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ON INFLATION WITH TACHYON FIELD IN THE HOLOGRAPHIC BRANEWORLD

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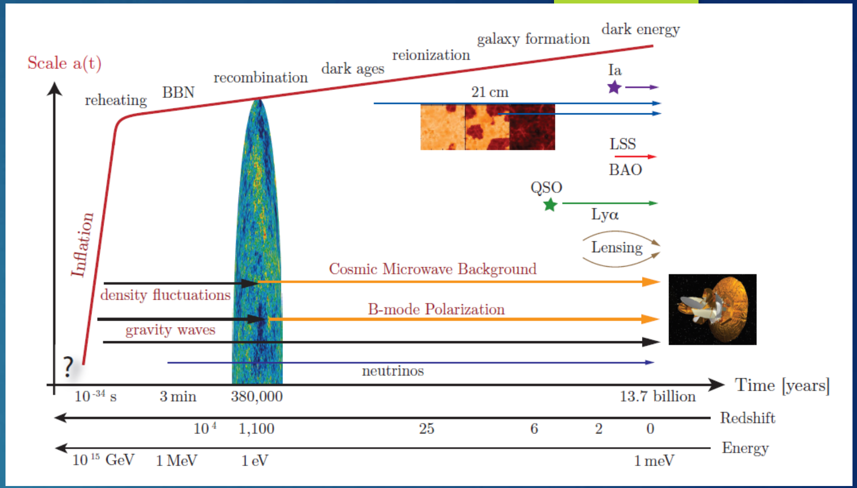
Inflation

- The ***inflation theory*** proposes a period of extremely rapid (exponential) expansion of the universe during the an early stage of evolution of the universe.
- The inflation theory predicts that during inflation (it takes about $10^{-34}\ s$) radius of the universe increased, at least $e^{60} \approx 10^{26}$ times.



Baumann, D. TASI Lectures on Inflation. (2009)

- FLRW metric $\quad ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2)$
- The Einstein equations $\quad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$
- The Friedman equations

$$T^\mu_\nu = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\dot H + H^2 = \frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

A condition for inflation

$$\frac{d^2 a}{dt^2} > 0 \quad \Leftrightarrow \quad \rho + 3p < 0$$

A negative pressure runs inflation!

- The most important way to test inflationary cosmological models is to compare the computed and measured values of the ***observational parameters.***

$a(t)$ - the scale factor
k – the spatial curvature parameter
$H(t)$ - the Hubble rate

Standard Single Field Inflation

- The simplest models - standard single scalar field inflation, a field ϕ – ***inflaton***

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- The Friedmann equation $\quad H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$
- Time evolution of homogeneous scalar field, for FLRW metric

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \qquad V' \equiv \frac{\partial V}{\partial \phi}$$

- Hubble hierarchy (slow-roll) parameters

$$\epsilon_{i+1} \equiv \frac{d\ln|\epsilon_i|}{dN} = \frac{d\ln|\epsilon_i|}{d\ln a}, \quad i \geq 0, \quad \epsilon_0 \equiv \frac{H_*}{H}$$

$$\epsilon_1 = -\frac{\dot{H}}{H^2} \qquad \epsilon_2 = 2\varepsilon_1 + \frac{\ddot{H}}{H\dot{H}}$$

- The end of inflation $\epsilon_1(t_{end}) \approx 1$
- The number of e-fold $(N \approx 60)$

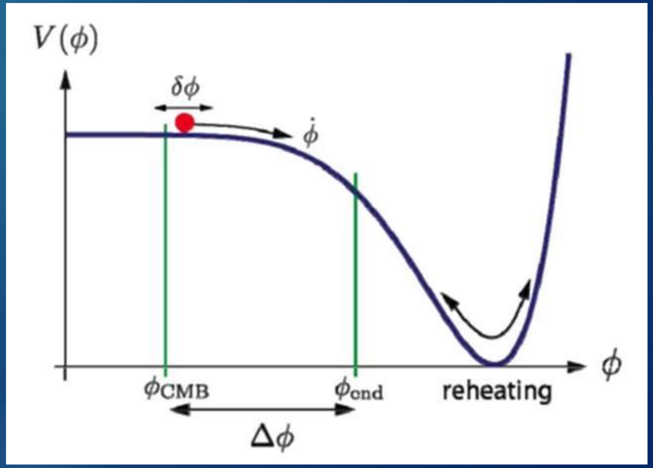
$$N = \ln\frac{a_{end}}{a} = \int_{t_{cmb}}^{t_{end}} d\ln a = \int_{t_{cmb}}^{t_{end}} H dt = \int_{\phi_{cmb}}^{\phi_{end}} \frac{H}{\dot{\phi}} d\phi$$

Slow-roll condition

$$\dot{\phi}^2 \ll V(\phi) \Rightarrow \begin{cases} H^2 \approx \frac{8\pi G}{3} V(\phi) \\ 3H\dot{\phi} + V' \approx 0 \end{cases}$$

In order for inflation to last long enough

$$\ddot{\phi} \ll |3H\dot{\phi}|$$
$$\ddot{\phi} \ll |V'|$$



Baumann, D. TASI Lectures on Inflation. (2009)

Tachyon inflation

- ***String theory***: states of quantum fields with imaginary mass (i.e., negative mass squared).
- It **was realized** that the imaginary mass creates an instability and tachyons spontaneously decay through the process known as ***tachyon condensation.***
- Effective tachyonic field theory was **proposed** by ***A. Sen***
- The corresponding Lagrangian and the Hamiltonian are

$$p \equiv \mathcal{L}(\dot{\theta},\theta) = -V(\theta)\sqrt{1-\dot{\theta}^2}$$

$$\rho \equiv \mathcal{H} = \frac{V(\theta)}{\sqrt{1-\dot{\theta}^2}}$$

- Properties of a tachyon potential

$$V(0) = const, \quad V'(\theta > 0) < 0, \quad V(|\theta| \to \infty) \to 0.$$

Dirac-Born-Infeld (DBI) Lagrangian

$$\mathcal{L}(X,\varphi) = -\frac{1}{f(\varphi)}\sqrt{1-2f(\varphi)X} - V(\varphi),$$

Examples:

$$V(\theta) = \omega\theta^{-n}$$

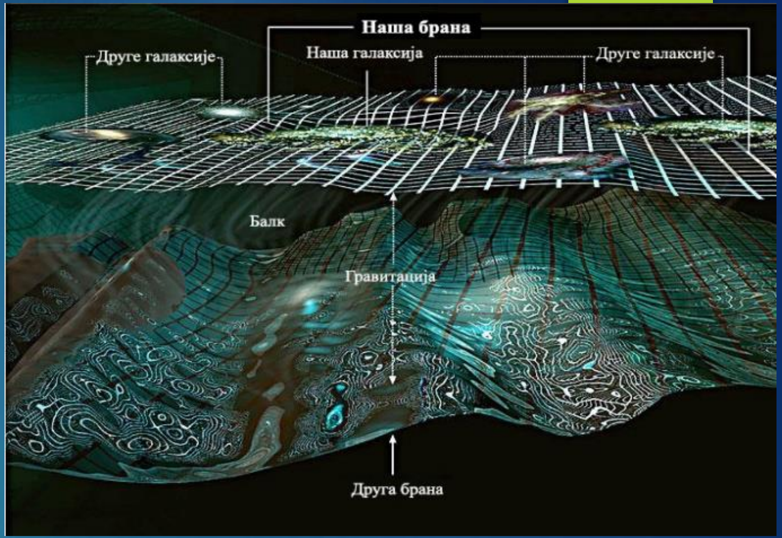
$$V(\theta) = e^{-\omega\cdot\theta}$$

$$V(\theta) = \frac{1}{\cosh(\omega\cdot\theta)}$$

D. Steer, F. Vernizzi, Tachyon inflation: *Tests and comparison with single scalar field inflation*, Phys. Rev. D. 70 (2004) 43527.

M. Milosevic, D.D. Dimitrijevic, G.S. Djordjevic, M.D. Stojanovic, *Dynamics of tachyon fields and inflation*, Serbian Astronomical Journal. 192 (2016).

Braneworld cosmology



- Braneworld universe is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk.
- Randall-Sundrum models

The RSII Model

- Observer is placed on the positive tension brane
- 2nd brane is pushed to infinity

$\sigma > 0$

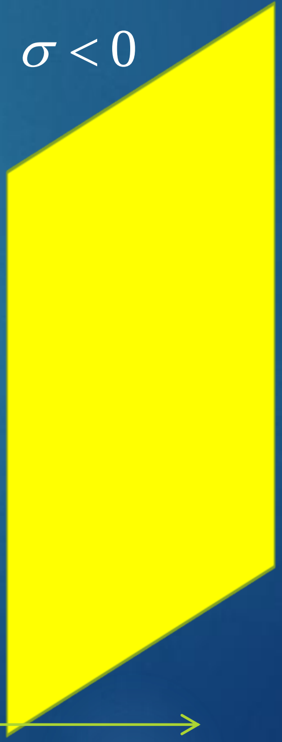
$y = 0$

$$ds^2_{(5)} = e^{-2y/\ell} g_{\mu\nu} dx^\mu dx^\nu - dy^2$$

x^μ

$x^5 \equiv y$

$\sigma < 0$



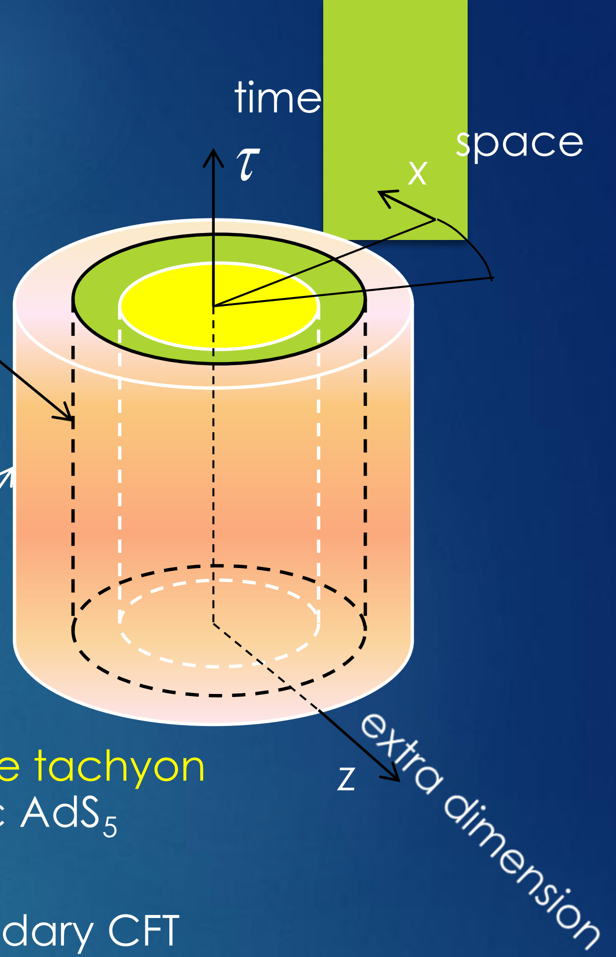
$y \to \infty$

Holographic braneworld

RSII brane at $z = z_{\text{br}}$

$$ds^2_{(5)} = \frac{\ell^2}{z^2}\left(g_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

Conformal boundary at $z = 0$



- Holographic braneworld - a cosmology based on the effective four-dimensional Einstein equations on the holographic boundary in the framework of anti de Sitter/conformal field theory (AdS/CFT) correspondence.
- The model is based on a holographic braneworld scenario with an effective tachyon field on a D3-brane located at the holographic boundary of an asymptotic AdS_5 bulk.
- The cosmology is governed by matter on the brane in addition to the boundary CFT
- The holographic Friedmann equations

$$h^2 - \frac{\ell^2}{4}h^4 = \frac{\kappa^2}{3}\ell^4\rho$$

$$\dot{h}\left(1 - \frac{\ell^2}{2}h^2\right) = -\frac{\kappa^2}{3}\ell^3(p+\rho)$$

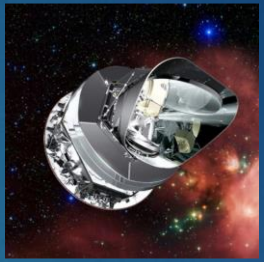
ℓ is the AdS curvature radius

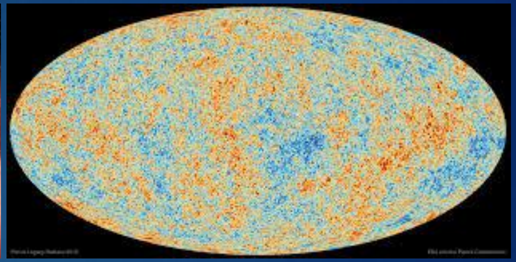
$h \equiv \ell H$ is a dimensionless expansion rate

$\kappa^2 = 8\pi G/\ell^2$ is the fundamental dimensionless coupling

N. Bilić, *Randall-Sundrum vs Holographic Cosmology*, IRB (2015)

Observational parameters (n_s, r)



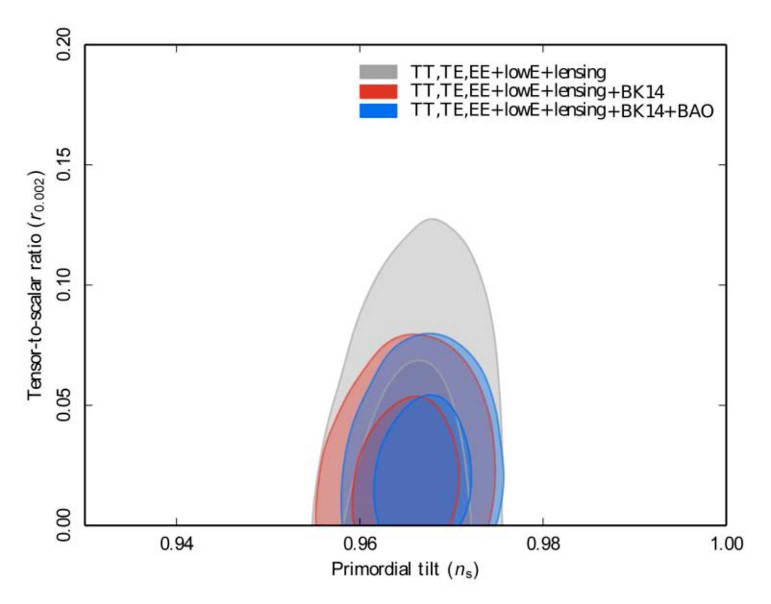


- Three independent observational parameters: amplitude of scalar perturbation A_s, tensor-to-scalar ratio r and scalar spectral index n_s
- Numerical calculation
- Holographic cosmology

$$n_s = 0.9649 \pm 0.0042$$
$$r < 0.056$$

$$r = 16\varepsilon_1\left[1 + C\varepsilon_2 + \frac{2(2-h^2)}{3(4-h^2)}\frac{pp_{,XX}}{p_{,X}^2}\varepsilon_1\right]$$

$$n_s = 1 - 2\varepsilon_1 - \varepsilon_2 - \left(2 + \frac{8h^2}{3(4-h^2)^2}\frac{pp_{,XX}}{p_{,X}^2}\right)\varepsilon_1^2 - \left(3 + 2C + \frac{2(2-h^2)}{3(4-h^2)}\frac{pp_{,XX}}{p_{,X}^2}\right)\varepsilon_1\varepsilon_2 - C\varepsilon_2\varepsilon_3$$



$$p_{,X} = \frac{\partial p}{\partial x}$$
$$X = \dot{\theta}^2$$

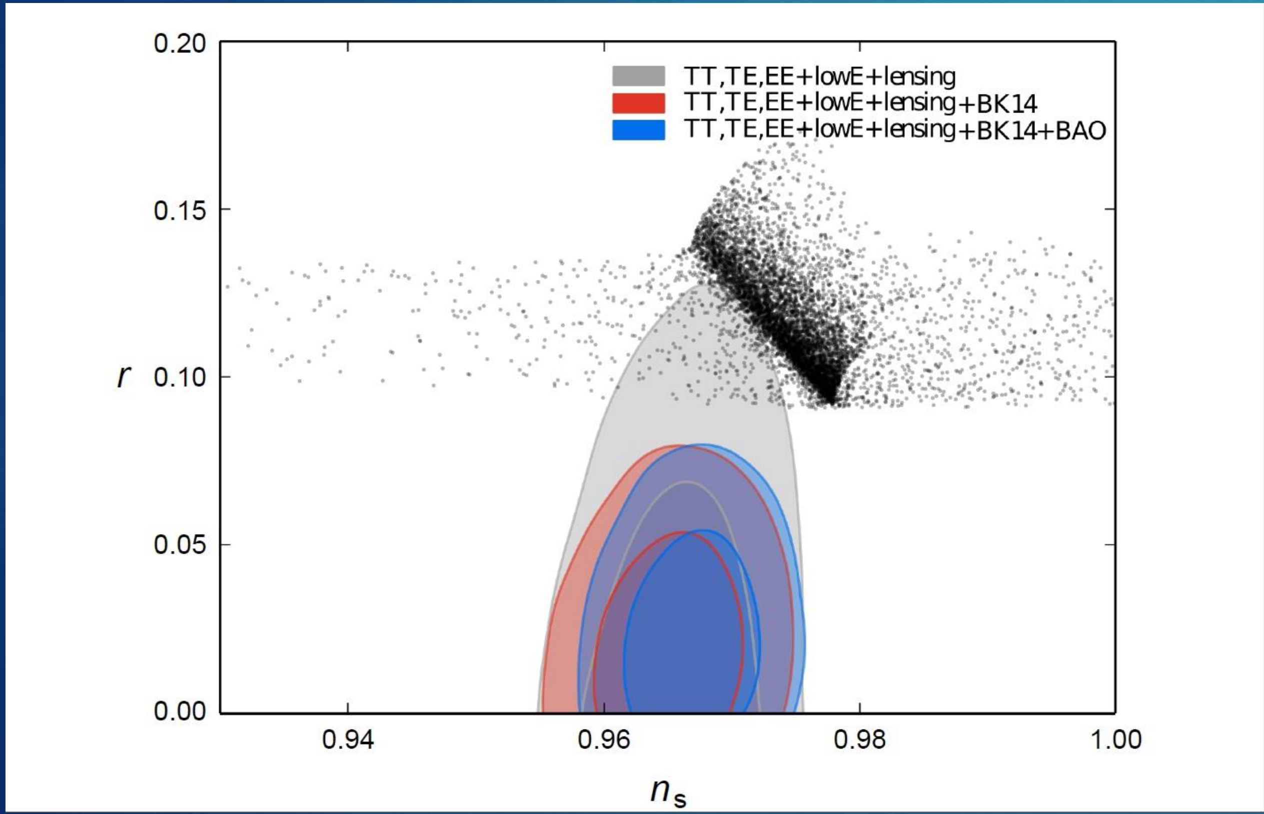
For $p = -V\sqrt{1-X}$ we have $\frac{pp_{,XX}}{p_{,X}^2} = -1$

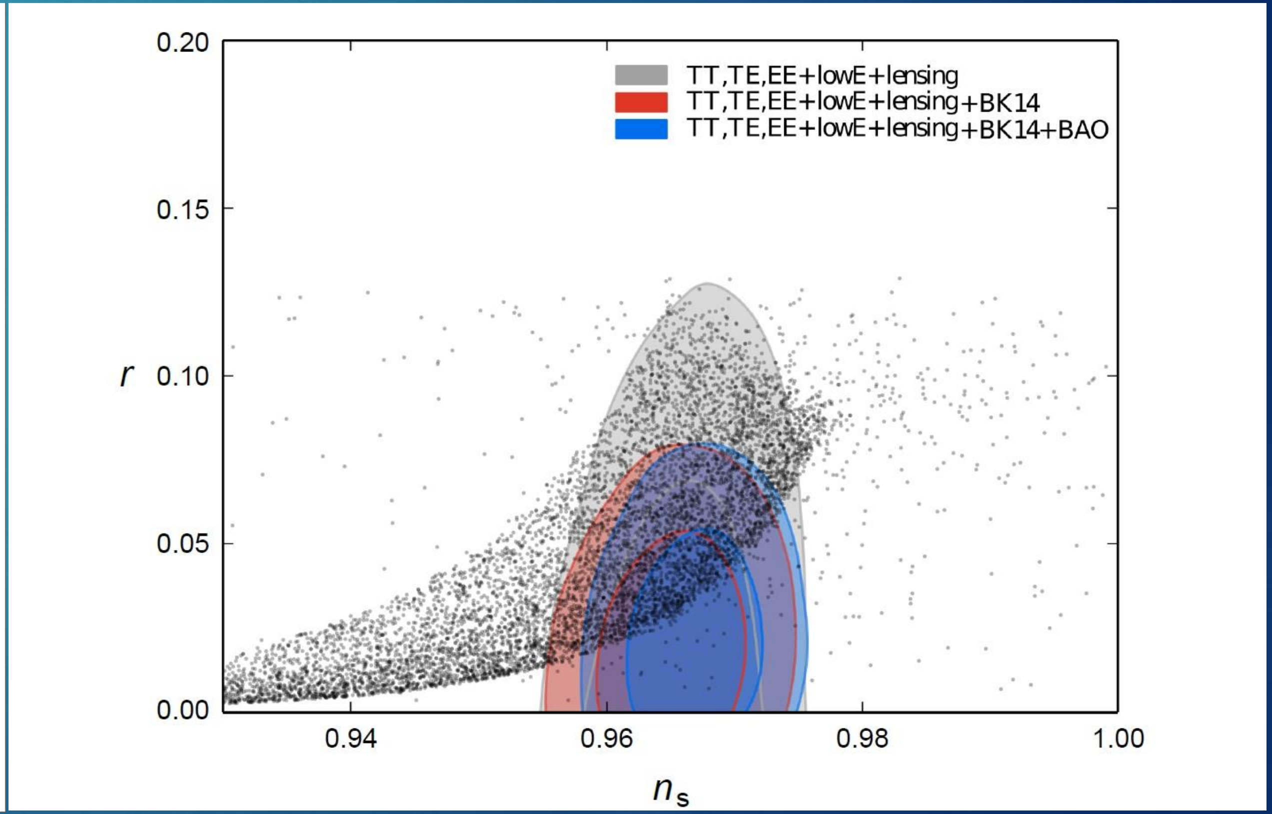
Planck 2018 results. X Constraints on inflation, arXiv:1807.06211 [astro-ph.CO]

Observational parameters (n_s, r)

$V(\theta) = e^{-\omega\cdot\theta}$

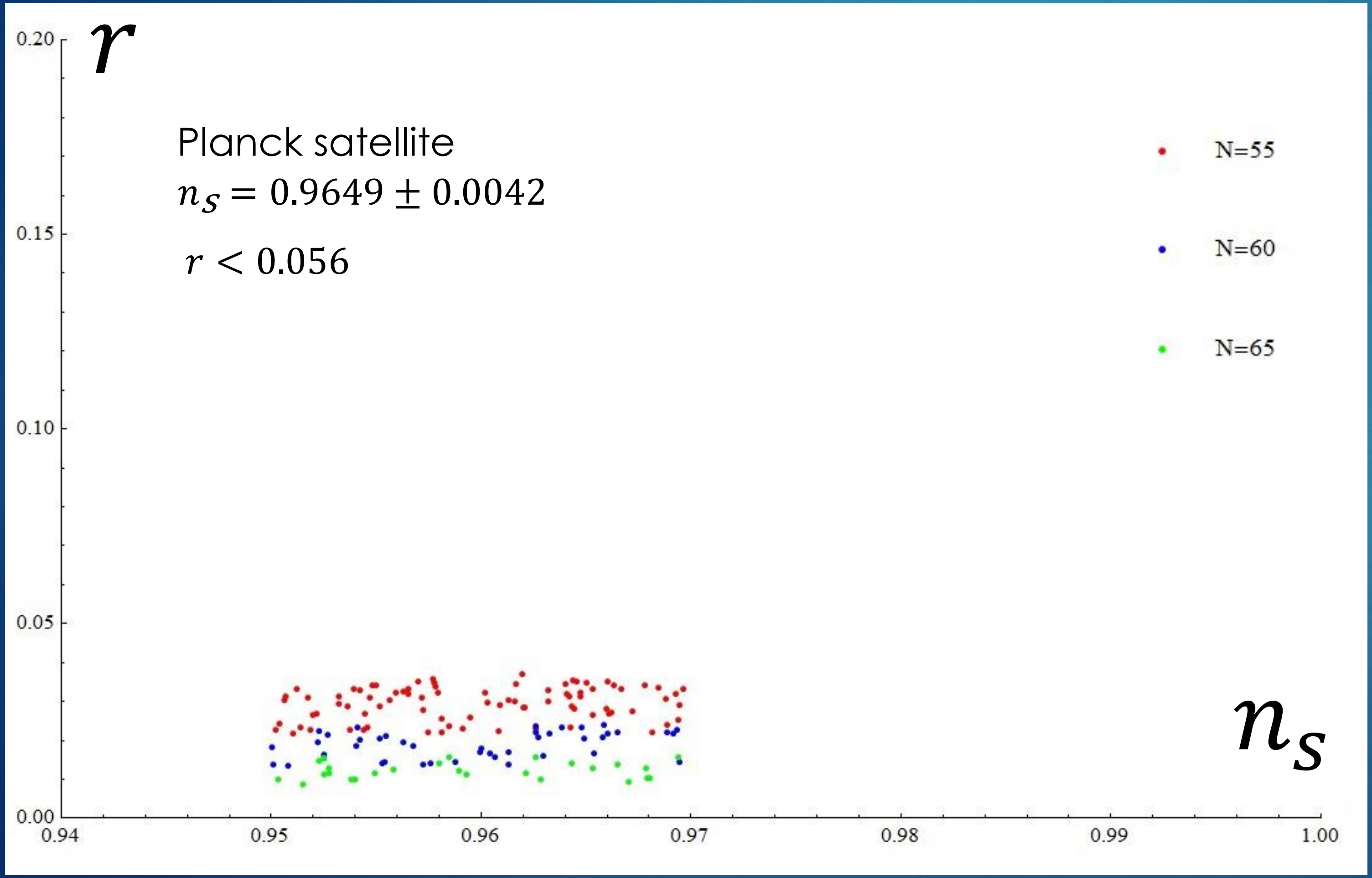
$V(\theta) = \dfrac{1}{\cosh(\omega\cdot\theta)}$





$60 < N < 90,\ \ 0 < \omega < 1$

Observational parameters (n_s, r)



The constant-roll inflation

$$\eta = \frac{\ddot{\theta}}{H\dot{\theta}} = \text{const}$$

$$0.08 < \eta < 0.085$$

Conclusion

- We discussed model of tachyon inflation based on a holographic braneworld scenario.
- We solve the evolution equations numerically and confront our result with the Planck data.
- The agreement of our model with the Planck observational data is good for holographic model with a higher number of e-folds.
- The constant-roll inflation in holographic cosmology gives significantly lower values for the number of e-fold, which are closer to to the typical value $N \simeq 60$.
- It would be of considerable interest to perform precise calculations for other types of tachyon potentials that are currently on the market.
- Preliminary results are promising and open good opportunity for further investigation (PBH).

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Thank you!