Correlation functions for open strings and the chaos bound

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Based on arXiv:2310.15697 w/ Mihailo Čubrović

ICTP Workshop on String Theory, Holography and Black Holes

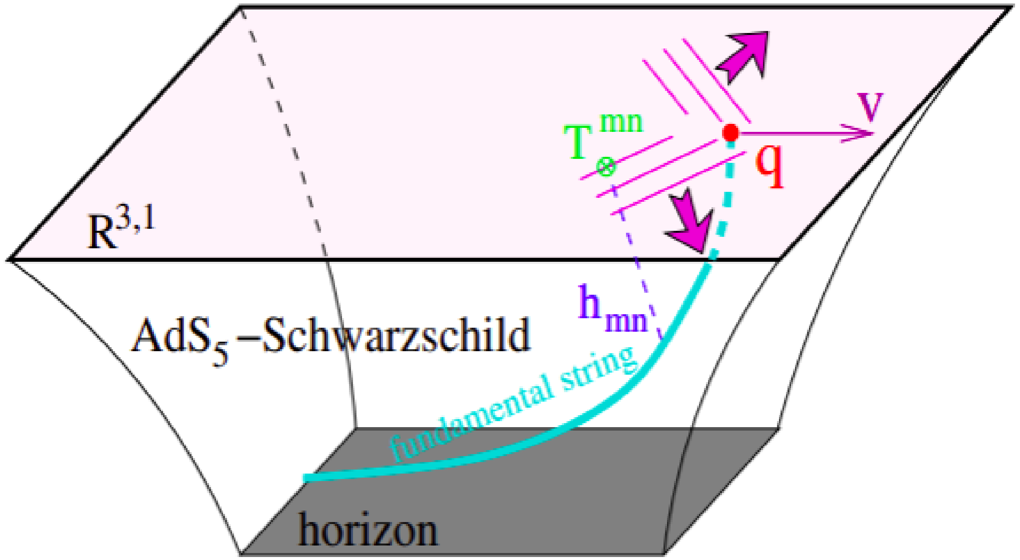
Gong Show

25/10/2023

Introduction

- Open string stretched from the boundary to the horizon of an asymptotically AdS Schwarzschild spacetime
- Holographic dual of a supersymmetric toy-model for dynamics of a heavy quark in quark-gluon plasma

Gubser 06', 09'



Introduction

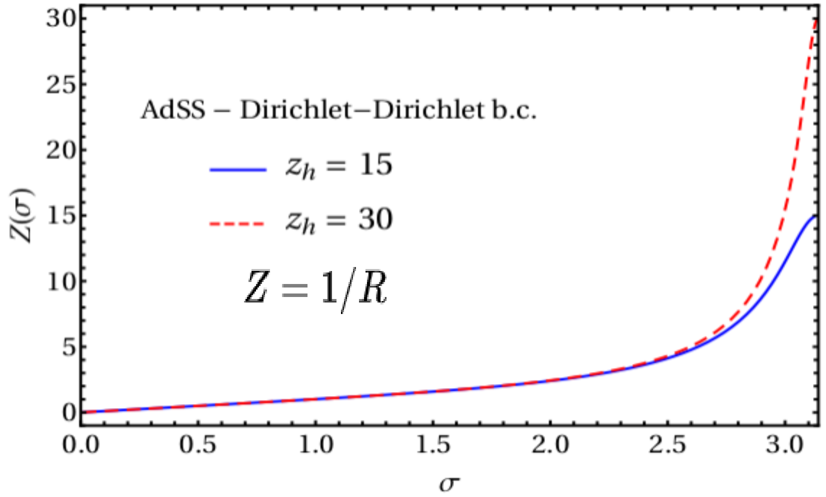
- **Maldacena, Shenker, Stanford 15'** cojectured a sharp bound on the rate of growth of chaos in thermal quantum systems with a large number of degrees of freedom.

- The chaos bound (MSS scale): $\lambda_L \leq 2\pi T$

Motivation

- What is the meaning of the Lyapunov exponent on the gravity side?
- Why would there ever be anything like the bound on chaos for dynamics of particles and strings in asymptotically AdS spacetimes? **Giataganas 19'**
- Literature is full of different setups that violate the MSS bound **Hashimoto, Tanahashi 17'; Čubrović 19'; Yu, Chen, Mu, He 23'**
- Nice setup for exploring these questions: open string stretched from the boundary to the horizon of BH and black branes in AdS, since there is a natural interpretation in holography.

Open static string in AdS-Schwarzschild



- Simple analytic method for estimating LE

$$S_{\text{Poly.}} = \frac{1}{\ell_s^2}\int d\tau d\sigma \sqrt{-\det h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X)$$

- Static string configuration

$$T(\tau,\sigma) = \tau, \quad R(\tau,\sigma) \equiv R(\sigma), \quad X(\tau,\sigma) \equiv X(\sigma)$$

- Virasoro constraint decouples “dynamics”

$$4h^3(R)R^3(\sigma) + h^2(R)h'(R)R^4(\sigma) + h'(R)R'^2(\sigma) - 2h(R)R''(\sigma) = 0$$

- Small perturbations near the horizon exhibit positive LE

$$\delta R'' - (D-1)^2 r_h^2 \delta R = 0 \qquad \delta R \propto e^{\pm 2\lambda_L \sigma} \qquad \Rightarrow \qquad \boxed{\lambda_L = 2\pi T}$$

- Fast Lyapunov indicator, i.e. finite time/range LE $0 \leq \sigma \leq \pi$
- MSS bound is saturated despite the fact that the system is integrable.
- Same result for a black D3 brane setup (UV deformed).

D1-D5-p black string

- Multi-charged higher-dimensional analogue of RN black holes from type-IIB supergravity compactified on $T^5 \equiv T^4 \times S^1$
- Bound state of D1 and D5 branes together with KK mode/momentum *p* along compactified S^1 shared with D1 string.
- Novelty here is the presence of the rotating horizon (IR deformation)

	Near-horizon extremal	**Near-horizon near-extremal**
Reissner-Nordstrom	$AdS_2 \times S^2$	$\mathrm{Rindler} \times S^2$
D1-D5-p black string	$AdS_3 \times S^3$	rotating $\mathrm{BTZ} \times S^3$

Open static string in D1-D5-p background

- Analytic estimate yields the following form of the Lyapunov exponent

"dilute gas" approximation

$$\lambda_L = \frac{r_0}{r_1 r_5}\sqrt{1+\frac{r_0^2(r_1^2+r_5^2)}{2r_1^2 r_5^2}\cosh(2\Sigma)} \approx \frac{r_0}{r_1 r_5} = \frac{2\pi T}{\sqrt{1-L^2\Omega^2}}, \quad L\Omega \in [0,1)$$

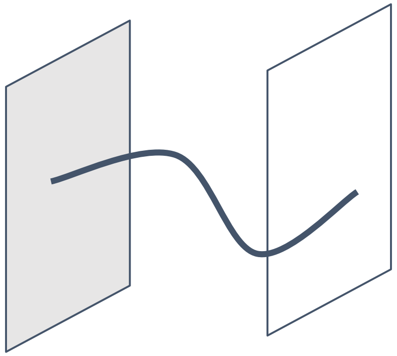
- We can compare this result to the one existing in the literature, obtained from OTOC calculations **Jahnke, Kim, Yoon 19'**
- Lyapunov spectrum: one value does not saturate, while the other violates it

$$\lambda_L^{\pm} = \frac{2\pi T}{1 \mp L\Omega}$$

- Our result is the exact geometric mean

$$\lambda_L^- < \lambda_L < \lambda_L^+$$

Retarded Green's function



- Let us now study dynamics of transverse fluctuations on the string.
- We will work in the NG formalism in the static gauge

$$\begin{aligned} t(\tau,\sigma) &= \tau \equiv t, \quad R(\tau,\sigma) = \sigma \equiv r, \\ \Psi(\tau,\sigma) &\equiv \psi(t), \quad \Theta(\tau,\sigma) \equiv \pi/2, \quad \Phi(\tau,\sigma) \equiv \phi(t), \quad X_5(\tau,\sigma) \equiv X(t,r) \end{aligned}$$

$$X(t,r) = \int \frac{d\omega}{2\pi} e^{-i\omega t} X_\omega(r)$$

- 2-point correlation function in the IR-region is obtained from the study of dynamics of open string in $\mathrm{BTZ} \times S^3$ geometry (no rotation)

$$\lambda_L = 2\pi T$$

$$\frac{h(r)}{r^4}\frac{d}{dr}\left(h(r) r^4 \frac{dX_\omega(r)}{dr}\right) + \frac{L^4\omega^2}{r^4} X_\omega(r) = 0 \qquad \xi = h(r) = 1 - r_0^2/r^2$$

$$X_\omega(\xi) = \widetilde{\mathcal{A}}\, \xi^{-i\alpha}\ {}_2F_1\left(a,b,c;\xi\right) \qquad r \gg r_0$$

$$X_\omega(r) \propto \widetilde{\mathcal{S}}\, r^{-d+\Delta} + \widetilde{\mathcal{F}}\, r^{-\Delta}, \quad d = \Delta = 3,$$

$$\mathcal{G}_R^{(T)}(\omega) \propto \widetilde{\mathcal{F}}/\widetilde{\mathcal{S}} = \frac{\Gamma\left(1 - i\frac{\omega}{2\lambda_L}\right)\Gamma\left(\frac{3}{2} - i\frac{\omega}{2\lambda_L}\right)}{\Gamma\left(-i\frac{\omega}{2\lambda_L}\right)\Gamma\left(-\frac{1}{2} - i\frac{\omega}{2\lambda_L}\right)}$$

Signals of instability

- Poles in the retarded Green's function give us a spectrum of quasi-normal modes

$$\mathcal{G}_R^{(T)}(\omega) \propto \widetilde{\mathcal{F}}/\widetilde{\mathcal{S}} = \frac{\Gamma\left(1 - i\frac{\omega}{2\lambda_L}\right)\Gamma\left(\frac{3}{2} - i\frac{\omega}{2\lambda_L}\right)}{\Gamma\left(-i\frac{\omega}{2\lambda_L}\right)\Gamma\left(-\frac{1}{2} - i\frac{\omega}{2\lambda_L}\right)}$$

$$\omega_{\mathfrak{m}} = -i(\mathfrak{m}+1)\lambda_L, \quad \mathfrak{m} = 1, 2, 3, \cdots .$$

Cardoso et. al. 09' "*Geodesic stability, Lyapunov exponents and quasinormal modes*"

- True interpretation of the Lyapunov exponent is as the timescale for the decay of instabilities on the fundamental strings.
- In dual field theory it gives us a thermalization timescale for a heavy quasiparticle in thermal plasma.

Conclusions

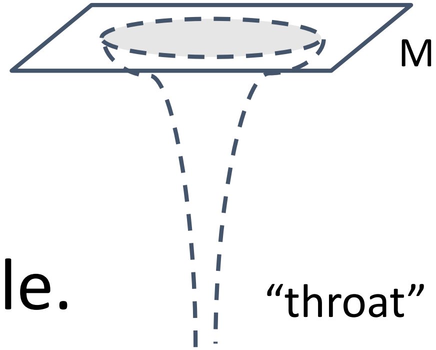
- We have shown how MSS scale gets modified in the presence of ***IR*** and ***UV*** deformations

  IR deformation: rotating thermal horizon
  UV deformation: asymptotically flat region

- MSS scale is in general an instability scale connected to the relaxation time in a dual field theory.
- Only for static, spherically symmetric, large *N*, etc. setups it becomes the same scale as the MSS bound scale.

Thank You
Merc
Dank Je
Grazie
Mamana
Kiitos
Juspaxc
Mochchakkeram
Spasibo
Multumesc
Obrigado
Terma Kasih
Maake
Asante
Chokrane
Kia Ora
Danko
Matondo
brigado
Dyakuyu
Arigato
Matur Nuwun
Raibh Maith Agat
Mercí

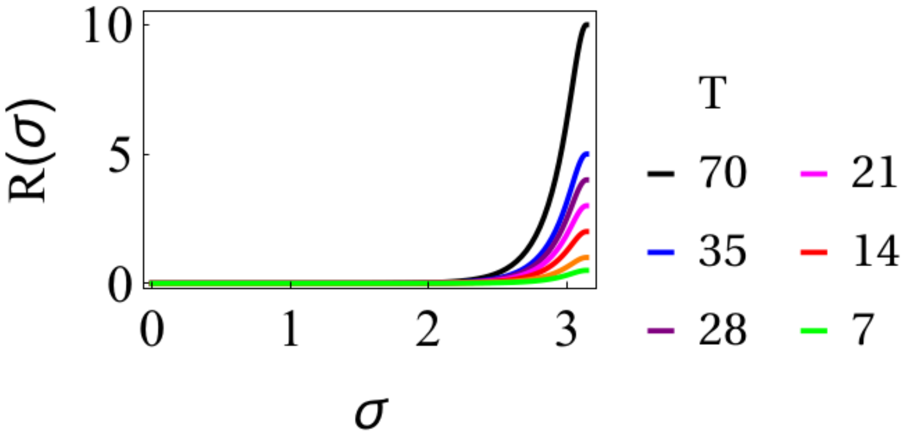
Black D3 brane

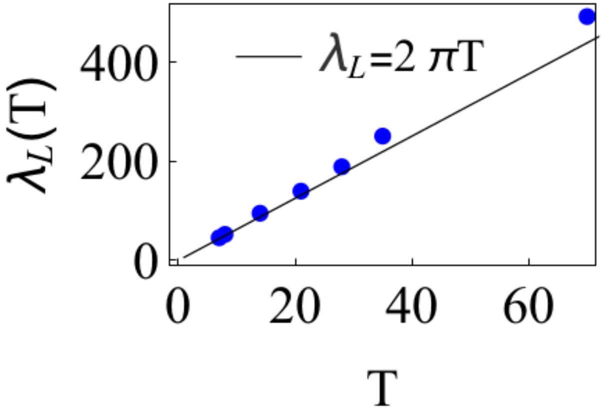


- We can preform the same analysis. Also integrable.
- Novelty is the existence of the asymptotically flat region (the one that decouples in the Maldacena's limit), that acts as an UV deformation.

$$2h(R)R(\sigma)R''(\sigma) - (2h(R) + R(\sigma)h'(R))R'^2(\sigma) + \\ + \frac{2v^2h^3(R)}{f^4(R)}\left(1 - R(\sigma)\left(\frac{h'(R)}{2h(R)} + 2\frac{f'(R)}{f(R)}\right)\right) = 0.$$

$$\delta R'' - \frac{16r_h^2}{Q + r_h^4}\delta R = 0$$





Thermal horizons are "fake" generators of chaos

- All of the systems we have analyzed here are integrable, our analytic calculations in the near-horizon region suggest that we are dealing with an unstable saddle point.
- MSS scale is the only dimensionful scale in the problem, what is its interpretation?
- We are working in the integrable sector of the theory. How than we have non-vanishing Lyapunov exponent? Counter-example: an inverted harmonic oscillator.
- Reducing the amount of symmetries (i.e. breaking of the rotational symmetry) modifies MSS scale. Furthermore, outside of "dilute gas" approximation we get higher-order in temperature corrections.