

# The Gauss-Bonnet Invariant in 5D, N=1 Gauged Supergravity

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Based on: [arxiv:2302.14295](https://arxiv.org/abs/2302.14295), [arxiv:2309.07637](https://arxiv.org/abs/2309.07637), [arxiv:2311.????](https://arxiv.org/abs/2311.????)

Gravity arises by gauging supersymmetry  
SUGRA

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## Why SUGRA?

Effective theory of low-energy string theory.

$$L_{\text{ST}}^{\text{low}} = L_{\text{SG}} + [D^p R^q \dots]$$

Playground for local SUSY.

Precision tests of AdS/CFT require  $R^2$  corrections.

Component construction of some invariants is daunting so not all were understood.

[[Bobev, Hristov, and Reys, 2021](#)]

[[Cassani, Ruipérez, and Turetta, 2022](#)]

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Superspace

$$W(x, \theta) = \mathcal{W}(x) + \mathcal{W}^{\alpha\beta}(x) \theta^{\alpha}\theta^{\beta} + \mathcal{W}^{\alpha\beta\gamma}(x) \theta^{\alpha}\theta^{\beta}\theta^{\gamma} + \dots$$

$$W|_{\theta=0} = \mathcal{W}(x)$$

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Necessary to parse upwards of 100,000 terms.

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# Computational Approach

Component reduction is largely algorithmic.

Cadabra can be used [Peeters, 2007].

## Python + Cadabra Reduction Class

```
ex2Der := \Lambda_{k}^{\hat{a}A_1} \nabla_{A_1} \{ \@ (ex1Der) \};
cleverSimplify = CleverSimplify(ex2Der)
cleverSimplify.main(15);
cleverSimplify.terms_per_iteration;
```

$$\Lambda_k^{\hat{a}A_1} \nabla_{A_1} (I \epsilon_{li} \lambda_{\hat{a}}^i \mathcal{W}^{-1})$$
$$- 4 I \epsilon_{ki} \epsilon_{lj} X^{ij} \mathcal{W}^{-1} - \epsilon_{ki} \epsilon_{lj} \epsilon^{\hat{\alpha}\hat{\beta}} \lambda_{\hat{\alpha}}^i \lambda_{\hat{\beta}}^j \mathcal{W}^{-2}$$

[1, 2, 2, 2, 5, 5, 3, 3]



# Results

Computationally generated all curvature-squared invariants in components.

$$S_{\text{HD}} = S_{\text{gSG}} + 1 S_{\text{Weyl}^2} + 2 S_{\text{log}} + 3 S_{R^2}$$

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## EOM of gravitational (Weyl) multiplet

$$\begin{aligned} J_{\text{log}} = 0 = & -\frac{3}{1024} WY - \frac{69}{1024} W^{ab} W_{ab} W + \frac{3}{32} W - \frac{3}{64} F_{ab} W^{ab} - \frac{3}{256} j X^j \\ & + \frac{3}{128} F^{ab} F_{ab} W^{-1} - \frac{9}{128} X^{ij} X_{ij} W^{-1} + \frac{3i}{32} ({}^a) W^{-1} i a i \\ & + \frac{3}{64} W^{-1} ({}^a W) a W - \frac{3i}{128} ({}^{ab}) F_{ab} i i W^{-2} - \frac{3i}{64} X^{ij} i j W^{-2} \\ & - \frac{3}{256} i i j j W^{-3} \end{aligned}$$

[Gold, Hutomo, Khandelwal, and Tartaglino-Mazzucchelli, 2023b]

[Gold, Hutomo, Khandelwal, and Tartaglino-Mazzucchelli, 2023c]

Computationally aided map to alternative gravitational multiplet and going on-shell yields Gauss-Bonnet combination:

$$(16 G)e^{-1}L_{2\partial+4\partial} = c_0 R + 12c_1 g^2 - \frac{1}{4}c_2 G_{ab}G^{ab} \\ + \frac{1}{12} \frac{1}{3} c_3^{abcde} C_a G_{bc} G_{de} + \dots L_{\text{GB/onshell}}$$

$$L_{\text{GB/onshell}} = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2 + \frac{1}{4}G^4 \\ - \frac{1}{2}W_{abcd}G^{ab}G^{cd} + \frac{1}{2} \frac{1}{3}^{abcde} C_a R_{bc}{}^{fg} R_{defg}$$

[Gold, Hutomo, Khandelwal, Ozkan, Pang, and Tartaglino-Mazzucchelli, 2023a]

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- G. Gold, J. Hutomo, S. Khandelwal, and G. Tartaglino-Mazzucchelli. Curvature-squared invariants of minimal five-dimensional supergravity from superspace. *Physical Review D*, 107(10), May 2023b.
- G. Gold, J. Hutomo, S. Khandelwal, and G. Tartaglino-Mazzucchelli. Components of curvature-squared invariants of minimal supergravity in five dimensions. 2023c.
- K. Peeters. Introducing cadabra: a symbolic computer algebra system for field theory problems, 2007.