Bootstrapping scattering amplitudes at high energies

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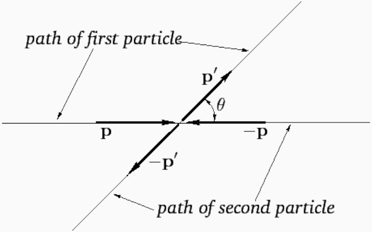
S-Matrix Bootstrap Basics

- **Bootstrap Philosophy:** Any 2-2 scattering amplitude must obey constraints arising from Lorentz invariance, causality and unitarity.
- **Definition:** $\mathcal{M}(s,t) \equiv$ 2-2 amplitude of identical massive scalars (e.g., $\pi^0\pi^0 \to \pi^0\pi^0$) where

$$s = (p_1 + p_2)^2 = 4E^2, \quad t = (p_1 - p_3)^2 = \frac{1}{2}(s-4)(\cos\theta - 1) \tag{1}$$

  Energy-Momentum conservation: $s + t + u = 4m^2$

- In the centre-of-mass frame



- **Problem:** Find bounds on the space of all functions of two complex variables consistent with the bootstrap/physical constraints

Beyond low energies

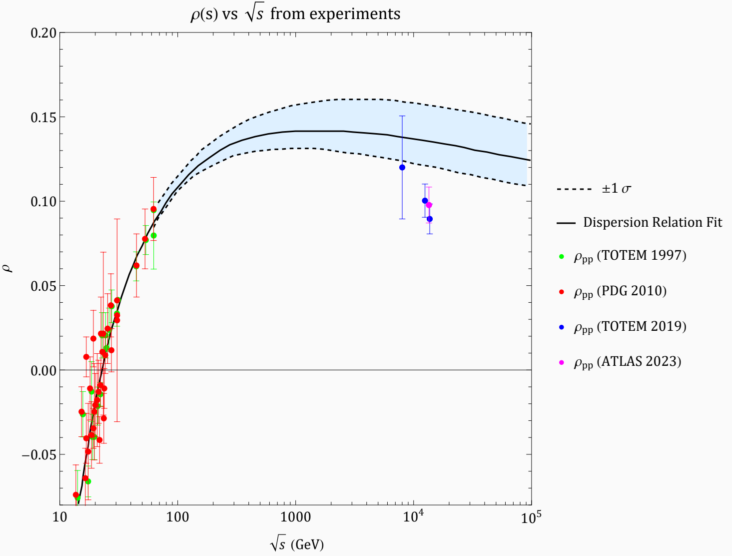
- **Why is this a good strategy?**
  - The expectation is that *physical theories of interest may saturate the bootstrap bounds* and therefore be solvable. Due to cues from the conformal bootstrap program.
  - Bootstrap bounds also allow us to test the validity of the basic QFT axioms. If experiments violate the bounds, one of the axioms has to go/be modified. Signs of new physics?
- Past work has focused on bounds on low-energy observables like the scattering lengths and the quartic coupling $\lambda \equiv \frac{1}{32\pi}\mathcal{M}\left(\frac{4m^2}{3}, \frac{4m^2}{3}\right)$. E.g. Sever, Guerreri ('21), Miro, Guerreri, et al ('22),

$$-8.02 \leq \lambda \leq 2.66 \tag{2}$$

- **Beyond low-energy observables:** We study the behaviour of $\rho(s) = \mathrm{Re}\mathcal{M}(s,0)/\mathrm{Im}\mathcal{M}(s,0)$ parameter at large energies. Khuri, Kinoshita ('65)
- Dispersion relations relate $\rho(s)$ to the total cross section at large energies. E.g., if $\sigma_{tot}(s) \sim \log^2(s)$, $\rho(s) \sim \pi/\log(s)$

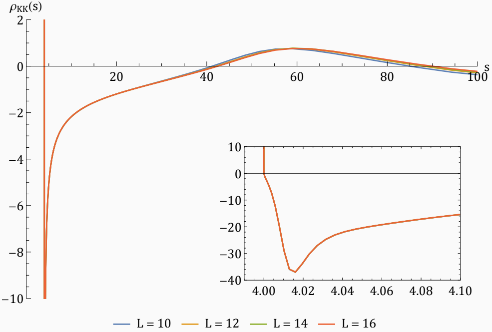
Experimental measurement of $\rho(s)$

- Experimental measurement of ρ for pp scattering by ATLAS (2023)



$\rho(s)$ from the bootstrap

- We study $\rho(s)$ for the amplitude with the minimum quartic coupling λ when S-wave (spin 0) and D-wave (spin 2) scattering lengths are fixed to the pion values.
- We observe that $\rho(s)$ crosses from negative to positive as in the *pp* scattering but then it crosses again and becomes negative.



- We set up the numerical bootstrap using the Primal approach & use the Crossing Symmetric Dispersive Representation for the amplitude.

THANK YOU!

Extra: PWE and Bootstrap Constraints

- **Partial Wave Equation**

$$\mathcal{M}(s,t) = 32\pi\sqrt{\frac{s}{s-4}}\sum_{\ell=0}^{\infty}(2\ell+1)f_\ell(s)P_\ell\left(z \equiv 1+\frac{2t}{s-4}\right) \quad (3)$$

- **Bootstrap Constraints**
  - **Unitarity** leads to the condition $|f_\ell(s)|^2 < \text{Im}f_\ell(s) < 1$ imposed as

$$\begin{pmatrix} 1+2\text{Re}f_\ell(s) & 1-2\text{Im}f_\ell(s) \\ 1-2\text{Im}f_\ell(s) & 1-2\text{Re}f_\ell(s) \end{pmatrix} \succeq 0, \quad s \geq 4\,. \quad (4)$$

  - **Crossing Symmetry:** $\mathcal{M}(s,t) = \mathcal{M}(t,u) = \mathcal{M}(u,s)$
  - **Maximal Analyticity:** Only branch cuts from $s \in (4,\infty)$ and simple poles for bound states at $s \in (0,4)$ and their images under crossing
  - **Real Analyticity:** $\mathcal{M}(s^*,t^*) = \mathcal{M}^*(s,t)$
  - **Polynomials boundedness:** $\lim_{|s|\to\infty}\left|\frac{\mathcal{M}(s,t)}{s^2}\right| \to 0$ for fixed t

Extra: Crossing Symmetric Dispersion relation

- **Crossing Symmetric Dispersion relation.** (Auberson, Khuri ('72), Sinha, Zahed ('21))

$$\mathcal{M}_0\left(s_1, s_2\right) = \alpha_0 + \frac{1}{\pi} \int_{\frac{8}{3}}^{\infty} \frac{d\tau}{\tau} \mathcal{A}_0\left(\tau; \widehat{s}_2\left(\tau, \beta\right)\right) \times H_0\left(\tau; s_1, s_2, s_3\right), \tag{5}$$

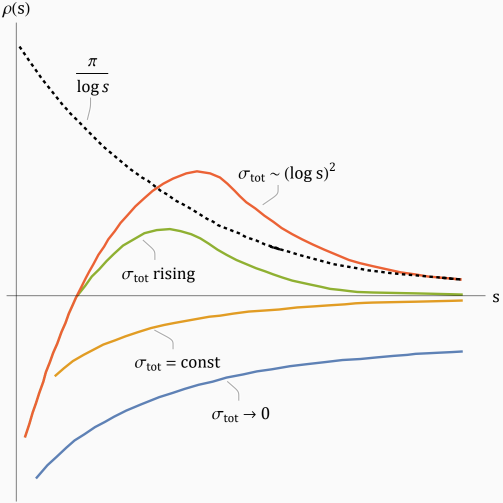
- $\alpha_0 = \mathcal{M}_0(0,0)$ is the subtraction constant, $\mathcal{A}_0\left(s_1; s_2\right)$ is the s-channel discontinuity and

$$H_0(\tau; s_1, s_2, s_3) \equiv \frac{s_1}{\tau - s_1} + \frac{s_2}{\tau - s_2} + \frac{s_3}{\tau - s_3},$$
$$\widehat{s}_2(\tau, \beta) \equiv \tau \frac{-1 + \sqrt{1 + 4\beta}}{2}, \quad \beta = \frac{a}{\tau - a}$$

- $s_1 = s - \frac{4m^2}{3}$ and $a = \frac{s_1 s_2 s_3}{s_1 s_2 + s_2 s_3 + s_3 s_1}$

Extra: Behavior of $\rho(s)$ for different $\sigma_{tot}(s)$

- From twice-subtracted dispersion relations, it can be shown (Khuri, Kinoshita ('65))



Extra: Ansatz for the numerical boostrap

- We choose the "wavelet" ansatz

$$\mathrm{Im} f_\ell(s) = b_0 \delta_{\ell,0} + \left(\frac{s-4}{s}\right)^{2\ell+\frac{1}{2}} \sum_{\kappa \in \Sigma} b_{\ell,\kappa} \mathrm{Im}\Delta_\kappa(s), \quad s > 4 \tag{6}$$

- Here, $\Delta_\kappa(s) = \rho_\kappa(s) \frac{\Gamma}{(s-\kappa)^2+\Gamma}$, $\rho_\kappa(s) = \frac{\sqrt{4-s}-\sqrt{\kappa-4}}{\sqrt{4-s}+\sqrt{\kappa-4}}$ and

$$\mathrm{Im}\Delta_\kappa(s) = \sin\left(2\arctan\sqrt{\frac{s-4}{\kappa-4}}\right) \frac{\Gamma}{(s-\kappa)^2+\Gamma}\,. \tag{7}$$

- Ansatz for spin two absorptive partial wave