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Teaser: A very strange EFT:

$$S = \int dt\, dx \left((\partial_t - \partial_x)\pi\right)^2 \Rightarrow (\partial_t - \partial_x)^2 \pi = 0$$

classical sols: $\pi = \sum_\omega \alpha_\omega e^{i\omega(t-x)} + \beta_\omega t\, e^{i\omega(t-x)}$

$\Rightarrow$ physical excitations with

$$\omega = k \underset{\substack{\downarrow \\ x \simeq x + 2\pi R}}{=} \frac{n}{R}, \quad n \in \mathbb{Z}.$$

(Contrast with $\omega = |k|$ for massless particles).

Instability for modes $\leftarrow$

stability for modes $\rightarrow$

but momentum is conserved so ok...

$\sum_i n_i = 0 \Rightarrow$ degenerate with vacuum.
same quantum numbers.

∞ degeneracy, ∞ entropy at $T=0$!

Imagine small correction:

$$\omega = \begin{cases} \frac{n}{R} - \delta^3 (n^3 - n) & n > 0 \\ \frac{n}{R} - \delta n & n < 0 \end{cases}, \quad \delta > 0.$$

This removes the ∞ entropy and leads to a stable g.s. Also it leads to a preference of single particle states.

How does this strange theory appear in physics?

Consider a CFT_d with some loc. operator $\mathcal{O}_\Delta(x)$, $D(\mathcal{O}(0)) = \Delta\, \mathcal{O}(0)$

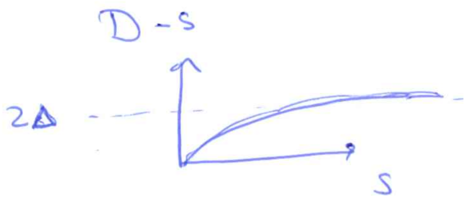
$$\mathcal{O}_\Delta(x)\, \mathcal{O}^\dagger_\Delta(0) \sim \dots + \sum_s C_s\, x^\alpha\, \mathcal{O}_\Delta \overleftrightarrow{\partial}^s \mathcal{O}^\dagger_\Delta(0)$$

$$\alpha = D\left(\mathcal{O}_\Delta \overleftrightarrow{\partial}^s \mathcal{O}_\Delta\right) - 2\Delta .$$

$$D\left(\mathcal{O}_\Delta \overleftrightarrow{\partial}^s \mathcal{O}^\dagger_\Delta\right) = 2\Delta + s - \frac{A}{s^{d-2}} + \dots$$

calculable.

$$A = \frac{d^2\, \Gamma(d+2)\, \Delta^2\, \Gamma^2(\Delta)}{2 C_T (d-1)^2\, \Gamma\left(\frac{d+2}{2}\right)^2 \Gamma^2\left(\Delta - \frac{d-2}{2}\right)}$$

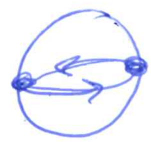


same story for

$$\mathcal{O}_{\Delta_1} \overleftrightarrow{\partial}^s \mathcal{O}_{\Delta_2}$$

$$D = \Delta_1 + \Delta_2 + s - A/s^{\tau_{min}}$$

"Theory becomes free @ large s $\leftrightarrow$ small angles."



Suppose now we have (Q, S)

$$D(\phi) = \Delta_\phi$$
$$Q(\phi) = 1.$$

$$\overrightarrow{\partial}^{a_1} \phi\, \overleftrightarrow{\partial}^{a_2} \phi \dots \overleftrightarrow{\partial}^{a_Q} \phi \qquad a_i \gg 1.$$

$$\sum_i a_i = S$$

$$D = Q\Delta_\phi + S + \underbrace{\dots}_{O(Q^3/S)}$$

Evidently we have to take $1 \ll Q^2 \ll s$. Then these are the lowest dim operators.

Note compare with

$$\partial^{b_1} \phi^2 \overleftrightarrow{\partial}^{b_2} \dots \overleftrightarrow{\partial}^{b_{Q-1}} \phi$$

$$D = (Q-2)\Delta_\phi + \Delta_{\phi^2} + S + \dots$$

$$\Delta_{\phi^2} > 2\Delta_\phi \rightarrow \text{finite gap.}$$

Let's therefore consider the excitations of

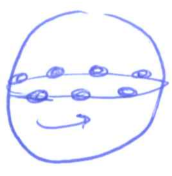
$$\vec{\partial}^{a_1} \phi \, \overleftrightarrow{\partial}^{a_2} \phi \dots \overleftrightarrow{\partial}^{a_Q} \phi$$

Since we only need to maintain $\sum a_i = s$, we can move derivatives back & forth!

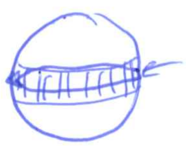
$\Rightarrow$ huge degeneracy $\quad a_i = \frac{s}{Q} + \delta a_i$

$$\sum \delta a_i = 0.$$

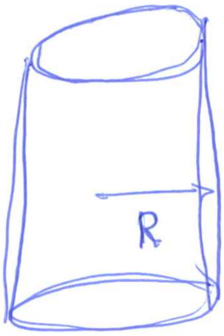
Very reminiscent of our strange EFT.

Idea:  for large Q, s,

turns into

 condensate.

Such a rotating condensate can be studied on the equator on S^{d-1} or in an actual trap of atoms (dilute)



N neutral atoms.
mutal repulsion with
scattering length a.

$$\hat{H} = \int dV \left[\psi^+ \left(\frac{-\hbar^2}{2M} \nabla^2 \right) \psi + \frac{1}{2} g \psi^+ \psi^+ \psi \psi \right\}$$

$$[\psi^+, \psi] = -\delta^{(d)}(r - r') \qquad ; \quad g = \frac{4\pi \hbar^2 a}{M}$$

$$S = \int dV dt \left[i\hbar \psi^+ \frac{\partial}{\partial t} \psi - \psi^+ \left(\frac{-\hbar^2}{2M} \nabla^2 \right) \psi - \frac{1}{2} g \psi^+ \psi^+ \psi \psi \right\}$$

constraint $\quad N = \int dV \psi^+ \psi$.

The solution: $\quad \psi = \frac{\sqrt{N}}{\sqrt{V}} e^{-i\mu t}$

$$\mu = g'/V$$

Fluctuations: $\quad \frac{1}{\sqrt{N}} \psi = \frac{1}{\sqrt{V}} e^{-i\mu t} + \delta\psi \, e^{-i\mu t}$.

$$\frac{1}{\hbar} S_{quadratic} = \frac{Ng'}{2V} \int dt + N \int dV dt \left[i \delta\psi^+ \frac{\partial}{\partial t} \delta\psi + \frac{\hbar}{2M} \delta\psi^+ \nabla^2 \delta\psi - \frac{g'}{2V} (\delta\psi + \delta\psi^+)^2 \right]$$

$$g' = \frac{4\pi\hbar N a}{M}$$

$$\omega(k) = \sqrt{\frac{g'\hbar}{VM}k^2 + \frac{\hbar^2}{4M^2}k^4}$$

$$\sim \sqrt{\frac{g'\hbar}{VM}}\,|k| \quad \Rightarrow \quad c_s = \sqrt{\frac{g'\hbar}{VM}} \sim \frac{\hbar}{M\xi}$$

ξ: healing length $\quad \xi = (na)^{-1/2}$

$R \gg \xi$.

- A way to analyze $\frac{1}{\hbar} S_{quadratic}$

$$-2\,\mathrm{Re}\,\delta\psi \frac{\partial}{\partial t}\mathrm{Im}\,\delta\psi - \frac{2g'}{V}(\mathrm{Re}\,\delta\psi)^2$$

$$\Rightarrow \mathrm{Re}\,\delta\psi = \frac{-V}{2g'}\frac{\partial}{\partial t}\mathrm{Im}\,\delta\psi$$

$$\Rightarrow$$

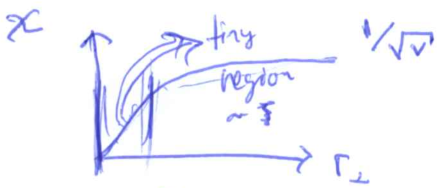
$$\frac{1}{\hbar} S_{quadratic} = \frac{Ng'}{2V}\int dt + \frac{N}{2}\int dV dt \left[\frac{V}{g'}\left(\frac{\partial}{\partial t}\mathrm{Im}\,\delta\psi\right)^2 - \frac{\hbar}{M}(\nabla \mathrm{Im}\,\delta\psi)^2 \right]$$

This is the usual homogenous BEC.
How about states w/ rotation?

$$\boxed{\frac{1}{\sqrt{N}}\psi = e^{-i\mu t + i l \varphi}\,\chi(r_\perp)}$$

$$-\frac{\hbar}{2M}\left(\frac{\partial^2}{\partial r_\perp^2} + \frac{1}{r_\perp}\frac{\partial}{\partial r_\perp} - \frac{l^2}{r_\perp^2}\right)\chi(r_\perp) + g'\chi^3(r_\perp) = \mu\chi(r_\perp)$$

$l = 1$:



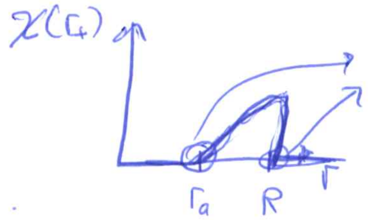
$\mu = g'/V$ + tiny corrections.

$V = \frac{\hbar}{M r_\perp} \cdot l$ (compare w/ rigid body).

But now take $\ell \gg 1$.

something nice happens:

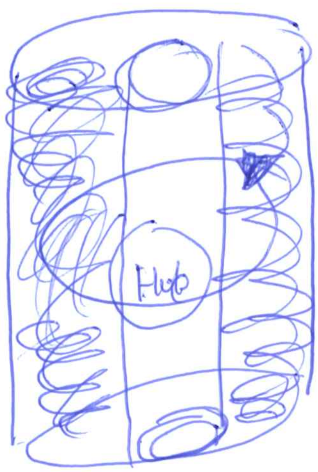
$$\chi(r_\perp) = \begin{cases} \ell\sqrt{\frac{\hbar}{2Mg'}}\sqrt{\frac{1}{r_a^2} - \frac{1}{r_\perp^2}} & r_a < r < R \\ 0 & 0 < r < r_a \end{cases}$$



$$r_a^2 = \frac{\hbar \ell^2}{2M\mu}. \qquad r_a = \ell \cdot \xi$$

$\mu = g'/V$ approximately. $1 \ll \ell \xrightarrow{\text{up to}} R/\xi$.

Giant Vortex.



Fluctuations:

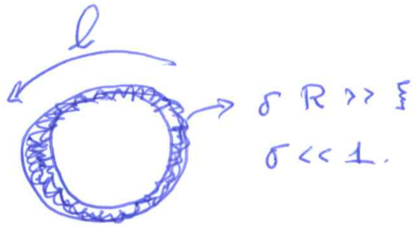
$$\frac{1}{\sqrt{N}}\Psi = e^{-i\mu t + i\ell\varphi + i\Pi}\left(\chi(r_\perp) + h\right)$$

Π: $\mathrm{Im}\,\delta\Psi$; h: $\mathrm{Re}\,\delta\Psi$

$$\frac{1}{\hbar} S_{\text{quadratic}} = S_0 + \frac{LN}{2g'}\int r_\perp dr_\perp\, d\varphi\, dt \left[\left(\frac{\partial\Pi}{\partial t} + \frac{\hbar\ell}{Mr^2}\frac{\partial\Pi}{\partial\varphi}\right)^2 + \frac{\hbar^2\ell^2}{2M^2}\left(\frac{1}{r_a^2} - \frac{1}{r_\perp^2}\right)\Pi\nabla^2\Pi\right]$$

Note the first term is similar to our weird EFT.

Now take



The solution $\Pi = J_0\left(\left| \frac{M\omega R^2}{\hbar \ell} + m \right| \sqrt{2\delta} \cdot \frac{1}{R\sqrt{\delta}} \cdot \sqrt{r^2 - r_a^2} \right)$

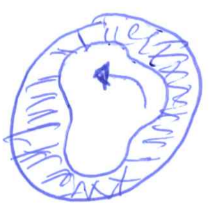
$$= J_0\left(\left| \frac{M\omega R^2}{\hbar \ell} + m \right| \frac{\sqrt{2}}{R} \sqrt{r^2 - r_a^2} \right)$$

The frequencies are fixed by the b.c.

$$\omega_{m,n} = \frac{\hbar \ell m}{MR^2} + x_n \frac{\hbar \ell}{\sqrt{2\delta}\, MR^2}$$

$$x_0 = 0, \quad x_1 = 3.8317, \quad \ldots \qquad J_0'(x_n) = 0.$$

This leads to density fluctuations moving together w/ the fluid



Preliminary indication in experiment...