Wormholes and surface defects in ensemble holography

Based on ongoing work with J. Raeymaekers

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Ensemble holography

Ensemble holographic principle:

$$Z_{grav^{d+1}} = \langle Z_{CFT^d} \rangle_{(G,\rho)} = \sum_{I \in G} \rho_I Z^I_{CFT},$$

G : CFT ensemble

$\{\rho_I\}_{I \in G}$: probability weights

(2d JT gravity, 3d gravity, Narain ensemble...)

The toy model: 2d RCFT vs. exotic 3d gravity

Rational (opp. to Narain) **CFT ensemble**:

Modular invariants $\{Z^{l}_{CFT}\}$ of $\widehat{u(1)}_k$ chiral algebra - 2d compact boson

$$R^2 \int d^2 z \partial X \bar{\partial} X \quad \text{at} \quad R^2 = k/(l^2),$$

parametrized by $l \in \mathrm{Div}(k) \Rightarrow$ finite ensemble!

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Tentative **bulk theory**, 1-boundary case:

- $U(1)_k \times U(1)_{-k}$ 3d CS on $D_2 \times \mathbb{S}^1$
- $+$ (anti-)holomorphic b.c.
- $+$ sum over geometries

$$Z^{(1)}_{grav}(\zeta) \propto \sum_{\gamma \in SL(2,Z)} |Z^{D_2 \times S^1}_{CS,k}(\gamma \cdot \zeta)|^2$$

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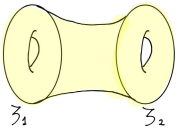
[Raeymaekers, 2021] $\rightarrow \{\rho_l\}_{l \in \mathrm{Div}(k)}$ ✓

Euclidean wormholes

2-boundary, connected case ("euclidean wormholes"):

$$Z_{wh}(\zeta_1, \zeta_2) =$$



$$= \langle Z_{CFT}(\zeta_1) Z_{CFT}(\zeta_2) \rangle_{(\rho)} - \langle Z_{CFT}(\zeta_1) \rangle_{(\rho)} \langle Z_{CFT}(\zeta_2) \rangle_{(\rho)}$$

$$\begin{cases} \neq 0 \text{ for generic level } k \\ \text{determined by } \{\rho_I\}_{I \in G} \text{ only} \end{cases}$$

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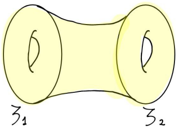

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From **bulk model**:

$$Z_{wh}(\zeta_1, \zeta_2) \overset{?}{\propto} \sum_{\gamma \in SL(2,Z)} |Z_{CS,k}^{T^2 \times I}(\gamma \cdot \zeta_1, \bar{\zeta}_2)|^2$$

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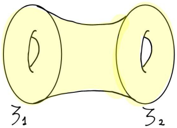
$Z_{wh}(\zeta_1, \zeta_2) =$ 

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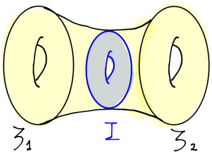
Findings: Z_{wh} match if $Z^{T^2 \times I}_{CS,k} = \sum_{I \in \mathrm{Div}(k)} c_I Z^I_{CFT}$ for some "wormhole coefficients" c_I (highly constrained by the ρ_I)!

- **Pure** $U(1)_k$ Chern-Simons on $T^2 \times I$ gives only the diagonal modular invariant, $c_I^{pure} = \delta_I^1 \neq c_I$.
- Not the full answer, need contribution from **"exotic" wormholes**!
- Bulk intepretation: CS path integral with insertion of topological **surface (condensation) defects**



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Exotic wormholes & surface defects

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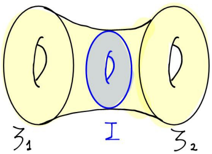

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Some questions:

- Coefficients c_I almost fixed (up to phases) by consistency. Geometric intepretation (higher gauging...)?
- For some k (e.g. $k = 1$) there is no ensemble. Exotic wormholes must cancel standard wormholes to give $Z_{wh} = 0$.
- Extension to multiple rational bosons or other RCFTs?

Thank you!