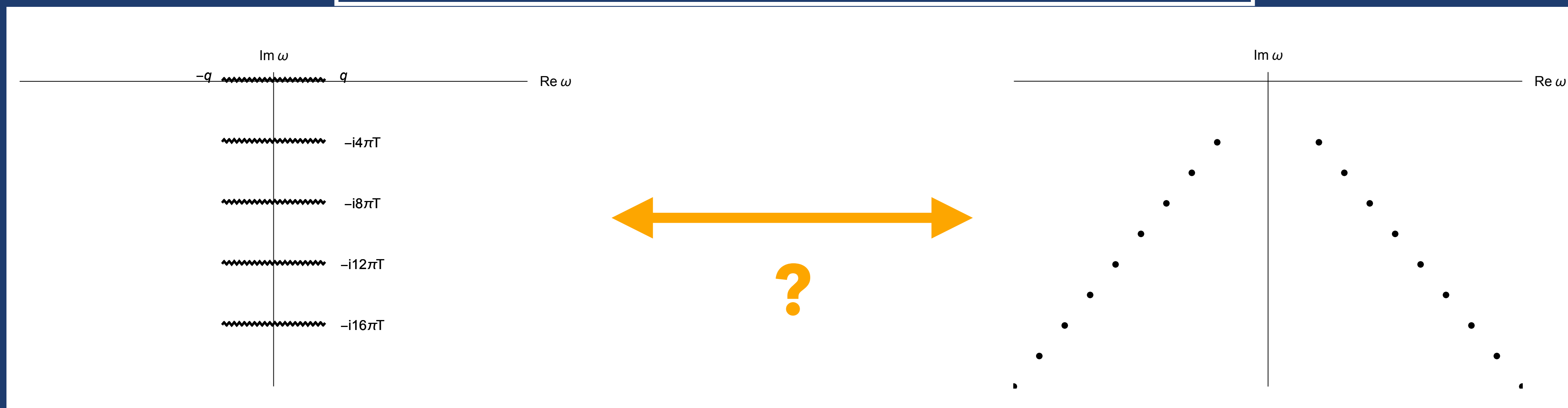
SAŠO GROZDANOV

SPECTRA OF THERMAL HOLOGRAPHIC CORRELATORS AND THEIR RECONSTRUCTIONS FROM POLE-SKIPPING

ICTP, 25.10.2023

ANALYTIC STRUCTURE OF CORRELATORS

$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R \sim \frac{b(\omega,q)}{a(\omega,q)}$$



$\lambda \to 0$

[Hartnoll, Kumar, (2005)]

$\lambda \to \infty$

holography

what is the structure of thermal correlators and what is the minimal information necessary to determine them completely

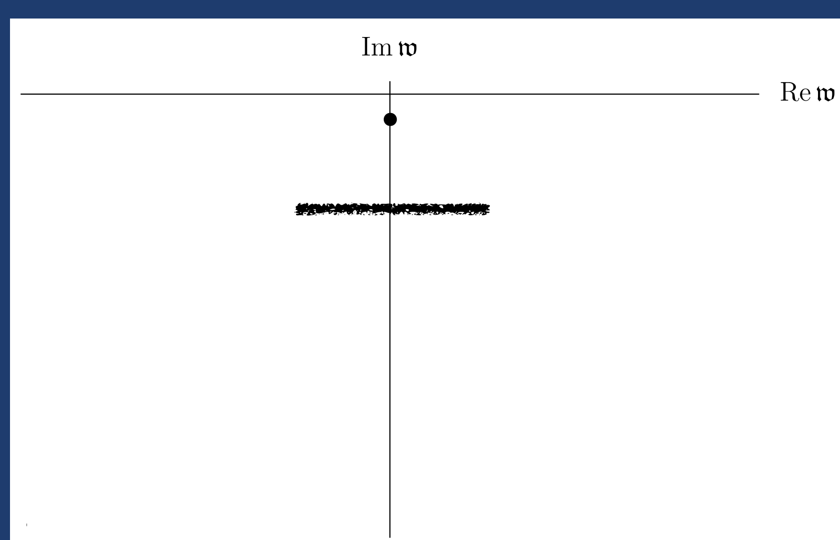
why do we like holography?
simplification of calculations in large-*N* theories;
instead of infinite # of Feynman diagrams, we solve ODEs

ANALYTIC STRUCTURE OF CORRELATORS

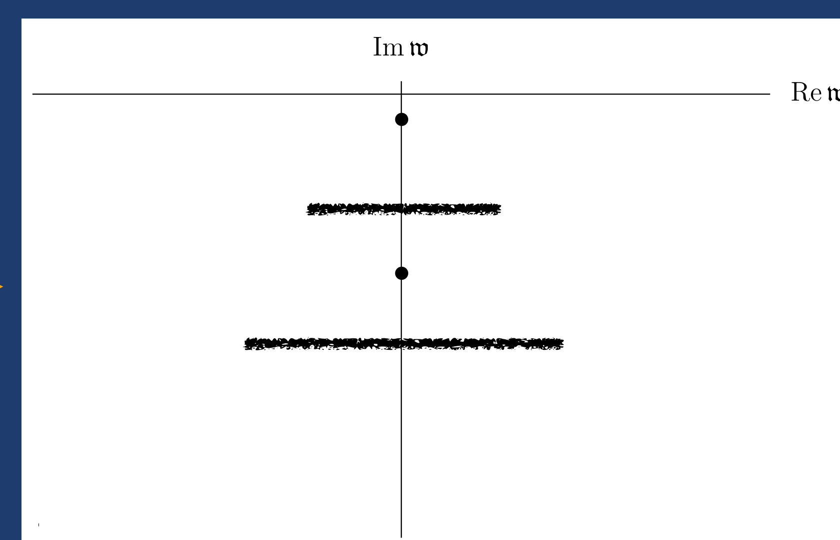
$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R \sim \frac{b(\omega,q)}{a(\omega,q)}$$

$\lambda \to 0$ $\lambda \to \infty$

poles from a cut?



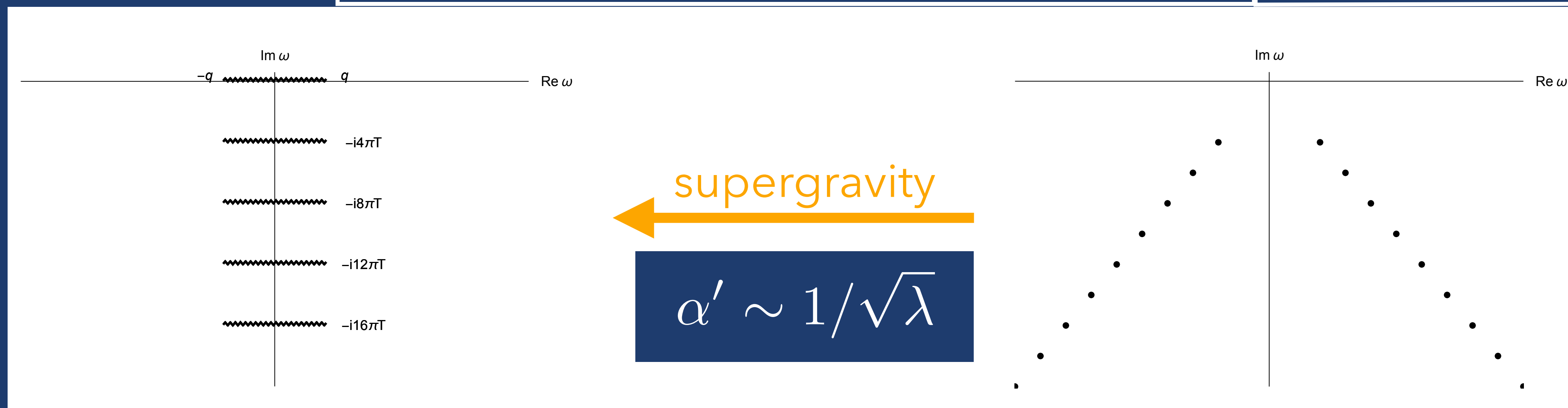
Boltzmann equation – RTA
[Romatschke, (2016)]



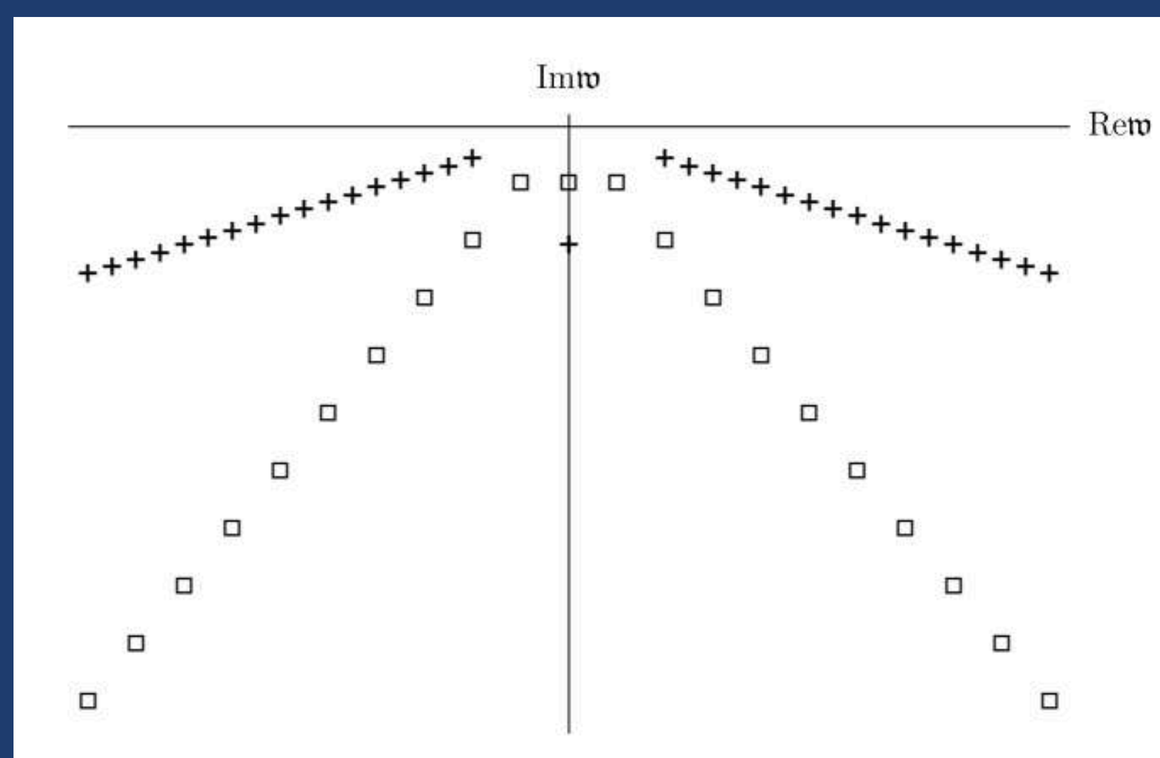
BBGKY hierarchy – RTA-like
truncations [SG, Soloviev, *wip*]

ANALYTIC STRUCTURE OF CORRELATORS

$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R \sim \frac{b(\omega,q)}{a(\omega,q)} = \frac{B(\omega,q)}{\prod_{i=0}^{\infty}(\omega-\omega_i(q))}$$



$\lambda \to 0$ | cut from poles | $\lambda \to \infty$



[SG, Starinets …, several papers]

ANALYTIC STRUCTURE OF CORRELATORS

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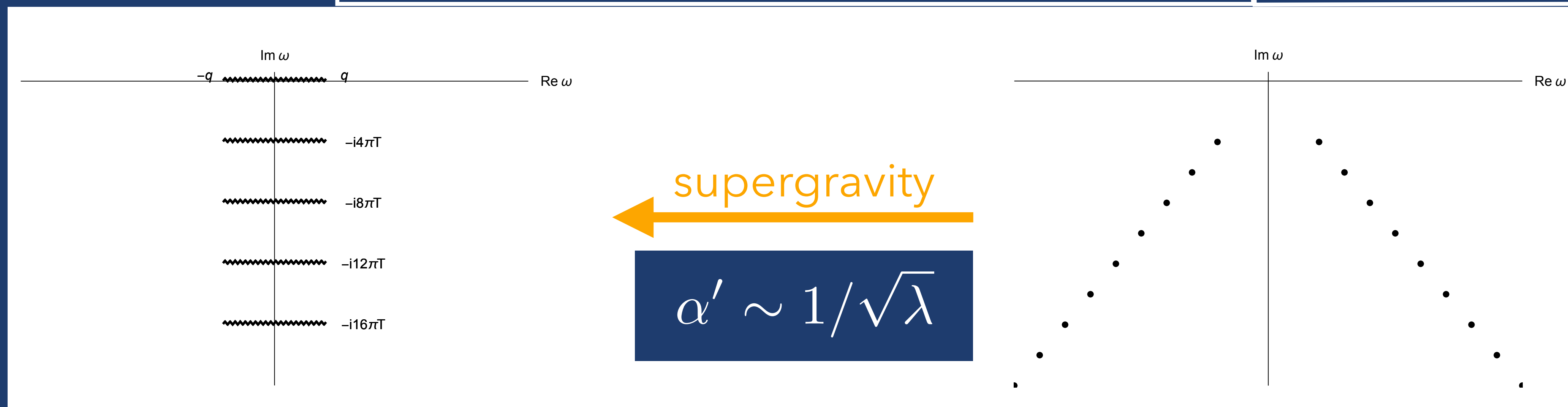
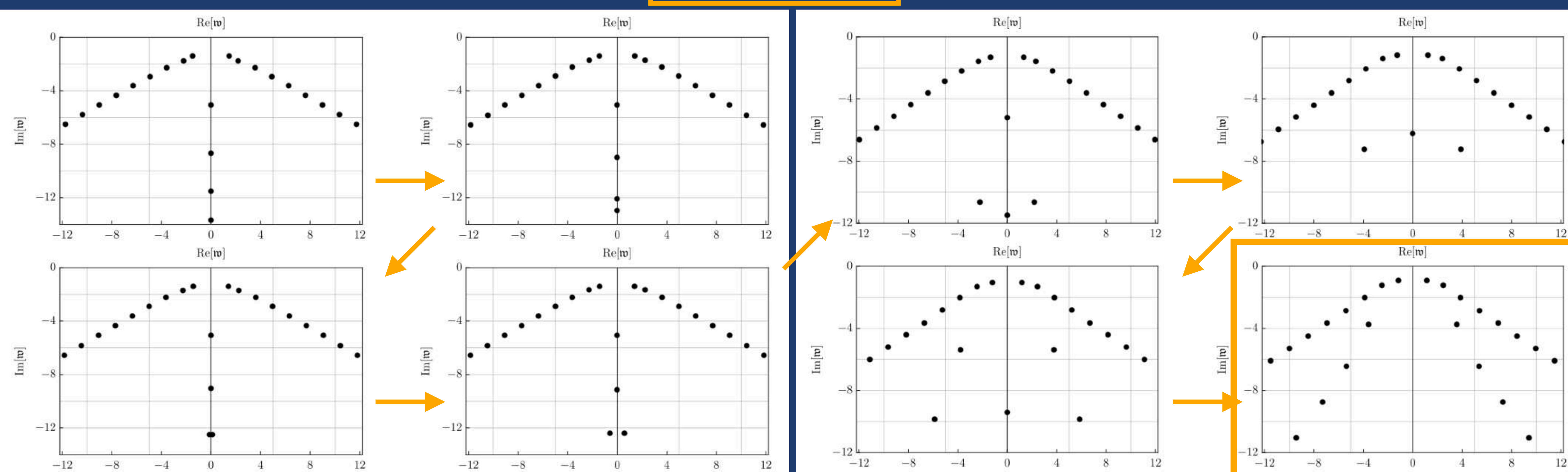


$\lambda \to 0$ cut from poles $\lambda \to \infty$



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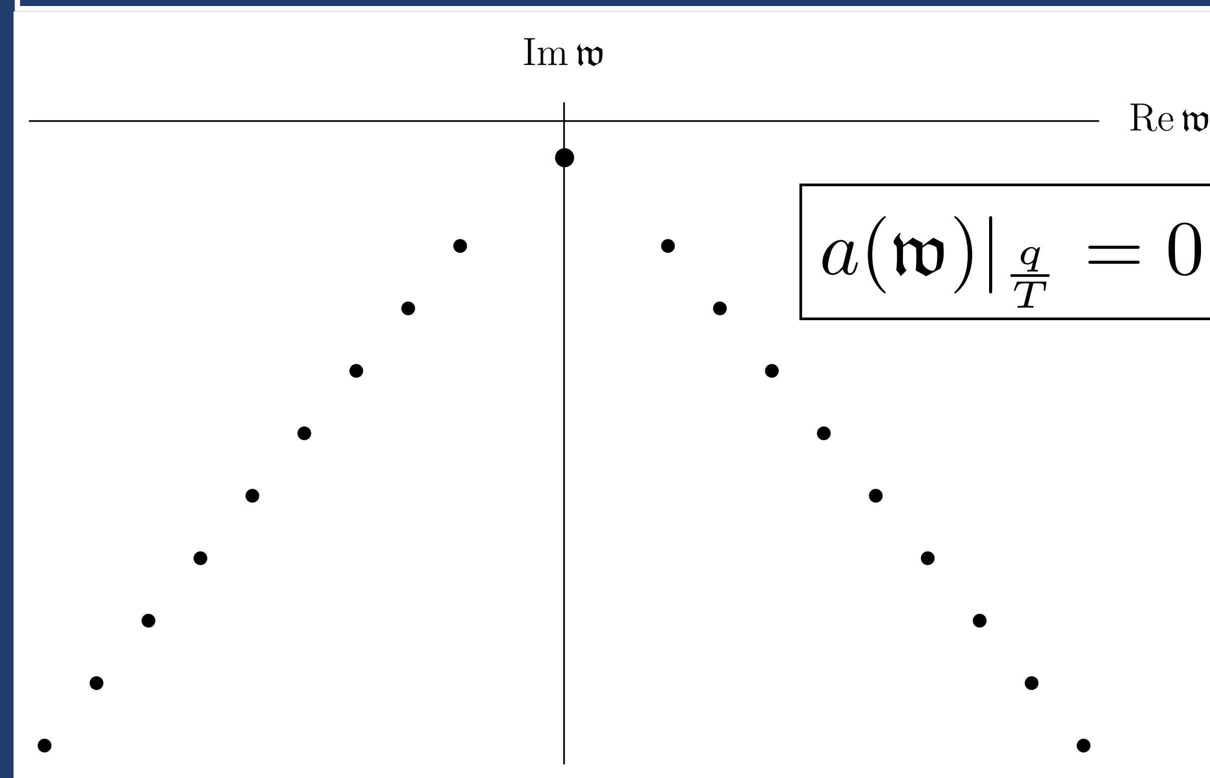

meromorphic momentum space correlator

quantum field theory

spectra of linear non-Hermitian operators

quasinormal mode spectrum of black holes

zeros of (algebraic) equations

ANALYTIC STRUCTURE OF CORRELATORS

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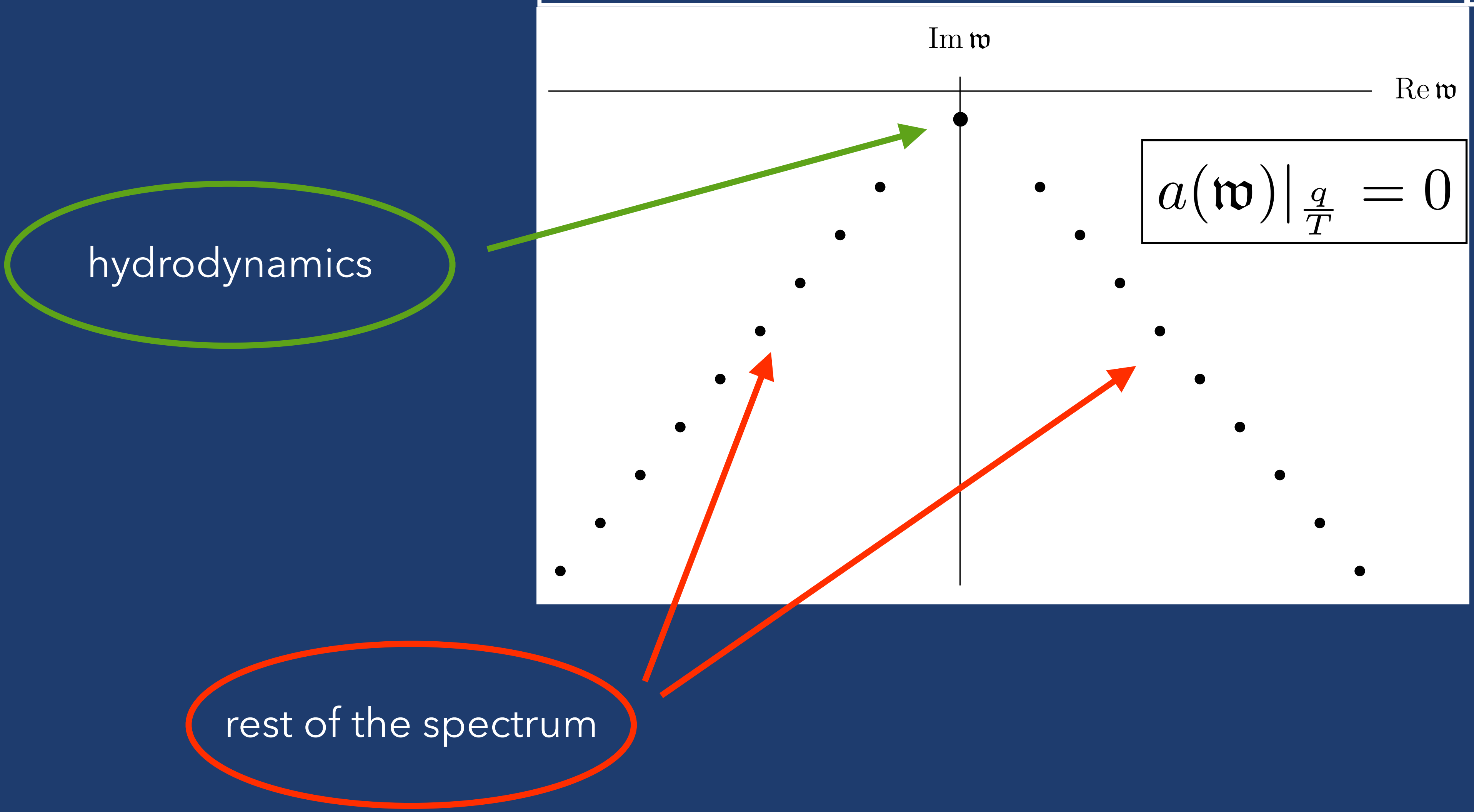

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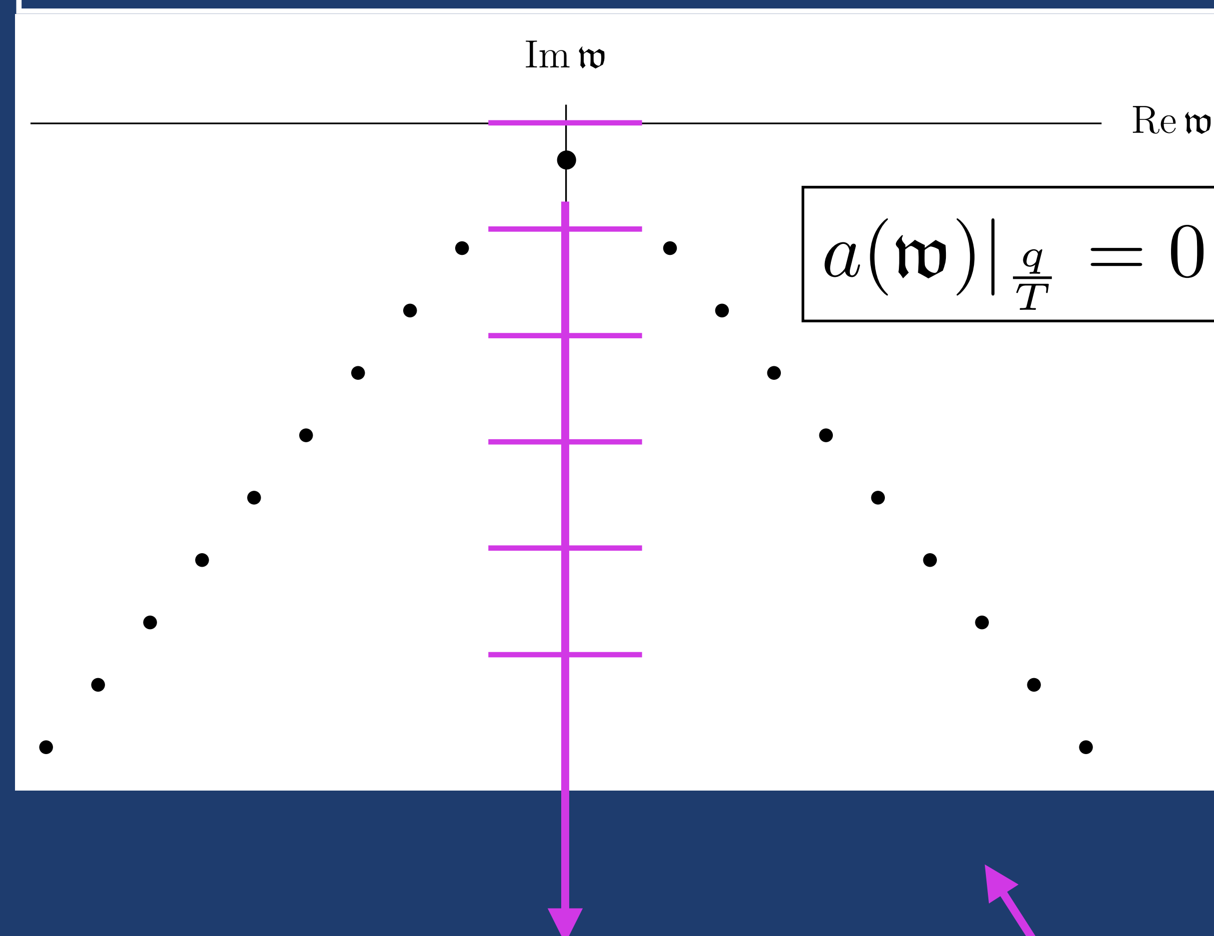


pole-skipping

reconstruction from pole-skipping

OUTLINE OF THE REST OF THE TALK

- hydrodynamics: holomorphic dispersion relations
- reconstruction of spectra
- pole-skipping and the reconstruction
- summary and future directions

HYDRODYNAMICS

HYDRODYNAMICS

- low-energy limit of QFTs – a Schwinger-Keldysh effective field theory
  [SG, Polonyi (2013); Crossley, Glorioso, Liu (2015); Haehl, Loganayagam, Rangamani (2015); …]

- conservation laws (equations of motion) of globally conserved operators

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu J^\mu = 0 \qquad \dots \nabla_\mu J^{\mu\nu} = 0$$

higher-form currents in MHD [SG, Hofman, Iqbal, PRD (2017)]

- tensor structures (symmetries, gradient expansions) and transport coefficients (QFT)

$$T^{\mu\nu} = \sum_{n=0}^{\infty} \left[\sum_i^N \lambda_i^{(n)} \mathcal{T}^{\mu\nu}_{(n)} \right] \quad \xrightarrow[u^\mu \sim T \sim e^{-i\omega t + iqz}]{\nabla_\mu T^{\mu\nu} = 0} \quad \omega(q) = \sum_{n=1}^{\infty} \alpha_n q^n$$

$$\partial u^\mu \sim \partial T \ll 1 \quad \longrightarrow \quad \omega/T \sim q/T \ll 1$$

equilibrium temperature

$$q = \sqrt{\mathbf{q}^2}$$

- dispersion relations:

shear diffusion	sound
$\omega = -iDq^2$	$\omega = \pm v_s q - i\Gamma q^2$

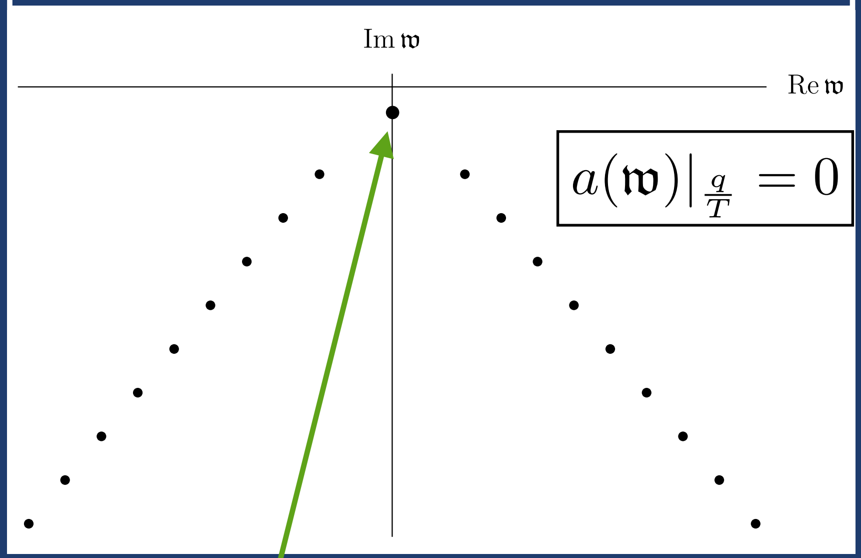
HYDRODYNAMICS FROM HOLOGRAPHY

- duality: theory *A* = theory *B*
- a result of string theory (quantum gravity) [Maldacena (1997)]

strongly coupled quantum theory (extremely hard) = *weakly coupled gravity* (much easier)

- perturbations of black holes (quasinormal modes) give spectra of QFT operators for $\mathfrak{w} \equiv \frac{\omega}{2\pi T} \in \mathbb{C}$
- large-*N* QFT calculations –> ODEs (usually Heun)
- invaluable explicit (toy) models: the $\mathcal{N}=4$ supersymmetric Yang-Mills theory [SG, Kovtun, Starinets, Tadić, JHEP (2019)]

$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R \sim \frac{b(\omega,q)}{a(\omega,q)}$$



sound:

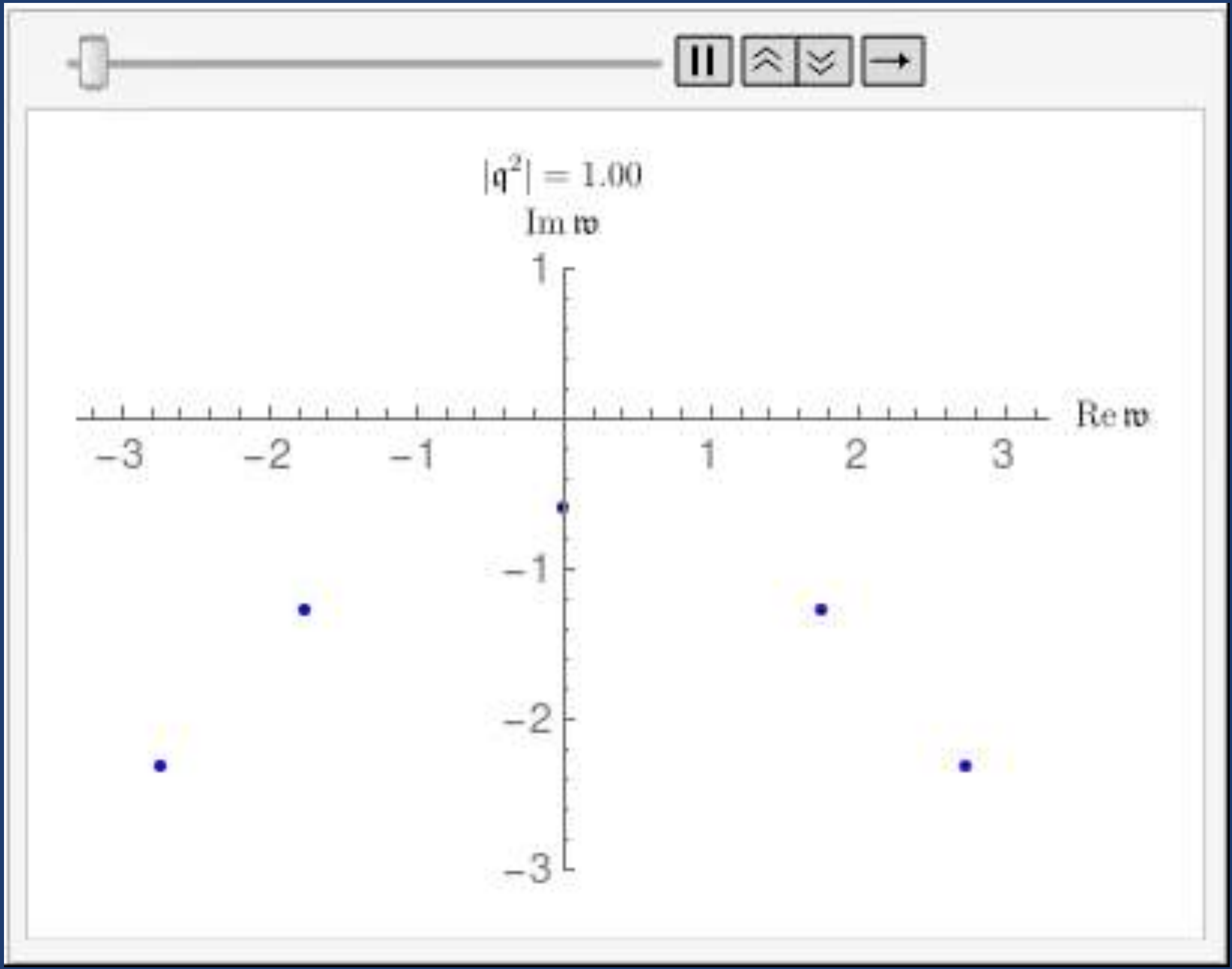
$$\omega = \pm\frac{1}{\sqrt{3}}q - \frac{i}{6\pi T}q^2 \pm \frac{3-2\ln 2}{24\sqrt{3}\pi^2 T^2}q^3 - \frac{i\left(\pi^2 - 24 + 24\ln 2 - 12\ln^2 2\right)}{864\pi^3 T^3}q^4 \pm \cdots$$

shear diffusion:

$$\omega = -\frac{i}{4\pi T}q^2 - \frac{i(1-\ln 2)}{32\pi^3 T^3}q^4 - \frac{i(24\ln^2 2 - \pi^2)}{96\,(2\pi T)^5}q^6 - \frac{i\left[2\pi^2(\ln 32 - 1) - 21\zeta(3) - 24\ln 2(1+\ln 2(\ln 32 - 3))\right]}{384\,(2\pi T)^7}q^8 + \cdots$$

HYDRODYNAMICS FROM COMPLEX SPECTRAL CURVE

- radius of convergence of $\mathfrak{w}(\mathfrak{q}) = \sum_{i=1}^{\infty} c_n \mathfrak{q}^n$, $|\mathfrak{q}| < \mathfrak{q}_*$, is set by the lowest momentum at which the hydro pole collides (level-crossing): $\boxed{\mathfrak{q}_* = \min\left[|\mathfrak{q}_{\text{collision}}|\right]}$



$\boxed{\mathfrak{q}^2 = |\mathfrak{q}^2|\, e^{i\theta}}$

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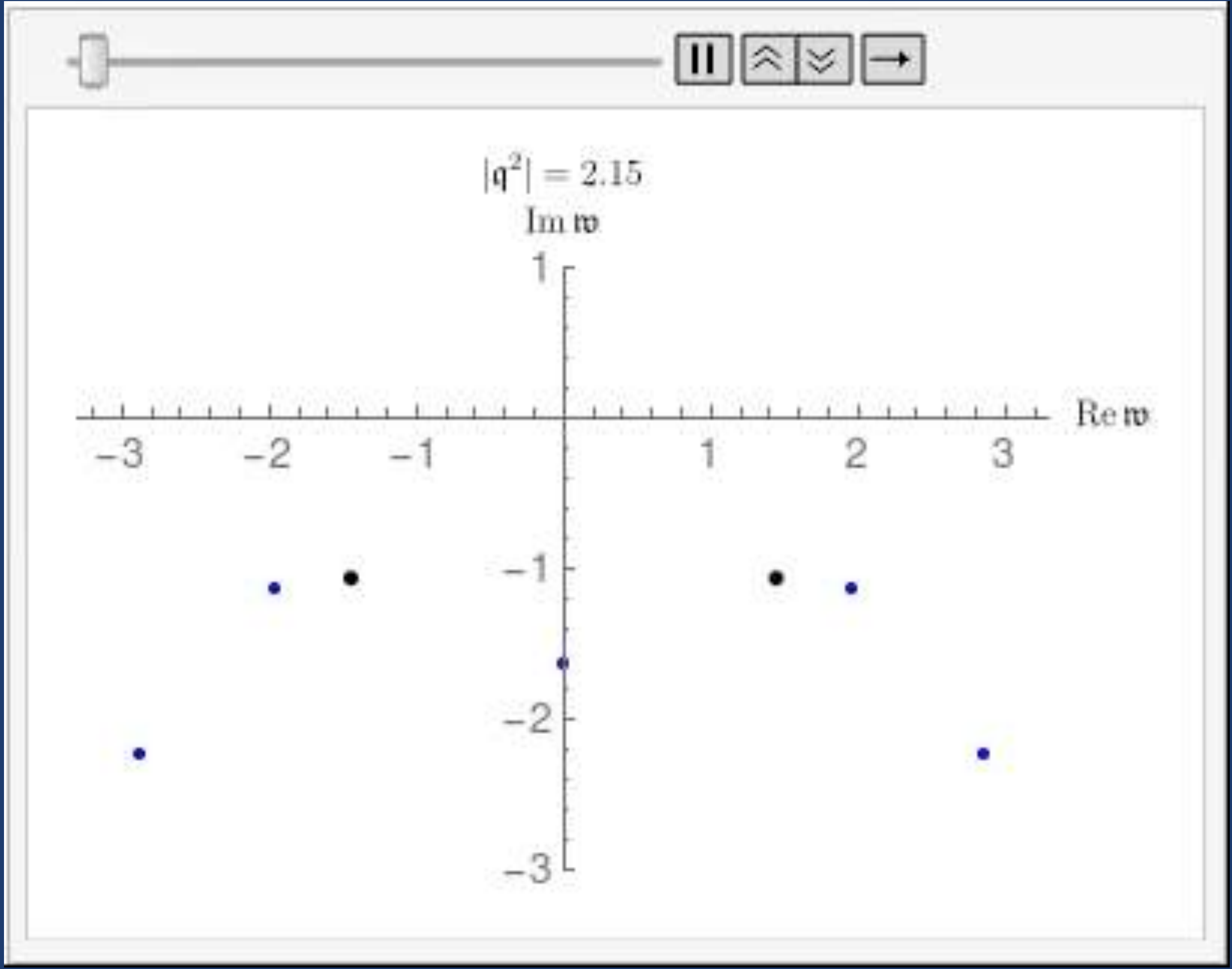

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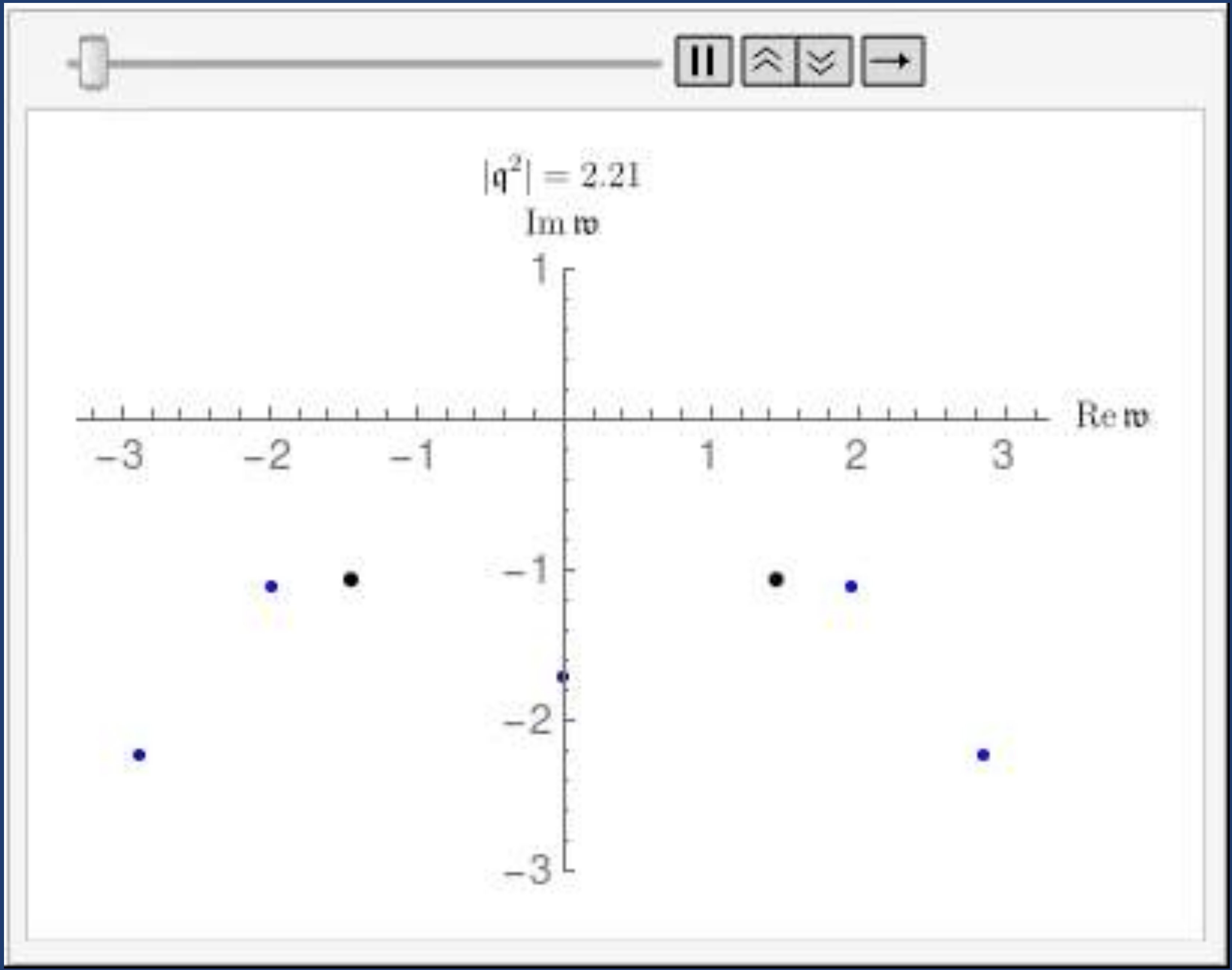

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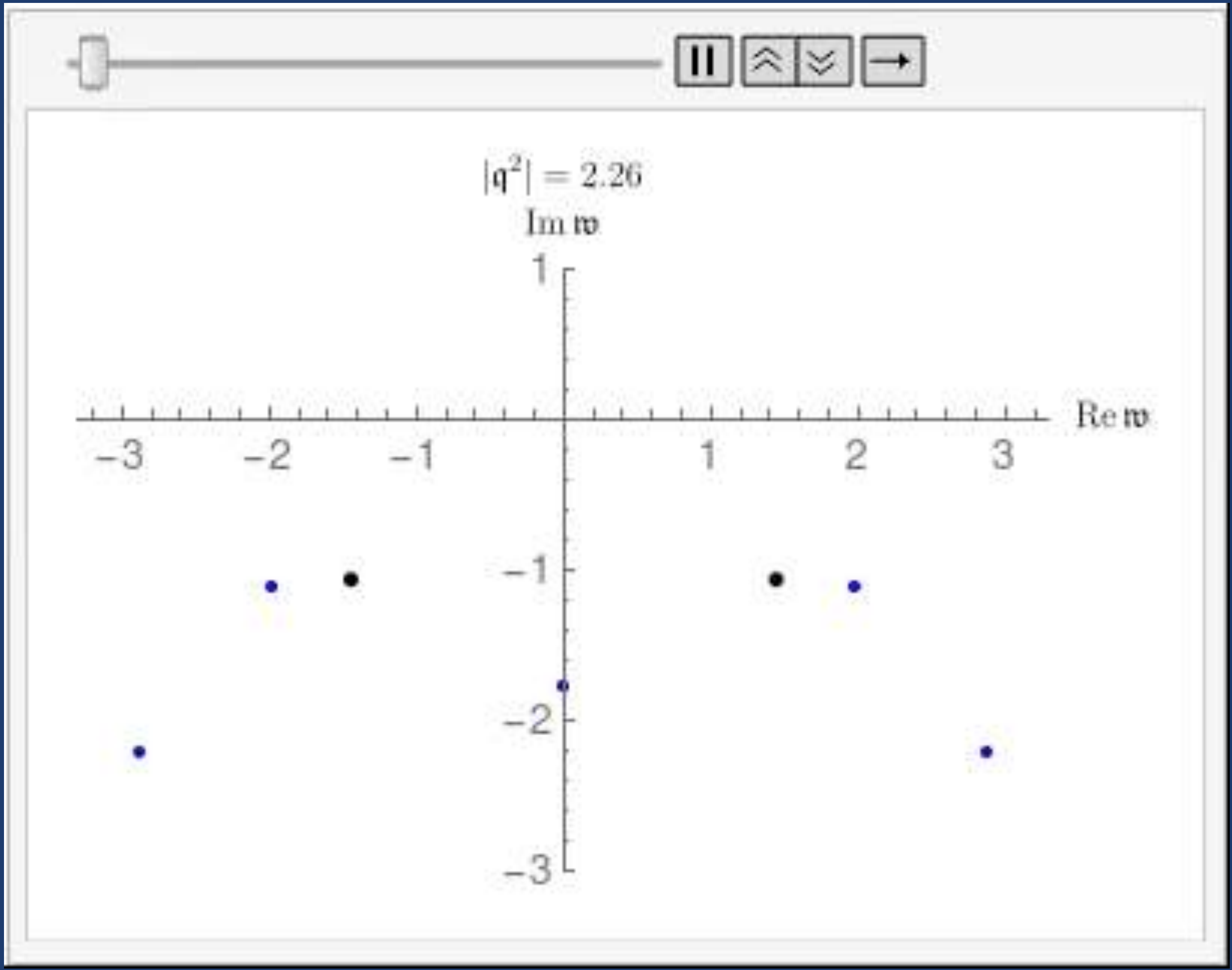


$\mathfrak{q}^2 = |\mathfrak{q}^2|\, e^{i\theta}$

HYDRODYNAMICS FROM COMPLEX SPECTRAL CURVE

- hydrodynamic series are convergent Puiseux series (shear *p=1*, sound *p=2*)
  [SG, Kovtun, Starinets, Tadić, PRL (2019); … ; see also Withers; JHEP (2018); Heller, et.al. (2020, …)]

$$\mathfrak{w}_{\rm shear} = -i \sum_{n=1}^{\infty} c_n \left(\mathfrak{q}^2\right)^n = -i\mathfrak{D}\mathfrak{q}^2 + \ldots$$

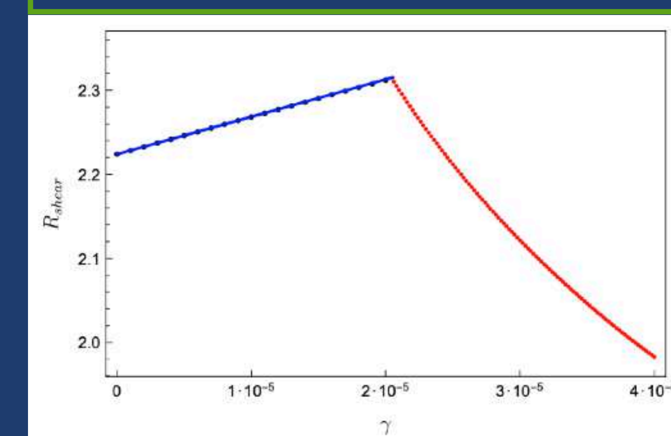
$$\mathfrak{w}_{\rm sound} = -i \sum_{n=1}^{\infty} a_n e^{\pm \frac{i\pi n}{2}} \left(\mathfrak{q}^2\right)^{n/2} = \pm v_s \mathfrak{q} - \frac{i}{2}\mathfrak{G}\mathfrak{q}^2 + \ldots$$

- dispersion relations are holomorphic in a disk

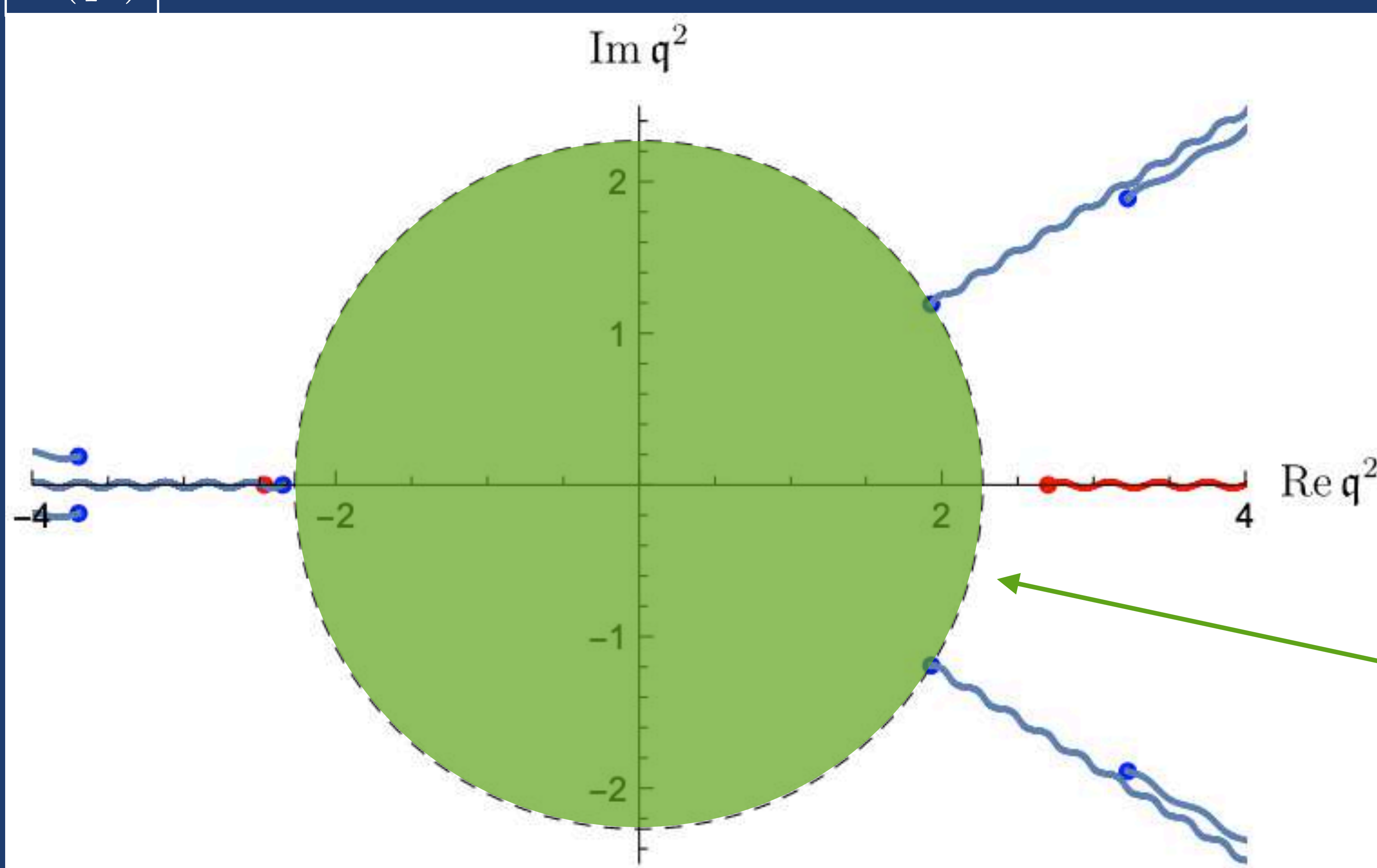
$$R_{\rm shear}(\lambda) = 2.22\left(1 + 674.15\,\lambda^{-3/2} + \cdots\right)$$

$$R_{\rm sound}(\lambda) = 2\left(1 + 481.68\,\lambda^{-3/2} + \cdots\right)$$

N=4 SYM radius convergence
[SG, Starinets, Tadić, JHEP (2021)]



$\omega(q^2)$



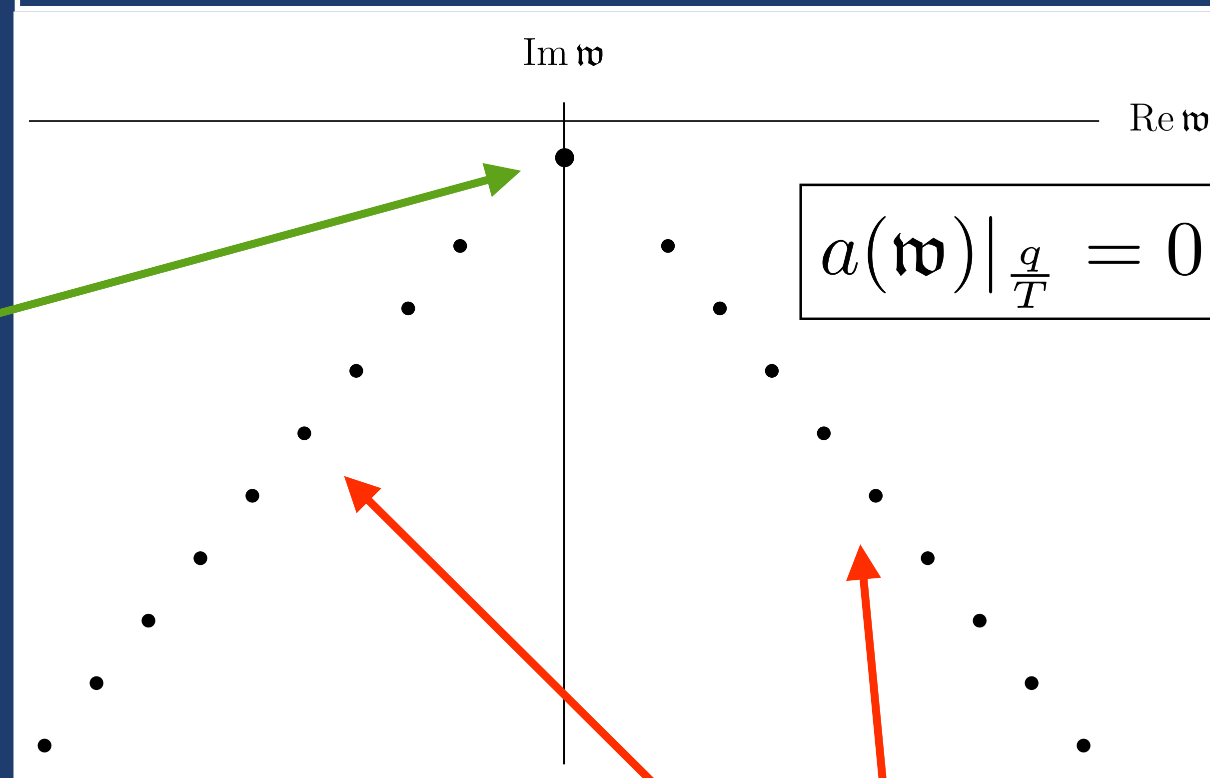
holomorphic disk

RECONSTRUCTION OF SPECTRA

[SG, Lemut, JHEP (2023)]

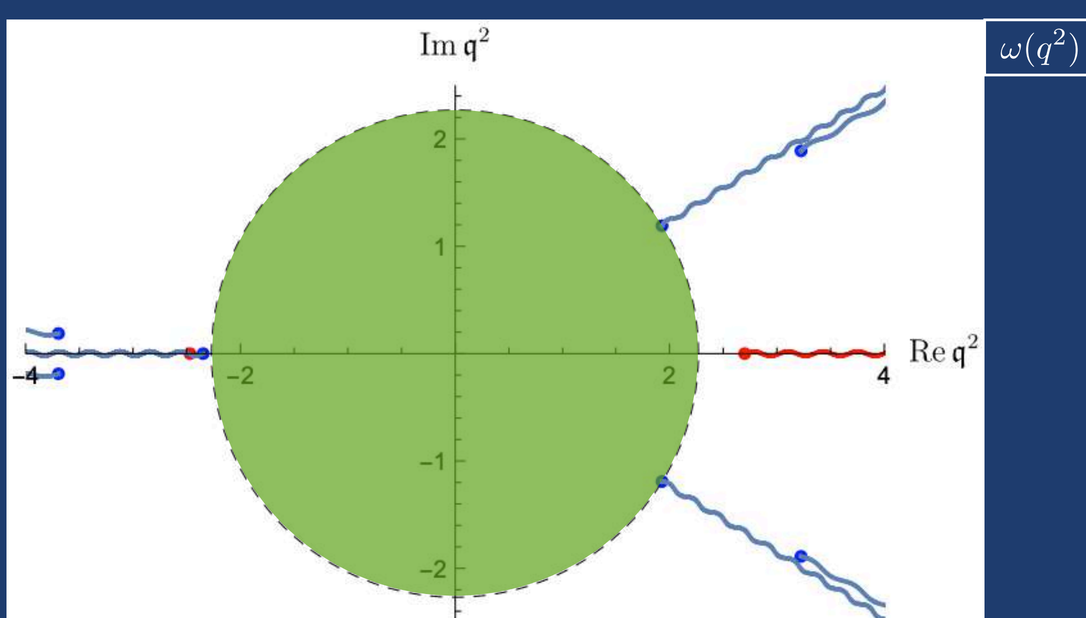
RECONSTRUCTION OF SPECTRA

$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R \sim \frac{b(\omega,q)}{a(\omega,q)} = \frac{B(\omega,q)}{\prod_{i=0}^{\infty}(\omega-\omega_i(q))}$$



hydrodynamics

rest of the spectrum



[see also Withers, JHEP (2019)]

PUISEUX AND DARBOUX THEOREMS

- *Puiseux theorem*

  Around a critical point of order *p*, we expect *p* branches of solutions

$$f(x_* = 0, y_* = 0) = 0,\ \partial_y f(0,0) = 0,\ \ \ldots,\ \partial_y^p f(0,0) \neq 0$$

$$y = Y_j(x) = \sum_{k \geq k_0}^{\infty} a_k x^{k/m_j}, \quad j = 1, \ldots, p$$

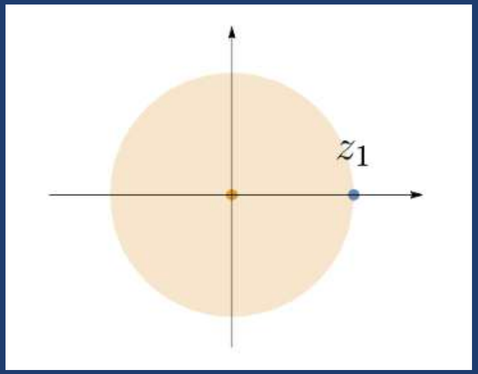
  If some $m_j > 1$, we necessarily have a family of m_j solutions

$$y = Y_l(x) = \sum_{k \geq k_0}^{\infty} a_k \left(e^{\frac{2\pi i l}{m_j}}\right)^k x^{k/m_j}, \quad l = 0, 1, \ldots, m_j - 1$$

- recall: sound

$$\mathfrak{w}_{\mathrm{sound}} = -i \sum_{n=1}^{\infty} a_n e^{\pm \frac{i\pi n}{2}} \left(\mathfrak{q}^2\right)^{n/2} = \pm v_s \mathfrak{q} - \frac{i}{2} \mathfrak{G} \mathfrak{q}^2 + \ldots$$

PUISEUX AND DARBOUX THEOREMS



- *Darboux theorem*

  Consider a power series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

that converges up to a critical point of order $\nu\,[= -1/p]$, which can be computed

$$f(z) \sim (z - z_1)^{-\nu\,[=1/2]}\, r(z) + q(z)$$

$$\nu = \lim_{n\to\infty}\left[z_1(n+1)\frac{a_{n+1}}{a_n} - n\right]$$

as well as all coefficients in the expansion and subleading (non-singular) terms

$$r(z) = \sum_{m=0}^{\infty} r_m\,(z - z_1)^m$$

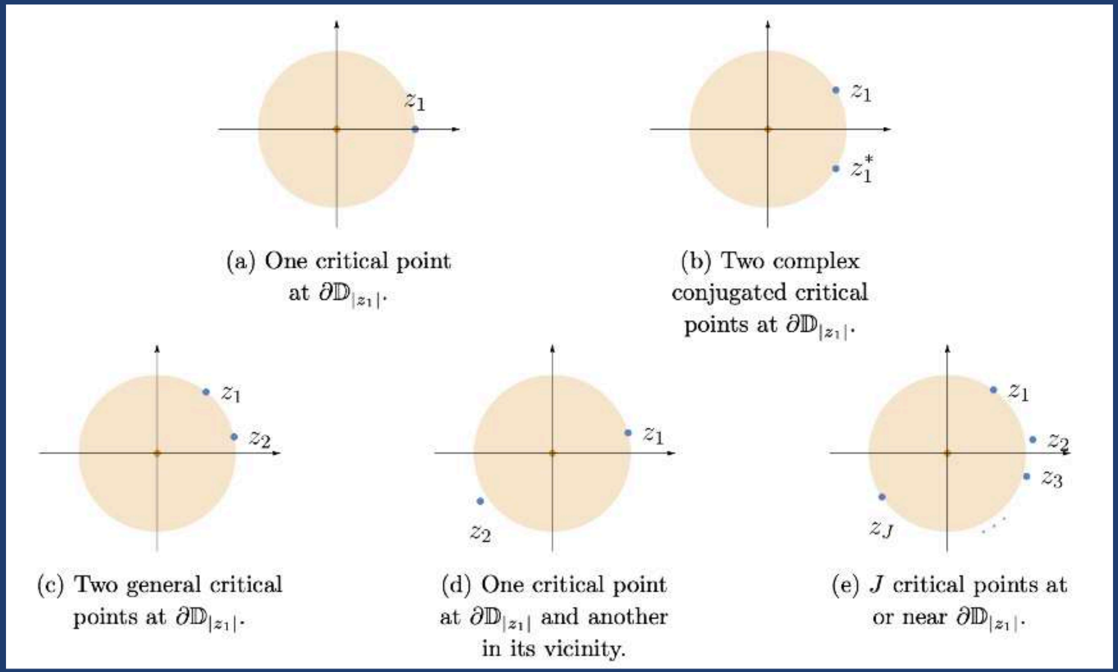
$$r_m = \lim_{n\to\infty}\left[\frac{(-1)^{m-\nu} n! z_1^{n-m+\nu} a_n}{(\nu - m)_n} - \sum_{k=0}^{m-1}\frac{(-1)^{m-k}(\nu - k)_n r_k}{(\nu - m)_n z_1^{m-k}}\right]$$

$$q_m = \lim_{n\to\infty}\left[\sum_{k=0}^{n}\frac{(-1)^{n+m-k} n! (\nu)_{n-k} a_k}{(-\nu - m)_n (n-k)! z_1^{m-k}} - \sum_{k=0}^{m-1}\frac{(-1)^{k-m}(-\nu - k)_n q_k}{(-\nu - m)_n z_1^{m-k}}\right]$$

PUISEUX AND DARBOUX THEOREMS

- *Darboux theorem*

  Need generalisation to different configurations of critical points



- Problem: need to know either the location of the critical point or the exponent… but this is resolved by Hunter and Guerrieri (1980), which we generalise

- Assume we only know a finite number of coefficients: $a_n, \; n = 0, \ldots, N$

$$X_n^0(\nu, z_1) = a_n$$

$$X_n^{m+1}(\nu, z_1) = X_n^m(\nu, z_1) - \frac{(n+\nu-2m-1)}{n z_1} X_{n-1}^m(\nu, z_1), \quad \text{for } m \geq 0$$

$$X_n^m(\nu, z_1) \sim \sum_{k=m}^{\infty} \frac{(-1)^{k+m-\nu} k!(\nu-k)_{n-m} r_k}{n!(k-m)! z_1^{n+\nu-k}} \sim O(n^{\nu-2m-1})$$

$$X_N^1 = 0, \; X_{N-1}^1 = 0 \xrightarrow{\text{iteration}} X_N^m = 0, \; X_{N-1}^m = 0 \longrightarrow z_1, \; \nu$$

PUISEUX AND DARBOUX THEOREMS

- *Darboux theorem*
- Similarly, define

$$Y^{0}_{\ell,n}(\nu,z_1) = a_n$$

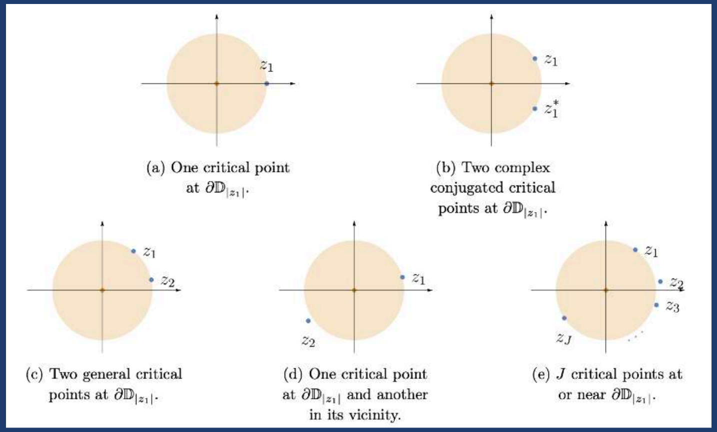
$$Y^{m+1}_{\ell,n}(\nu,z_1) = Y^{m}_{\ell,n}(\nu,z_1) - \frac{(n+\nu-2m-\ell-2)}{nz_1}Y^{m}_{\ell,n-1}(\nu,z_1), \quad \text{for } m \geq 0$$

$$Y^{m}_{\ell,n} \sim \sum_{k=0}^{\ell} \frac{(-1)^{k-\nu}(m+\ell-k)!(\nu-k)_{n-m}r_k}{n!(\ell-k)!z_1^{n+\nu-k}} + \mathcal{O}(n^{\nu-2m-\ell-2})$$

$$r_\ell = \lim_{n\to\infty}\left[\frac{(-1)^{\ell-\nu}n!z_1^{n+\nu-\ell}}{m!(\nu-\ell)_{n-m}}Y^{m}_{\ell,n} - \sum_{k=0}^{\ell-1}\binom{m+\ell-k}{m}\frac{(-1)^{\ell-k}(\nu-k)_{n-m}r_k}{(\nu-\ell)_{n-m}z_1^{\ell-k}}\right]$$

subleading parts of the function (recall: *q*) follow in an analogous way

- Extended to several critical points



RECONSTRUCTION OF '*ALL*' UV MODES

claim: systematic reconstruction of *all* modes connected via *level-crossing* is possible by exploration (analytic continuations) of the Riemann surface connecting physical modes

- statement should hold for spectra that are 'sufficiently complicated' – Heun function
- momentum space analogue of resurgence in position space – everything is convergent

$$\omega_0(z) = \sum_{n=0}^{\infty} a_n z^n$$

$$\omega_0(z) = -i \sum_{n=0}^{\infty} e^{\frac{i\pi n}{2}} b_n (z - z_1)^{n/2}$$

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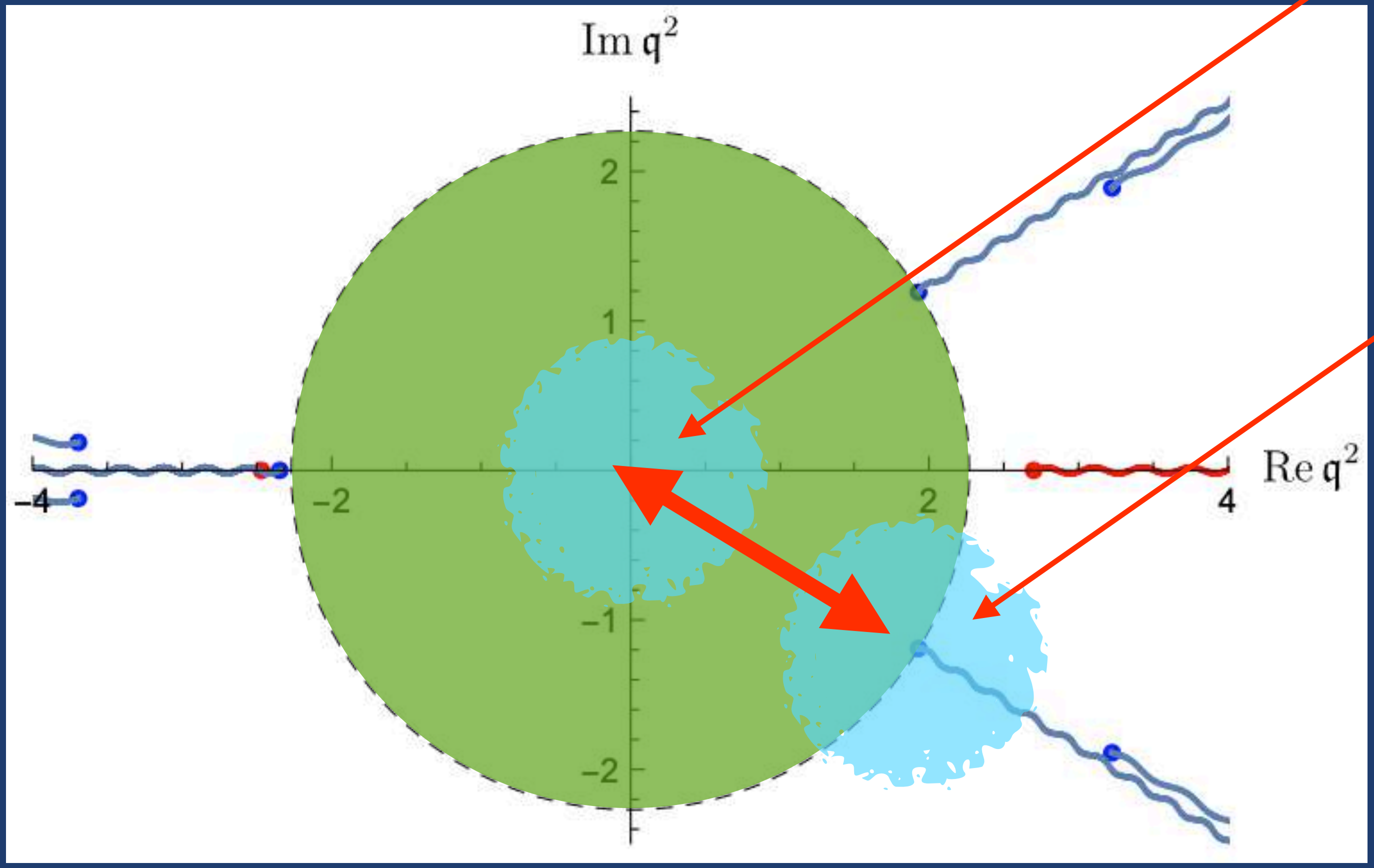
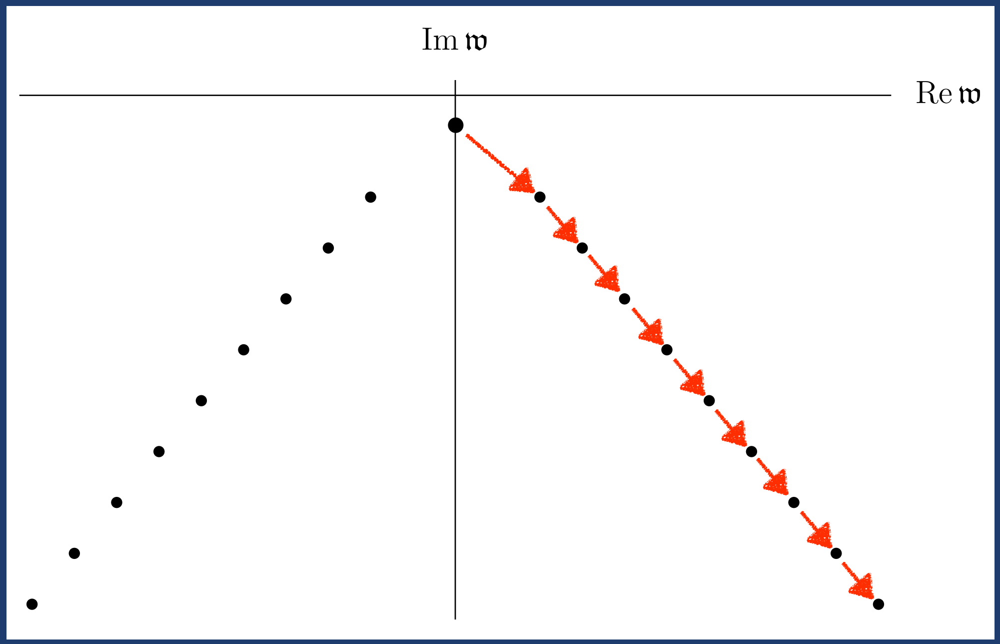




all UV modes from one IR mode

EXAMPLE: MOMENTUM DIFFUSION OF M2 BRANES

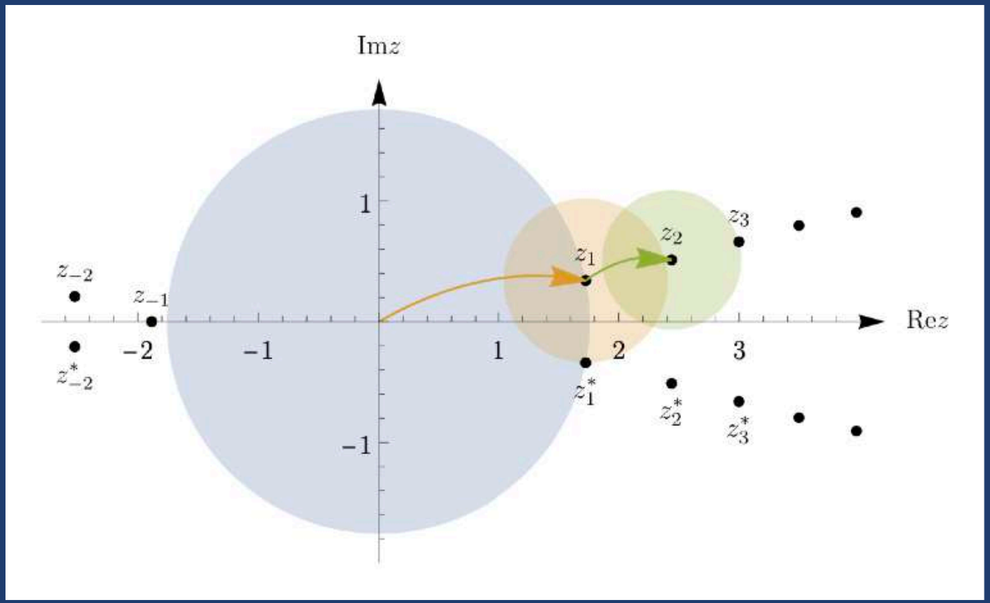
- start from 300 hydrodynamic coefficients $\omega_0(z) = \sum_{n=0}^{\infty} a_n z^n$
- use algorithm with 2 c.c. critical points, 'recover' 12 coefficients and compute the gap with analytic continuation on the same sheet (Padé approximant, …)

$$\mathfrak{w}_1(z) = \sum_{n=0}^{(N_1=12)-1} b_n (z-z_1)^{n/2} \quad \longrightarrow \quad \begin{aligned} \mathfrak{w}_1^{\text{calc}}(0) &= 1.23506 - 1.76338i \\ \mathfrak{w}(0) &= 1.23455 - 1.77586i \end{aligned}$$

- (re)compute the first 300 coefficients, use algorithm with 2 general critical points, 'recover' 12 coefficients and compute the gap

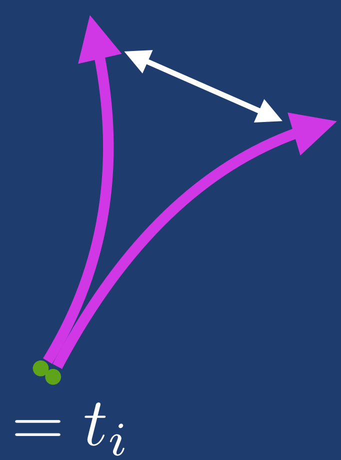
$$\mathfrak{w}_2(z) = \sum_{n=0}^{(N_2=12)-1} c_n (z-z_2)^{n/2} \quad \longrightarrow \quad \begin{aligned} \mathfrak{w}_2^{\text{calc}}(0) &= 2.16275 - 3.25341i \\ \mathfrak{w}_2(0) &= 2.12981 - 3.28100i \end{aligned}$$

- … exploration continues …
- conceptually useful and instructive, practically not quite (yet)…



POLE-SKIPPING AND THE RECONSTRUCTION

CHAOS



Lyapunov exponent   butterfly velocity

$$|\Delta Z(t,\mathbf{x})| \approx |\Delta Z(t_i,\mathbf{x}_i)|\, e^{\lambda_L (t-|\mathbf{x}|/v_B)}$$

- classical chaos means extreme sensitivity to initial conditions

- "what is quantum chaos?"
  a measure: "out-of-time-ordered" correlation functions [Larkin, Ovchinnikov; Kitaev]

$$C(t,\mathbf{x}) = \left\langle [W(t,\mathbf{x}),V(0,\mathbf{0})]^\dagger [W(t,\mathbf{x}),V(0,\mathbf{0})] \right\rangle_T \sim \epsilon\, e^{\lambda_L (t-|\mathbf{x}|/v_B)}$$

'quantum' Lyapunov exponent   butterfly velocity

- the Maldacena-Shenker-Stanford bound on exponential Lyapunov chaos

OTOC of $\mathcal{O}(t,x)$

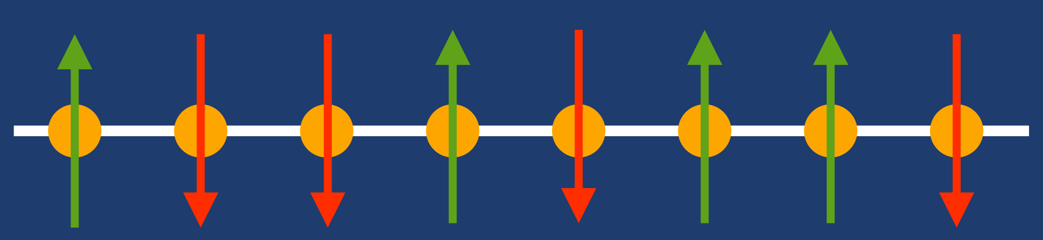
$$C(t,x) \sim \epsilon\, e^{\lambda_L (t-x/v_B)}$$

$$\lambda_L \leq 2\pi T/\hbar$$

- in finite-*N* systems, quantum chaos spreads polynomially with a bounded rate of growth – weak quantum chaos [Kukuljan, SG, Prosen, PRB (2017)]

OTOC of $\int d^d x\, \mathcal{O}(t,x)$

$$c(t) \leq A t^{3d}$$



CHAOS FROM HYDRODYNAMICS: POLE-SKIPPING

- precise analytic connection between ‘low-energy’ hydrodynamics and quantum chaos
  [SG, Schalm, Scopelliti, PRL (2017); Blake, Lee, Liu, JHEP (2018); Blake, Davison, SG, Liu, JHEP (2018); SG, JHEP (2019)]

- resumed all-order hydrodynamic series (e.g. sound)

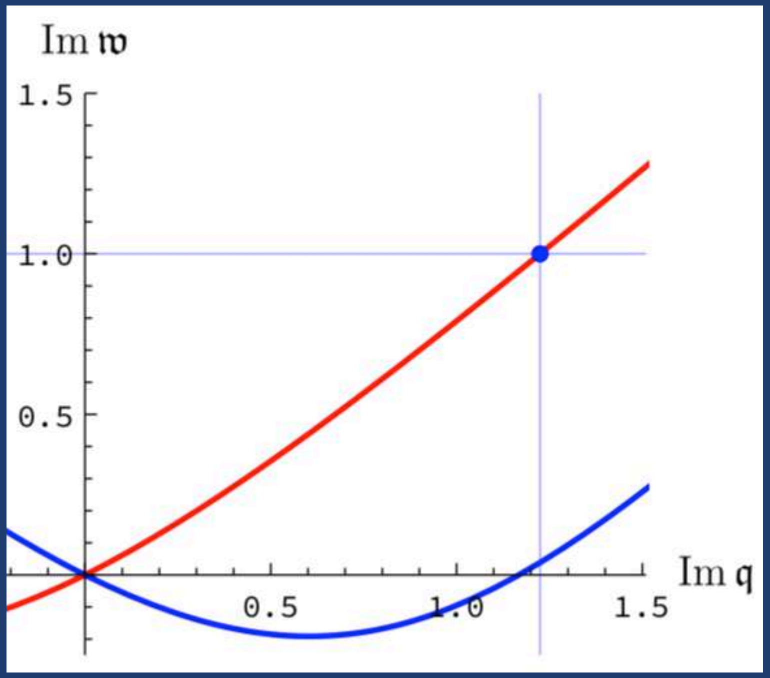
$$\omega(q) = \sum_{n=1}^{\infty} \alpha_n \left(T, \mu_i, \langle \mathcal{O}_j \rangle, \lambda\right) q^n$$

  passes through a “chaos point” at imaginary momentum

$$\omega(q = i\lambda_L / v_B) = i\lambda_L = 2\pi T i$$

  where the associated *2*-pt function is “0/0”:

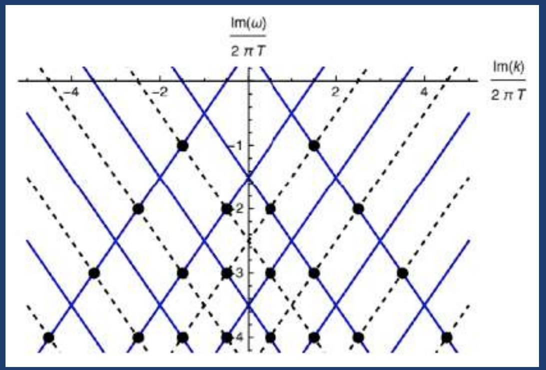
$$G_R = \frac{0}{0} = N(\delta\omega/\delta q)$$



- triviality of Einstein’s equations at the horizon [Blake, Davison, SG, Liu, JHEP (2018)]

- infinite constraints on correlators
  [SG, Kovtun, Starinets, Tadić, JHEP (2019); Blake, Davison, Vegh, JHEP (2019)]

$$\omega_n(q_n) = -2\pi T i n$$



[from Blake, Davison, Vegh, JHEP (2019)]

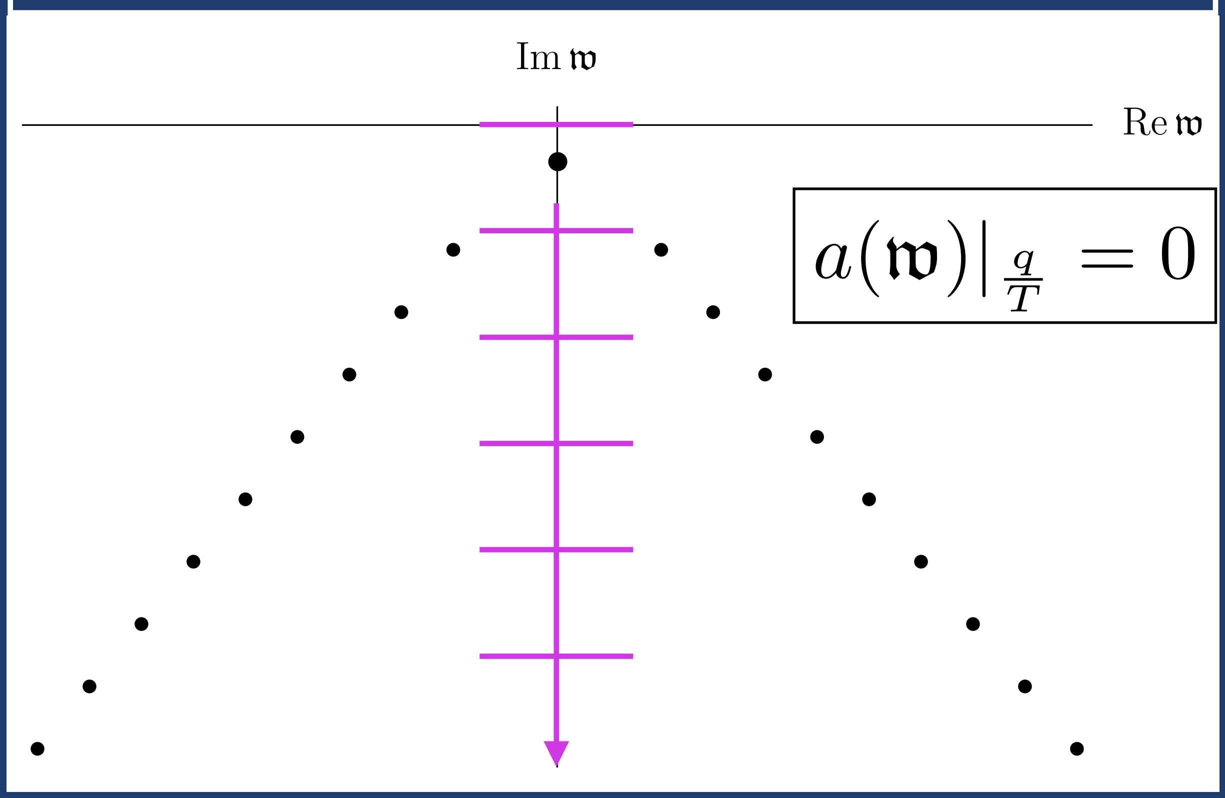
DIFFUSION AND SPECIAL POLE-SKIPPING POINTS

- consider diffusion in a neutral 3*d* CFT dual to AdS$_4$-Schwarzschild

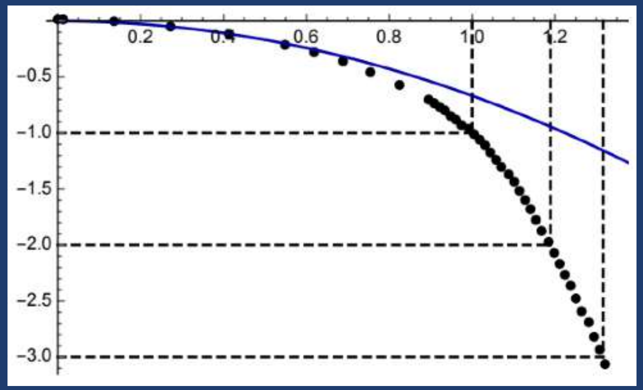
$$\mathfrak{w}_{\text{shear}} = -i \sum_{n=1}^{\infty} c_n \left(\mathfrak{q}^2\right)^n = -i\mathfrak{D}\mathfrak{q}^2 + \ldots$$

$$\omega_n(q_n) = -2\pi T i n$$

$$\langle T_{\mu\nu}(-\omega,-q), T_{\rho\sigma}(\omega,q)\rangle_R \sim \frac{b(\omega,q)}{a(\omega,q)}$$



for increasing real q



[plot from Blake, Davison, Vegh, JHEP (2019)]

analytic result known for 4*d* bulk [SG, PRL (2021)]

$$q_n = \frac{4\pi T}{\sqrt{3}} n^{1/4}, \quad n = 0, 1, 2, \ldots$$

algebraically special points

4*d* odd-even duality: Darboux transform

[Chandrasekhar (1983); SG, Vrbica (2023)]

RECONSTRUCTION FROM POLE-SKIPPING

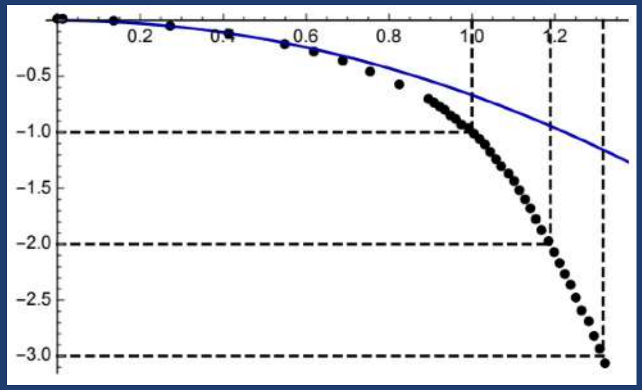
- how much information is required to reconstruct a QFT spectrum?
  so far in the talk: the knowledge of one dispersion relation

new claim: in holographic theories of the type discussed here (N=4 SYM, M2, M5, …), the entire spectrum can be computed from only a discrete set of pole-skipping points

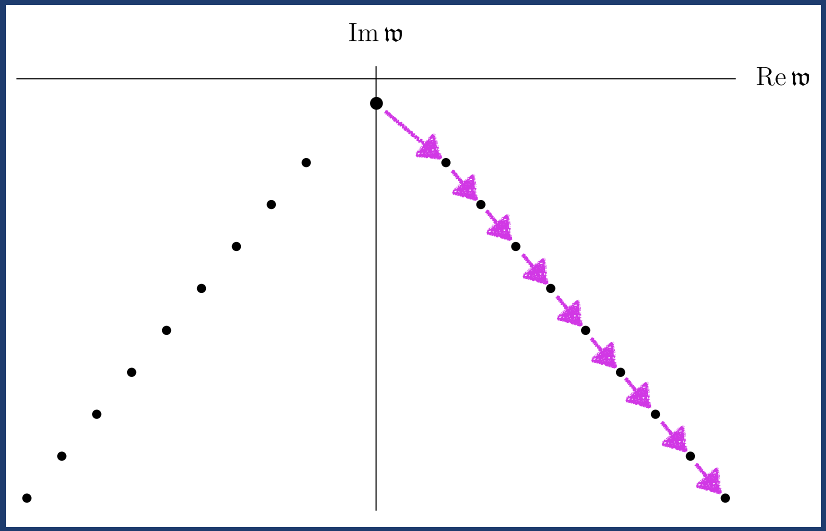
[SG, Lemut, Pedraza, PRD (2023)]

$$\omega_n(q_n) = -2\pi T i n$$

$$\mathfrak{w}_{\text{shear}} = -i \sum_{n=1}^{\infty} c_n \left(\mathfrak{q}^2\right)^n = -i\mathfrak{D}\mathfrak{q}^2 + \dots$$



[plot from Blake, Davison, Vegh, JHEP (2019)]



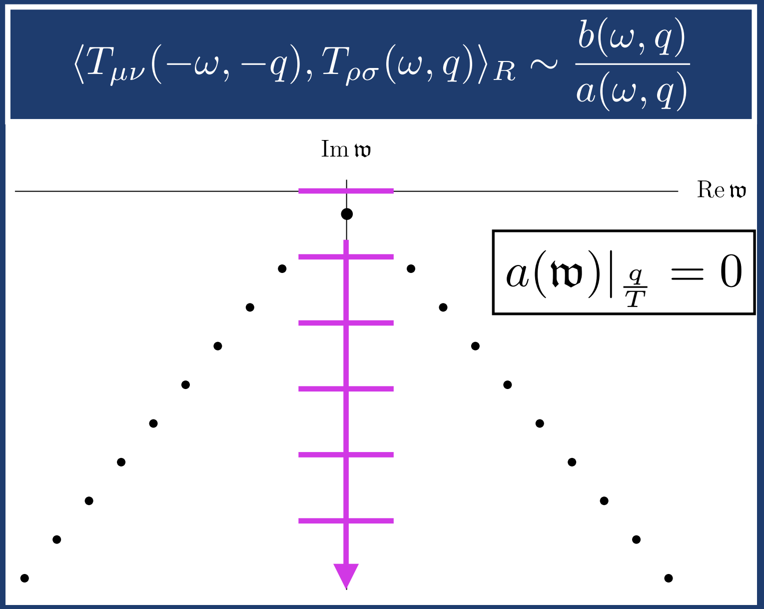
RECONSTRUCTION FROM POLE-SKIPPING

- interpolation problem:

$$\omega_n(q_n) = -2\pi T i n$$

$$\downarrow$$

$$\mathfrak{w}_{\text{shear}} = -i \sum_{n=1}^{\infty} c_n \left(\mathfrak{q}^2\right)^n = -i\mathfrak{D}\mathfrak{q}^2 + \ldots$$



for increasing real q

- unique solutions to interpolation problems are extremely hard in the absence of special known properties of the function (e.g. Hadamard, Nevanlinna-Pick, …)
- trick: ‘analytic continuation’ to *d* spacetime dimensions and expansion around infinite *d*
- general relativity in large *d* drastically simplifies [review by Emparan, Herzog, 2003.11394]

$$V \sim 1/r^d$$

- recall: large-*d* limit of quantum mechanics allows a solution of the Helium problem
- convergence of such series depends on the details

RECONSTRUCTION FROM POLE-SKIPPING

- interpolation: $\omega_n(q_n) = -2\pi T i n$ → $\mathfrak{w}_{\text{shear}} = -i \sum_{n=1}^{\infty} c_n \left(\mathfrak{q}^2\right)^n = -i\mathfrak{D}\mathfrak{q}^2 + \dots$

- 'analytic continuation' to *d* spacetime dimensions and expansion around infinite *d*

$$\omega_0(q) = -i\left(\frac{q}{\sqrt{d}}\right)^2 - i\sum_{m=2}^{\infty}\frac{1}{d^m}\sum_{j=2}^{m} c_{m,n}\left(\frac{q}{\sqrt{d}}\right)^{2j}$$

$$\frac{q_n}{\sqrt{d}} = \sqrt{\frac{nd}{2}}\left(1+\sum_{m=1}^{\infty}\frac{b_{n,m}}{d^m}\right)$$

→

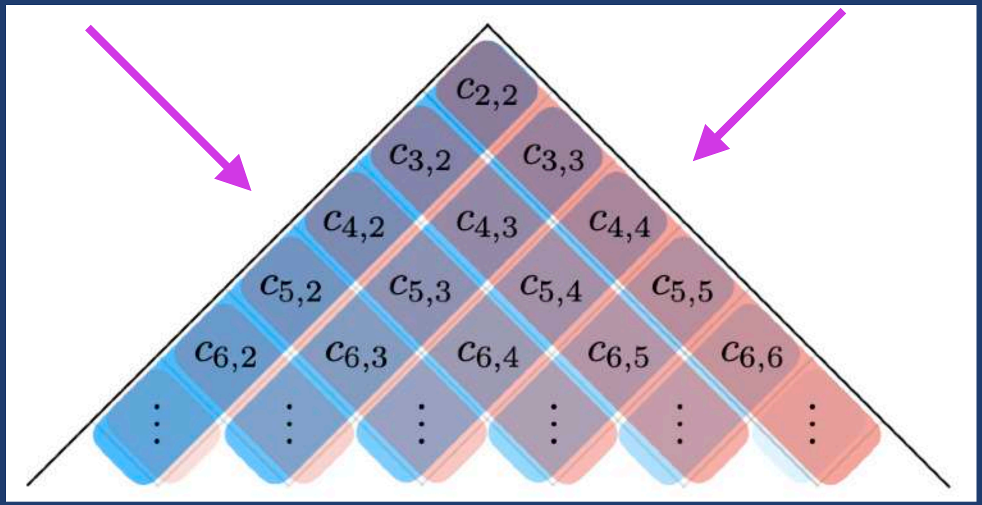
$$b_{n,1} = -\sum_{m=2}^{\infty}\frac{n^{m-1}c_{m,m}}{2^m}$$

$$b_{n,2} = -\frac{b_{n,1}^2}{2} - \sum_{m=2}^{\infty}\frac{n^{m-1}\left(c_{m+1,m}+2mb_{n,1}c_{m,m}\right)}{2^m}$$

second analytic continuation

hydrodynamics

pole-skipping



$$c_{m,m} = -\frac{2^m}{(m-1)!}\partial_x^{m-1}b_1(0)$$

$$c_{m+1,m} = -\frac{2^m\partial_x^{m-1}b_2(0)}{(m-1)!} + \sum_{j=2}^{m-1}\left(j-\frac{1}{4}\right)c_{j,j}c_{m-j+1,m-j+1}$$

generating functions

RECONSTRUCTION FROM POLE-SKIPPING

$$\omega_0(q) = -i\left(\frac{q}{\sqrt{d}}\right)^2 - i\sum_{m=2}^{\infty}\frac{1}{d^m}\sum_{j=2}^{m} c_{m,n}\left(\frac{q}{\sqrt{d}}\right)^{2j}$$

$$\frac{q_n}{\sqrt{d}} = \sqrt{\frac{nd}{2}}\left(1+\sum_{m=1}^{\infty}\frac{b_{n,m}}{d^m}\right)$$

- first level: $b_{n,1} = -\frac{1}{2}H_n = -\frac{1}{2}\sum_{k=1}^{n}\frac{1}{k}$ → $H_n \to H(x) = \sum_{k=1}^{\infty}\frac{x}{k(x+k)}$

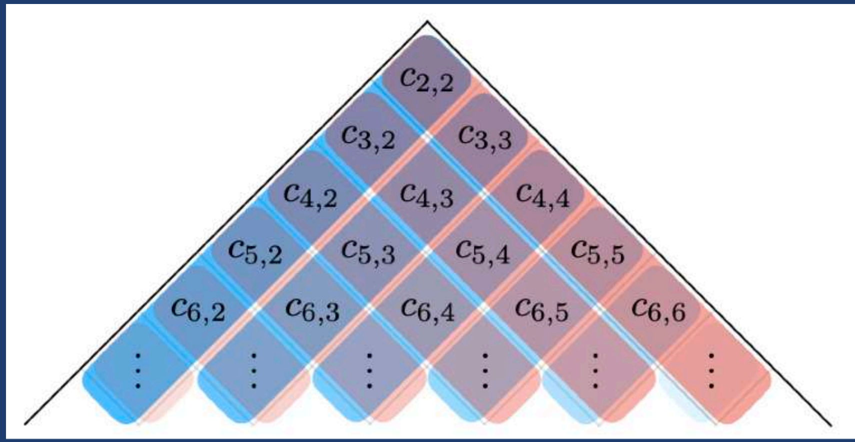
→ $c_{m,2} = c_{m,m} = (-1)^m 2^{m-1}\zeta(m)$

$$\omega_0(q) = -i\bar{q}^2 - i\frac{\bar{q}^2}{d}H_{2\bar{q}^2/d} + \ldots$$

partial (convergent) resummation

- higher levels:

$$c_{2,2} = 2\zeta(2),$$
$$c_{3,2} = -4\zeta(3), \quad c_{3,3} = -4\zeta(3),$$
$$c_{4,2} = 8\zeta(4), \quad c_{4,3} = 7\times 8\zeta(4), \quad c_{4,4} = 8\zeta(4)$$



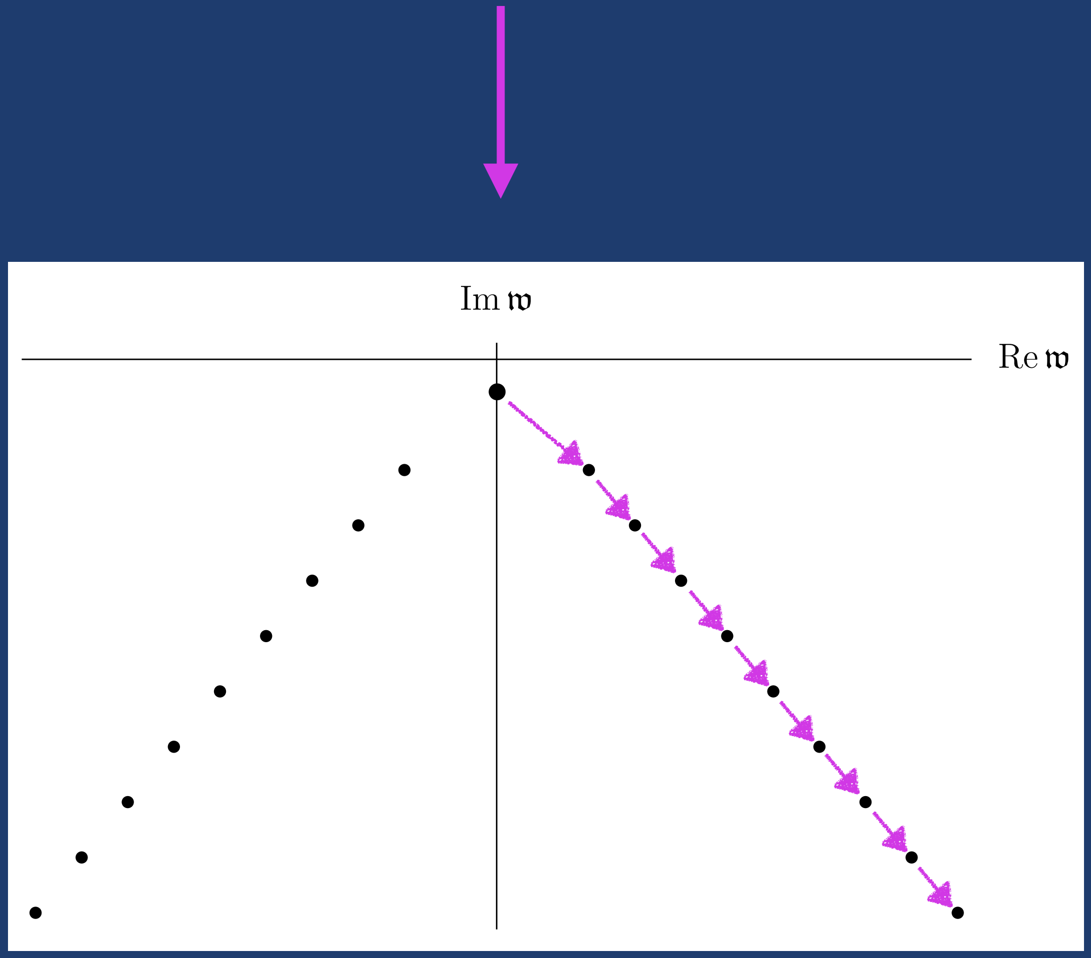
$$c_{3,2} \approx -1.000\times 4\zeta(3),$$
$$c_{4,3} \approx 7.001\times 8\zeta(4),$$
$$c_{5,4} \approx -15.548\times 16\zeta(5),$$
$$c_{6,5} \approx 27.546\times 32\zeta(6)$$

$$c_{4,2} \approx 1.000\times 8\zeta(4),$$
$$c_{5,3} \approx -15.502\times 16\zeta(5)$$

- symmetries
- polylogs

RECONSTRUCTION FROM POLE-SKIPPING

- the rest of the spectrum follows from a reconstruction discussed before



- complete reconstruction of the spectrum using only algebraic near-horizon manipulations (local analysis)
- full correlator follows from poles – recent work by Dodelson, et.al., 2304.12339

SUMMARY AND FUTURE DIRECTIONS

SUMMARY AND FUTURE DIRECTIONS

- complex analytic structures of transport are a powerful tool for exploring physics
- claim: in some QFTs reconstruction of a spectrum is possible all the way from IR to UV
- in momentum space we can deal with convergent series, but, ‘morally’, this is equivalent to resurgence in position space
- useful not only in QFTs but also for QNM reconstructions and other similar problems
- improve practical aspects of reconstructions given a limited number of known coefficient
- can these techniques be used in realistic QFTs (Euler-Heisenberg, chiral Lagrangian)?

- new ‘classification’ of pole-skipping points in 3*d* CFTs
- claim: reconstruction is possible from an infinite discrete set of pole-skipping points
- large-*N* QFT calculation → differential equations → algebraic manipulations

THANK YOU!