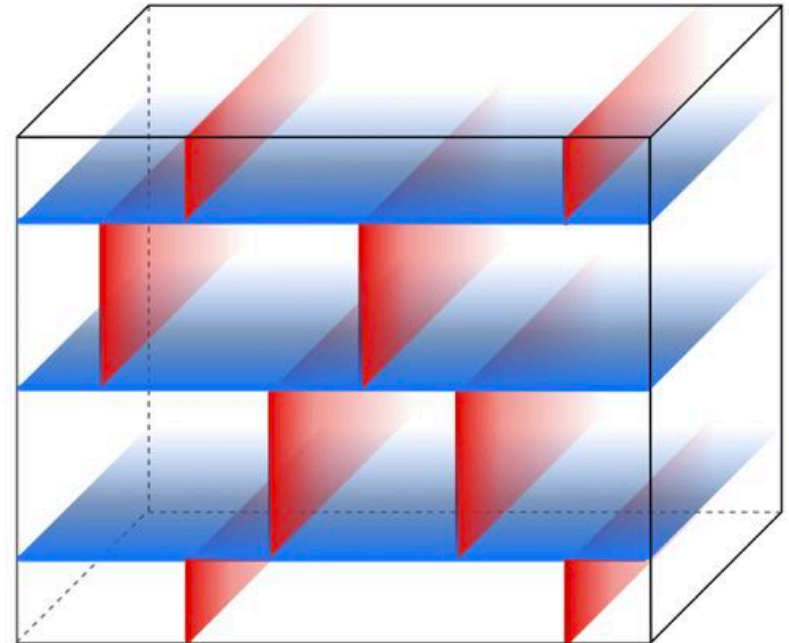


# The Amazing Super-Maze

Iosif Bena

IPhT, CEA

Université Paris-Saclay



with **Dimitrios Toulikas**, Anthony Houppe, Yixuan Li,  
Nejc Čeplak, Shaun Hampton and Nick Warner



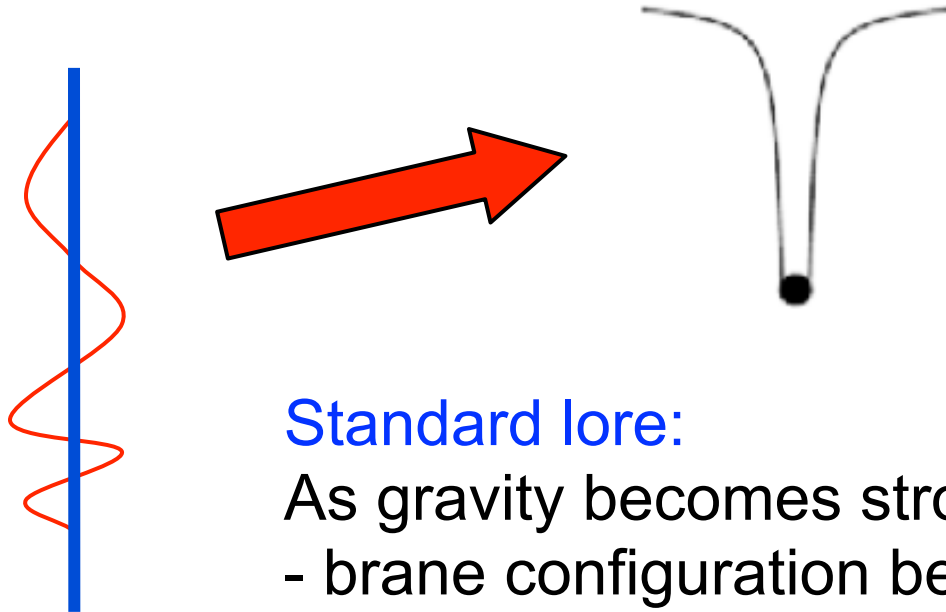
An amazing success of String Theory

*Count Black Hole Microstates* (branes + strings)

Correctly match B.H. entropy !!!

Zero Gravity

One Particular Microstate at **Finite Gravity**:



**Standard lore:**

As gravity becomes stronger,

- brane configuration becomes smaller
- horizon develops and engulfs it
- recover standard black hole

Susskind  
Horowitz, Polchinski  
Chen, Maldacena, Witten

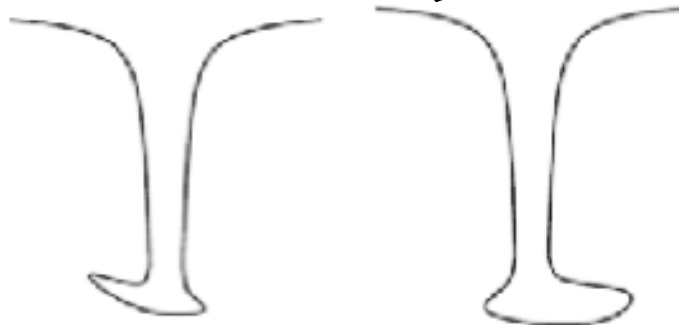
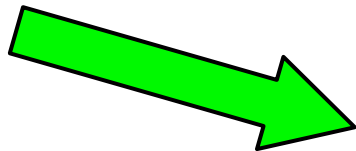
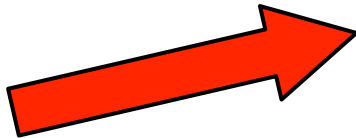
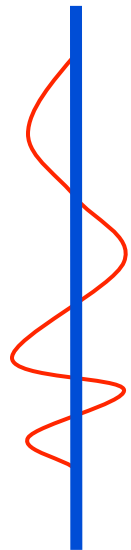
An amazing success of String Theory

*Count Black Hole Microstates* (branes + strings)

Correctly match B.H. entropy !!!

Zero Gravity

One Particular Microstate at **Finite Gravity**:



Identical to black hole far away.

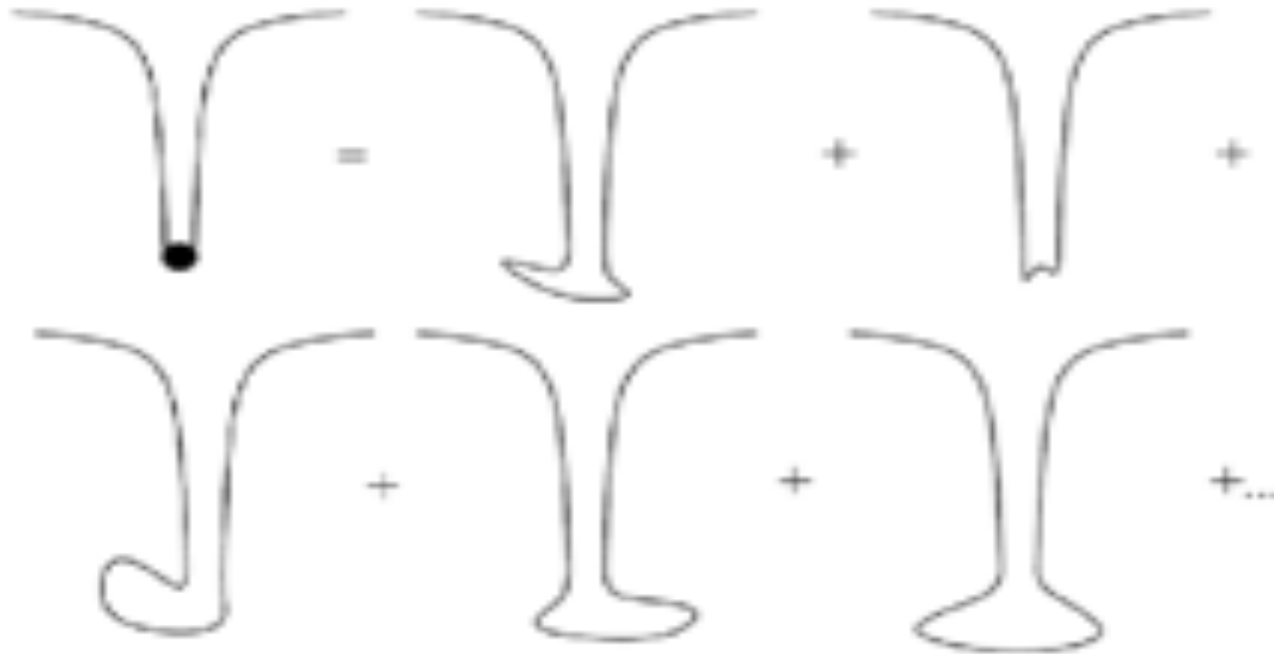
Horizon → Smooth cap

past 20 years

**BIG QUESTION:** Are *all* black hole microstates becoming geometries with no horizon ?

# Black hole = ensemble of horizonless microstate configurations

Mathur 2003



(Only?) *reasonable* way to solve the **Information Paradox**

Mathur 2009, Almheiri, Marolf, Polchinski, Sully 2012

Other options:

- **ER=EPR, Islands**  $\Rightarrow$  wormholes + nonlocalities at scales  $M_{BH}^3$   
(for solar-mass BH this is  $10^{80}$  m =  $10^{53} \times$  size of observable universe)

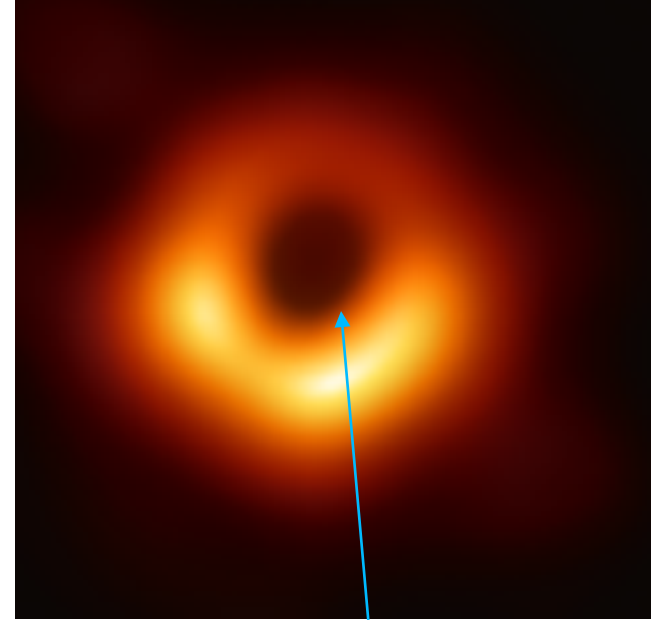
- **State-dependent operators** (non-Copenhagen)

Papadodimas, Raju

# Structure@horizon

in vogue these days  
(ECO)

- Gravastars
- Quark-stars
- Boson-stars
- Gas of wormholes (ER=EPR)
- Quantum Black Boxes
- BMS / Soft hair @ horizon
- Mirrors floating on Pixie Dust
- Modified gravity
- Bose-Einstein condensate of gravitons
- Infinite-density firewall hovering just above horizon



*Here Be Microstructure*

# But ...

# 1. Growth with $G_N \leftrightarrow$ BH size for **all** masses

Horowitz

- Normal objects shrink; BH horizon grows
- **microstate geometries** have BH size for all masses
- D-branes = solitons,  $m \sim 1/g_s$  lighter as  $G_N = g_s^2$  increases



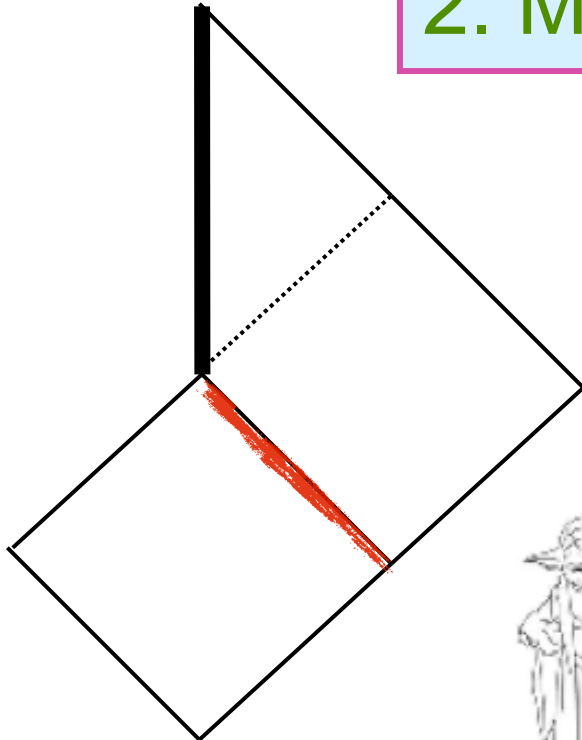
*To build structure@horizon, non-perturbative degrees of freedom you must use !*

## 2. Mechanism not to fall into BH

**GR Dogma:**

**Thou shalt not put anything at the horizon !!!**

- Null  $\rightarrow$  speed of light.
- **Normal stuff** falls in
- Cannot hide behind *quantum*



*If support mechanism have you not, wrong physics are you doing*

# Quantum Coyote Principle



**GRAVITY DOES NOT WORK  
`TILL YOU LOOK DOWN ....**



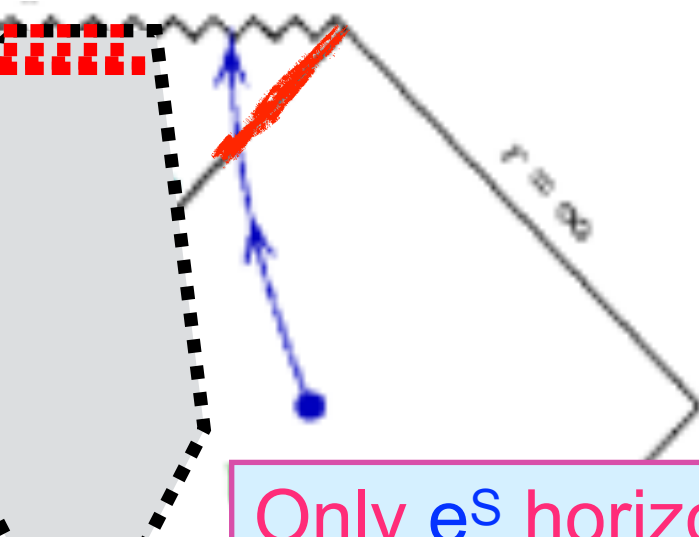
Such is the fate of  
*Firewalls, quantum black boxes, Mirrors & their brothers*



### 3. Avoid forming a horizon

- Collapsing shell forms horizon @ low curvature  
Oppenheimer and Snyder (1939)
- By the time shell becomes **curved-enough** for quantum effects to become **important**, horizon in causal past (180 hours for TON618 BH)

Backwards in time - **illegal** !



BH has  $e^S$  microstates with no horizon

Small tunneling probability =  $e^{-S}$

Shell tunnels with probability **ONE** !!!

Kraus, Mathur; Bena, Mayerson, Puhm, Vercocke

Only  $e^S$  horizon-sized microstates can do it !

*Black hole entropy the structure must have*

Rules out gravastars & almost everything else



# Supersymmetric Microstate Geometries:

- Only construction with 3 properties - 2.5 rather 😊
- **Largest** family of solutions known to mankind

Arbitrary fns. of **3** variables:  $\infty \times \infty \times \infty$  parameters !

Cohomogeneity-**5** !

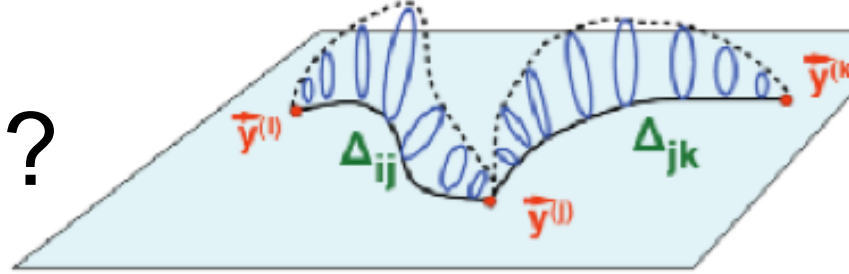
$$\begin{aligned}
 \omega_{\theta}^{(0)} &= \frac{1}{\sqrt{2}} d\theta + \sqrt{\frac{2}{\Sigma}} d\phi, \\
 \omega_{\psi}^{(0)} &= \frac{2}{\sqrt{2}} (dr + \beta) \left[ \frac{1}{2} (dr + \beta) + \frac{\Sigma}{2} (dr + \beta) \right] = \sqrt{2} d\psi, \\
 \omega_{\phi}^{(0)} &= \frac{2}{\sqrt{2}} d\phi, \\
 \omega_{\rho}^{(0)} &= \frac{2}{\sqrt{2}} \left[ \frac{1}{p} (dr + \omega) \wedge (dr + \beta) + q_1 \wedge (dr + \beta) + q_2 \right], \\
 \omega_{\tau}^{(0)} &= \frac{2}{\sqrt{2}} \left[ \frac{1}{p} (dr + \omega) \wedge (dr + \beta) + q_1 \wedge (dr + \beta) + \tau \right], \\
 \omega_{\nu}^{(0)} &= \frac{2}{\sqrt{2}} \left[ \frac{1}{p} (dr + \omega) \wedge (dr + \beta) + q_1 \wedge (dr + \beta) + \nu \right], \\
 \omega_{\mu}^{(0)} &= \frac{2}{\sqrt{2}} \left[ \frac{1}{p} (dr + \omega) \wedge (dr + \beta) + q_1 \wedge (dr + \beta) + \mu \right].
 \end{aligned}$$

$$\begin{aligned}
 \omega_{\theta}^{(1)} &= -\frac{Rr}{\sqrt{2}k_2(m_1^2 - 1)} \frac{m_1(k_2 + m_1 + 1)\Delta_{k_2+m_1-1, m_1-1} + (k_2 + m_1 - 1)\Delta_{k_2+m_1-1, m_1}}{(r^2 - \alpha^2)^2}, \\
 \omega_{\psi}^{(1)} &= \frac{R}{\sqrt{2}k_2(m_1^2 - 1)\alpha^2 \sin\theta \cos\theta} \left[ 2(m_1 - 1)\Delta_{k_2+m_1-1, m_1-1} \right. \\
 &\quad + (m_1 - 1)(m_1 - 2)\Delta_{k_2+m_1-1, m_1-1} + m_1(k_2 - 2)\Delta_{k_2+m_1-1, m_1-1} \\
 &\quad \left. - m_1(m_1 - 1)\Delta_{k_2+m_1-1, m_1-1} + (m_1^2(k_2 - 1) + 1)\Delta_{k_2+m_1-1, m_1-1} \right], \\
 \omega_{\phi}^{(1)} &= \frac{R}{\sqrt{2}} \frac{\Delta_{k_2+m_1-1, m_1-1}}{\Sigma} \sin^2\theta = \frac{R}{\sqrt{2}k_2(m_1^2 - 1)\alpha^2} \left[ 2(m_1 - 1)\Delta_{k_2+m_1-1, m_1-1} \right. \\
 &\quad + (m_1^2 - 2m_1 + k_2 - 1)\Delta_{k_2+m_1-1, m_1-1} + m_1(k_2 - 2)\Delta_{k_2+m_1-1, m_1-1} \\
 &\quad \left. + m_1(k_2 - m_1 - 1)\Delta_{k_2+m_1-1, m_1-1} + (k_2(m_1^2 + m_1 - 1) - m_1(m_1 + 1))\Delta_{k_2+m_1-1, m_1-1} \right], \\
 \omega_{\rho}^{(1)} &= \frac{R}{\sqrt{2}} \frac{\Delta_{k_2+m_1-1, m_1-1}}{\Sigma} \cos^2\theta = \frac{R}{\sqrt{2}k_2(m_1^2 - 1)\alpha^2} \left[ (k_2 - 1)(m_1 - 1)\Delta_{k_2+m_1-1, m_1-1} \right. \\
 &\quad \left. - 2(m_1 - 1)\Delta_{k_2+m_1-1, m_1-1} - (m_1 - 1)(m_1 - 2)\Delta_{k_2+m_1-1, m_1-1} \right. \\
 &\quad \left. + (m_1 - 1)(k_2 - 3)\Delta_{k_2+m_1-1, m_1-1} + m_1(m_1 - 1)\Delta_{k_2+m_1-1, m_1-1} \right. \\
 &\quad \left. + m_1(k_2 - 1)\Delta_{k_2+m_1-1, m_1-1} \right].
 \end{aligned}$$

## Habemus Superstratum !!!

Many features of **typical** microstates: **mass gap** =  $\frac{1}{N_1 N_5}$

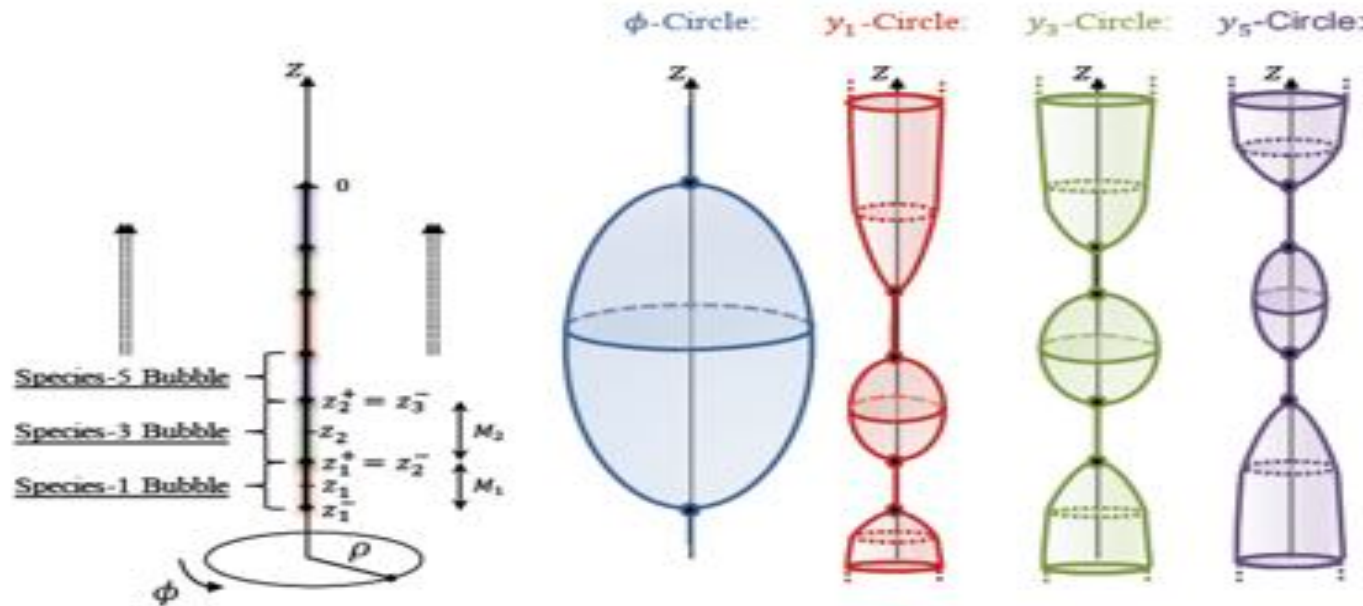
# Why not collapsing ?



- 5(+6)d : smooth solutions + **quantized** magnetic flux on topologically-nontrivial **2-cycles**
  - cycles smaller  $\rightarrow$  increases energy
  - bubbling = **only** mechanism to avoid collapse Gibbons, Warner

Works for nonextremal black holes as well

Bah, Heidmann



**First  
Schwarzschild  
microstates !!!**

Bah, Heidmann, Weck '22

# 20 years of microstate geometries

- Huge number of smooth horizonless solutions
  - Bubbling geometries, superstrata
  - Largest known class of solutions to Einstein's equations
  - Many features of **typical** microstates (mass gap)
  - $S \sim (Q_1 Q_5)^{1/2} (Q_p)^{1/4} < S_{\text{BH}} \sim (Q_1 Q_5 Q_p)^{1/2}$  Mayerson, Shigemori '20
- Link with D1-D5 states that count BH entropy ?
  - Only known for a few solutions
  - ***Needs Elvish Medicine*** (precision holography)
  - momentum modes giving D1-D5 BH entropy are quantized in units of  $1/R_y N_1 N_5$  - ***fractionated***
  - Duals of states with fractionated momentum carriers are very hard to build in supergravity

Bena, Martinec, Turton, Warner '16; Shigemori '21, '22

# The Painful Reality

- We have **not** succeeded to track *typical* D1-D5 Strominger-Vafa microstates from the **zero-gravity regime** to the **finite-gravity regime** where BH exists
- *Fundamental* limitation or *technical* problem ? we can only build superstrata as fibrations on  $\mathbb{R}^4$  **base**
- Bubbling solutions - more general hyper-Kähler base
  - but no holographic dual
  - superstrata-building techniques fail
  - most generic base - not even hyper-Kähler
  - fractionated modes - missing magical ingredient ?

**Do not pray to the saint who  
does not help you !**

Romanian proverb

# Instead of D1-D5 look at D2-D4 (or F1-NS5 in type IIA)

One F1 inside  $N_5$  NS5 branes  $\rightarrow N_5$  little strings.

Dijkgraaf, Verlinde, Verlinde

- Visible as **M2 brane strips** in M-theory
- **Total**  $N_1 N_5$  independent **momentum carriers**
- each has **4 oscillation directions** ( $T^4$ ) + **4 fermionic partners**

$$S = 2\pi \sqrt{\frac{4+2}{6} N_1 N_5 N_p} = S_{BH}$$

**M2** along y, 11

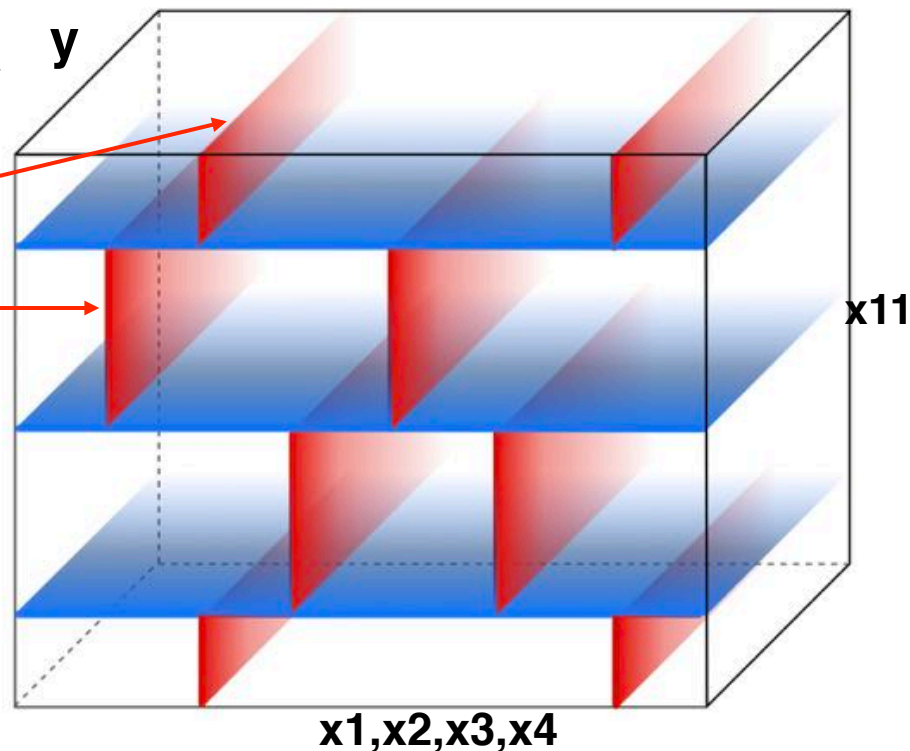
**M5** along y, 1234

P along y

D1-D5: fractionated P

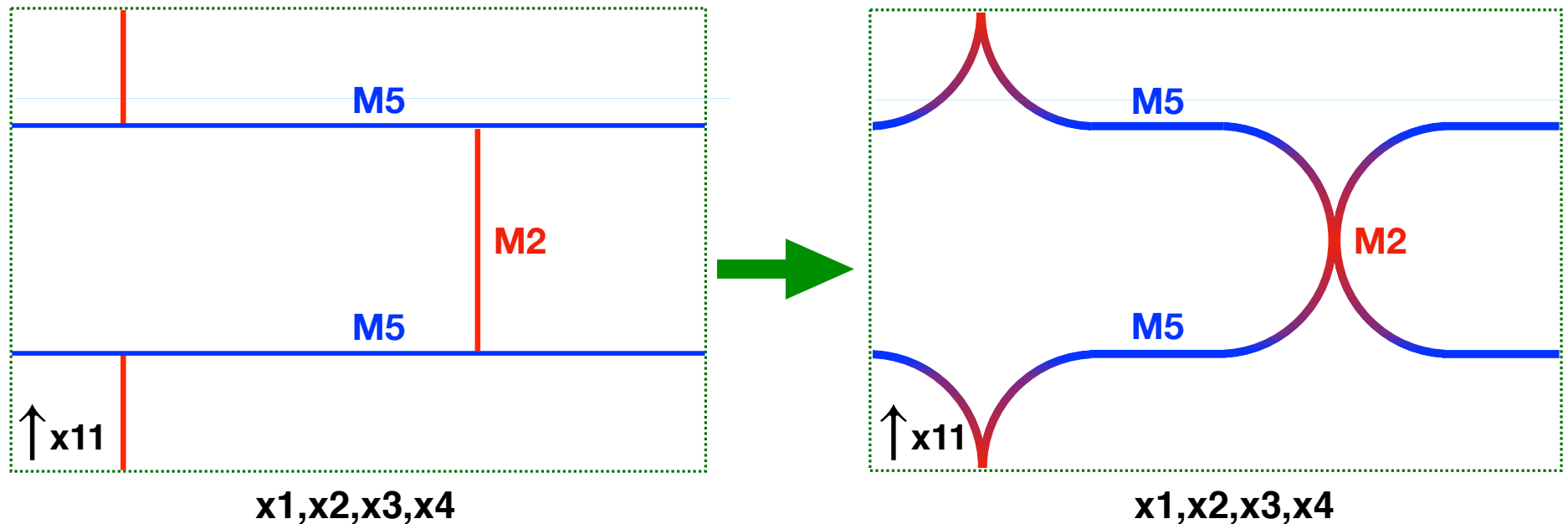
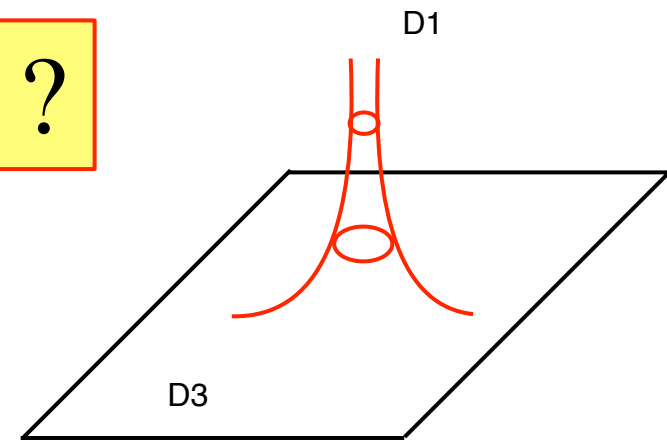
F1-NS5: fractionated F1

zero-coupling picture

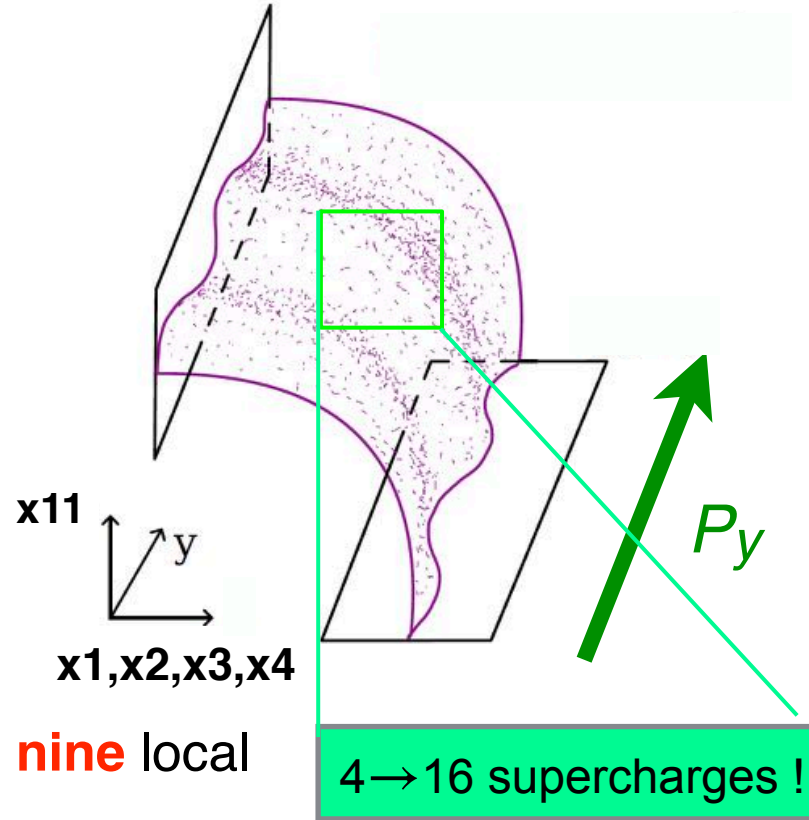
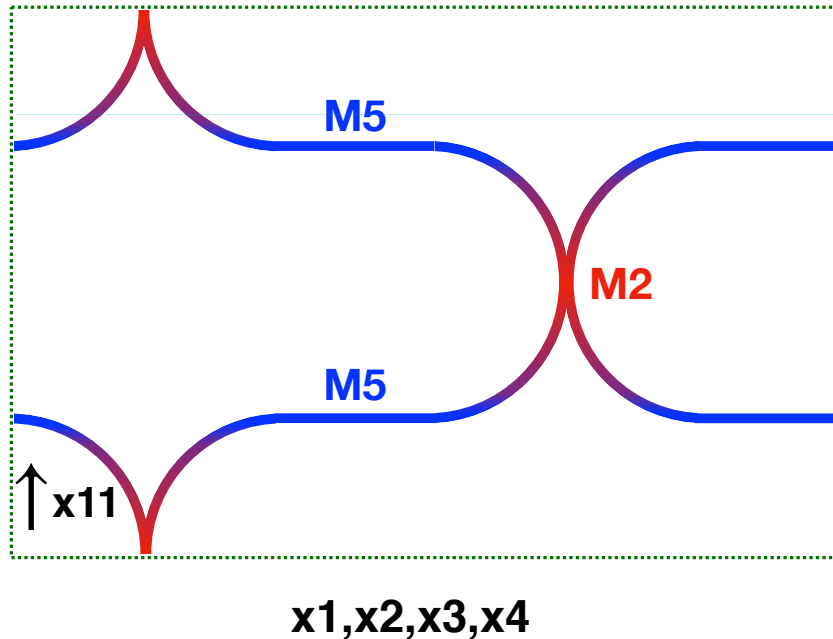


# What about finite coupling ?

- Reminder:  
*Callan-Maldacena spike* formed by D1 pulling on an orthogonal D3
- M2 branes also pull on the M5 branes



Except that the spike is a *furrow* carrying momentum waves along  $y$



Zoom in on the furrow carrying momentum: **nine** local brane charges:  $M2_{x_{11}, y}$   $M5_{y, x_1, x_2, x_3, x_4}$   $P_y$

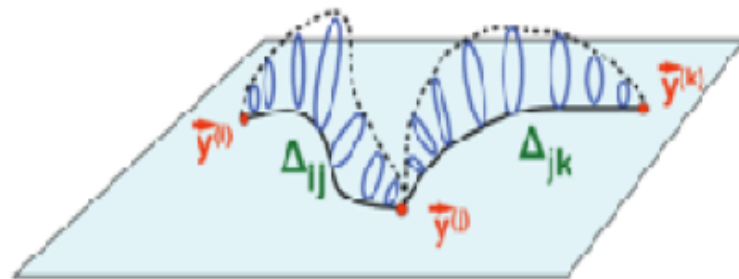
$M2_{x_1, x_{11}}$   $M5_{x_{11}, y, x_2, x_3, x_4}$   $M2_{x_1, y}$   $M5_{x_{11}, x_1, x_2, x_3, x_4}$   $P_{x_{11}}$   $P_{x_1}$

4  $\rightarrow$  16 supercharges !

Smoking gun of smooth horizonless solutions

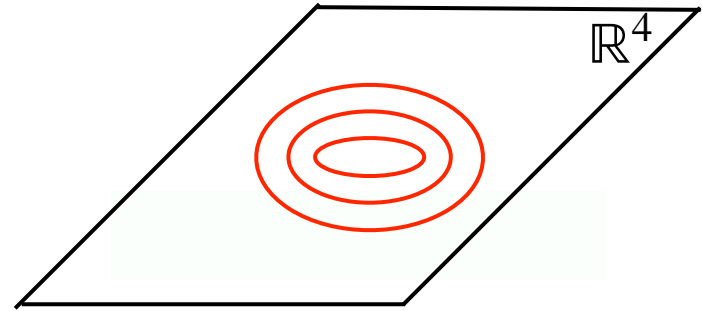
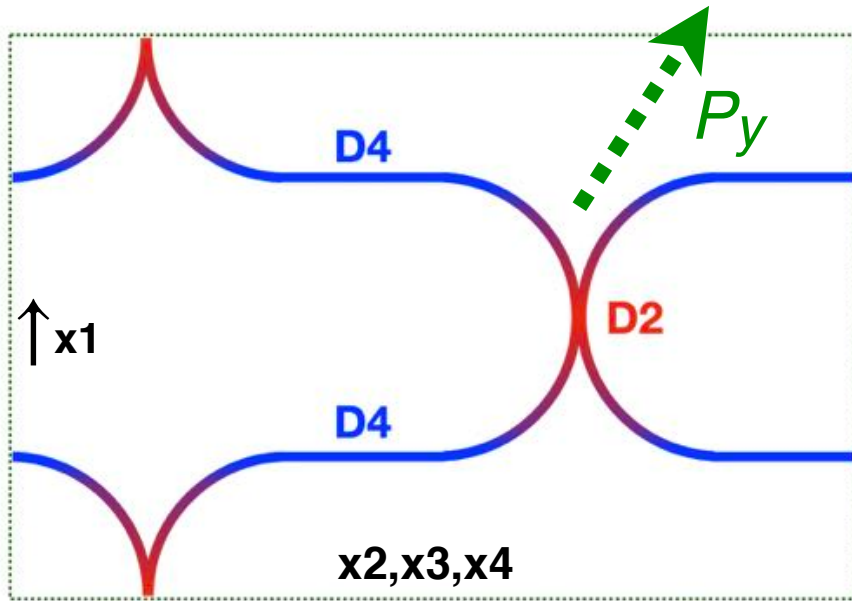


# Some history



- First microstate geometries
  - Bubbling solutions with GH centers. Bena, Warner '06
  - Smooth in all duality frames. Horizonless
  - Multicenter fluxed D6 branes Balasubramanian & al '06
  - **16 susy** at every center, **4 globally**
  - Entropy much smaller than BH de Boer & friends
- Microstate geometries with supertubes
  - Functions of one variable Bena, Bobev, Giusto, Ruff, Warner '10
  - Smooth  $\Leftrightarrow$  **16 susy** when zooming on supertube
- Superstrata. conjectured in Bena, de Boer, Shigemori, Warner '11
  - Fns. of 2 variables; **16 susy locally**, **4 globally**
  - **HABEMUS**: Smooth. Bena, Giusto, Russo, Shigemori, Warner '15
- Pattern: **smooth horizonless sols  $\Leftrightarrow$  brane configurations: 16 susy locally, 4 globally**

# Super-Maze entropy



spherically symmetric in  $\mathbb{R}^4$  ( $x_5, x_6, x_7, x_8$ )  
 same **spacetime  $SO(4)$**  symmetry as BH

**$SO(4)$  invariant solutions:**

momentum carried by **waves** on fractionated strings (inside  $T^4$ ) =

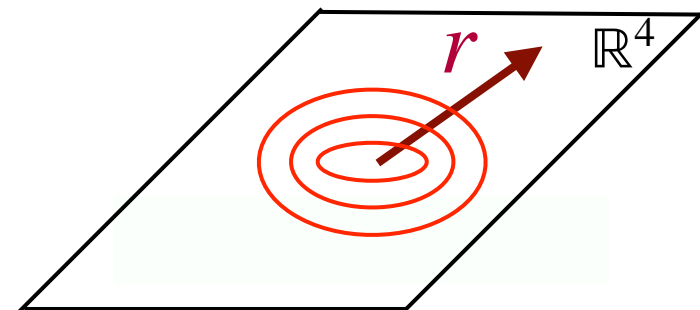
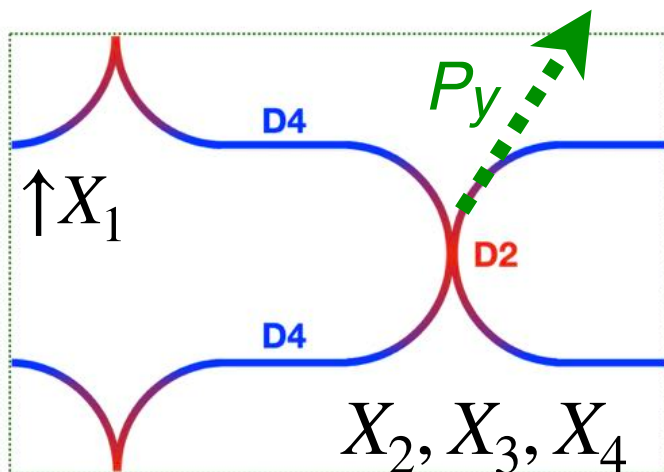
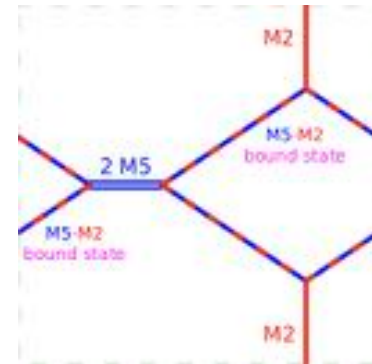
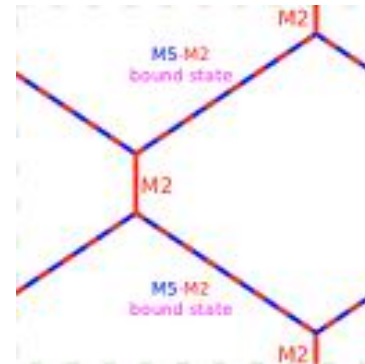
**bosonic** d.o.f. :  $S_{bosonic} = 2\pi\sqrt{\frac{4}{6}N_1N_5N_p}$

+ 2 **fermionic** d.o.f. preserving  $SO(4) \Rightarrow S_{SO(4) \text{ invariant}} = 2\pi\sqrt{\frac{5}{6}N_1N_5N_p}$

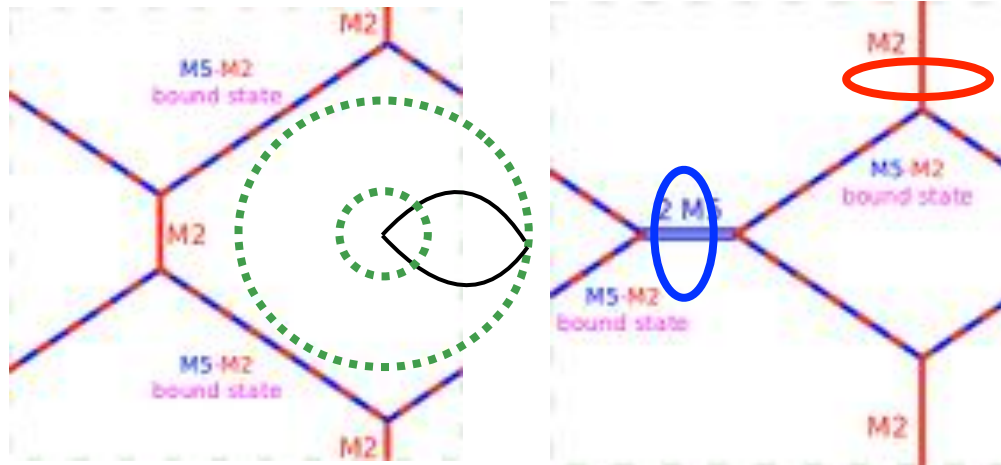
Remaining 2 **fermionic** d.o.f. **break  $SO(4)$**   $\Rightarrow S_{SO(4) \text{ breaking}} = 2\pi\sqrt{\frac{1}{6}N_1N_5N_p}$

# How will the $SO(4)$ -invariant solution look like ?

- **Two-charge solutions:**
- Monge-Ampère equation
- solution at least *cohomog-3*
- smeared on  $T^3 \Rightarrow$  string web:
- Singular brane sources  $\Rightarrow$  solution exists (singular)
- **Three-charge solutions** with  $D2_{y_1} + D4_{y_{234}} + P_y$  at least *cohomogeneity-4*  $(X_1, X_2, r, y)$



# How will the $SO(4)$ -invariant solution look like ?



- *16-susy locally*  $\Rightarrow$  *no horizon*
- Branes wrapping compact contractible cycles  $\Rightarrow$  **Geometric transition**  $\Rightarrow$  Bubbles wrapped by fluxes on internal dimensions.
- Smooth bubbling sources: can we construct it ?
- can we show in principle that the solution exists ?
- Expectation based on earlier work:
  - **backreaction** will make bubbles **large**
  - *irrespective* of  $T^4$  size at infinity

# Holography of $SO(4)$ -invariant solutions

- Microstate geometry differs from BH by  $T^4$  KK modes:
- Asympt.  $\mathbb{R}^{4,1} \times S^1 \times T^4$  : *exponentially-decay*
- Asympt.  $AdS_3 \times S^3 \times T^4$  : *power-law decay*
  - High-dimension operators:  $\Delta^2 \sim Q_5 n_{\text{mode}}^2 / L_1^2$
  - Official '97 Dogma: *not surviving in decoupling limit*
  - *Νέα θεολογία*: anything asymptotic to  $AdS_3 \times S^3 \times T^4 \in CFT$  & can tunnel to anything else
  - Operator dimension depends on  $T^4$  moduli. SUSY?
  - Is operator visible at free-orbifold point ?
  - Can CFT distinguish different supermaze solutions ?

# How will the **generic** solution look like ?

- Generic microstates will contain  
 $SO(4)$  breaking modes +  $SO(4)$  invariant ( $T^4$  modes)

## 2-charge systems revisited:

- when both  $T^4$  and  $SO(4)$  breaking modes are present
- $S_{\text{total}} = 2\pi\sqrt{2N_1N_5}$
- Smearing on  $T^4$  does not lose info. Can get  $S_{\text{total}}$  from  $T^4$ -invariant solutions Kanitscheider, Taylor, Skenderis
- If only  $T^4$  dependent modes present:
- $S_{SO(4) \text{ invariant}} = 2\pi\sqrt{N_1N_5}$
- smearing on  $T^4$  erases information  $\Rightarrow$  one obtains the naïve D1-D5 solution: singular, small horizon

# How will the **generic solution** look like ?

## 3-charge story ?

- $SO(4)$ -breaking strands:  $(+,+),(-,-),(+,-),(-,+)$
- $T^4$ -dependent strands:  $(\dot{a}b + \dot{b}a), \dot{a}a, \dot{b}b, (\dot{a}b - \dot{b}a) = (00)$
- **Superstrata** = 6D supergravity solutions **smear**ed on  $T^4$ 
  - When  $SO(4)$ -breaking  $(++)$  strands are present, superstrata can capture  $T^4$  strands:  $(00)$
  - When no  $(++)$  strands are present, superstrata collapse into naïve solution with a horizon

**We get horizons only when smearing too much**

- **Q1:** Could the presence of  $SO(4)$ -breaking modes in generic supermaze allow  $T^4$  smearing without info loss ?
- **Q2:** Would  $T^4$ -dependent supermaze information be lost upon smearing, even when  $SO(4)$ -breaking modes exist ?

# How will the **generic solution** look like ?

## Big fat 3-charge generic beast ?

Combination of  $SO(4)$ -breaking modes and  $T^4$ -dependent modes

### **Themelia:**

General idea:

**Global charges**

dipole charges = **Glue**

needed for 16 susy

| Object              | Coefficient                  |       | Object               | Coefficient                  |       |
|---------------------|------------------------------|-------|----------------------|------------------------------|-------|
| F1(y)               | $\alpha_1$                   | $x_1$ | F1( $\psi$ )         | $\alpha_5$                   | $x_2$ |
| NS5(y1234)          | $\alpha_2$                   |       | NS5( $\psi$ 1234)    | $\alpha_6$                   |       |
| P(y)                | $\alpha_3$                   |       | P( $\psi$ )          | $\alpha_7$                   |       |
| KKm(y1234; $\psi$ ) | $\alpha_4$                   | $z_1$ | KKm( $\psi$ 1234;y)  | $\alpha_8$                   | $z_2$ |
| D2(y1)              | $\alpha_9$                   | $u_1$ | D2( $\psi$ 1)        | $\alpha_{11}$                | $u_2$ |
| D4(y234)            | $\alpha_{10} = -\alpha_9$    |       | D4( $\psi$ 234)      | $\alpha_{12} = -\alpha_{11}$ |       |
| D0                  | $\alpha_{13}$                | $v_1$ | D2(y $\psi$ )        | $\alpha_{15}$                | $v_2$ |
| D4(1234)            | $\alpha_{14} = -\alpha_{13}$ |       | D6(y $\psi$ 1234)    | $\alpha_{16} = -\alpha_{15}$ |       |
| F1(1)               | $\alpha_{17}$                | $w_1$ | NS5(y $\psi$ 234)    | $\alpha_{19}$                | $w_2$ |
| P(1)                | $\alpha_{18} = -\alpha_{17}$ |       | KKm(y $\psi$ 234; 1) | $\alpha_{20} = -\alpha_{19}$ |       |

$$\begin{aligned}
 u_1 + iu_2 &= s_1 s_2 e^{i\varphi_1}, \\
 v_1 + iv_2 &= s_2 c_2 e^{i(\varphi_1 - \varphi_2 - \varphi_3)} (e^{-2i\varphi_4} - c_1) \\
 w_1 + iw_2 &= s_1 c_2 e^{i\varphi_2}, \quad x_1 + ix_2 = c_1 e^{i\varphi_3} \\
 y_1 + iy_2 &= e^{i(2\varphi_2 + \varphi_3)} (c_1 c_2^2 + s_2^2 e^{-2i\varphi_4}), \\
 z_1 + iz_2 &= e^{i(2\varphi_1 - \varphi_3)} (c_2^2 e^{2i\varphi_4} + c_1 s_2^2),
 \end{aligned}$$

**Most generic beast with 16 supercharges locally**



**The big hope:** Track **each and every** BH microstate from zero-gravity regime to fully-backreacted solution

DVV microstates

$$S = S_{\text{BH}}$$

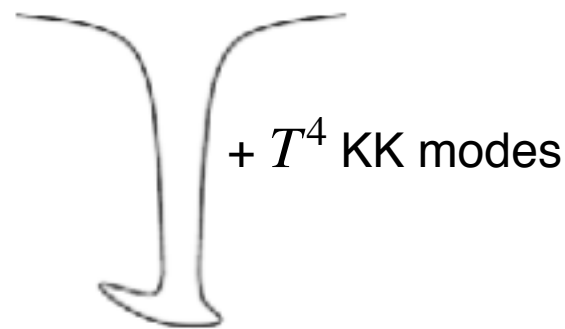
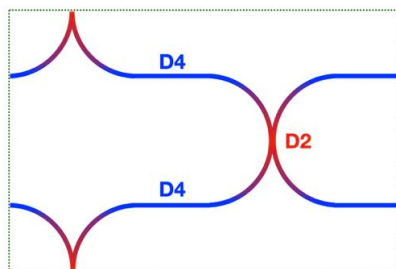
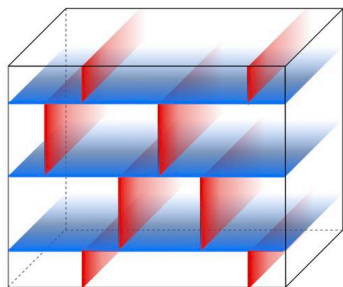
**SUPERMAZE**

branes pull & merge  
16 susy locally !

New Microstate Geometries

$$S = S_{\text{BH}}$$

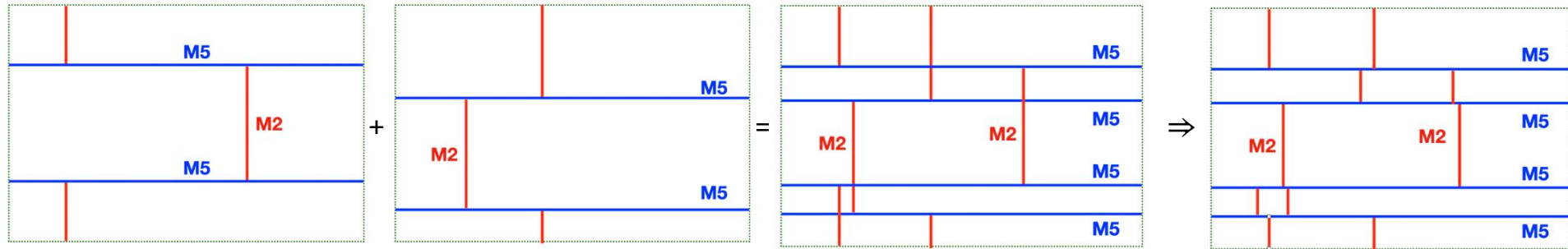
Effective coupling ( $g_s$ )



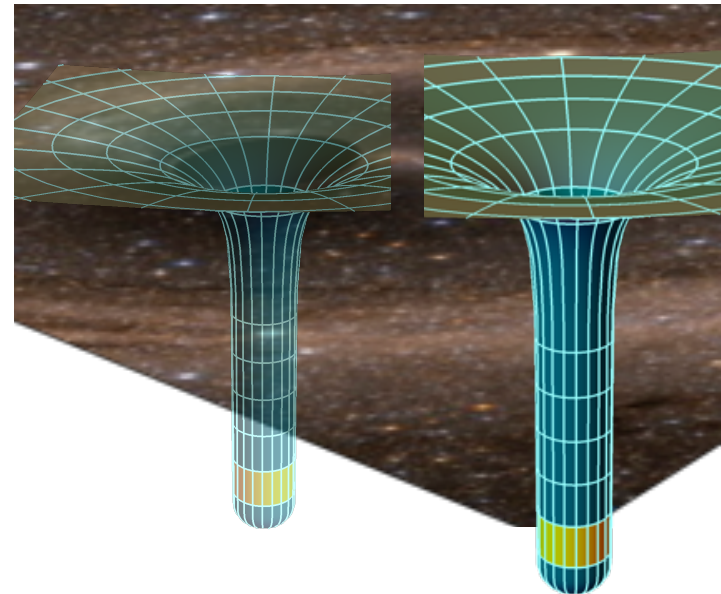
- Need to build supergravity solution !
- Precision holography for supermaze with  $T^4$ -dependent modes ?  
 $\langle \Psi_{\text{supermaze}} | \mathcal{O}_{T^4\text{-dependent}} | \Psi_{\text{supermaze}} \rangle \neq 0$
- Most generic beast: is 6D sugra enough? or one needs 10D?
- Flat space: supermaze fields decay exponentially. Universal ?

# How to black holes merge ?

- **GR Dogma:** horizons join, new horizon forms, irreversible
- *Νέα θεολογία* : microstates - KK modes/internal directions



- Some of these modes shed off
- KK charge = 0  $\Rightarrow$  gravity
- KK charge  $\neq$  0  $\Rightarrow$  Stand. Model
- *Electromagnetic counterpart ?*
- *Experimental constraints?*
- Calculate for 2-charge





After 20 years

# A NEW HOPE

The Supermaze

**Stay tuned for the supergravity solution  
and the new holographic insights**