LOW D SINGULARITIES

Marija Tomašević

University of Amsterdam

OUTLINE

(Quantum) BTZ black holes

3D singularities

2D singularities

WHY SINGULARITIES?

- Singularities come in several flavors: spacelike, null and timelike.
- Usually in physics, we are used to dealing with timelike singularities: they indicate the need for a ‘higher resolution’, that is, more fundamental dofs.
- Null singularities can be argued to always lead to spacelike singularities.
- And spacelike singularities represent singular moments in time.

Understanding their nature and resolution is one of the biggest problems in theoretical physics.

WHY 3D AND 2D SINGULARITIES?

- In this talk, we will deal with low-dimensional singularities only.
- This is because the description of black holes in general becomes much more tractable as we lower the number of dimensions.
- Tractability indicates the hope that we can say something about the nature of singularities (although this is still not clear).
- This would be done through the holographic correspondence.

The simple example we will start from is the Bañados-Teitelboim-Zanelli (BTZ) black hole.

Bañados, Teitelboim, Zanelli '92
Carlip '95
Birmingham, Sachs, Sen '01

BTZ BLACK HOLES

- These are 3D black holes in AdS spacetimes, obtained through a discrete quotient of the AdS_3 spacetime

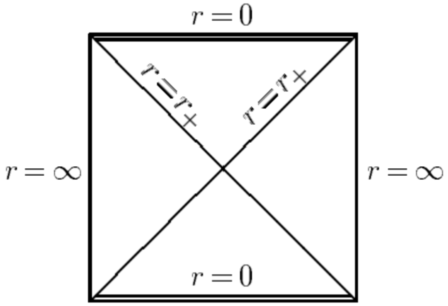
$$ds^2 = -\left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right) dt^2 + \left(-M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)^{-1} dr^2 + r^2 \left(d\phi - \frac{J}{2r^2} dt\right)^2$$

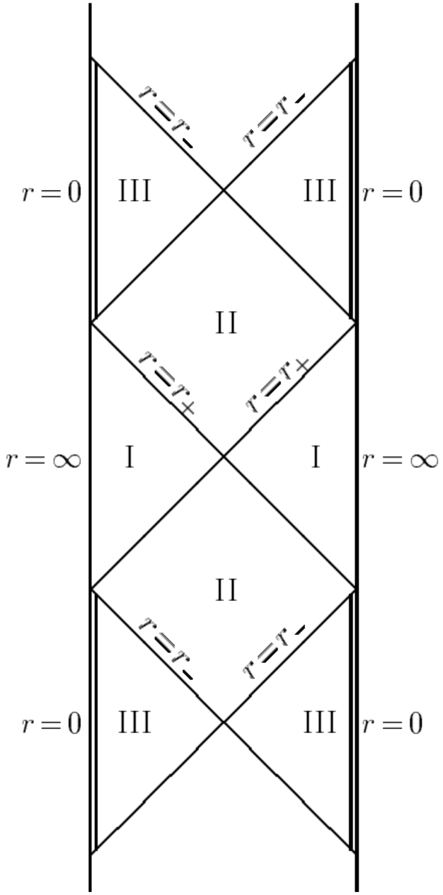
$$\Lambda = -1/l^2 \qquad M = \frac{r_+^2 + r_-^2}{l^2}, \quad J = \frac{2r_+ r_-}{l} \qquad r_\pm^2 = \frac{Ml^2}{2}\left(1 \pm \sqrt{1 - \frac{J^2}{M^2 l^2}}\right)$$

BTZ ‘92
Emparan, Tomašević ‘21

CAUSAL STRUCTURE OF BTZ

- The casual structure of J = 0 solutions is usually depicted as a “square” Penrose diagram.
- However, the surface r = 0 is a **null** surface, and a boundary between chronologically regular and pathological region (one with closed timelike curves).
- One can show this by mapping the solution to the Misner-AdS3 solution.
- The rotating solution has the standard Kerr-like causal structure.





BTZ '92
Emparan, Tomašević '21

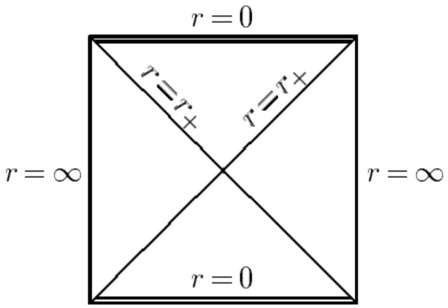
CAUSAL STRUCTURE OF BTZ

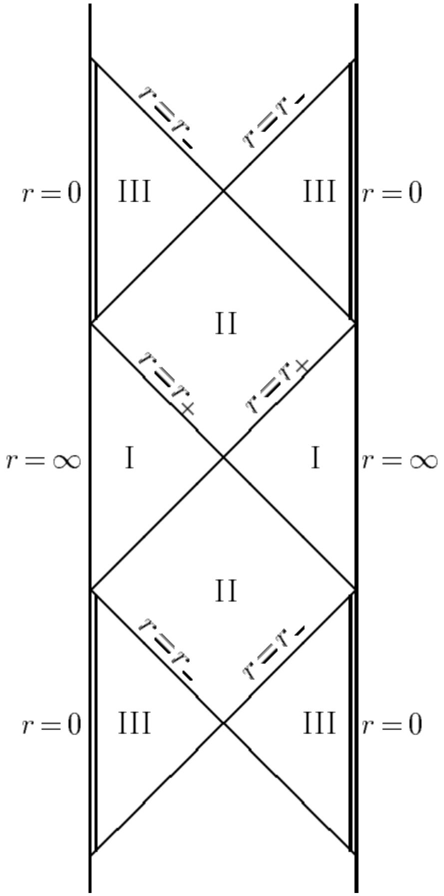
- The r = 0 surface is also not properly singular—it is only quasi-regular.
- One can see that by taking the r → 0 limit:
- In the BTZ case, one obtains a locally flat solution (so one can extend beyond it).

$$\sim -dr^2 + r^2 d\phi^2$$

- In contrast, the near-singularity limit for the Schwarzschild solution gives a Kasner form

$$\sim -r dr^2 + r^2 d\phi^2$$





Strominger, Vafa ‘96
Birmingham, Sachs, Sen ‘01

BTZ FEATURES

- BTZ black holes are very useful.
- They are simpler, but they capture some essential features of real-life black holes—they have event horizons and obey laws of thermodynamics.
- They are also very useful for understanding the black hole entropy: a generic feature of all string theory black holes for which an exact counting of microstates is possible, is that the near horizon geometry of these solutions is that of a BTZ black hole.

$$S(E) = 2\pi\sqrt{\frac{c}{3}E}$$

Achucarro, Ortiz '93
Ghosh, Maxfield, Turiaci '19

BTZ FEATURES

- Another interesting feature of BTZ black holes lies in their dimensional reduction.
- Usually for black holes with spin or charge, one takes the near-extremal limit (large spin or charge), and obtains a special geometry, (w)AdS2 x compact mfd.
- For rotating BTZ, the same limit gives AdS2 x S1.

$$r = \tfrac{1}{2}(r_+ + r_-) + \tfrac{1}{2}(r_+ - r_-)\rho \qquad T \sim \frac{r_+ - r_-}{\pi \ell_3} \to 0$$

$$ds^2 \sim \frac{\ell_3^2}{4}\left[-(\rho^2-1)(2\pi T)^2 dt^2 + \frac{d\rho^2}{(\rho^2-1)}\right] + r_+^2\left(d\varphi - dt + \tfrac{\ell_3}{r_+}\rho\,\pi T\,dt\right)^2$$

- But dimensionally reducing the static black hole also gives the AdS2 geometry.

Saad, Shenker, Stanford ’19
Turiaci, Usatyuk, Weng ’20

review: Mertens, Turiaci ’22

JT GRAVITY

- The reason why this AdS_2 geometry is special is due to its connection with JT gravity.
- JT gravity encompasses the physics of its higher-dimensional parent black holes.
- More than that, it is a simple solvable model of quantum gravity in 2D.
- And this solvability has led to the conclusion that classical, extremal, non-SUSY black holes don’t actually exist.
- This is due to low-temperature (long-throat) quantum gravity corrections that render the classical solution meaningless—quantum gravity fluctuations simply become too large.

- Crucially, JT gravity (and some modifications of it) have been mapped to certain matrix integrals, providing a new way of looking at the bulk quantum gravity theory.

PROBLEMS WITH BTZ

- They don’t really represent true black holes in our universe, mainly because of their causal structure.
- They allow for CTCs behind the inner horizons, and they don’t have curvature singularities.

But the solution changes completely once we add matter.

ADDING MATTER

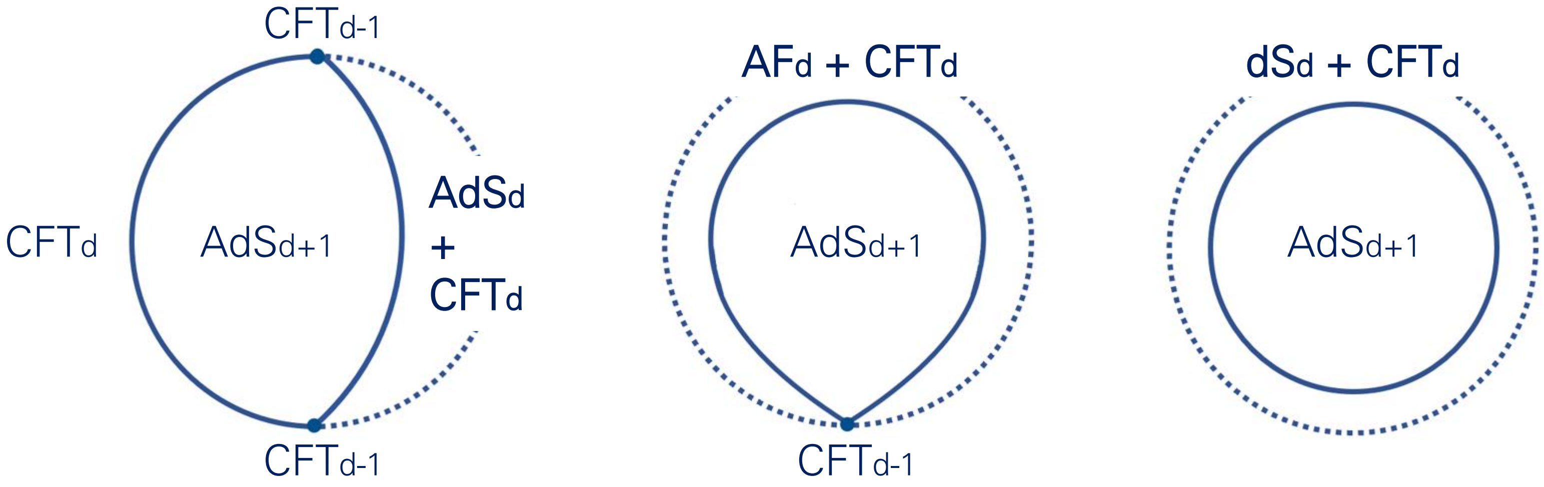
- There are several ways in which we can add matter.
- The simplest way is to put some fields on the black hole background and solve for the backreacted geometry.
- This is, of course, the standard way of perturbing the solution.

- However, there exists a framework in which we can do better—double holography.

Randall, Sundrum '99
Karch, Randall '01

DOUBLE HOLOGRAPHY

- The framework of braneworld/RS-KR/double holography refers to the gravity on the brane.
- In AdS, we can embed lower-dimensional manifolds with any asymptotics.



de Haro, Skenderis, Solodukhin '00
Bueno, Emparan, Llorens '22

DOUBLE HOLOGRAPHY

- The basic idea: a CFT with a cut-off corresponds to an AdS bulk with a cut-off brane.
- Integrating out the UV dofs of the CFT leads to an induced action on the brane—gravity is dynamical, and the brane theory is a CFT coupled to dynamical gravity.
- Classical dynamics in an AdS_{d+1} bulk with a d-dim brane holographically encodes the quantum dynamics of the dual d-dim CFT coupled to a d-dim gravitational theory on the brane.

Crucially, brane metrics **encode the quantum backreaction of the holographic CFT.**

This allows us to study quantum effects that would otherwise require us to solve iteratively the backreaction problem

DOUBLE HOLOGRAPHY

- The full action consists of two terms, $I + I_b$
- Written in this way, this is a **bulk view** of the action: it is the action of a finite gravitational system with Einstein-Hilbert dynamics, plus a brane, in d + 1 dimensions

- We can rewrite these same two terms in a **brane view**, $I_{bgrav} + I_{CFT}$
- Now, the same sum is recast in the form of a higher curvature gravitational theory in d dimensions, coupled to a cut-off CFT_d, which backreacts on the metric g_{ij}

$$I + I_b = I_{\mathrm{bgrav}} + I_{\mathrm{CFT}}$$

Emparan, Horowitz, Myers ‘99
Emparan, Fabbri, Kaloper ‘02
Emparan, Frassino, Way ‘20

QUANTUM BTZ BLACK HOLES

- To obtain a quantum-corrected BTZ black hole on the brane, we start with a specific solution in the bulk, the AdS_4 C-metric, and slice it along x = 0.

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left(-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right)$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}\,,$$
$$G(x) = 1 - \kappa x^2 - \mu x^3$$

QUANTUM BTZ BLACK HOLES

- To obtain a quantum-corrected BTZ black hole on the brane, we start with a specific solution in the bulk, the AdS_4 C-metric, and slice it along x = 0.

$$ds^2 = -H(r)dt^2 + \frac{dr^2}{H(r)} + r^2 d\phi^2$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} ,$$

$$G(x) = 1 - \kappa x^2 - \mu x^3$$

QUANTUM BTZ BLACK HOLES

- This black hole is a solution to Einstein-Hilbert gravity in 3D + CFT3 that lives on the brane + higher curvature corrections.

$$ds^2 = -H(r)dt^2 + \frac{dr^2}{H(r)} + r^2 d\phi^2$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r},$$
$$G(x) = 1 - \kappa x^2 - \mu x^3$$

- Here μ is a parameter set by bulk regularity: it sets the stress tensor of the brane CFT—this is why it is a quantum correction to the metric.
- Parameter ℓ determines the brane tension, and so, the distance of the brane to the would-be boundary.

STATIC 3D SINGULARITIES

- The new term in the blackening factor provides a proper curvature singularity, at least in the static case.

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \sim \frac{\mu^2\ell^2}{r^6}$$

- This then reinforces chronology protection, and the Penrose diagram is no longer a "square": the singularity bends down.
- This quantum BTZ black hole is much more analogous to Sch-AdS_4 black hole.
- Therefore, it provides a more tractable (since it's 3D) model of a real-life black hole (in AdS).

STRONG COSMIC CENSORSHIP

- The fact that a curvature singularity prevented us from crossing to the pathological side is one instance of the strong cosmic censorship conjecture, which claims that

all Cauchy horizons are singular.

- It has been conjectured by Penrose in the 70s to prevent various dubious things from happening.
- Experiencing time machines is one of them, but we would also be allowed to see the timelike singularity, and we would be able to cross into another universe.

Hollands, Wald, Zahn '19

ROTATING BTZ BLACK HOLE

- The SCC conjecture has been shown to be upheld whenever a quantum field stress tensor is evaluated at the inner horizon.
- This stress tensor has a universal form.

$$\langle T_{vv} \rangle_\Psi \sim \frac{C}{|v|^2} + subleading\ terms, \quad v \to 0^- \,, \; C \sim O(\hbar)$$

- However, a notable outlier to this picture has been the rotating BTZ black hole, for which that constant C is exactly zero.

Emparan, Tomašević '20
Kolanowski, Tomašević '23

QUANTUM ROTATING BTZ BLACK HOLES

- As for the static case, one can also obtain the quantum version of the rotating solution

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} - 1 - \frac{\mu\ell}{r} + \frac{a^2}{r^2}\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 1 - \frac{\mu\ell}{r} + \frac{a^2}{r^2}} + r^2\left(d\phi - \frac{a}{r^2}dt\right)^2$$

- Turns out, adding the new term in the quantum case is not enough to render the inner horizon singular: the geometry is highly symmetric, leading to non-trivial cancelations for the constant C.
- But adding another quantum scalar to probe it **restores** the strong cosmic censorship, by breaking the highly symmetric nature of the BTZ solution and making the constant C non-zero.

Emparan, Tomašević '20
Kolanowski, Tomašević '23

QUANTUM ROTATING BTZ BLACK HOLES

- One can do the calculation explicitly, but we can also argue without calculations that this will hold true.
- Namely, via double holography, we can see that the quantum rotating BTZ solution is a particular section of a Kerr-AdS4 black hole.
- In fact, in the limit of very small tension, we obtain the Kerr-AdS4 solution.
- Since the two black holes share the interior, whatever happens for the inner horizon of Kerr-AdS4 will happen for quantum rotating BTZ.
- And indeed, adding matter to the Kerr black hole renders its inner horizon singular.

QUANTUM EFFECTS PRESERVE STRONG COSMIC CENSORSHIP

Kolanowski, Tomašević '23

2D SINGULARITIES

- Let us go back to the static case. For the standard BTZ black hole, one could reduce the solution to AdS_2 and JT gravity.
- What happens if we do the same with the quantum-corrected solution?
- We obtain a modified, "quantum" JT gravity (some dilaton-gravity).
- But the interesting part is that now this quantum JT incorporates spacelike curvature singularities. And we can map it to a specific dilaton potential.

$$S = \frac{1}{16\pi G}\int d^2x\sqrt{-g}\,(\phi R - U(\phi)) \qquad R = U'(\phi) \qquad U(\phi) = -\frac{2}{L^2}\phi - \frac{\ell\mu}{\phi^2} + \frac{2a^2}{\phi^3}$$

Martinez, Zanelli '96
Kolanowski, Tomašević '23

2D SINGULARITIES

- We can also obtain this potential from EH3 + conf. scalar, where we integrate out the scalar,

$$I = I_{EH} + I_{\text{matter}},$$

$$I_{EH} = \frac{1}{16\pi\bar{G}} \int_{\mathcal{M}} d^D x \sqrt{\bar{g}} \left(\bar{R} - 2\lambda\right) + \frac{1}{8\pi\bar{G}} \int_{\partial\mathcal{M}} d^{D-1} x \sqrt{\bar{h}} \bar{K},$$

$$I_{\text{matter}} = \frac{1}{2} \int_{\mathcal{M}} d^D x \sqrt{\bar{g}} \left(\xi \bar{R} \psi^2 + (\partial\psi)^2\right),$$

where bars denote 3D quantities.

The only caveat is the reduced set of parameters that we can use.

2D SINGULARITIES

- Namely, the solution one finds

$$H(r) = \frac{r^2}{\ell_3^2} - a - \frac{b}{r} \qquad b = \frac{2a}{3} = \frac{2B^2}{\ell_3^2}, \qquad B \geq 0, \qquad \psi = \frac{A}{\sqrt{r+B}}, \qquad A = \sqrt{\frac{B}{\pi \bar{G}}}.$$

- So we see that we cannot take the BTZ limit for instance, nor do we have control over the parameter ranges.
- Nevertheless, since we only care about having a singularity at r = 0, this model is good enough.

2D SINGULARITIES

- Another way one could try to obtain the potential is through the quantum BTZ action.
- But we don't know what is the exact CFT on the brane.
- Moreover, it would be a quantum CFT, necessitating a much more complicated procedure.

- A way around this is to use the doubly-holographic setup in our favor and rewrite the action in terms of the classical 4D bulk.
- Unfortunately, the action is not reducible in that case.

2D SINGULARITIES

- There is a warping factor that depends on the angle.

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left(-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right) \qquad \bar{g}_{\mu\nu} = \omega^2 g_{\mu\nu}, \qquad \omega(x,r) = \frac{\ell}{\ell + xr}$$

- So, our action can be written as below, but it cannot be reduced further

$$\frac{1}{16\pi G_D}\int \sqrt{-g_{\mu\nu}}\sqrt{-\gamma_{ab}}\,\omega^4 e^{2\phi}\,(R_F - 2\lambda) + \frac{1}{8\pi G_D}\int \sqrt{-h_{\mu\nu}}\sqrt{-\gamma_{ab}}\,\omega^3 e^{2\phi} K_F,$$

$$R_F = \omega^{-2}\left(R^{(2)} - 4\Box\phi - 6(\partial\phi)^2 + 2e^{-2\phi}\right) - \frac{6}{\omega}\Box\omega \qquad K_F = \omega^{-1}\left(K^{(1)} + 2n^\alpha\partial_\alpha\phi\right) + \frac{3}{\omega^2}n^\alpha\partial_\alpha\omega$$

2D SINGULARITIES

- Regardless of the way we obtain the action, we have a 2D model with spacelike singularities, with a physically well-motivated dilaton potential.
- This opens the door for studying spacelike curvature singularities through the lens of dilaton-gravities.
- In particular, one could hope for obtaining some sort of a matrix dual to the singularity, possibly shedding light on its resolution (at least in 2D).

SUMMARY

- BTZ black holes have proven to be very useful for a variety of different things.
- Adding matter to them gives us new possibilities for studying their duals and general properties.
- We have already seen that quantum corrections create curvature singularities and preserve strong cosmic censorship.
- Furthermore, they remain at the 2D level after dimensionally reducing the theory, allowing for the possibility to study spacelike singularities with more control.

Stay tuned!

THANK YOU!