Superconformal index for quiver quantum mechanics

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Based on:

- D. Mirfendereski, J.Raeymaekers, C.S., D.V.d.Bleeken, *The geometry of gauged (super)conformal mechanics*, JHEP **08** (2022), [arXiv: 2203.10167 [hep-th]].
- J.Raeymaekers, C.S., D.V.d.Bleeken, *Superconformal index in $\mathcal{N} = 2B$ quantum mechanics*, 2311.xxxxx.

Susy mechanics of D-particles

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* $(d = 1, \mathcal{N} = 4)$ Effective dynamics: $(x_i, \lambda_\alpha, \bar{\lambda}^\alpha, D)$, (Denef,2002).

$$L_B = \frac{1}{2}\left(\mu + \frac{\kappa}{4|\vec{x}|^3}\right)\left(\dot{\vec{x}}^2 + D^2\right) - \left(f + \frac{\kappa}{2|\vec{x}|}\right) D - \kappa \vec{A}^d \cdot \vec{x}$$

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* 3-particles: $x^A := x^{ia}$, $A = 1, \cdots, 9$. **(scaling limit $\rightarrow \mu_a = f_a = 0$)** (Anninos, Anous, de Lange, Konstantinidis, 2013).

$$L_B = \frac{1}{2} G_{AB} \dot{x}^A \dot{x}^B + A_A \dot{x}^A - V(x)$$

* $SL(2, \mathbb{R})$ invariant: $L_\xi G_{AB} = -G_{AB}$, $L_\xi V = V$, (dilatations)

$\xi = -\frac{1}{2} dK$, $i_\xi F = 0$ (special conformal).

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Gauged Model on $\mathcal{M}$: $\delta_\lambda x^A = \lambda^I k_I,$

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Conclusion

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3. computed in simpler ($\mathcal{N}=2$) cases (J. Raeymaekers, C.S., D.V.d.Bleeken, 2311.xxxxx):

$$\mathrm{tr}[(-1)^F y^{L_0+J}] = \frac{1}{N!}\left(\frac{1}{2\pi}\right)^N \int \check{F}^{\pm} \wedge \check{F}^{\pm} \wedge \ldots \wedge \check{F}^{\pm}$$

$\mathcal{N} = 2$ superconformal index

* $\mathcal{N} = 2$ superconformal algebra:

$$[L_m, L_n] = (m-n)L_{m+n}$$

$$[L_0, \mathcal{G}_{\pm\frac{1}{2}}] = \mp\frac{1}{2}\mathcal{G}_{\pm\frac{1}{2}}, \qquad [L_{\mp 1}, \mathcal{G}_{\pm\frac{1}{2}}] = \mp\mathcal{G}_{\mp\frac{1}{2}}, \quad [R, \mathcal{G}_{\pm\frac{1}{2}}] = \mathcal{G}_{\pm\frac{1}{2}}$$

$$\{\mathcal{G}_{\pm\frac{1}{2}}, \mathcal{G}^\dagger_{\pm\frac{1}{2}}\} = 2L_0 \pm R, \qquad \{\mathcal{G}_{\pm\frac{1}{2}}, \mathcal{G}^\dagger_{\mp\frac{1}{2}}\} = 2L_{\pm 1}, \quad \{\mathcal{G}_\alpha, \mathcal{G}_\beta\} = 0$$

* lowest weight state: $|h, r\rangle$

$$\mathcal{G}_{+1/2}\,|h, r\rangle = \mathcal{G}^\dagger_{-1/2}\,|h, r\rangle = 0$$

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$\mathcal{N} = 2$ superconformal index

* (unitary, lowest-weight, infinite-dimensional) Irreps:

$$\text{chiral}: \qquad (h)_r + \left(h + \frac{1}{2}\right)_{r-1}, \qquad r = 2h$$

$$\text{anti}-\text{chiral}: \qquad (h)_r + \left(h + \frac{1}{2}\right)_{r+1}, \qquad r = -2h$$

$$\text{long}: \quad (h)_r + \left(h + \frac{1}{2}\right)_{r-1} + \left(h + \frac{1}{2}\right)_{r+1} + (h+1)_r \,, \quad |r| < 2h$$

$\mathcal{N} = 2$ superconformal index

* Well-defined '(refined-)superconformal-index':

$$\Omega_{\pm}(\zeta) = \mathrm{tr}(-1)^F e^{-\beta \mathcal{H}_{\pm}} \zeta^{\pm J} \ , \qquad \mathcal{H}_{\pm} = \omega \{ \mathcal{G}_{\pm\frac{1}{2}}, \mathcal{G}^{\dagger}_{\pm\frac{1}{2}} \} = \omega(2L_0 \pm R)$$

which counts

$$\begin{aligned} \Omega_{+}(\zeta) &= \sum_h \left(N^{+}_{\mathrm{anti-chiral}}(h) - N^{-}_{\mathrm{anti-chiral}}(h) \right) \zeta^{J}, \\ \Omega_{-}(\zeta) &= \sum_h \left(N^{+}_{\mathrm{chiral}}(h) - N^{-}_{\mathrm{chiral}}(h) \right) \zeta^{-J} \end{aligned}$$

* Computable through supersymmetric localization of the path integral:

$$\Omega_{\pm}[e^{\beta\lambda}] = (\pm 1)^N \prod_{i=1}^{N} \left(e^{\frac{\beta\lambda\omega_i}{2}} - e^{\frac{\beta\lambda\omega_i}{2}} \right)^{-1} .$$

Results and Open Questions

*** Results**

1. Coulomb quiver mechanics (in the scaling limit) is described by the (4,4,0) gauged super(conformal) mechanics, which provides a geometric description for the susy ground states.
2. An index is defined and computed for a general $\mathcal{N} = 2B$ superconformal quantum mechanics.

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*** Open Questions**

1. Geometric interpretation for the index
2. Generalization to (gauged) $\mathcal{N} = 4B$ superconformal mechanics
3. Identify these ground states in quiver D-brane quantum mechanics to obtain a description of the black hole microstates in sugra
4. Comparison with the Higgs branch results
5. Contributions from higher order terms in superpotential

THANK YOU !