

Fission reactions

S. Hilaire

CEA, DAM, DIF

Content

- Introduction

- General features about nuclear reactions

- Time scales and associated models
- Types of data needed
- Data format = f (users)

- Nuclear Models

- Basic structure properties
- Optical model
- Pre-equilibrium model
- Compound Nucleus model

- Model ingredients

- Level densities
- Gamma-ray strengths
- Fission transmission coefficients

- Fission reactions

- Generalities about fission
- Fission neutrons and gammas
- Fission yields
- Fission cross sections

- Prospects

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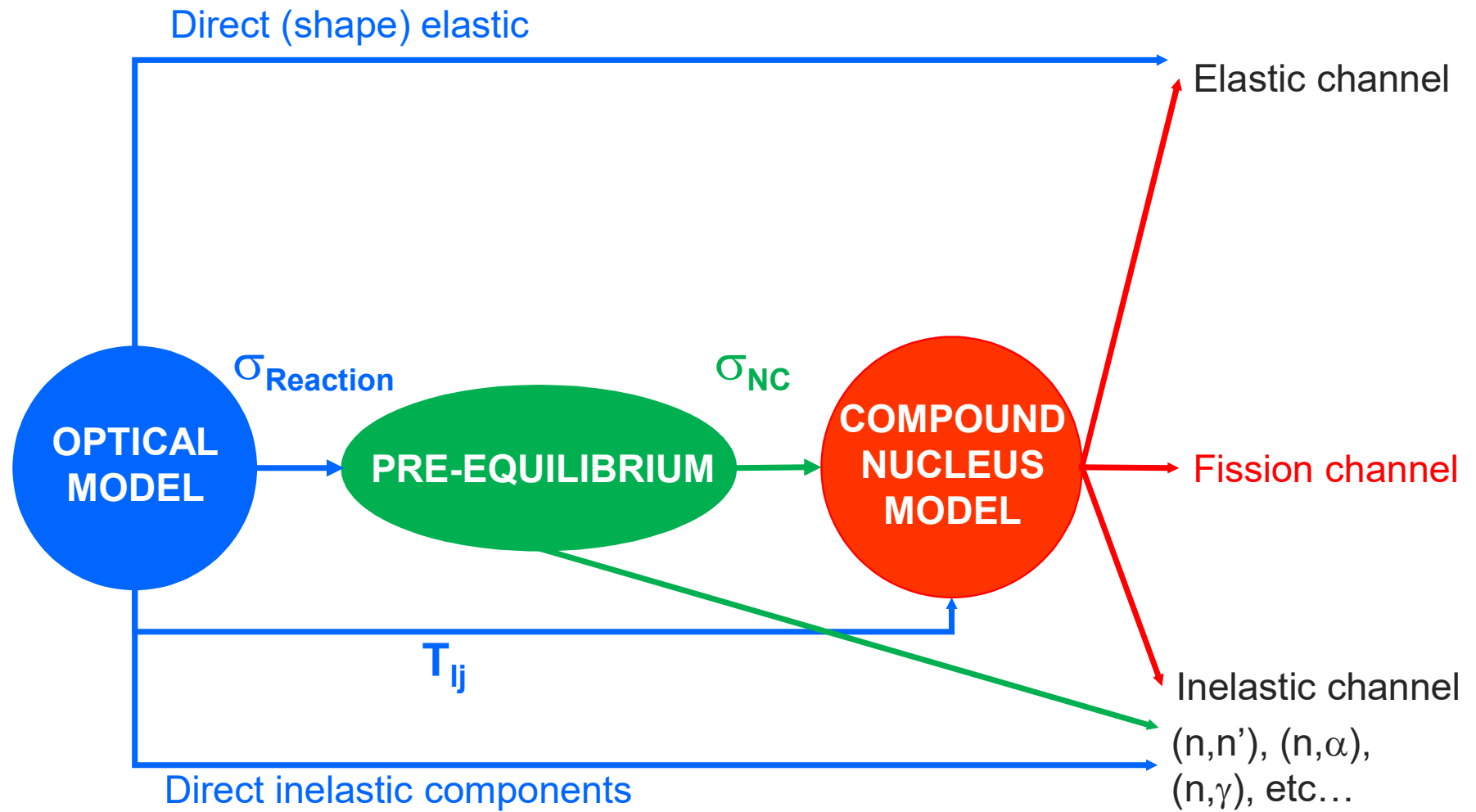
- Generalities about fission
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- Fission cross sections

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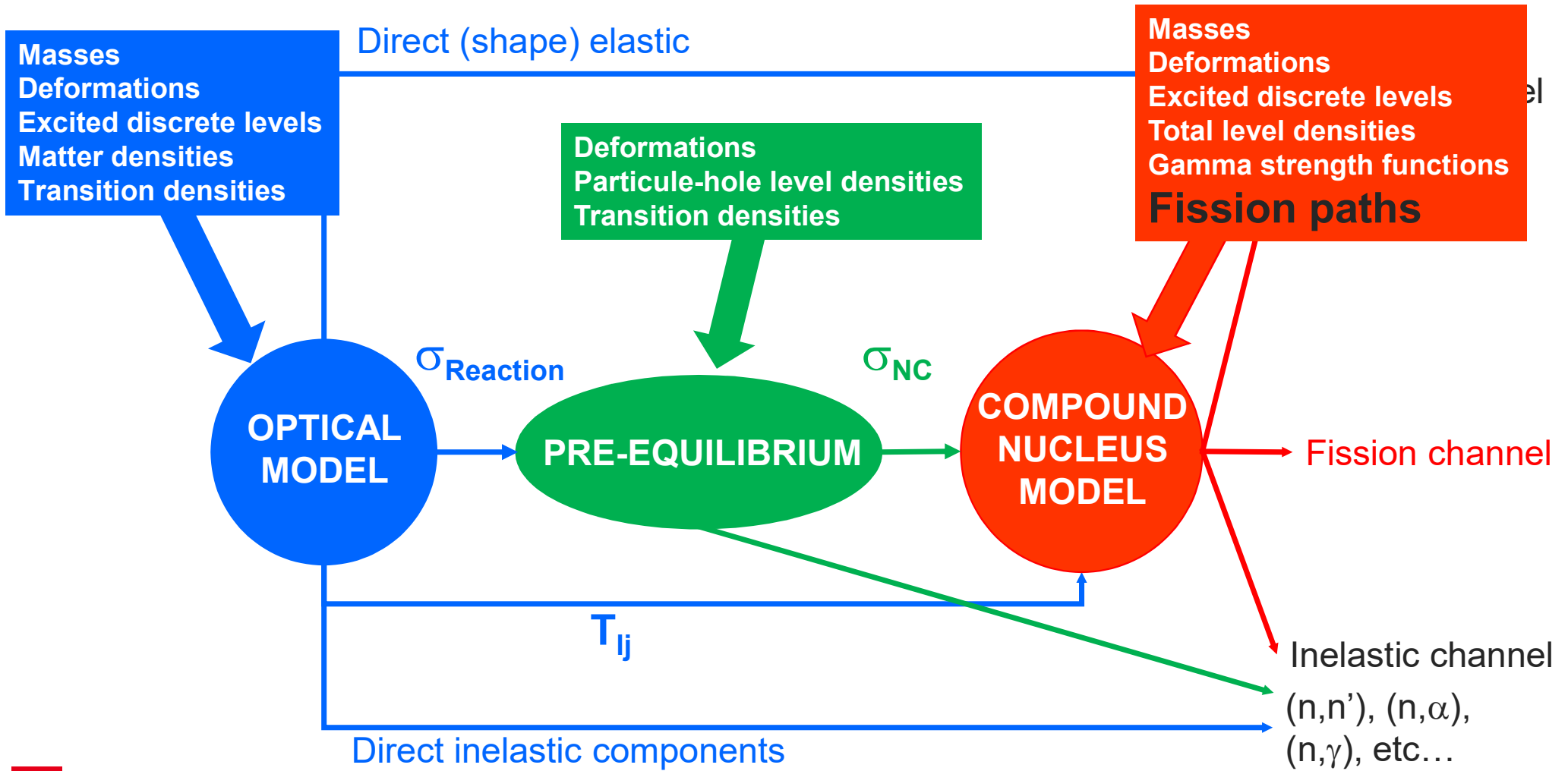


1. FEW REMINDERS

Models sequence and required ingredients

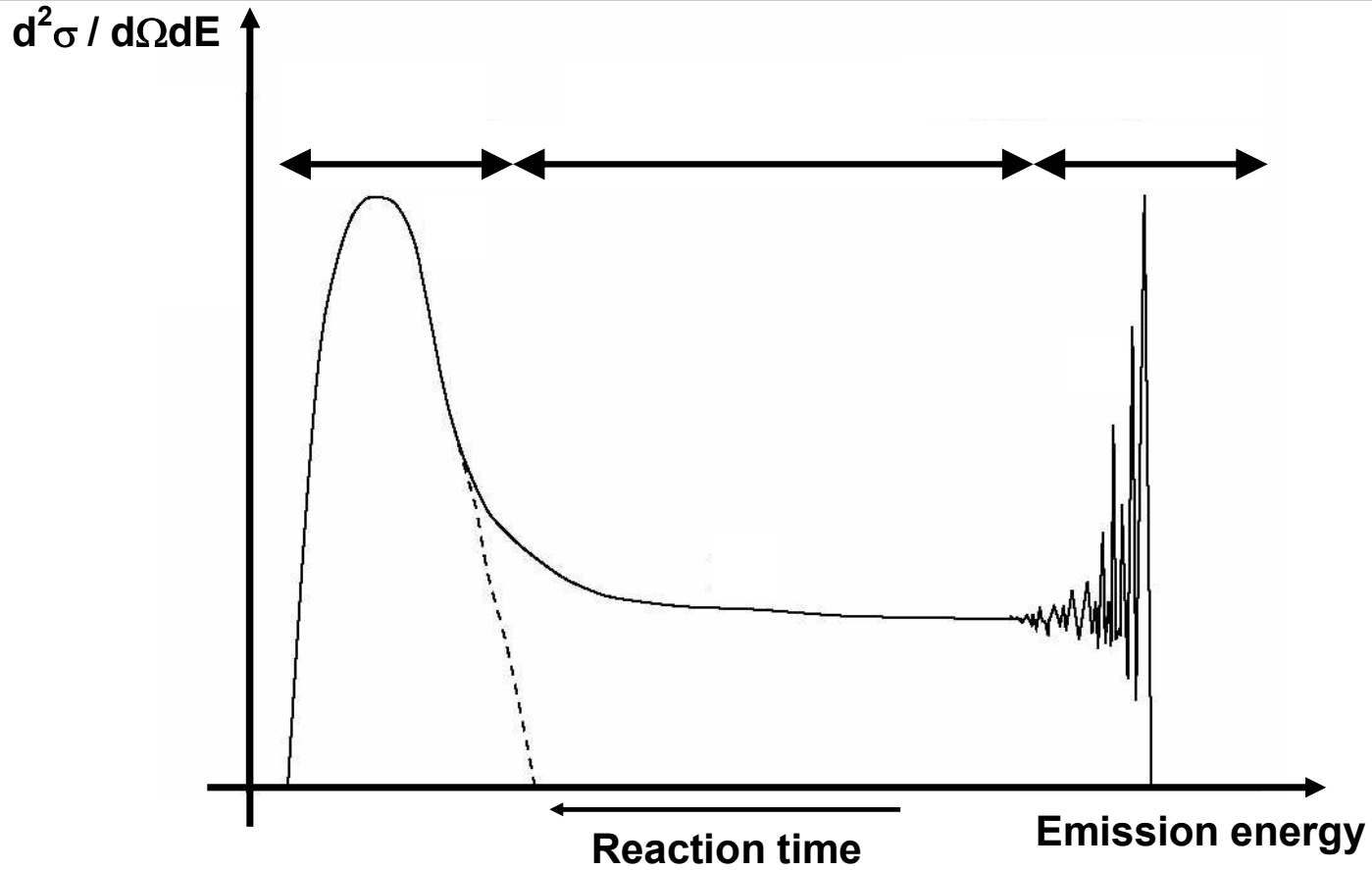


Models sequence and required ingredients





Time scales and associated models



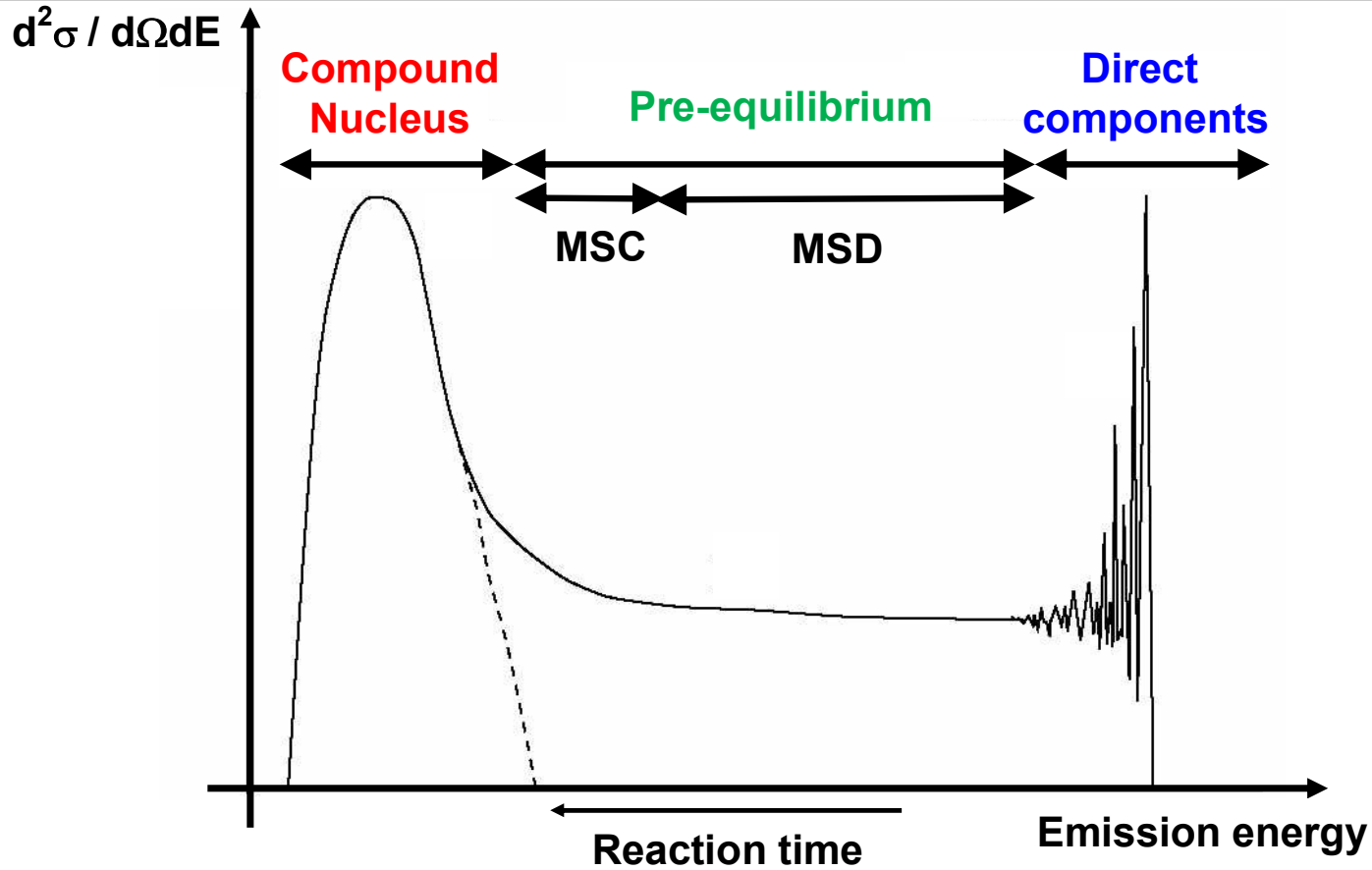
Real scale : 10^{-15} s
Human scale : year

10^{-22} s
s





Time scales and associated models



Real scale : 10^{-15} s
Human scale : year

10^{-22} s
s





Fission reactions

- Generalities about fission

Induced, spontaneous, energy balance, fission yields, PFNS, neutron multiplicities, cross sections, fission chances, Kinetic energies,

- Fission yields

- GEF model
- SPY model
- Microscopic approach

- Neutrons and gammas from fission

- Madland-Nix model
- GEF model
- FIFRELIN
- Microscopic approach

- Fission cross sections

- Phenomenological approaches
- Coherent fission cross sections
- Microscopic approaches
- Integral benchmark sensitivity

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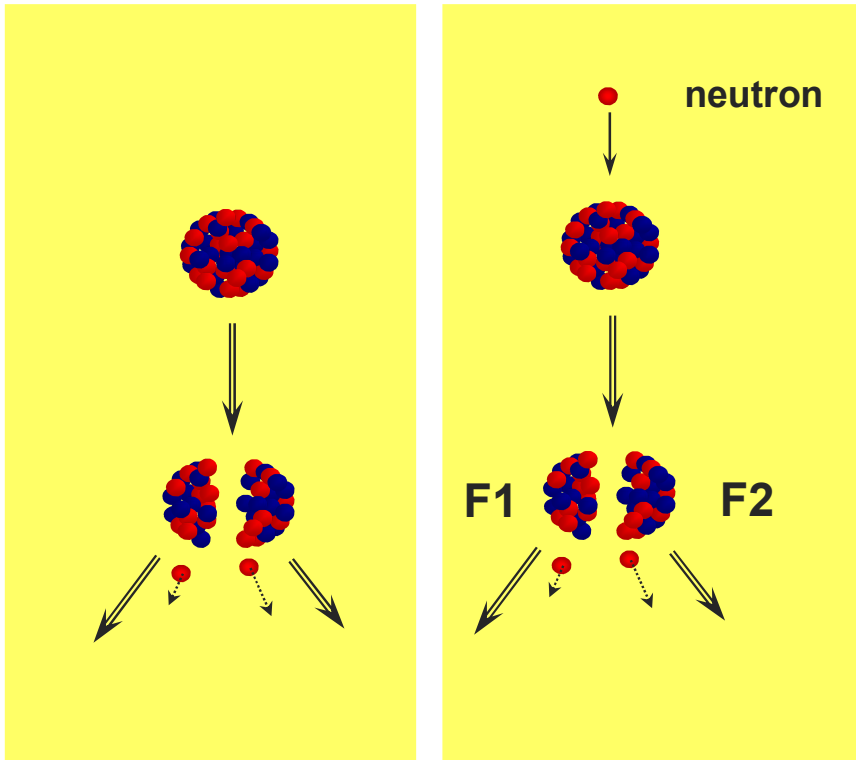
2. GENERALITIES



The fission process

spontaneous

induced



Fragments ($2+\epsilon$)

Prompt neutrons (ν)

Delayed neutrons ($\nu/100$)

Prompt gammas

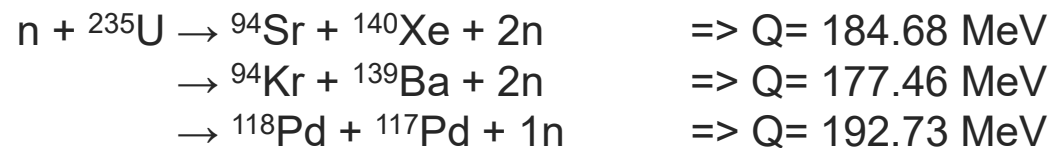
Delayed gammas

..... Energy (200 MeV)



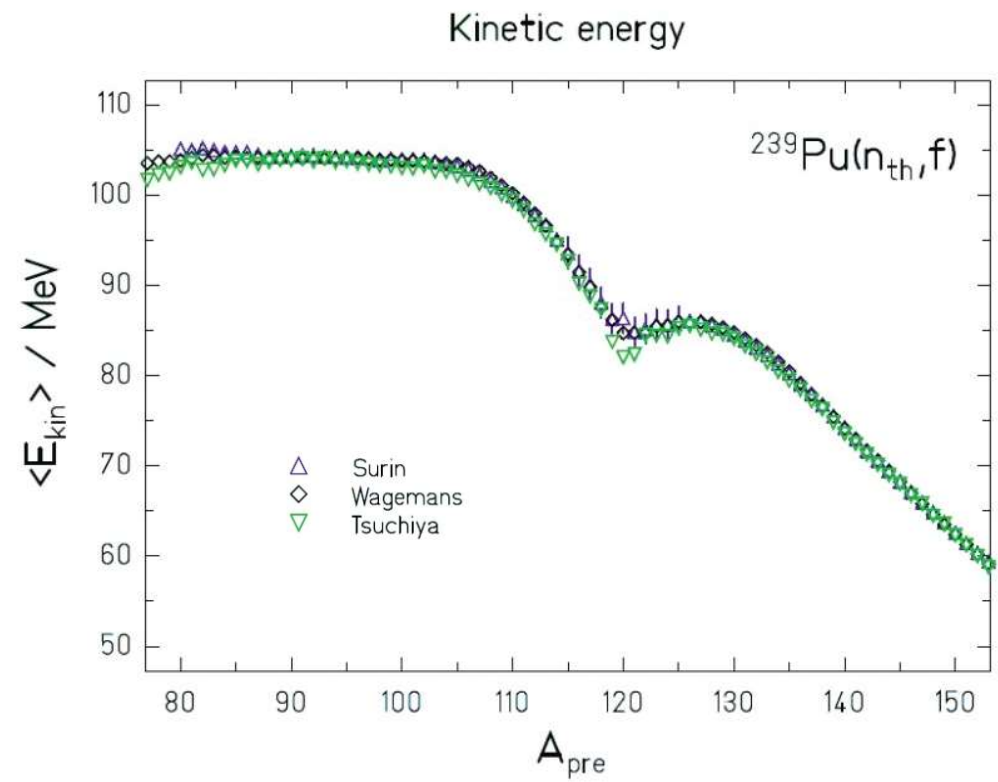
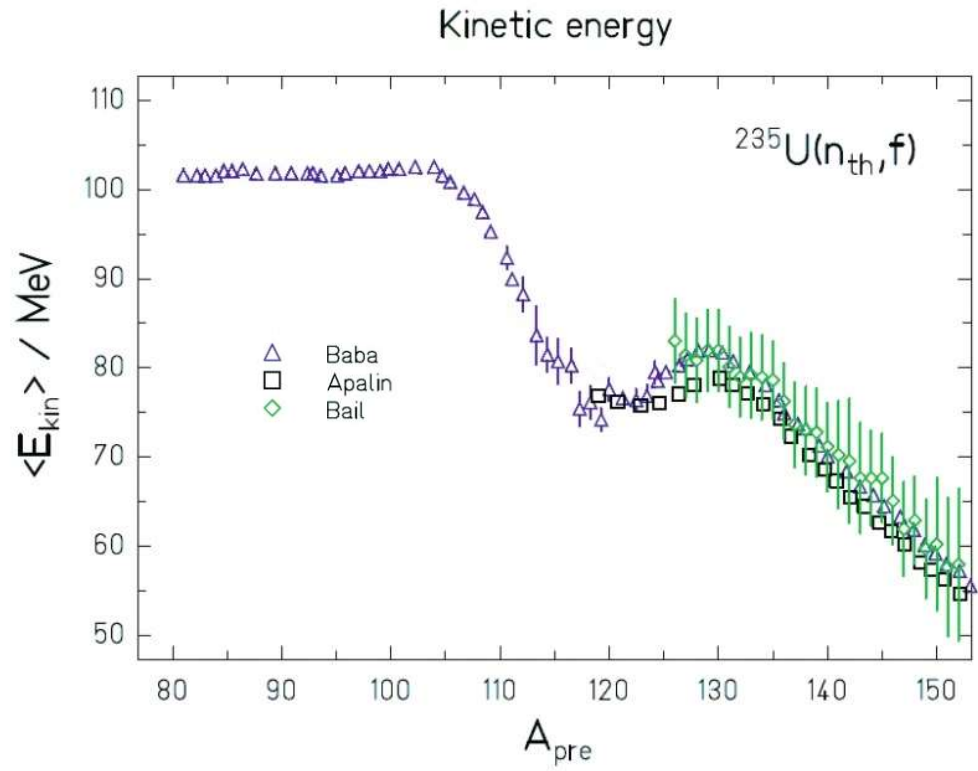
The fission process : typical energy balance

Fragments kinetic energy	165 MeV
prompt γ	8 MeV
β decay	19 MeV
delayed γ	7 MeV
prompts neutrons	5 MeV
TOTAL :	204 MeV



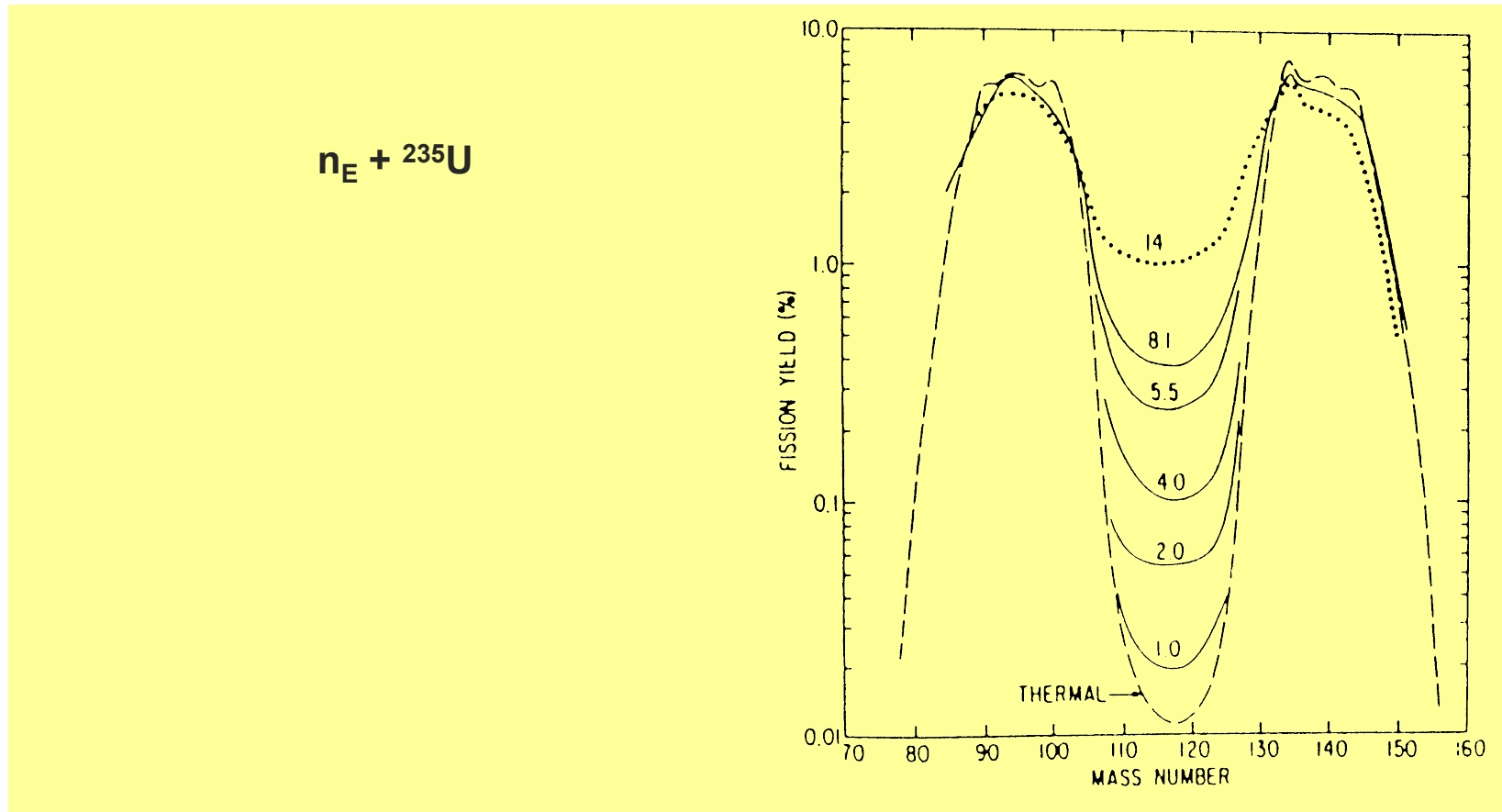


The fission process : fragments kinetic energies





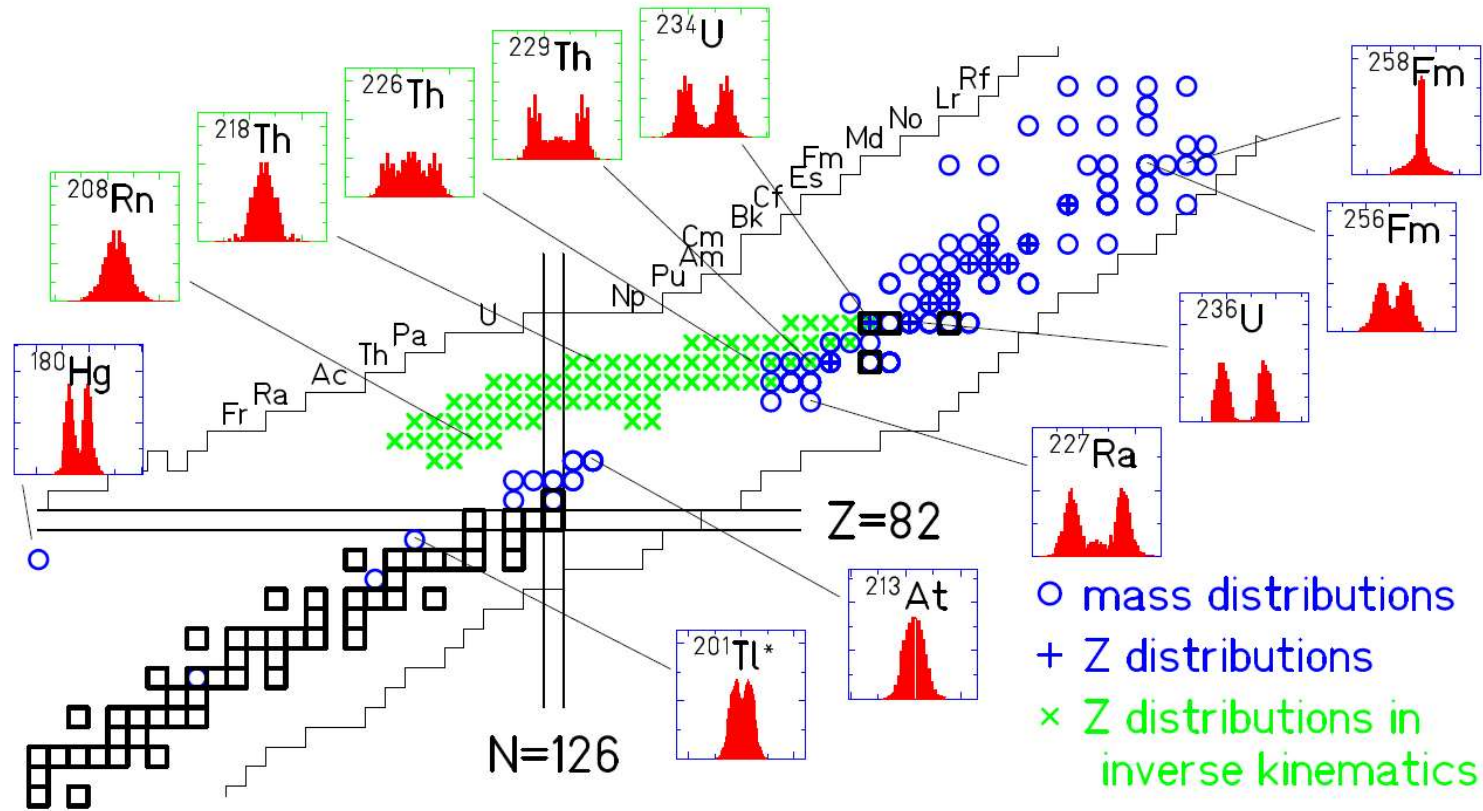
The fission process : yields variations with energy



⇒ Well filled with increasing incident energy



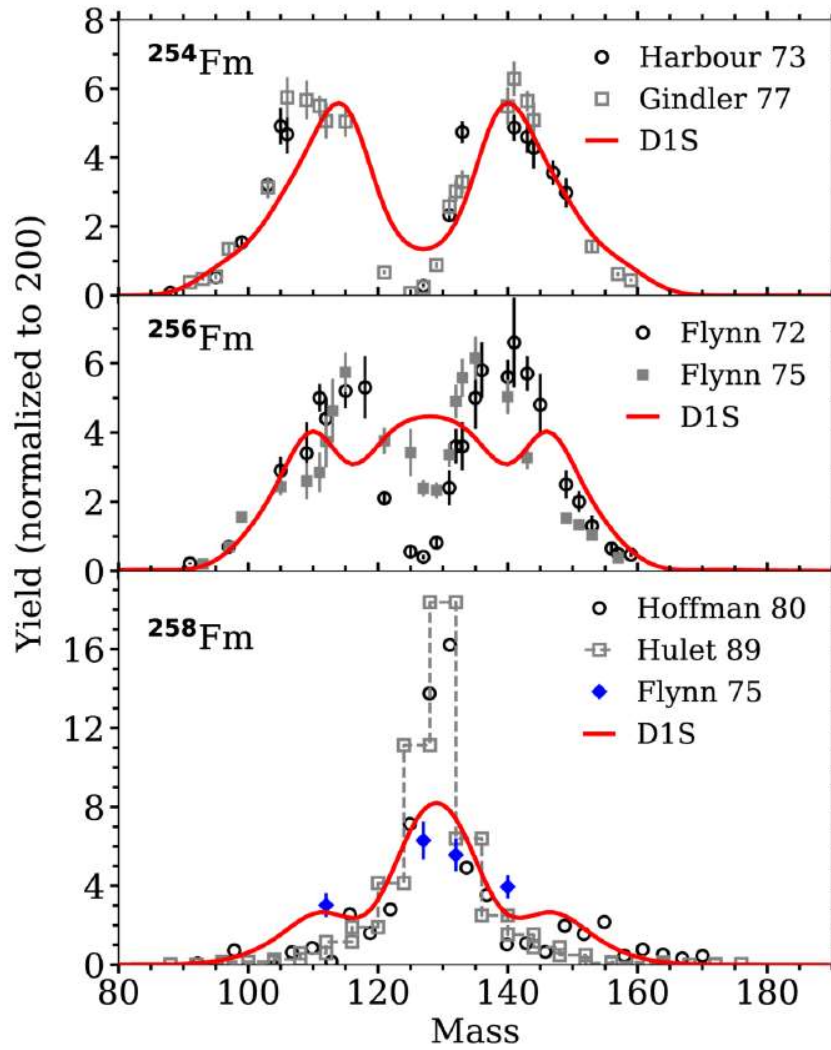
The fission process : yields variations with targets



⇒ FF distribution strongly modified with different targets

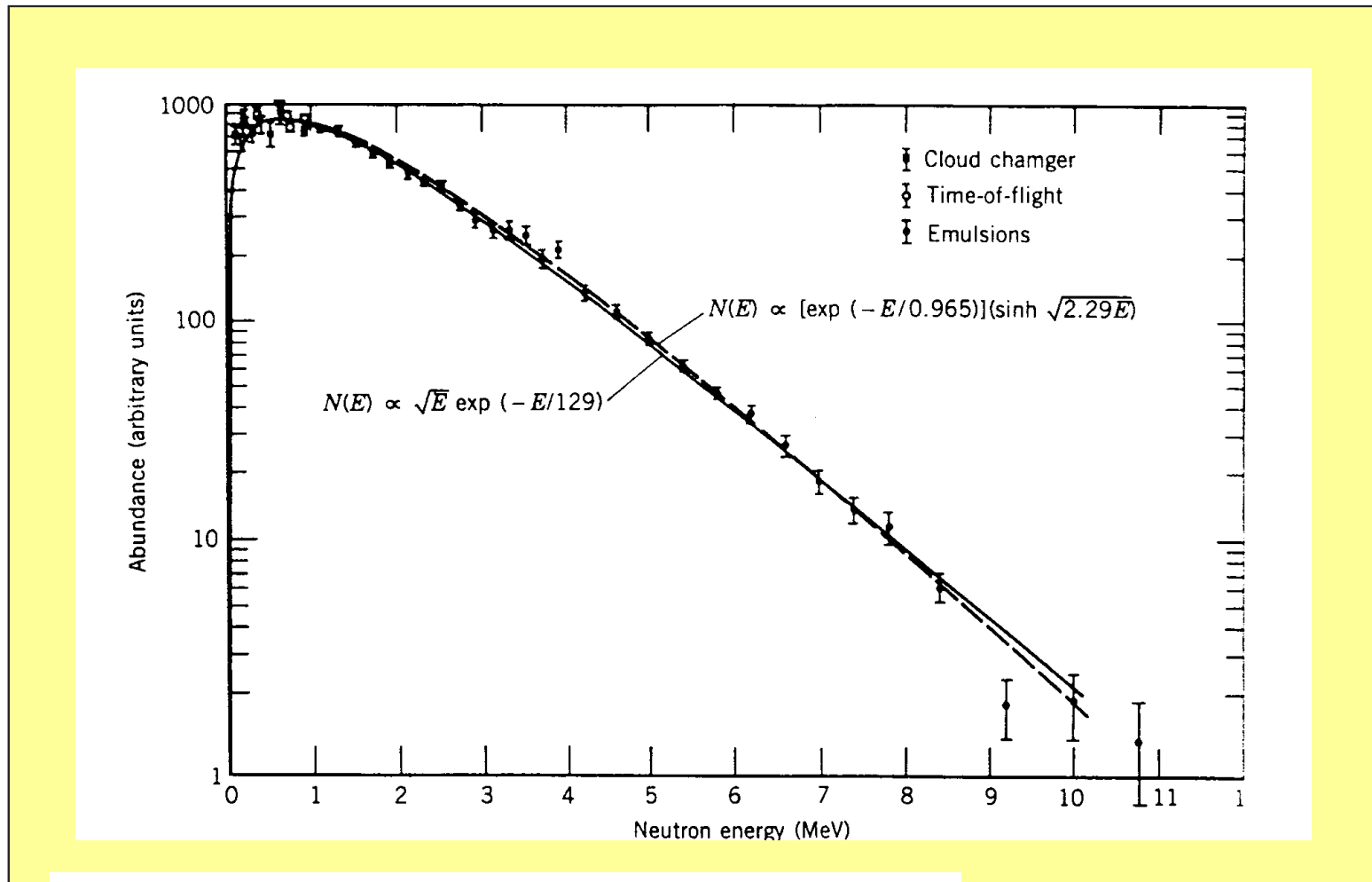


The fission process : rapid yield variations





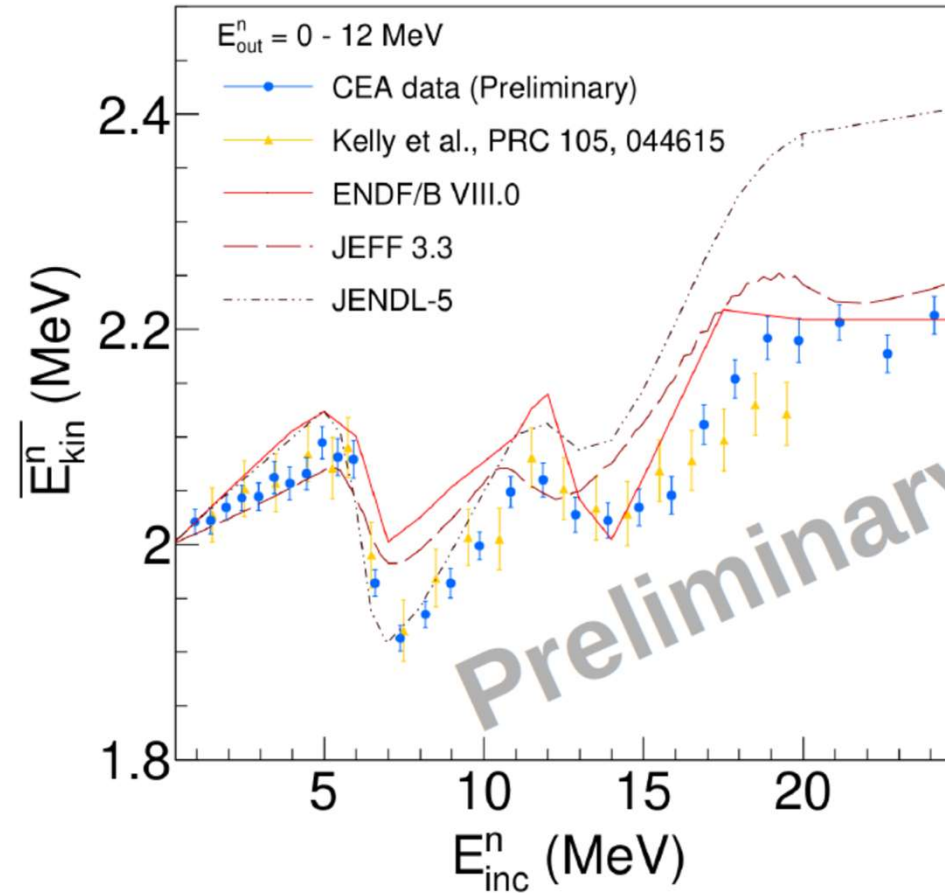
The fission process : emitted neutrons spectrum



⇒ Maxwellian (evaporation) spectrum



The fission process : mean energy of emitted neutrons



^{235}U



The fission process : neutron multiplicities

	$\bar{\nu}$		
^{240}Pu	2.257	+/-	.045
^{238}Pu	2.33	+/-	.08
$^{235}\text{U}+\text{n}$	2.47	+/-	.03
^{242}Cm	2.65	+/-	.09
^{244}Cm	2.82	+/-	.05
^{252}Cf	3.86	+/-	.07
^{242}Pu	2.18	+/-	.09
$^{233}\text{U}+\text{n}$	2.585	+/-	.062

The fission process : neutron multiplicities and incident nrj

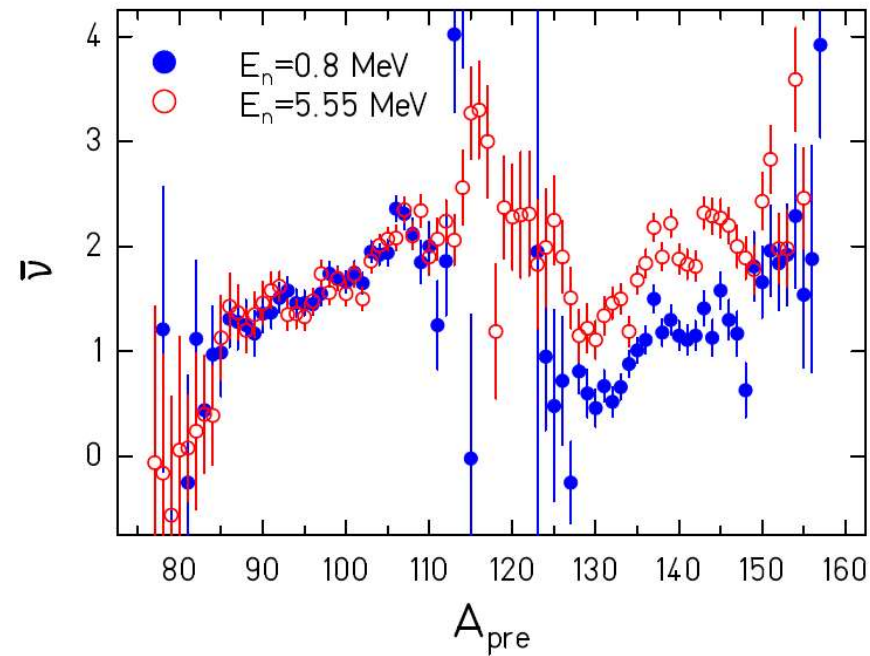
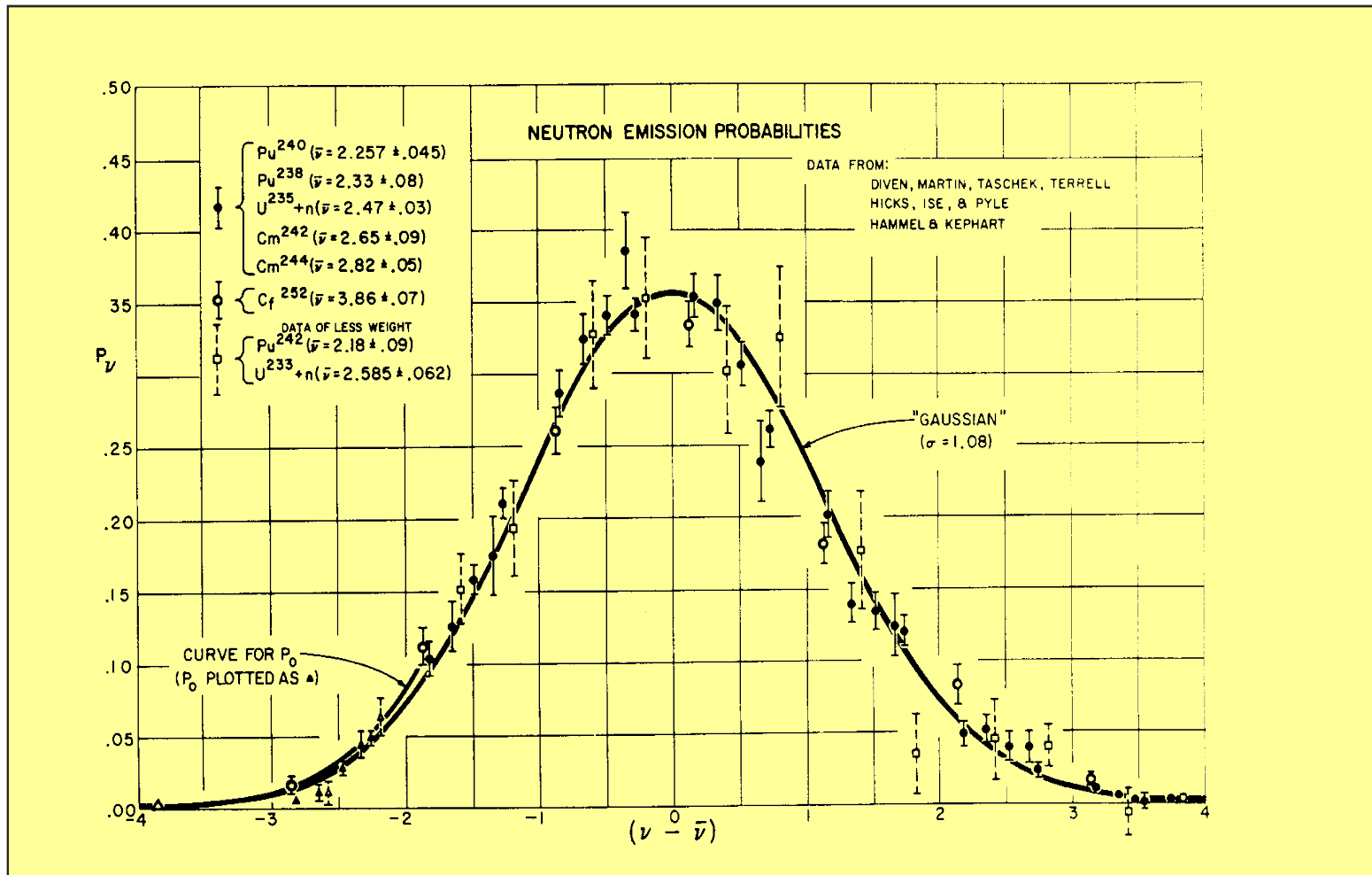


Figure 13: Prompt-neutron multiplicity as a function of the pre-neutron fragment mass for the system $^{237}\text{Np}(n,f)$ for $E_n = 0.8$ MeV and 5.55 MeV

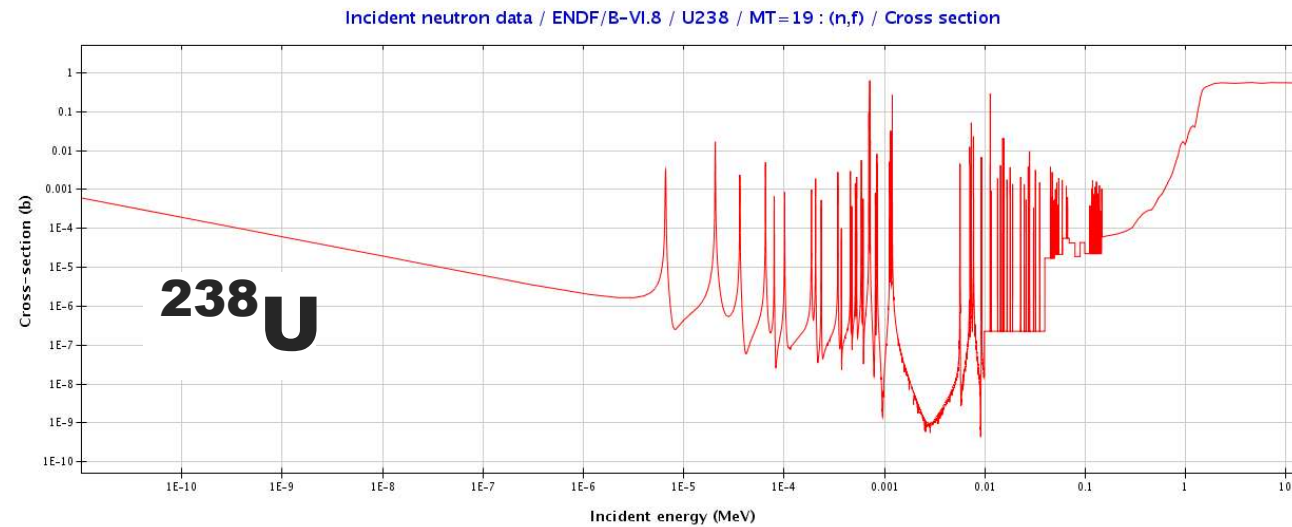
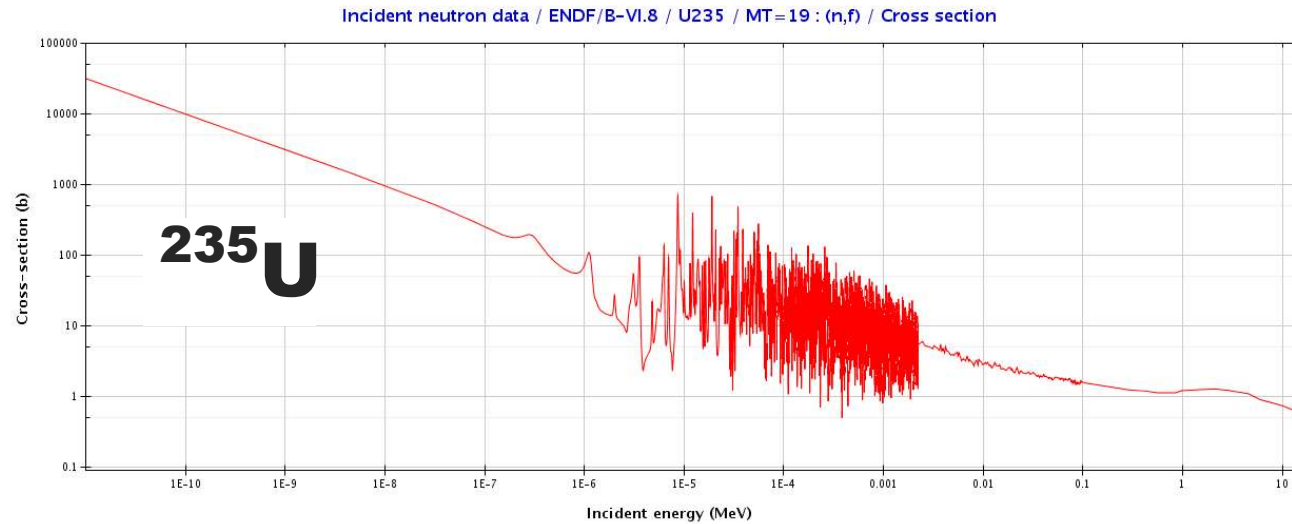


The fission process : sequence for induced fission



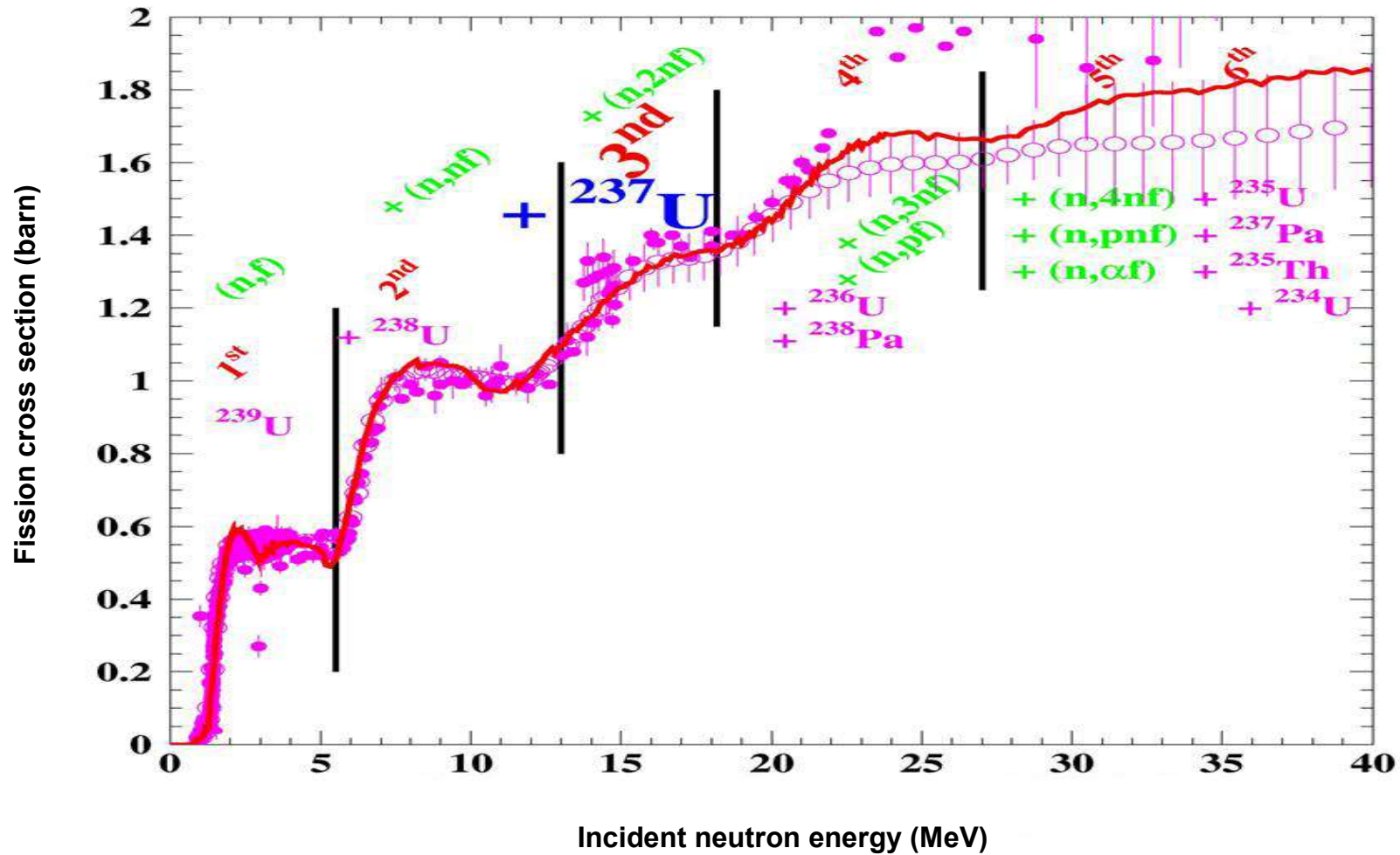


The fission process : fissile vs fertile





The fission process : fission chances





The fission process : sequence for induced fission

Describing all previously mentioned data is a real challenge for theoretical models.

Several phenomenological approaches are usually adopted to describe each type of data because of the flexibility they offer for measured nuclei.

- ⇒ Many models and parameters : extrapolation at your own risks !**
- ⇒ Clear lack of coherence or deep understanding of the underlying physics !**



3. FISSION YIELDS



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GEF model

Details in Nucl. Data Sheets 131 (2016) 107-221
Code at www.khs-erzhausen.de/home.html

Phenomenological approach not intended to compete with fundamental microscopic approach, but aiming at producing data with the accuracy required for industrial application

⇒ many empirical laws fitted to data



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The available measured fission barriers were used to deduce the following empirical function, which describes the difference between the inner and the outer barrier height:

$$E_A - E_B = 5.40101 - 0.00666175 \cdot Z^3/A + 1.52531 \cdot 10^{-6} \cdot (Z^3/A)^2. \quad (10)$$



GEF model

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Phenomenological approach not intended to compete with fundamental microscopic approach, but aiming at producing data with the accuracy required for industrial application

⇒ **many empirical laws fitted to data**

⇒ **range of application restricted close to experimentally accessible regions**

GEF model



Details in
Code at v

Phenomenological
microscopic
the accuracy

⇒ many errors
⇒ range of

According to the concept of the GEF model, the range of validity is not strictly defined. Technically, the code runs for any heavy nucleus. However, the results of the model are more reliable for nuclei which are not too far from the region where experimental data exist. It is recommended not to use the code outside the range depicted in figure 1 on the chart of the nuclides.

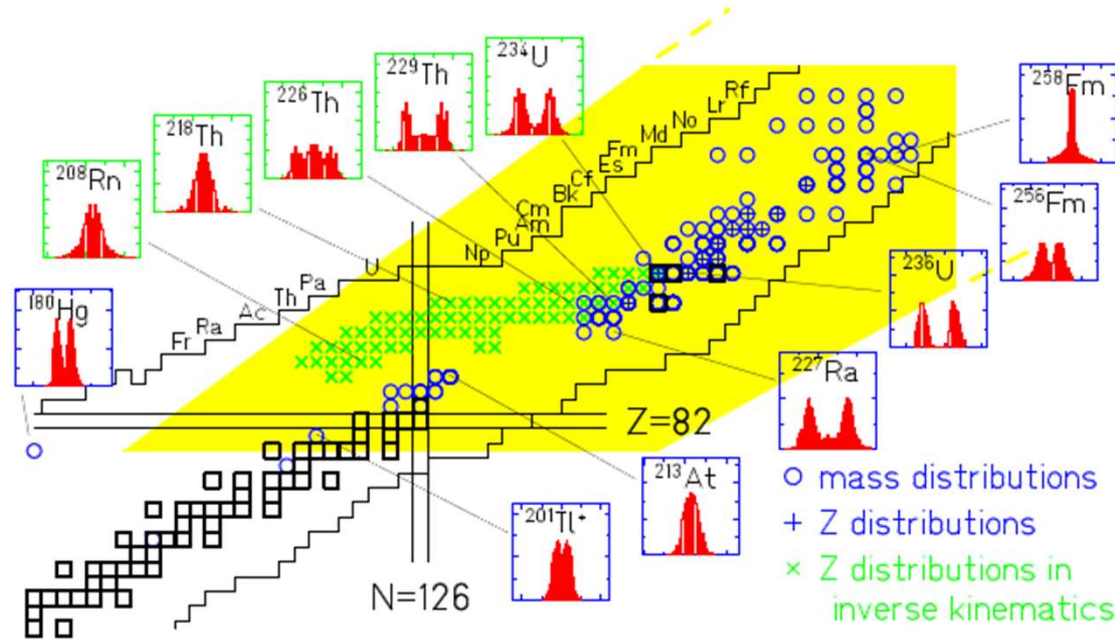


Figure 1: Validity range of the GEF model on a chart of the nuclides, marked in yellow. For a detailed description of the figure see figure 6.

GEF model



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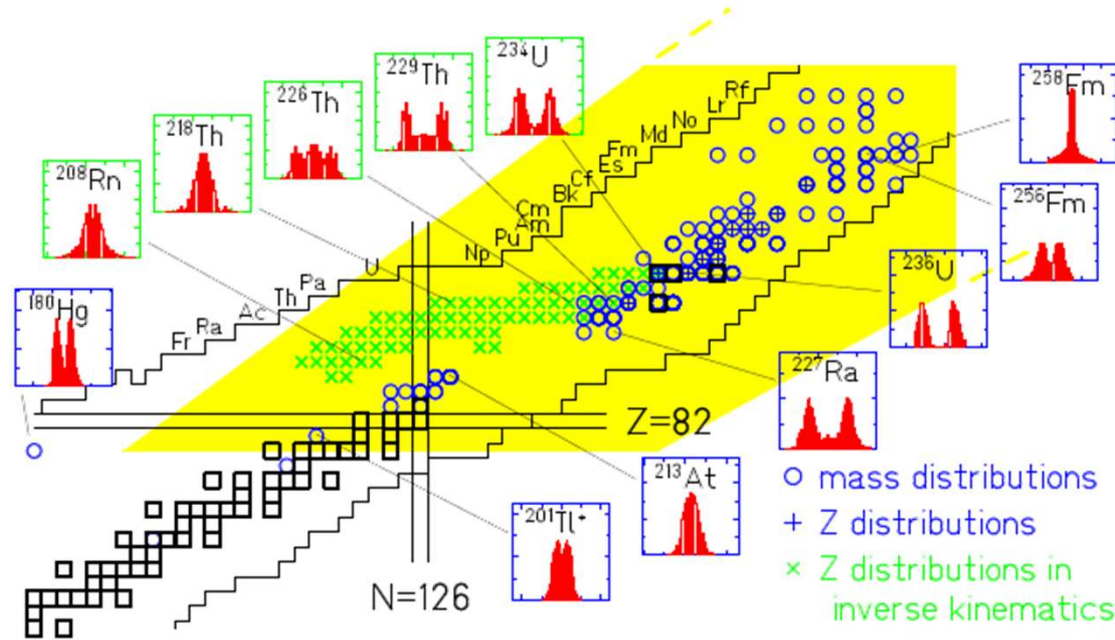


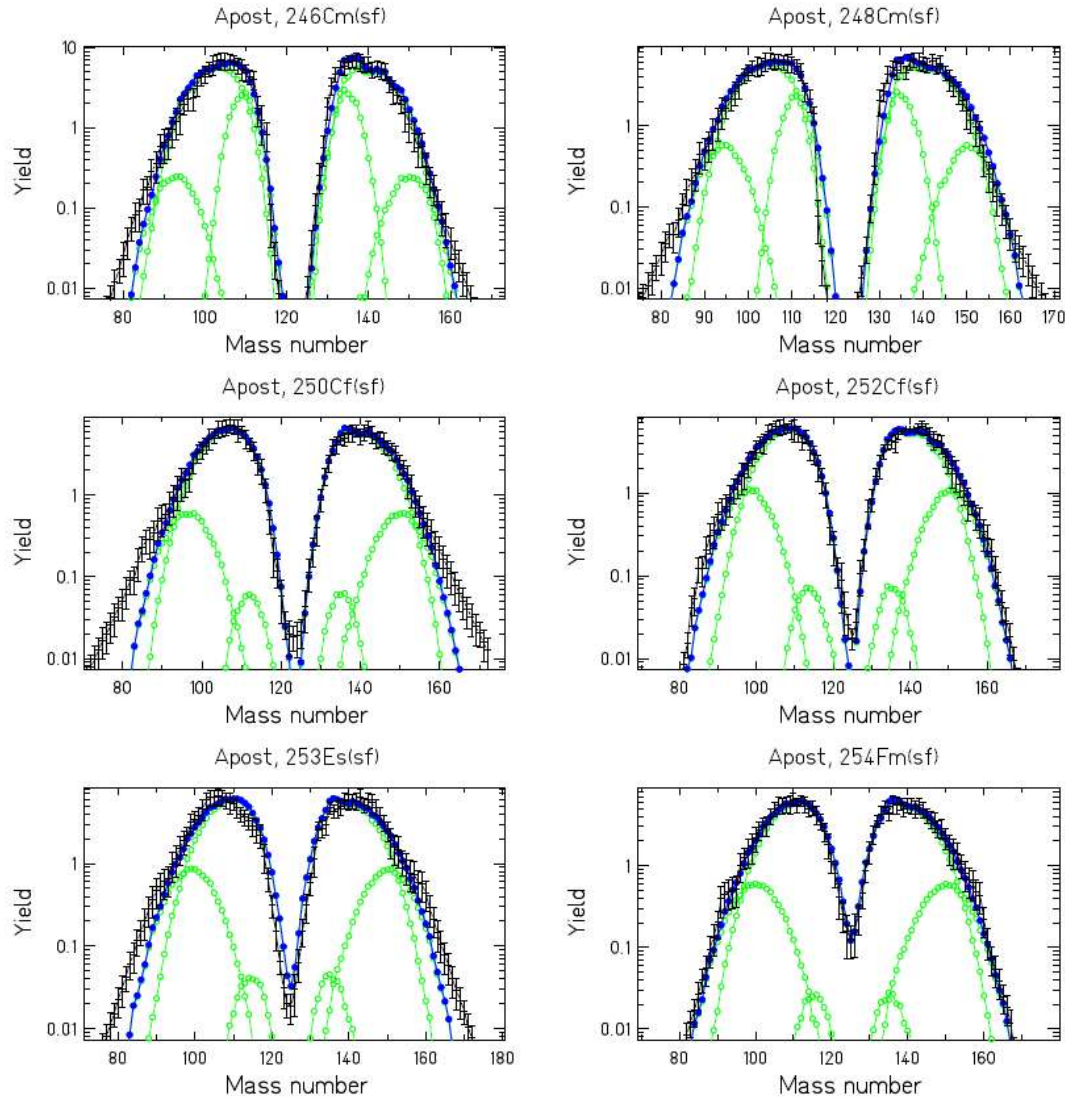
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The fission process : GEF results



Spontaneous fission

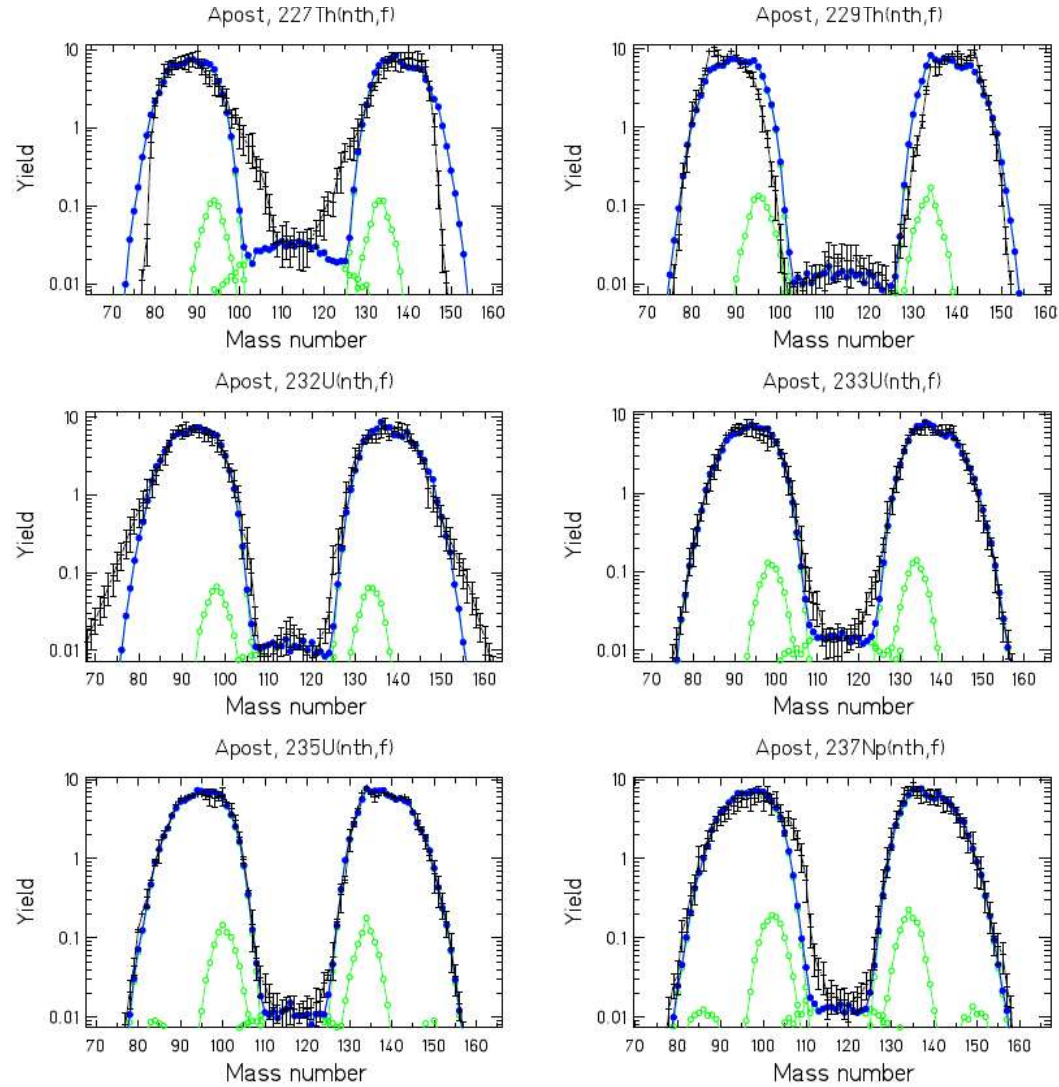


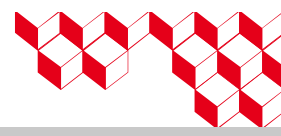


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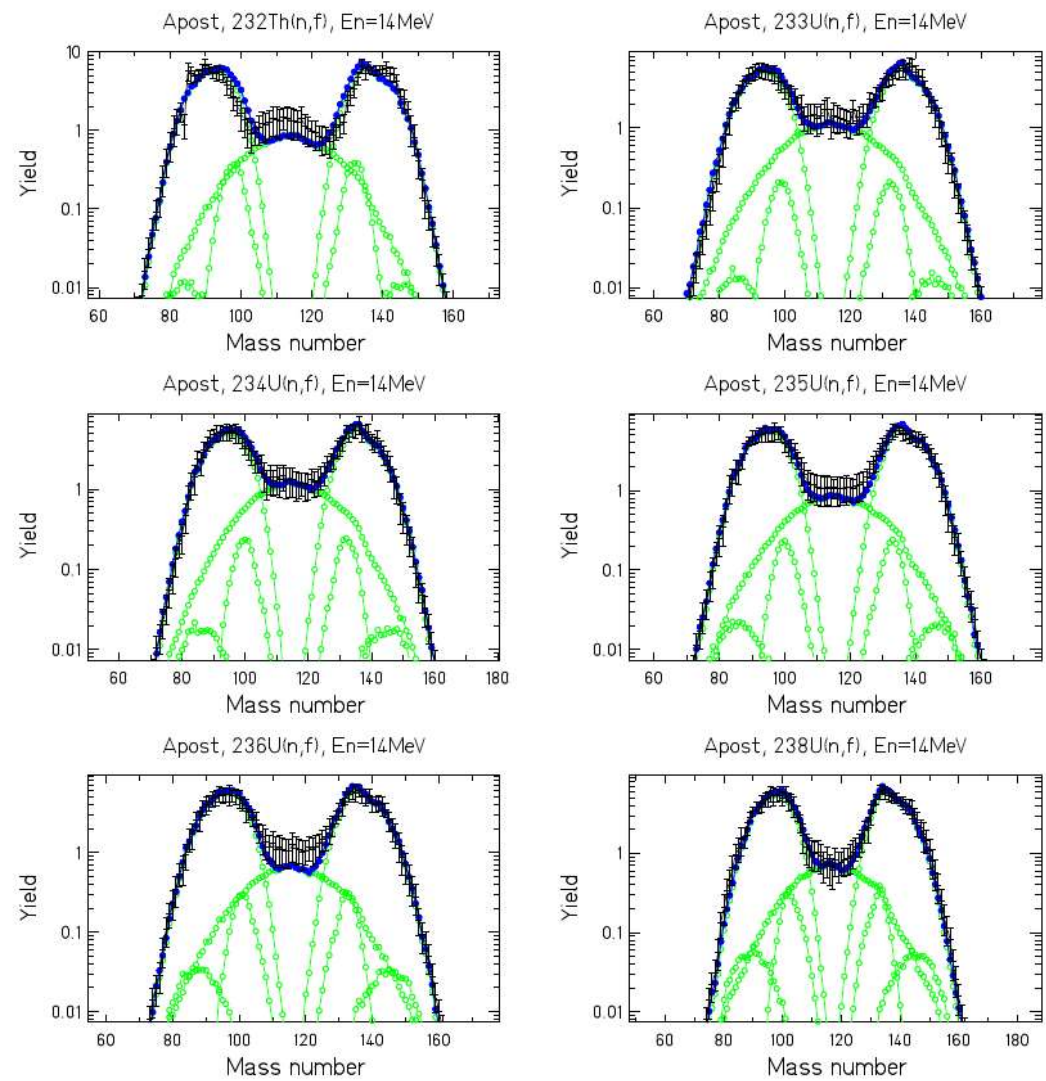


thermal neutrons induced fission





fast neutrons induced fission





SPY model

Details in Phys. Rev. C92 (2015) 034617 & Phys. Rev. C99 (2019)

Approach based on absolute energy balance based on HFB potential energy surfaces as function of axial deformation

⇒ Available energy at scission

Available energy (AE) shared between fragments : x (AE) and $(1-x)$ (AE)

$$\pi(Z_1, N_1, Z_2, N_2, \tilde{q}_1, \tilde{q}_2, x) = \rho_1(x|AE|) \rho_2((1-x)|AE|) \delta E^2$$

$$\Pi(Z_1, N_1, Z_2, N_2, \tilde{q}_1, \tilde{q}_2) = \int_0^1 \pi(Z_1, N_1, Z_2, N_2, \tilde{q}_1, \tilde{q}_2, x) dx$$

$$P(Z_1, N_1, Z_2, N_2) = \iint \Pi(Z_1, N_1, Z_2, N_2, \beta_1, \beta_2) d\tilde{q}_1 d\tilde{q}_2$$

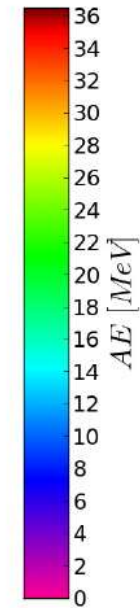
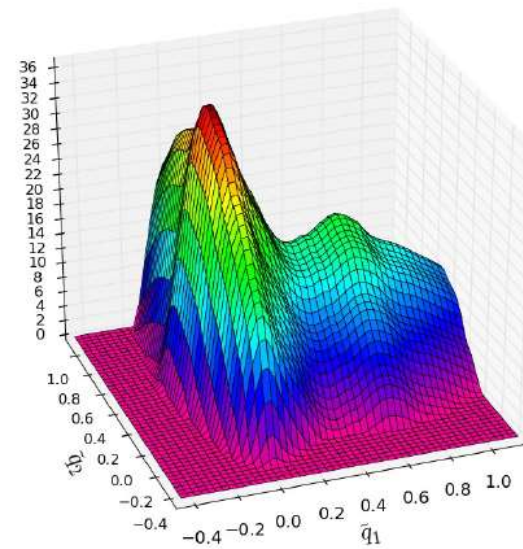
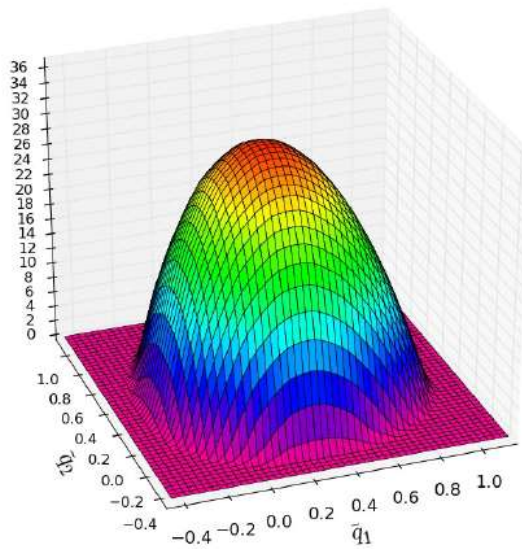


SPY : available energy

Individual energy ($^{132}\text{Sn}+^{104}\text{Mo}$)

liquid drop model (macroscopic)

Amedee data base : HFB+Gogny (microscopic)



Courtesy J.F. Lemaître

$$AE = |E_{\text{ind1}} + E_{\text{ind2}} + E_{\text{coul}} + E_{\text{nucl}} - E_{\text{CN}}|$$

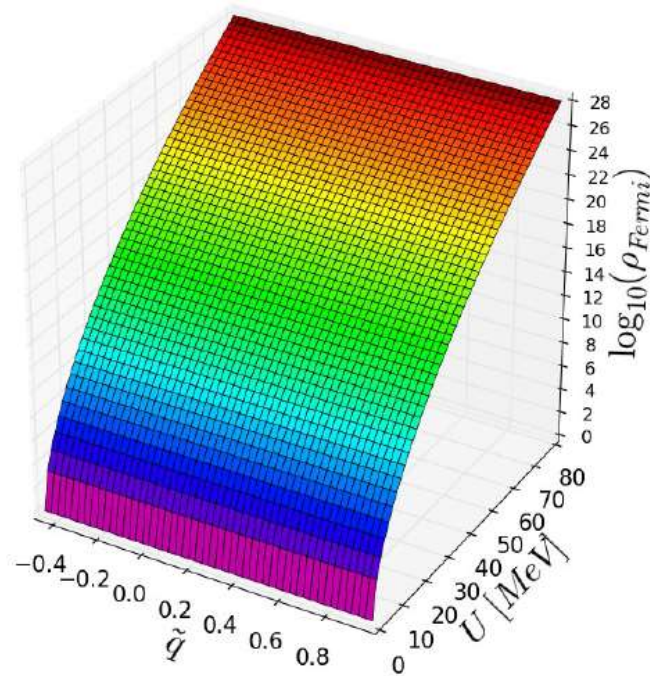
1



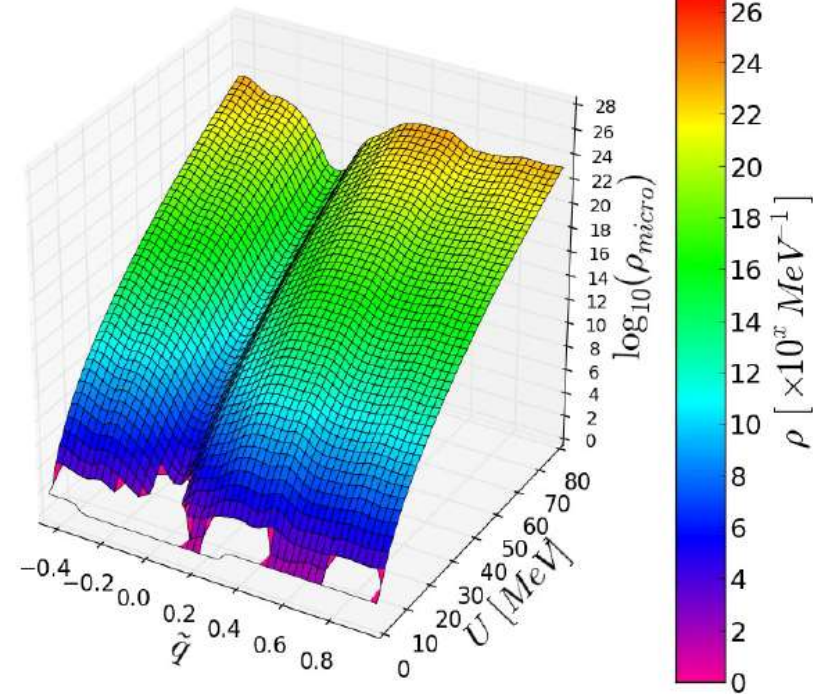
The fission process : energy sharing & level densities

State density of ^{132}Sn ; $\rho \propto e^{2\sqrt{aU}}$

Fermi gas model (macroscopic)



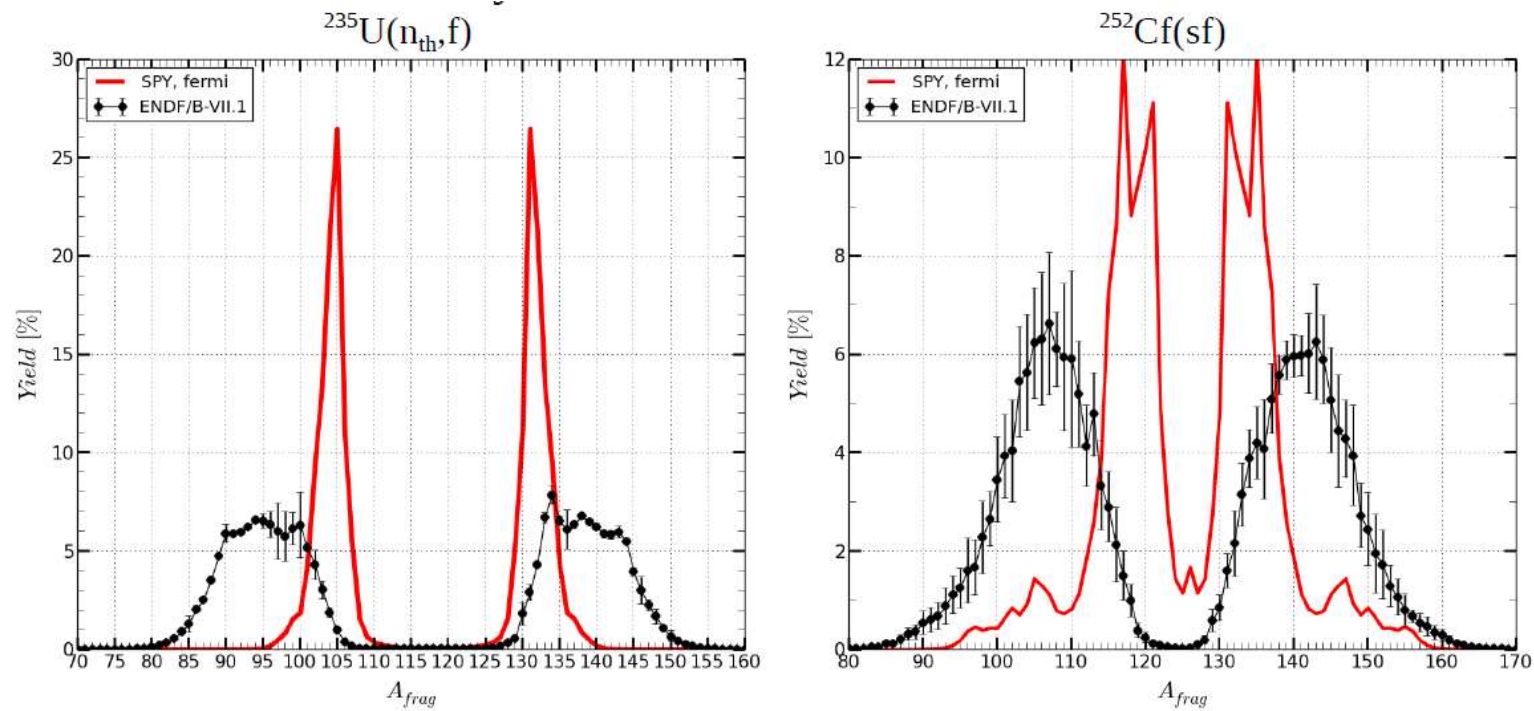
combinatorial method (microscopic)



Courtesy J.F. Lemaître



SPY model : raw results

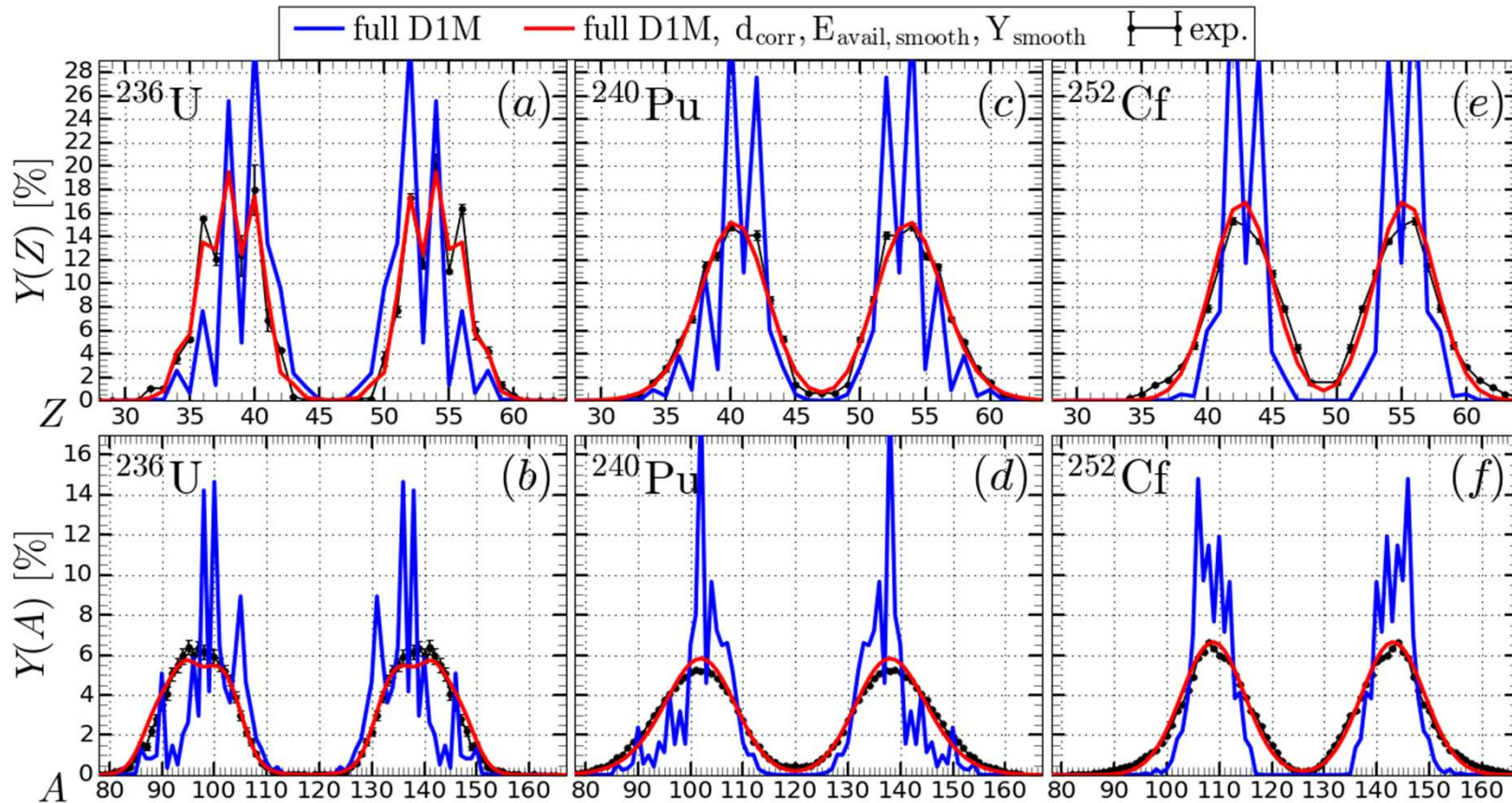


⇒ Less accurate than GEF (only one parameter fixed !)

SPY model : raw results

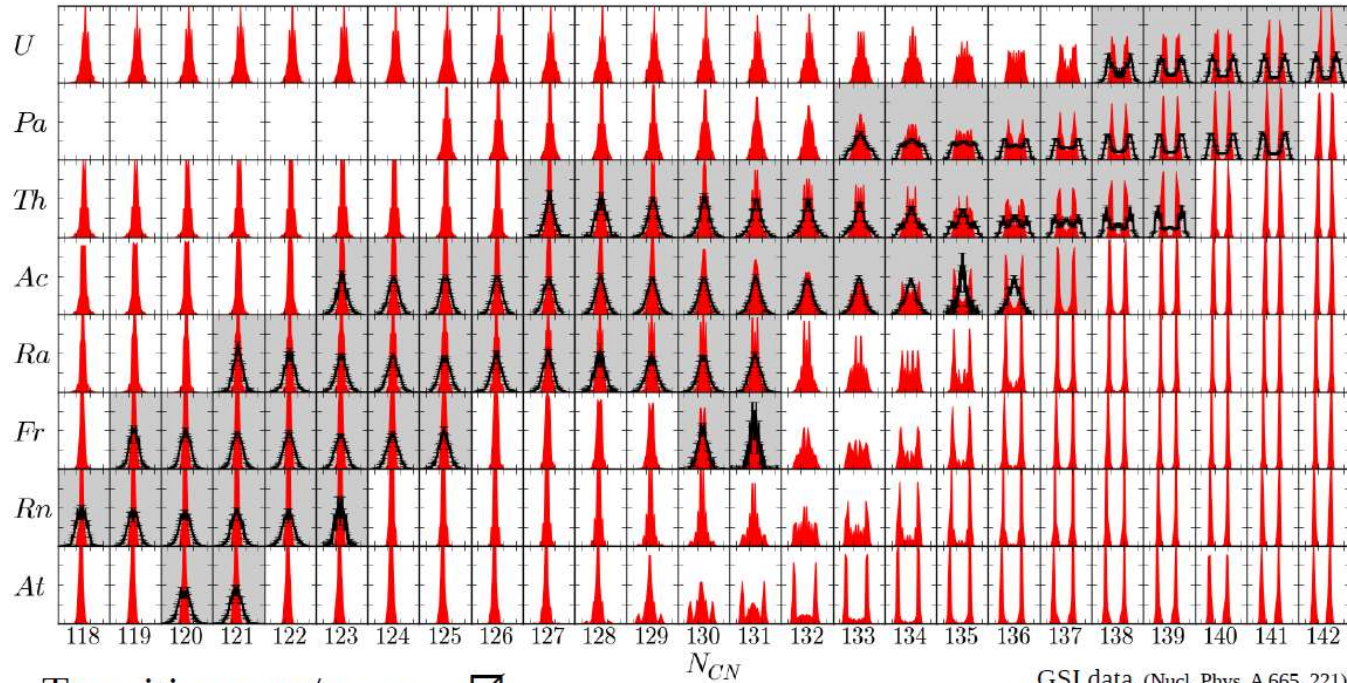


⇒ Can be improved with smoothing methods (much more parameters)





SPY model : systematic predictions rather easy



Transition sym/asym

Good tendency, with shift of sym/asym transition of few charges

Peak width

Too narrow due to strong influence of nuclear structure

GSI data, (Nucl. Phys. A 665, 221)
SPY model

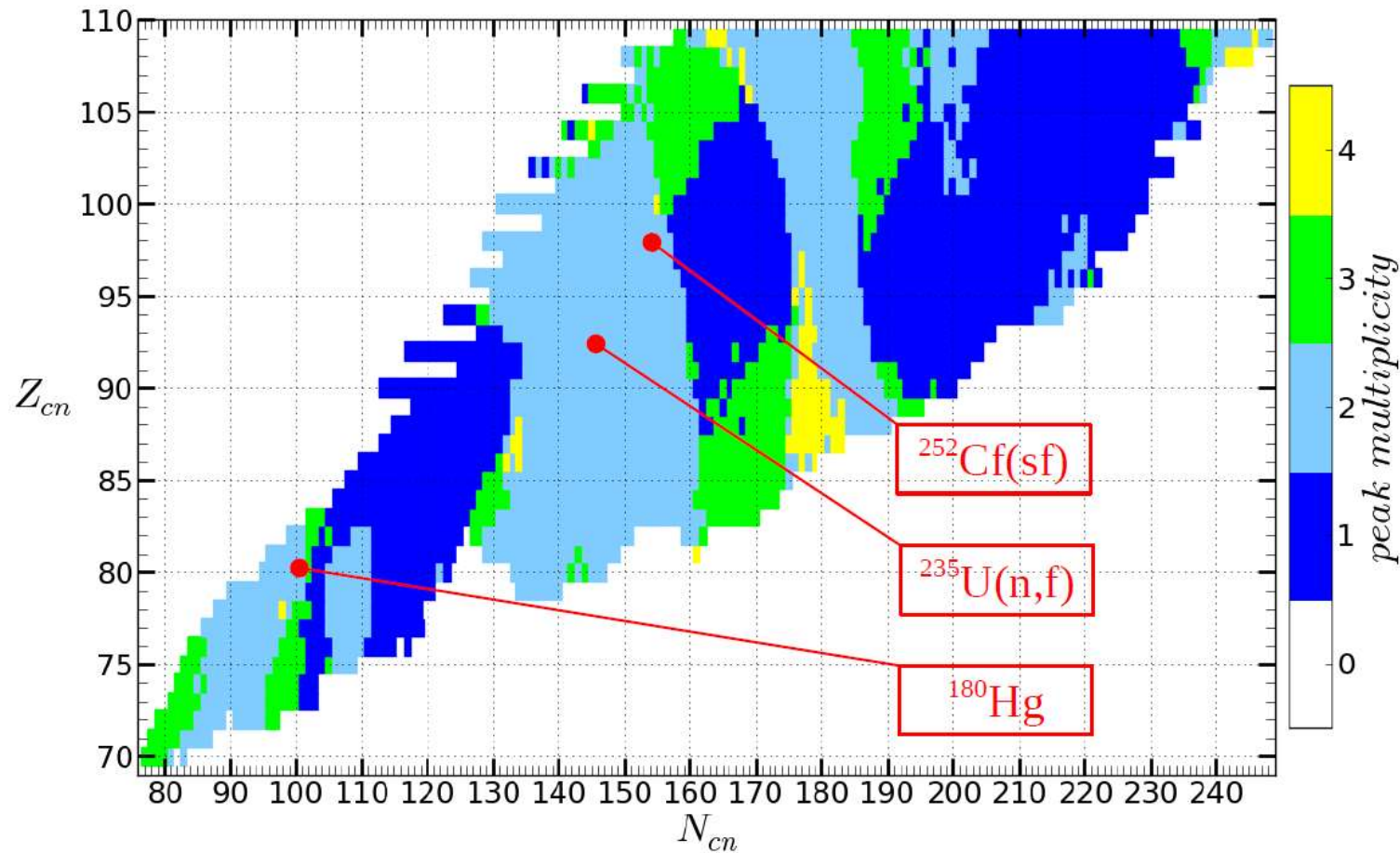
Courtesy J.F. Lemaitre

21

⇒ Rather good qualitative description



SPY model : systematic predictions rather easy



Courtesy J.F. Lemaître

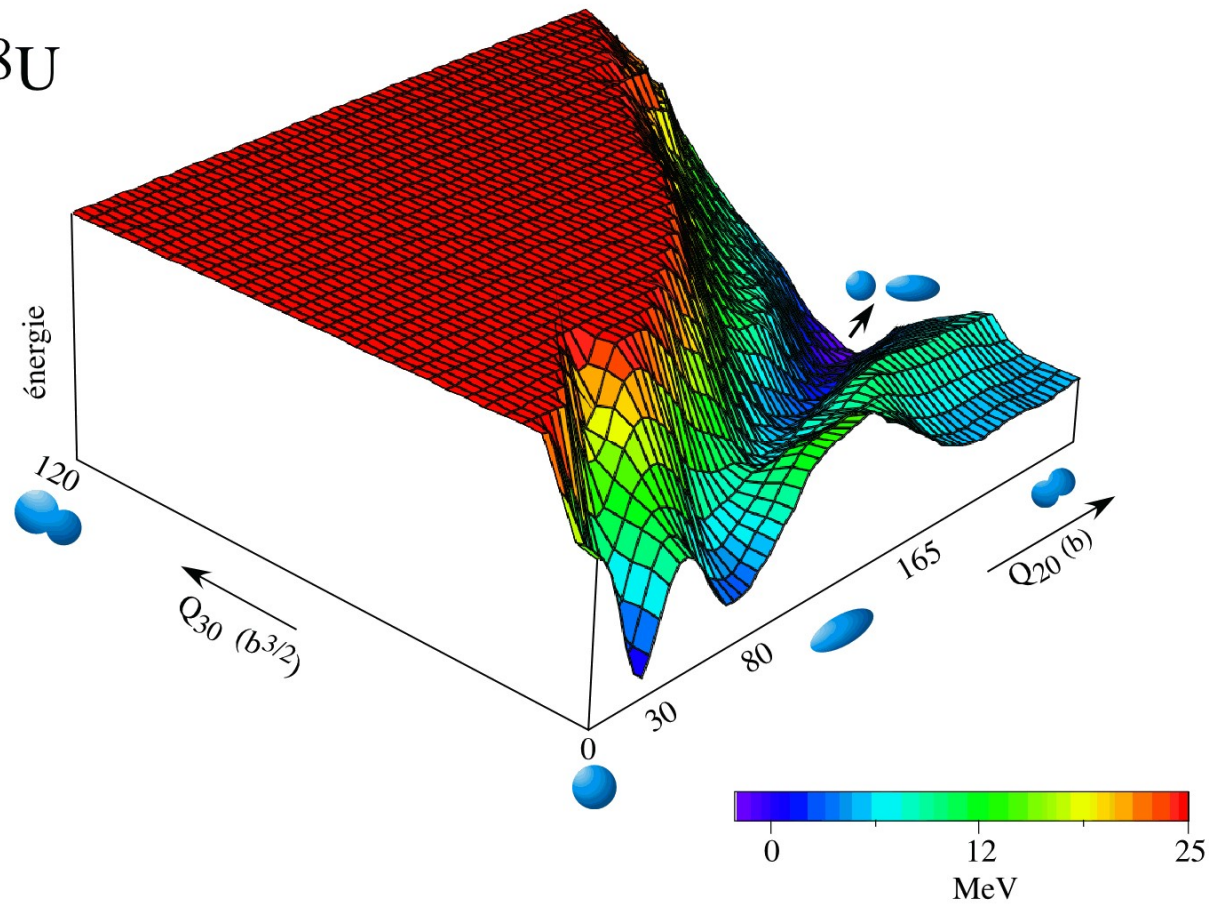
Based on microscopic grounds \Rightarrow predictive power even far from stability
 \Rightarrow region with 4 peaks predicted !



Microscopic approach

1) PES calculation as function of elongation-asymmetry

^{238}U





Microscopic approach

- 1) PES calculation as function of elongation-asymmetry
- 2) Quantum mechanical Wave packet propagation in the computed PES (FELIX code)



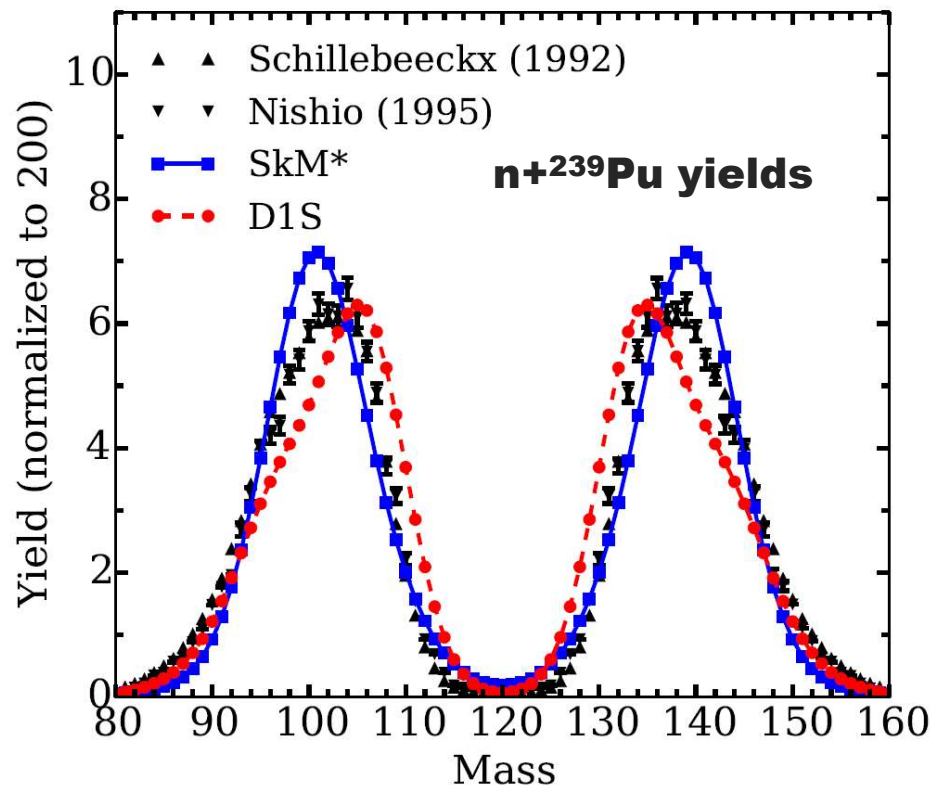
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- 1) PES calculation as function of elongation-asymmetry
- 2) Quantum mechanical Wave packet propagation in the computed PES (FELIX code)



Microscopic approach

- 1) PES calculation as function of elongation-asymmetry
- 2) Quantum mechanical Wave packet propagation in the computed PES (FELIX code)
- 3) Extraction of the flux through a scission line (whose definition is not trivial)



- ⇒ Not accurate enough for applications
- ⇒ Time consuming (10000 h per nucleus on single CPU)
- ⇒ Extrapolations and systematics manageable with HPC
- ⇒ Limited to even-even



3 ■ **Neutrons and gammas from fission**



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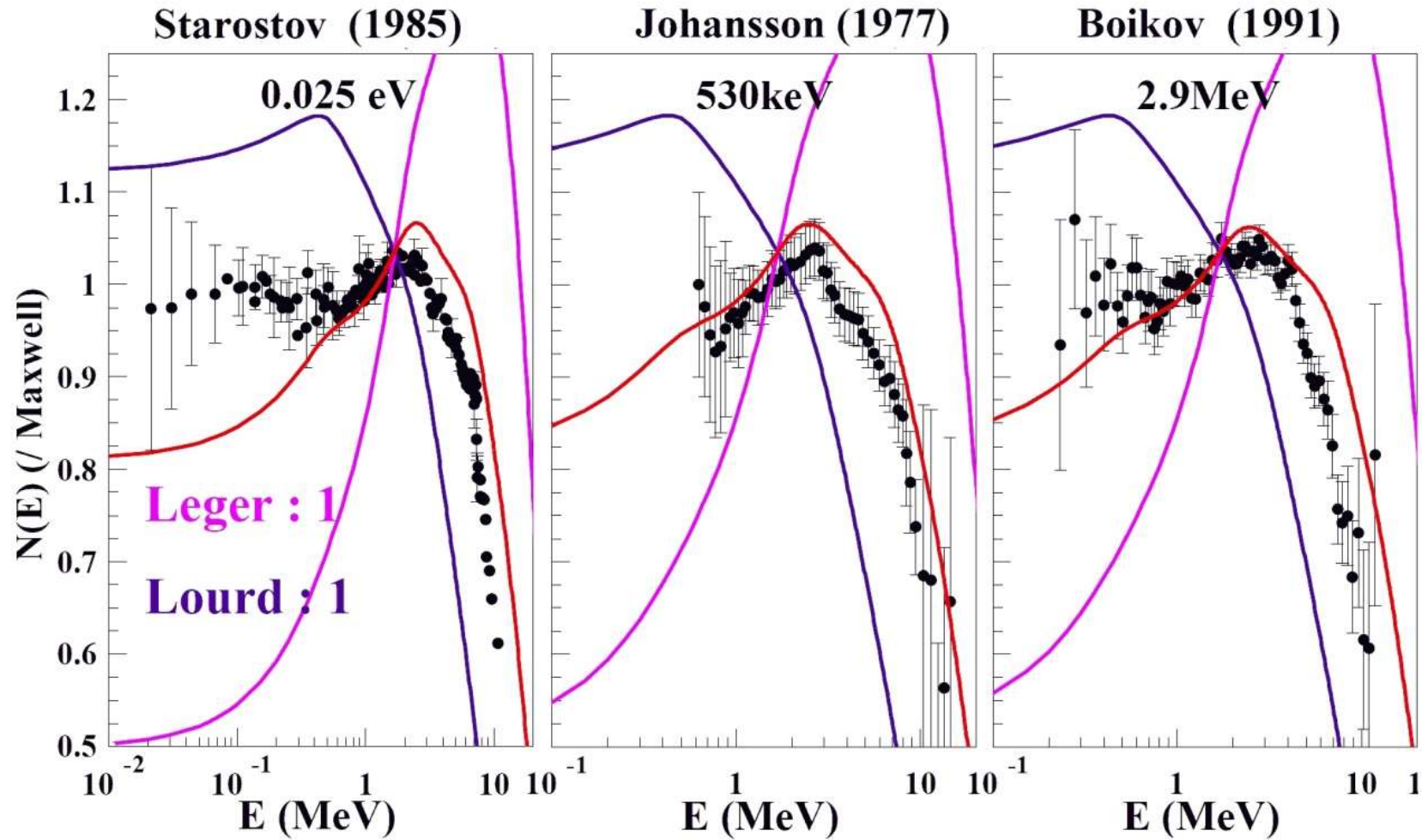
Madland-Nix model : PFNS

Details in Nucl. Sci. Eng. 81 (1982) 213.

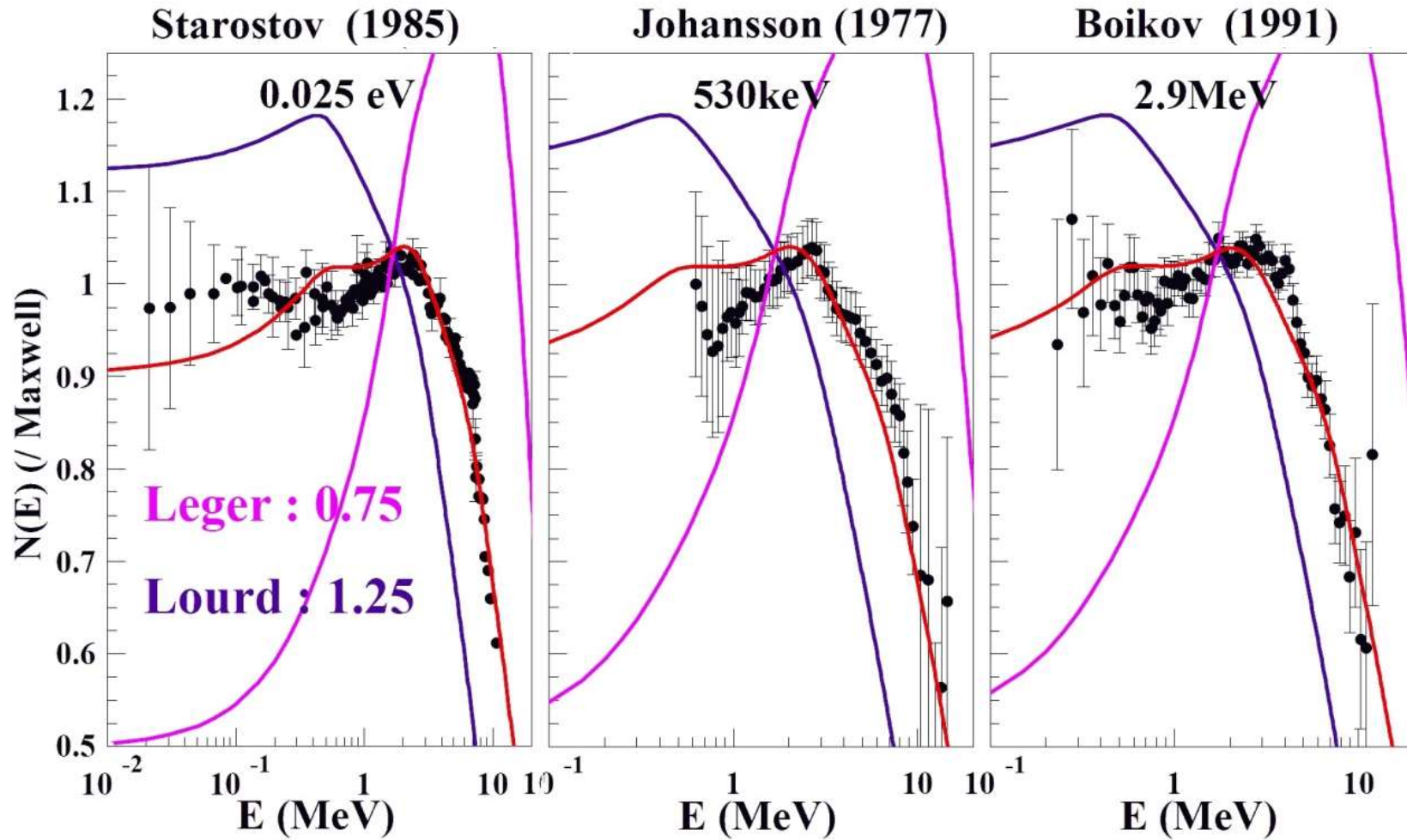
- Fission yields reduced to a light and a heavy fragment whose temperatures are distributed according to a triangular law
- For each temperature, each fragment neutron emission is modeled by the corresponding Weisskopf spectrum normalized by the fragment formation cross section by induced neutron reaction
- Multiple fission chances accounted for using fission cross sections
- Final neutron spectrum defined by an average of light and heavy fragment spectra

⇒ model with parameters designed to fit data

Madland-Nix model : PFNS



Madland-Nix model : PFNS





Madland-Nix model : PFNS

mean gamma energy parameterized (several options)

⇒ model with parameters designed to fit data



Madland-Nix model : PFNS

mean gamma energy parameterized (several options)

$$\overline{\nu_{pi}} = \frac{\langle E_i^* \rangle - \langle E_{\gamma}^{tot} \rangle}{\langle S_{ni} \rangle + \langle \epsilon_i \rangle}$$

$\langle E_{\gamma}^{tot} \rangle$ mean prompt gamma energy
 $\langle S_n \rangle$ mean neutron binding energy
 $\langle \epsilon \rangle$ mean emitted neutron energy

⇒ model with parameters designed to fit data



Madland-Nix model : PFNS

mean gamma energy parameterized (several options)

$$\bar{\nu}_{pi} = \frac{\langle E_i^* \rangle - \langle E_{\gamma}^{tot} \rangle}{\langle S_{ni} \rangle + \langle \epsilon_i \rangle}$$

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 $\langle S_n \rangle$ mean neutron binding energy
 $\langle \epsilon \rangle$ mean emitted neutron energy

$$\bar{\nu}_p = \sum_{i=1}^N \frac{\sigma_{fi}}{\sigma_f} (i - 1 + \bar{\nu}_{pi})$$

σ_{fi} : i^{th} fission chance cross section.

⇒ model with parameters designed to fit data

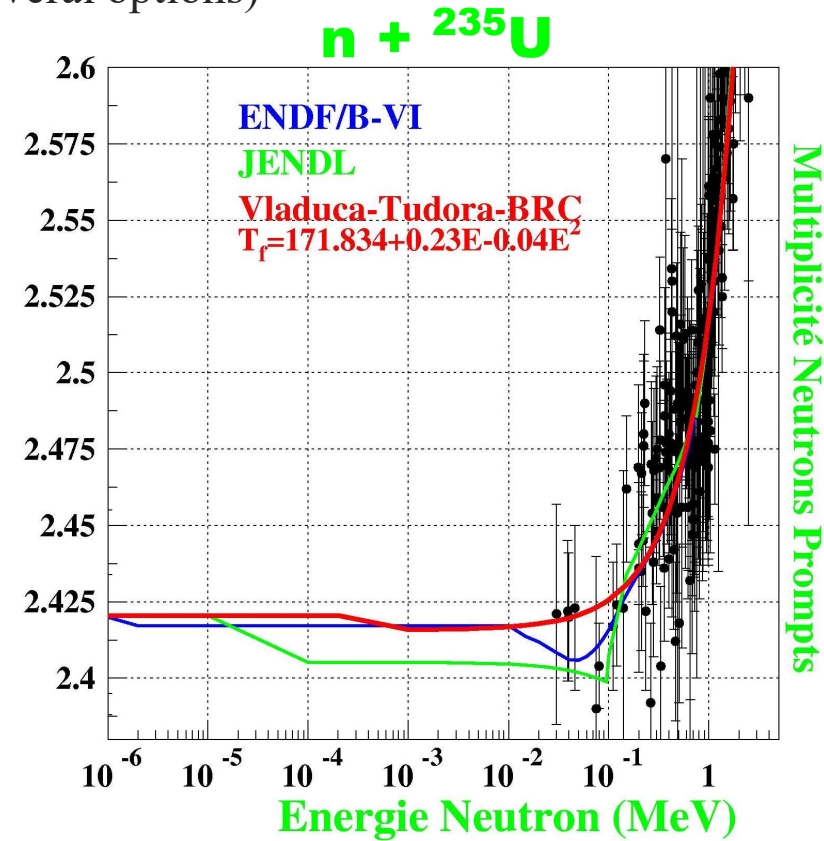


Madland-Nix model : PFNS

mean gamma energy parameterized (several options)

$$\overline{\nu_{pi}} = \frac{\langle E_i^* \rangle - \langle E_{\gamma}^{tot} \rangle}{\langle S_{ni} \rangle + \langle \epsilon_i \rangle}$$

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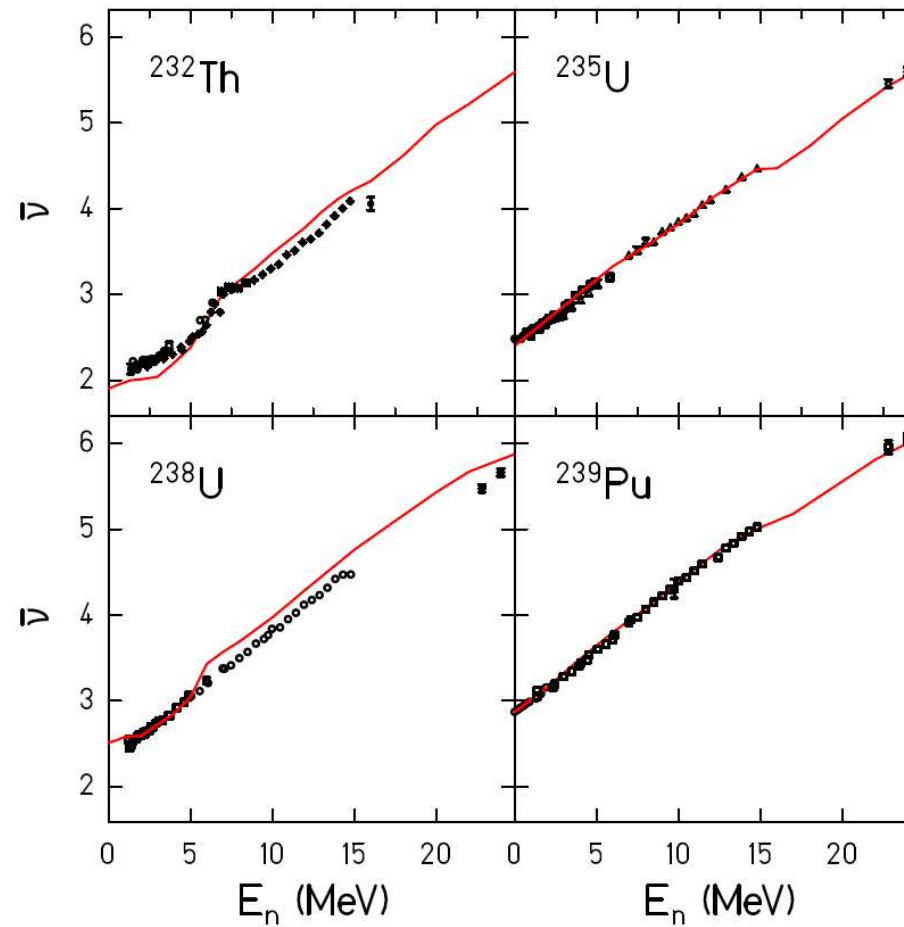
⇒ model with parameters designed to fit data



GEF model : neutron multiplicities

Details in Nucl. Data Sheets 131 (2016) 107-221

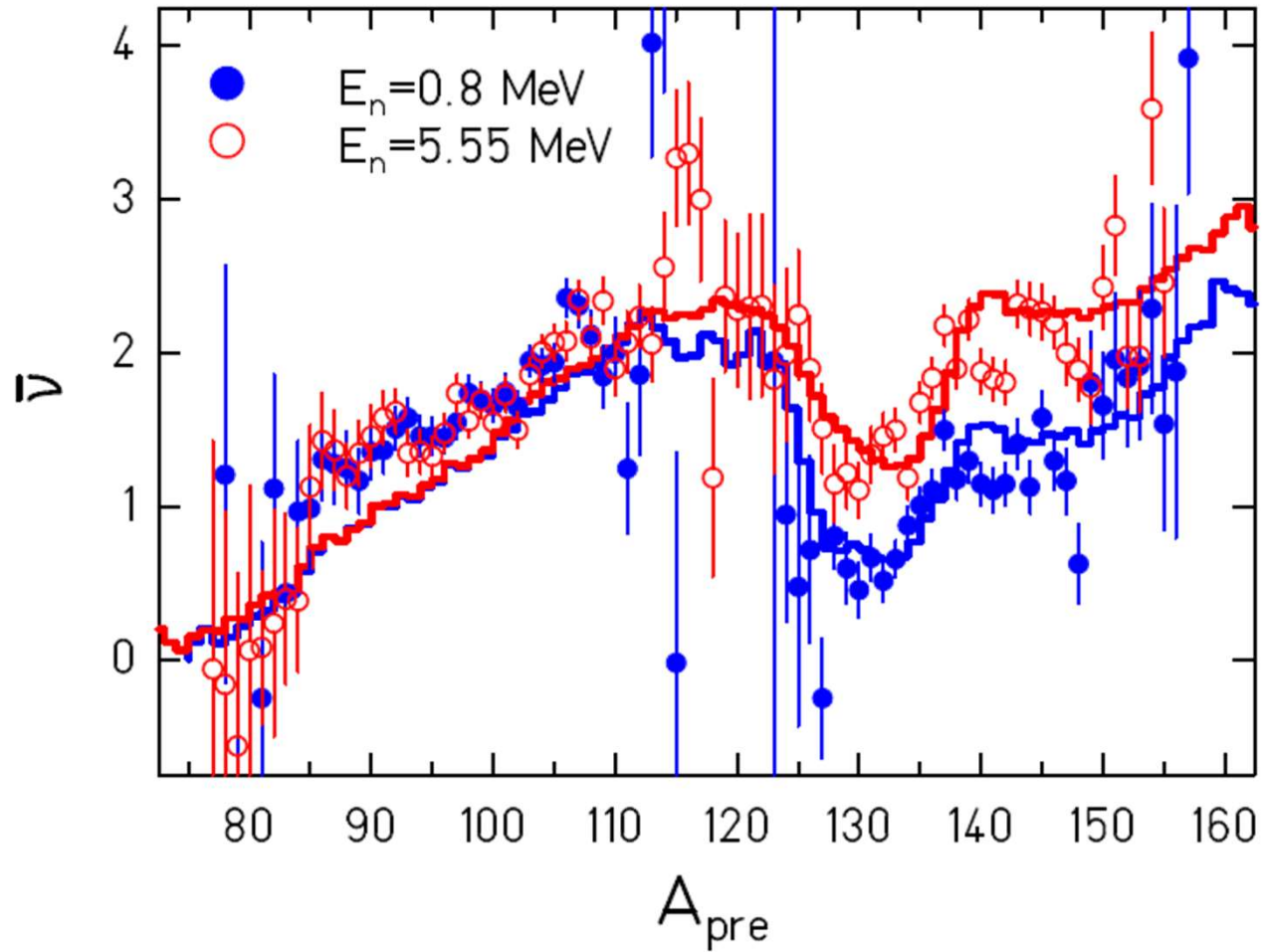
Code at www.khs-erzhausen.de/home.html



GEF model



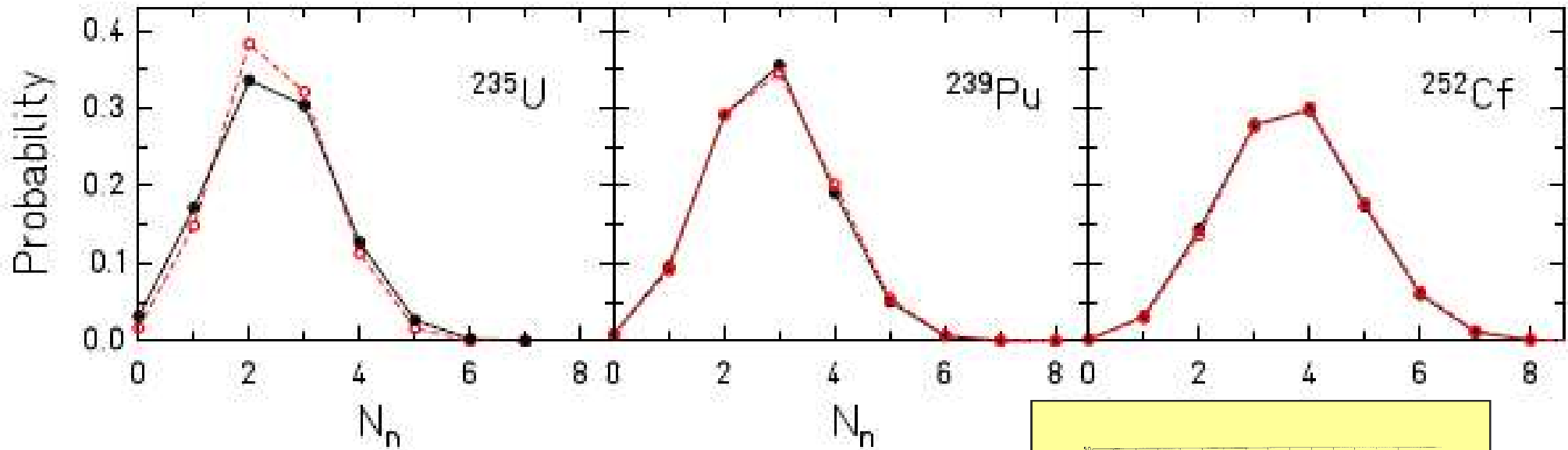
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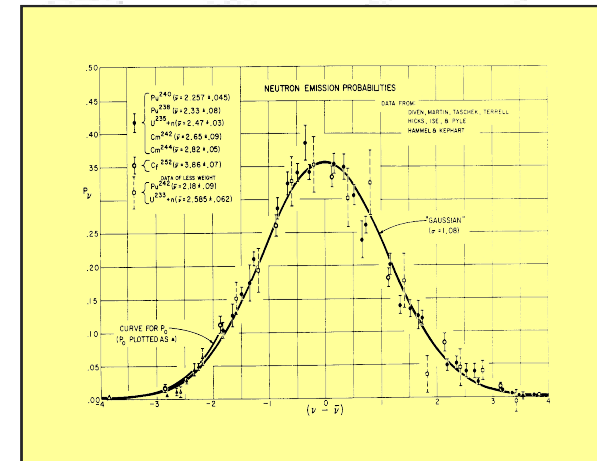
GEF model : neutron multiplicities distributions



Details in Nucl. Data Sheets 131 (2016) 107-221
Code at www.khs-erzhausen.de/home.html



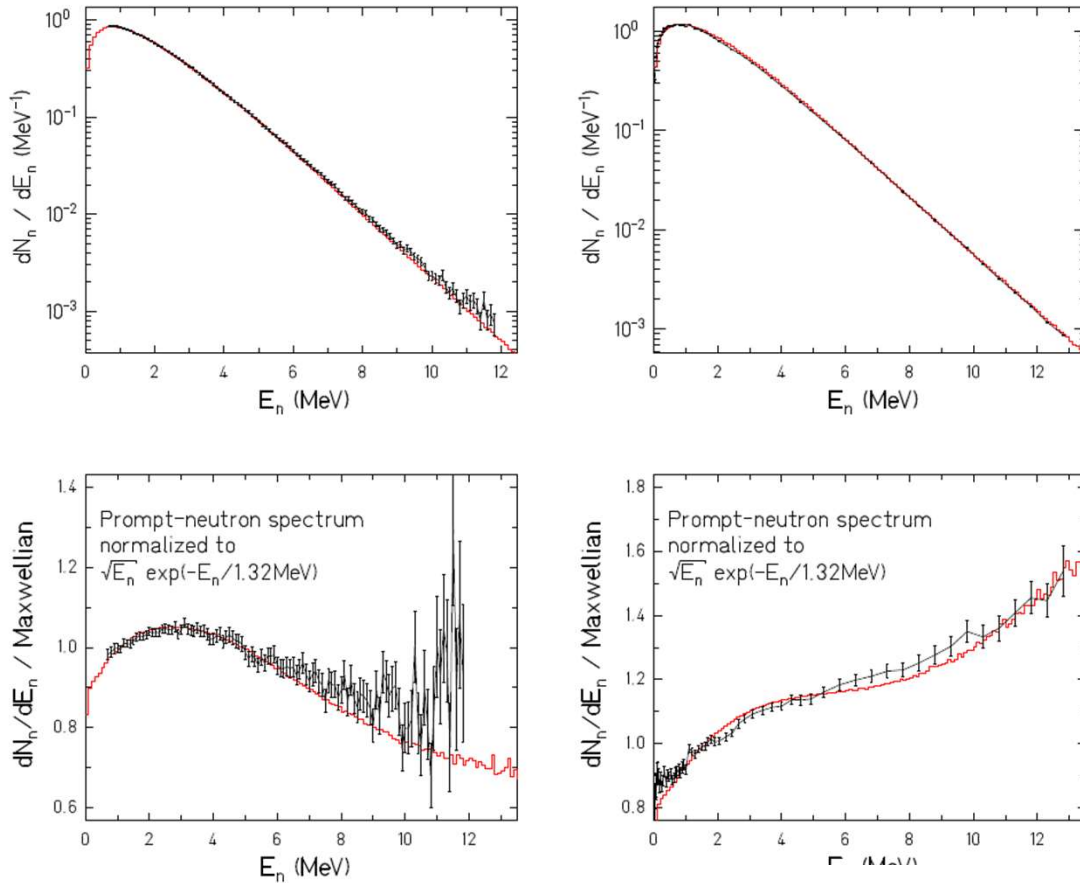
Derivative of a Gaussian
(remember slide 18)



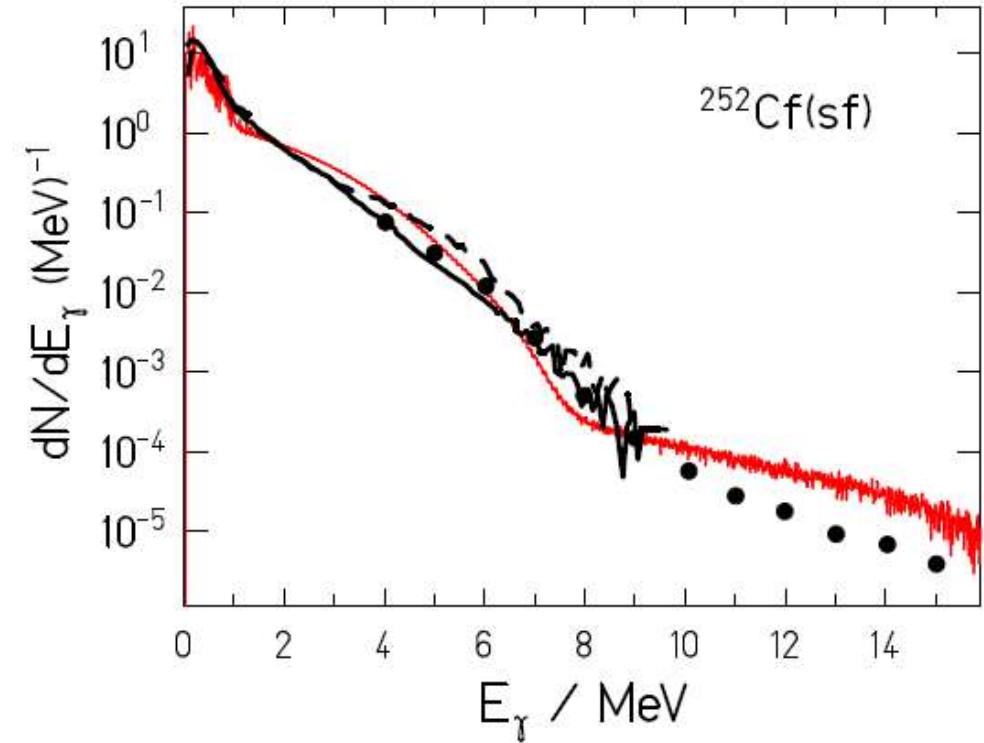
GEF model : neutron and gamma spectra



PFNS



PFGS

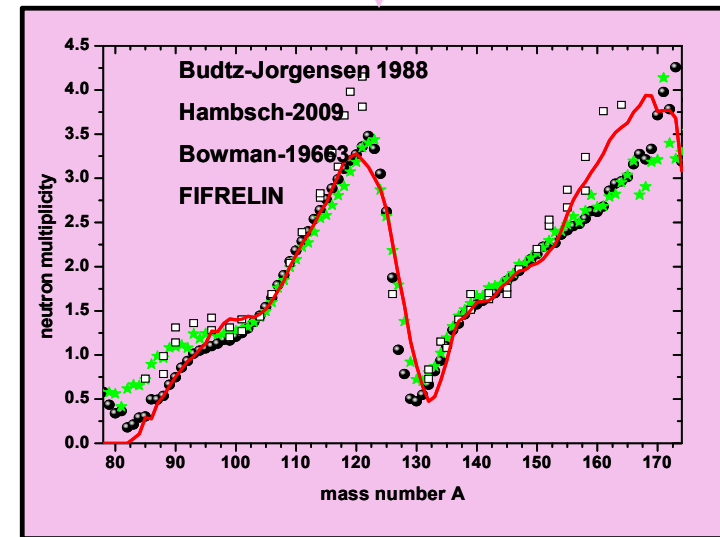
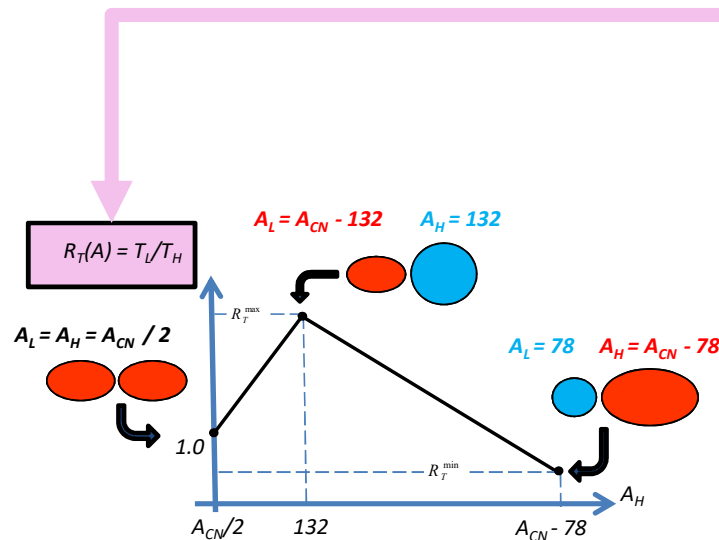


⇒ **GEF very efficient to fit data and fill the gaps for applications**



Monte Carlo approach designed to deal with fission fragment decay

- ⇒ fragment's mass sampled from exp. or theory (GEF)
- ⇒ fragment's kinetic energy sampled from exp. or theory
- ⇒ fragment's charge sampled from Wahl model ($Z=Z_{CN}/A_{CN} * A$)
- ⇒ fragment's spin distribution sampled from level density law
- ⇒ excitation energy sharing following temperature ratio law adjusted on saw tooth

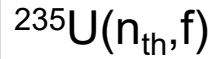
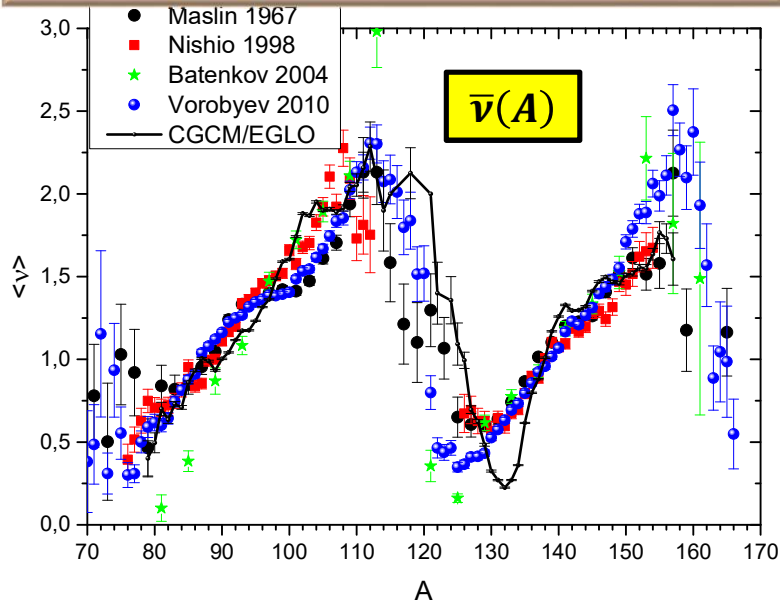


Details in Phys. Rev. C 82 (2010) 054616.

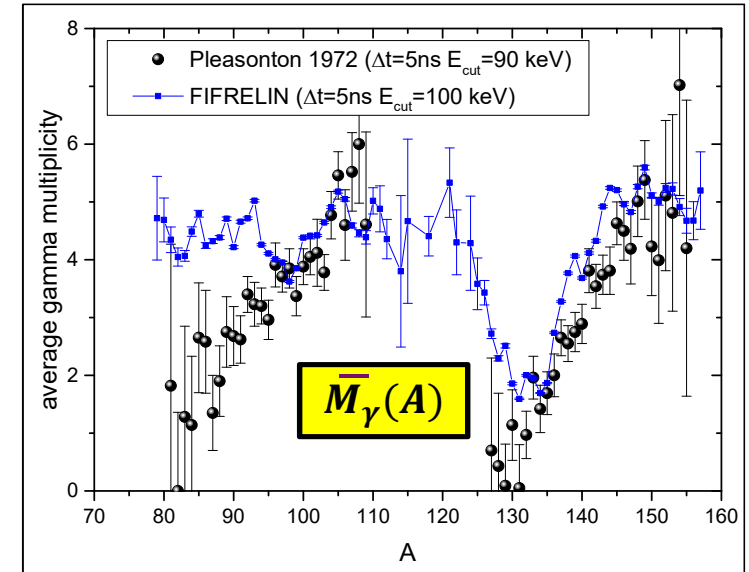
FIFRELIN : neutron multiplicities



Average prompt **neutron** multiplicity as a function of pre-neutron fragment mass



Average prompt **gamma** multiplicity as a function of pre-neutron fragment mass



Neutron average quantities

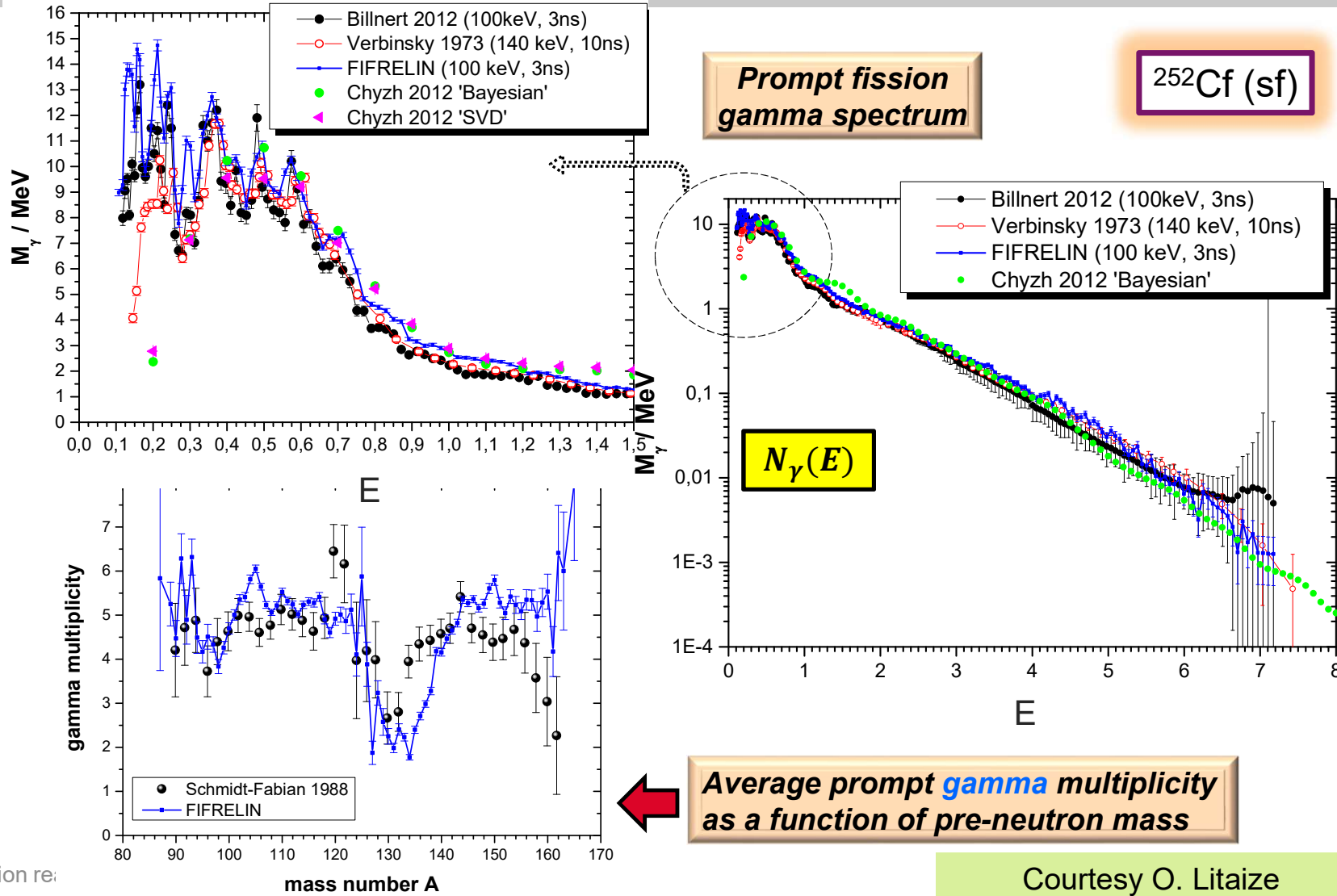
	$\langle \nu_L \rangle$	$\langle \nu_H \rangle$	$\langle \nu \rangle$
Nishio 2004	1.42	1.01	2.43 ± 0.03
FIFRELIN	1.41 ± 0.001	1.02 ± 0.001	2.43 ± 0.001

Gamma average quantities

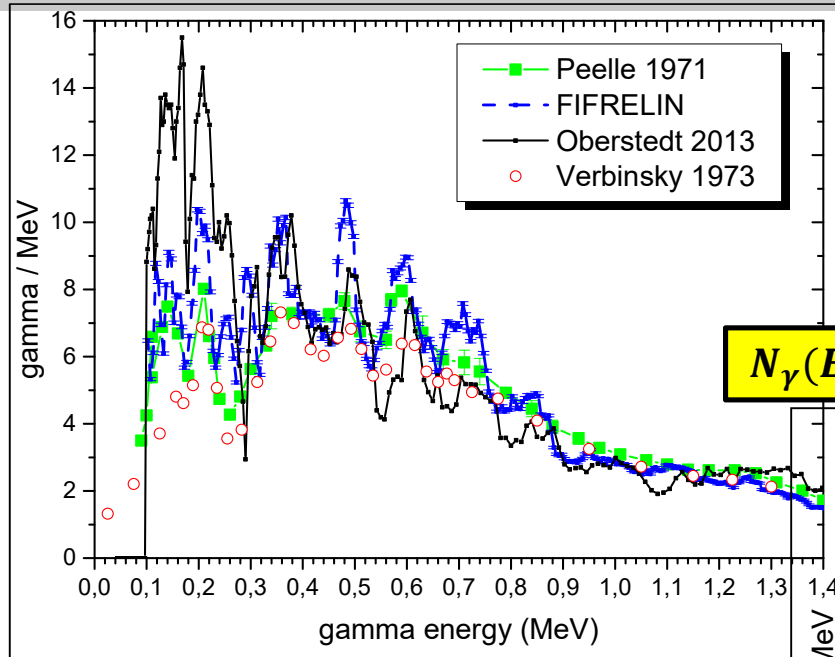
	Threshold	ΔT	$\langle M_\gamma \rangle$ (γ/f)	$\langle E_\gamma^{\text{tot}} \rangle$ (MeV)	$\langle \epsilon_\gamma \rangle$ (MeV)
Oberstedt 2013	100 keV	5 ns	8.19 ± 0.11	6.92 ± 0.09	0.85 ± 0.02
FIFRELIN	100 keV	5 ns	8.04 ± 0.01	7.02 ± 0.01	0.875 ± 0.001



FIFRELIN : PFGS and spontaneous fission



FIFRELIN : PFGS and neutron induced fission

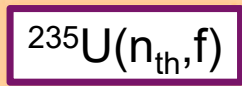
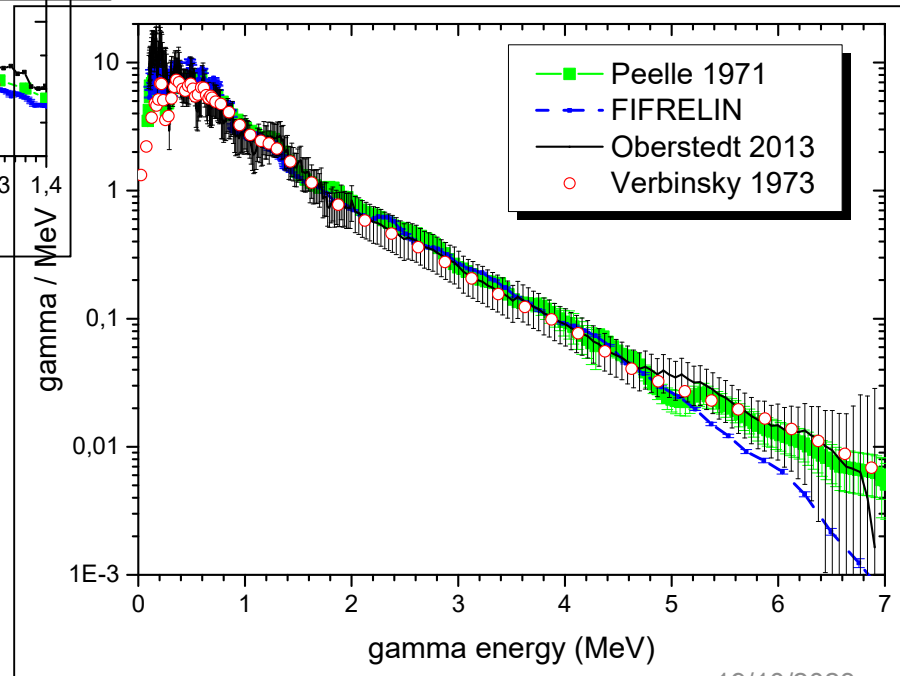


Prompt fission gamma spectrum

$N_\gamma(E)$

➤ Lower strength in the calculation above 6 MeV ?

➤ Position of the structures at low energy is reproduced by the calculation



Courtesy O. Litaize

FIFRELIN : PFNS and neutron induced fission



**Prompt fission
neutron spectrum**

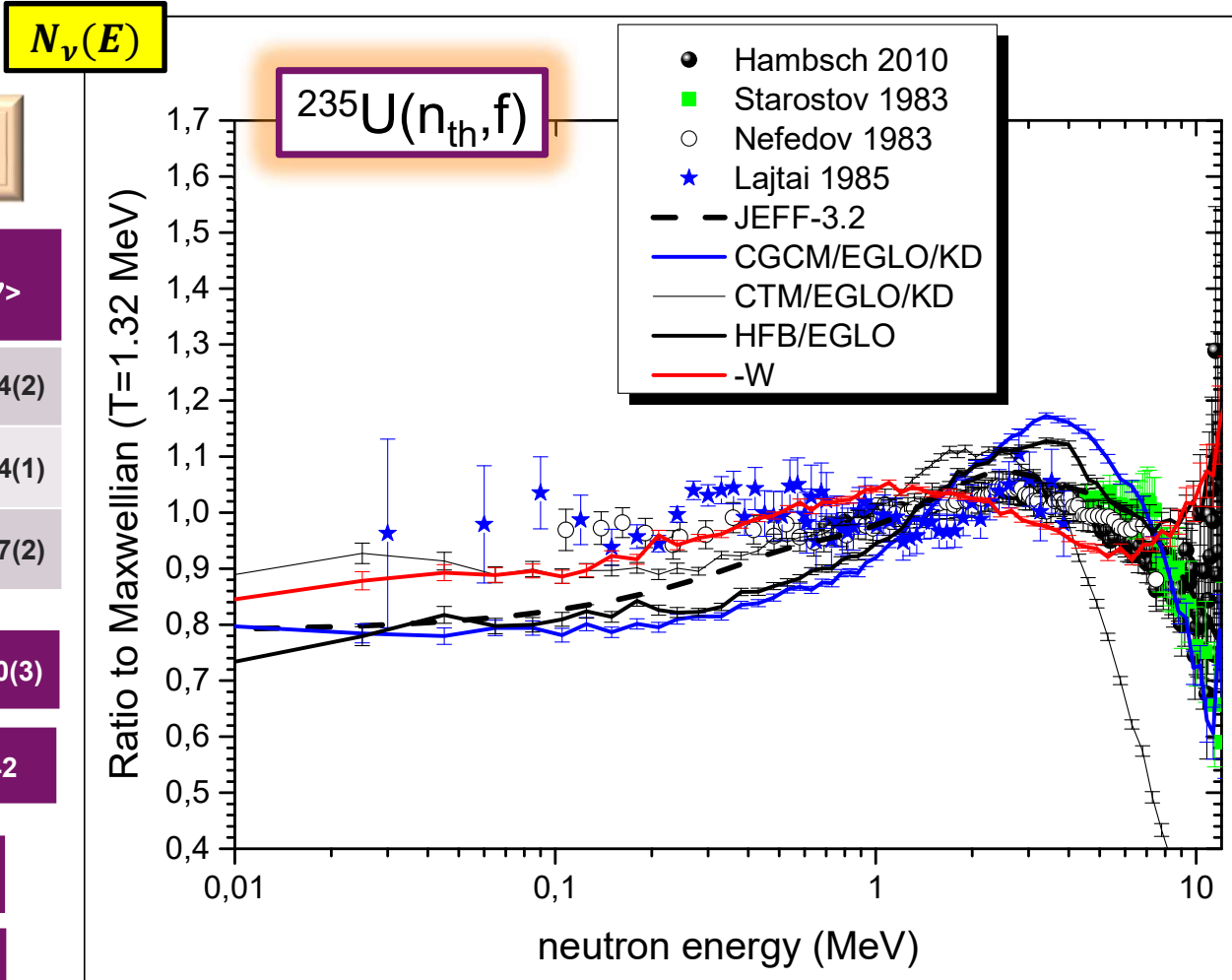
FIFRELIN -HF (LDM)	$\langle E \rangle$ MeV	$\langle \nu \rangle$
(CGCM)	2.102(2)	2.424(2)
(CTM)	1.891(1)	2.444(1)
(HFB)	2.079(2)	2.397(2)

FIFRELIN -W	1.945(2)	2.430(3)
----------------	----------	----------

JEFF-3.2	2.03	2.42
----------	------	------

Micro data	1.974(2)
------------	----------

Macro data	2.03
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FIFRELIN : neutron multiplicities and incident energy



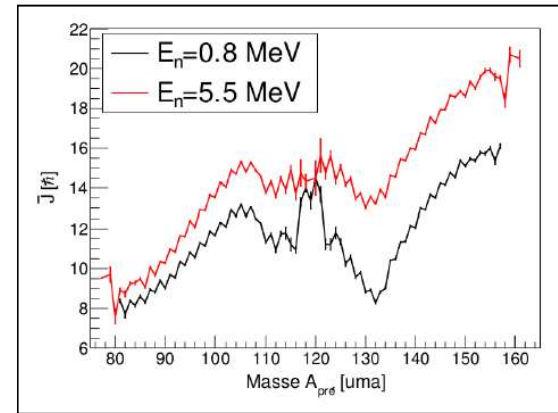
Constant temperature ratio

$R_T=1$ @ 0,8 MeV

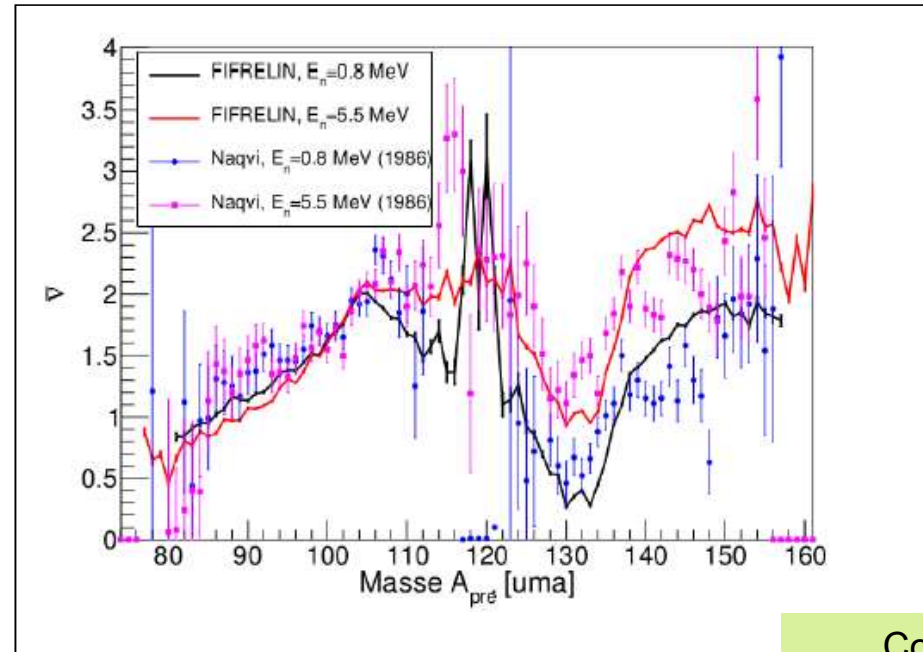
$R_T=0.8$ @ 5,5 MeV



Energy dependent spin cut-off for initial spin



$^{237}\text{Np}(n,f)$



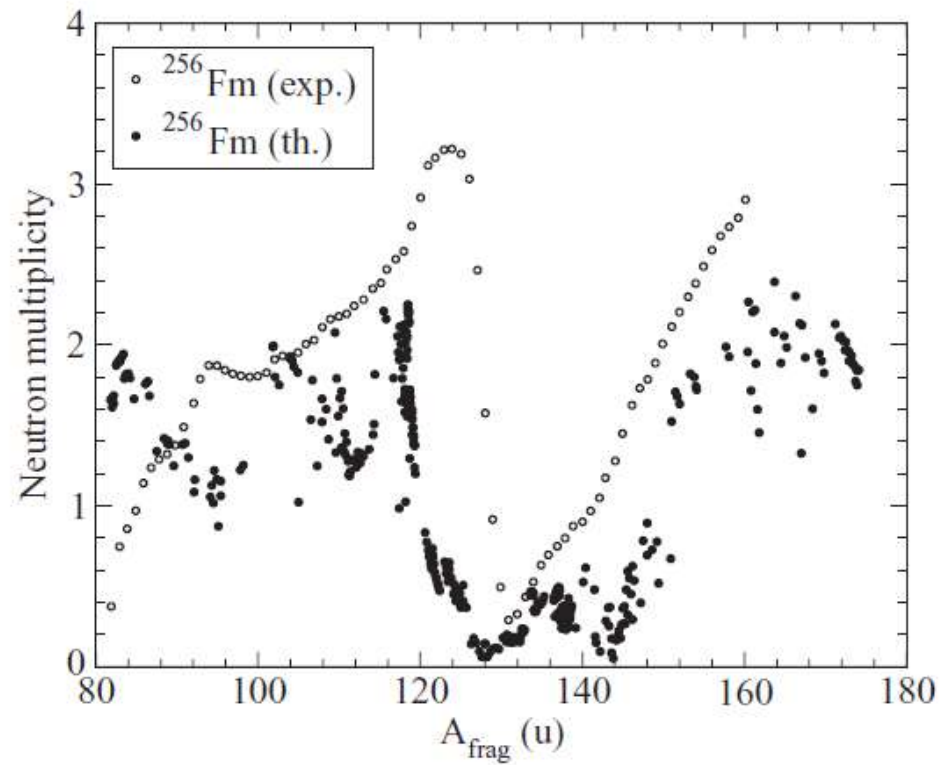
Courtesy L. Thulliez



MICROSCOPIC APPROACH



Details in Phys. Rev. C 77 (2008) 014310.



⇒ Not yet at the level

FIG. 15. ^{256}Fm . Neutron multiplicity versus fragment mass. Comparison between predictions (solid symbols) and data [47] (empty symbols).



4. Fission cross sections



Fission reactions

- Generalities about fission

Induced, spontaneous, energy balance, fission yields, PFNS, neutron multiplicities, cross sections, fission chances, Kinetic energies,

- Fission yields

- GEF model
- SPY model
- Microscopic approach

- Neutrons and gammas from fission

- Madland-Nix model
- GEF model
- FIFRELIN
- Microscopic approach

- Fission cross sections

- Phenomenological approaches
- Coherent fission cross sections
- Microscopic approaches
- Integral benchmark sensitivity

- Conclusions



Fission reactions

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Induced, spontaneous, energy balance, fission yields, PFNS, neutron multiplicities, cross sections, fission chances, Kinetic energies,

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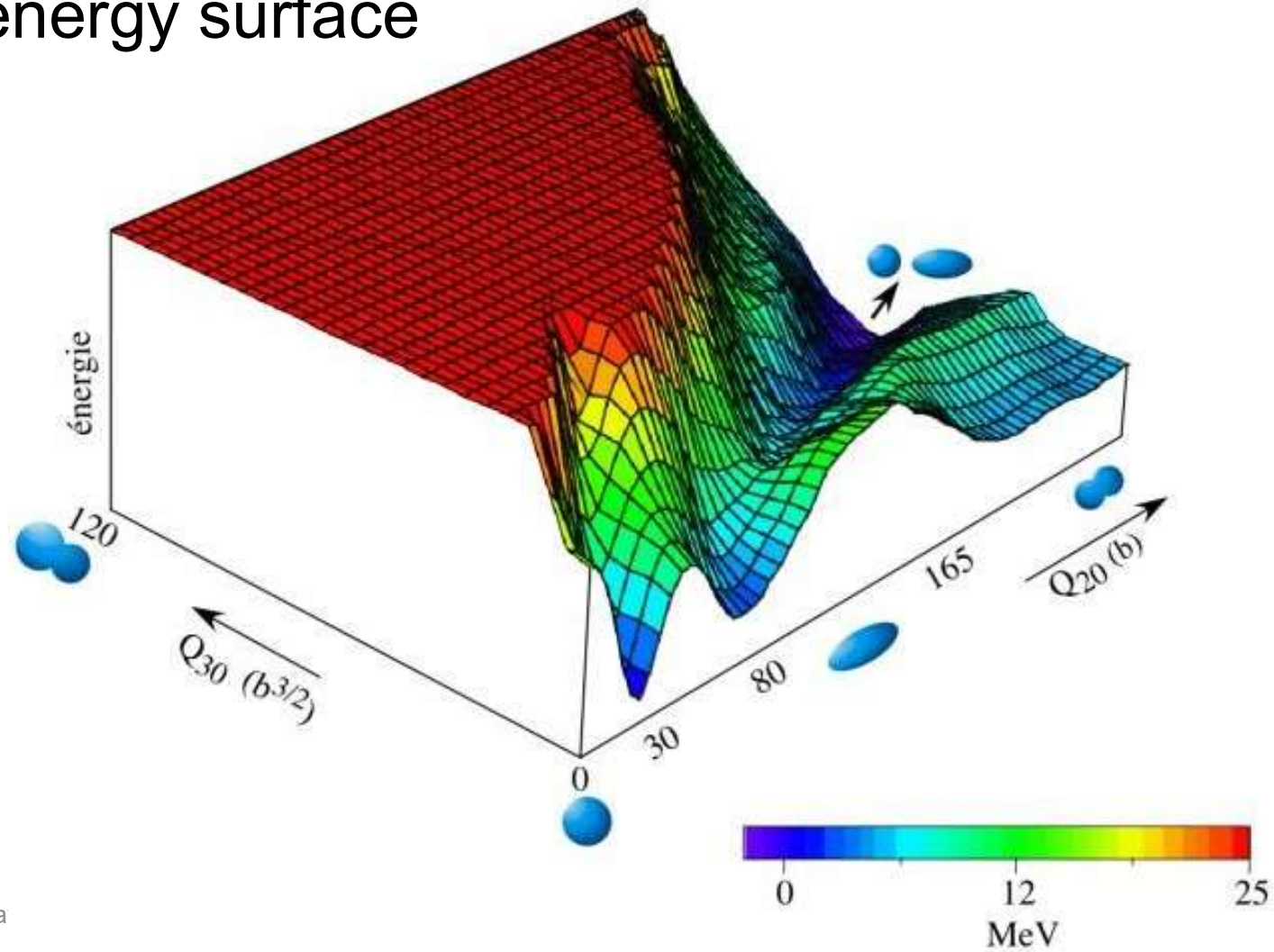
- Phenomenological approaches
- Coherent fission cross sections
- Microscopic approaches
- Integral benchmark sensitivity

- Conclusions

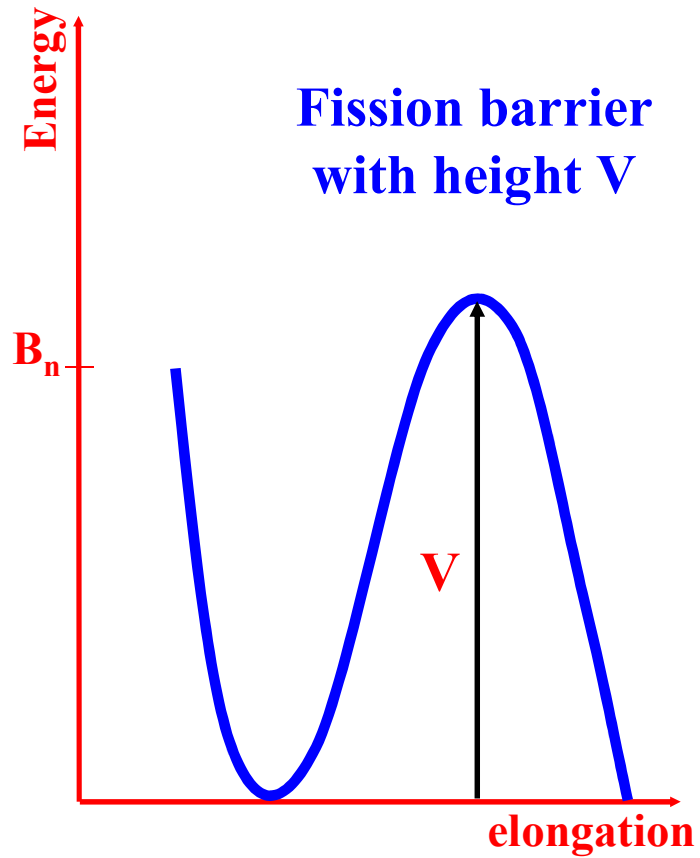


Fission barriers and fission paths

^{238}U Potential energy surface

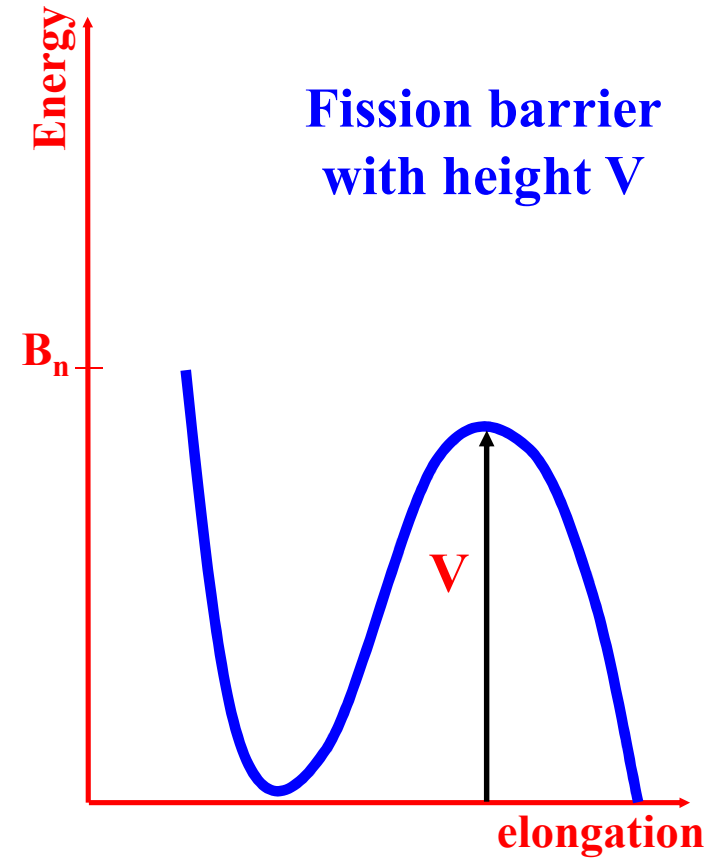


Fissile or fertile



$$B_n < V$$

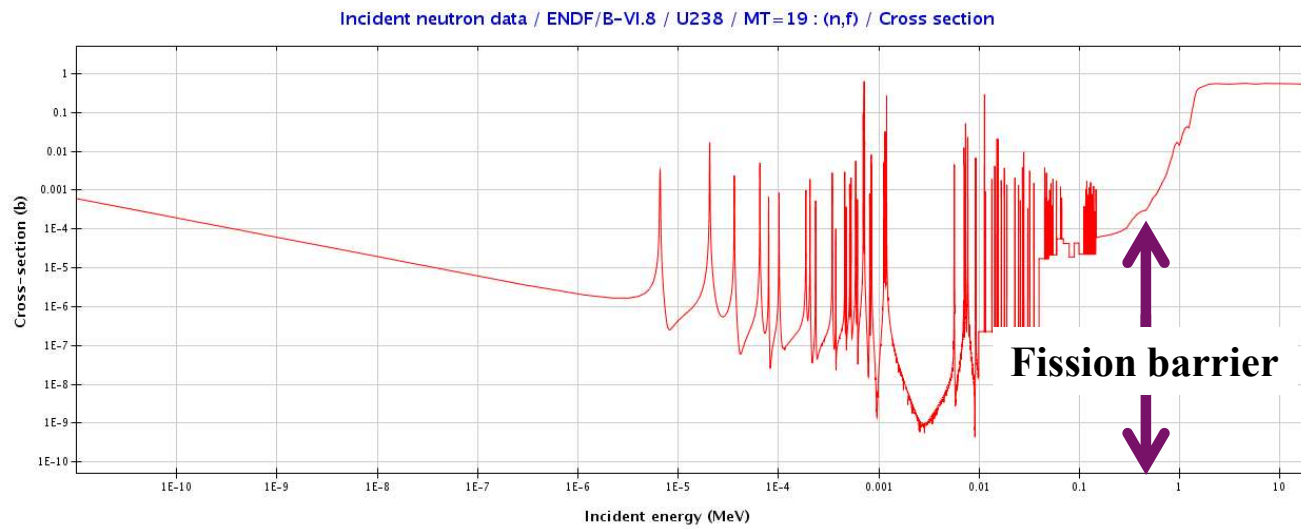
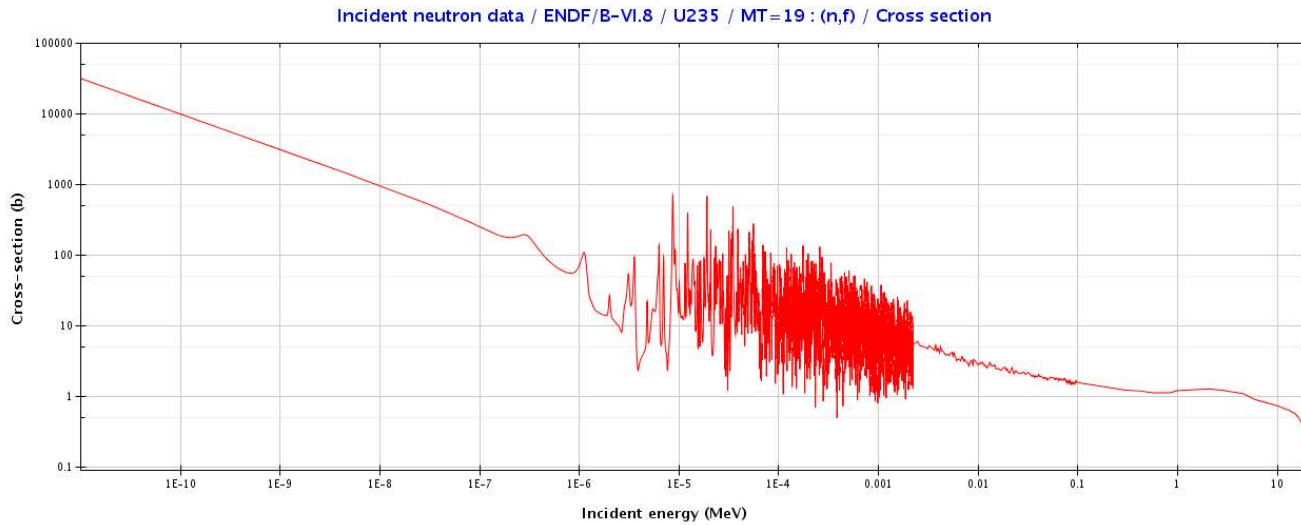
Fertile target (^{238}U)



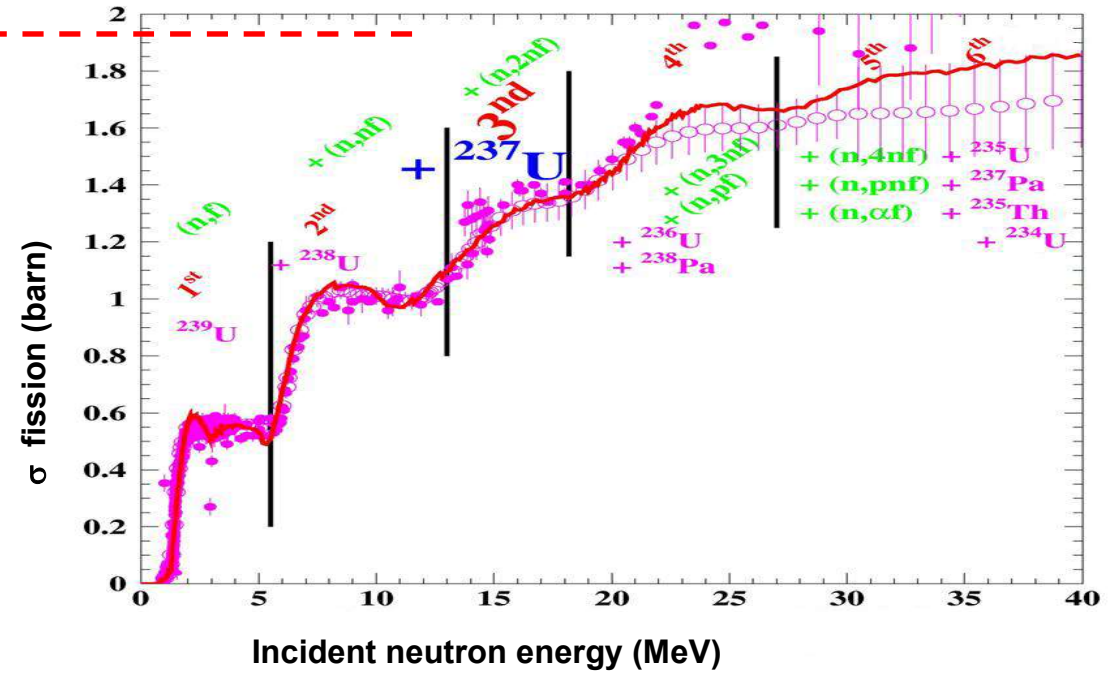
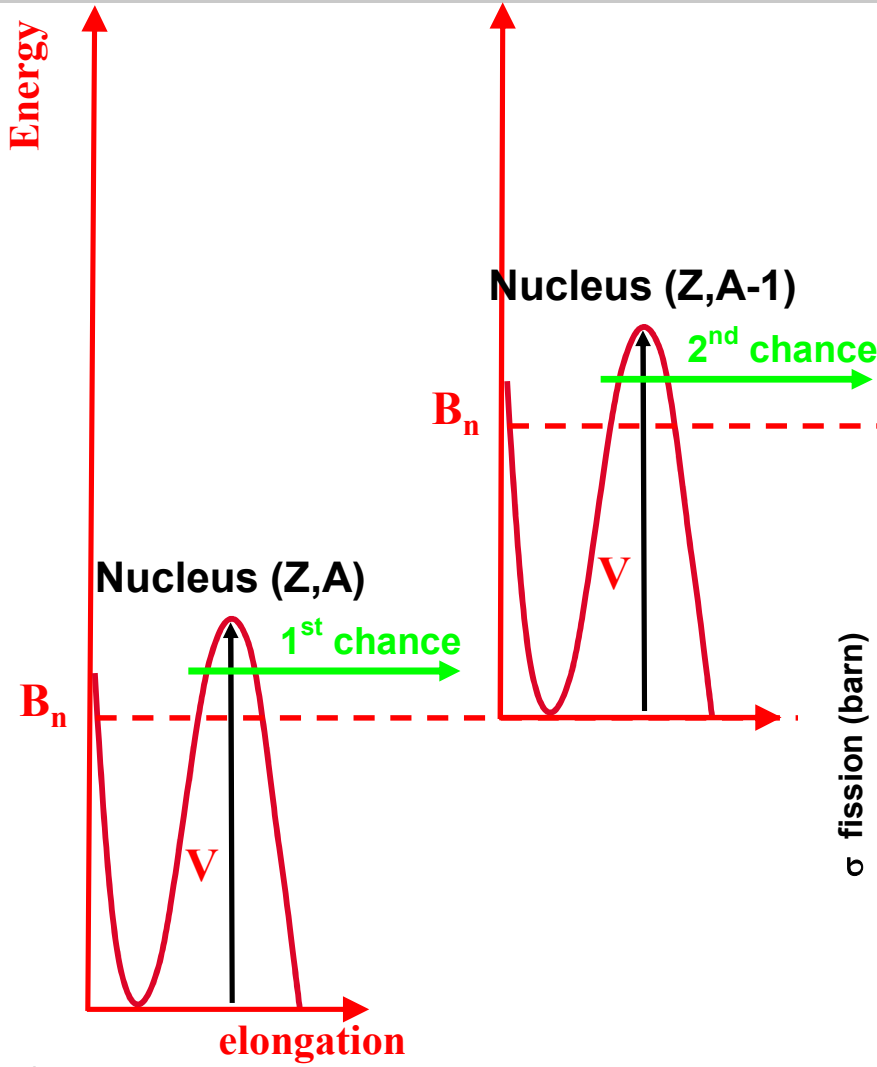
$$B_n > V$$

Fissile target (^{235}U)

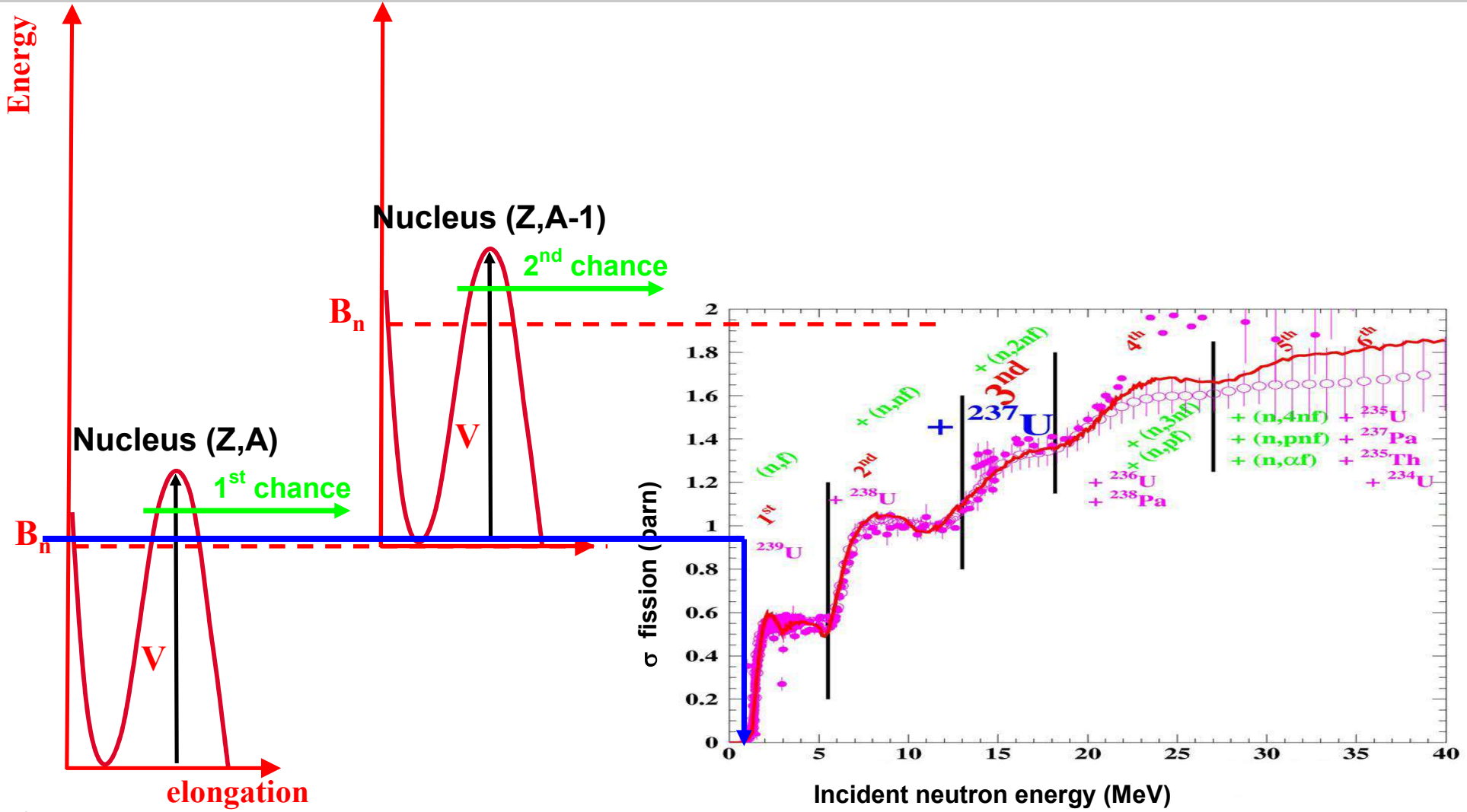
Fissile or fertile



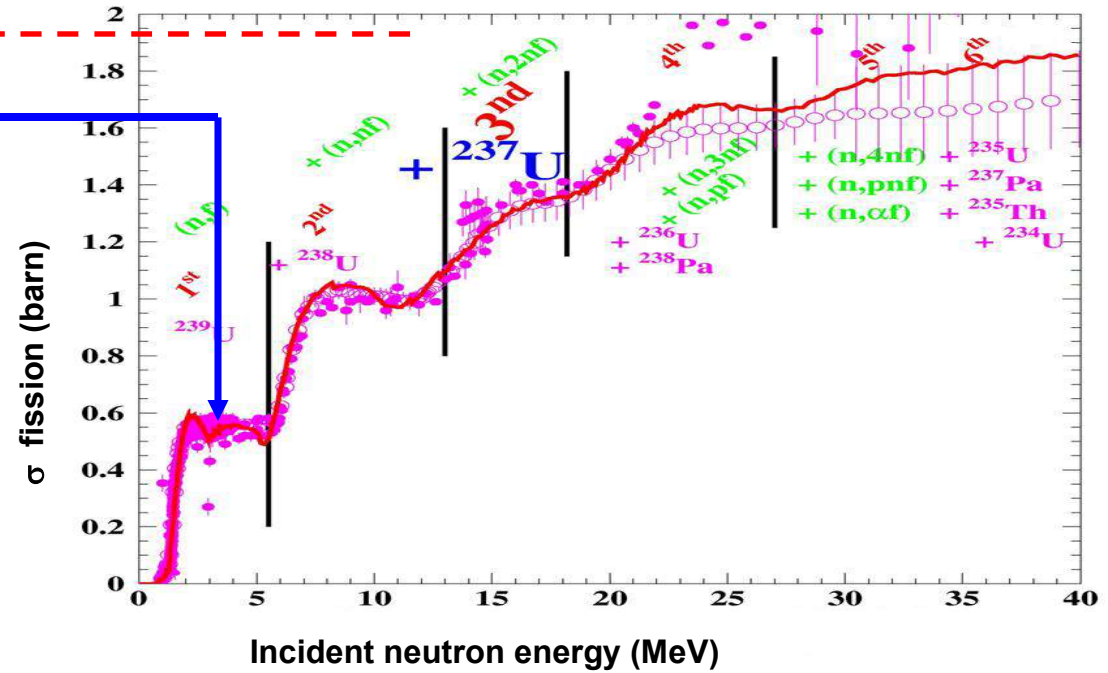
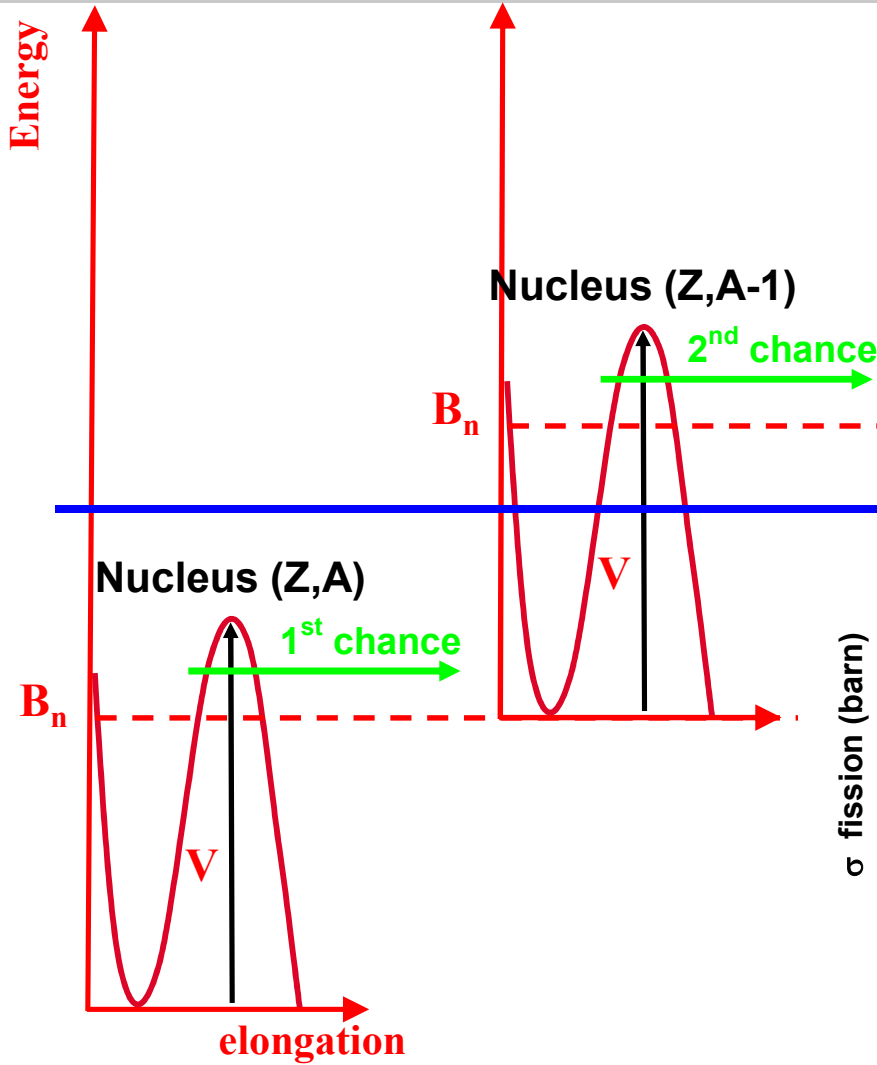
Multiple fission chances



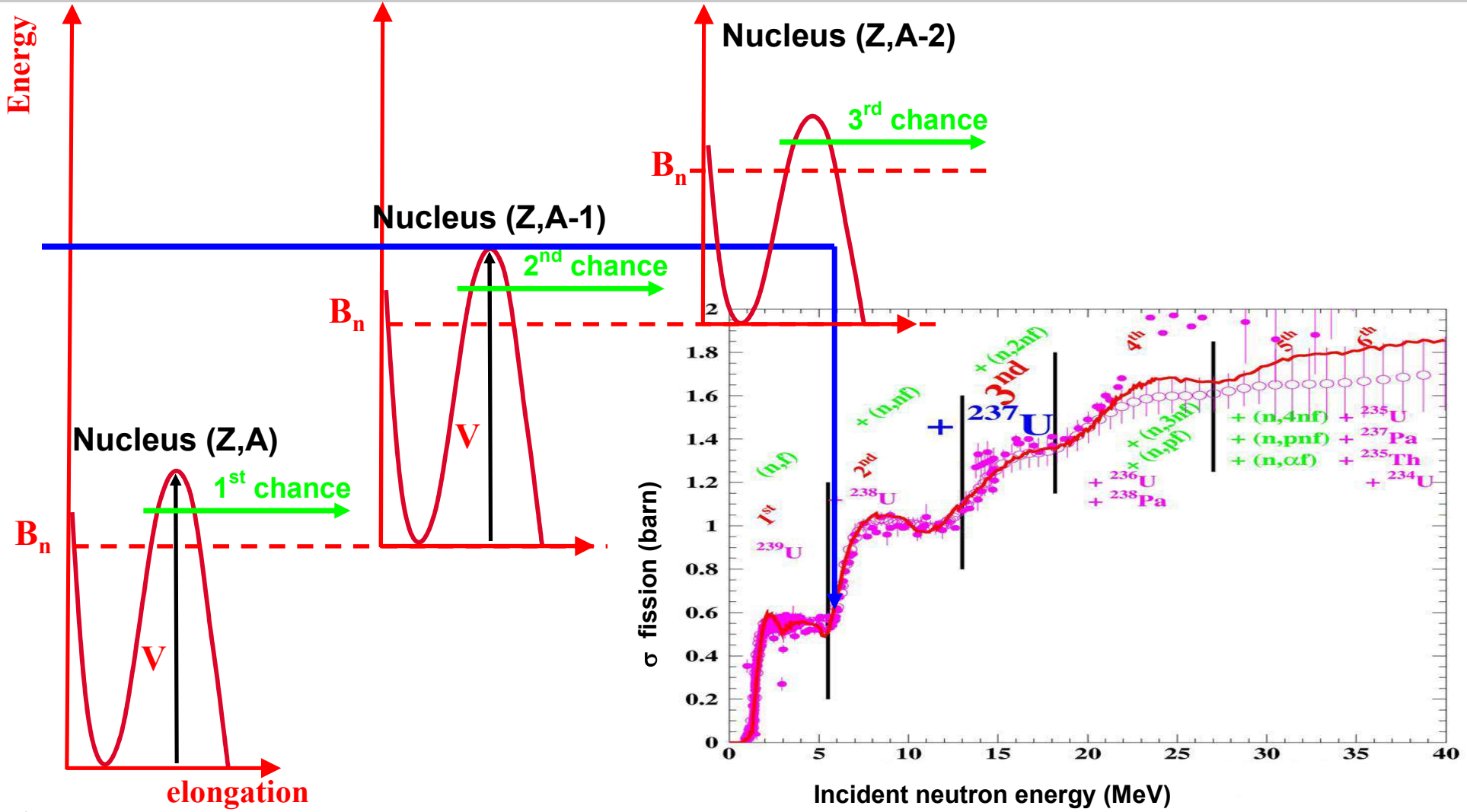
Multiple fission chances



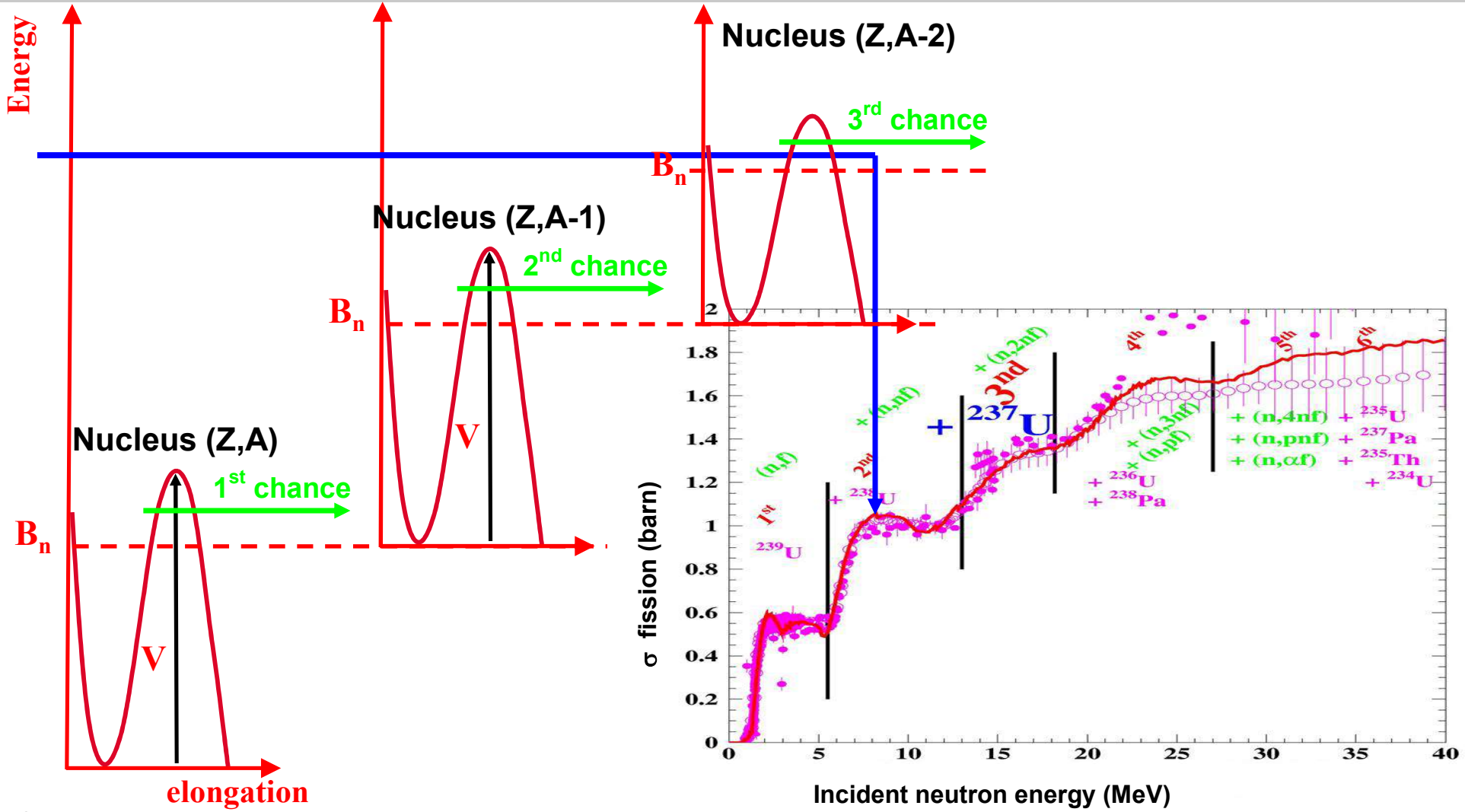
Multiple fission chances



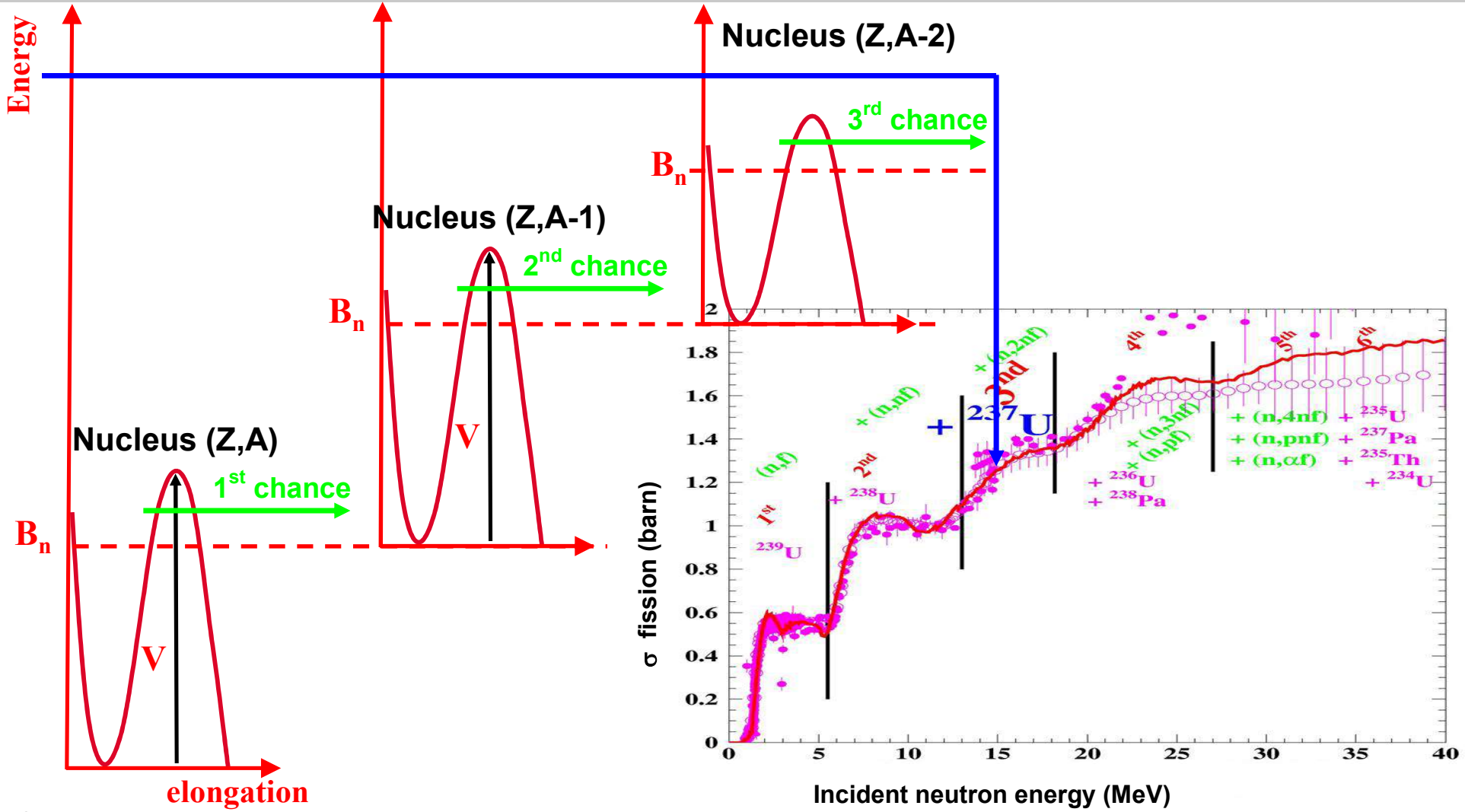
Multiple fission chances



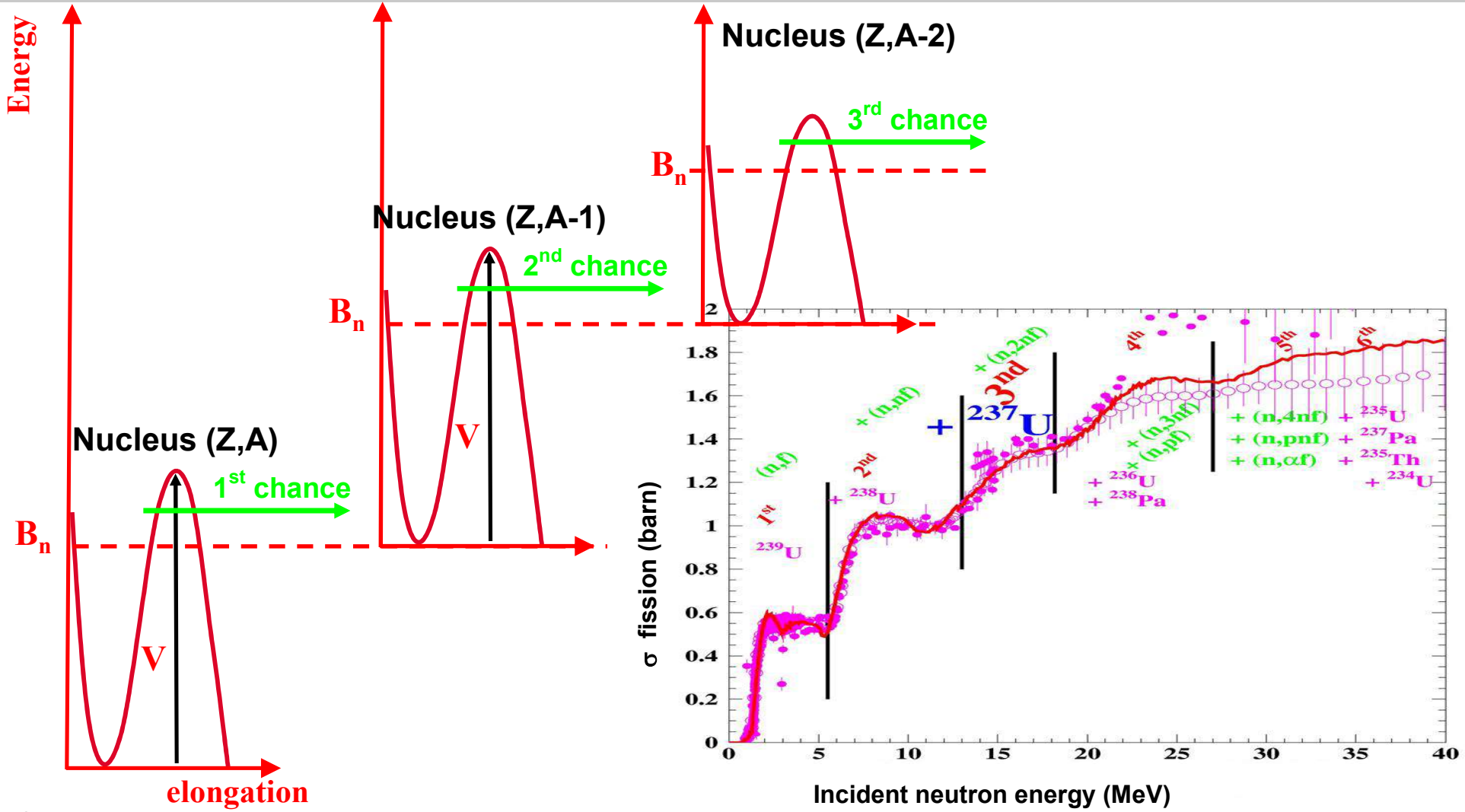
Multiple fission chances



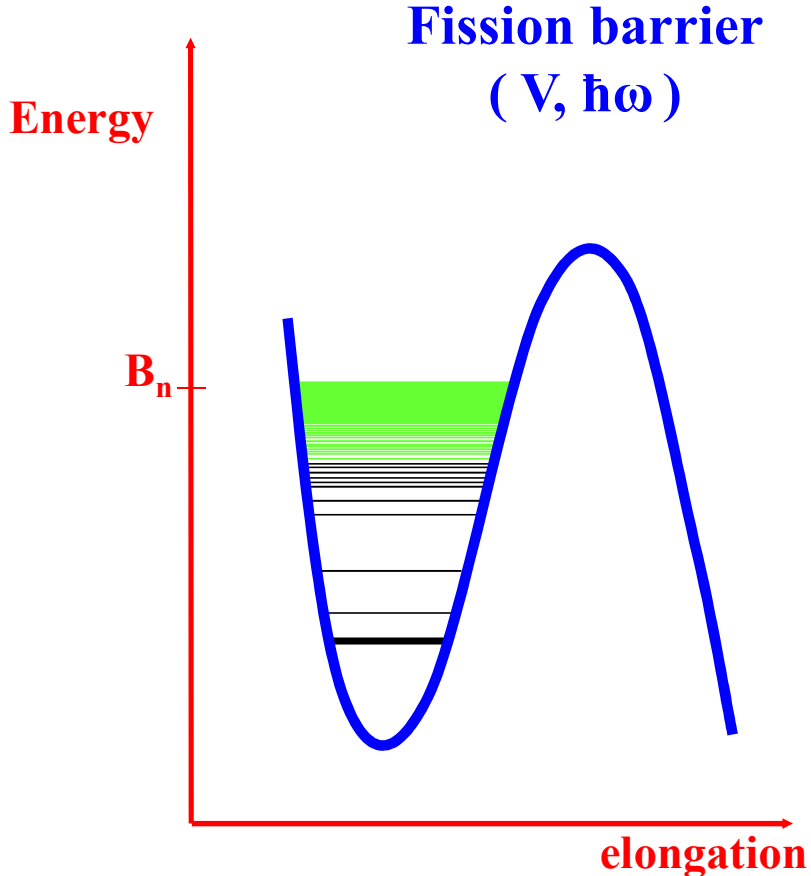
Multiple fission chances



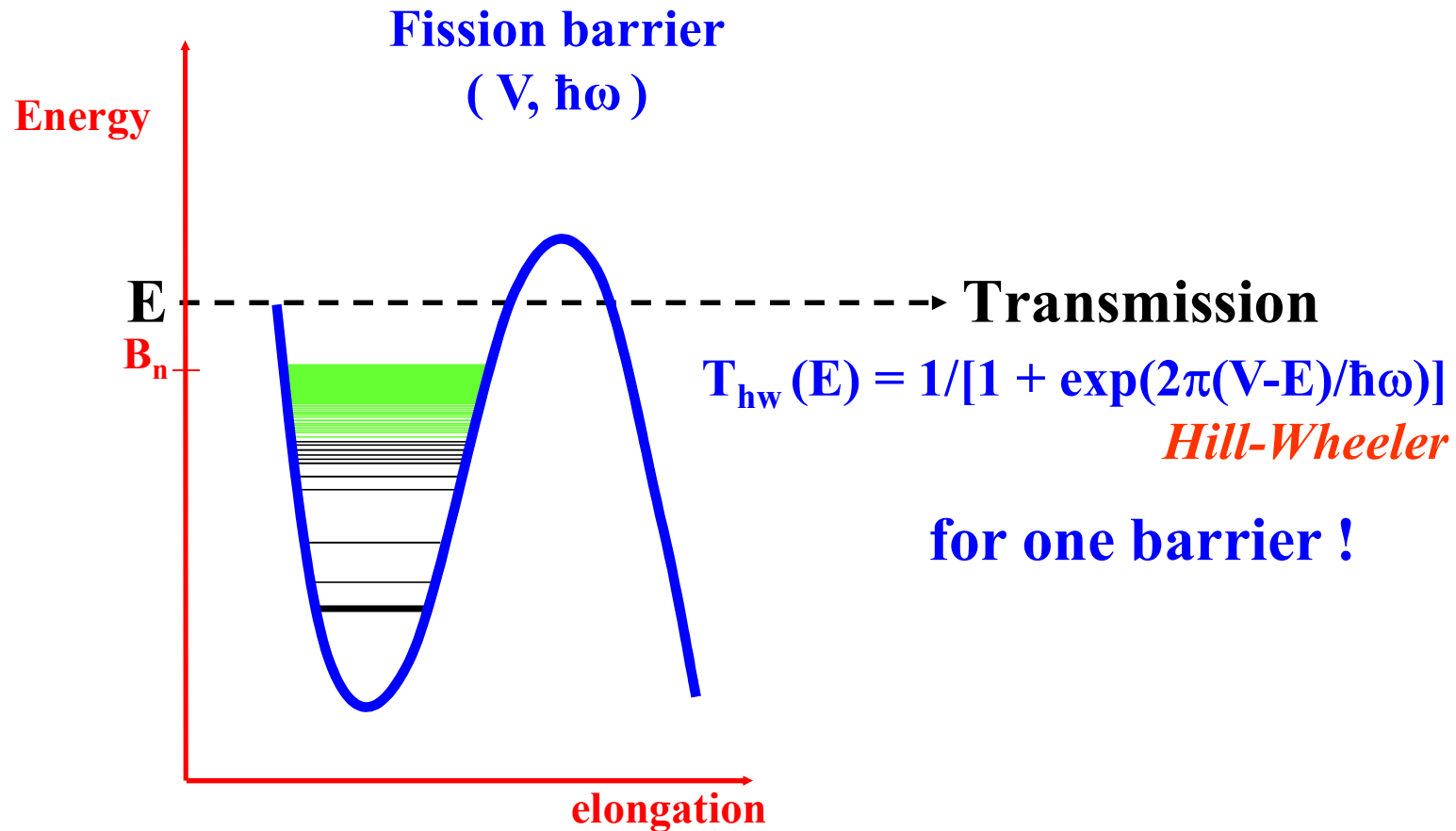
Multiple fission chances



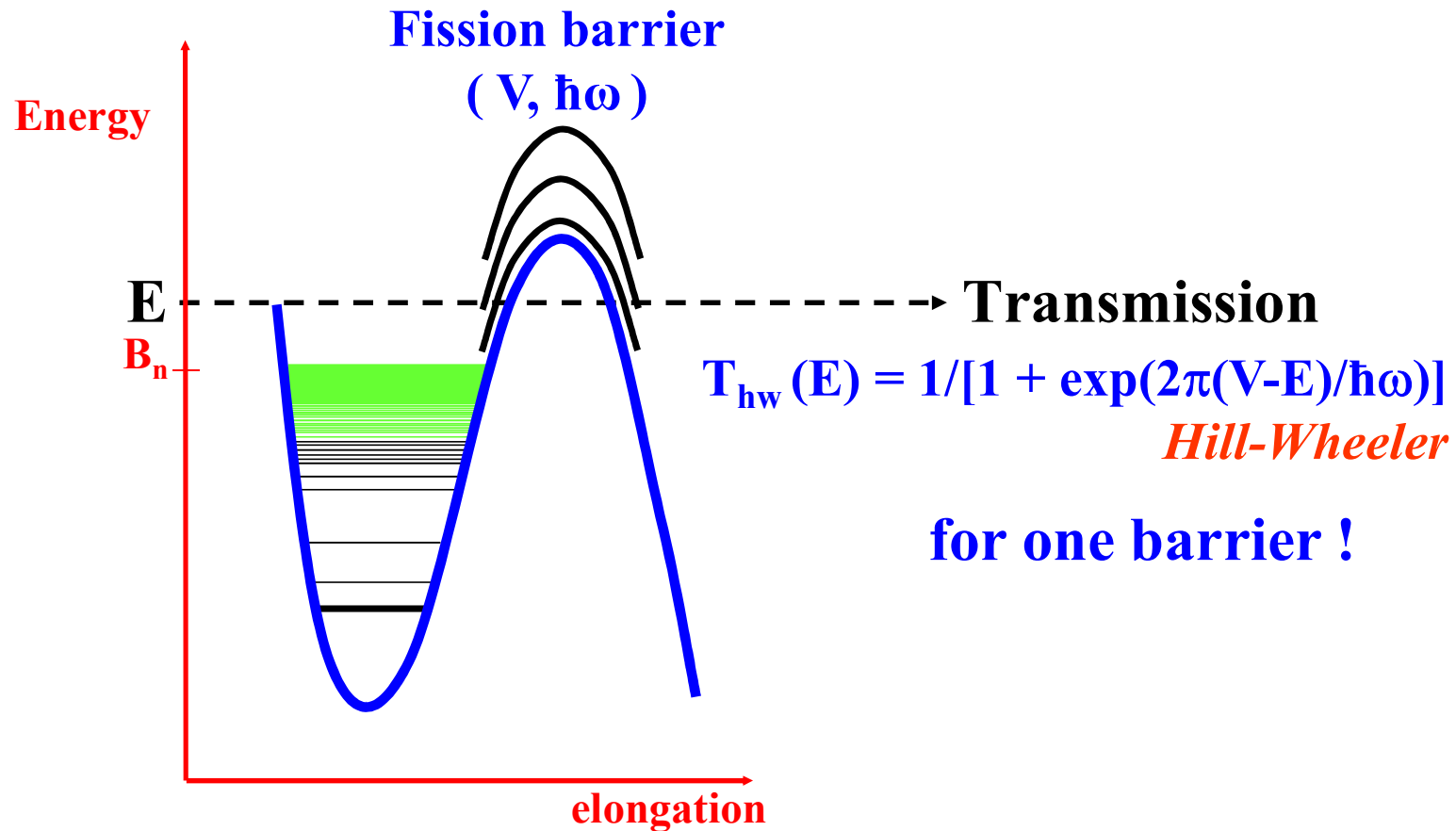
Fission transmission coefficient :Hill-Wheeler penetrability



Fission transmission coefficient : Hill-Wheeler penetrability



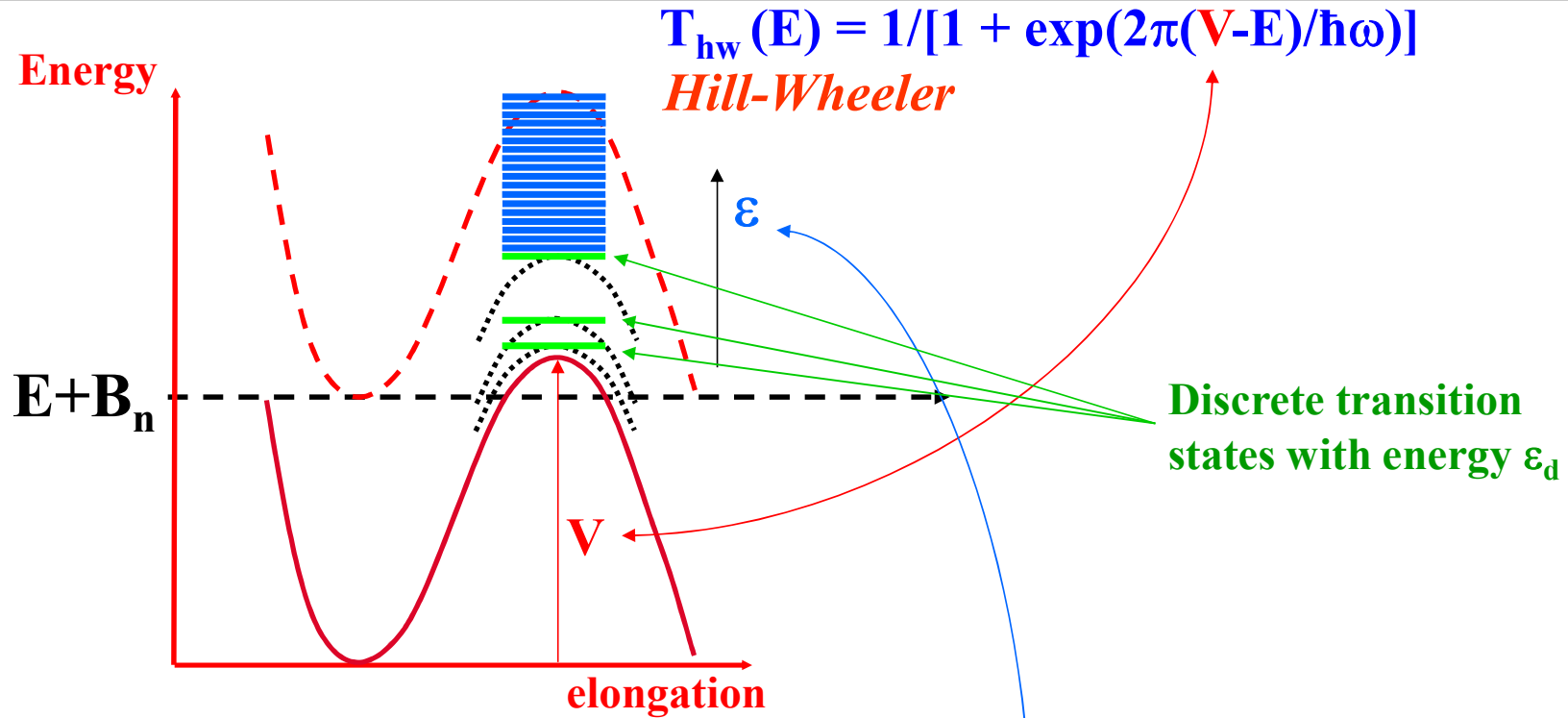
Fission transmission coefficient :Hill-Wheeler penetrability



+ transition state on top of the barrier !

Bohr hypothesis

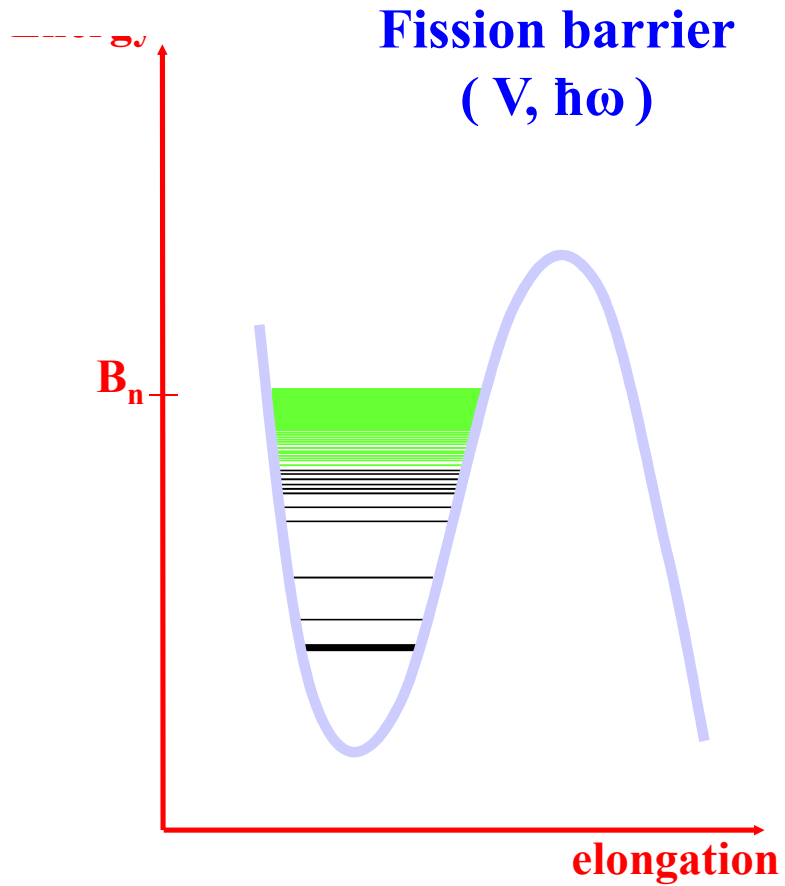
Fission transmission coefficient : single humped barrier



$$T_f(E, J, \pi) = \sum_{\substack{\text{discrete} \\ J, \pi}} T_{hw}(E - \epsilon_d) + \int_{E_s}^{E+B_n} \rho(\epsilon, J, \pi) T_{hw}(E - \epsilon) d\epsilon$$

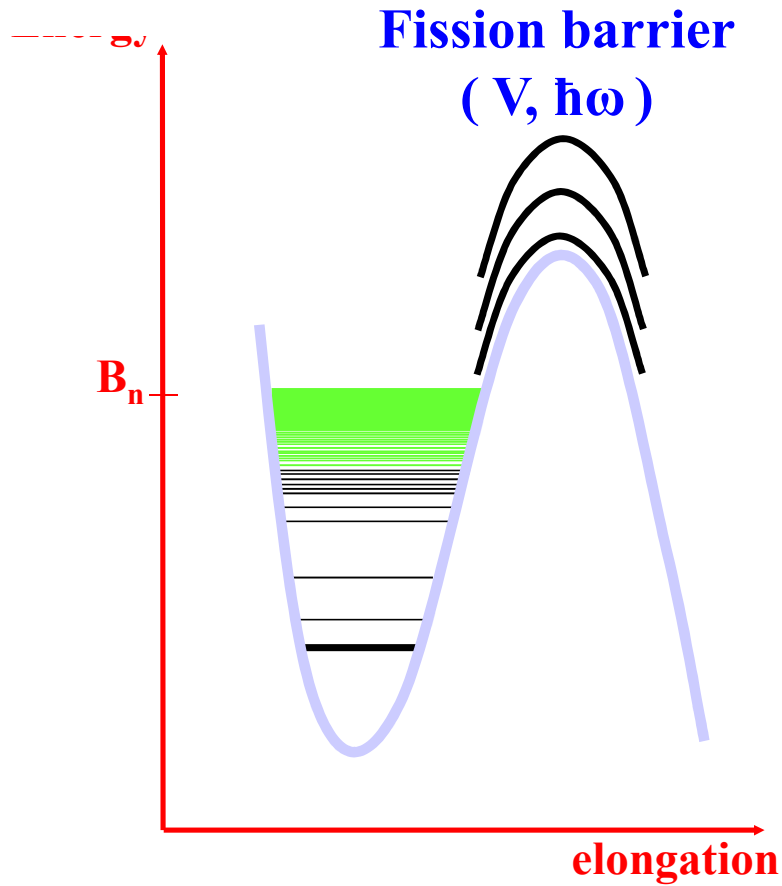


Fission transmission coefficient : penetrability for double humped barrier





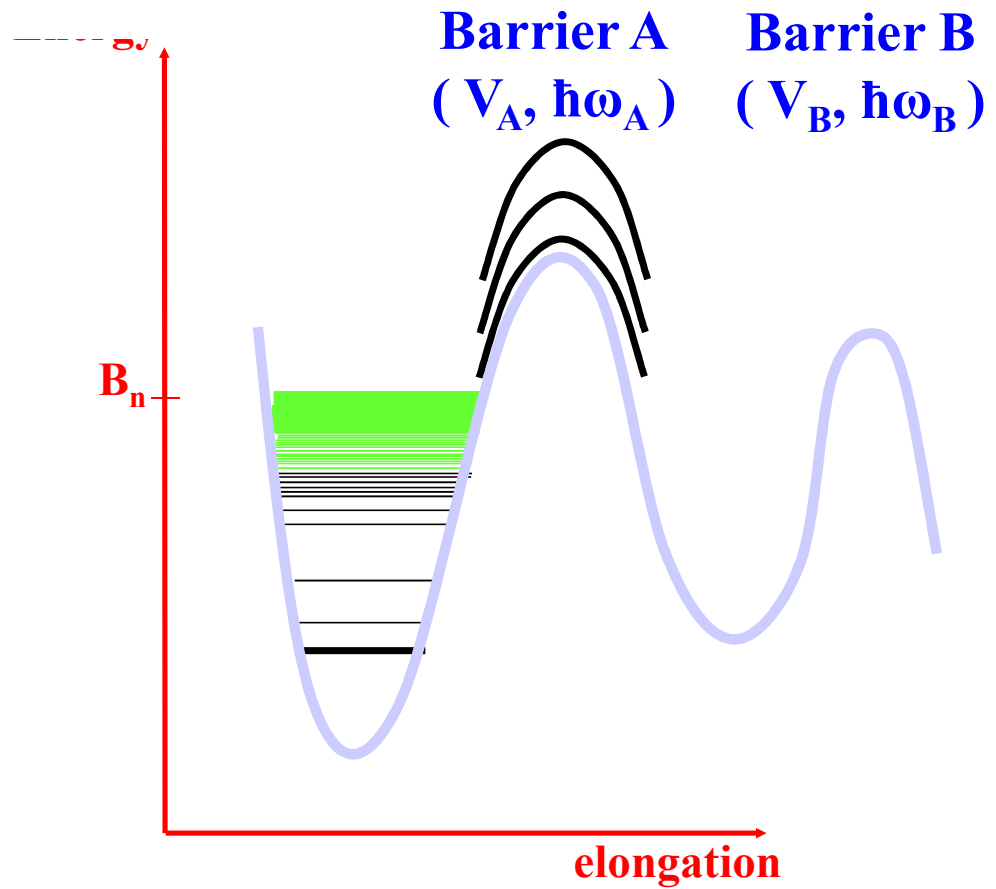
Fission transmission coefficient : penetrability for double humped barrier



+ transition states on top of the barrier !



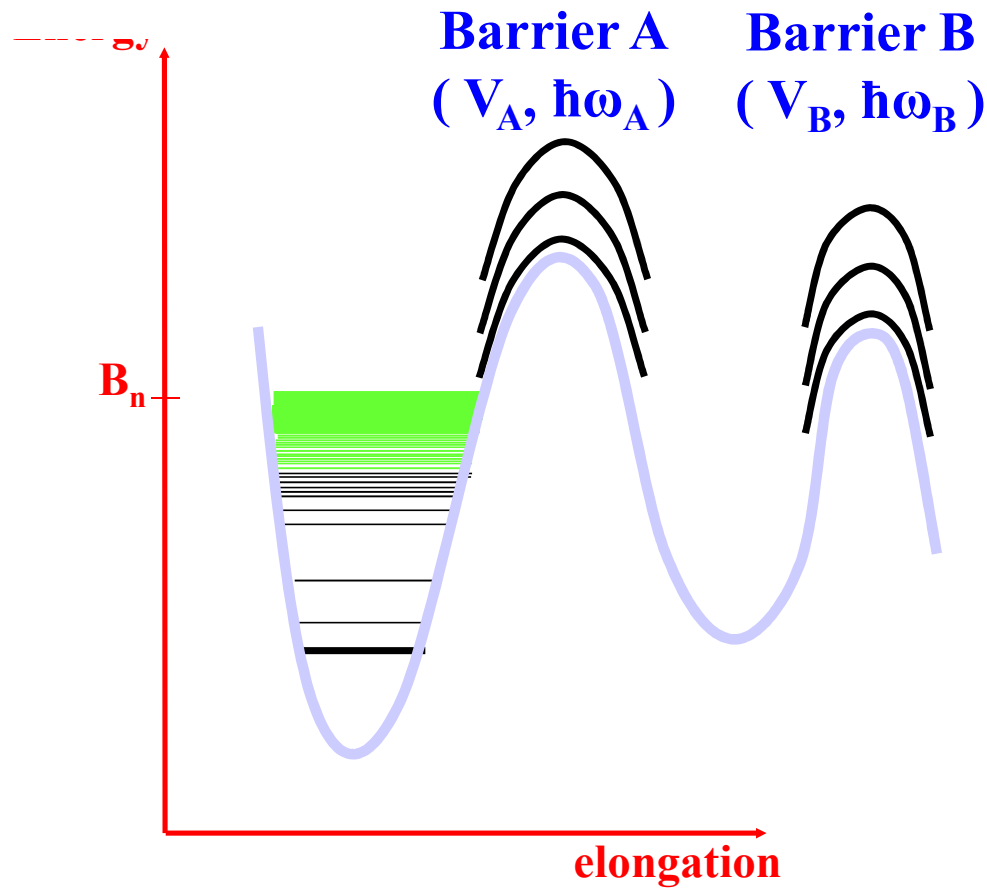
Fission transmission coefficient : penetrability for double humped barrier



+ transition states on top of the barrier !



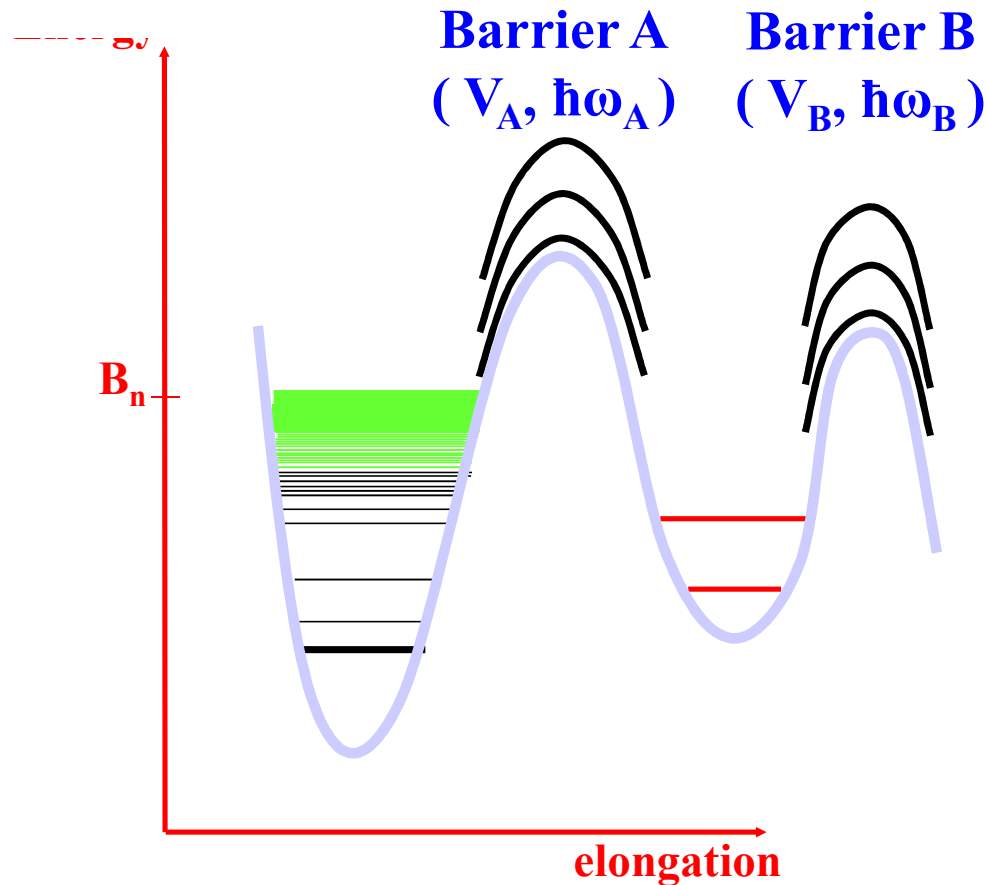
Fission transmission coefficient : penetrability for double humped barrier



+ transition states on top of each barrier !



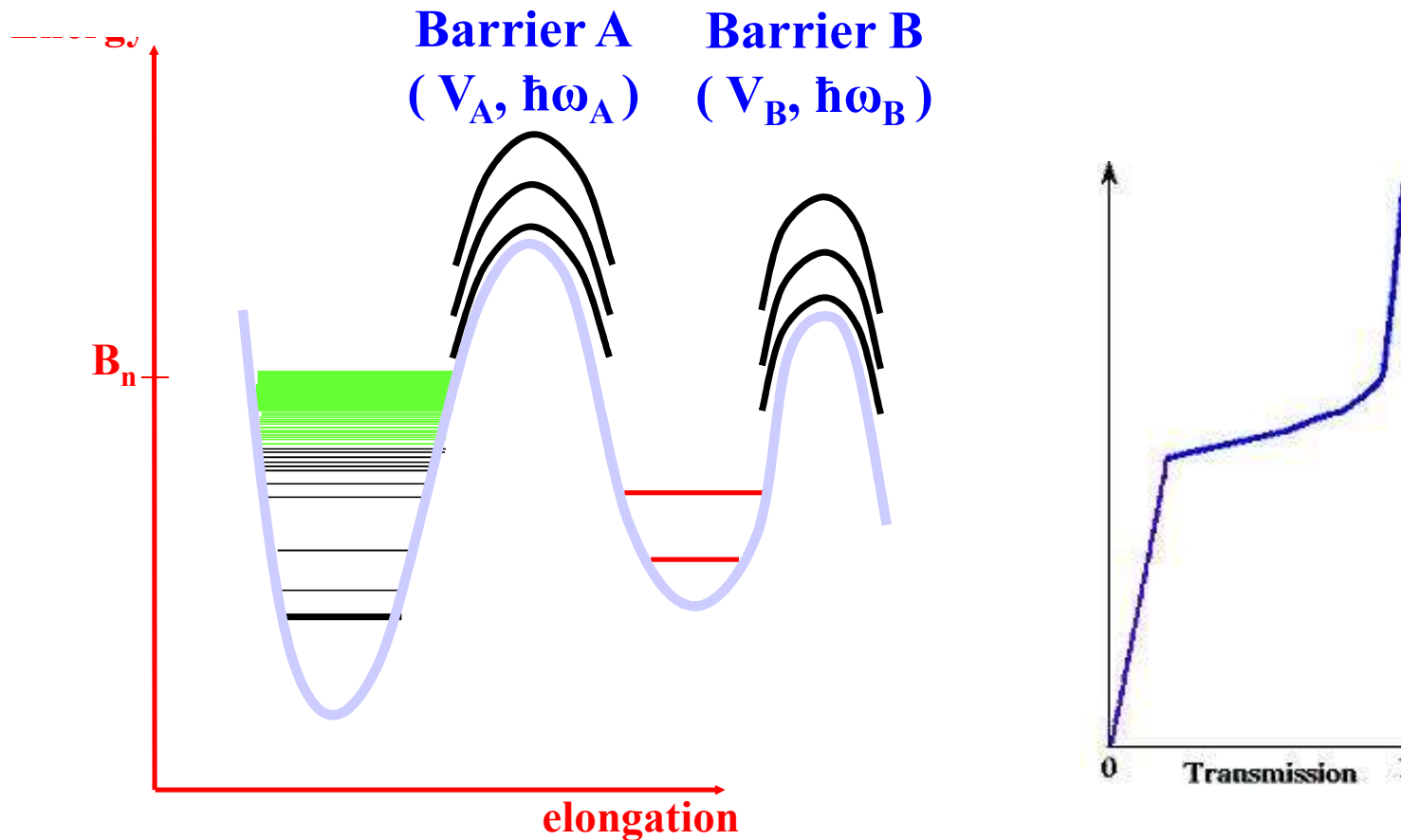
Fission transmission coefficient : penetrability for double humped barrier



- + transition states on top of each barrier !
- + class II states in the intermediate well !



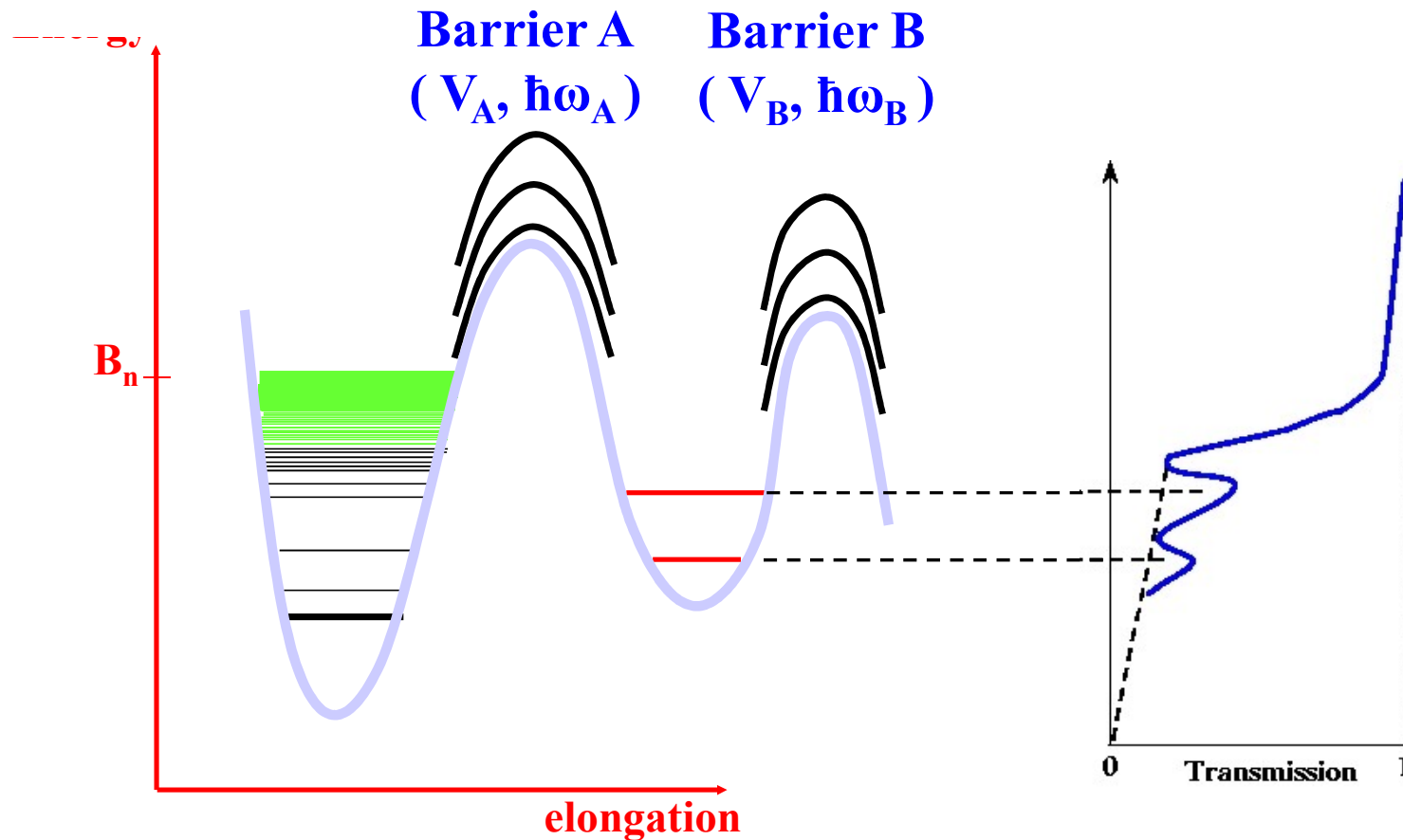
Fission transmission coefficient : penetrability for double humped barrier



- + transition states on top of each barrier !
- + class II states in the intermediate well !



Fission transmission coefficient : penetrability for double humped barrier



- + transition states on top of each barrier !
- + class II states in the intermediate well !



Fission transmission coefficient : double/triple humped barrier

Two barriers A et B

$$T_f = \frac{T_A T_B}{T_A + T_B}$$



Fission transmission coefficient : double/triple humped barrier

Two barriers A et B

$$T_f = \frac{T_A T_B}{T_A + T_B}$$

Three barriers A, B and C

$$T_f = \frac{\frac{T_A T_B}{T_A + T_B} \times T_C}{\frac{T_A T_B}{T_A + T_B} + T_C}$$



Fission transmission coefficient : double/triple humped barrier

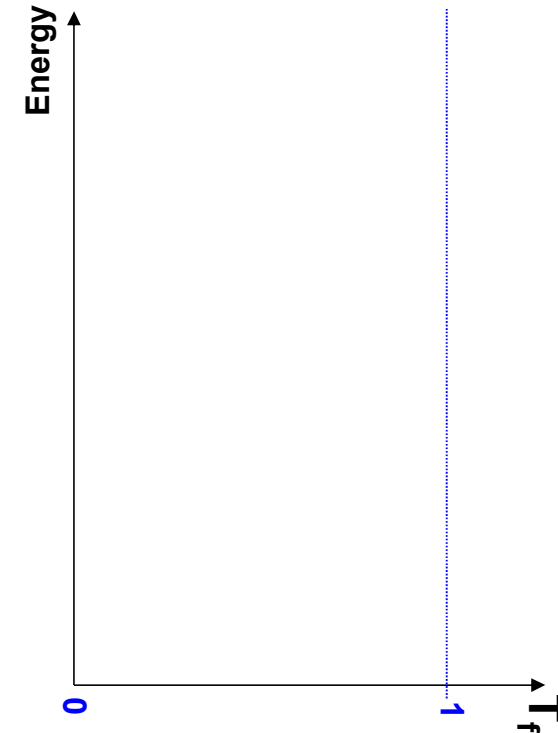
Two barriers A et B

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Three barriers A, B and C

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Resonant transmission





Fission transmission coefficient : double/triple humped barrier

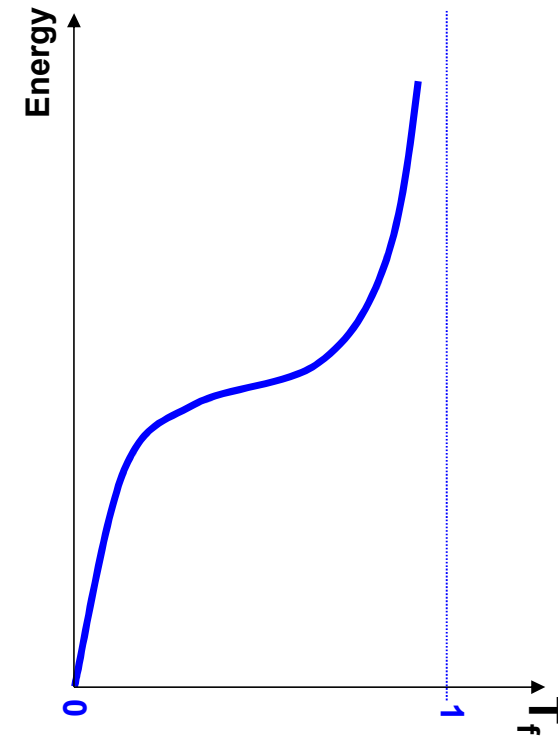
Two barriers A et B

$$T_f = \frac{T_A T_B}{T_A + T_B}$$

Three barriers A, B and C

$$T_f = \frac{\frac{T_A T_B}{T_A + T_B} \times T_C}{\frac{T_A T_B}{T_A + T_B} + T_C}$$

Resonant transmission



$$T_f =$$

Fission transmission coefficient : double/triple humped barrier



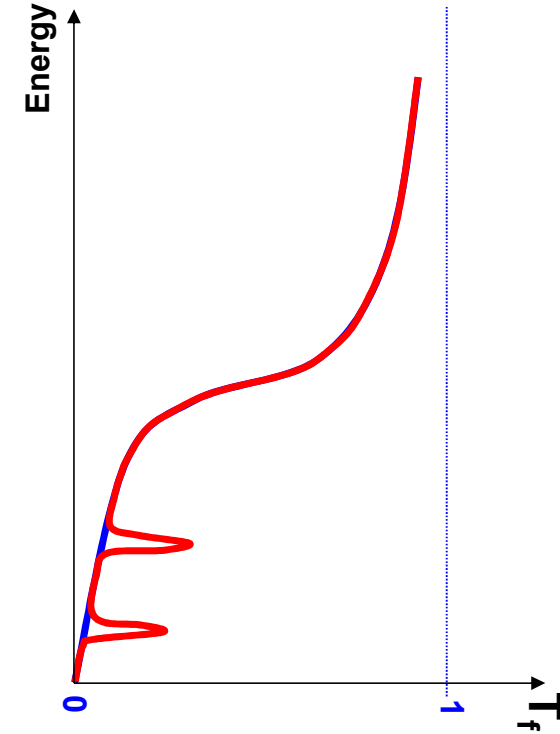
Two barriers A et B

$$T_f = \frac{T_A T_B}{T_A + T_B}$$

Three barriers A, B and C

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Resonant transmission

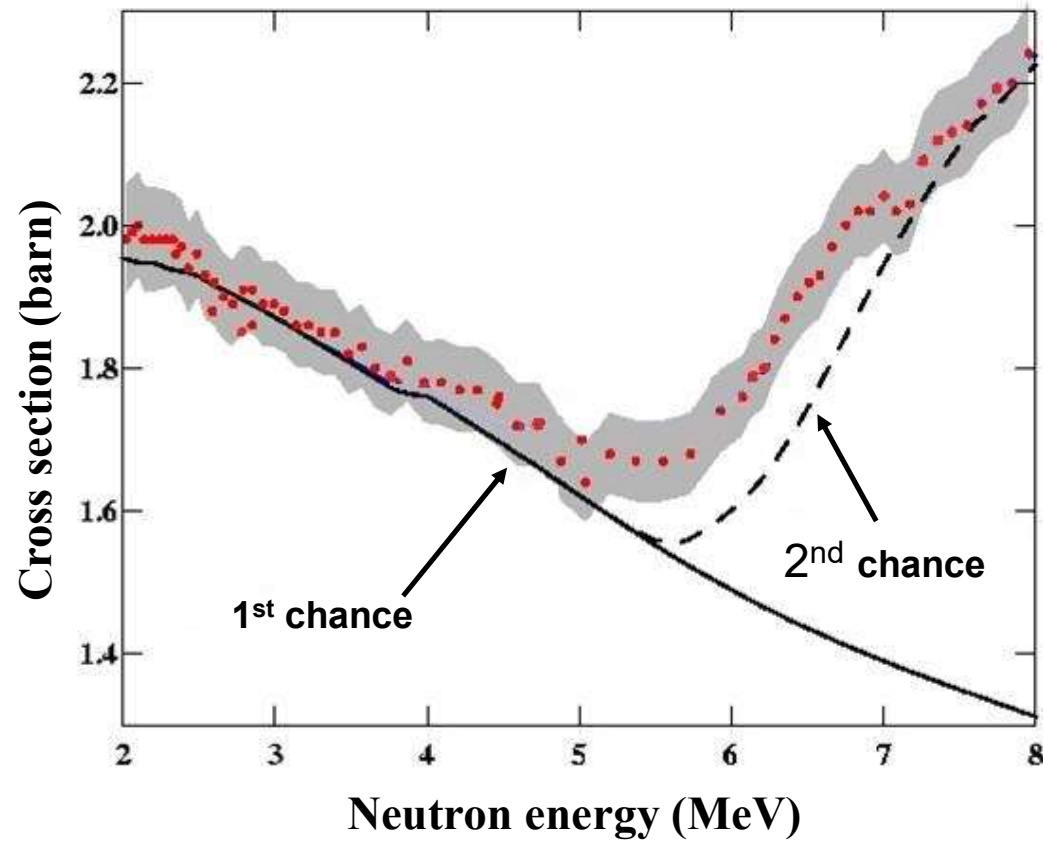


$$T_f =$$

Fission transmission coefficient : role of class II states



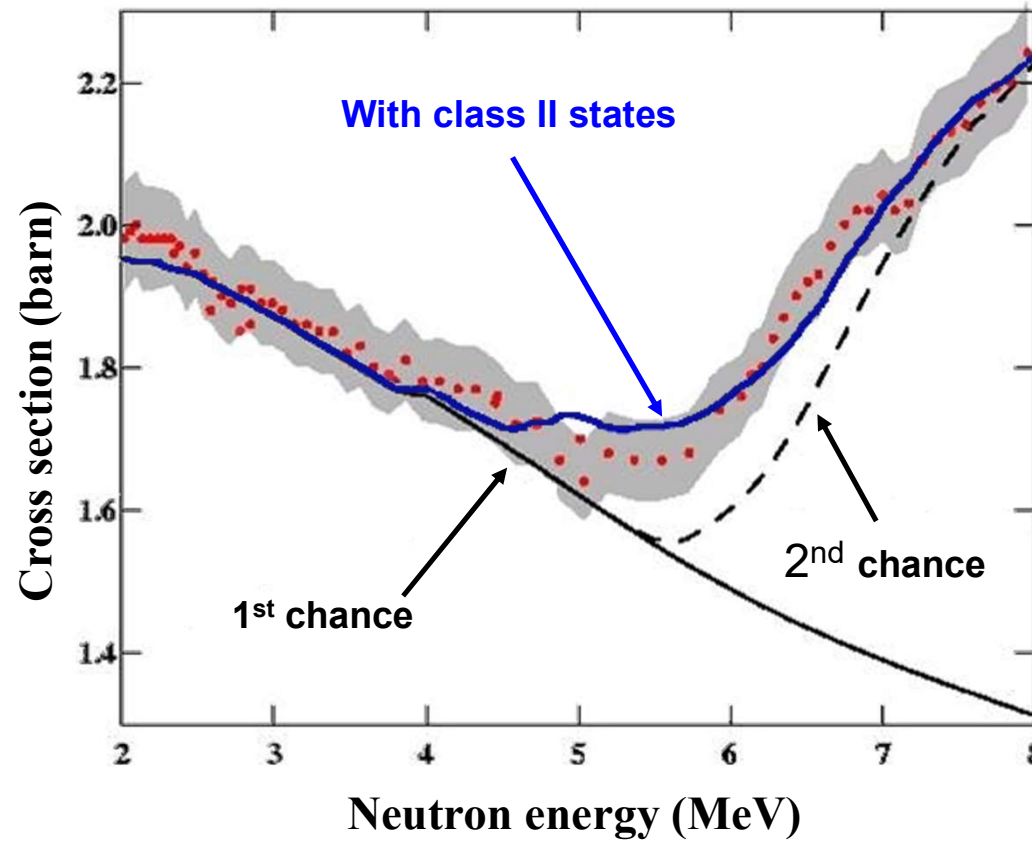
$^{239}\text{Pu} (n,f)$



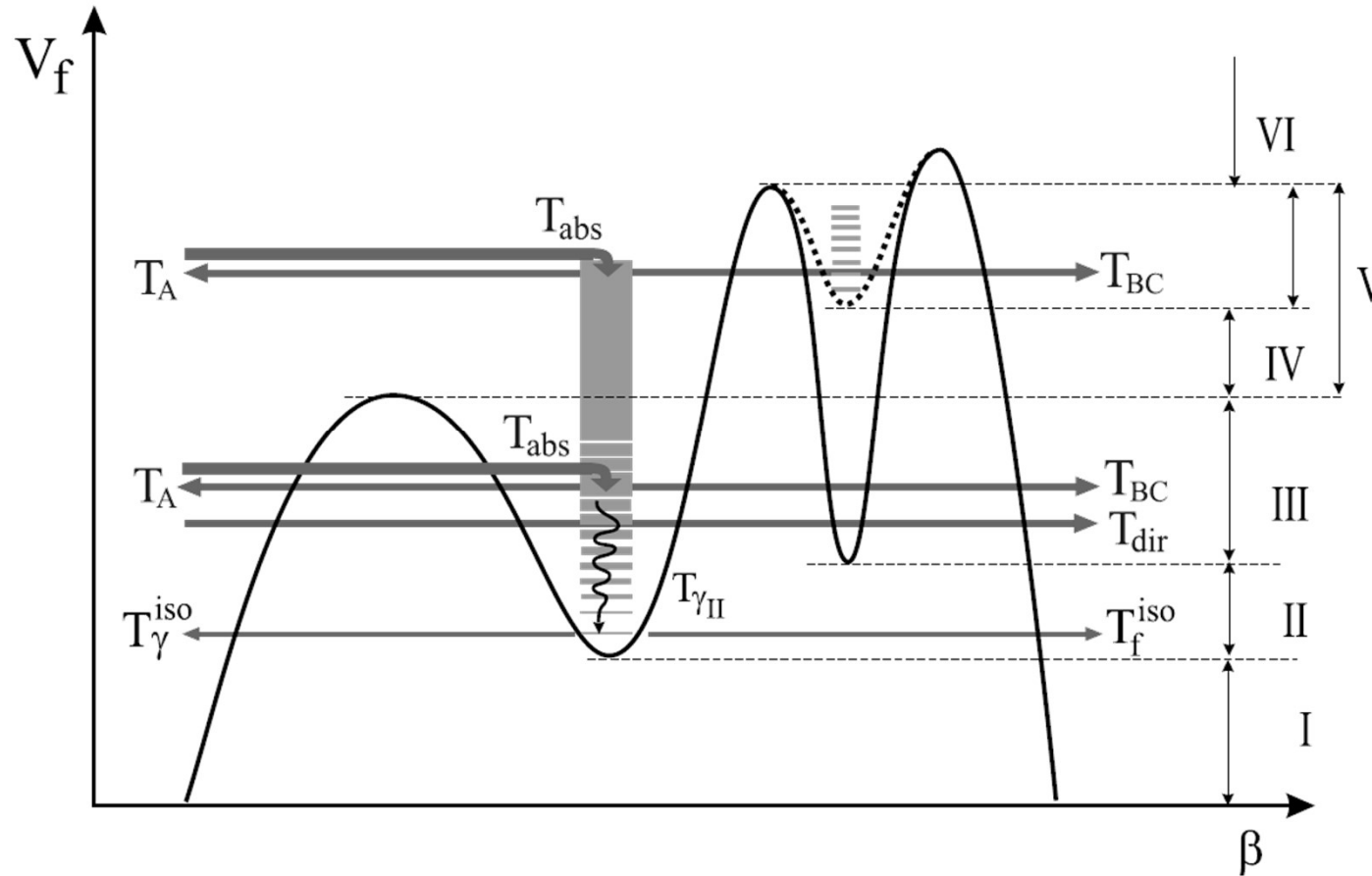
Fission transmission coefficient : role of class II states



$^{239}\text{Pu} (n,f)$



Fission transmission coefficient : maximum complexity



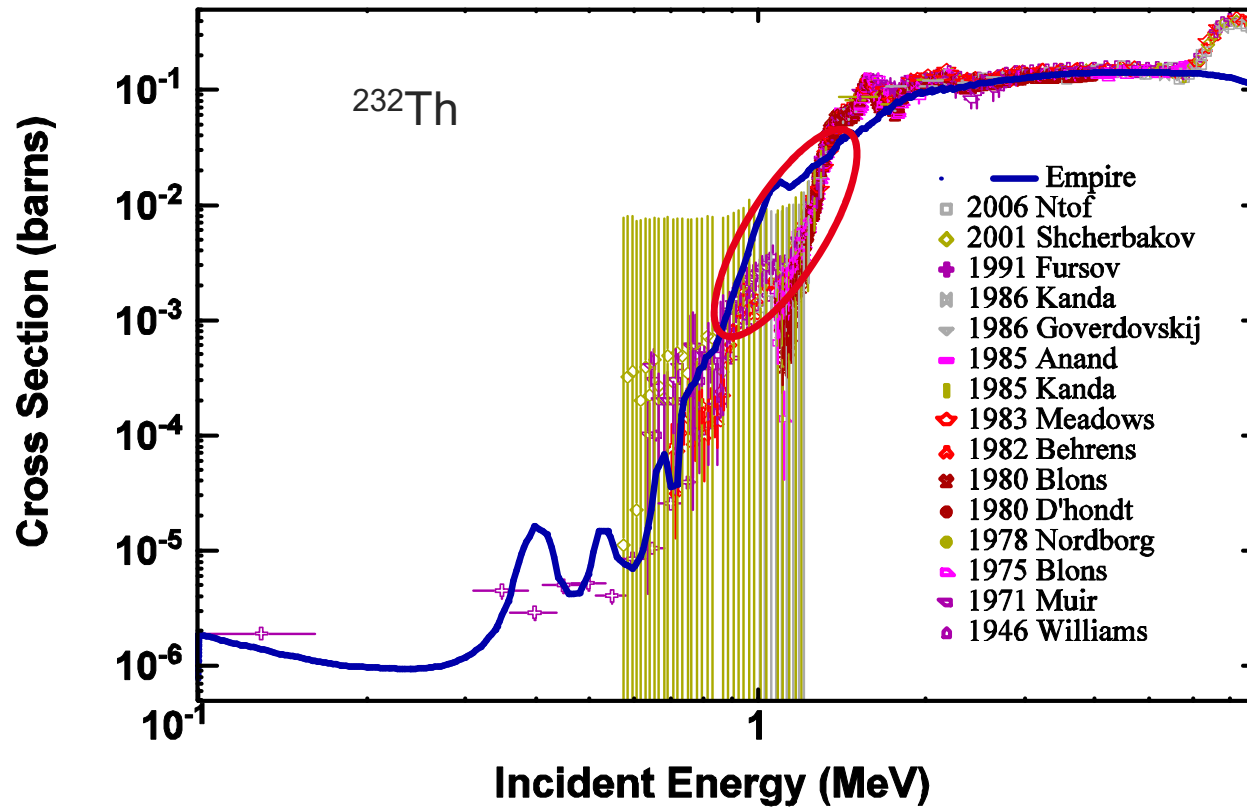
See in Sin et al., PRC 74 (2006) 014608

Bjornholm and Lynn, Rev. Mod. Phys. 52 (1980) 725.



Case of a fertile nucleus

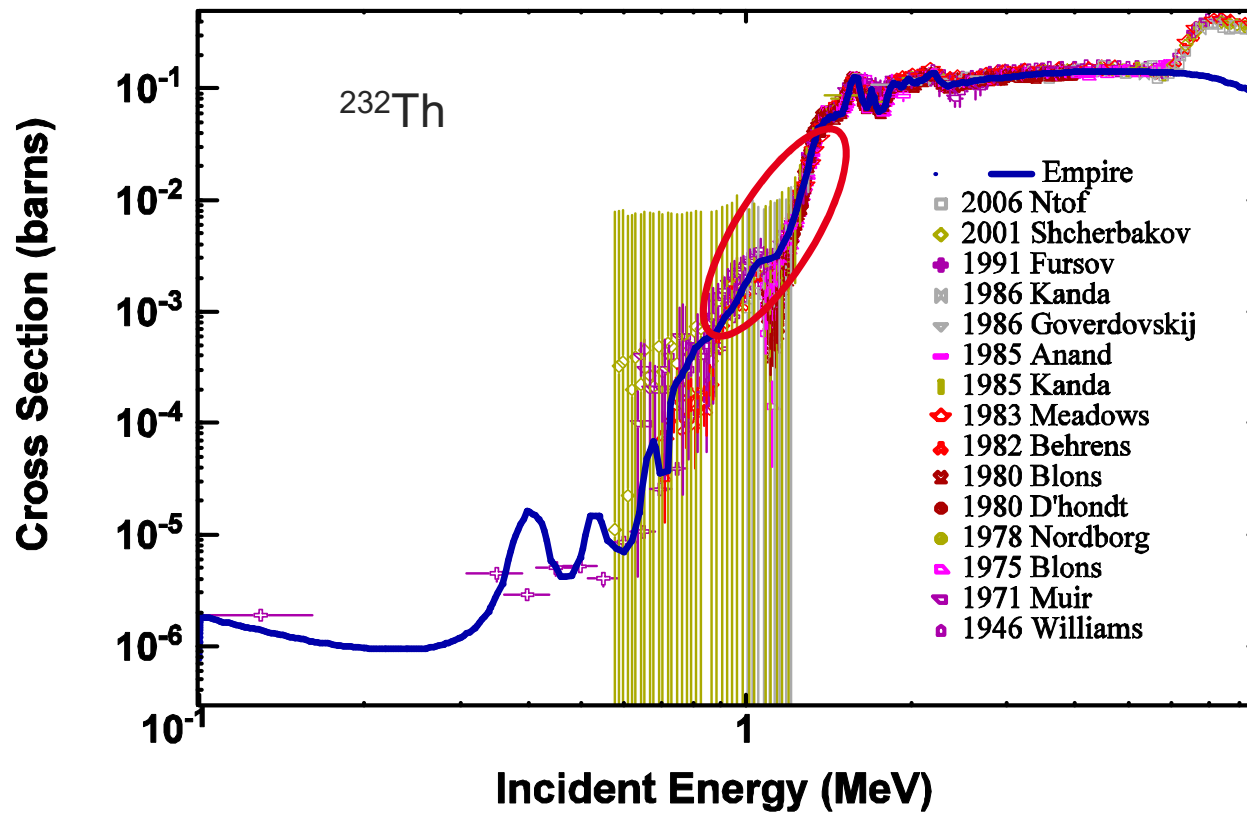
Partially damped class II states. No class III states





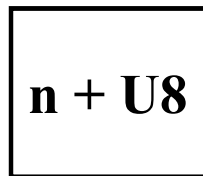
Case of a fertile nucleus

Class II + III states. Partial damping.





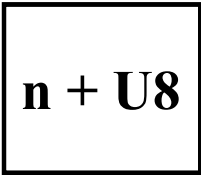
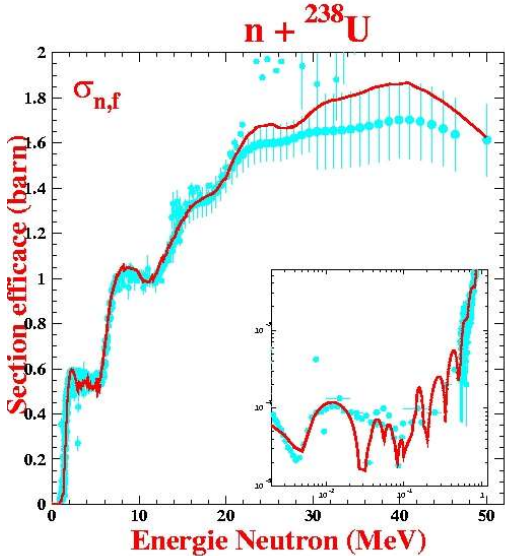
Coherent fission modeling : single target / several fissions



—————→ MeV

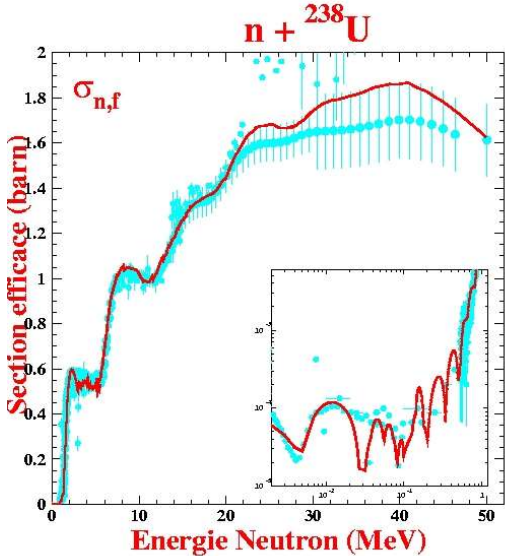


Coherent fission modeling : single target / several fissions



→ MeV

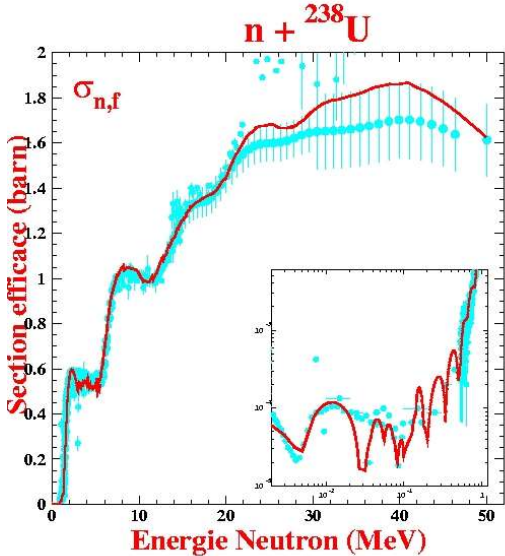
Coherent fission modeling : single target / several fissions



U9

→ MeV

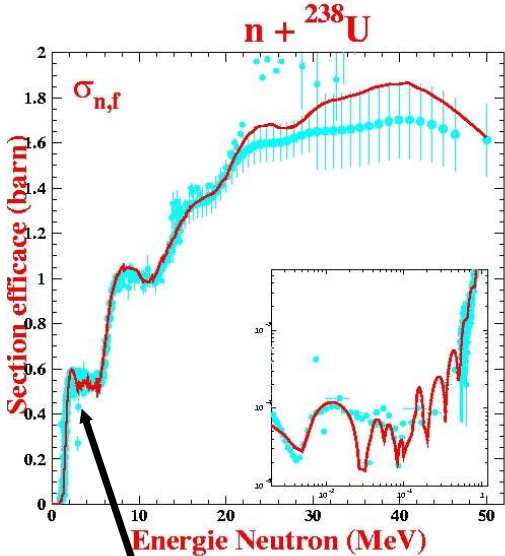
Coherent fission modeling : single target / several fissions



U9



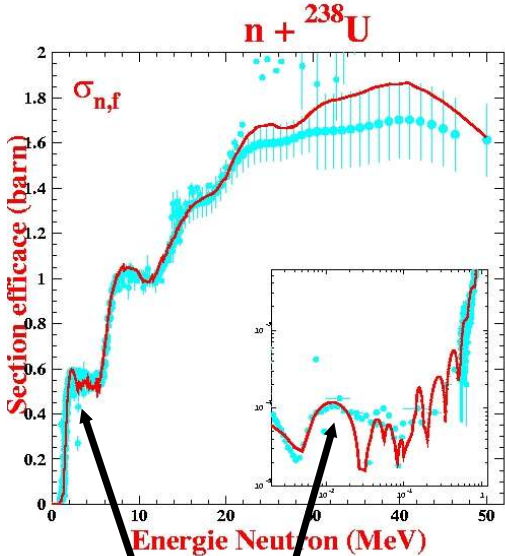
Coherent fission modeling : single target / several fissions



U9



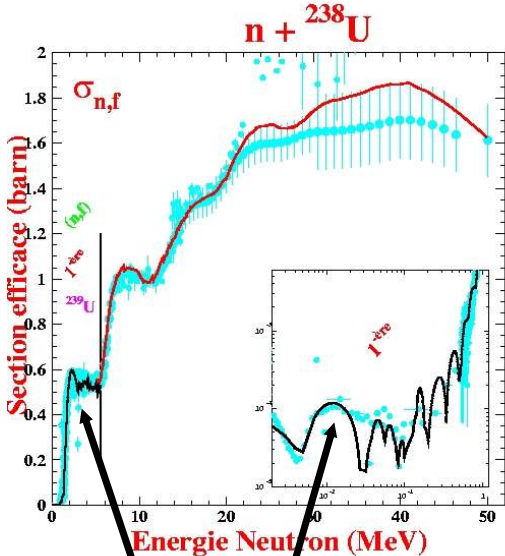
Coherent fission modeling : single target / several fissions



U9



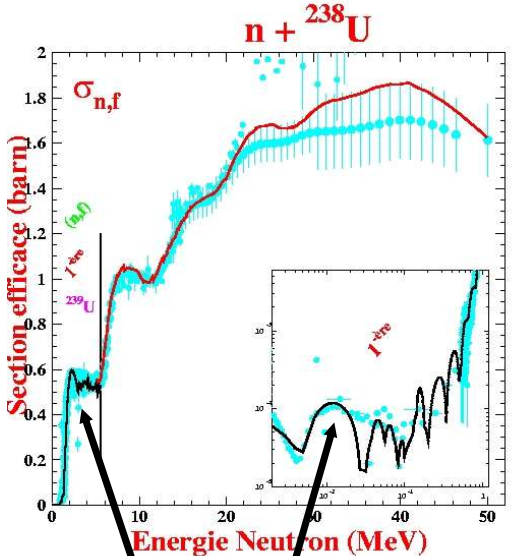
Coherent fission modeling : single target / several fissions



U9



Coherent fission modeling : single target / several fissions



U9

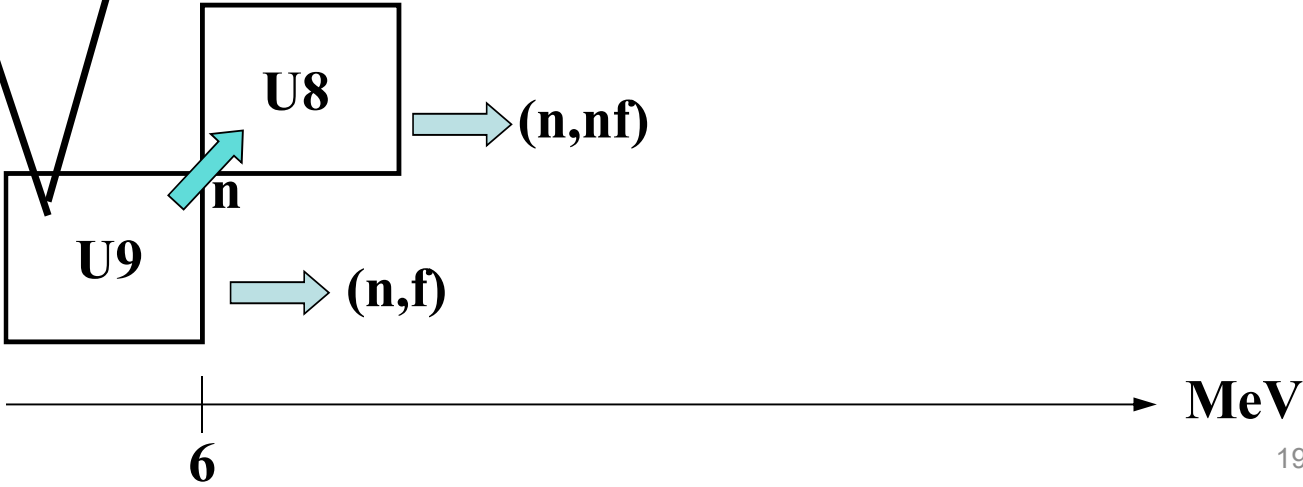
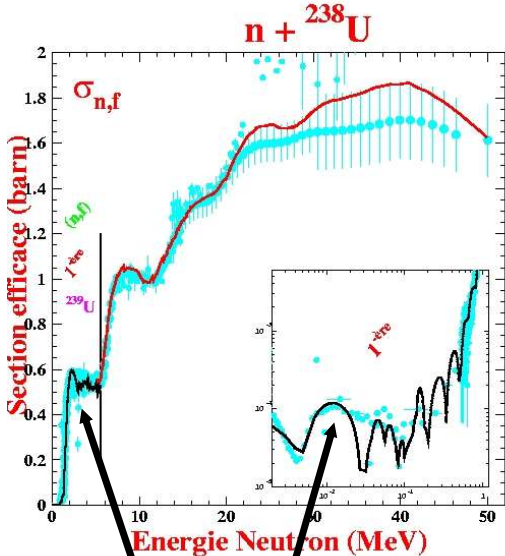
(n,f)

MeV

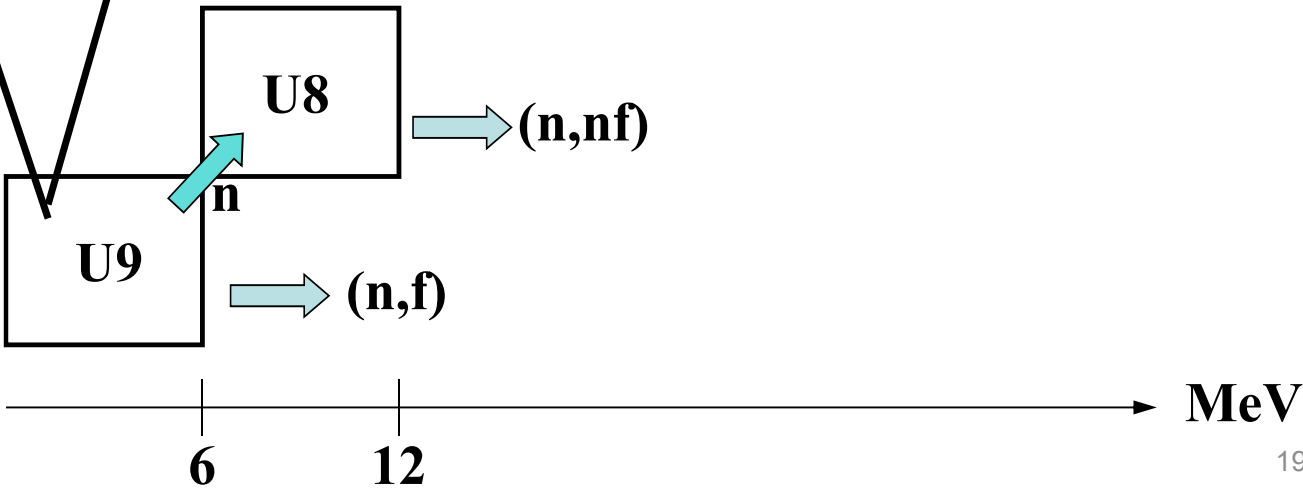
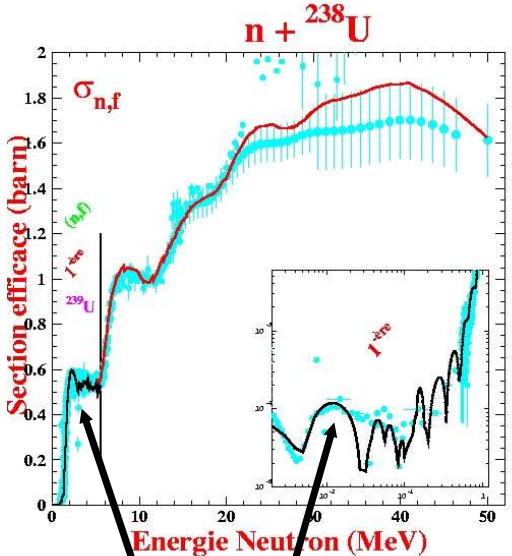
6



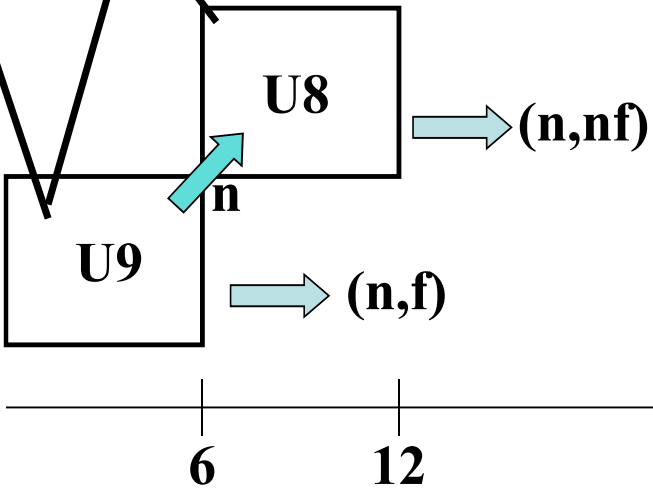
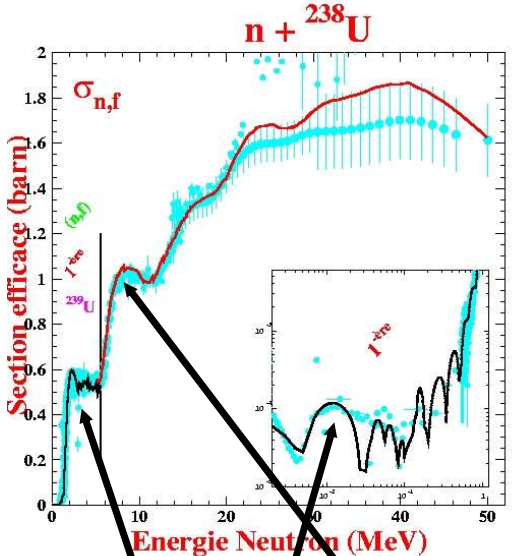
Coherent fission modeling : single target / several fissions



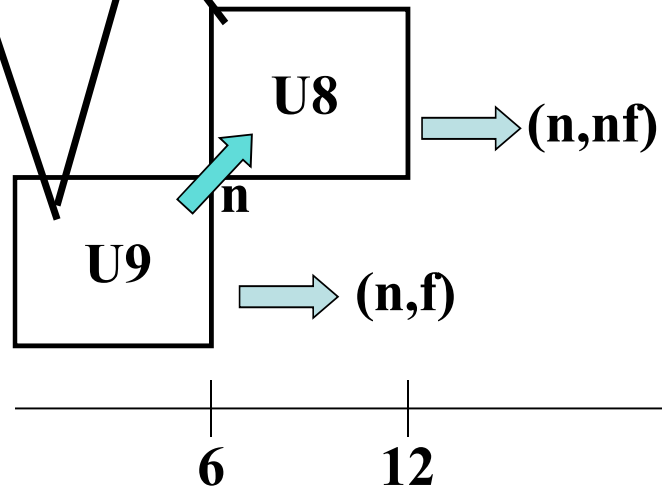
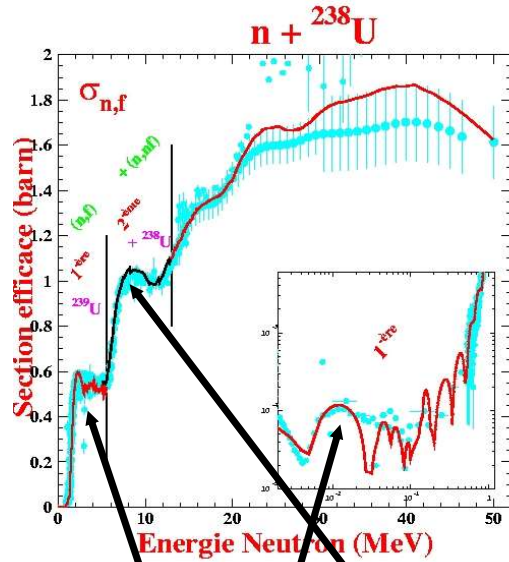
Coherent fission modeling : single target / several fissions



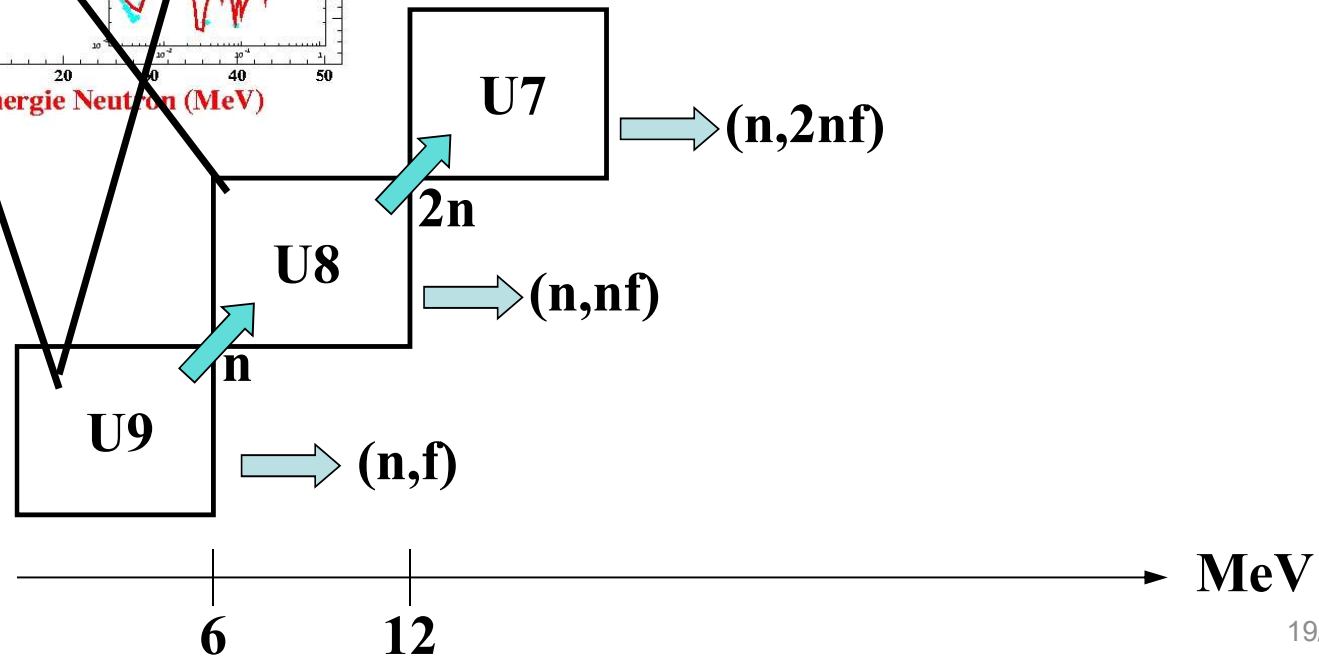
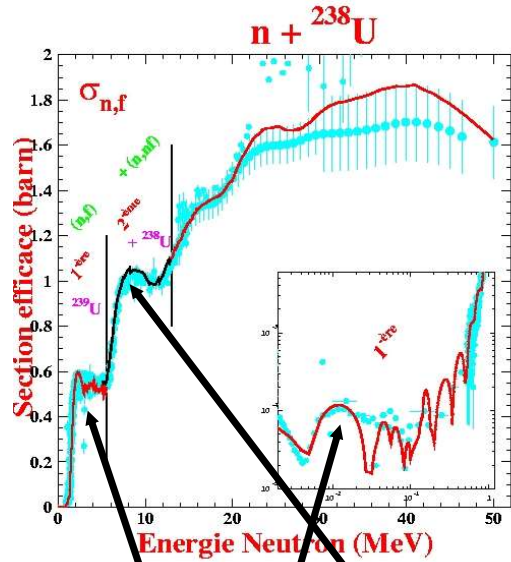
Coherent fission modeling : single target / several fissions



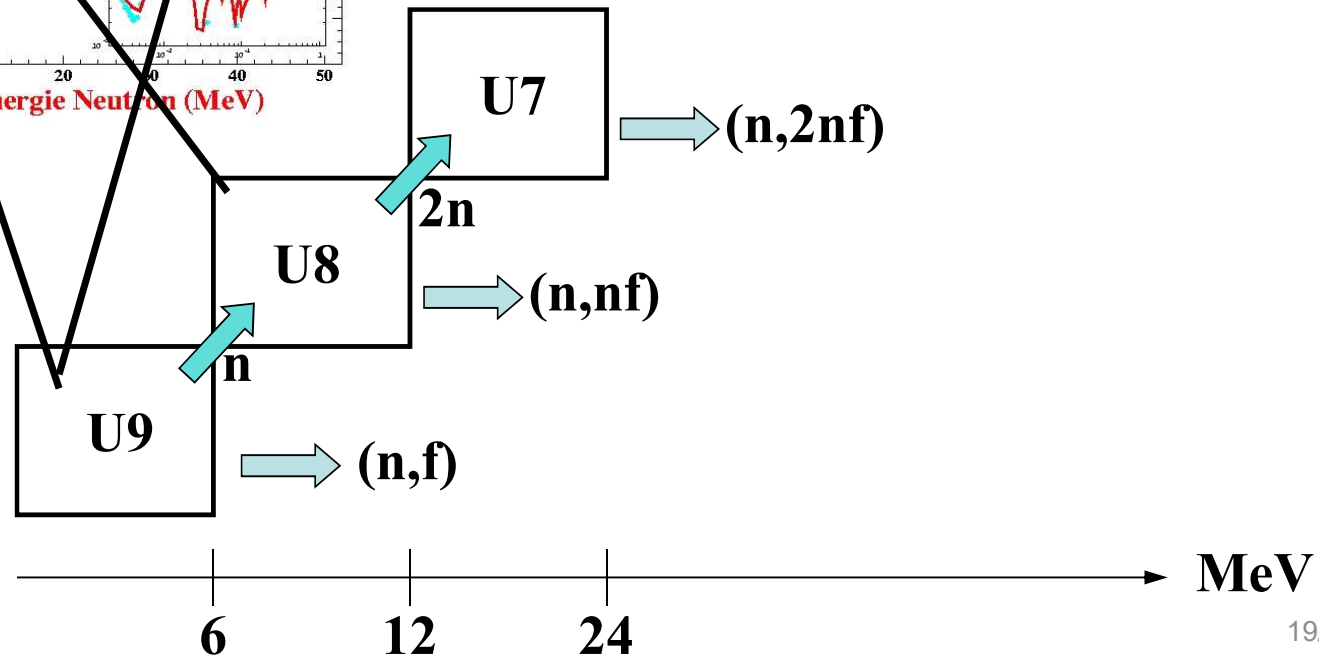
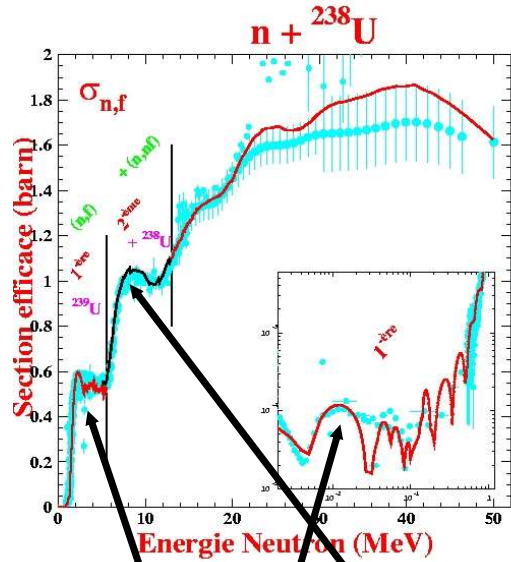
Coherent fission modeling : single target / several fissions



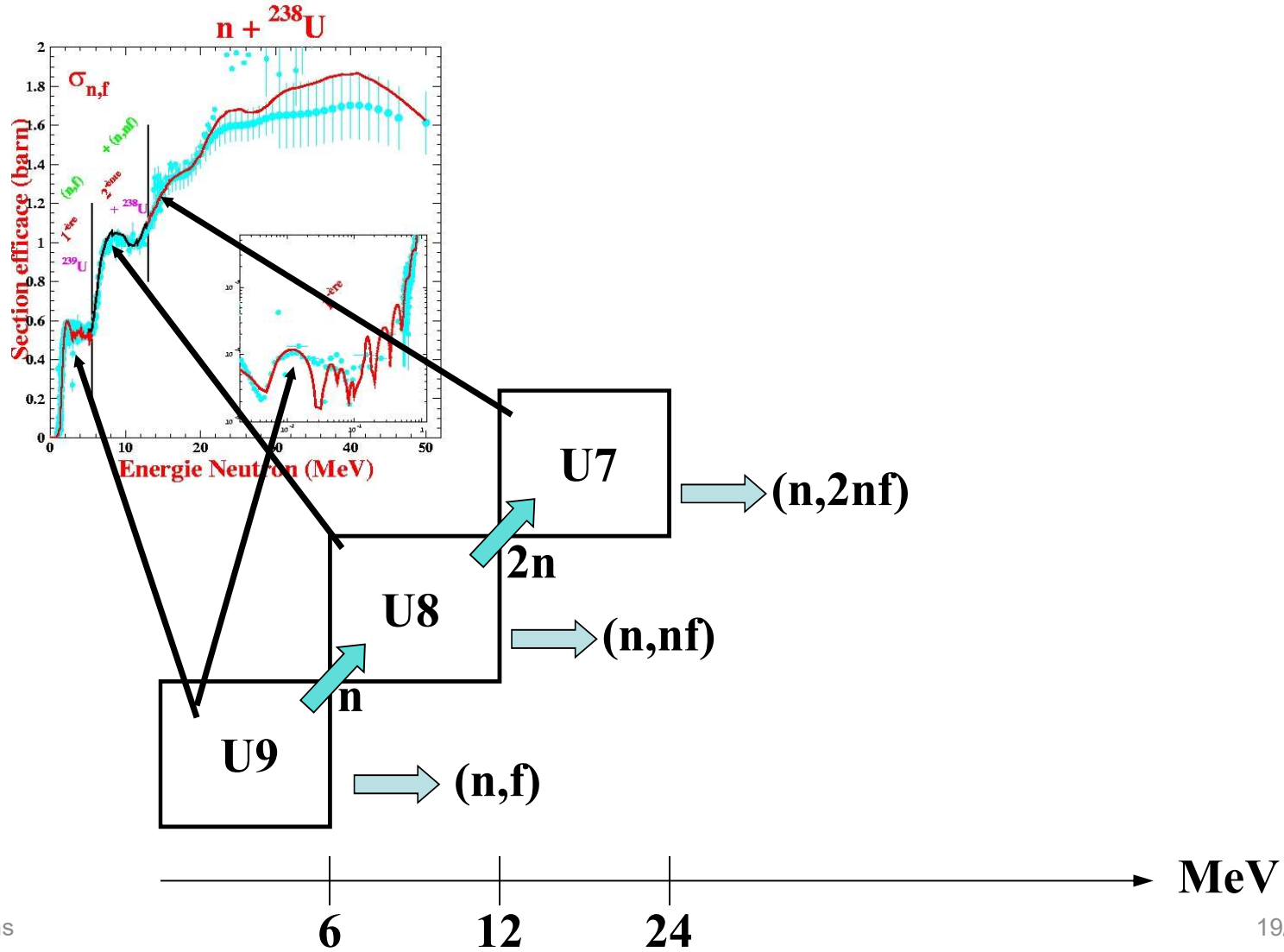
Coherent fission modeling : single target / several fissions



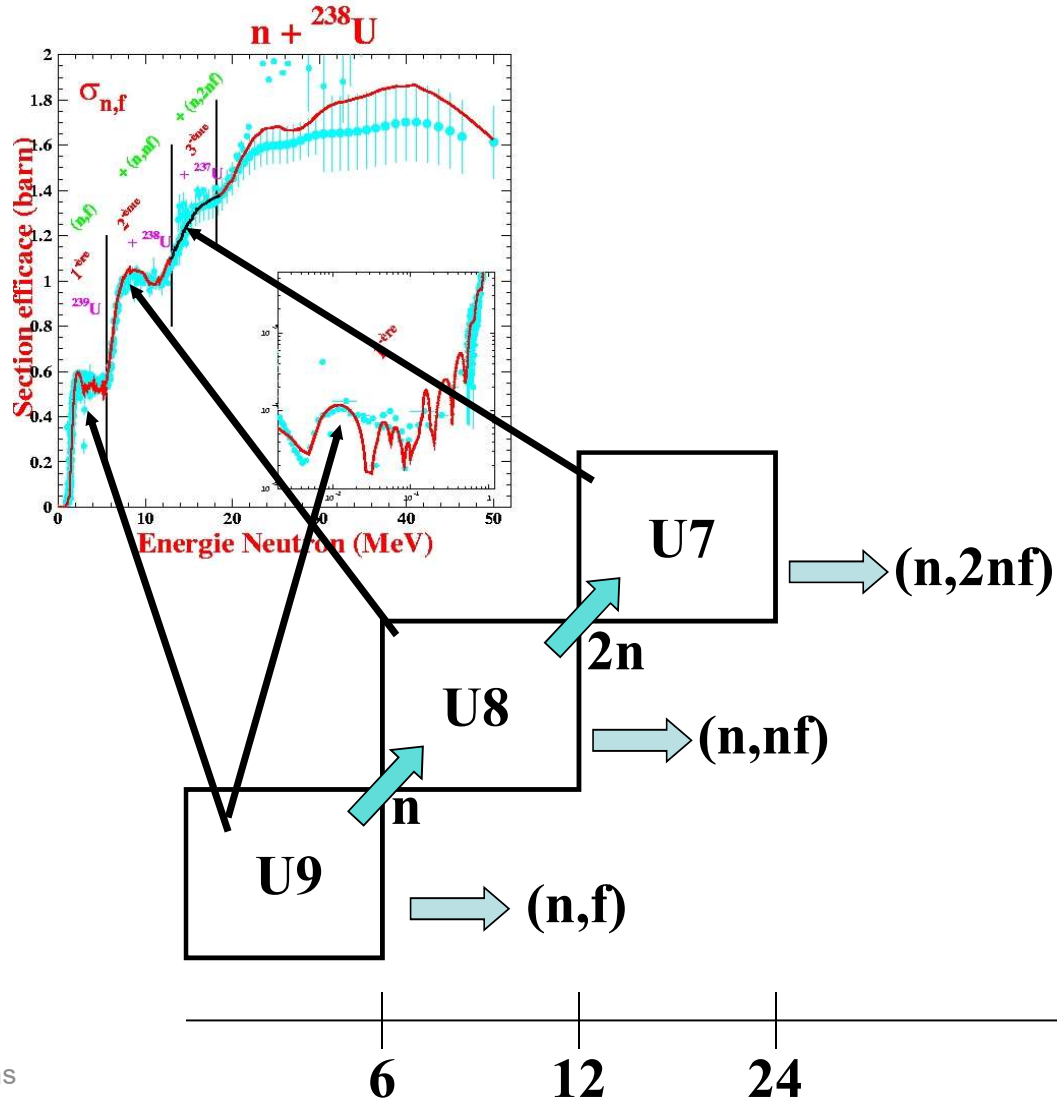
Coherent fission modeling : single target / several fissions



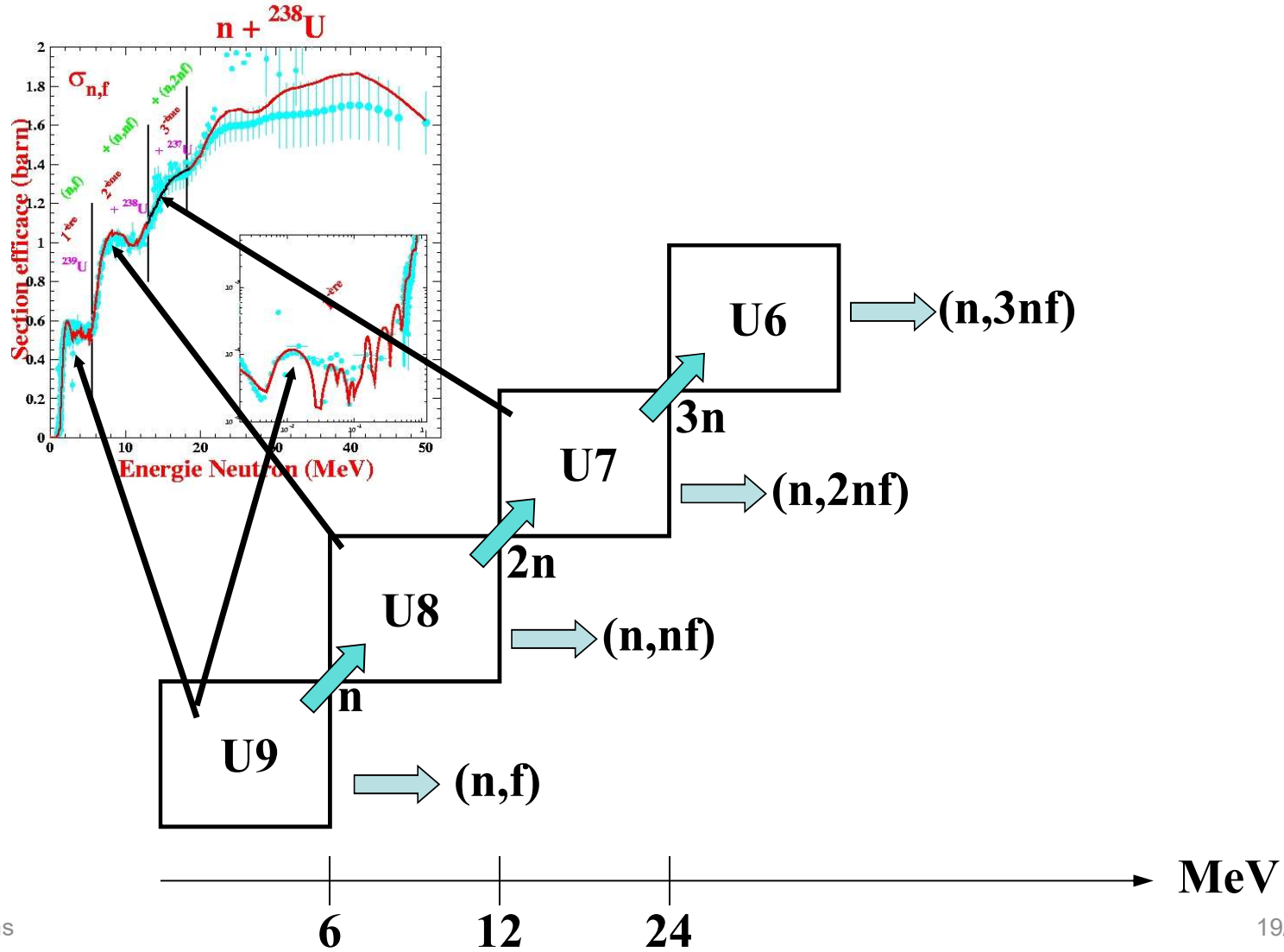
Coherent fission modeling : single target / several fissions



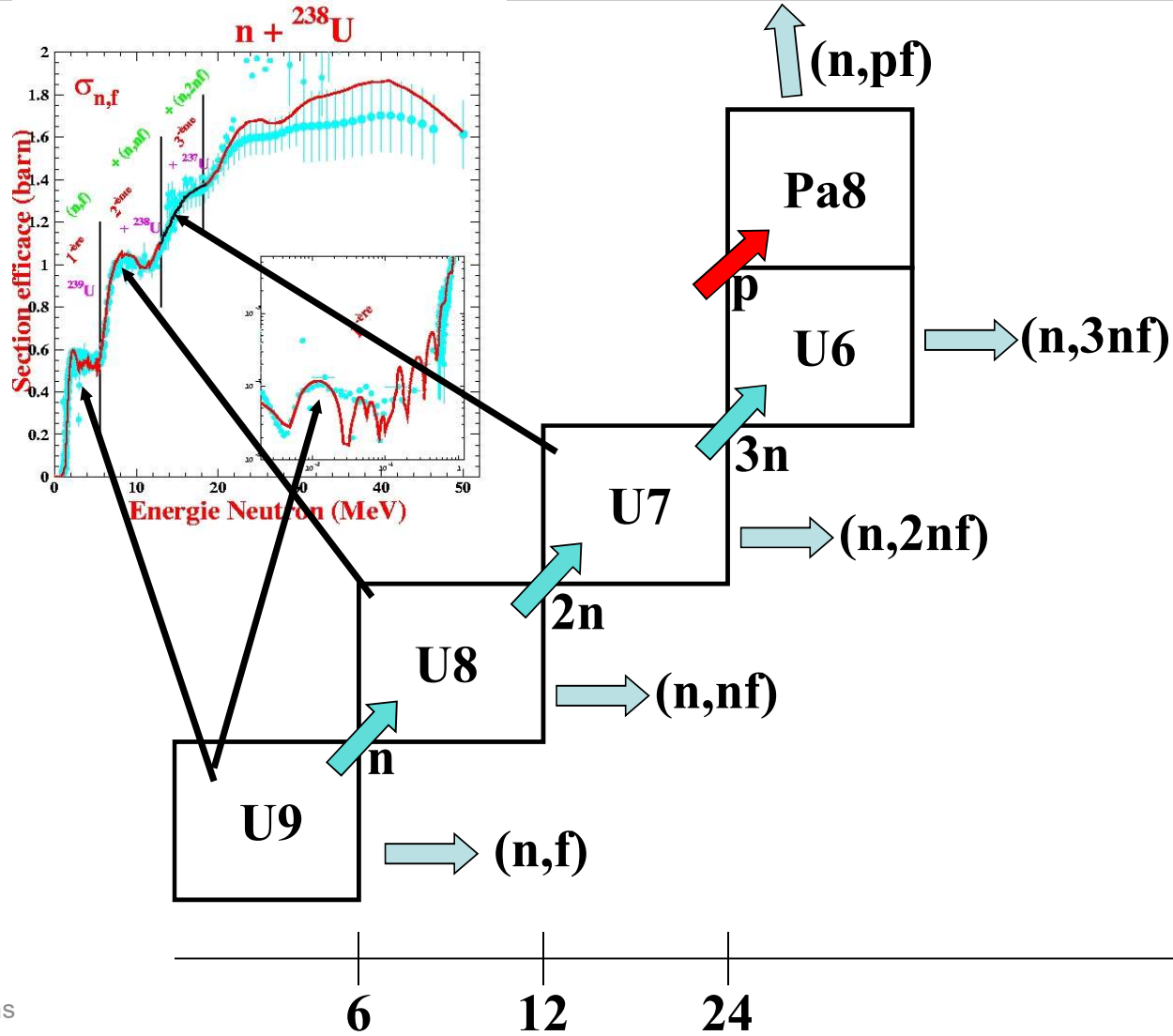
Coherent fission modeling : single target / several fissions



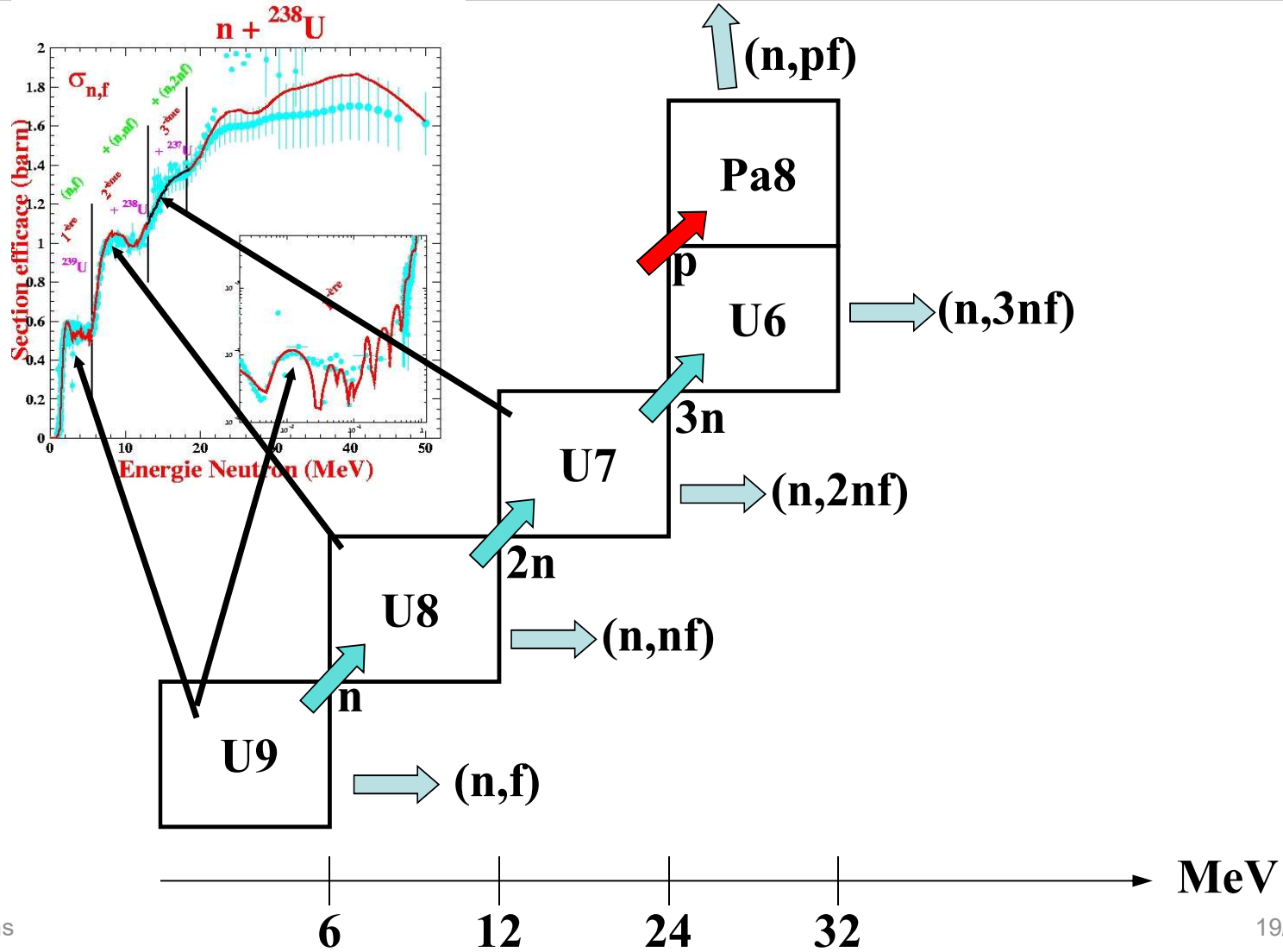
Coherent fission modeling : single target / several fissions



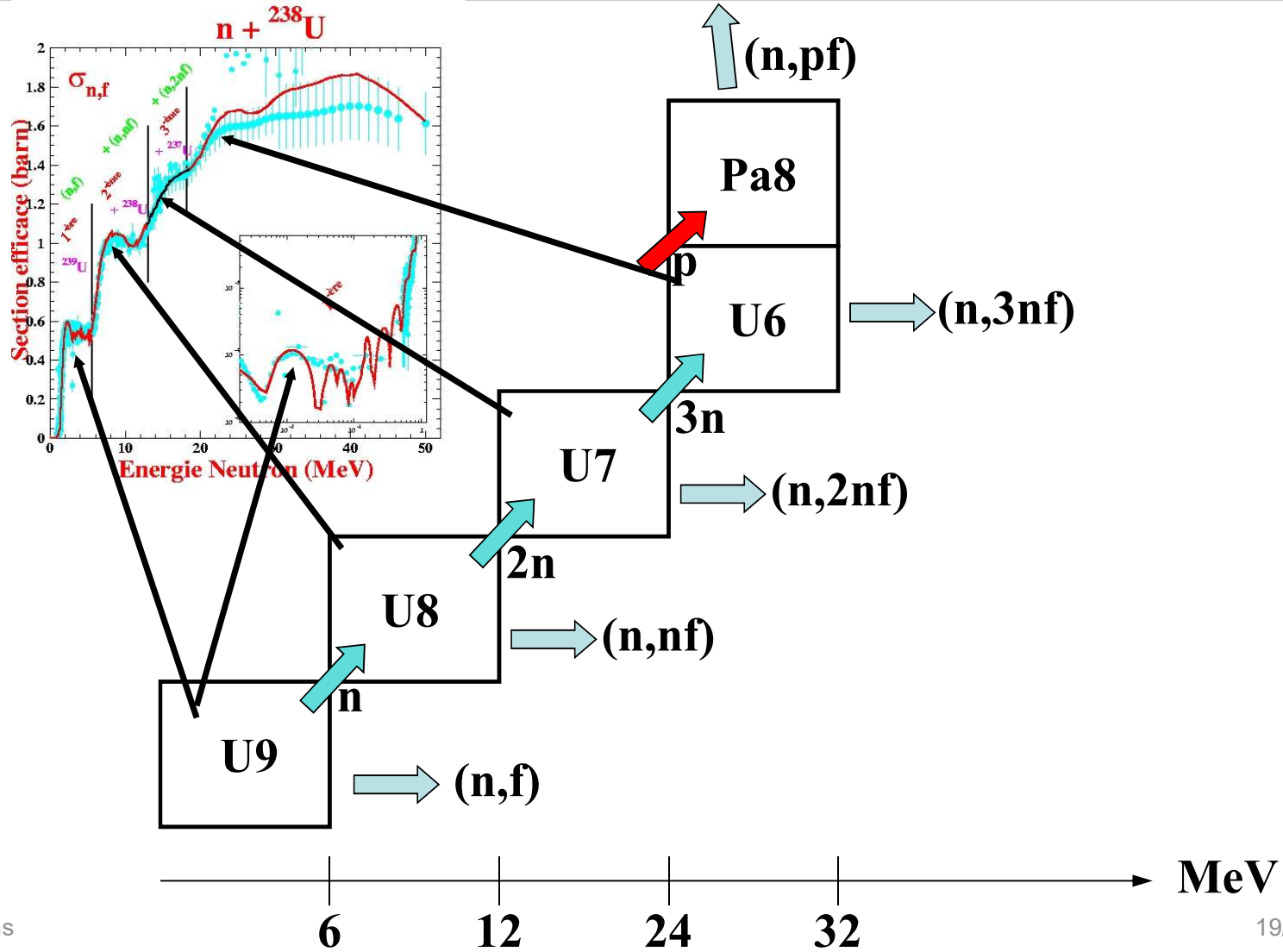
Coherent fission modeling : single target / several fissions



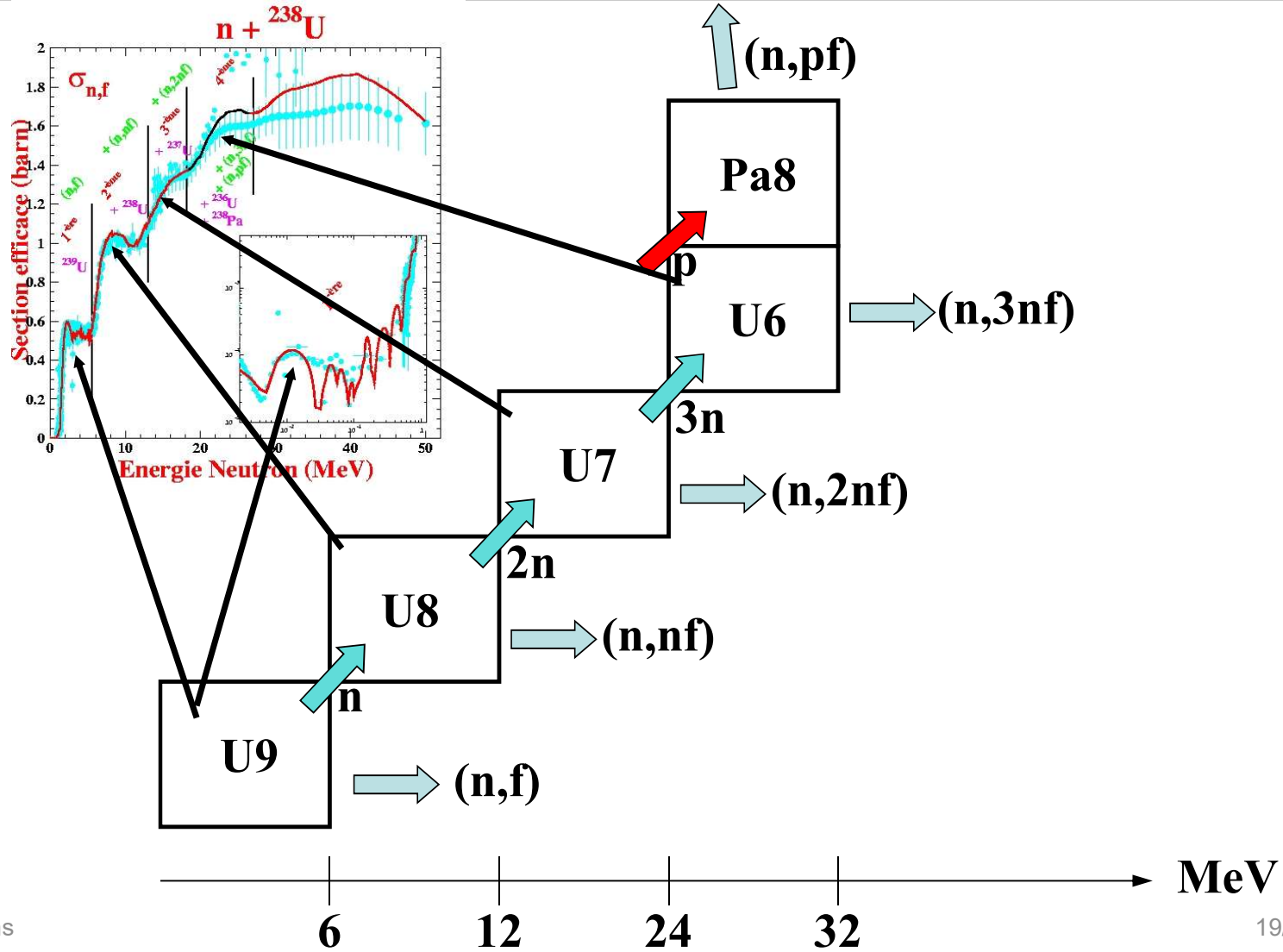
Coherent fission modeling : single target / several fissions



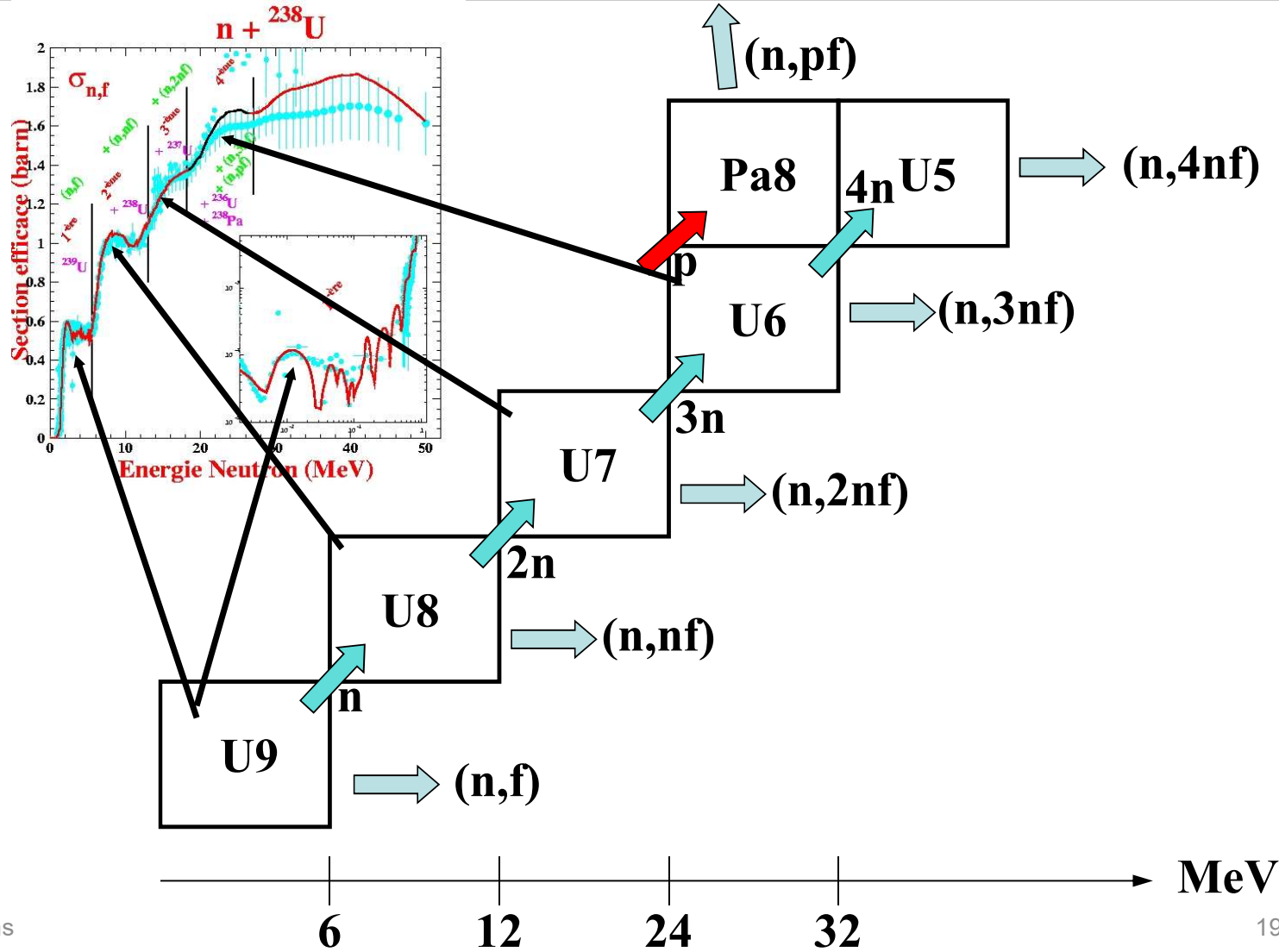
Coherent fission modeling : single target / several fissions



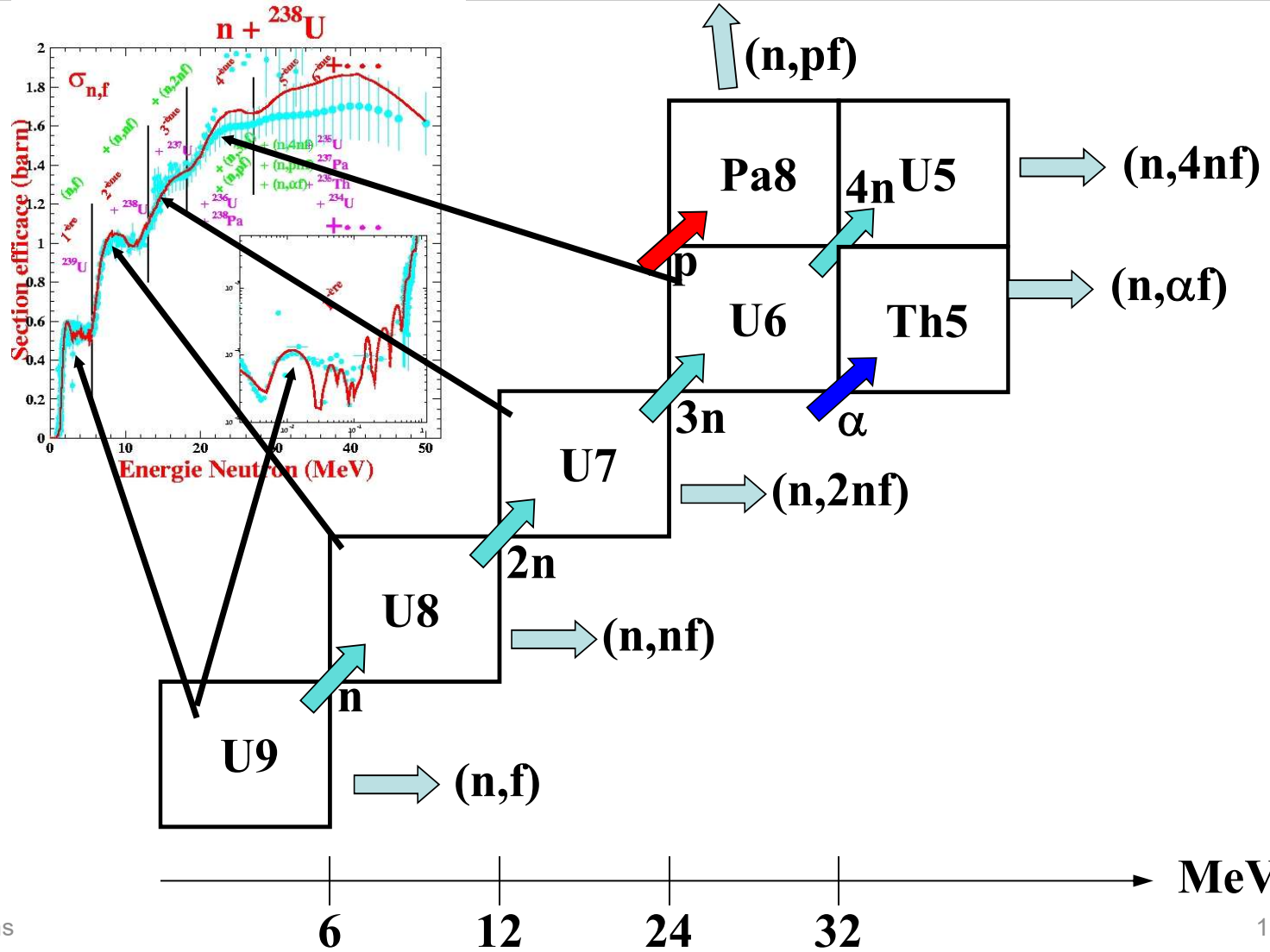
Coherent fission modeling : single target / several fissions



Coherent fission modeling : single target / several fissions

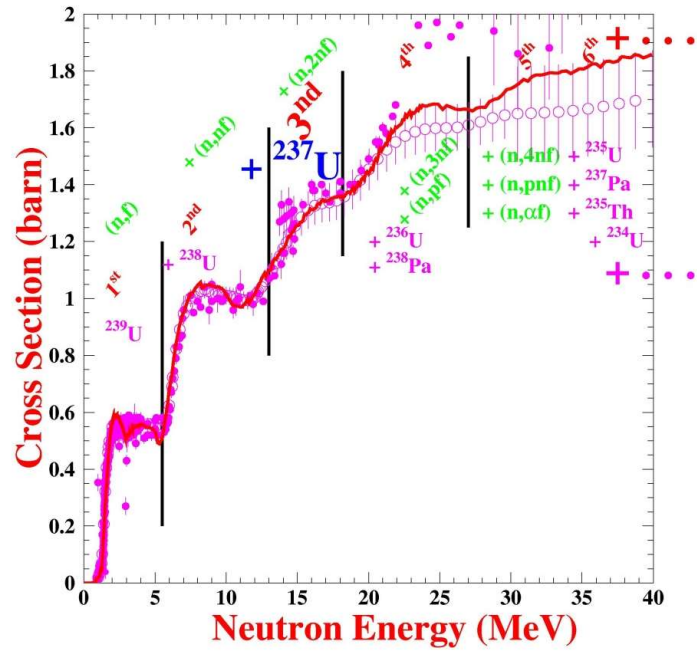


Coherent fission modeling : single target / several fissions

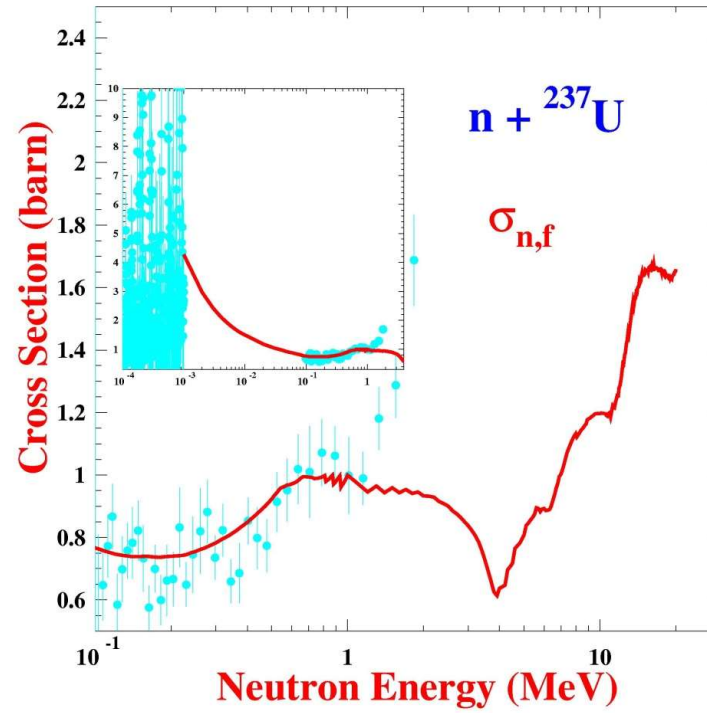
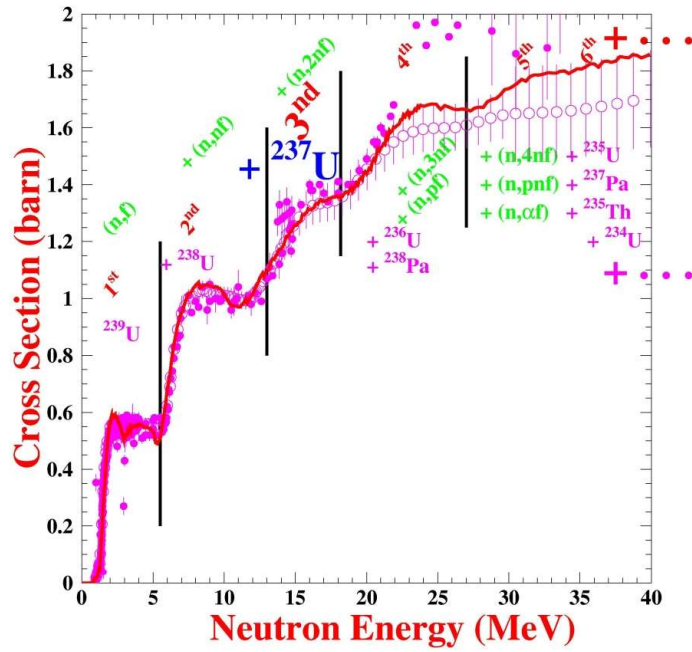




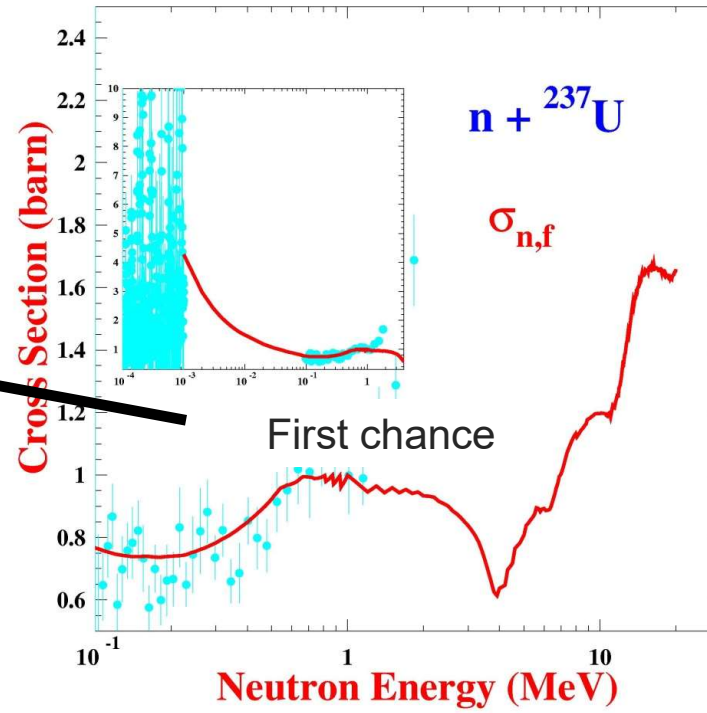
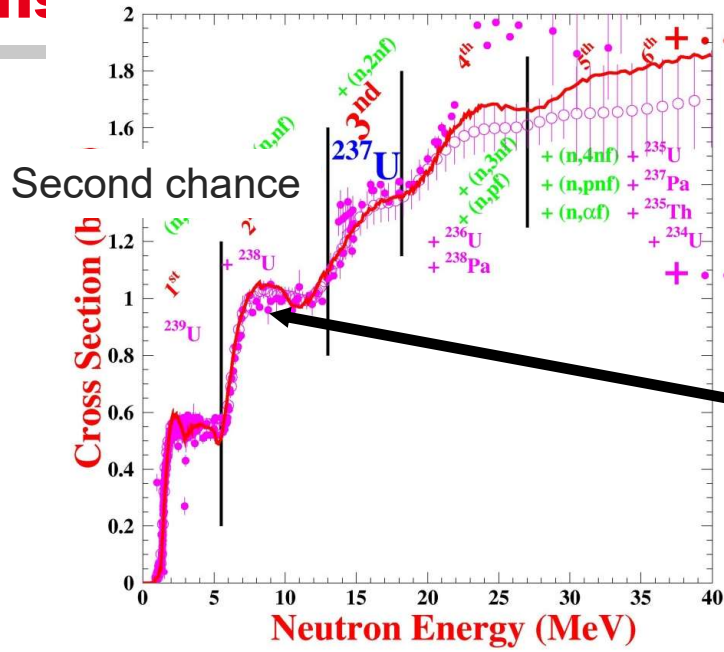
Coherent fission modeling : several targets / strong constraints



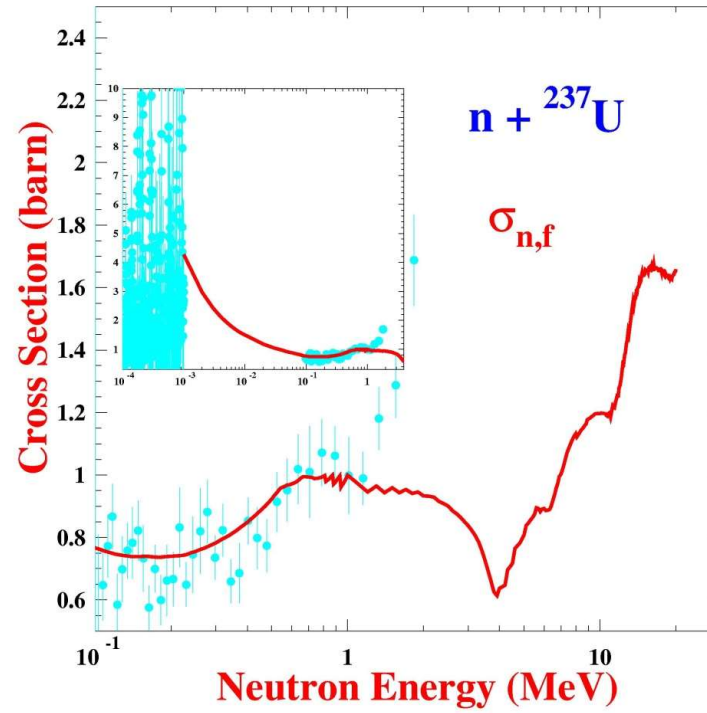
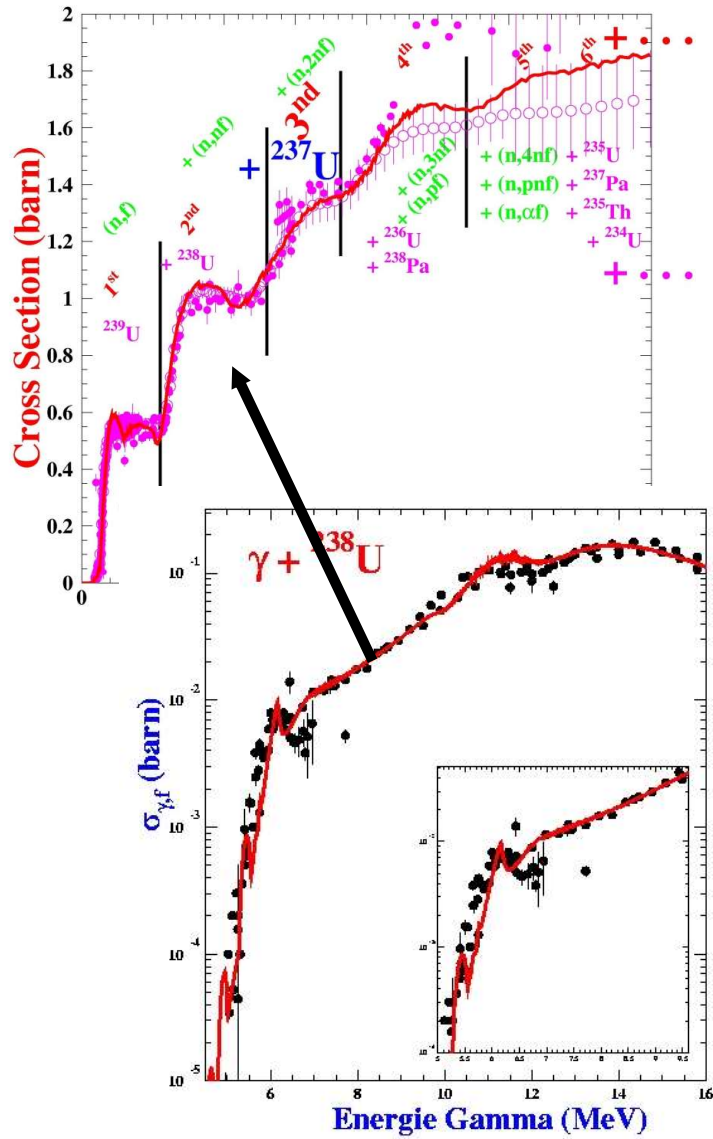
Coherent fission



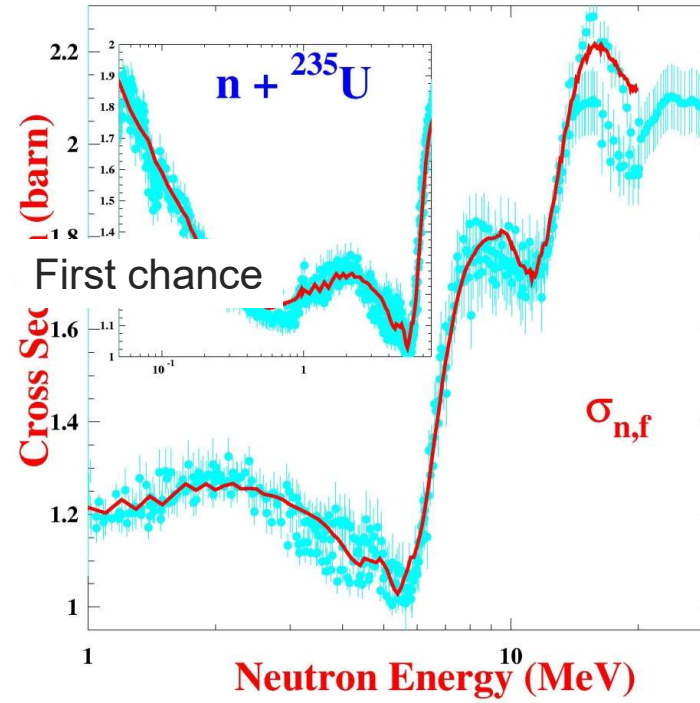
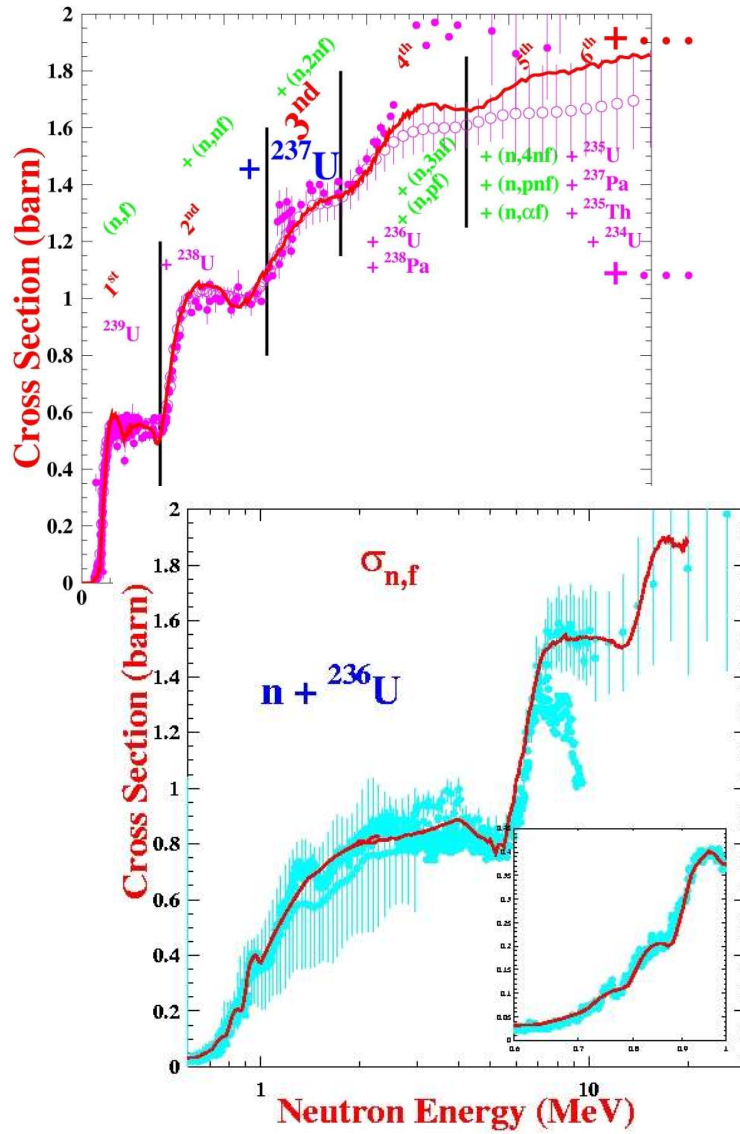
Coherent fission



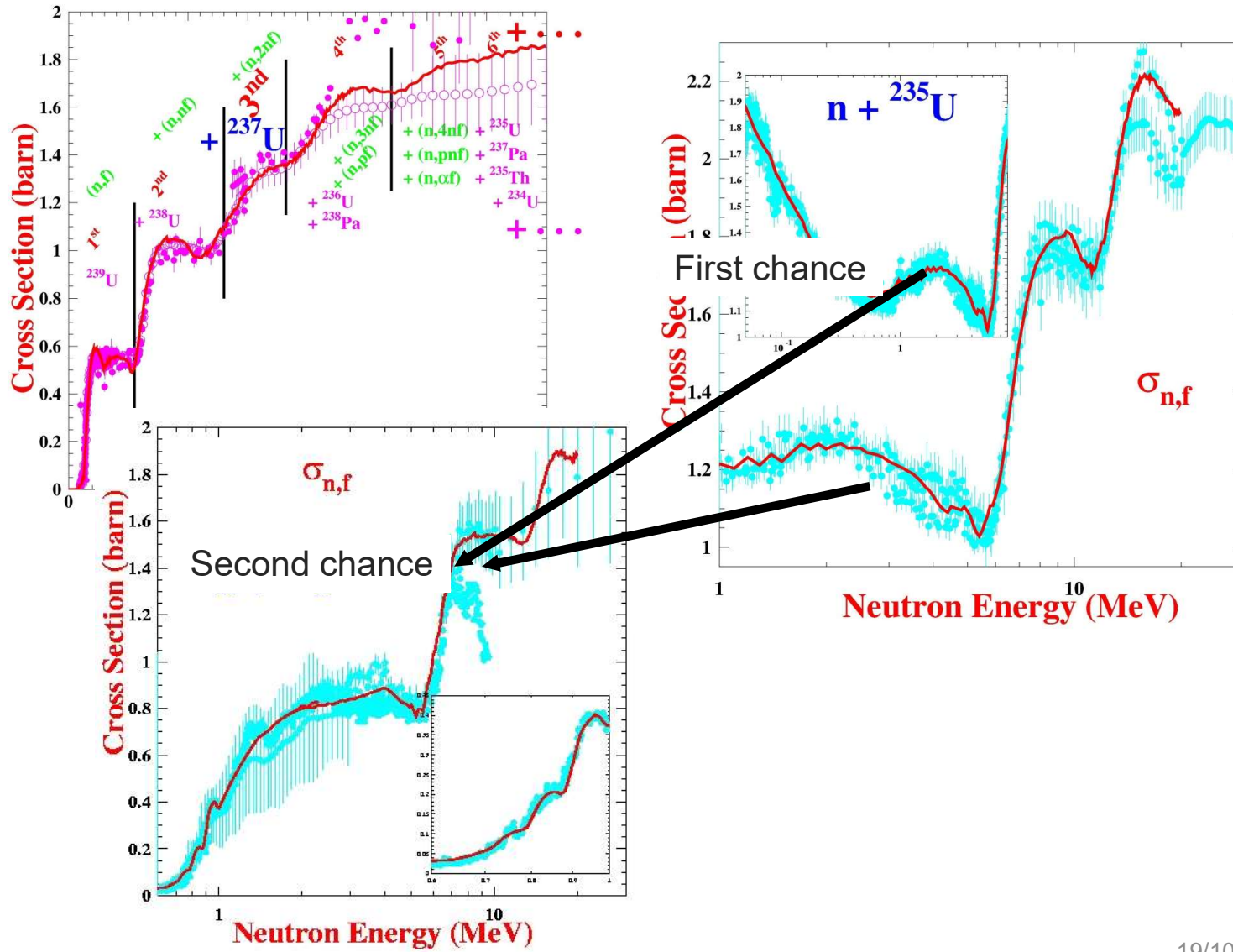
Coherent fission



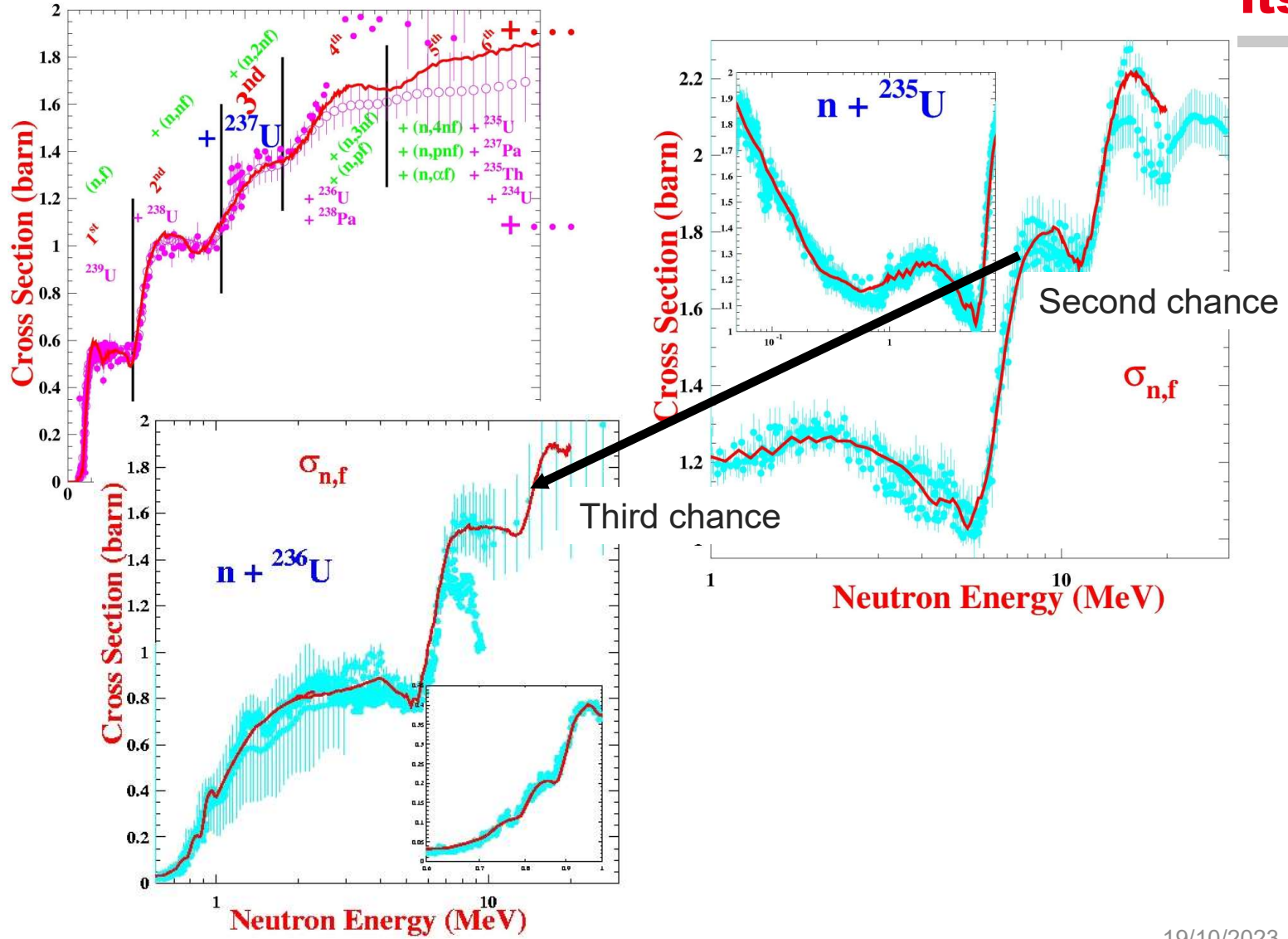
Coherent fis



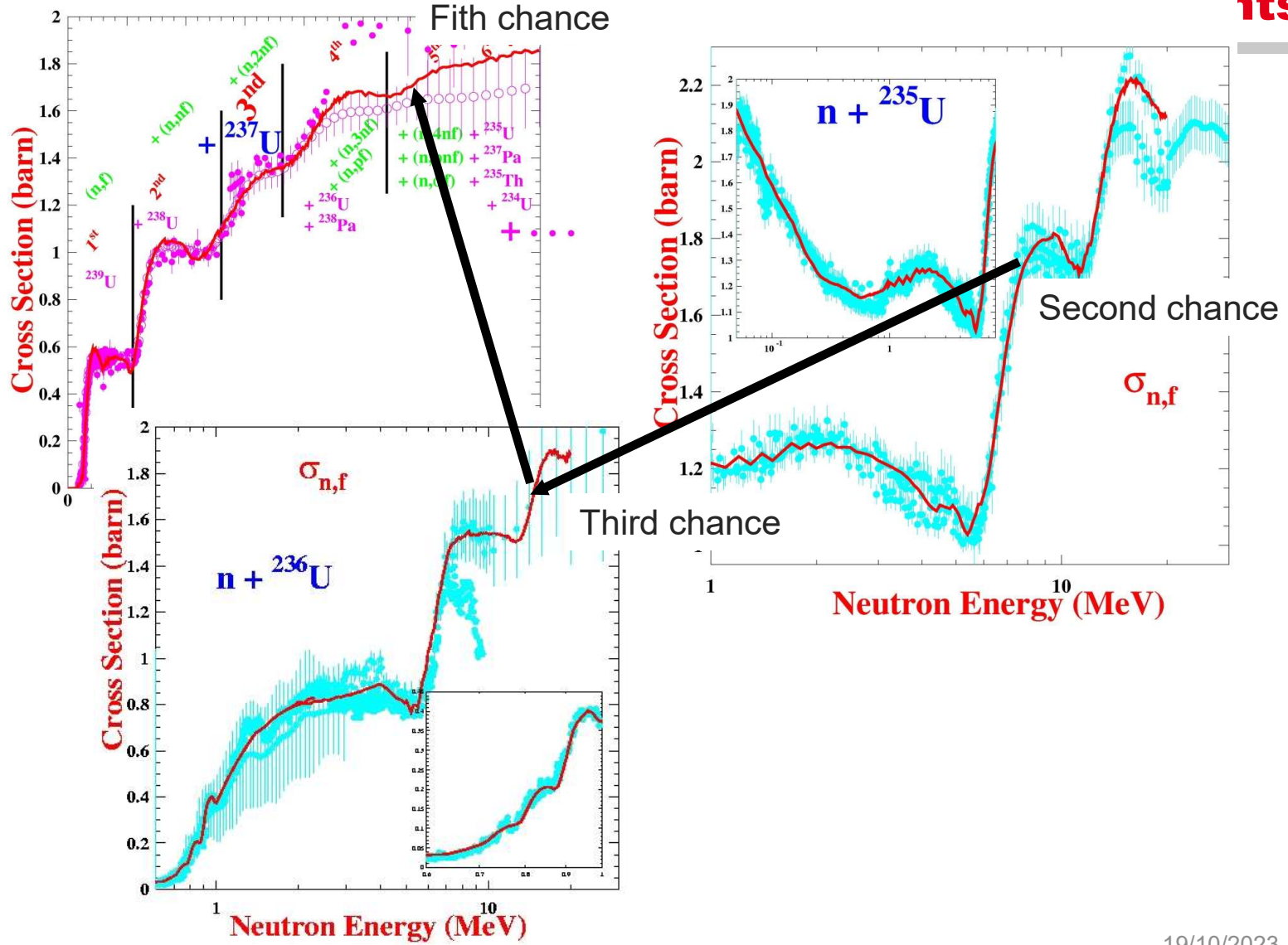
Coherent fis



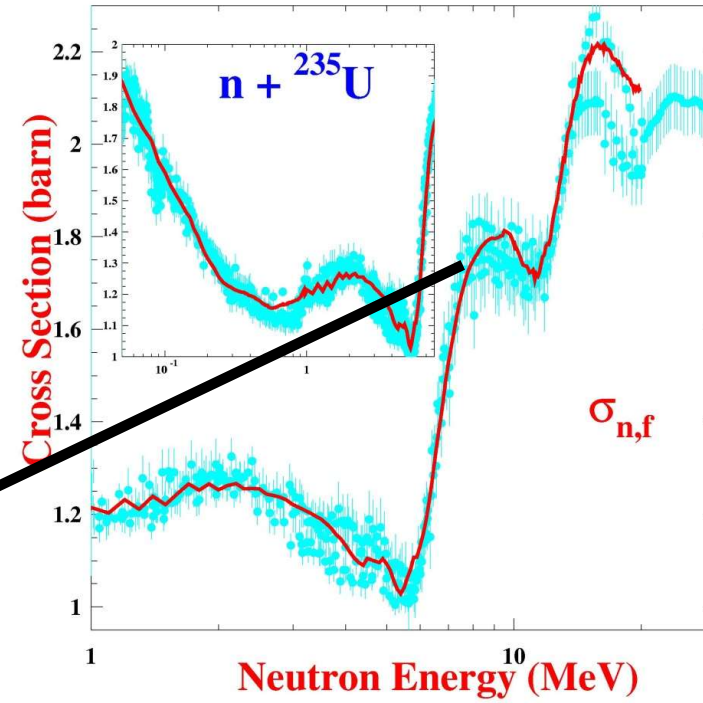
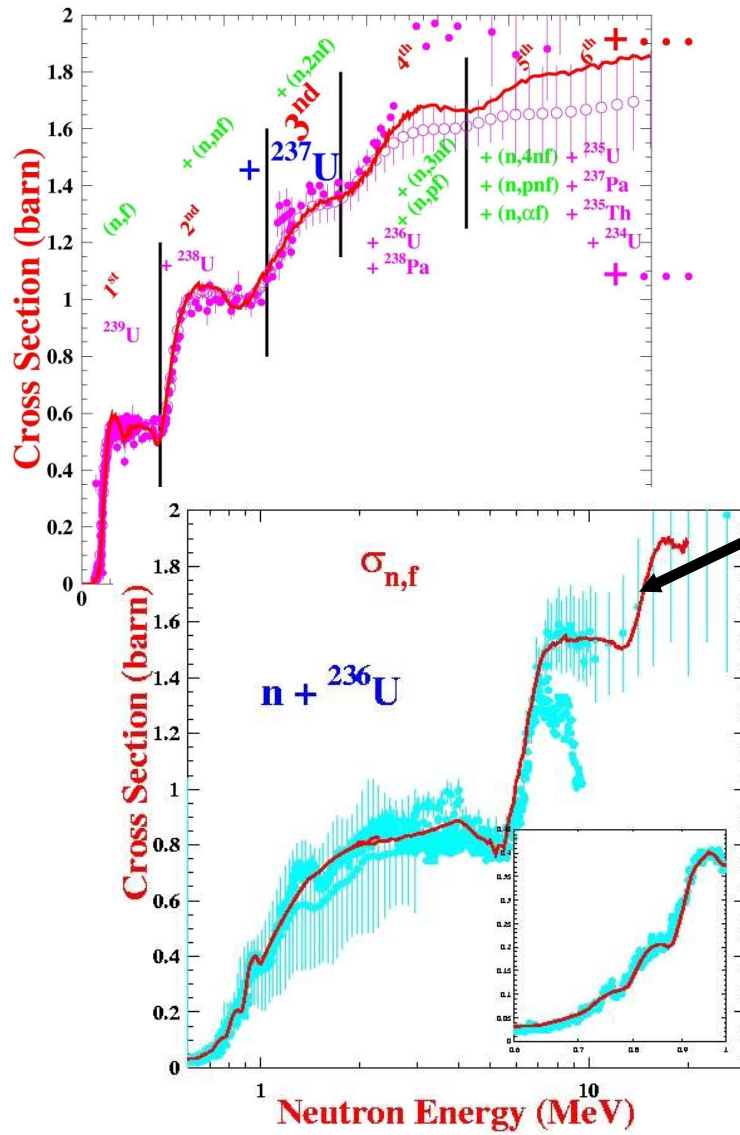
Coherent fis



Coherent fission



Coherent fission





Microscopic approach

Ingredients of relevance to estimate fission properties

$$T(E, J, \pi) = \int_0^E P(E - \varepsilon) \rho(\varepsilon, J, \pi) d\varepsilon \begin{cases} P(E) = \frac{1}{1 + \exp(2K)} \\ K = \pm \int_a^b [2\mu(E - V(\beta))/\hbar^2]^{1/2} d\beta \end{cases}$$

$$\text{Hill-Wheeler approximation: } P^{HW} = \frac{1}{1 + \exp[2\pi(V_0 - E)/\hbar\omega]}$$

Fundamental ingredients:

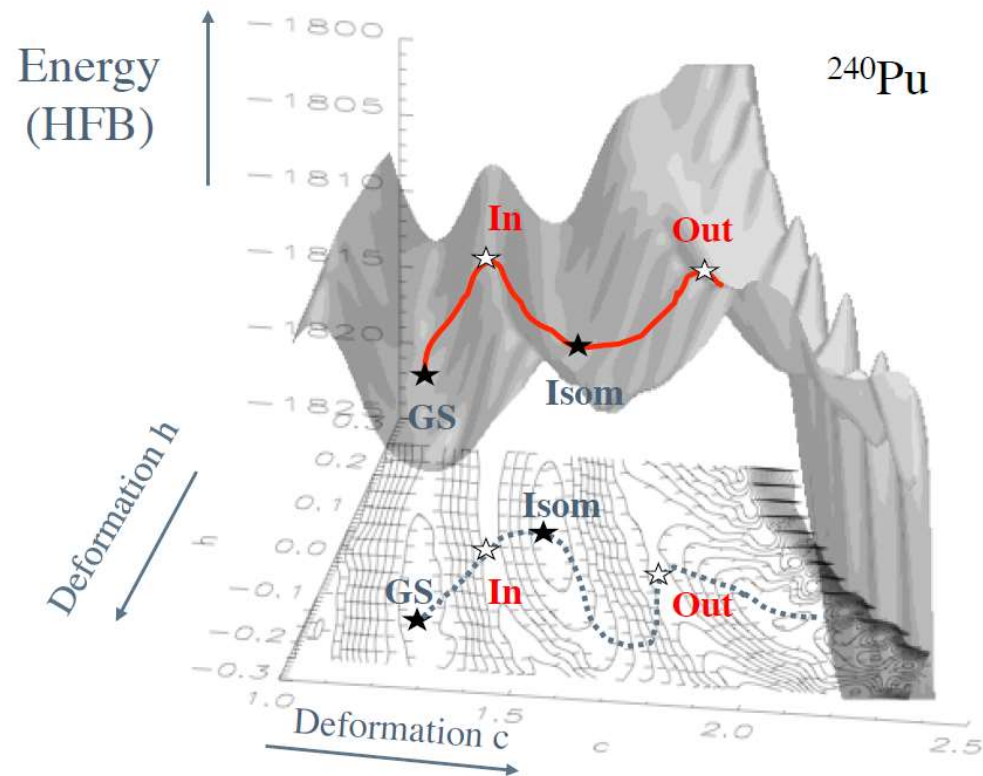
- Fission barrier heights
 - Fission barrier widths
 - *Nuclear Level Densities at saddle points*
- } Fission path

MAJOR CHALLENGE: COHERENT PREDICTIONS OF ALL INPUTS



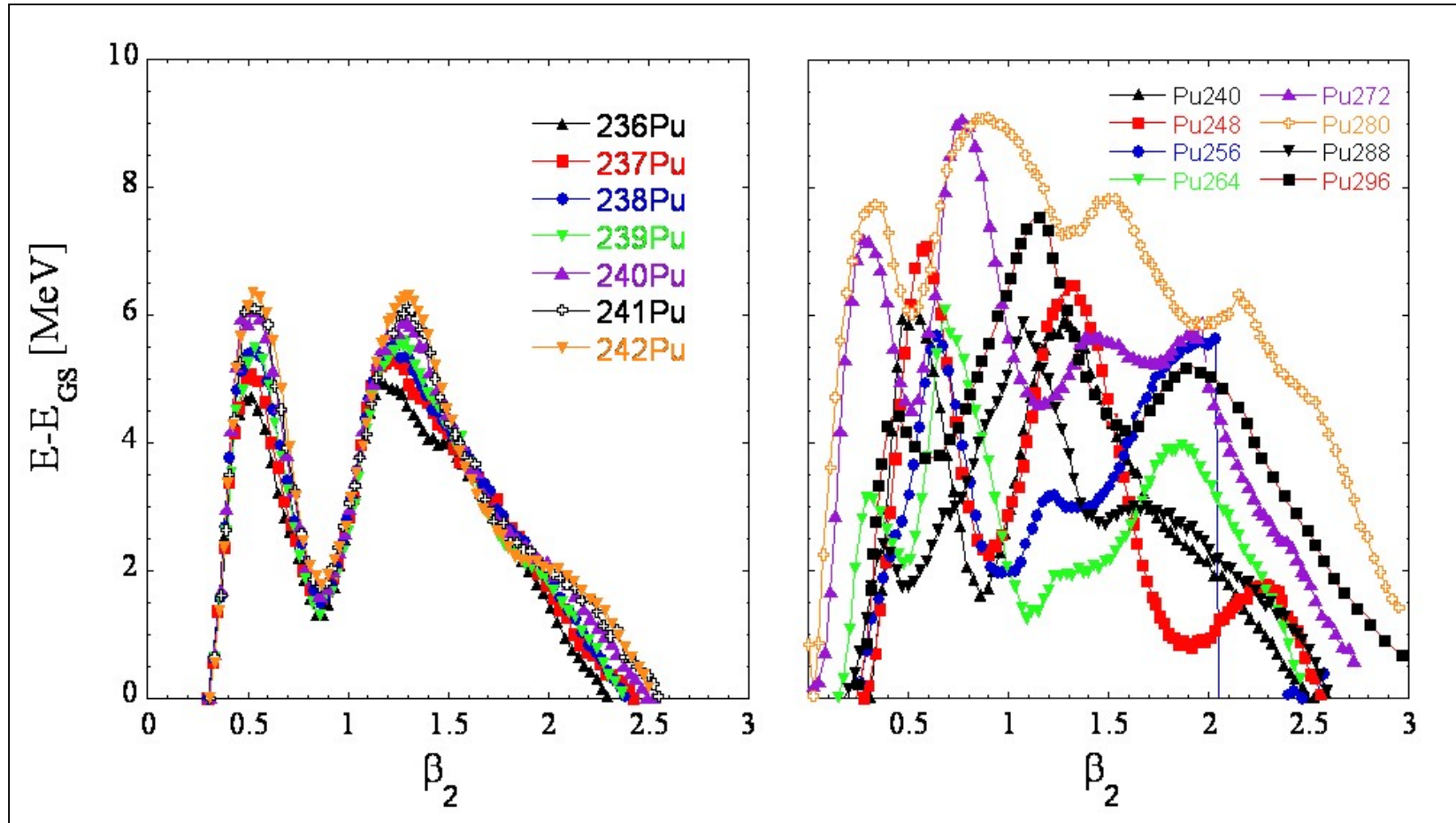
Microscopic approach

Determination of the fission path performing HFB calculation as function of appropriate deformation (collective) variables using ideally an effective interaction also adjusted on experimental masses



Also use the same effective interaction to calculate level densities (GS and top of each barrier)

Microscopic approach : fission paths



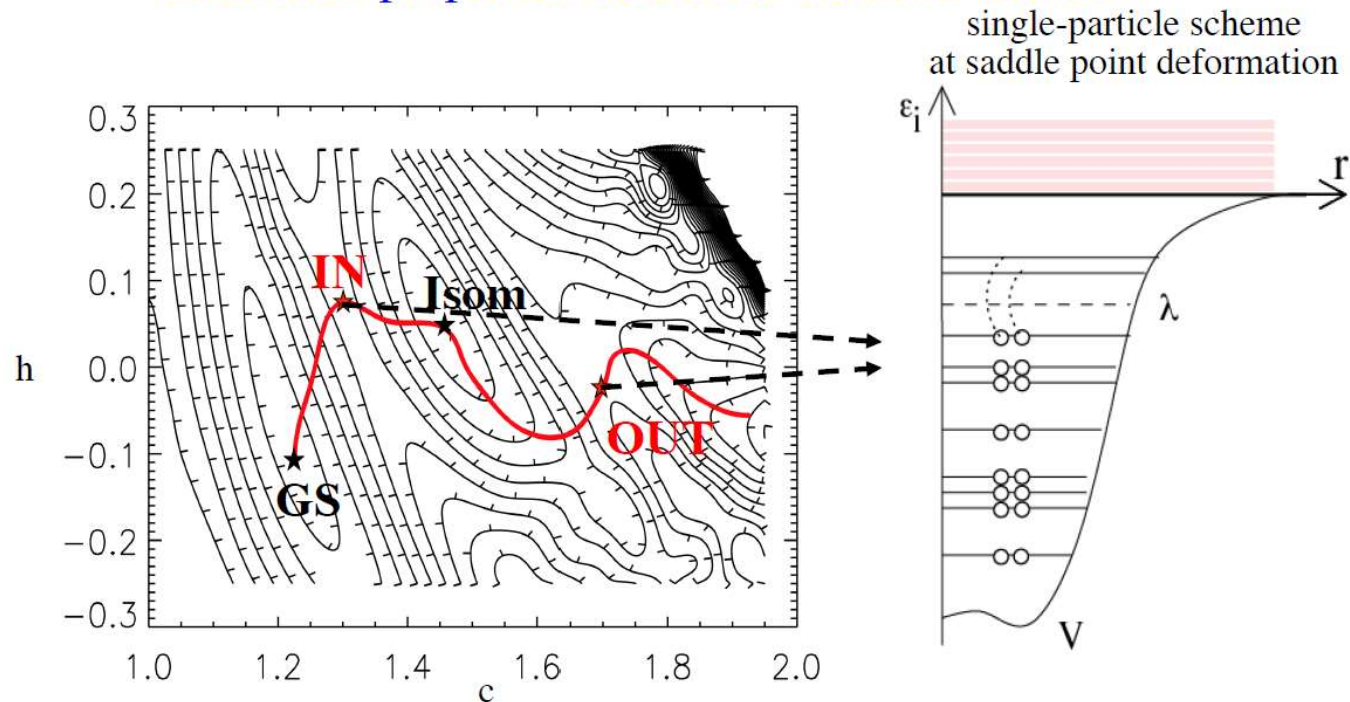
⇒ For exotic nuclei : strong deviations from Hill-Wheeler.



Microscopic approach : Fission level densities

Nuclear level densities at the saddle points

HFB model provides at each deformation (including saddle points)
all nuclear properties needed to estimate the NLD



Possibility to estimate NLD at the saddle point within the HFB+Combinatorial model



Microscopic approach : summary

Nuclear Level Density at Saddle Points

- **Fission Barriers** and saddle point deformations (Q,O,H) determined within HFB method
- **Nuclear properties** (spl, pairing) at the inner and outer saddle points with constrained HFB model
- **NLD** in the framework of the microscopic combinatorial model based on HFB single-particle level and pairing predictions at the HFB saddle points (plus collective rotational and vibrational enhancement)

All ingredients described on the basis of the
same Skyrme effective interaction (BSk14) at GS and Saddle Points

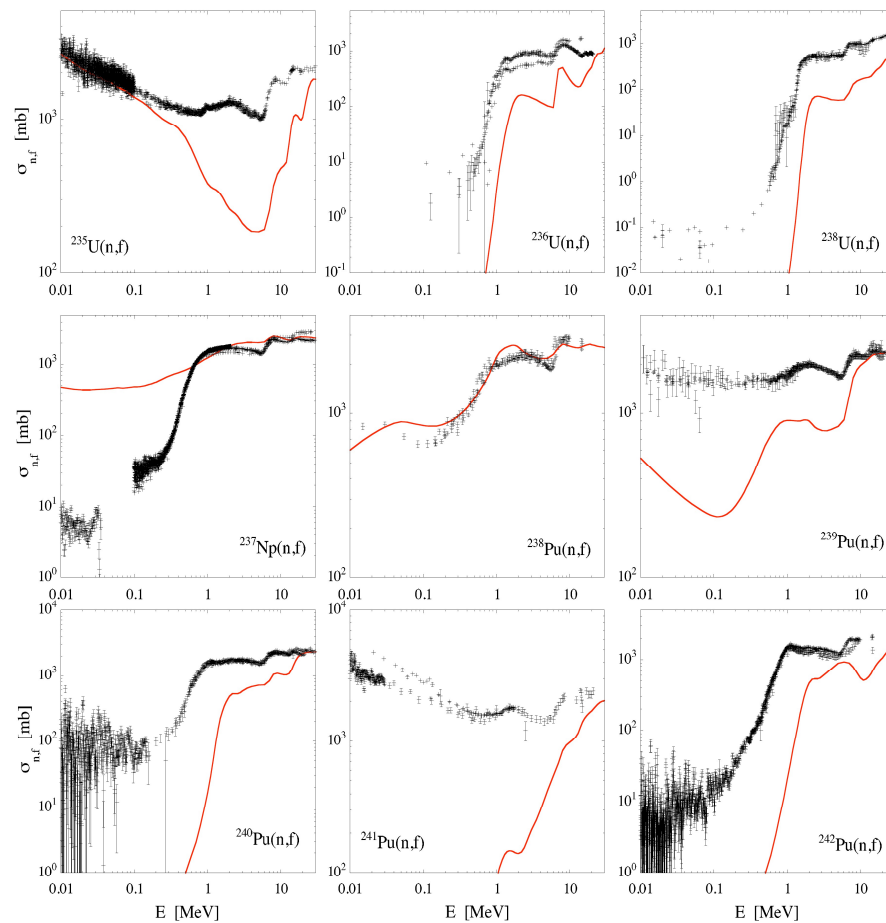
→ NLD in a table format at inner and outer saddle points
(~2000 nuclei : 2/3 saddle points & 1/2 shape isomers)

For inner barrier, usually predicted to be triaxial: $\rho_{triax} = \sqrt{\frac{\pi}{2}} \sigma_{\perp} \times \rho_{Comb}$ Bjornholm & Lynn (1980)

For outer barrier, usually predicted to be left-right asymmetric: $\rho_{asym} = 2 \times \rho_{Comb}$

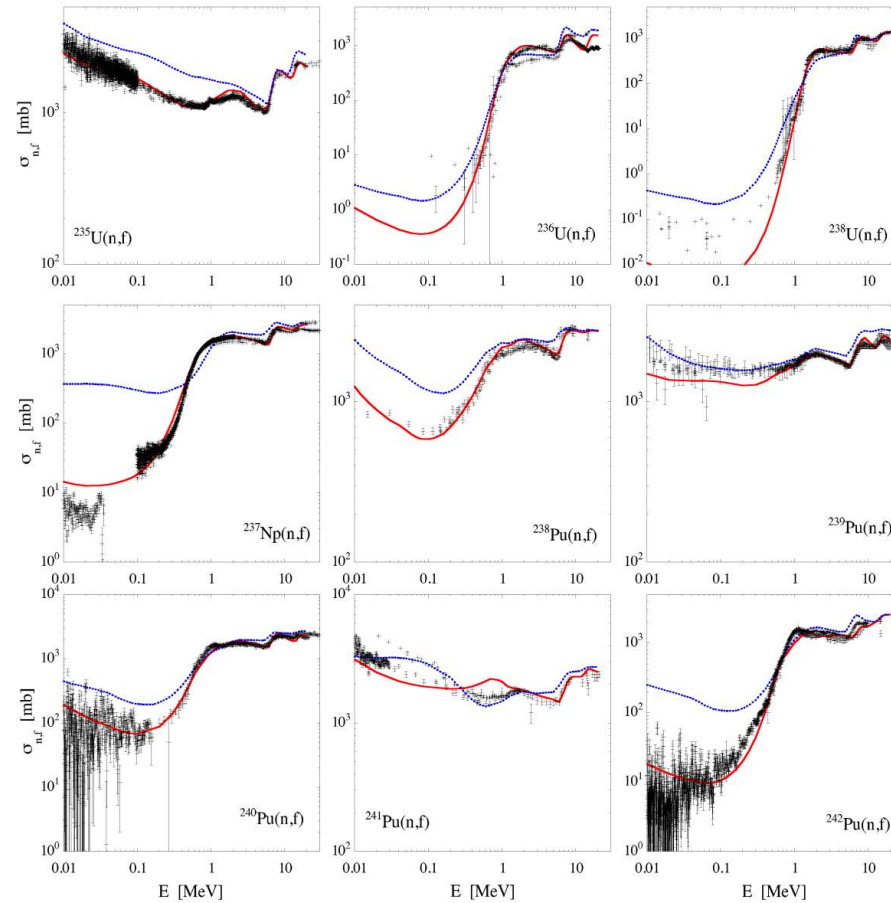


Microscopic approach : results



⇒ **Default calculations not sufficient for applications.**

Microscopic approach : results



Fission barriers
adjusted for
each target

Fission barriers
adjusted for
each type of target

- odd-odd
- odd-even
- even-odd
- even-even

⇒ Not ridiculous after few adjustments.



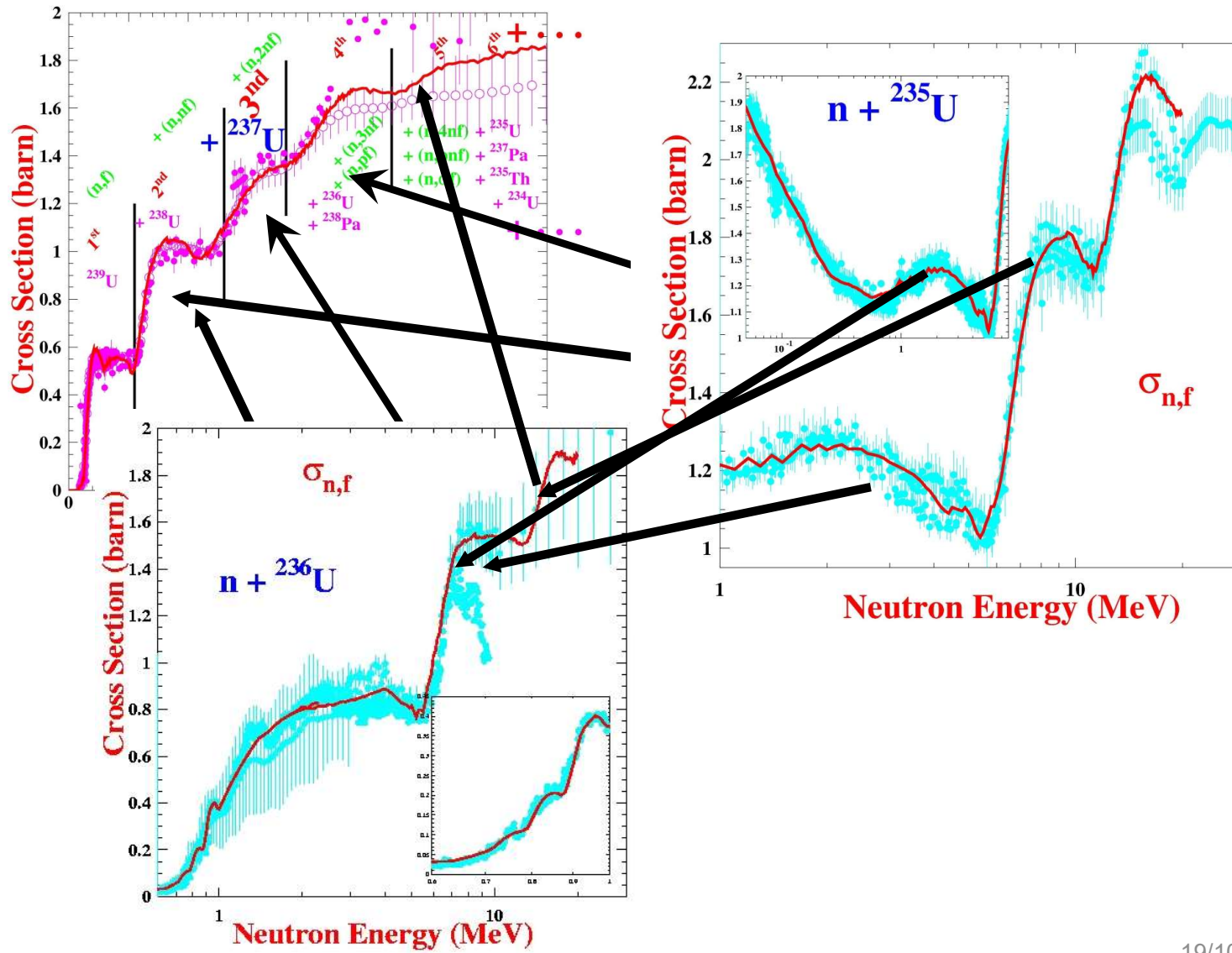
Microscopic approach

Coherent fission cross sections
with phenomenological approach

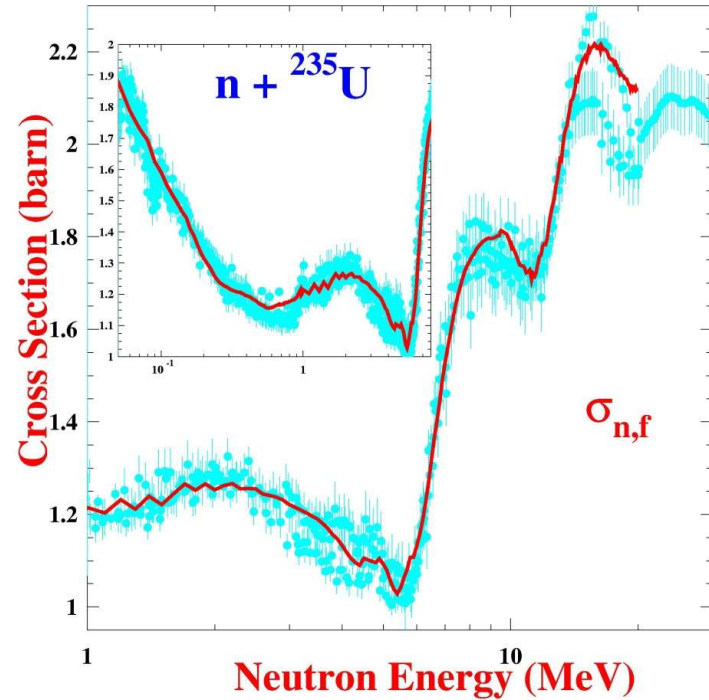
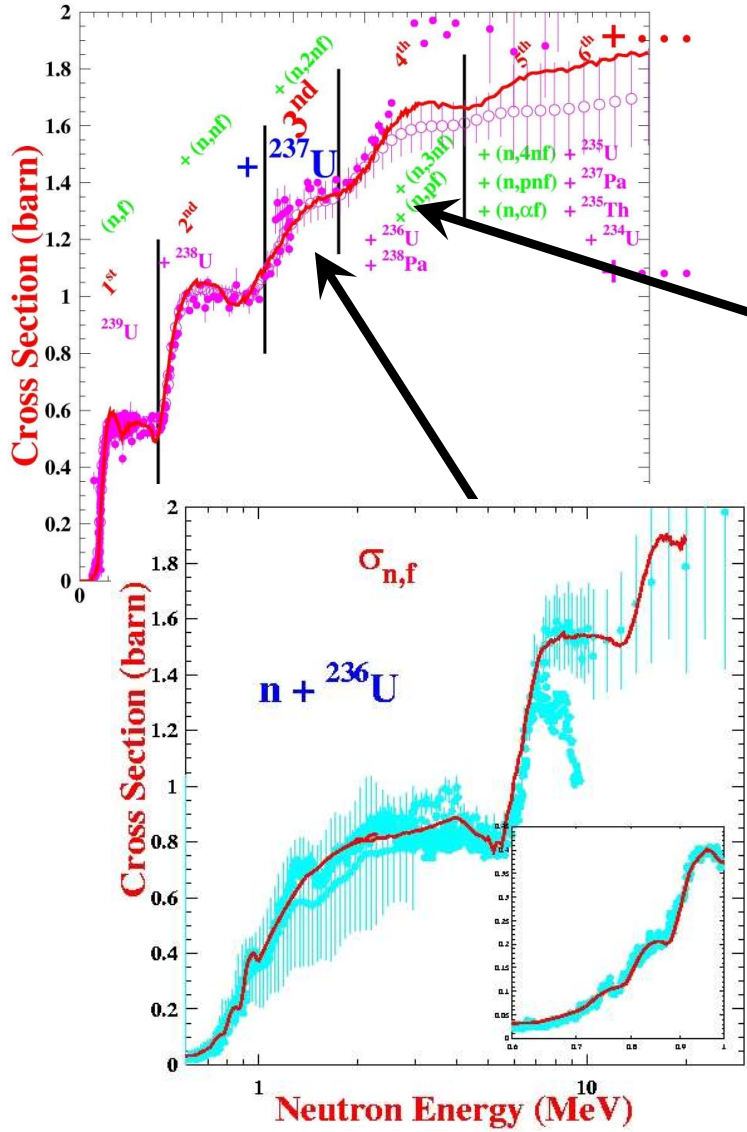
Neutron induced fission on ^{238}U

- several hundreds of parameters
- unique set for all fission chances or U targets

Microscopic



Microscopic





**Can we do the same
with microscopic ingredients ?**



Microscopic approach





Microscopic approach

HFB-14 predictions of fission barriers and NLD at saddle points, including renormalization (max 5 parameters) of

- fission path height: $B_f'(\beta_2) = B_f(\beta_2) \times v_{corr}$
- NLD at 1st and 2^d saddle points:

$$\rho'(U, J, P) = \rho(U - \delta, J, P) e^{\alpha\sqrt{U-\delta}}$$

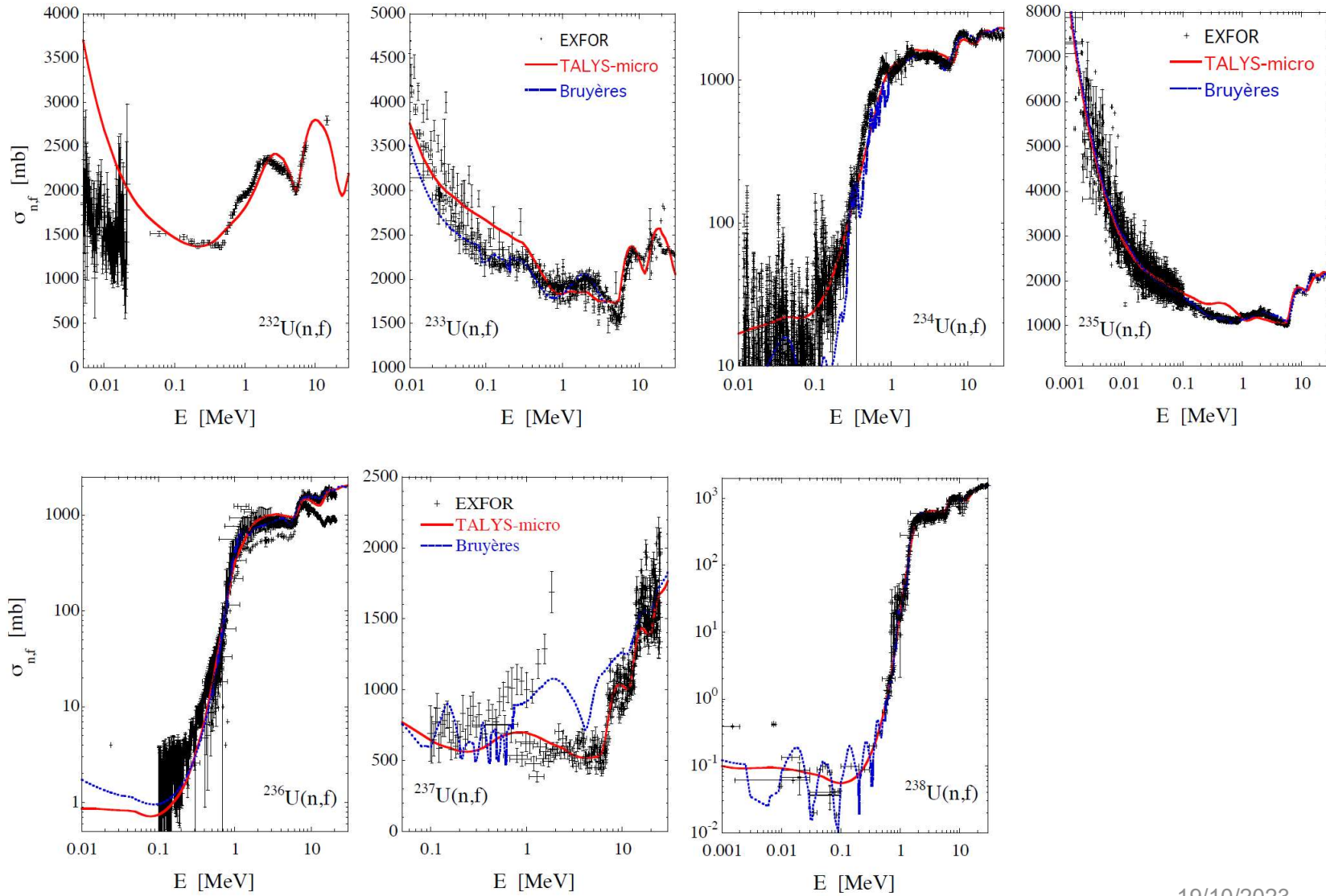
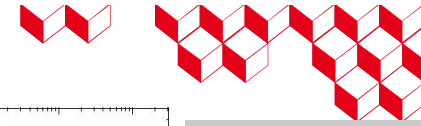
Additional nuclear inputs:

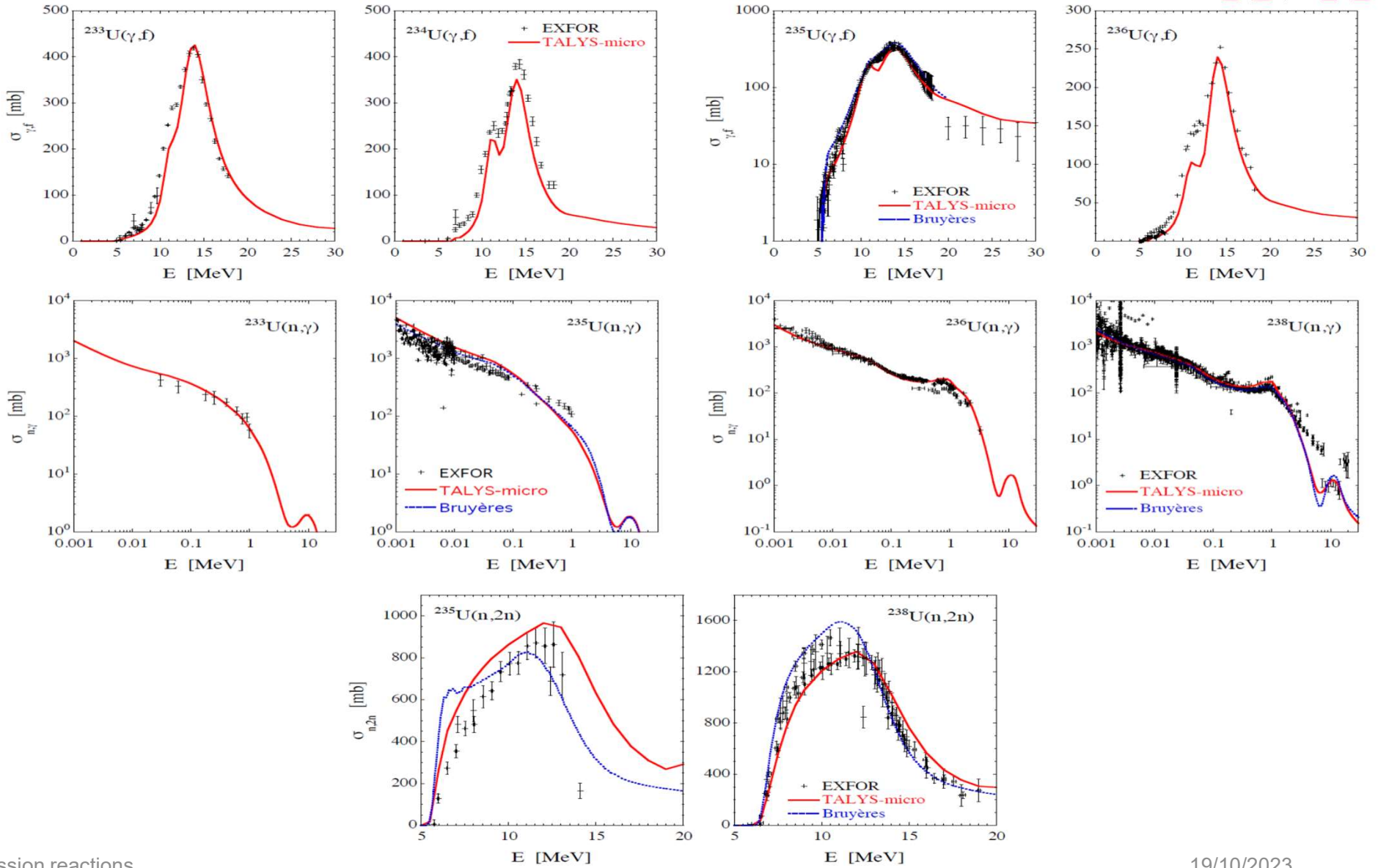
- Nuclear structure properties: HFB-14 (Goriely et al. 2007)
- Optical potential: Soukhovitskii et al. (2004)
- γ -ray strength: Hybrid model (Goriely, 1998)
- NLD: HFB-14 plus combinatorial model (Goriely et al., 2008)
normalized on s-wave spacings and discrete excited levels

Note:

- **1 UNIQUE set of nuclear ingredients for all U isotopes**
- no class 2 states included
- no discrete transition states included

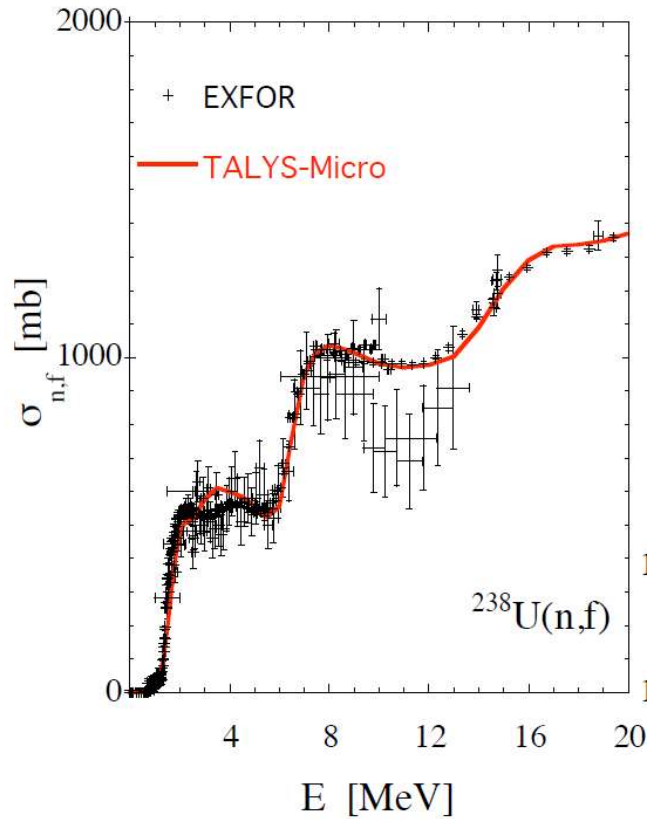
Microscopic approach



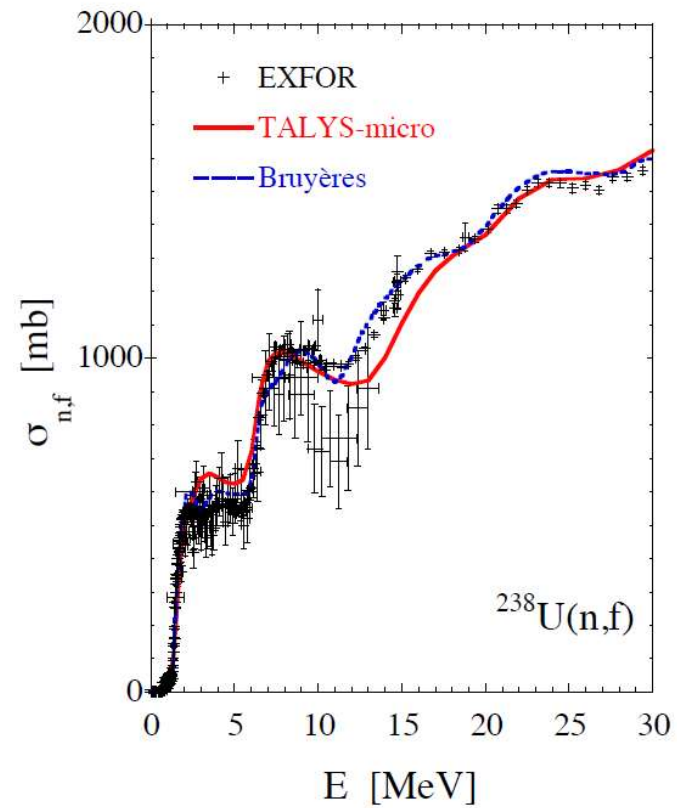




Without Coherence



With Coherence



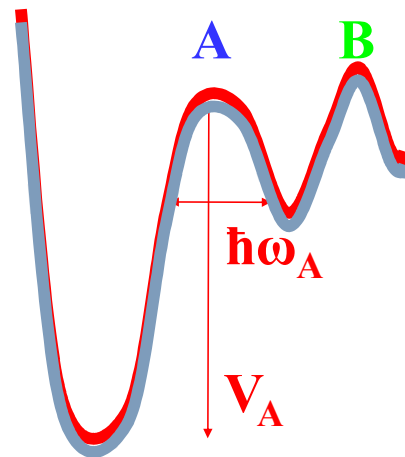
Coherence = more constraints = slightly worse fit



Integral benchmark / fission / parameter sensitivity

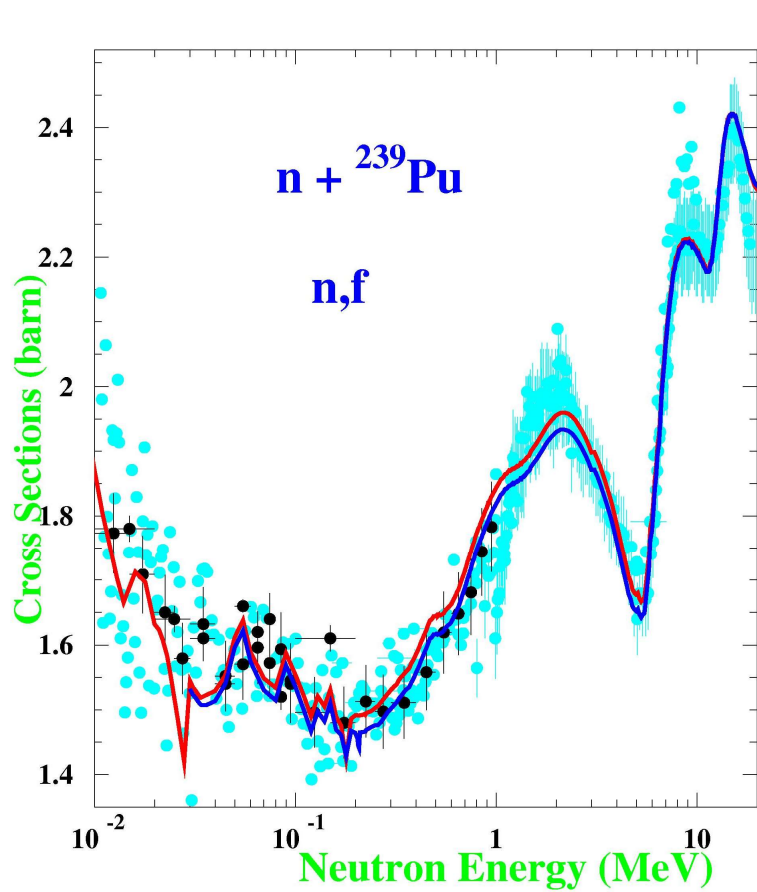


$^{239}\text{Pu} (n,f)$

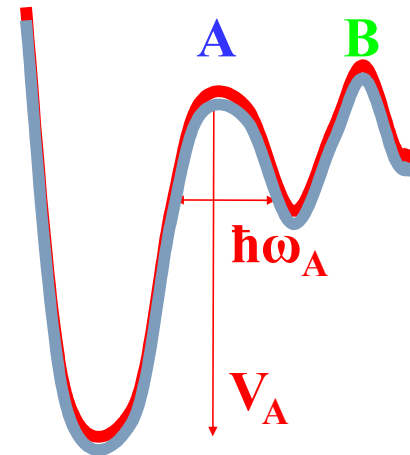


-20 keV sur V_A
 $\approx 0.34\%$!!

Integral benchmark / fission / parameter sensitivity

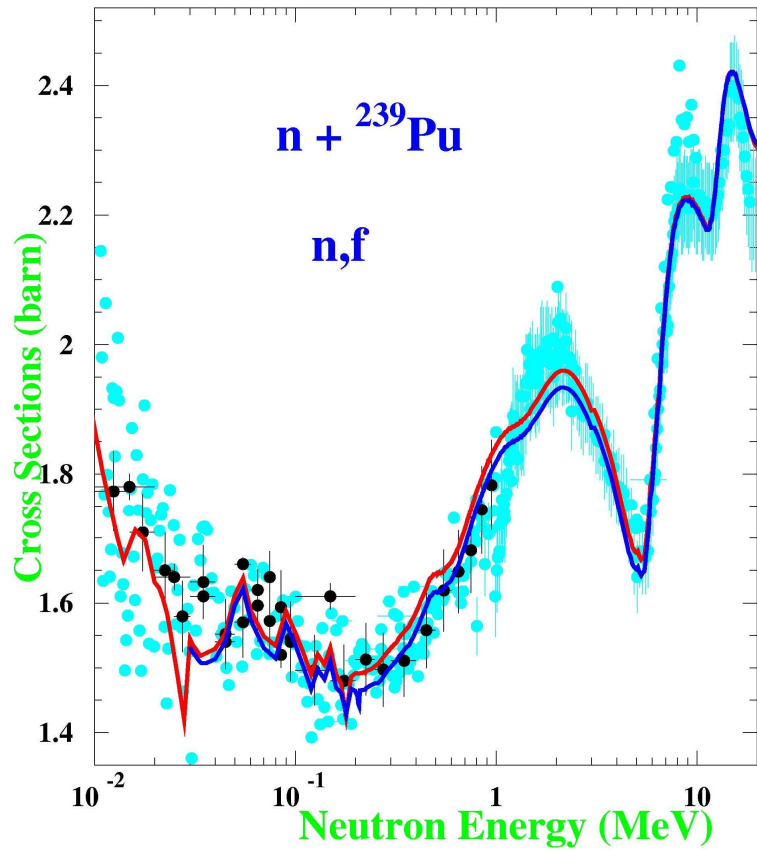


${}^{239}\text{Pu} (n, f)$

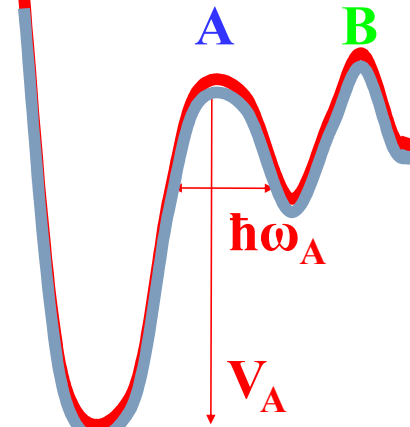


-20 keV sur V_A
 $\approx 0.34\% !!$

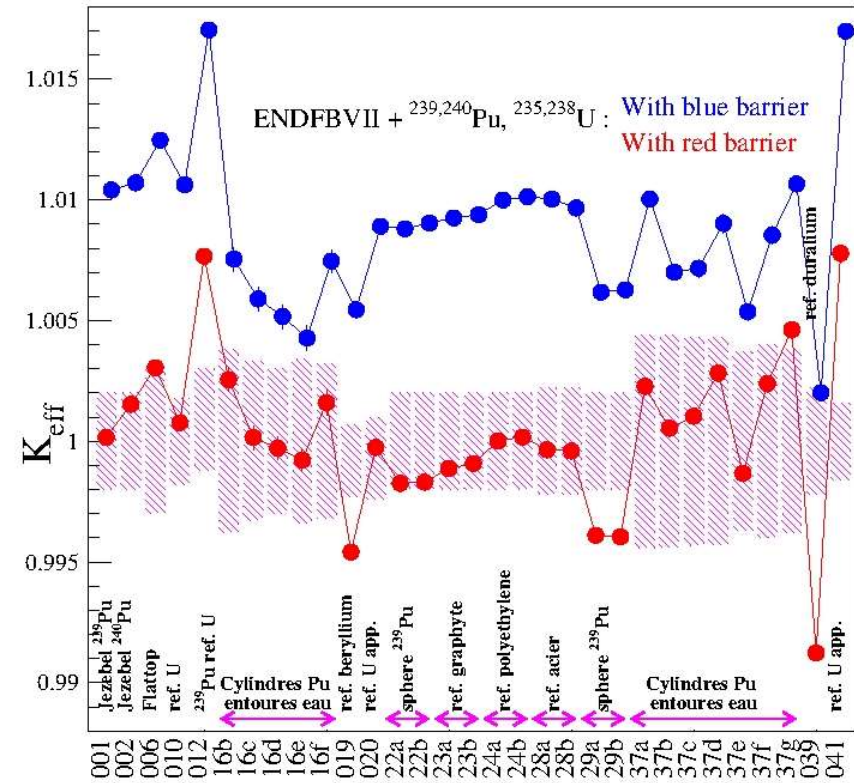
Integral benchmark / fission / parameter sensitivity



$^{239}\text{Pu} (n,f)$



-20 keV sur V_A
 $\approx 0.34\%$!!





4. Conclusions and Prospects



Nuclear reaction modeling : 2 complementary paths



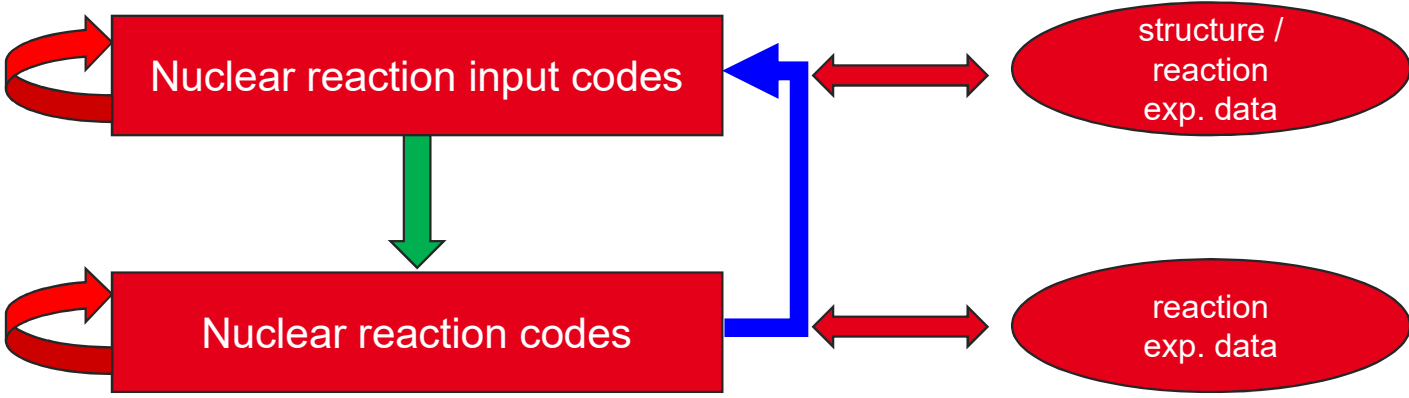
Empirical / Analytical approaches

- Good (very) fitting power
 - Weak (modest) computing time
 - Weak predictive power
 - Important human optimization
- ⇒ accurate evaluated files for applications (ENDF, JEFF, JENDL ...)

Microscopic (semi-) approaches

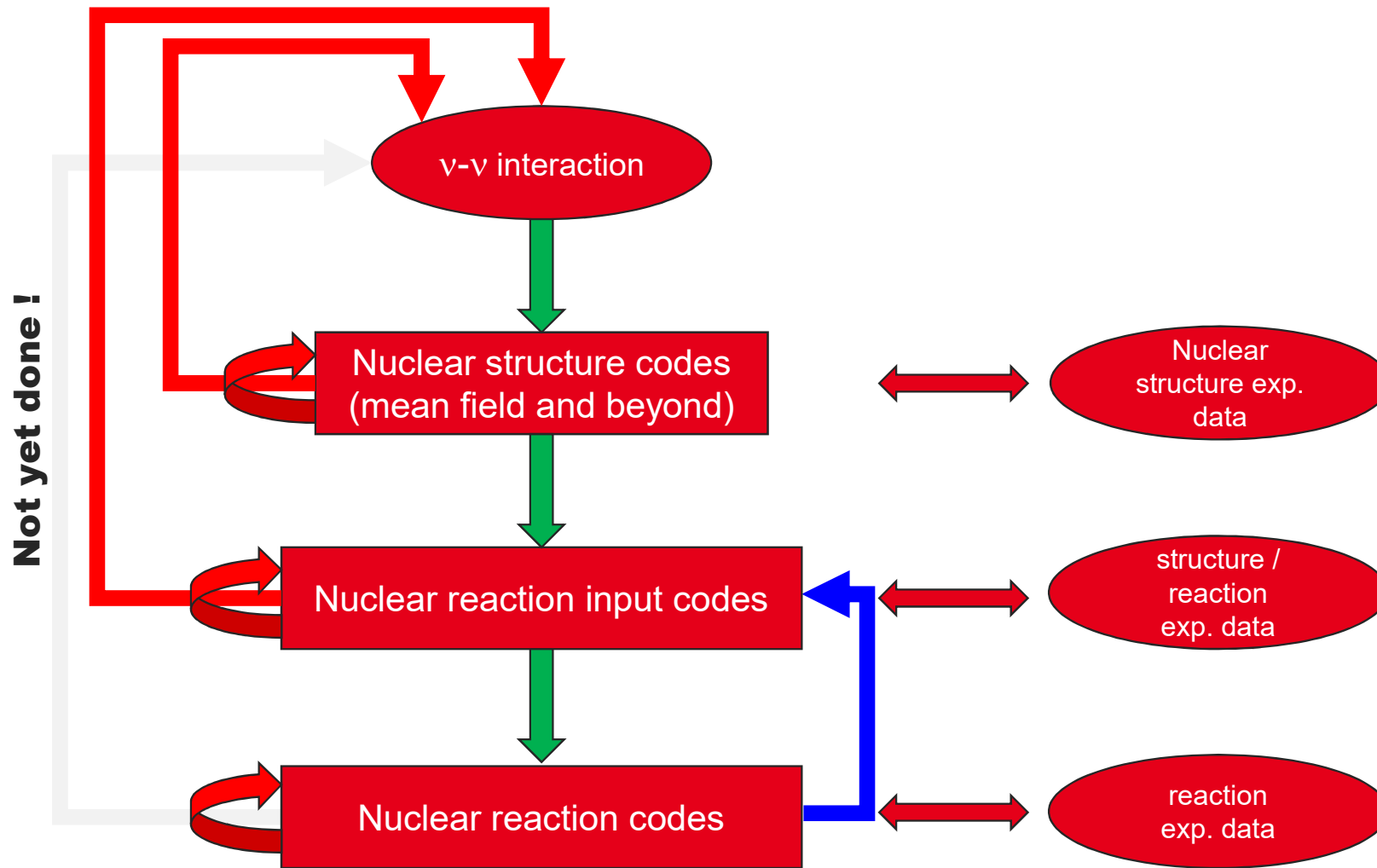
- Weak fitting power
 - Important computing time
 - Good predictive power
 - Weak human optimization
- ⇒ astrophysical applications
⇒ fundamental research
⇒ guide for empirical approaches

Phenomenological approach : fitting loop

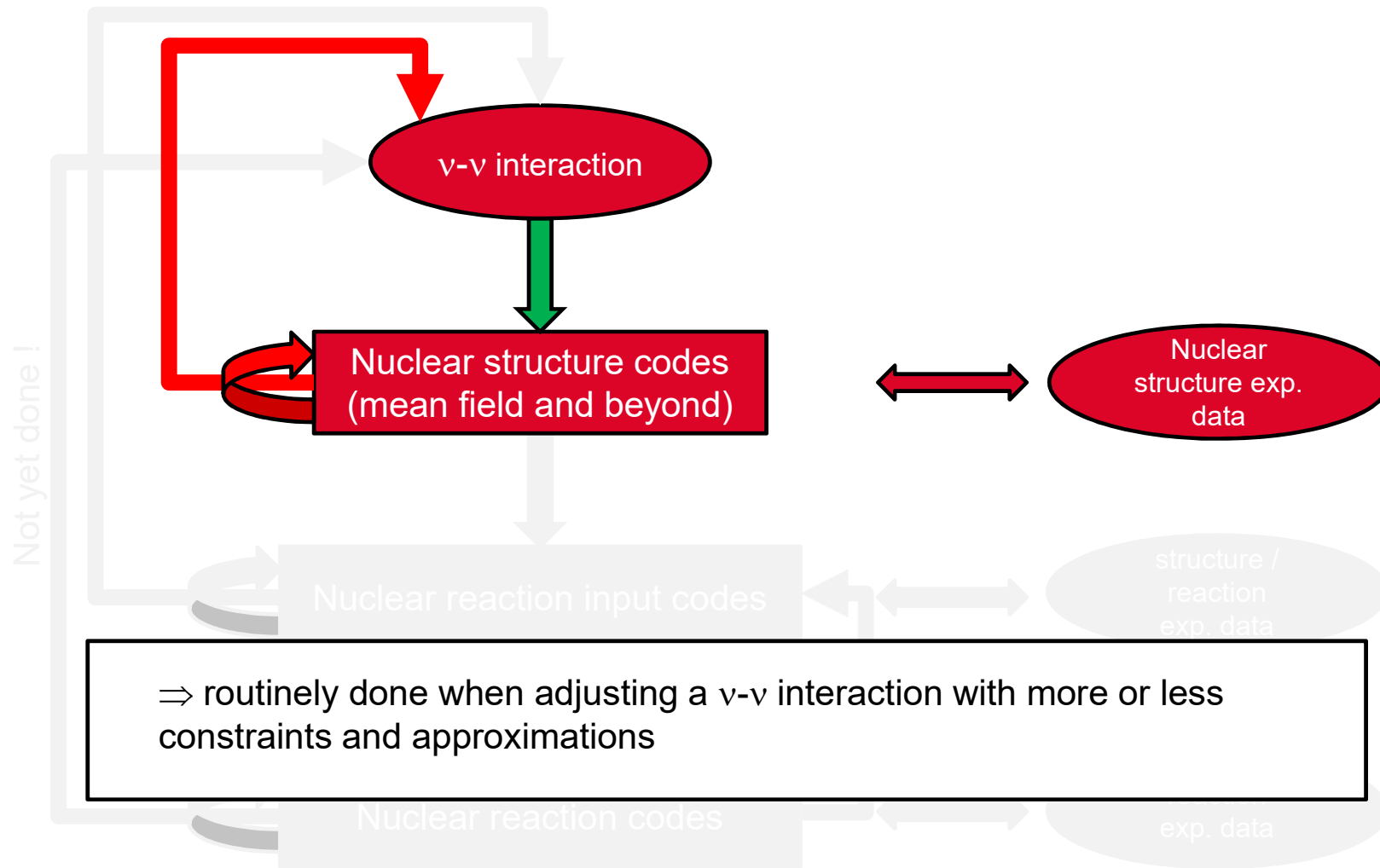




Microscopic approach : multiscale fitting loop

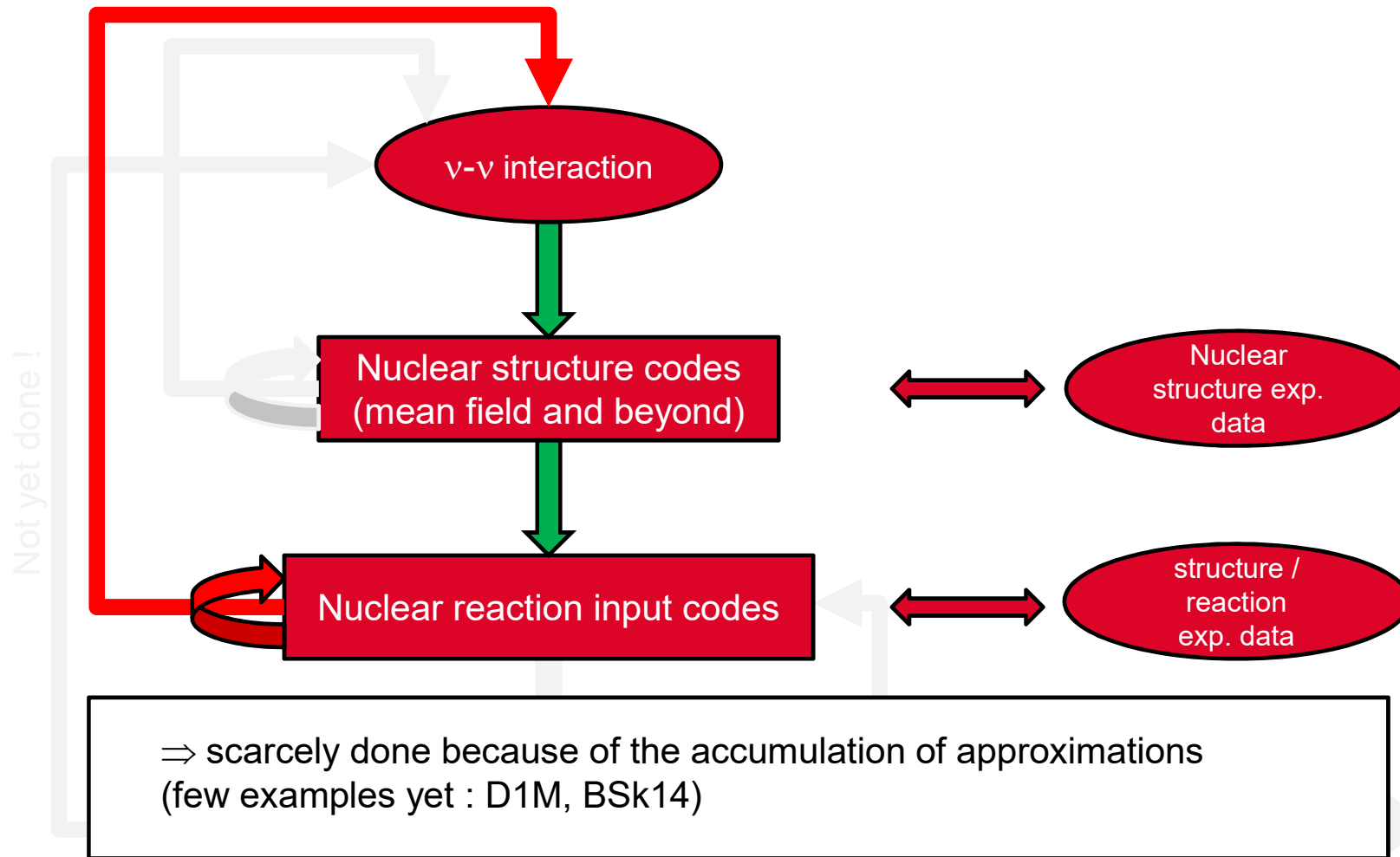


Microscopic approach : multiscale fitting loop

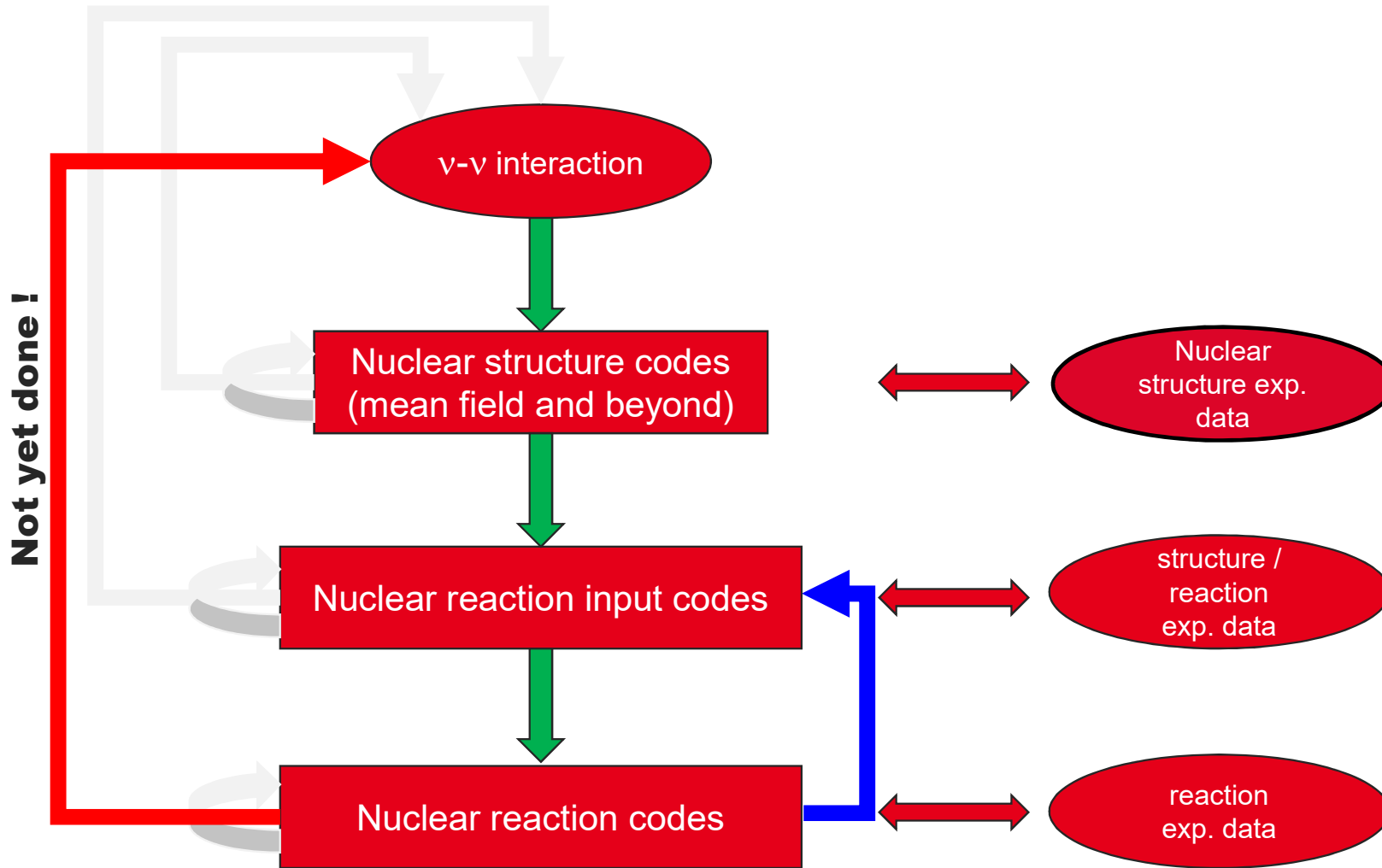




Microscopic approach : multiscale fitting loop



Microscopic approach : multiscale fitting loop





Microscopic approach : multiscale fitting loop

