Nontrivial zeros of the Riemann zeta function on the celestial circle

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[arXiv:2308.15743]

Gong Show, SMR 3888

The Riemann hypothesis

The Riemann zeta function:

$$\zeta(s) := \sum_{n=1}^{\infty} \frac{1}{n^s}, \ \mathrm{Re}(s) > 1.$$

---> Analytic continuation to whole complex plane

Trivial zeros: -2, -4, -6, …

Riemann hypothesis: All nontrivial zeros on the critical line $Re(s) = \frac{1}{2}$

[E. Bombieri, Problems of the Millennium : the Riemann Hypothesis. (2000).]

Celestial CFT

Asymptotic symmetry & AdS/CFT & scattering amplitude

Reviews from the Amp:

[Laura Donnay, arXiv:2310.12922]

[Tristan McLoughlin, Andrea Puhm, Ana-Maria Raclariu, arXiv:2203.13022]

[Sabrina Pasterski, Monica Pate, Ana-Maria Raclariu, arXiv:2111.11392]

[Ana-Maria Raclariu, arXiv:2107.02075]

[Sabrina Pasterski, arXiv:1905.10052]

[Andrew Strominger, arXiv:1703.05448]

Celestial CFT

conformal primary wavefunction [arXiv:1705.01027]

$$\varphi^{\pm}_{\Delta}(X^{\mu};\vec{w}) \equiv \int_0^{\infty} d\,\omega\omega^{\Delta-1} e^{\pm i\omega q(\vec{w})\cdot X-\epsilon\omega} = \frac{(\mp i)^{\Delta}\Gamma(\Delta)}{(-q(\vec{w})\cdot X \mp i\epsilon)^{\Delta}},$$

$$X \in R^{1,1+d}, q(\vec{w}) \in R^{d}$$

->bulk-to-boundary propagator [arXiv:1609.00732, hep-th/9802150]

Hyperbolic slicing: Euclidean AdS_3 [hep-th/0303006]

->principal continuous series

$$\Delta = \frac{d}{2} + i\lambda, \quad \lambda \in R.$$

S1: chain of points $\{-q_n(\vec{w})\cdot X = n, \quad n \in Z_+\}$
--> Fix bulk point X, a discretized cycle on the boundary
--> fix a boundary point $q(\vec{w}) \in R^d$, a discretized cycle in the bulk
--> Or a mixture: pair of discretized cycle in bulk & boundary
triangulation, topology & geometry, Bulk & Boundary together?
S2: Sum the propagator over this chain *Meaning of this sum?*

$$\sum_{n=1}^{\infty} \frac{(\mp i)^{\Delta}\Gamma(\Delta)}{(n \mp i\epsilon)^{\Delta}} = (\mp i)^{\Delta}\Gamma(\Delta)\zeta(\Delta), \epsilon \to 0$$

S3: analytic continuation from $d > 2$ to $\boldsymbol{d = 1}$ *Meaning?*

$$= (\mp i)^{\frac{1}{2}+i\lambda}\, \Gamma\left(\frac{1}{2} + i\lambda\right)\zeta\left(\frac{1}{2} + i\lambda\right)$$

S4: Nontrivial zeros *Meaning of the special dimension* $\Delta^{\star} = \frac{1}{2} + i\lambda^{*}$*?*

$$\zeta\left(\frac{1}{2} + i\lambda^{*}\right) = 0$$