M5-brane precision holography

ICTP Workshop on String Theory, Holography, and Black Holes

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The AdS/CFT master formula

$$\underbrace{Z_{\text{string- or M-theory}} \text{ on } \mathrm{AdS}_d \times \mathcal{M}_{10-d \text{ or } 11-d}}_{\text{sugra + quantum corrections}} \quad = \quad \underbrace{Z_{\text{SCFT}} \text{ on } S^1 \times \mathcal{N}_{d-2}}_{\text{susy localization/Cardy @ large-N}}$$

Field theory examples

The superconformal index of a generic 4d $\mathcal{N}=1$ SCFT on $S^1 \times S^3$ goes as

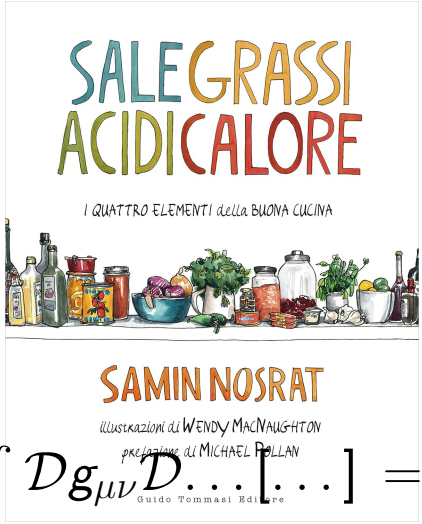
$$\log \mathcal{I} = \frac{(\omega_1+\omega_2-2\pi\mathrm{i})^3}{\omega_1\omega_2}\frac{\mathrm{Tr}\, R^3}{48} - \frac{(\omega_1+\omega_2-2\pi\mathrm{i})(\omega_1^2+\omega_2^2-4\pi^2)}{\omega_1\omega_2}\frac{\mathrm{Tr}\, R}{48} + \dots$$

[Cassani, Komargodski, 2021] [Arabi Ardehali, Murthy, 2021] ...

$\mathcal{N}=4$ SYM	$\mathrm{Tr}\, R^3 = \#N^2$	$\mathrm{Tr}\, R = 0$
$\mathcal{N}=1$ Leigh-Strassler SCFT's	$\mathrm{Tr}\, R^3 = \#N^2$	$\mathrm{Tr}\, R = \#$
$\mathcal{N}=1$ $Y^{p,q}$ quiver gauge theories	$\mathrm{Tr}\, R^3 = \#N^2$	$\mathrm{Tr}\, R = \#$
$\mathcal{N}=1$ theories of class $\mathcal{S}$	$\mathrm{Tr}\, R^3 = \#N^3 + \boxed{\#N} + \dots$	$\mathrm{Tr}\, R = \boxed{\#N} + \dots$

... [Bah, Beem, Bobev, Wecht, 2012]

A recipe for the quantum gravity path integral



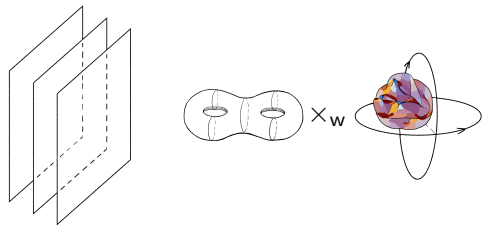
$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}\ldots[\ldots] = ?$$

A recipe for the quantum gravity path integral

1. Find a suitable consistent truncation from 11d or 10d sugra down to a d-dim sugra
2. Find the dominant saddle of the d-dim consistent truncation
3. Compute its Euclidean on-shell action
4. Include the following quantum gravity corrections:
   - higher derivative corrections
   - perturbative loop effects
   - KK modes
   - subdominant saddles

Step 1: Consistent truncation

A stack of N M5-branes wrapping a Riemann surface of genus $\mathfrak{g}$ inside a CY_3

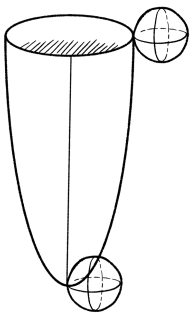


M-theory on $\mathrm{AdS}_5 \times_w \Sigma_\mathfrak{g} \times_w S^4 \iff \mathcal{N} = 1$ SCFT of class $\mathcal{S}$ on $S^1 \times_w S^3$

11d sugra $\xrightarrow{S^4}$ 7d U(1) × U(1)-gauged sugra $\xrightarrow{\Sigma_\mathfrak{g}}$ 5d minimal sugra

[Donos, Gauntlett, Kim, Varela, 2010] … [Bobev, VD, Vekemans, 2022]

Steps 2 & 3: Dominant saddle & on-shell action



CCLP most general BH solution to 5d minimal sugra
[Chong, Cvetic, Lu, Pope, 2005]

UV divergence holoren or background subtraction
... [Bobev, VD, Vekemans, 2022]

IR divergence non-susy $\to$ susy $\to$ susy + extremal
[Cabo-Bizet, Cassani, Martelli, Murthy, 2018]

Step 4: Higher derivative corrections

[Bobev, VD, Vekemans, Reys, 2022]

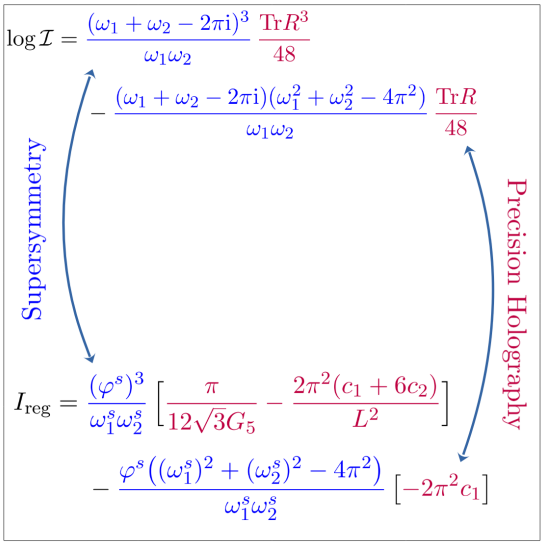
- Supersymmetry fixes the 4∂ Lagrangian of 5d minimal sugra up to two unknown coefficients: (c_1, c_2)
  [Bobev, Hristov, Reys, 2022]
- $I_{4\partial}[\,\mathrm{CCLP}_{4\partial}\,] = I_{2\partial}[\,\mathrm{CCLP}_{2\partial}\,] + I_{2\partial}[\,\delta(\mathrm{CCLP})\,] + \delta(I)[\,\mathrm{CCLP}_{2\partial}\,] + \mathcal{O}(c_1^2, c_2^2)$
  [Cassani, Ruiperez, Turetta, 2023]
- Use empty AdS_5 to fix
  [Baggio, Halmagyi, Mayerson, Robbins, Wecht, 2014]

  $(c_1, c_2) \to$ the conformal anomalies (a, c)

- Use the superconformal symmetry of the SCFT to fix

  $(a, c) \to$ the 't Hooft anomalies $\left(\mathrm{Tr}\, R^3, \mathrm{Tr}\, R\right)$

Summary


$\log \mathcal{I} = \frac{(\omega_1+\omega_2-2\pi\mathrm{i})^3}{\omega_1\omega_2}\frac{\mathrm{Tr}R^3}{48}$
$-\frac{(\omega_1+\omega_2-2\pi\mathrm{i})(\omega_1^2+\omega_2^2-4\pi^2)}{\omega_1\omega_2}\frac{\mathrm{Tr}R}{48}$
Supersymmetry
Precision Holography
$I_{\mathrm{reg}} = \frac{(\varphi^s)^3}{\omega_1^s\omega_2^s}\left[\frac{\pi}{12\sqrt{3}G_5} - \frac{2\pi^2(c_1+6c_2)}{L^2}\right]$
$-\frac{\varphi^s\left((\omega_1^s)^2+(\omega_2^s)^2-4\pi^2\right)}{\omega_1^s\omega_2^s}\left[-2\pi^2 c_1\right]$

Thank you!