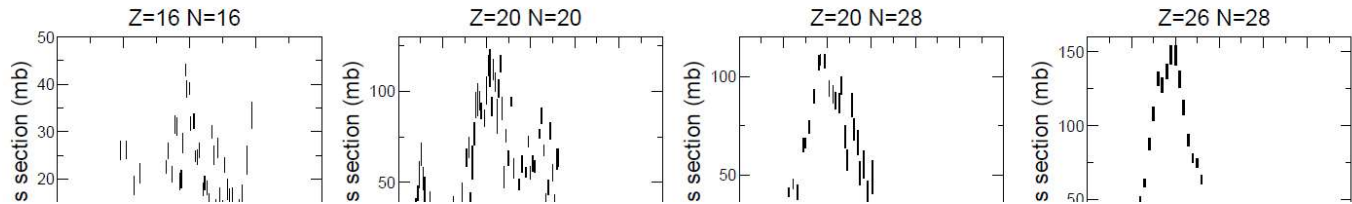
⇒ photoabsorption cross section peaks around 10-20 MeV

⇒ peak energy decreases with mass





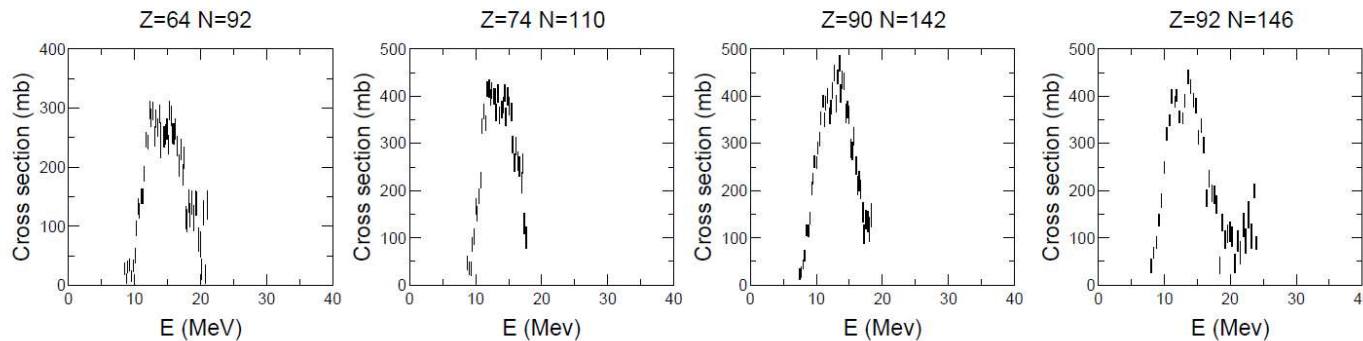
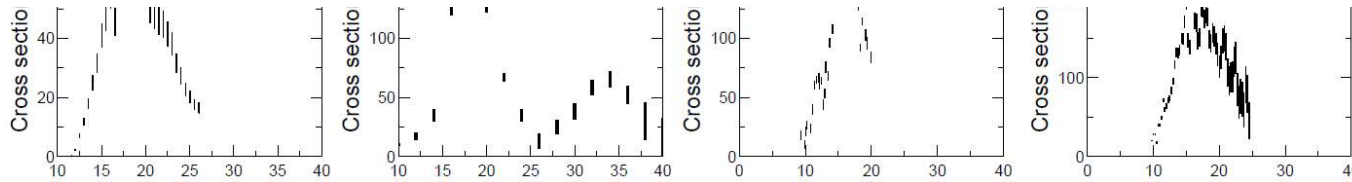
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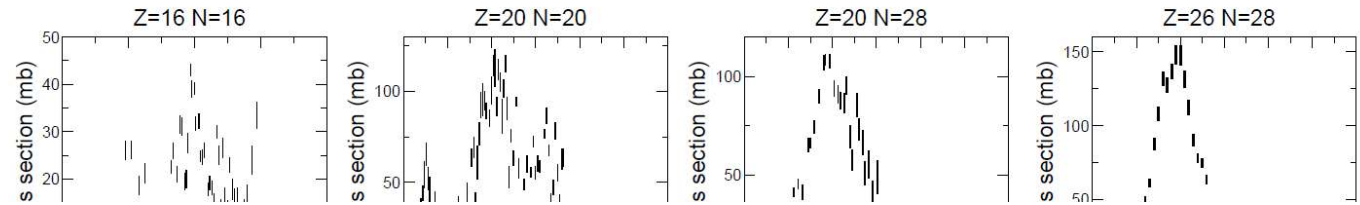
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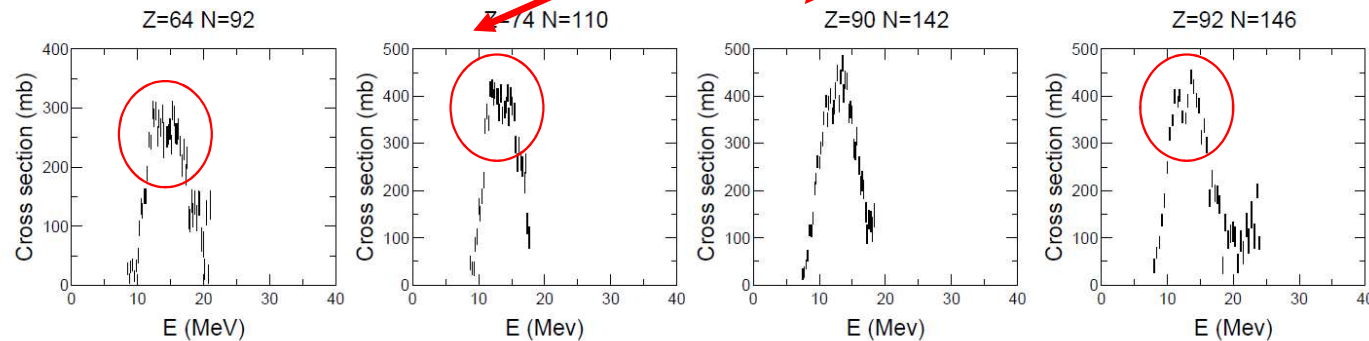
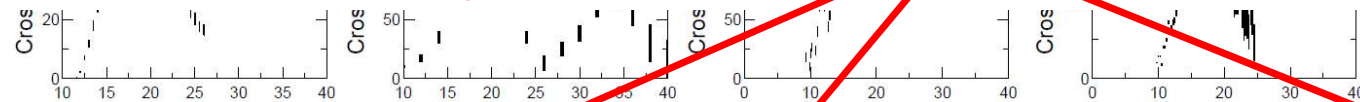


⇒ photoabsorption cross section peaks around 10-20 MeV

⇒ peak energy decreases with mass

⇒ peak height increases with mass

⇒ two peaks usually appear for deformed nuclei





# Gamma-ray strengths : Brink-Axel hypothesis

Two types of strength functions :

- the « upward » related to photoabsorption

$$\vec{f}_{\text{XL}}(\epsilon_\gamma) = \frac{\epsilon_\gamma^{-2L+1} \langle \sigma_{\text{XL}}(\epsilon_\gamma) \rangle}{(\pi \hbar c)^2 (2L+1)}$$

- the « downward » related to g-decay

$$\overleftarrow{f}_{\text{XL}}(\epsilon_\gamma) = \epsilon_\gamma^{-(2L+1)} \frac{\langle \Gamma_{\text{XL}}(\epsilon_\gamma) \rangle}{D_l}$$

Spacing of states from which the decay occurs

$D_l$

## Standard Lorentzian (SLO)

[D.Brink, PhD Thesis(1955); P. Axel, PR 126(1962)]

$$\overleftarrow{f} = \overrightarrow{f} \sim \frac{E_\gamma \Gamma_r^2}{(E_\gamma^2 - E_r^2)^2 + E_\gamma \Gamma_r^2} \Rightarrow 0 \quad E_\gamma \rightarrow 0$$



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# Gamma-ray strengths : transmission coefficients

$$T^{k\lambda}(E, \varepsilon_\gamma) = 2\pi \int_E^{E+\Delta E} f(k, \lambda, \varepsilon_\gamma) \varepsilon_\gamma^{2\lambda+1} \rho(E-\varepsilon_\gamma) dE$$

$k$  : transition type (E or M)

$\lambda$  : transition multipolarity

$\varepsilon_\gamma$  : outgoing gamma energy

$f(k, \lambda, \varepsilon_\gamma)$  : gamma strength function (several models)

Decay selection rules  $S(k, \lambda, J_i^{\pi_i}, J_f^{\pi_f})$  from a level  $J_i^{\pi_i}$  to a level  $J_f^{\pi_f}$ :

For  $E\lambda$ :  $\pi_f = (-1)^\lambda \pi_i$

For  $M\lambda$ :  $\pi_f = (-1)^{\lambda+1} \pi_i$

$$|J_i - \lambda| \leq J_f \leq J_i + \lambda$$

(E1  $\approx$  10 – 100 M1)

(XL  $\approx$  10<sup>-3</sup> XL-1)

Renormalisation method for thermal neutrons

$$\langle T_\gamma \rangle = C \sum_{J_i, \pi_i} \sum_{k\lambda} \sum_{J_f, \pi_f} \int_0^{B_n} T^{k\lambda}(\varepsilon) \rho(B_n - \varepsilon, J_f, \pi_f) S(k, \lambda, J_i, \pi_i, J_f, \pi_f) d\varepsilon = 2\pi \langle \Gamma_\gamma \rangle \frac{1}{D_0}$$

experiment



# Gamma-ray strengths

## - Qualitative features

### - Analytical approaches

### - Microscopic approaches

- HFBCS-RPA
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- Shell Model

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# Gamma-ray strengths : analytical approaches

## Improved analytical expressions :

- 2 Lorentzians for deformed nuclei
- Account for low energy deviations from standard Lorentzians for E1
  - . Kadmenskij-Markushef-Furman model (1983)
    - ⇒ Enhanced Generalized Lorentzian model of Kopecky-Uhl (1990)
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  - ⇒ Modified Lorentzian model of Plujko et al. (2002)
  - ⇒ Simplified Modified Lorentzian model of Plujko et al. (2008)
- Update of SMLO in 2019
  - ⇒ Temperature dependence
  - ⇒ Adding extra-M1 strength at low energies
  - ⇒ guided by microscopic results



# Gamma-ray strengths : Brink-Axel & Kopecky-Uhl

## Brink-Axel (option 2 in TALYS)

$$f_{X\ell}(E_\gamma) = K_{X\ell} \frac{\sigma_{X\ell} E_\gamma \Gamma_{X\ell}^2}{(E_\gamma^2 - E_{X\ell}^2)^2 + E_\gamma^2 \Gamma_{X\ell}^2} \quad \text{with} \quad K_{X\ell} = \frac{1}{(2\ell + 1)\pi^2 \hbar^2 c^2}$$

## Kopecky-Uhl (for E1) (option 1 in TALYS)

$$f_{E1}(E_\gamma, T) = K_{E1} \left[ \frac{E_\gamma \tilde{\Gamma}_{E1}(E_\gamma)}{(E_\gamma^2 - E_{E1}^2)^2 + E_\gamma^2 \tilde{\Gamma}_{E1}(E_\gamma)^2} + \frac{0.7\Gamma_{E1}4\pi^2 T^2}{E_{E1}^3} \right] \sigma_{E1} \Gamma_{E1}$$

$$\text{with } \tilde{\Gamma}_{E1}(E_\gamma) = \Gamma_{E1} \frac{E_\gamma^2 + 4\pi^2 T^2}{E_{E1}^2} \quad \text{and} \quad T = \sqrt{\frac{E_n + S_n - \Delta - E_\gamma}{a(S_n)}}$$

- ⇒ Deformed nuclei : incoherent sum of two Lorentzians
- ⇒ Parameters taken from experimental fit of data (RIPL-III) for measured nuclei
- ⇒ From global systematics otherwise

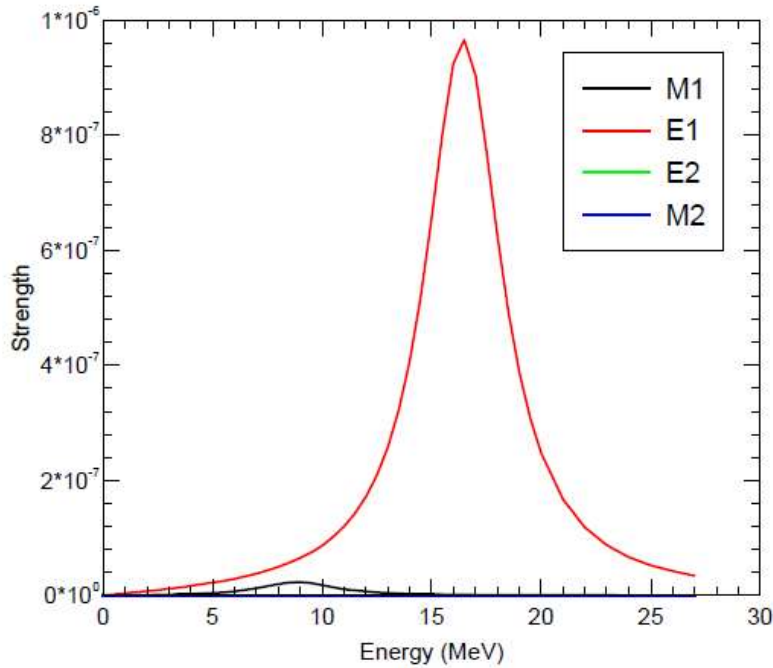
$$\sigma_{E1} = 1.2 \times 120NZ / (A\pi\Gamma_{E1}) \text{ mb}, \quad E_{E1} = 31.2A^{-1/3} + 20.6A^{-1/6} \text{ MeV}, \quad \Gamma_{E1} = 0.026E_{E1}^{1.91} \text{ MeV}.$$

$$\sigma_{E2} = 0.00014Z^2 E_{E2} / (A^{1/3}\Gamma_{E2}) \text{ mb}, \quad E_{E2} = 63.A^{-1/3} \text{ MeV}, \quad \Gamma_{E2} = 6.11 - 0.012A \text{ MeV}.$$

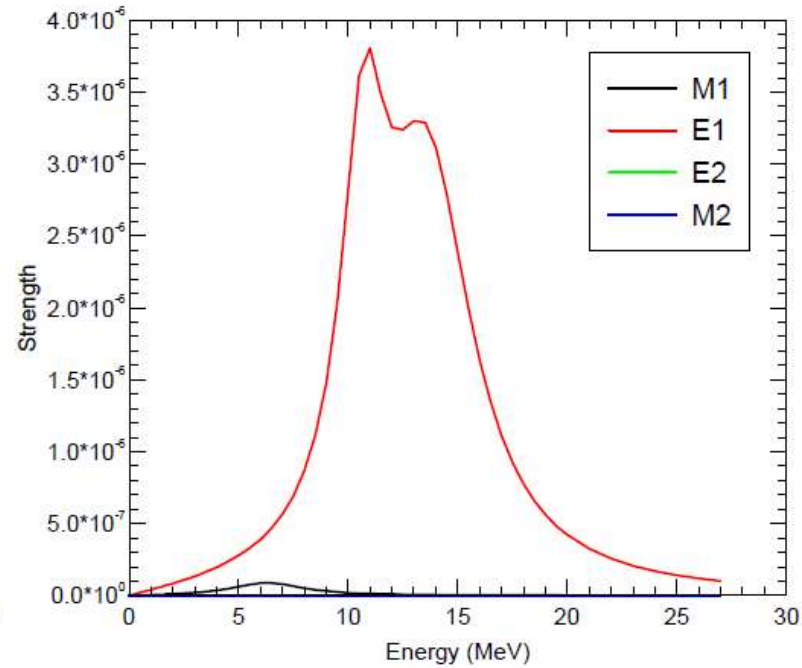


# Gamma-ray strengths : Brink-Axel

$^{90}\text{Zr}$  (spherical)



$^{238}\text{U}$  (deformed)

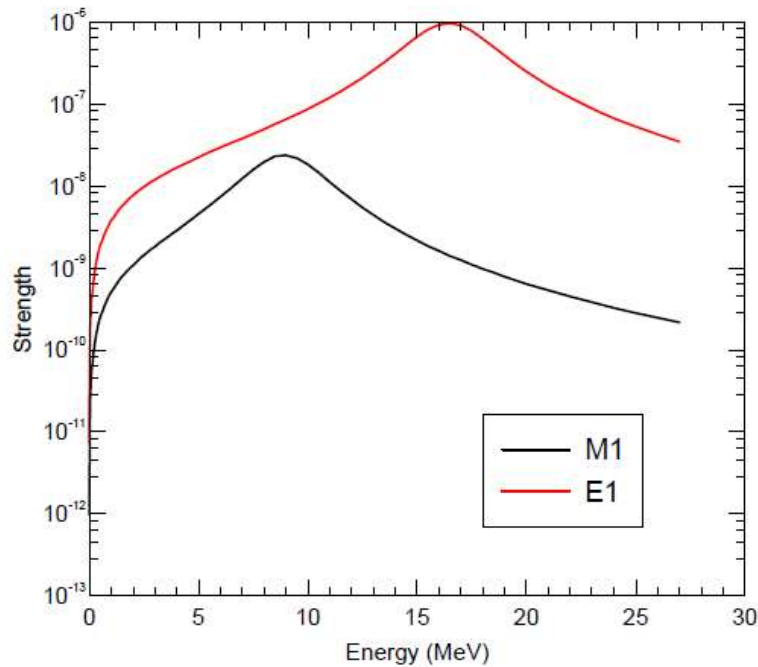


- ⇒ Deformed nuclei : two Lorentzians = two peaks
- ⇒ Lorentzian centroid energy decreasing with A
- ⇒ M1 much weaker than E1 ⇒ **log scale**

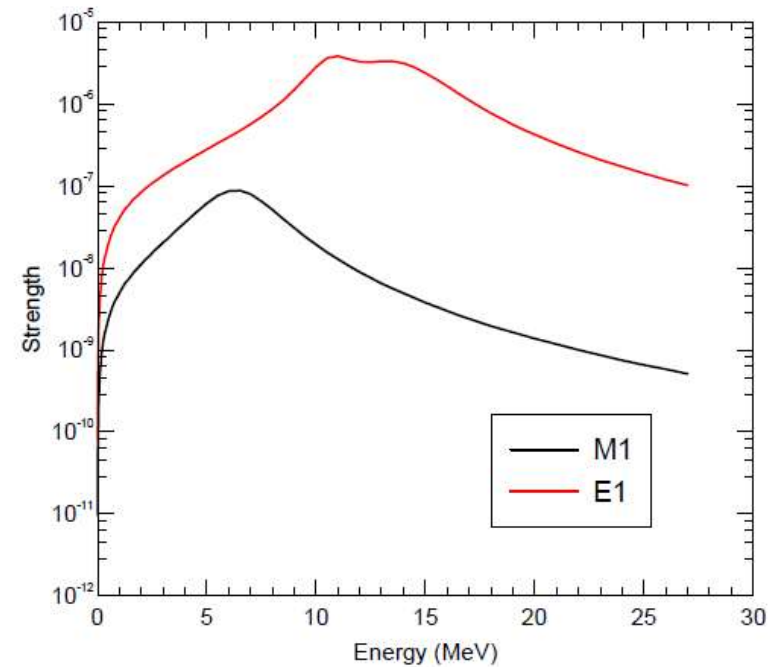


# Gamma-ray strengths : Brink-Axel in log scale

$^{90}\text{Zr}$  (spherical)



$^{238}\text{U}$  (deformed)

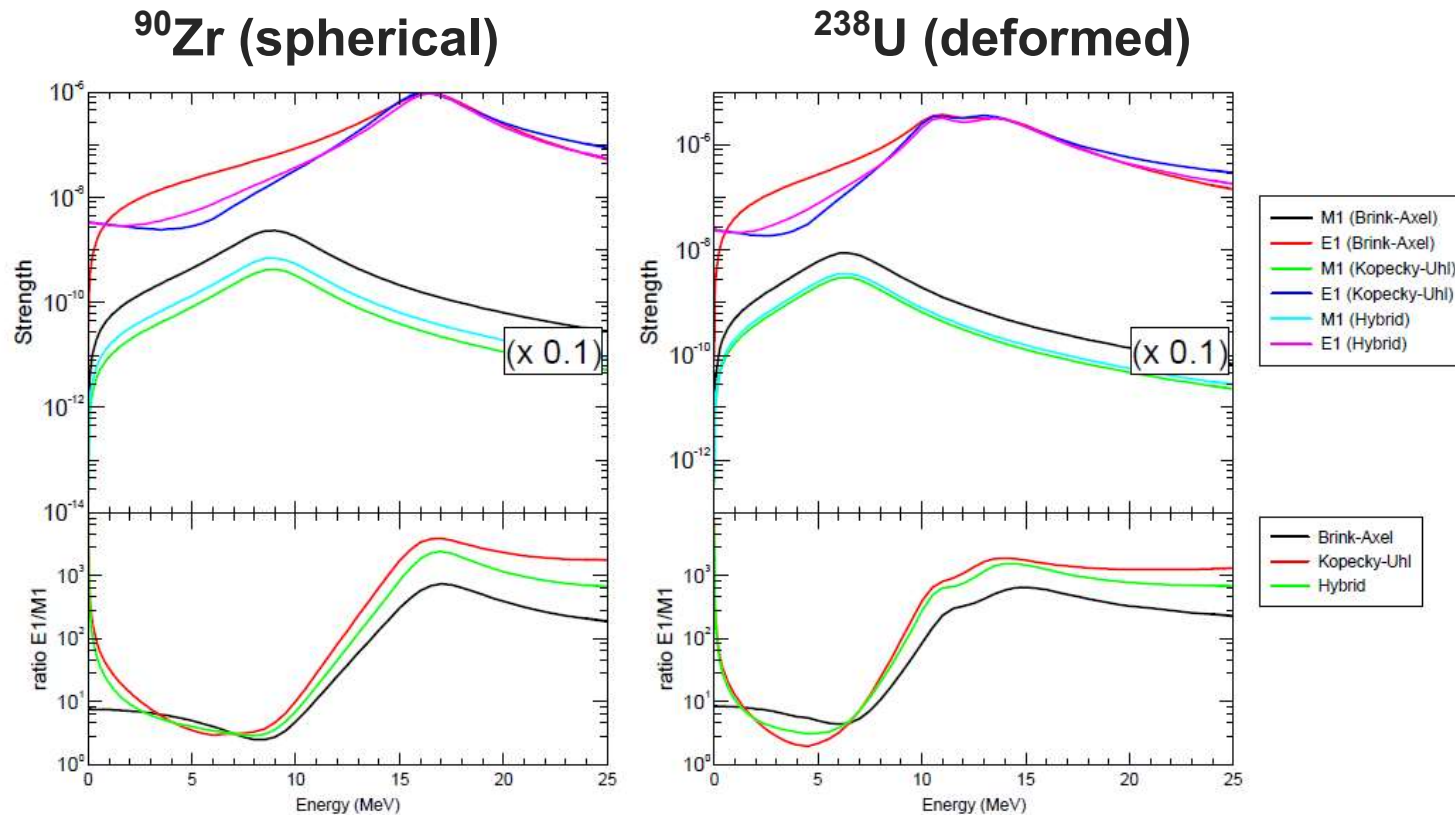


- ⇒ Deformed nuclei : two Lorentzians = two peaks
- ⇒ Lorentzian centroid energy decreasing with  $A$
- ⇒ Strength  $\rightarrow 0$  for  $E \rightarrow 0$  (ok for gamma absorption but not for gamma decay)





# Gamma-ray strengths : various models in TALYS



- ⇒ Deformed nuclei : two Lorentzians = two peaks
- ⇒ Lorentzian centroid energy decreasing
- ⇒ E1 = (10 – 100) M1 « where it counts »
- ⇒ Kopecky-Uhl or Hybrid model correct low energy behavior of Brink-Axel when considering gamma decay rather than gamma absorption



# Gamma-ray strengths : analytical approaches

## Improved analytical expressions :

- 2 Lorentzians for deformed nuclei
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⇒ **Many choices and parameters : extrapolation at your own risks !**

- Re

**Except maybe the latest SMLO !**

- Update of SMLO in 2019
  - ⇒ Temperature dependence
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# Gamma-ray strengths : SMLO 2019

## The newly proposed Simplified M1 Lorentzian Model (SMLO)

$$\begin{aligned} \overrightarrow{f}_{M1}(\varepsilon_\gamma) &= \frac{1}{3\pi^2 \hbar^2 c^2} \sigma_{sc} \frac{\varepsilon_\gamma \Gamma_{sc}^2}{(\varepsilon_\gamma^2 - E_{sc}^2)^2 + \varepsilon_\gamma^2 \Gamma_{sc}^2} && \text{Scissors mode for deformed nuclei} \\ &+ \frac{1}{3\pi^2 \hbar^2 c^2} \sigma_{sf} \frac{\varepsilon_\gamma \Gamma_{sf}^2}{(\varepsilon_\gamma^2 - E_{sf}^2)^2 + \varepsilon_\gamma^2 \Gamma_{sf}^2} && \text{Spin-Flip mode} \end{aligned}$$

where the SMLO M1 properties are inspired from the D1M+QRPA predictions

$$\overleftarrow{f}_{M1}(\varepsilon_\gamma) = \overrightarrow{f}_{M1}(\varepsilon_\gamma) + C \exp(-\eta \varepsilon_\gamma) \quad \text{M1 upbend for de-excitation}$$

where the upbend properties are inspired from the Shell Model predictions

$$C = 3.5 \cdot 10^{-8} \exp(-6\beta_2) \text{ MeV}^{-3}$$

$$\eta = 0.8$$

Schwengner et al. 2017

Sieja 2017

Midtbø et al. 2018

...

# Gamma-ray strengths : SMLO 2019

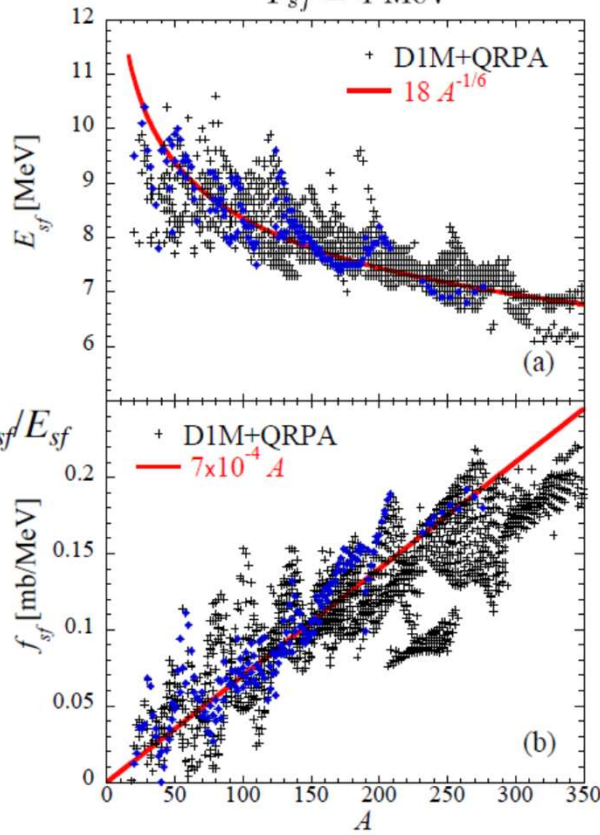


## Spin-Flip mode

$$\sigma_{sf} = 0.03A^{5/6} \text{ mb}$$

$$E_{sf} = 18A^{-1/6} \text{ MeV}$$

$$\Gamma_{sf} = 4 \text{ MeV}$$

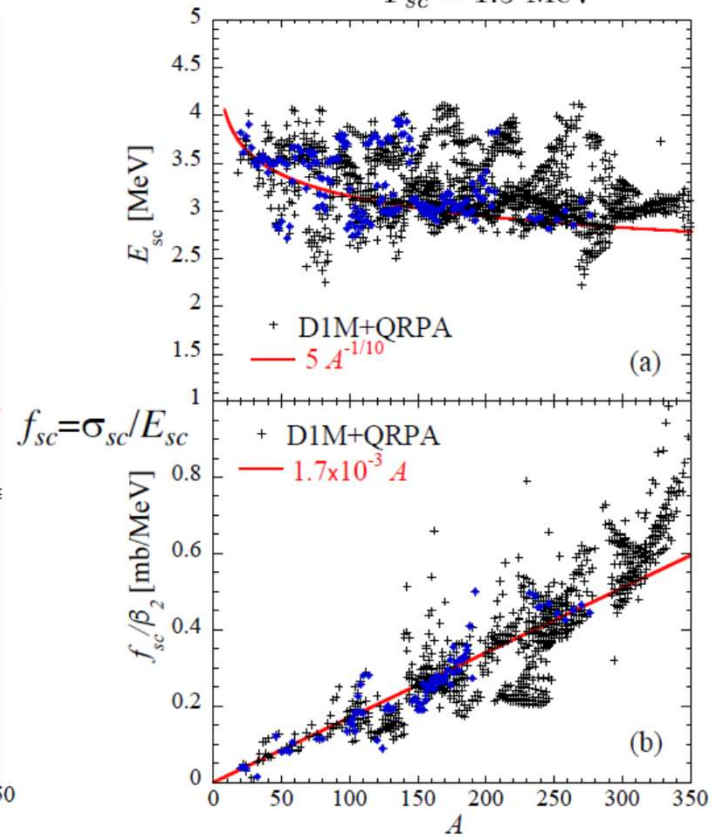


## Scissors mode

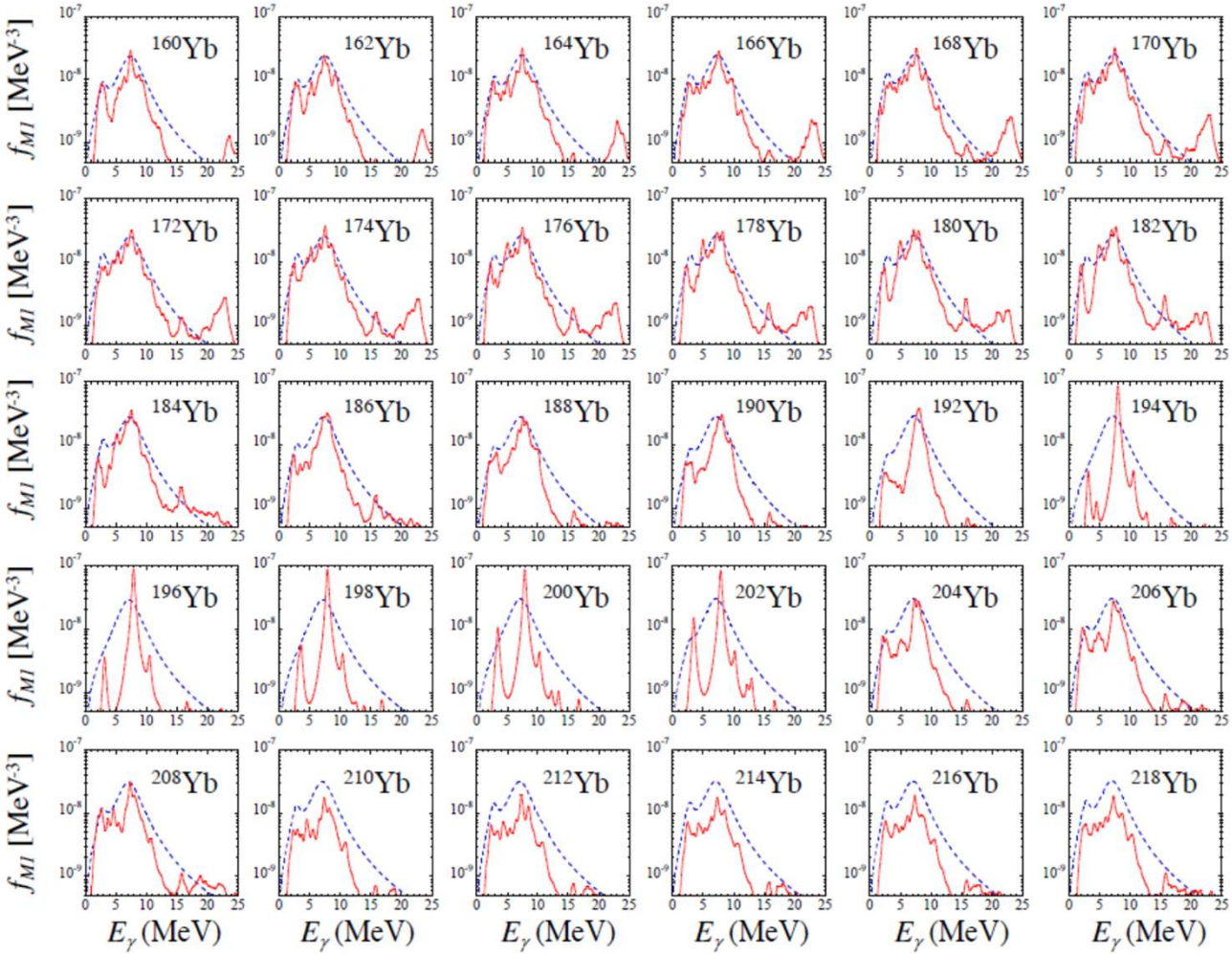
$$\sigma_{sc} = 10^{-2} |\beta_2| A^{9/10} \text{ mb}$$

$$E_{sc} = 5 \times A^{-1/10} \text{ MeV}$$

$$\Gamma_{sc} = 1.5 \text{ MeV}$$



# Gamma-ray strengths : SMLO 2019





# Gamma-ray strengths

## - Qualitative features

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## - Microscopic approaches

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# Gamma-ray strengths : microscopic approaches

## Systematic approaches : all nuclei feasible

« *Those who know what is (Q)RPA don't care about details, those who don't know don't care either* », private communication

⇒ Systematic QRPA with Skm/RMF forces

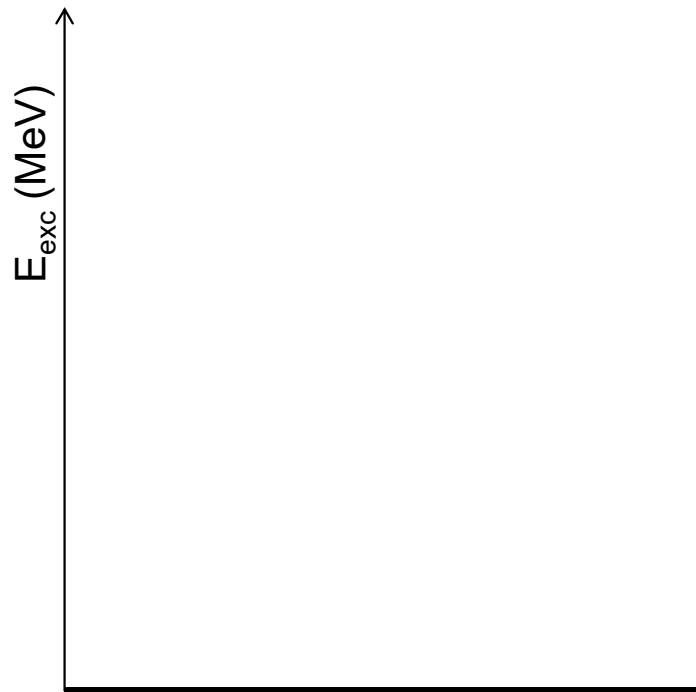
⇒ Systematic QRPA with Gogny force

## Local approaches : regional study only

⇒ Shell Model approach

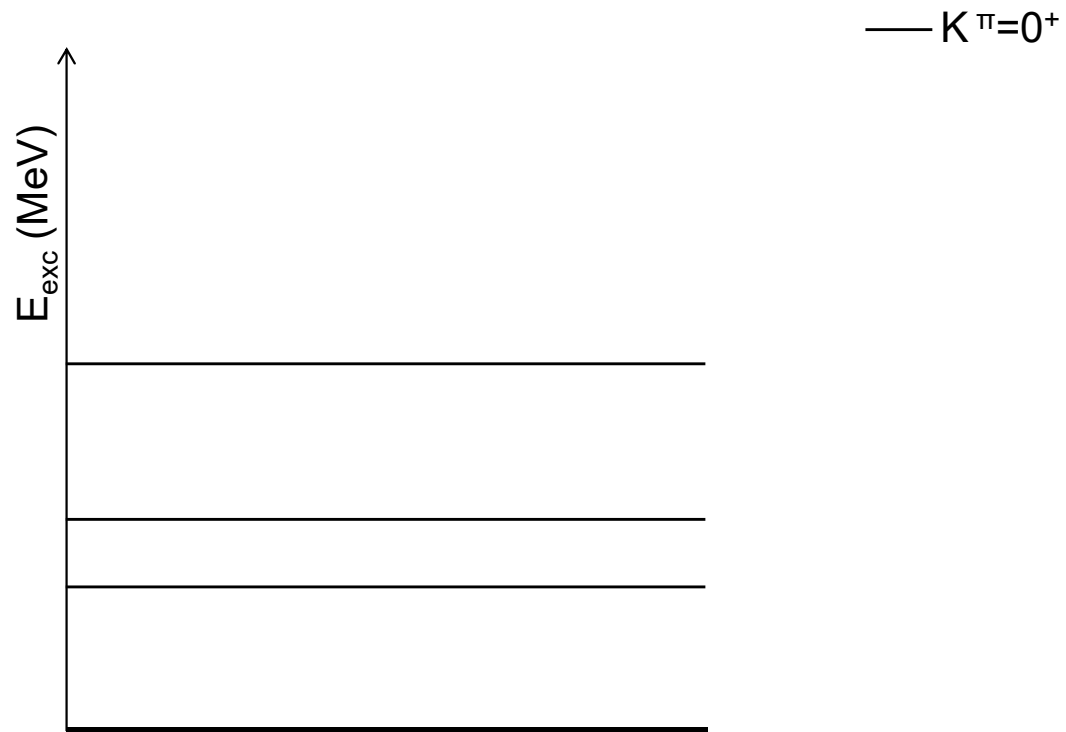


# Gamma-ray strengths : microscopic approaches principle

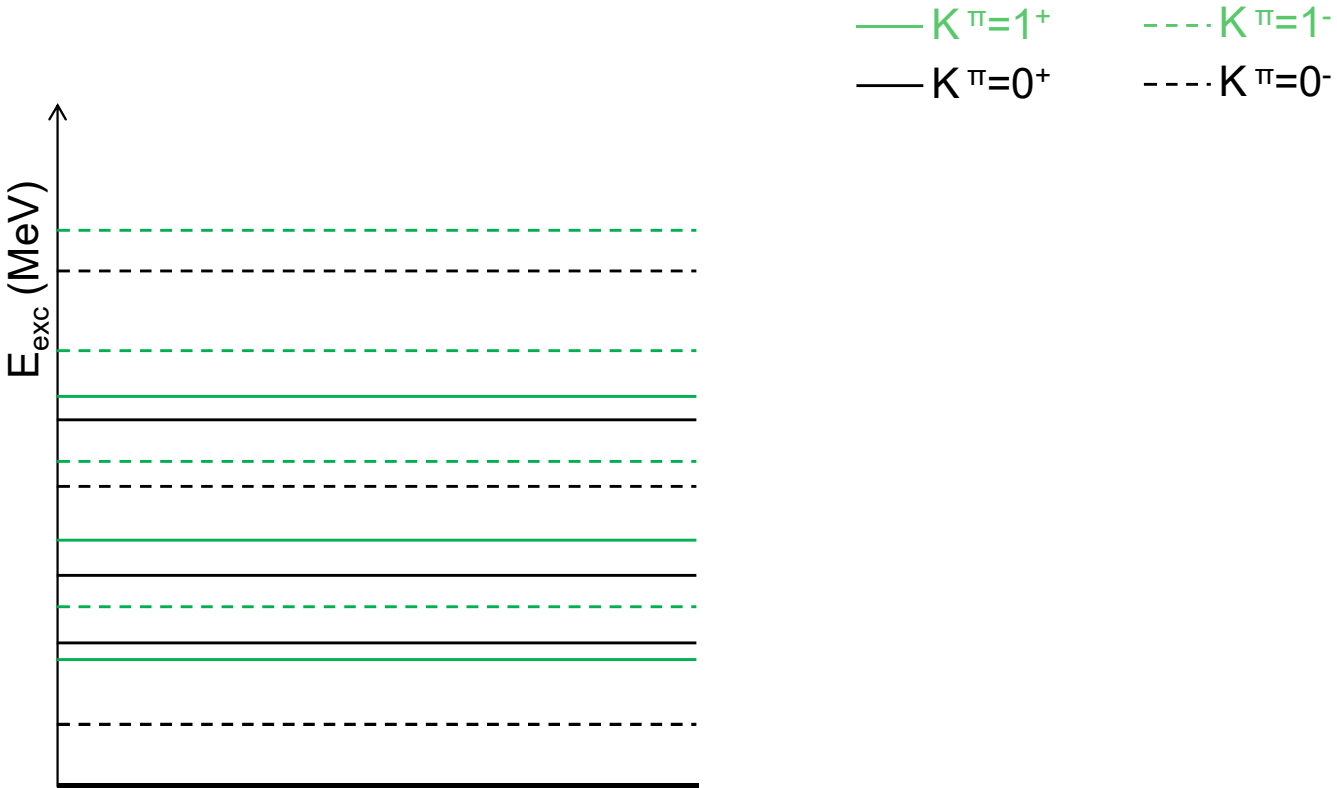




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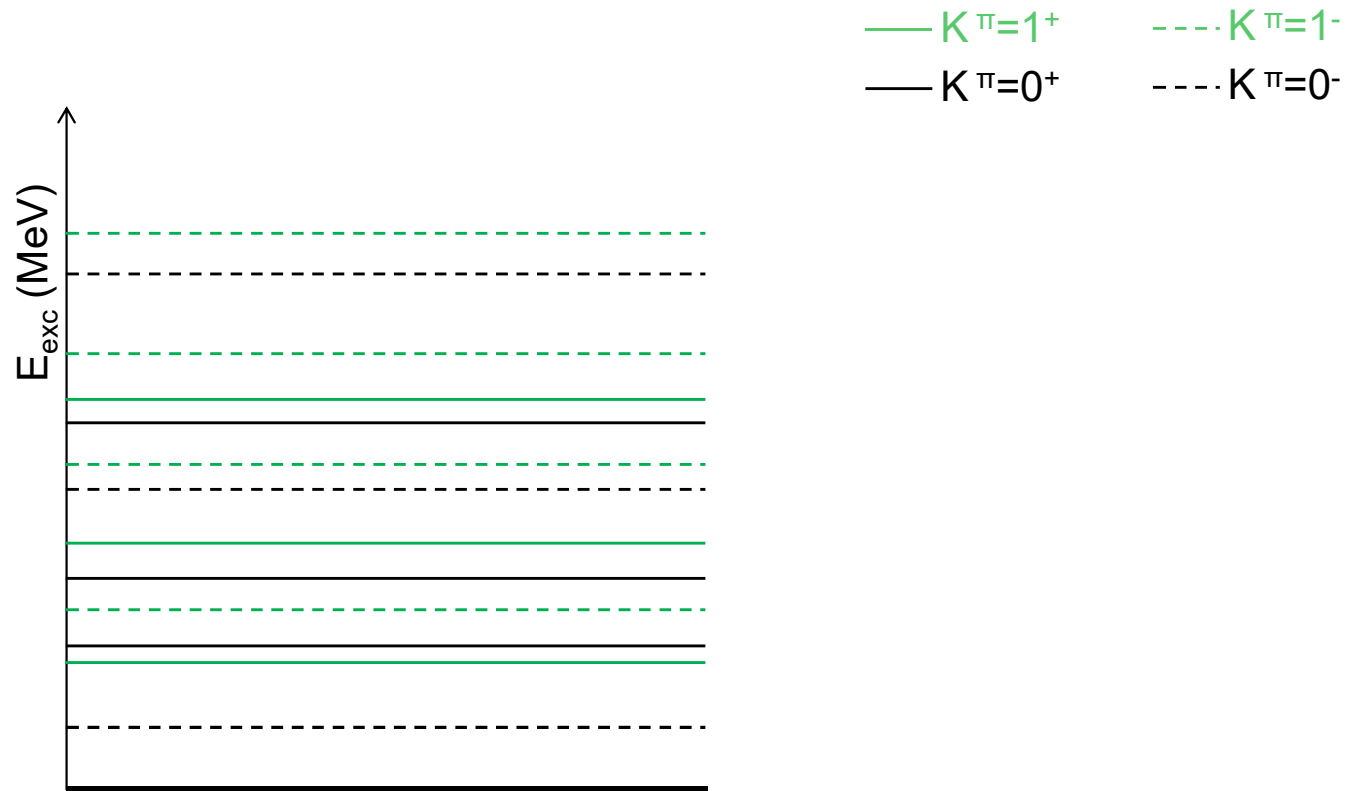
# Gamma-ray strengths : microscopic approaches principle





# Gamma-ray strengths : microscopic approaches principle

Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$

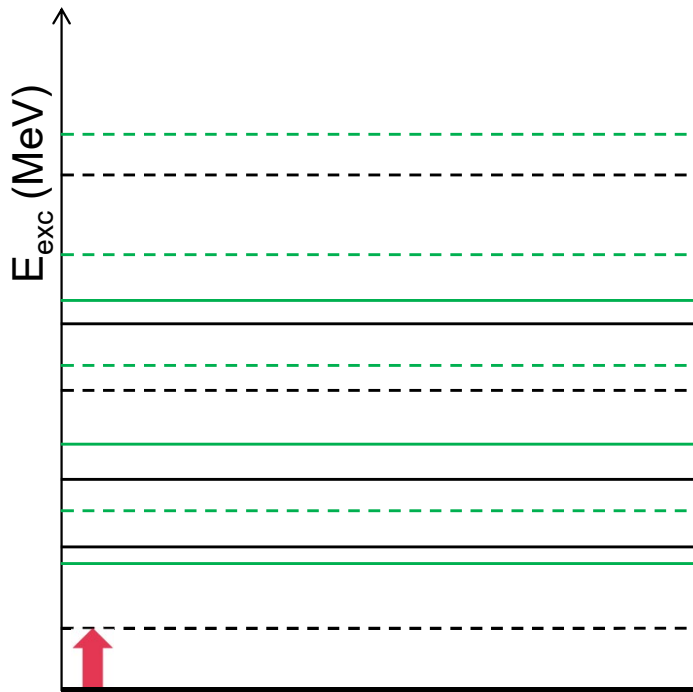


# Gamma-ray strengths : microscopic approaches principle



Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$

—  $K^\pi=1^+$       - - - -  $K^\pi=1^-$   
—  $K^\pi=0^+$       - - - -  $K^\pi=0^-$

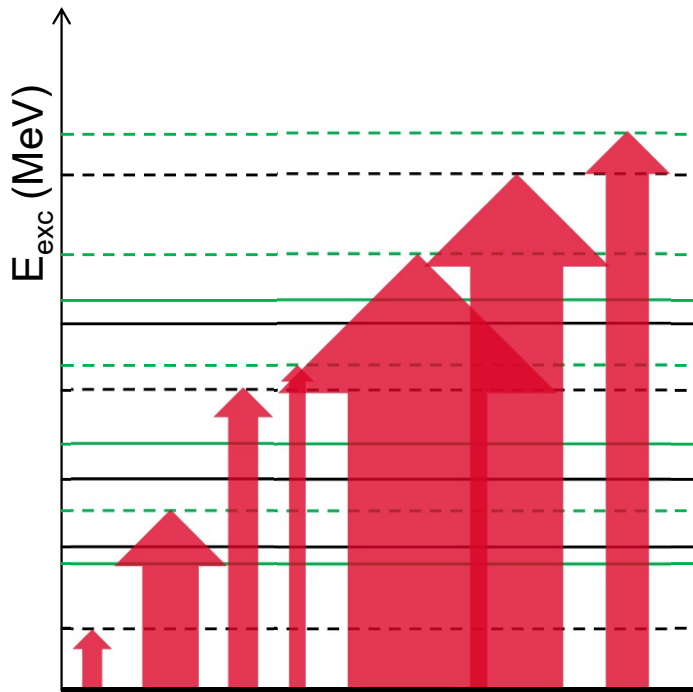


# Gamma-ray strengths : microscopic approaches principle



Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$

—  $K^\pi=1^+$       - - -  $K^\pi=1^-$   
—  $K^\pi=0^+$       - - -  $K^\pi=0^-$

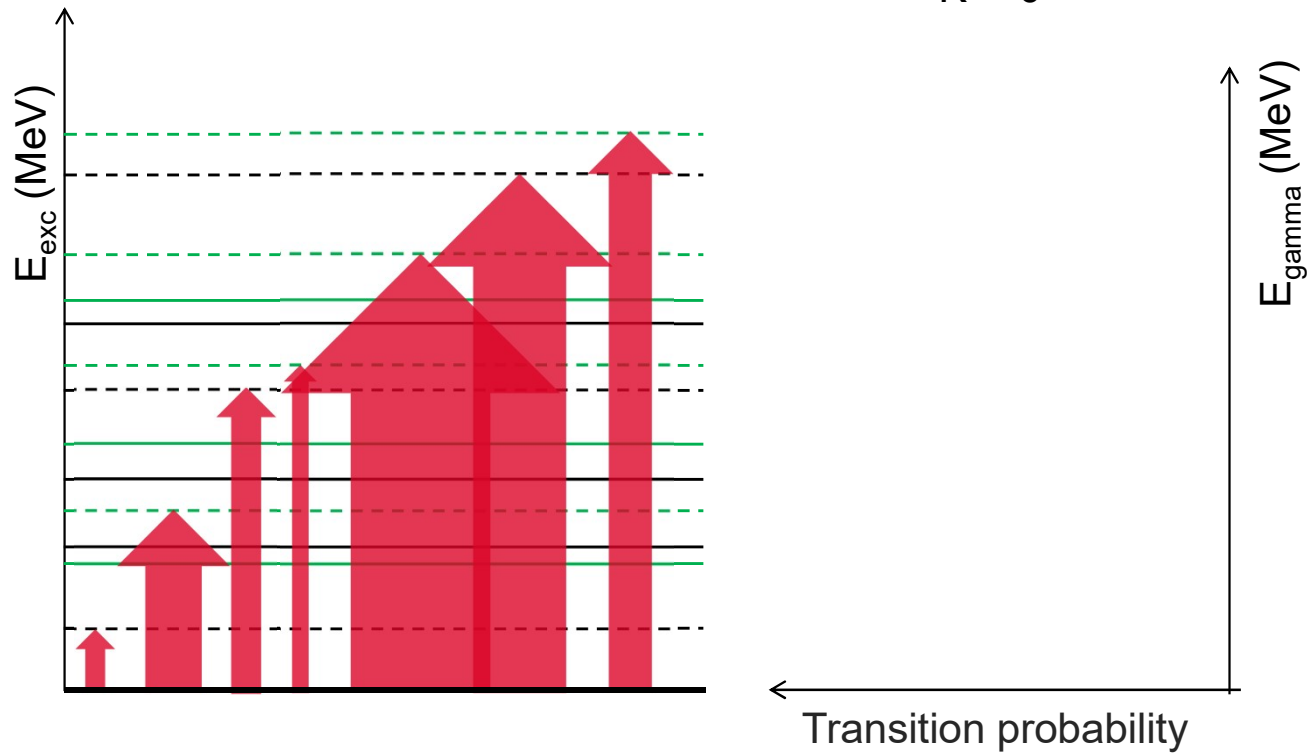


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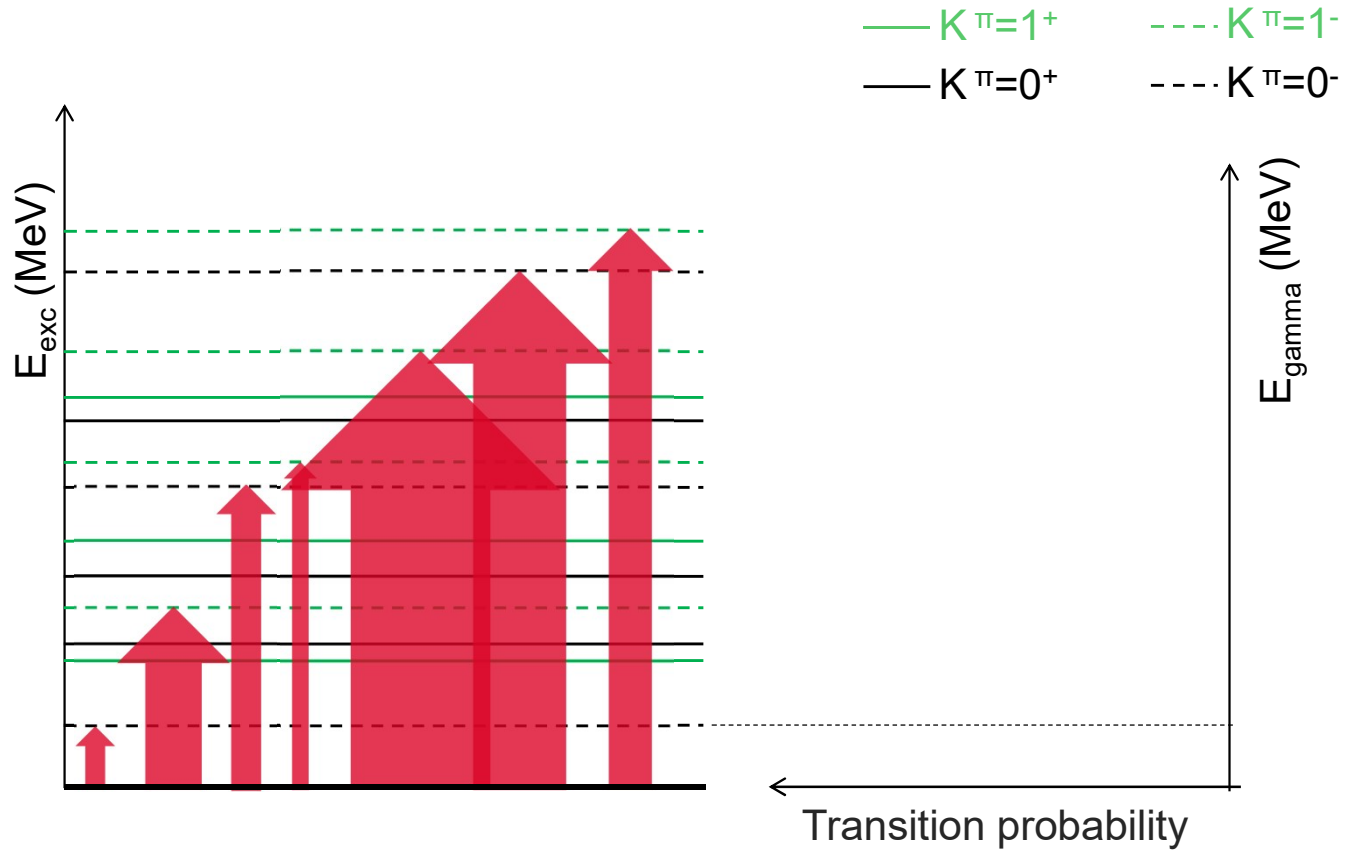




# Gamma-ray strengths : microscopic approaches principle



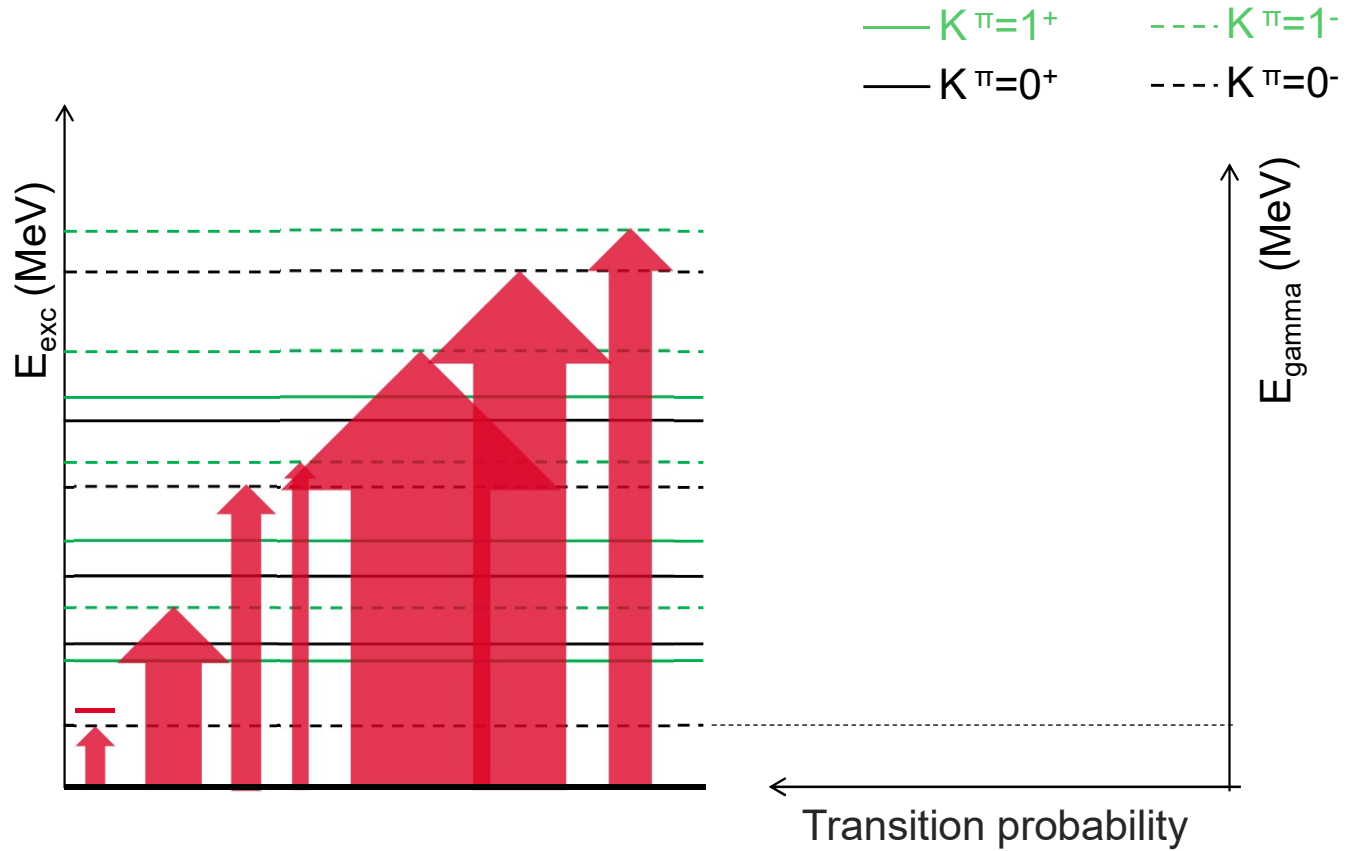
Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$



# Gamma-ray strengths : microscopic approaches principle



Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$

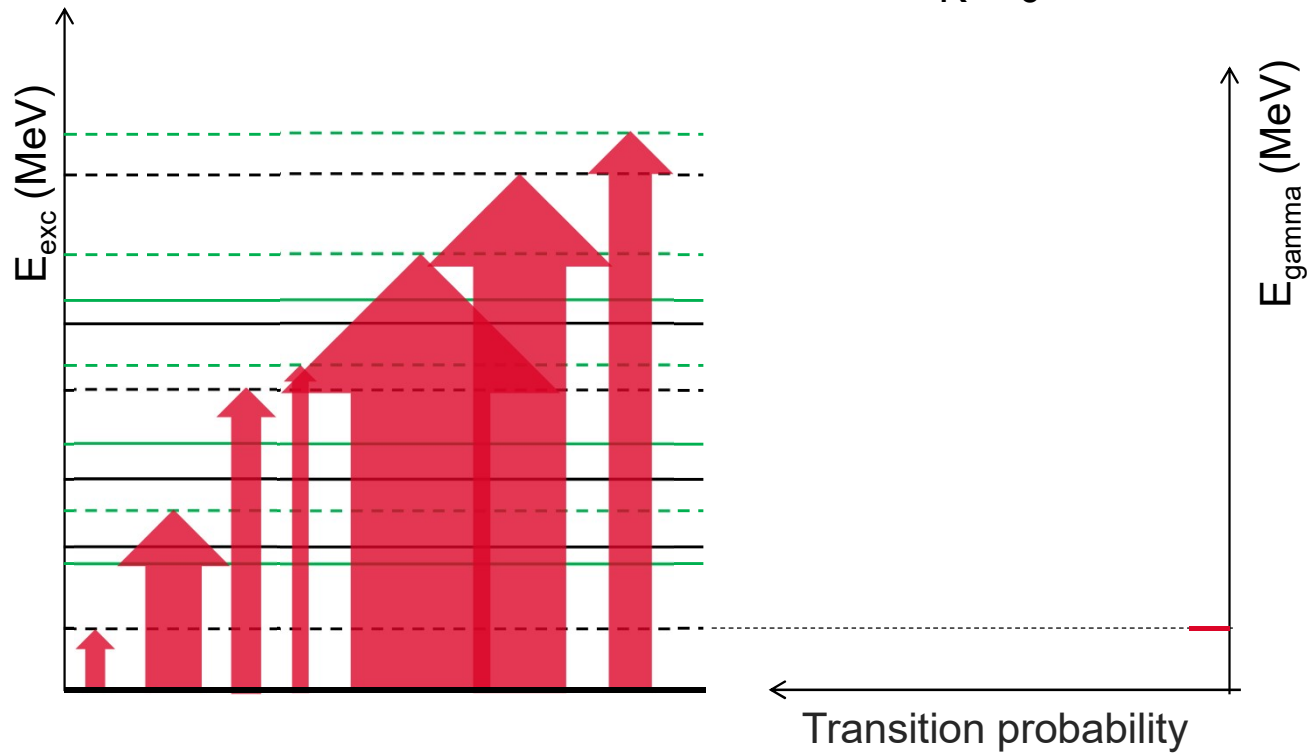


# Gamma-ray strengths : microscopic approaches principle



Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$

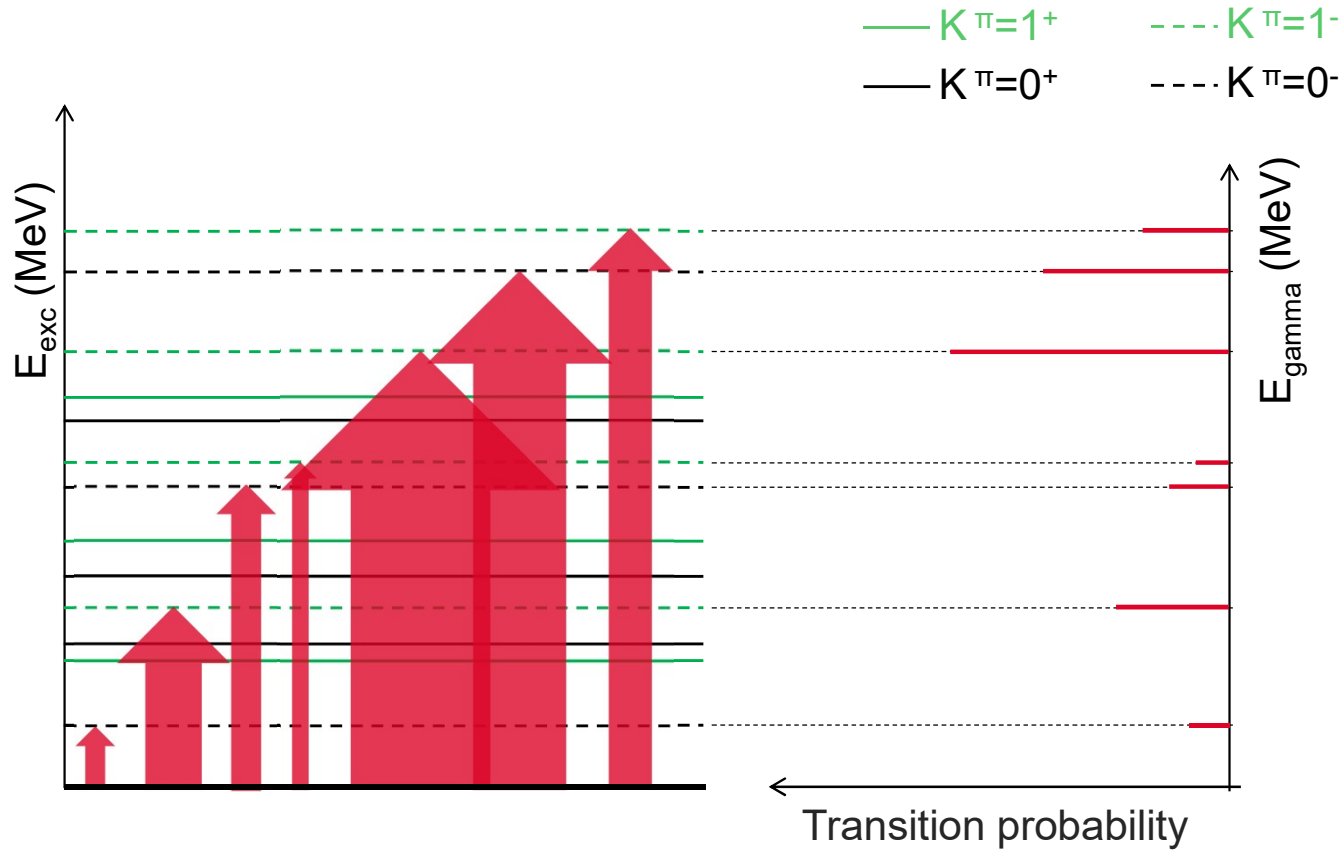
—  $K^\pi=1^+$       - - -  $K^\pi=1^-$   
—  $K^\pi=0^+$       - - -  $K^\pi=0^-$



# Gamma-ray strengths : microscopic approaches principle



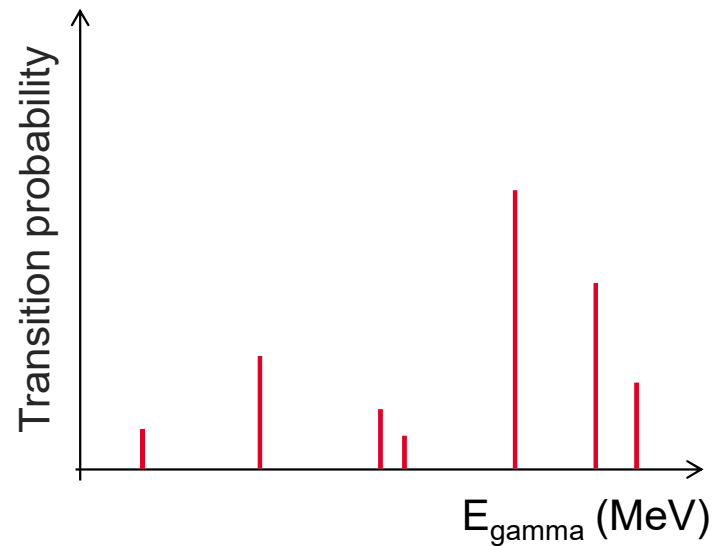
Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$





# Gamma-ray strengths : microscopic approaches principle

Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$



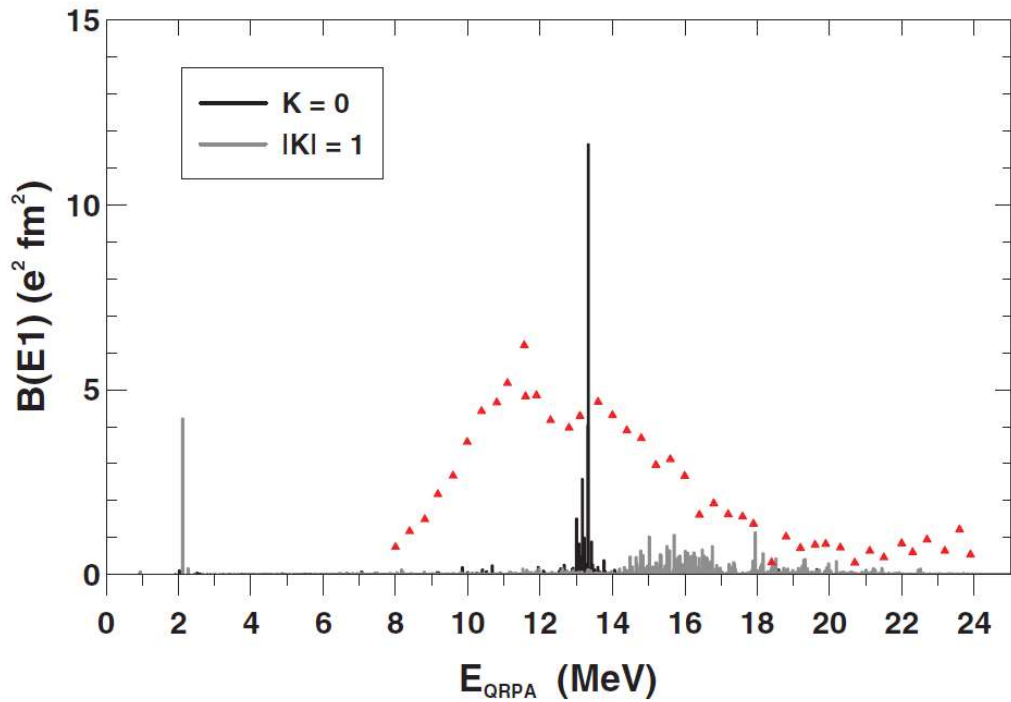
+ Broadening with a Lorentzian

$$S_{E_1}(E) = \sum_i \frac{1}{\pi} \frac{\Gamma E^2}{[E^2 - (\omega_i - \Delta(\omega_i))^2]^2 + \Gamma^2 E^2} B_{E_1}(\omega_i)$$

# Gamma-ray strengths : microscopic approaches principle



Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$

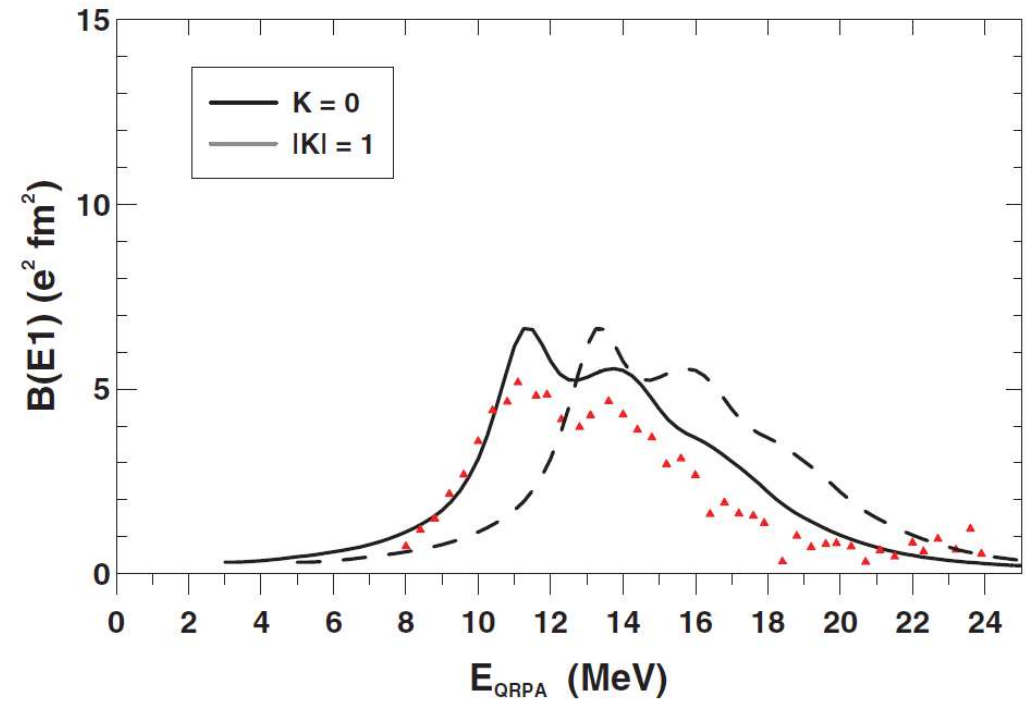
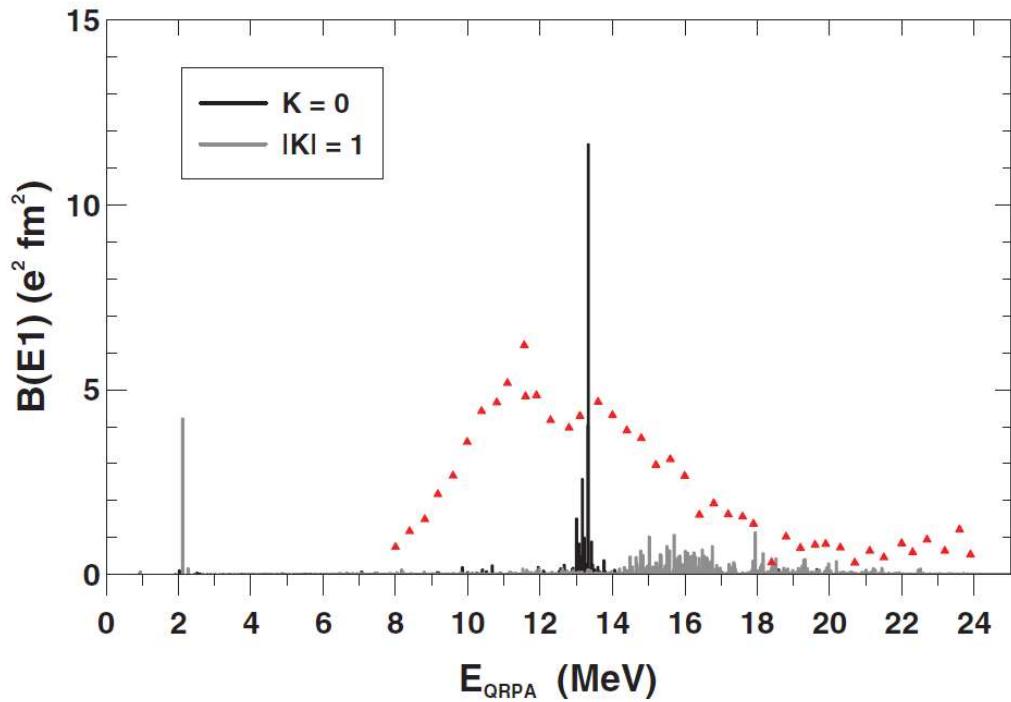


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Photoabsorption : E1 transitions dominate  $0^+ \Rightarrow 0^-, 1^-$

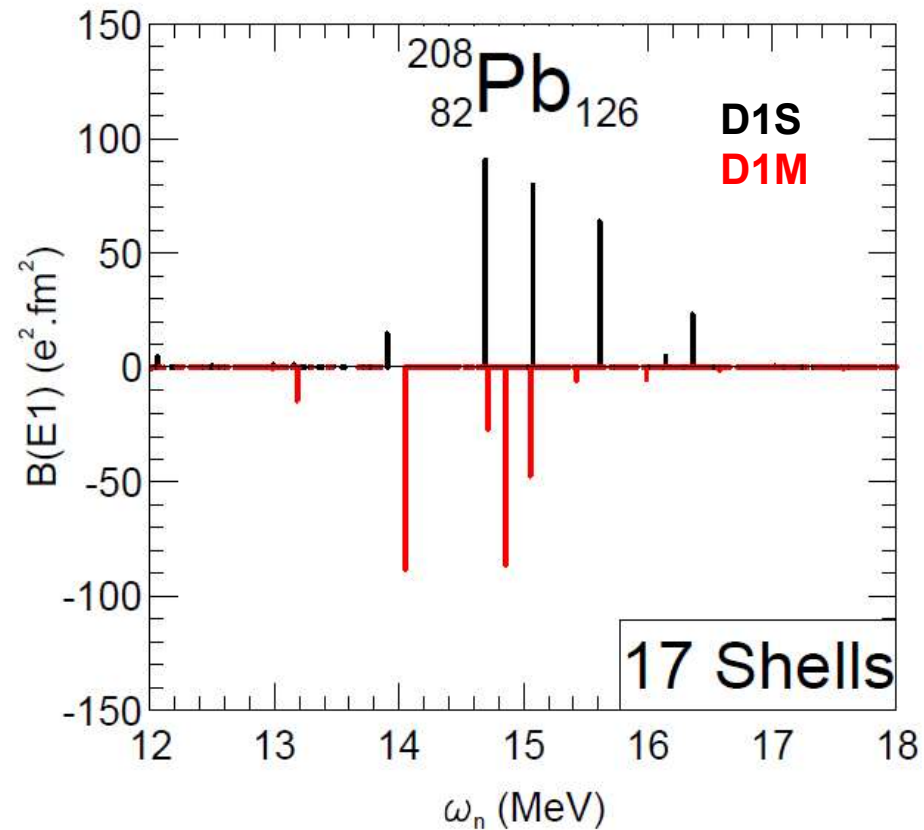


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# Gamma-ray strengths : QRPA raw results

QRPA provides with emission probability between an excited state and the GS



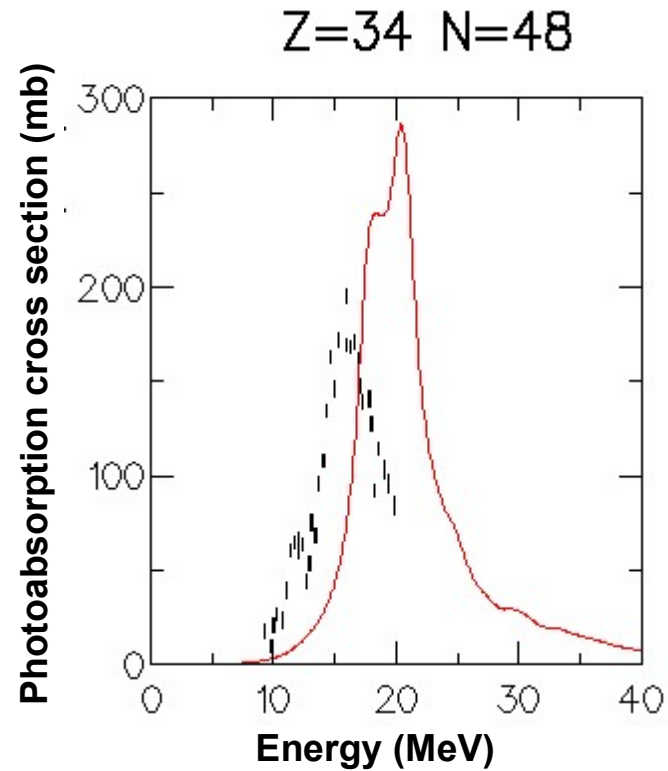
⇒ Broadening necessary to account for damping of collective motion





# Gamma-ray strengths : QRPA broadened results

QRPA provides with emission probability between an excited state and the GS



- ⇒ Shift to account for phonon couplings + beyond 1p-1h approximation
- ⇒ Peak normalization to improve experimental data fitting



## Gamma-ray strengths : QRPA peak normalization

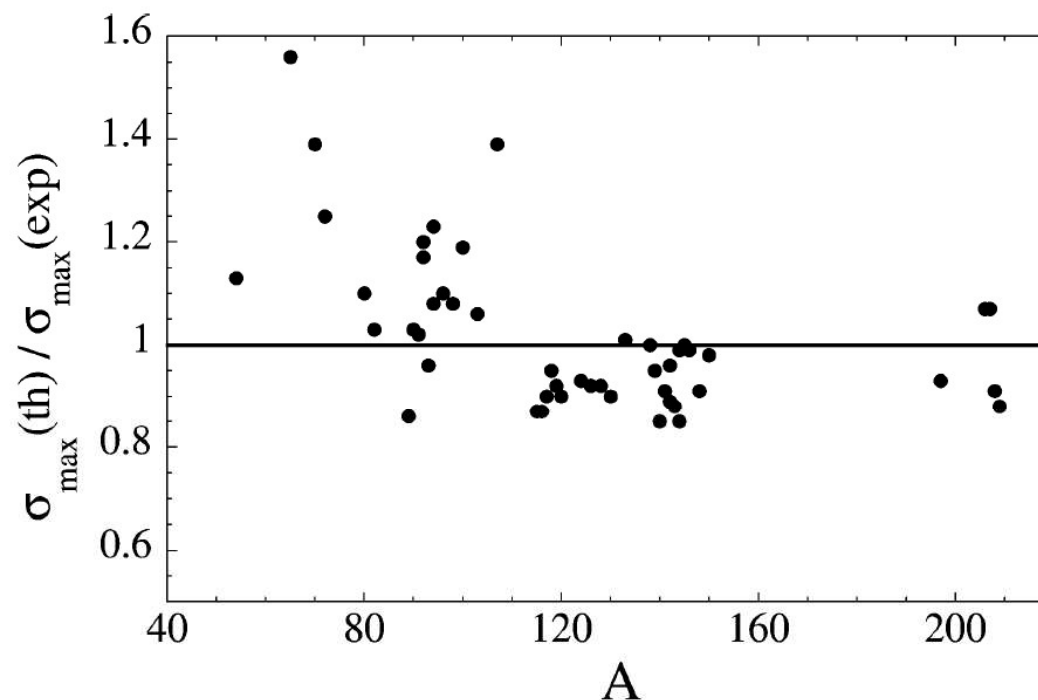


Fig. 4. Ratio of the peak cross section  $\sigma_{\max}(\text{th})$  estimated within the HFB + QRPA model with the BSk7 Skyrme force to the experimental value  $\sigma_{\max}(\text{exp})$  for the 48 spherical nuclei as a function of the mass number  $A$ .

See S. Goriely & E. Khan, *NPA* 706 (2002) 217.

S. Goriely et al., *NPA* 739 (2004) 331.



## Gamma-ray strengths : deformed nuclei

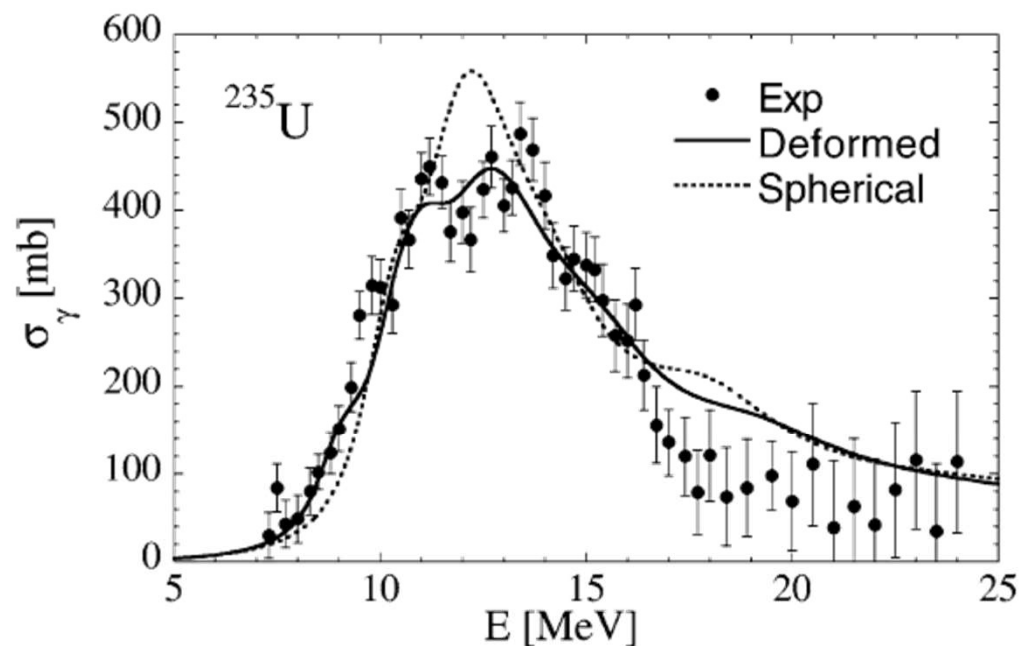


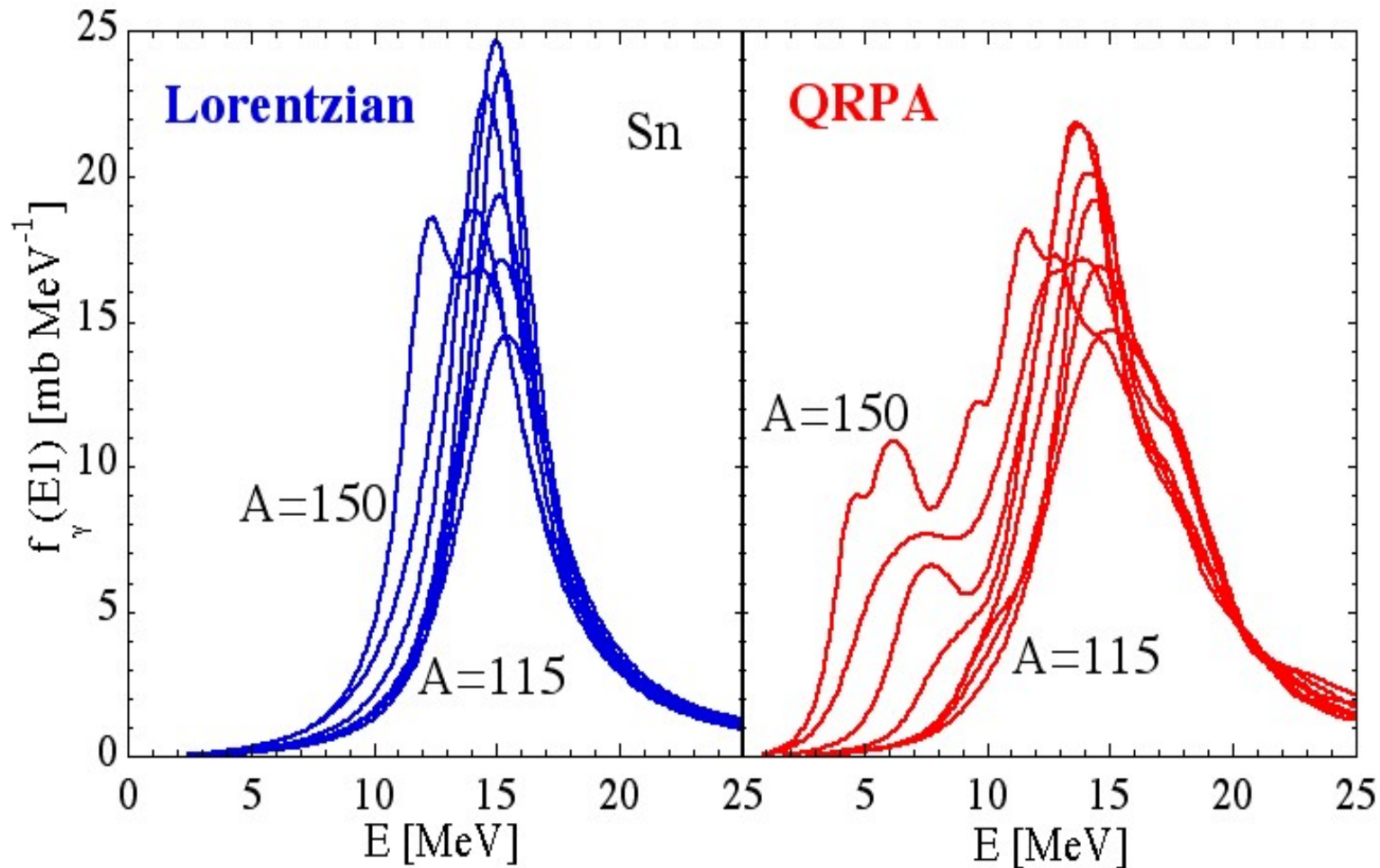
Fig. 5. Photoabsorption cross section for  $^{235}\text{U}$ . The dots correspond to experimental data [16]. The dotted line is the HFB + QRPA calculation obtained with the BSk7 force in the spherical approximation (applying the damping method) and the full line when applying in addition our **phenomenological procedure to describe deformation effects**. Both cross sections have been shifted by 0.5 MeV upwards to reproduce the low energy tail.

See S. Goriely & E. Khan, *NPA* 706 (2002) 217.

S. Goriely et al., *NPA* 739 (2004) 331.



# Gamma-ray strengths : QRPA for exotic nuclei





## Gamma-ray strengths : beyond spherical approximation

QRPA calculations can accurately reproduce experimental data, provided empirical corrections are made, *i.e.*

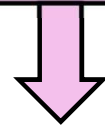
- Empirical damping of collective motions → broadening
- Empirical Energy shift (beyond 1p-1h excitations and phonon couplings)
- Empirical deformation effects for spherical calculations



## Gamma-ray strengths : beyond spherical approximation

QRPA calculations can accurately reproduce experimental data, provided empirical corrections are made, *i.e.*

- Empirical damping of collective motions → broadening
- Empirical Energy shift (beyond 1p-1h excitations and phonon couplings)
- Empirical deformation effects for spherical calculations



**Can be removed within the axial Gogny QRPA framework  
but high computational cost**



# Gamma-ray strengths : axial Gogny QRPA approach

**Extremely high computational cost !**

QRPA calculations performed to

- 1) perform sensitivity analyses w.r.t :
    - effective interaction (D1S vs D1M)
    - nuclear deformation
    - quasiparticle energy cut-off  $\epsilon_c$
    - number of major shells  $N_{sh}$
- ↪ **compromise accuracy vs computing time**

computing time for a given  $K^\pi$  with 1024 cpu

$N_{sh}$	No cut	$\epsilon_c = 100$ MeV	$\epsilon_c = 60$ MeV	$\epsilon_c = 30$ MeV
9	5'	5'	4'	38"
11	2 h	2 h	1h	5'
13	42 h	26 h	6 h	30'
15	21 d	8 d	30 h	2h
17	286 d	63 d	7 d	8h

- 2) compute QRPA strengths for all nuclei included in the IAEA RIPL-3 database
- 3) compute low energy collective states
- 4) Add “global” corrections to theoretical predictions to fit data
- 5) Produce tables for all nuclei



## Gamma-ray strengths : adjustment method

folded strength



raw strength



$$S_{E1}(E) = \sum_n L(E, \omega_n) B_{E1}(\omega_n)$$

with

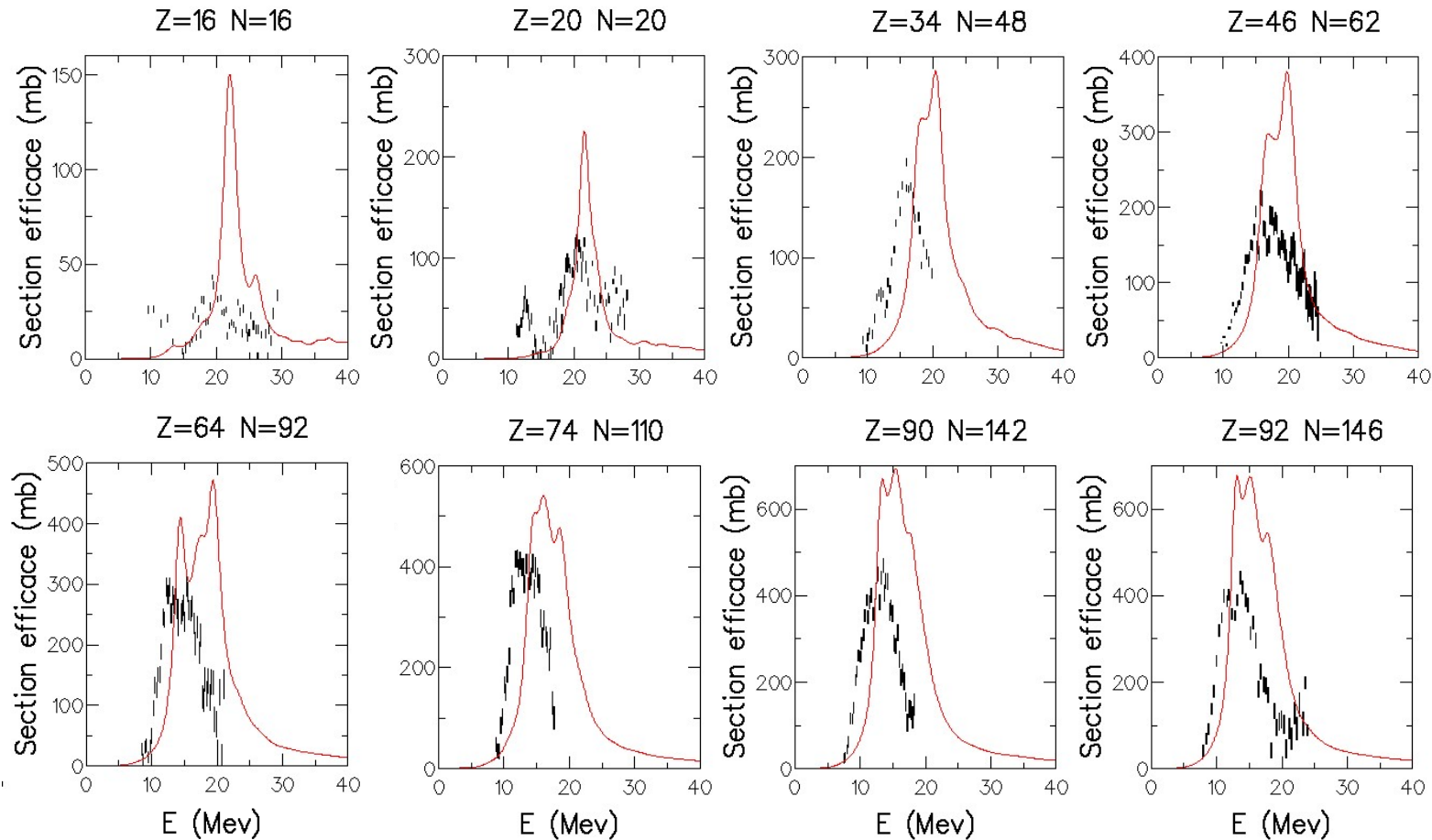
$$L(E, \omega) = \frac{K}{\pi} \frac{\Gamma E^2}{[E^2 - (\omega - \Delta)^2]^2 + \Gamma^2 E^2}$$

where  $K$ ,  $\Delta$  and  $\Gamma$  can be adjusted





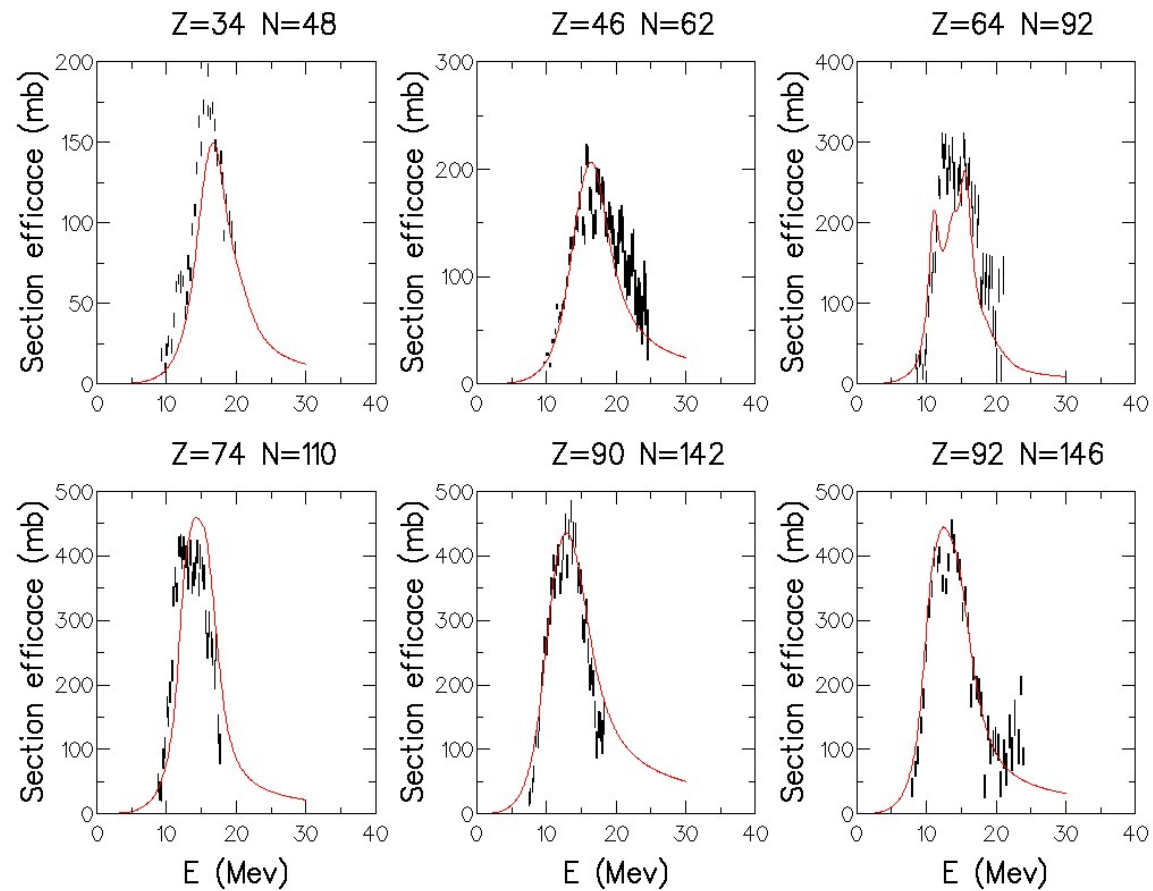
# Gamma-ray strengths : broadening of 2 MeV only



- ⇒ Shift to account for phonon couplings + beyond 1p-1h approximation
- ⇒ Peak normalization to improve experimental data fitting

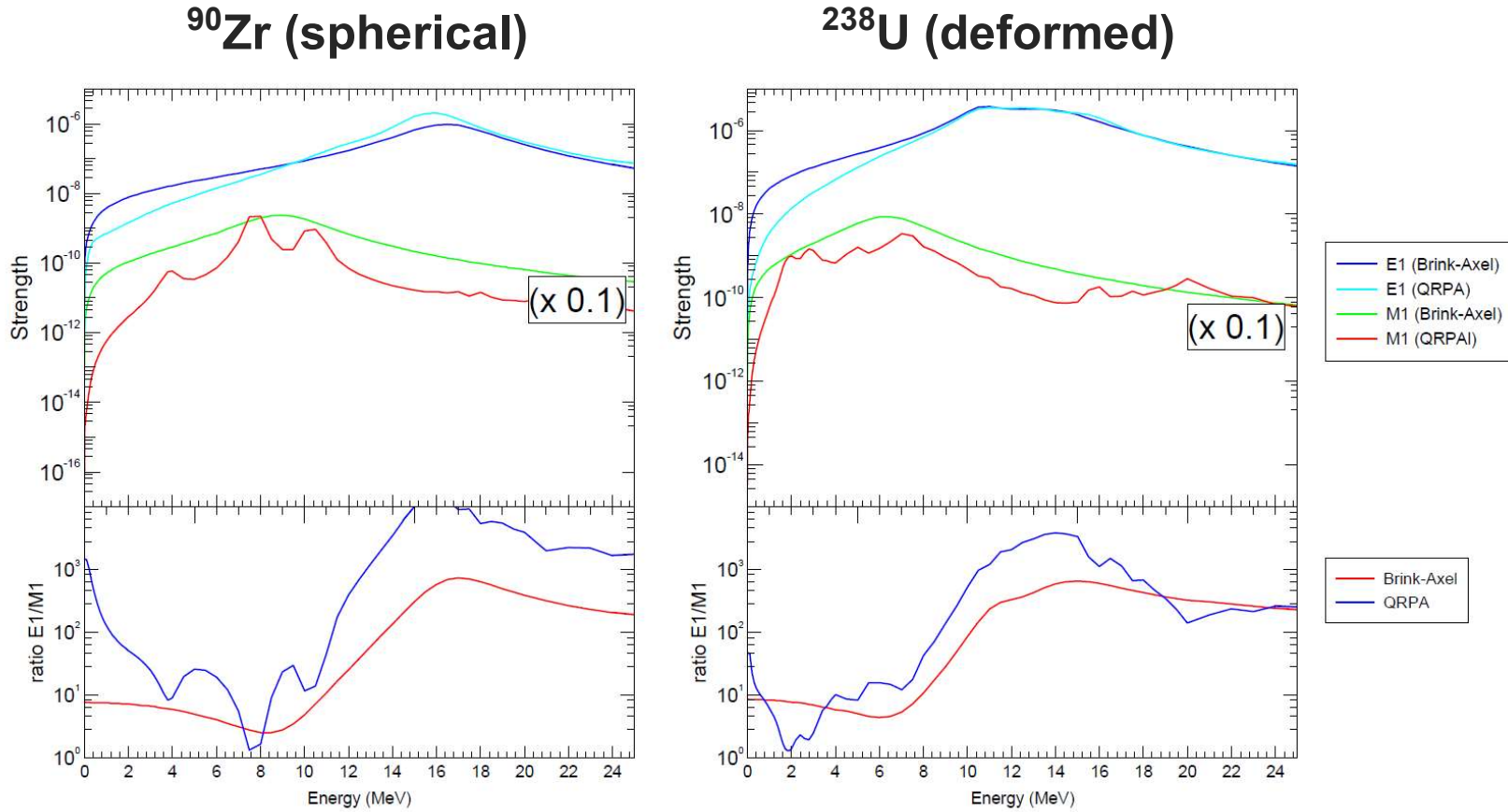


# Gamma-ray strengths : all parameters adjusted



- ⇒ Good agreement with data
- ⇒ Systematic predictions can be performed

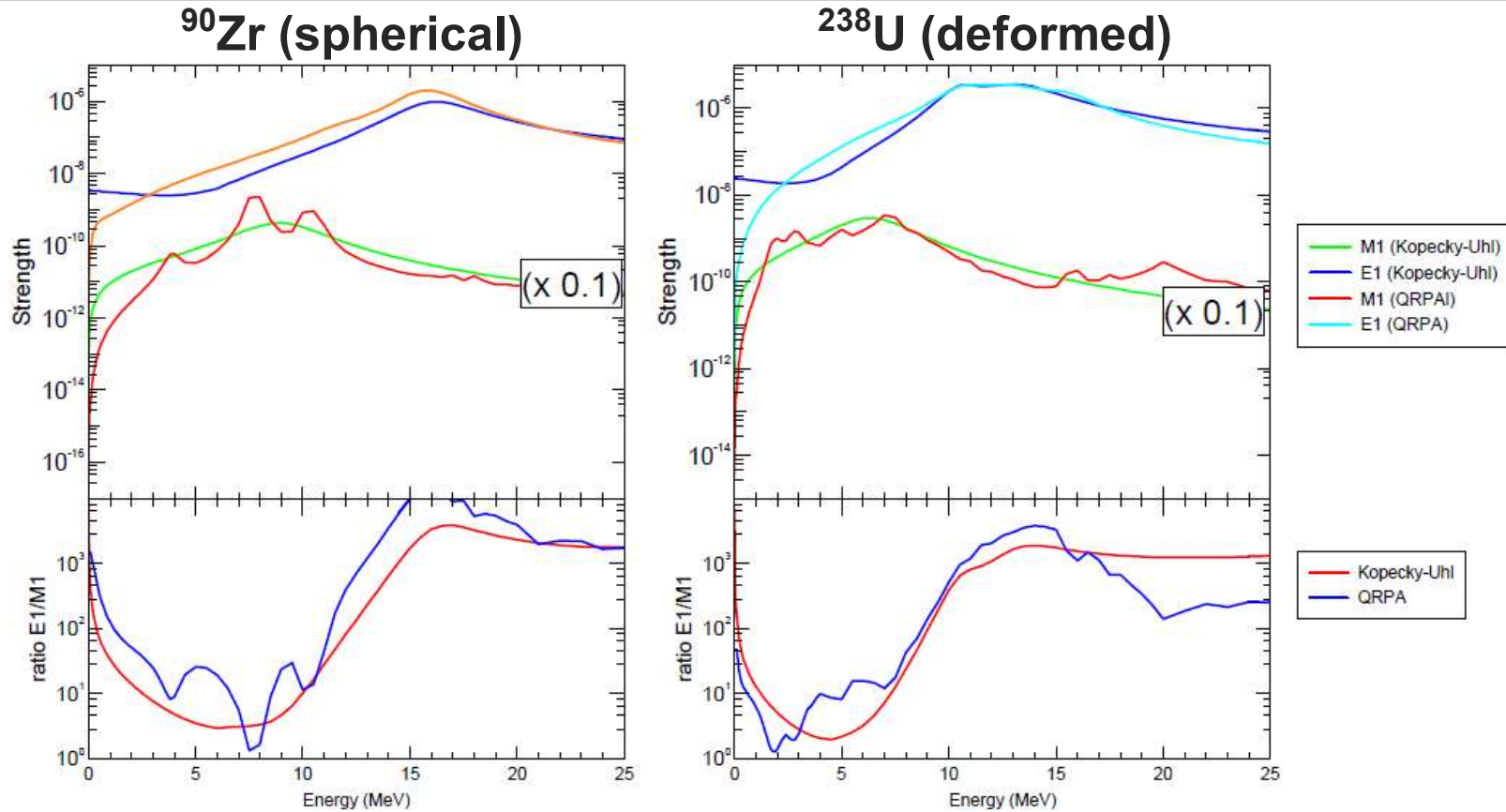
# Gamma-ray strengths : deformed QRPA vs Brink-Axel



- ⇒ OK for photoabsorption
- ⇒ Significant structure for M1 transitions



# Gamma-ray strengths : deformed QRPA vs Koepcky-Uhl



- ⇒ Missing low energy strength for E1
- ⇒ Significant structure for M1 transitions



# Gamma-ray strengths : shell model

- **Shell Model approach**

*E. Caurier et al., Rev. Mod. Phys. 77 (2005) p410-427*

- ⇒ Very precise
- ⇒ Even-even, odd-A, odd-odd nuclei treated on the same footing
- ⇒ Possibility to predict within the same framework
  - spectra
  - transitions between **any** excited state
  - weak decays (beta, double-beta, ...)
  - pairing, deformation, ...

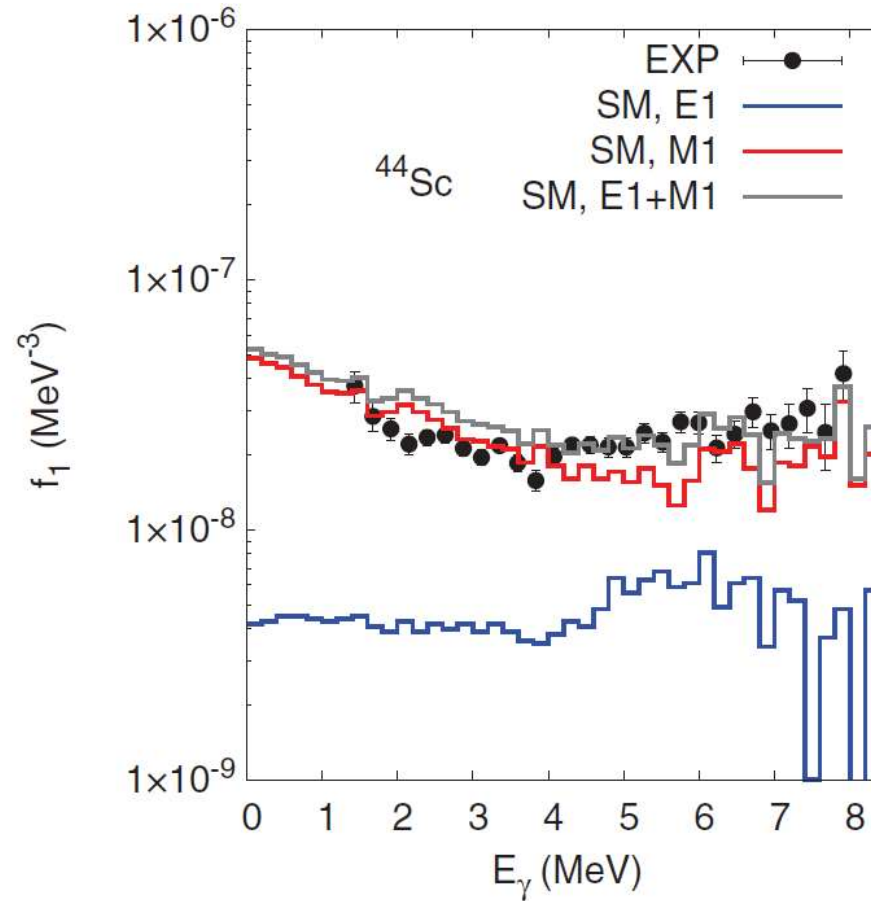
Courtesy K. Sieja

## But

- ⇒ local (parameters adjusted on exp. data for each mass region)
- ⇒ Not applicable everywhere due to the dimension of the matrices to diagonalize when large valence spaces are required



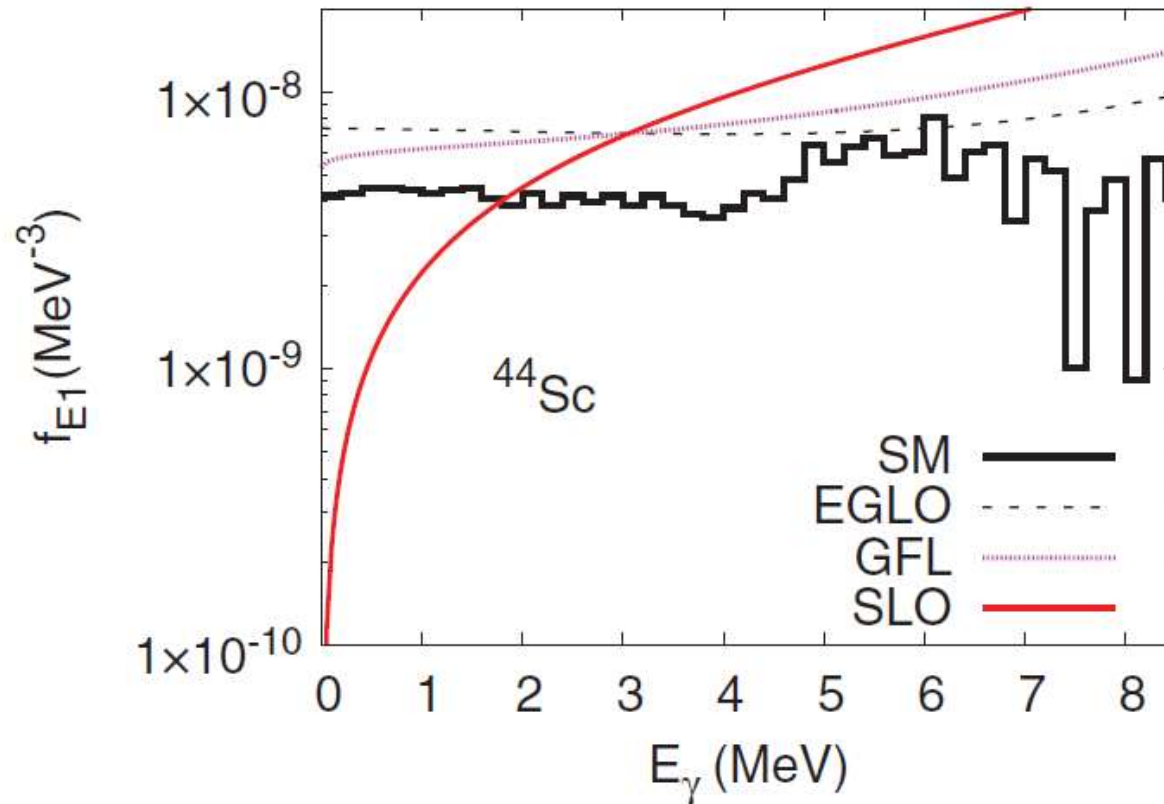
# Gamma-ray strengths : shell model



Courtesy K. Sieja

⇒ Shell model : first microscopic model reproducing low energy experimental data related to gamma decay

# Gamma-ray strengths : shell model vs analytical approaches

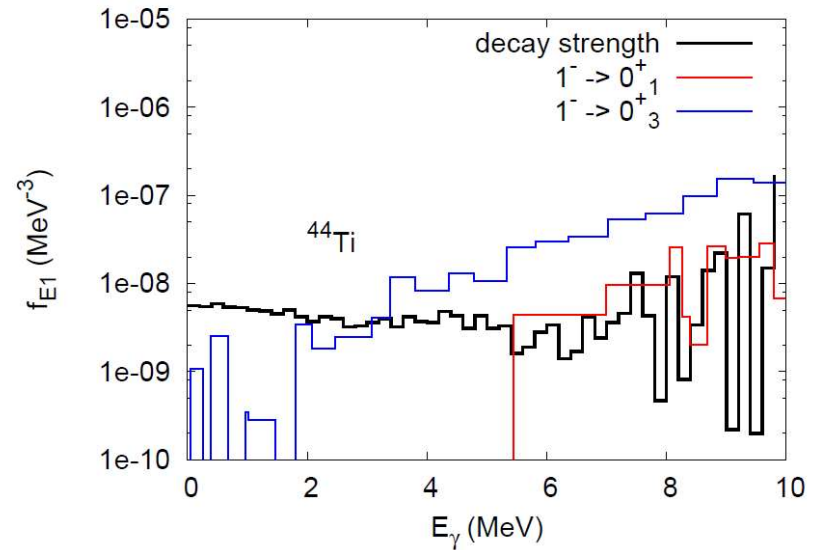
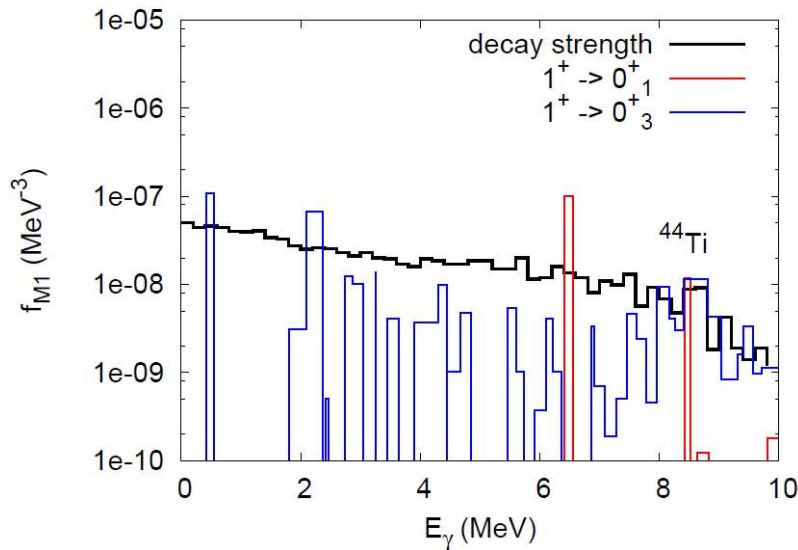


Courtesy K. Sieja

⇒ Shell model validates the non-vanishing of the strength at low energy as phenomenologically introduced in some analytical formulae



# Gamma-ray strengths : shell model lesson



⇒ Shell model shows that both E1 and M1 non vanishing low energy strength stem from intra-band transitions.





# Gamma-ray strengths

## - Qualitative features

## - Analytical approaches

## - Microscopic approaches

- HFBCS-RPA
- HFB+QRPA
- Shell Model

## - Impacts on cross sections

- Normalizations
- Exotic nuclei
- Hot topics



# Gamma-ray strengths

## - Qualitative features

## - Analytical approaches

## - Microscopic approaches

- HFBCS-RPA
- HFB+QRPA
- Shell Model

## - Impacts on cross sections

- Normalizations
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- Hot topics

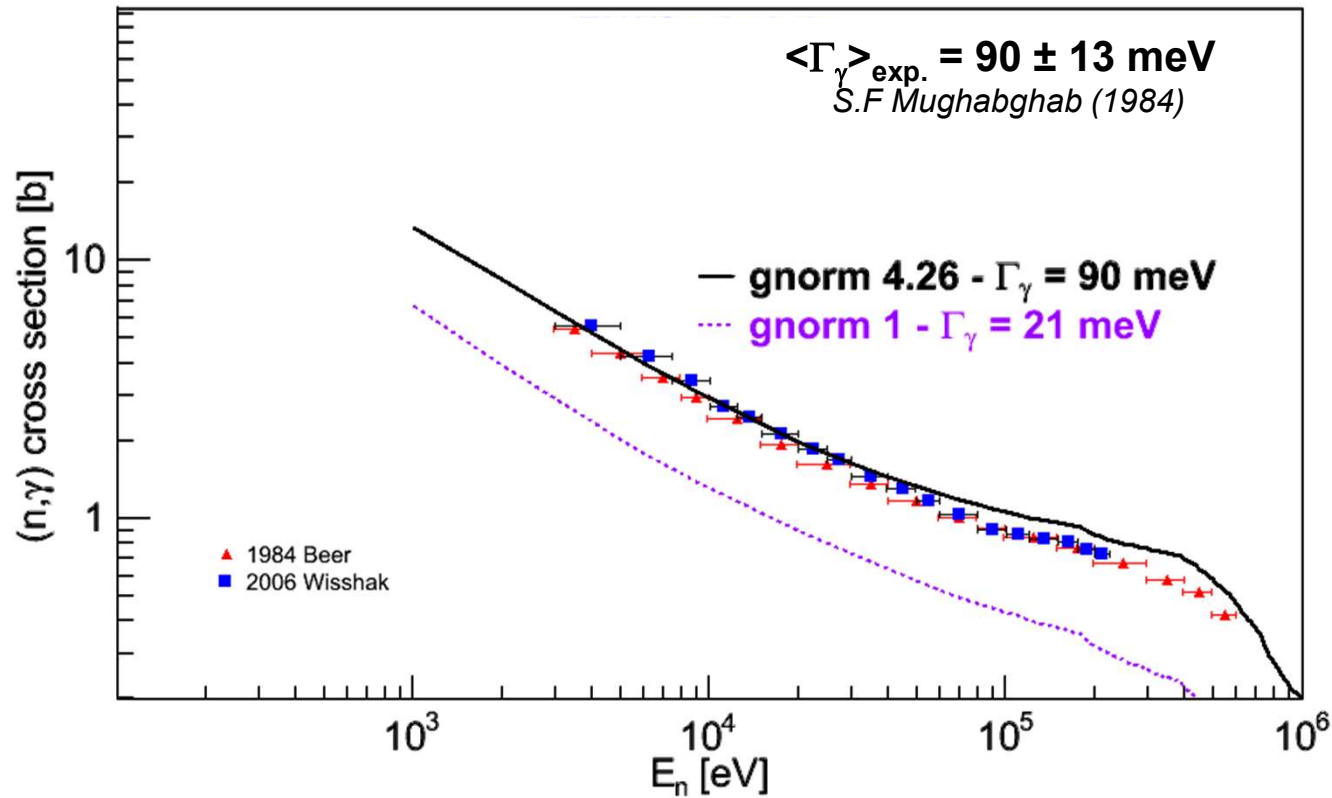
# Gamma-ray strengths : normalizations



Normalisation method for thermal neutrons

$$\langle T_\gamma \rangle = C \sum_{J_i, \pi_i} \sum_{k\lambda} \sum_{J_f, \pi_f} \int_0^{B_n} T^{k\lambda}(\varepsilon) \rho(B_n - \varepsilon, J_f, \pi_f) S(k, \lambda, J_i, \pi_i, J_f, \pi_f) d\varepsilon = 2\pi \langle \Gamma_\gamma \rangle \frac{1}{D_0}$$

experiment



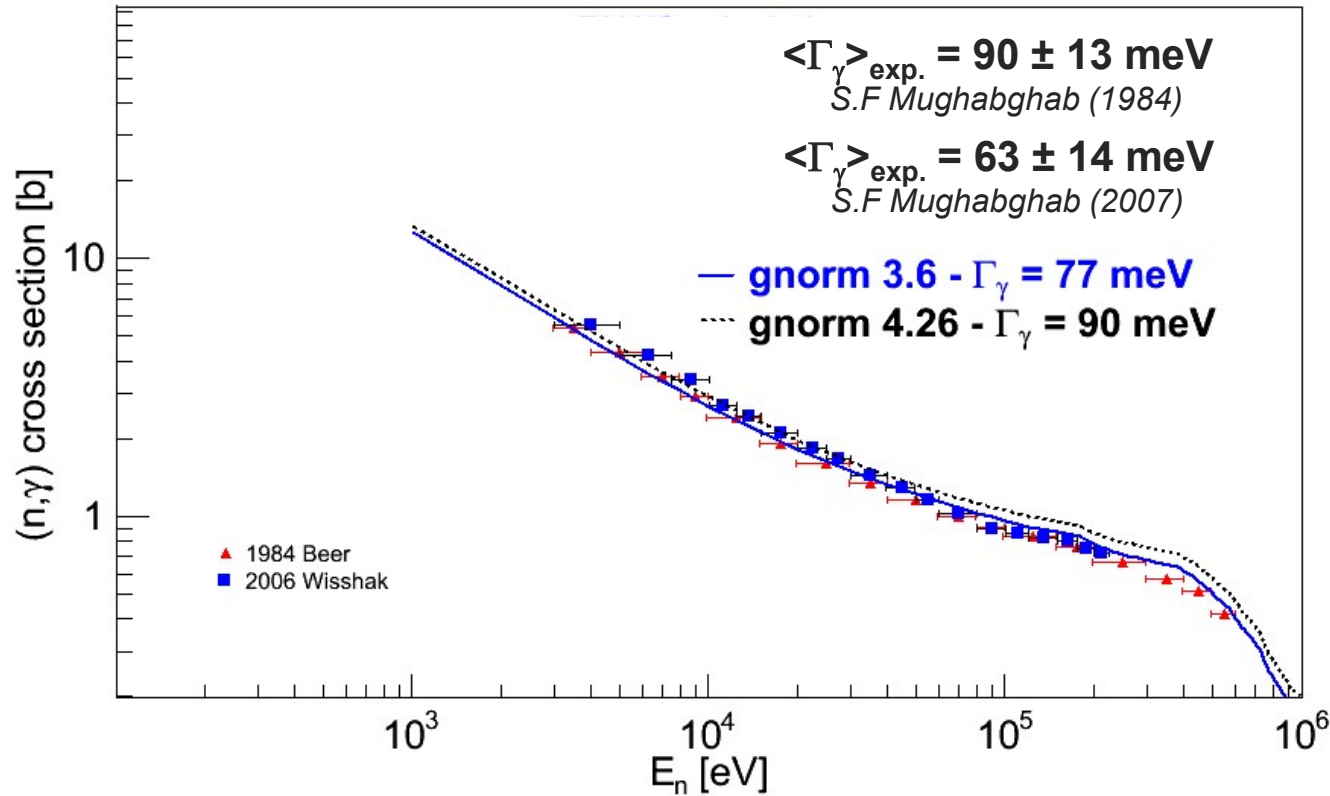
# Gamma-ray strengths : normalizations



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experiment



Experiment  
revisited

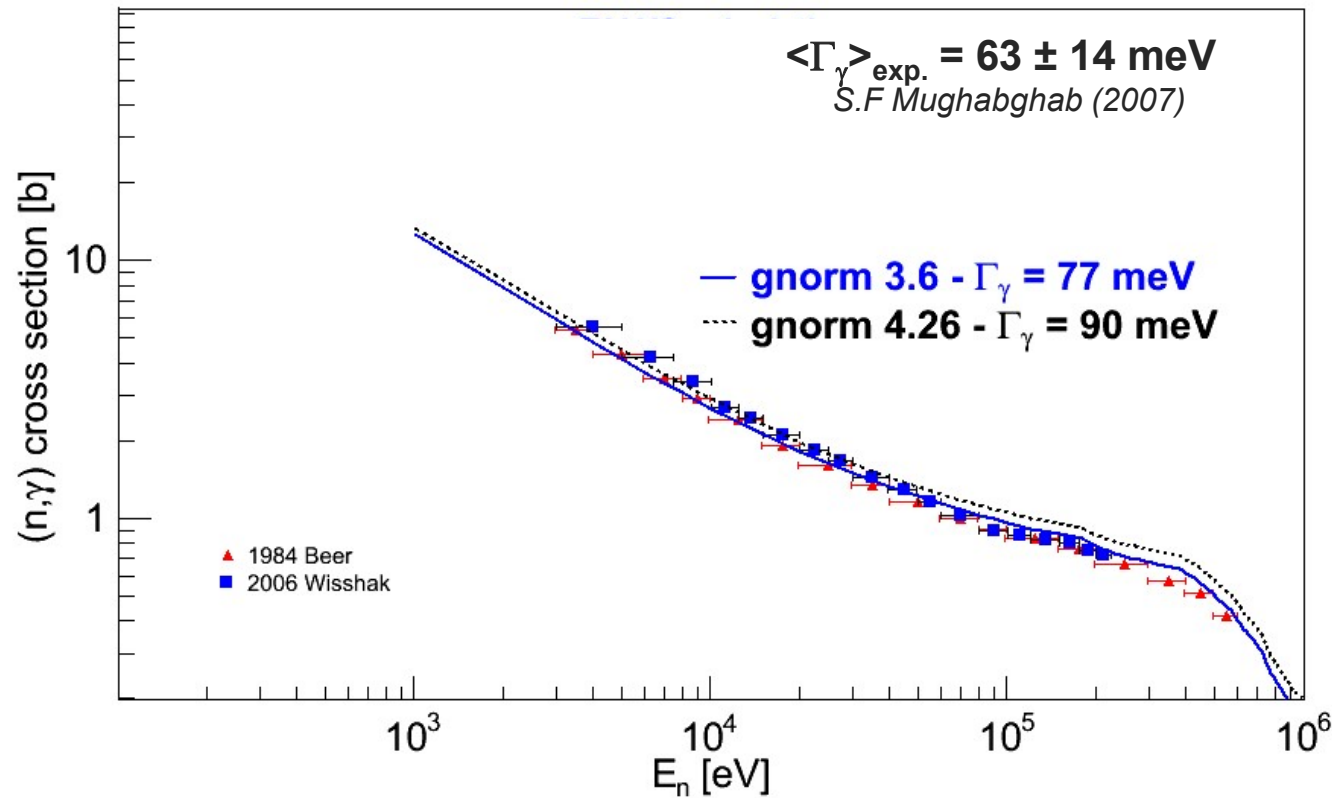


# Gamma-ray strengths : normalizations

Normalisation method for thermal neutrons

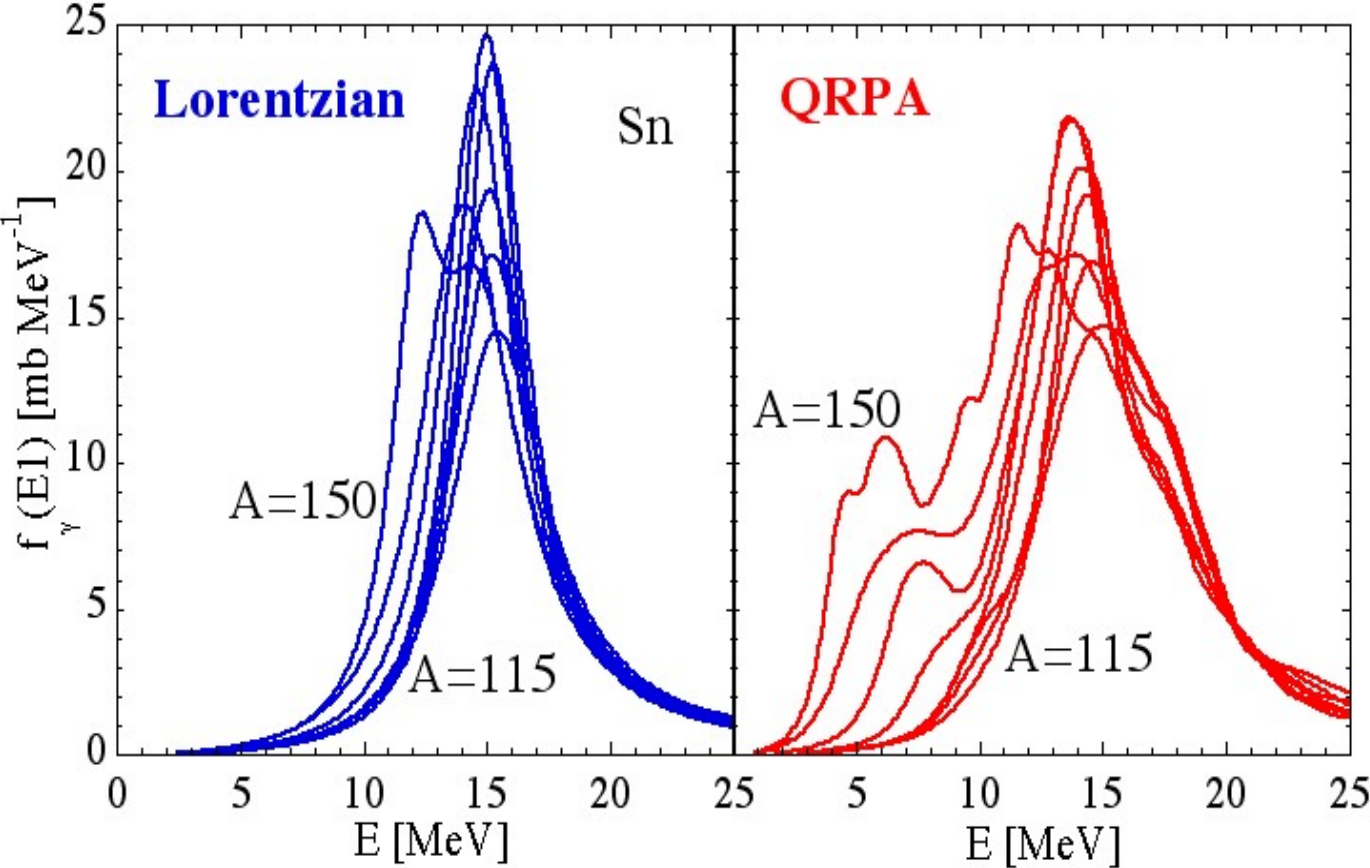
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experiment



Experiment  
revisited

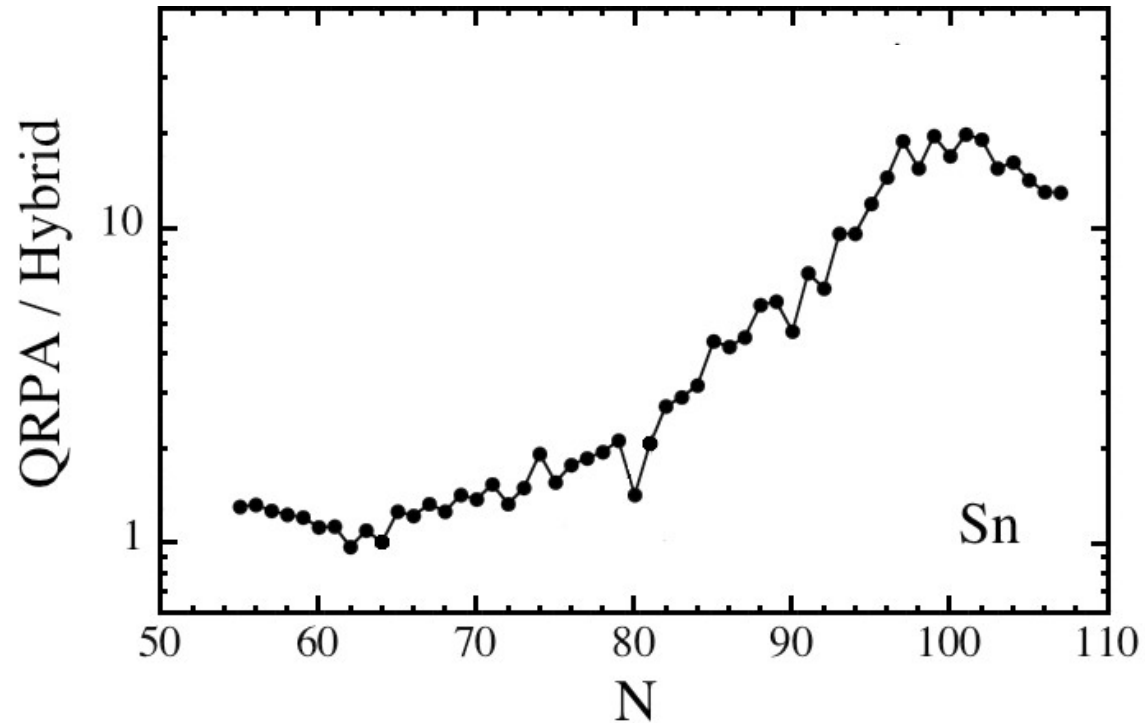
# Gamma-ray strengths : exotic nuclei



# Gamma-ray strengths : exotic nuclei



Capture cross section @  $E_n=10$  MeV for Sn isotopes



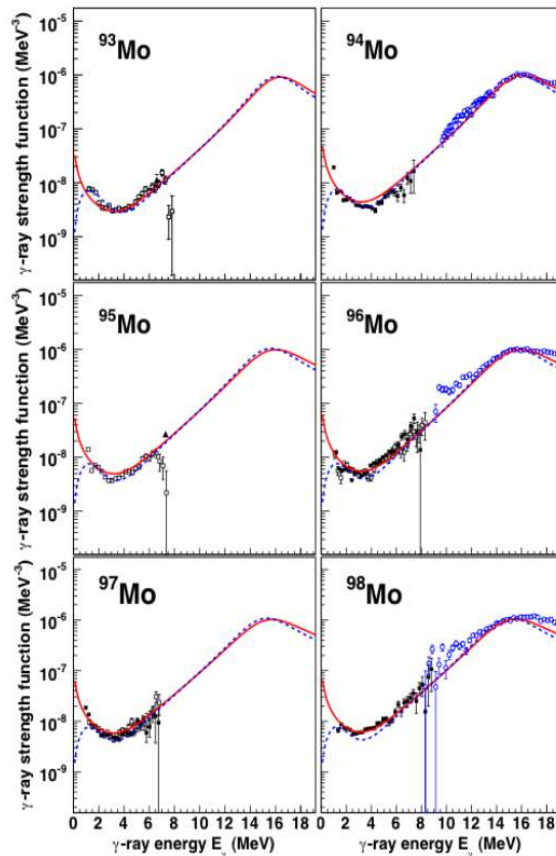
⇒ Weak impact close to stability but large for exotic nuclei

# Gamma-ray strengths : Hot topics



## Low energy upbend of gamma-ray strength observed in several experiment

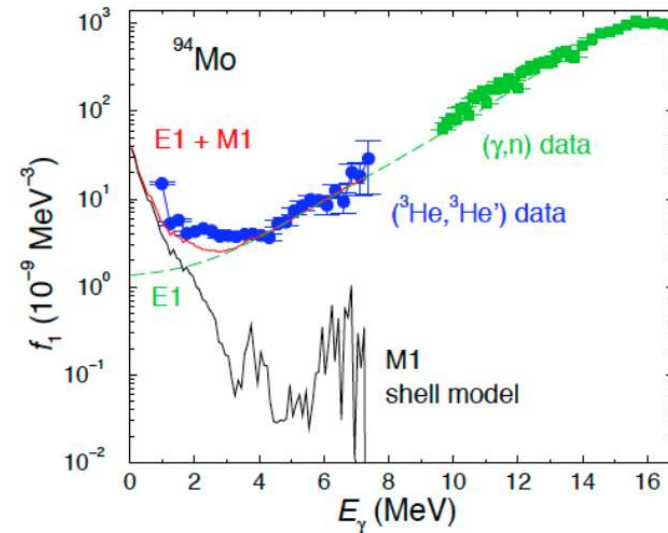
particle- $\gamma$  coincidence in the ( $^3\text{He}, \alpha\gamma$ ) & ( $^3\text{He}, ^3\text{He}'\gamma$ ) reactions



A.-C. Larsen et al. (2009)

Upbend observed for  $^{44,45}\text{Sc}$ ,  $^{50,51}\text{V}$ ,  $^{56,57}\text{Fe}$ ,  $^{73-74}\text{Ge}$ ,  $^{93-98}\text{Mo}$ , Sm but not (yet) for Sn, Dy, Er or Yb

The  $M1$  character of the upbend seems to be confirmed by shell model calculations (though an  $E1$  character cannot be excluded yet)



R. Schwengner et al. (2013); Brown & Larsen (2014); Sieja (2016)



# Gamma-ray strengths : Hot topics



Low energy upbend of gamma-ray strength observed in several experiment

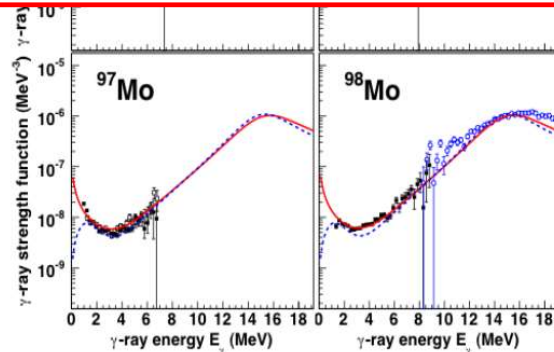
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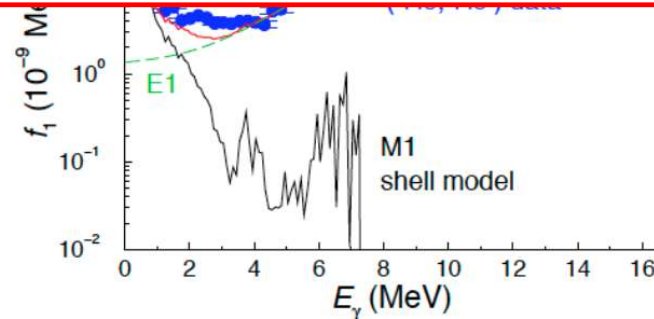
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Upbend interpreted by Shell model as transitions between excited states (intra-band) rather than between excited states and ground state.

Could be calculated within QRPA framework provided a few more developments and “much more calculation”



A.-C. Larsen et al. (2009)



R. Schwengner et al. (2013); Brown & Larsen (2014); Sieja (2016)

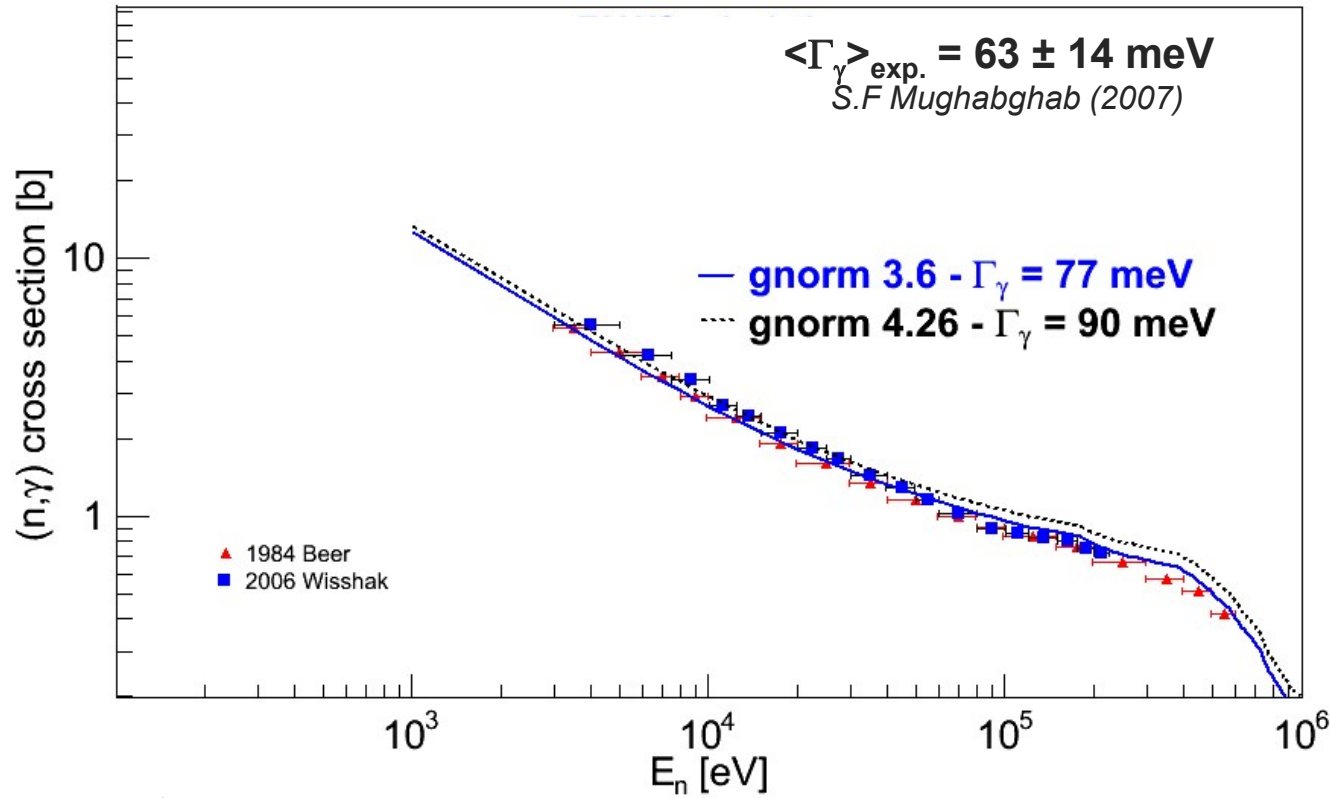
# Gamma-ray strengths : low energy missing strength



Normalisation method for thermal neutrons

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experiment

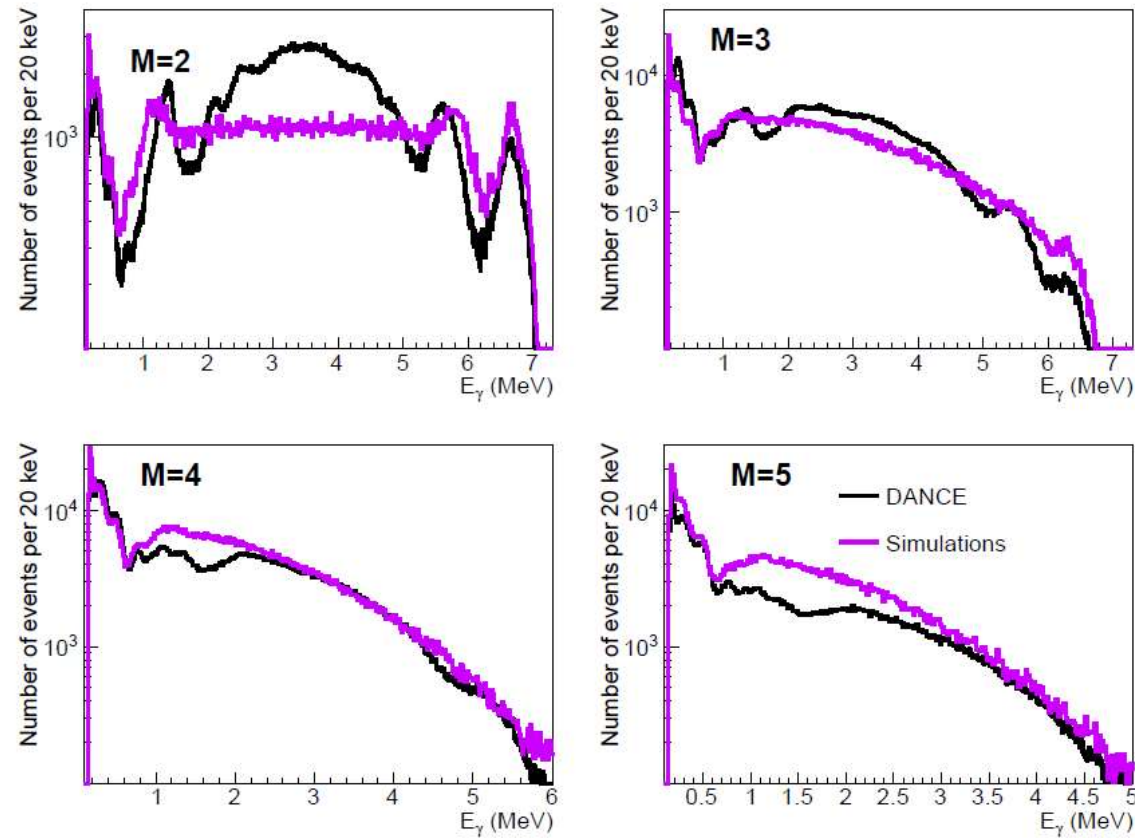


Experiment  
revisited



# Gamma-ray strengths : low energy missing strength

Capture cross section OK but gamma spectra constrained by multiplicity not reproduced !





# Gamma-ray strengths : low energy missing strength

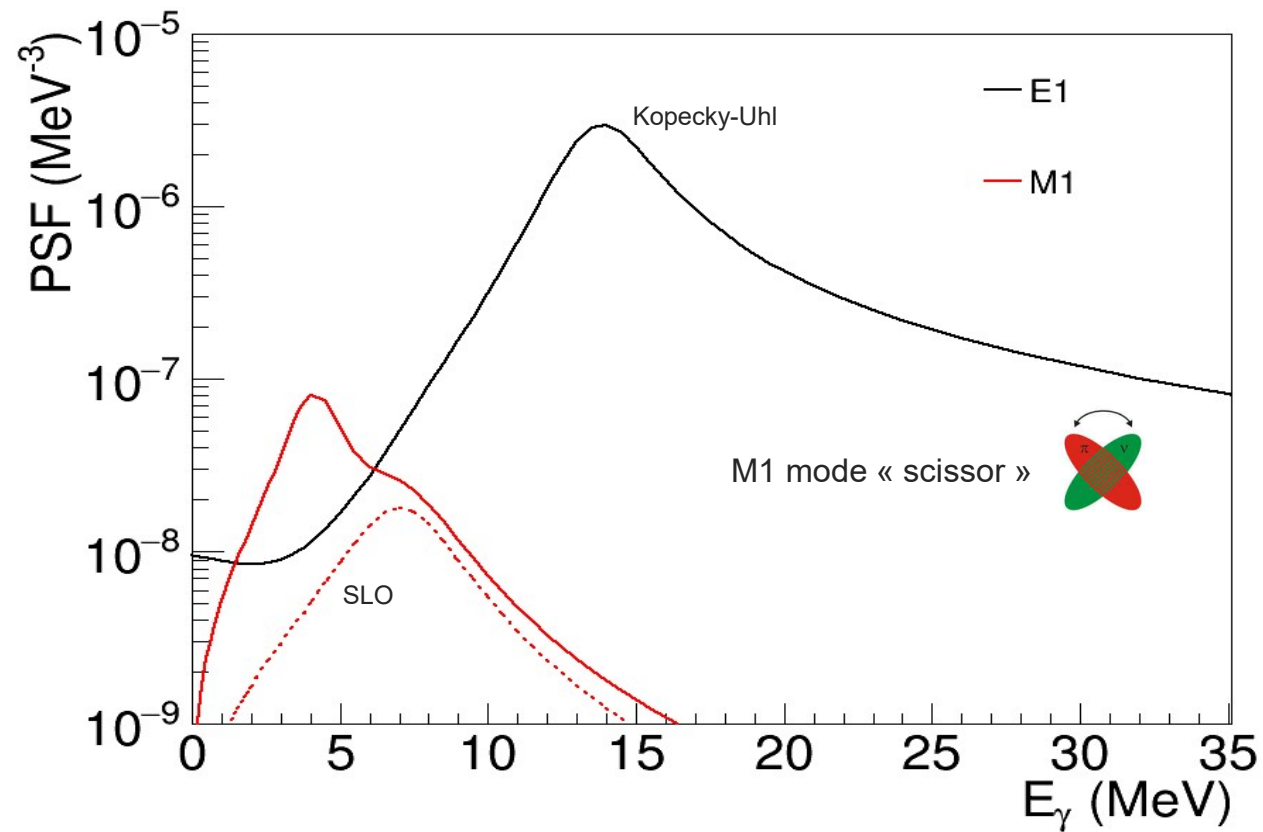
Capture cross section OK but gamma spectra constrained by multiplicity not reproduced !



# Gamma-ray strengths : low energy missing strength

Capture cross section OK but gamma spectra constrained by multiplicity not reproduced !

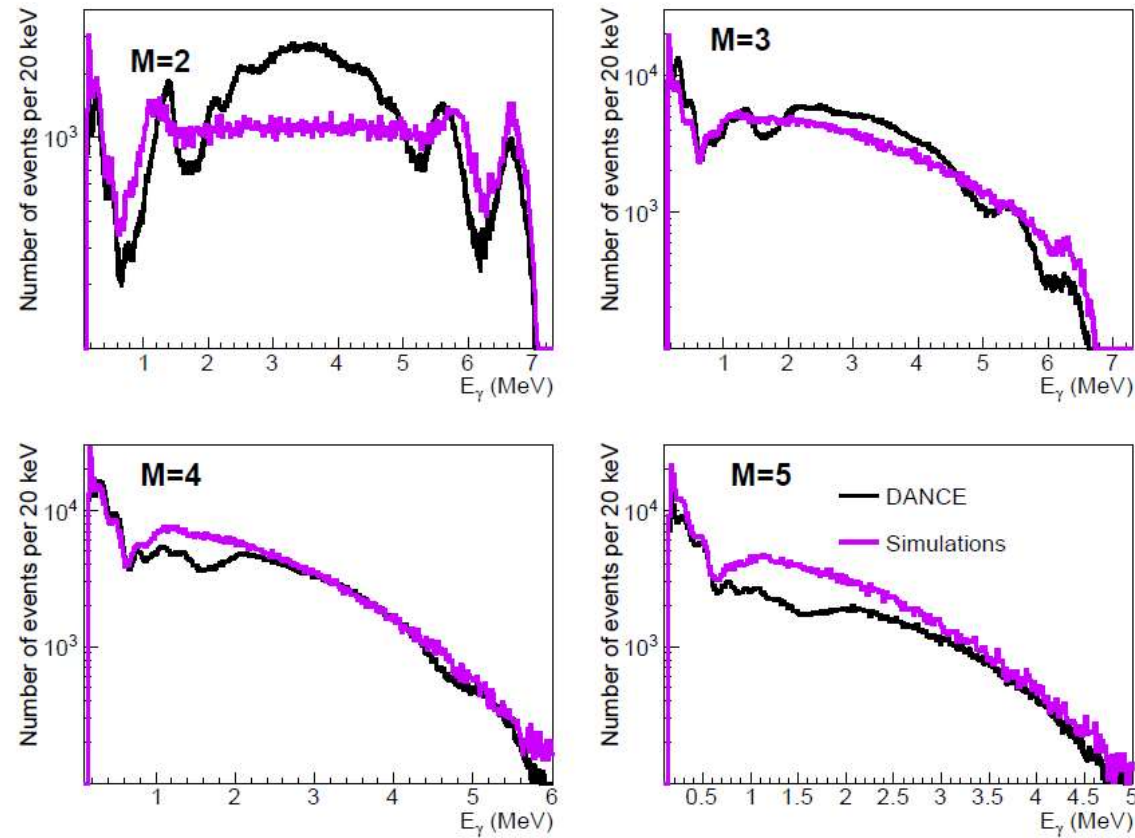
⇒ New resonance added at energies around 4 MeV (E1 or E2 pygmy resonance or M1 scissor mode)





# Gamma-ray strengths : low energy missing strength

Capture cross section OK but gamma spectra constrained by multiplicity not reproduced !

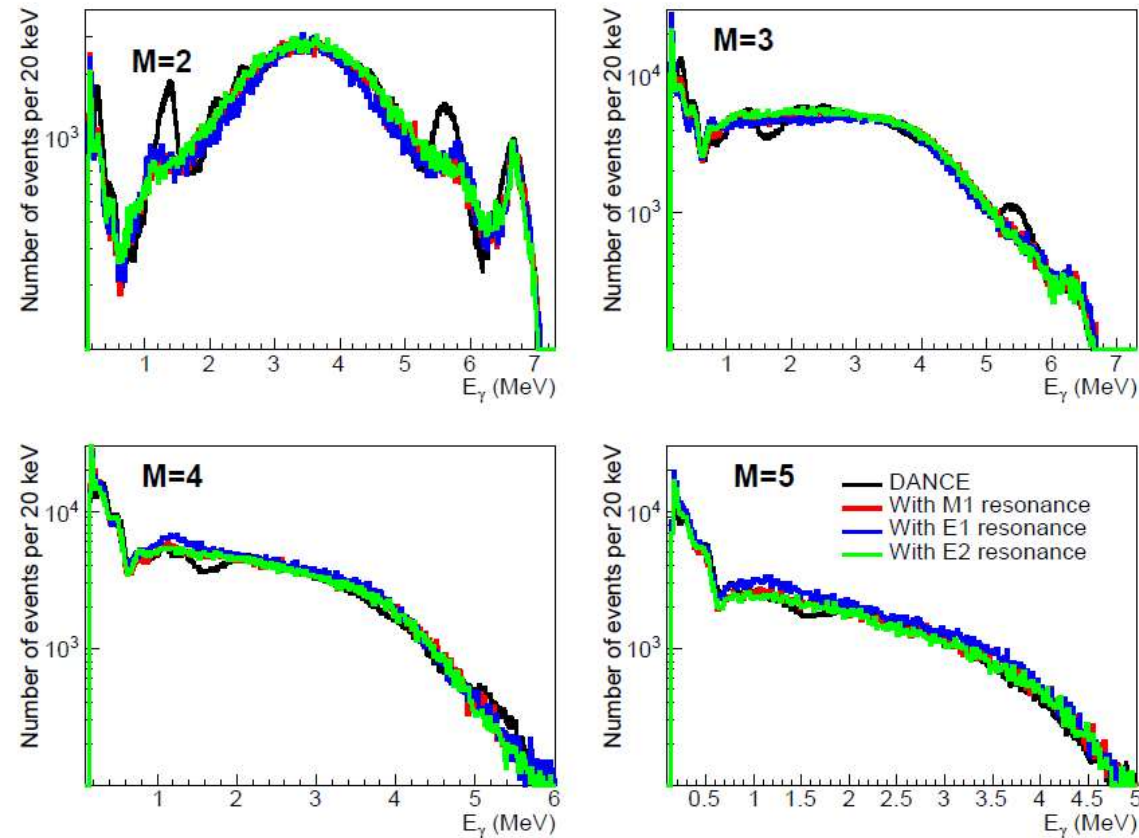




# Gamma-ray strengths : low energy missing strength

Capture cross section OK but gamma spectra constrained by multiplicity not reproduced !

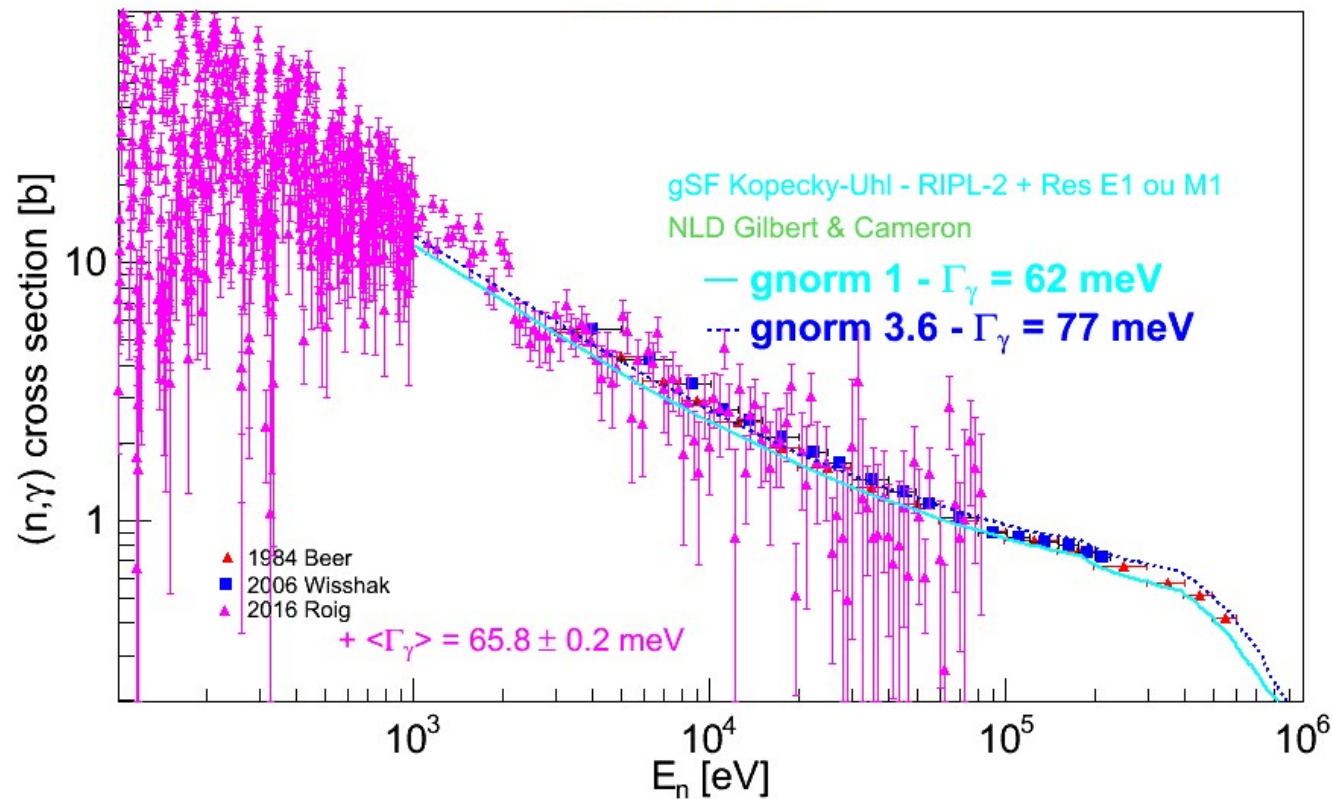
⇒ New resonance added at energies around 4 MeV (E1 or E2 pygmy resonance or M1 scissor mode)





# Gamma-ray strengths : low energy missing strength

Capture cross section OK + gamma spectra OK and no more arbitrary normalization

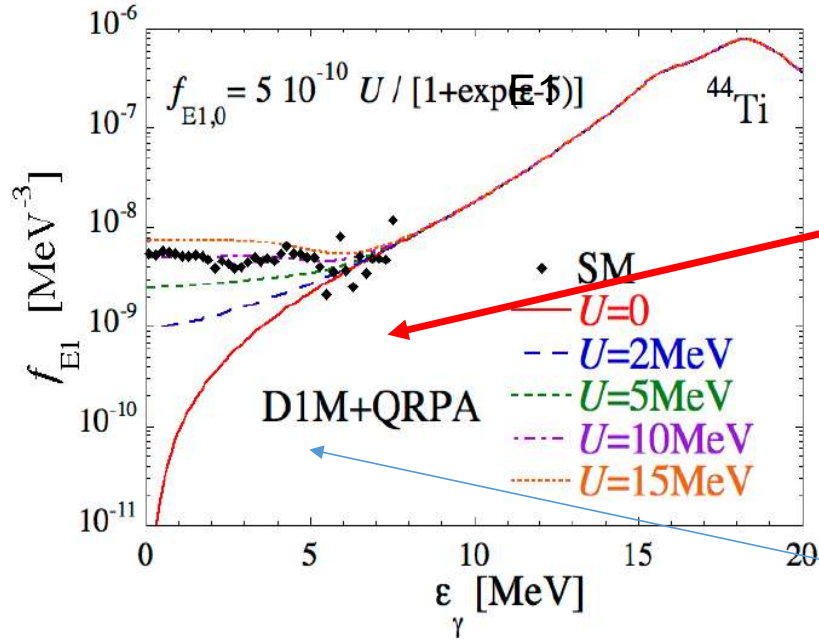






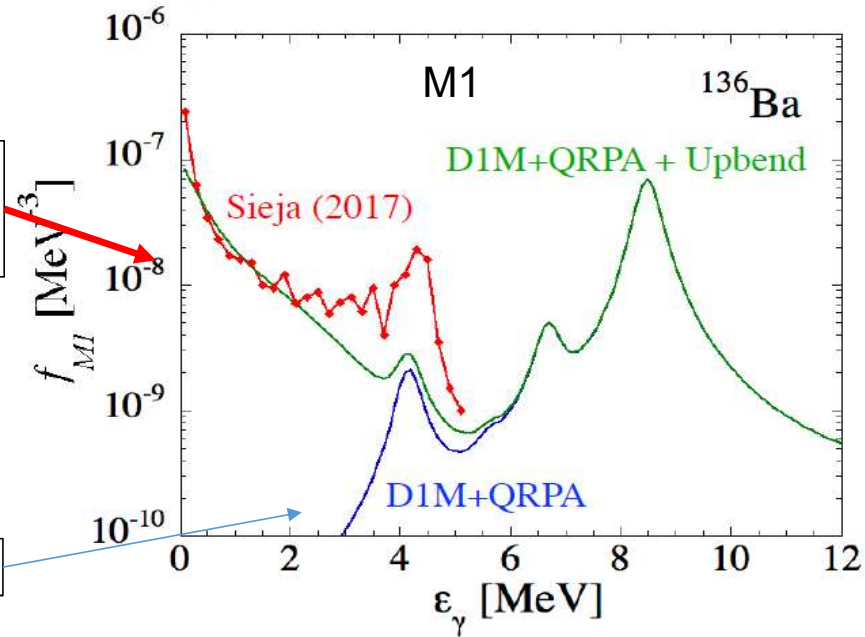
# Gamma-ray strengths : low energy missing strength

⇒ Shell model based correction added to QRPA predictions



QRPA + SM fit

QRPA

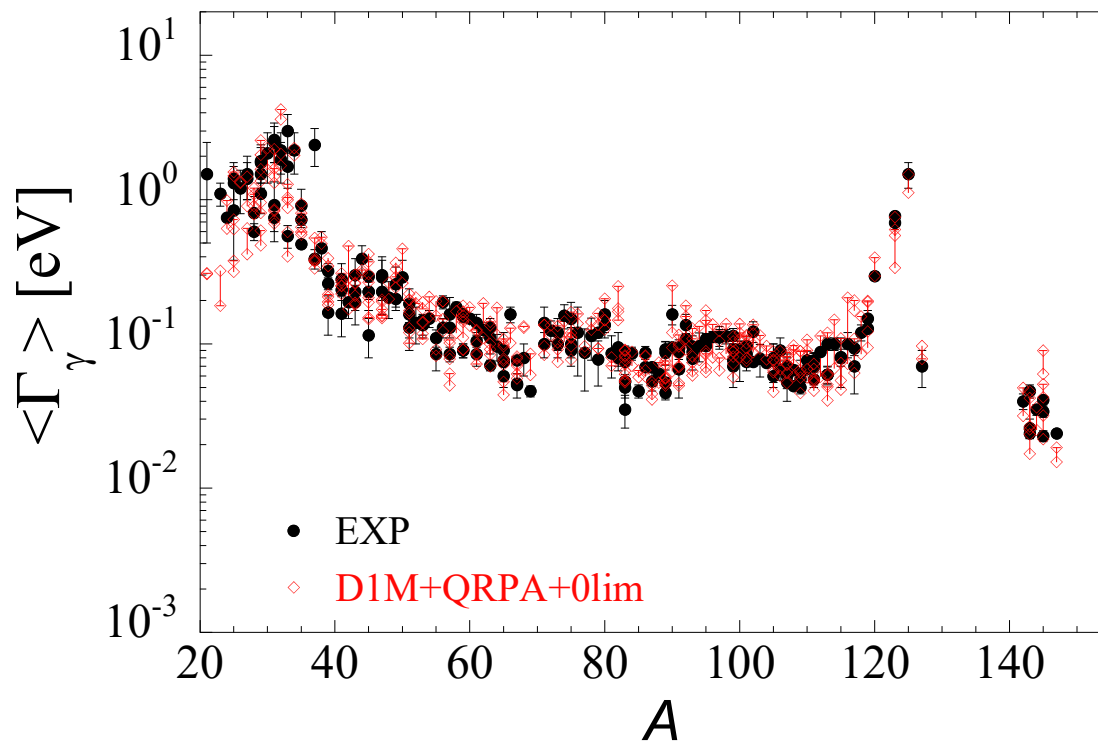
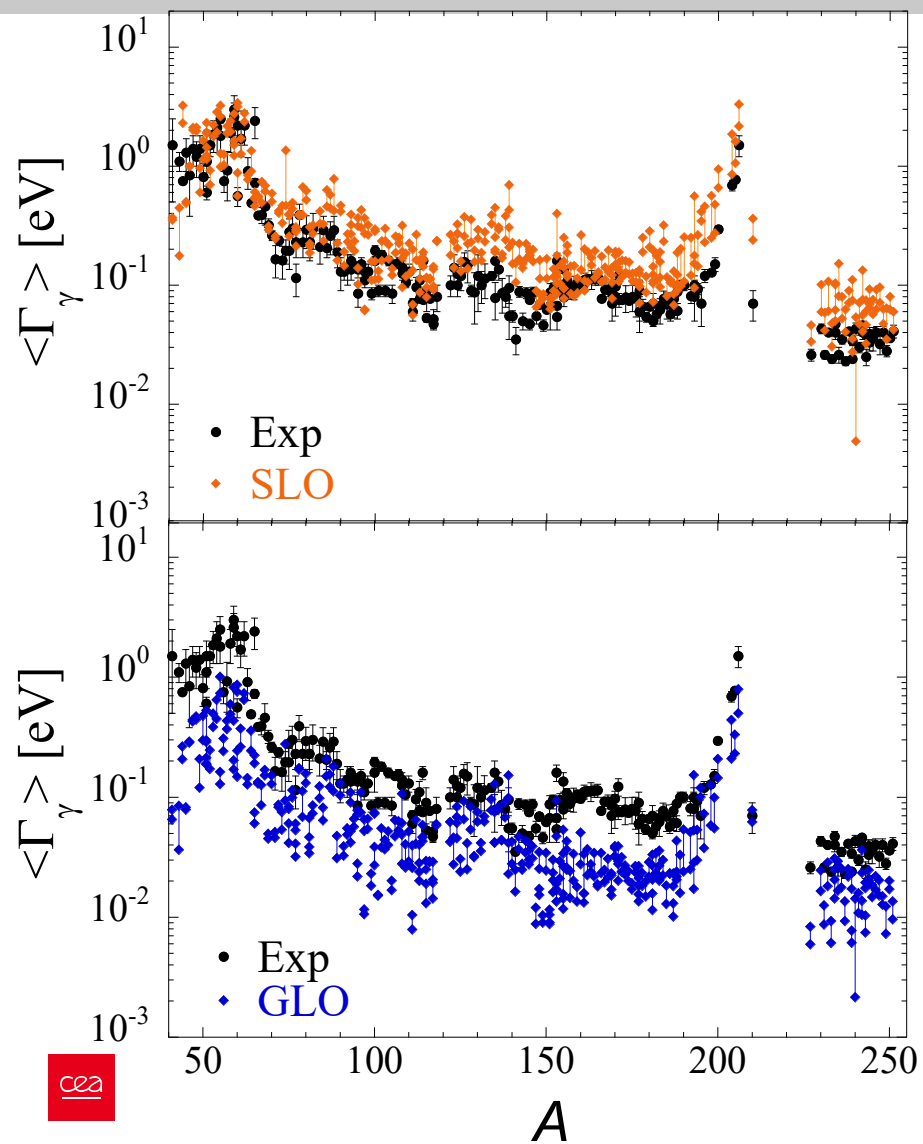


$$f_{E1}^{QRPA}(E\gamma) = f_{E1}^{QRPA}(E\gamma) + \frac{f_0 U}{1 + e^{(E\gamma - E_0)}}$$

$$f_{M1}^{QRPA}(E\gamma) = f_{M1}^{QRPA}(E\gamma) + C e^{-\eta E\gamma}$$



# Gamma-ray strengths : Gogny QRPA vs analytical

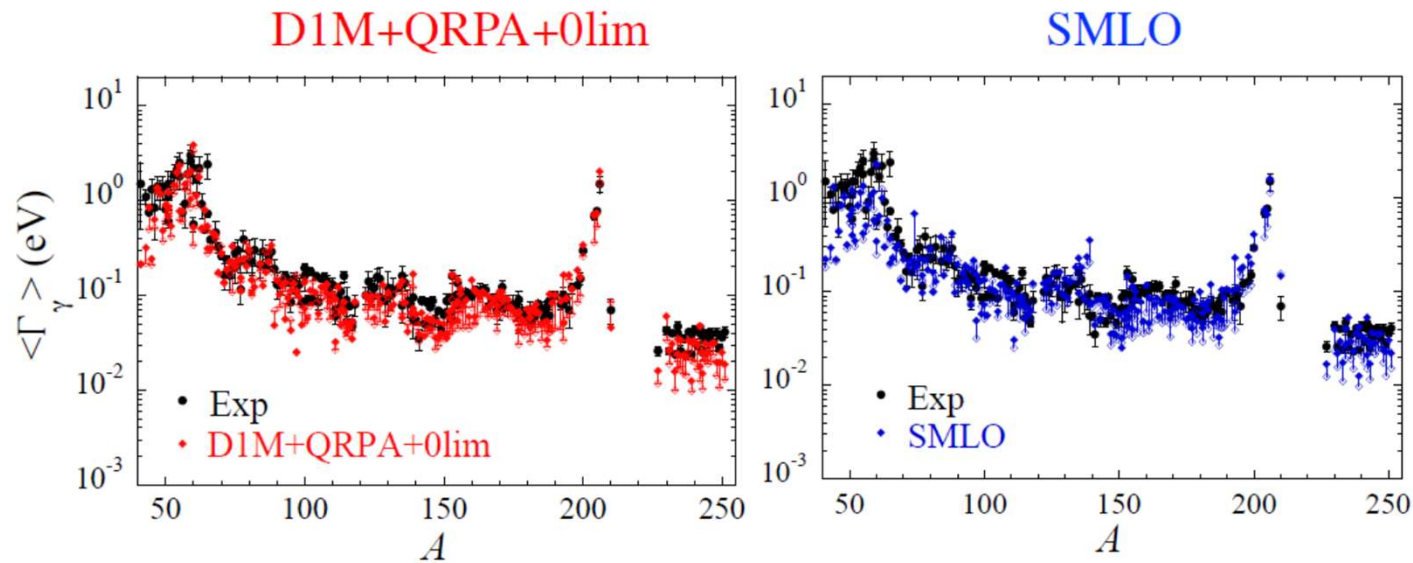




# Gamma-ray strengths : Gogny QRPA vs analytical

Comparison of **D1M+QRPA+0lim** and **SMLO** with  $\langle \Gamma_\gamma \rangle$  data

$$\langle \Gamma_\gamma \rangle = \frac{D_0}{2\pi} \sum_{X,L,J,\pi} \int_0^{S_n+E_n} T_{XL}(\varepsilon_\gamma) \times \rho(S_n + E_n - \varepsilon_\gamma, J, \pi) d\varepsilon_\gamma$$



Open diamonds = CT + BSFG  
Full diamonds = HFB + Combinatorial

Both PSF models reproduce  $\sim 230 \langle \Gamma_\gamma \rangle$  within  $\sim 30-50\%$



## Gamma-ray strengths : various options in TALYS

projectile n

element u

mass 278

energy 1.

strength 1 → 10

- strength = 1 : GLO model (Kopecky & Uhl 1990)
- strength = 2 : SLO model
- strength = 3 : Skyrme-HFBCS + QRPA
- strength = 4 : Skyrme-HFB + QRPA
- strength = 5 : Hybrid model
- strength = 6 :  $T$ -dependent Skyrme-HFB + QRPA
- strength = 7 :  $T$ -dependent RMF-HFB + QRPA
- strength = 8 : Gogny-HFB + QRPA
- strength = 9 : SMLO 2019
- strength = 10 :  $T$ -dependent Bsk27-HFB + QRPA

With many options to modify/adjust the strength parameters

- Analytical formulas (strength=1,2,5):  $\sigma_i$ ,  $\Gamma_i$ ,  $E_p$  ... for GR and PR
- Microscopic formulas (strength=3-4,6-8): “etable”, “ftable”, “wtable”

# Gamma-ray strengths : summary



International Atomic Energy Agency  
**Nuclear Data Services**  
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## Reference Input Parameter Library (RIPL-3)

R. Capote, M. Herman, P. Oblozinsky, P.G. Young, S. Goriely, T. Belgya, A.V. Ignatyuk, A.J. Koning, S. Hilaire, V.A. Plujko, M. Avrigeanu, O. Bersillon, M.B. Chadwick, T. Fukahori, Zhigang Ge, Yinlu Han, S. Kailas, J. Kopecky, V.M. Maslov, G. Reffo, M. Sin, E.Sh. Soukhovitskii and P. Talou

Nuclear Data Sheets - Volume 110, Issue 12, December 2009, Pages 3107-3214

RIPL discrete levels database should be corrected for +X,... levels, new release soon.

Introduction | **MASSES** | LEVELS | RESONANCES | OPTICAL | DENSITIES | **GAMMA** | FISSION | CODES | Contacts

### Introduction

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# Gamma-ray strengths : summary



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## Gamma-ray strength (formulae, tables)

- experimental gamma width
- theoretical GDR
- microscopic tables

# Gamma-ray strengths : summary



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# **2.** FISSION TRANSMISSION COEFFICIENTS





# FISSION TRANSMISSION EVENTS



To be discussed on thursday !