Thermodynamics of AdS black holes and AdS/CFT correspondence

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Basic idea and motivation

Since we do not live in Anti de Sitter (AdS) spacetime, AdS black holes do not exist in our universe. However, if we adopt the braneworld idea that our universe is a 3-brane embedded in a d+1 dimensional bulk, e.g., 4+1 dim. AdS spacetime, a black hole in the bulk may affect the physics of the brane. For example, in the RSII model or in the holographic braneworld model (brane located at the AdS boundary), a Schwarzschild BH in the bulk induces the so-called dark radiation: non-thermal production of massless particles the temperature of which is related to the BH mass.

S. Mukohyama, T. Shiromitzu, K. Maeda, PRD (2000)
P. S. Apostolopoulos, G. Siopsis and N. Tetradis, PRL (2009);
N. B., PRD (2016)

AdS Schwarzschild black holes have a curious thermodynamic properties: 1. The horizon temperature of a large AdS Schwarzschild BH increases with increasing horizon radius in contrast to an ordinary Schwarzschild BH, the temperature of which decreases with horizon radius. 2. A large AdS Schwarzschild BH can be well approximated by an AdS planar BH.

E.Witten, Adv. Theor. Math. Phys. (1998)

Planar BHs have recently attracted attention in non-gravitational contexts: interesting applications in condensed matter physics (S. A. Hartnoll, CQG (2009)), 2+1-dimensional superconductor (N. Bobev, A. Kundu, K. Pilch ,and N. P. Warner, JHEP (2012), T. Albash and C. V. Johnson, JHEP(2012), A. Chakraborty, CQG (2020), N. B. and J. C. Fabris, CQG (2022)), and acoustic geometry (N. B. and H. Nikolic, CQG (2018))

As we will see, the horizon of an AdS planar BH translates when a planar BH metric undergoes a rescaling transformation. Because of that, the horizon location of an AdS planar BH is not uniquely defined. Following V.E. Hubeny, D. Marolf, and M. Rangmani, CQG (2010), we will refer to this property as the "translational invariance" of the BH horizon.

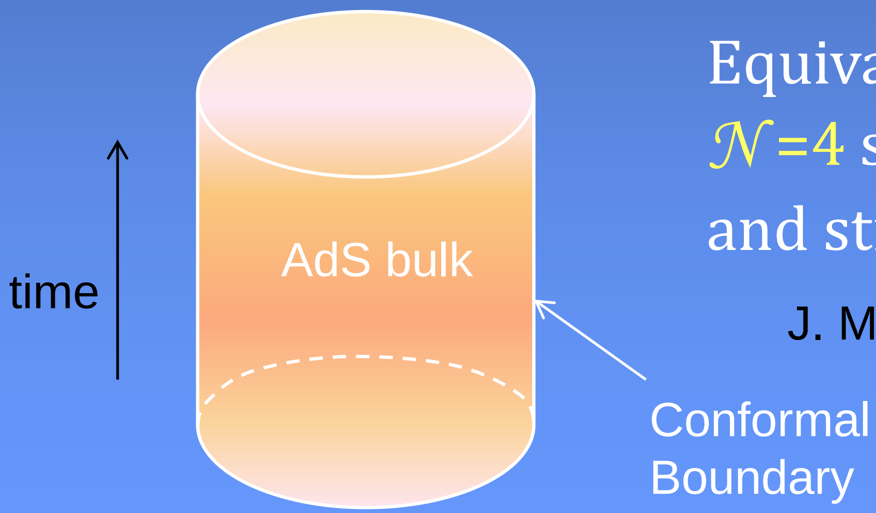
A while ago, based on this property, it was argued that the high local temperatures associated with large values of AdS Schwarzschild BH horizon temperatures are not locally observable (S. Hemming and L. Thorlacius, JHEP (2007) and R. Gregory, S. F. Ross and R. Zegers, JHEP (2008)).

Using a similar reasoning, Hubeny et al argued that large Schwarzschild-AdS black holes, although very hot, locally appear at most only lukewarm.

Basic idea

We will invoke the AdS/CFT holography to relate the temperatures T_{Sch} and T_{pl} of the Schwarzschild and planar BHs to the conformal fluid at the boundary of AdS spacetime.

AdS/CFT correspondence is a holographic duality between gravity in d+1-dim space-time and quantum CFT on the d-dim boundary. The original formulation stems from string theory:



Equivalence of 3+1-dim $\mathcal{N}=4$ supersymmetric YM Theory and string theory in $AdS_5 \times S_5$

J. Maldacena, Adv. Theor. Math. Phys. **2** (1998)

Outline

Our scheme consists of the following steps

1. We will show that we can fix the horizon temperature of a planar BH by the requirement of the entropy area law.
2. By invoking the AdS/CFT correspondence conjecture, we will calculate the entropy of the boundary CFT fluid.
3. Using this and thermodynamic equations, we will relate the horizon temperature T_{pl} of the AdS planar BH to the temperature T_{Sch} of the large AdS Schwarzschild BH near the AdS boundary.

Based on
N.B and J.C. Fabris, "Thermodynamics of AdS planar black holes and holography", JHEP 11 (2022) 013

Horizon temperature – periodicity in Euclidean time

Consider a $d+1$ dimensional **Euclidean** spacetime metric of the form

$$ds^2 = f(r)d\tau^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{d-1}$$

If $f(r)$ is a smooth function with no zeros, the time coordinate τ can be assumed periodic with an arbitrary period β (including $\beta = \infty$). The manifold supports the standard thermal ensemble at an arbitrary non-zero temperature $T = 1/\beta$ with partition function $\mathrm{Tr}e^{-\beta H}$. This temperature is measured by asymptotic observers. The local temperature is $T_{\mathrm{loc}} = T/\sqrt{g_{00}}$ required by the Tolman law for thermal equilibrium in a gravitational field.
The topology of the manifold is $\mathbf{S}^1 \times \mathbf{R}^d$.

If $f(r)$ has a simple zero at $r = r_0$, i.e., if there is a horizon at $r = r_0$, the function can be expanded in powers of $r - r_0$

$$f = (r - r_0)f_0 + O\big((r - r_0)^2\big), \qquad f_0 = \left.\frac{df}{dr}\right|_{r=r_0}.$$

The near-horizon metric becomes

$$ds^2 = (r - r_0) f_0 d\tau^2 + \frac{1}{(r - r_0) f_0} dr^2 + r_0^2 d\Omega_{d-1}$$

For $r \geq r_0$, a coordinate transformation to a new radial coordinate ρ

$$r - r_0 = \rho^2$$

yields

$$ds^2 = 4 f_0 \underbrace{\left(d\rho^2 + \frac{f_0}{4} \rho^2 d\tau^2 \right)}_{\text{conical singularity}} + r_0^2 d\Omega_{d-1}.$$

$$d\rho^2 + \rho^2 \left(\frac{|f_0| d\tau}{2} \right)^2 = d\rho^2 + \rho^2 d\varphi^2$$

The conical singularity is removed if φ is periodic with period 2π. This means the time coordinate τ is periodic with period

$$\beta = \frac{4\pi}{|f_0|}.$$

In this case, the original topology $\mathbf{S}^1 \times \mathbf{R}^d$ goes to $\mathbf{R}^2 \times \mathbf{S}^{d-1}$.

Note: the periodicity β is observed by an asymptotic observer. Local observers perceive different periodicity β_{loc} since their proper time is $\sqrt{g_{00}}\,\tau$. Hence, $\beta_{\text{loc}} = \sqrt{g_{00}}\beta$.
This corresponds to the Tolman temperature law for a thermal equilibrium in a gravitational field with $T_{\text{loc}} = T/\sqrt{g_{00}}$.

For the Schwarzschild spacetime

$$f = 1 - \frac{2MG}{r}$$

$$f_0 = \frac{1}{2MG}$$

From this we obtain the well known horizon temperature of the Schwarzschild BH

$$T = \frac{1}{\beta} = \frac{1}{8\pi MG}$$

The horizon temperature of an AdS Schwarzschild BH

The metric of an AdS Schwarzschild black hole in $d+1$ dimensions is usually written in the form

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega_{d-1}.$$

$$f(r) = \frac{r^2}{\ell^2} + 1 - \mu\left(\frac{\ell}{r}\right)^{d-2},$$

ℓ is the curvature radius of AdS$_{d+1}$ and the dimensionless parameter μ is related to the black-hole mass via

$$\mu = \frac{16\pi G_{d+1} M_{\text{bh}}}{(d-1)\ell^{d-2}\Omega_{d-1}} = \left(\frac{r_+}{\ell}\right)^d \left(1 + \frac{\ell^2}{r_+^2}\right)$$

r_+ is the largest root of the equation $f(r) = 0$, G_{d+1} is a d+1-dimensional Newton's constant, and

$$\Omega_{d-1} = \frac{2\pi^{d/2}}{\Gamma(d/2)} = \begin{cases} (2\pi)^{d/2}/(d-2)!!, & \text{for even } d, \\ 2(2\pi)^{(d-1)/2}/(d-2)!!, & \text{for odd } d, \end{cases}$$

Defining the periodicity of the Euclidean time to avoid the conical singularity, one finds the inverse horizon temperature

$$\beta = \frac{1}{T_{\rm Sch}} = \frac{4\pi}{f_+} = \frac{4\pi \ell^2 r_+}{d r_+^2 + (d-2)\ell^2}.$$

Note: The horizon temperature $T_{\rm Sch} \to \infty$ when either $r_+ \to 0$ (small BH) or $r_+ \to \infty$ (large BH), unlike the Schwarzschild BH in an asymptotically flat background where $T_{\rm Sch} \to 0$ for a large BH. In particular, for a large AdS Schwarzschild BH, we have

$$T_{\rm Sch} = \frac{d r_+}{4\pi \ell^2}.$$

AdS planar black hole

As pointed out by Witten, in the limit of large AdS Schwarzschild BH the topology goes from $\mathbf{S}^1 \times \mathbf{S}^{d-1}$ to $\mathbf{S}^1 \times \mathbf{R}^{d-1}$, and the Schwarzschild BH is approximated by a planar BH with a translationally invariant horizon.
To see this, consider a large Schwarzschild-AdS black hole with $\mu \gg 1$ ($r_+ \gg \ell$). By rescaling $r \to 1/\rho$, the function f becomes

$$f = \mu^{2/d}\left(\frac{\ell^2}{\rho^2} - \frac{\rho^{d-2}}{\ell^{d-2}} + \mu^{-2/d}\right)$$

Then, neglecting this term and rescaling the time $t = \mu^{-1/d}\tau$, the line element squared becomes

$$ds^2 = \frac{\ell^2}{\rho^2}\left[\left(1 - \frac{\rho^d}{\ell^d}\right)d\tau^2 - \left(1 - \frac{\rho^d}{\ell^d}\right)^{-1} d\rho^2 - \mu^{2/d}\ell^2 d\Omega_{d-1}^2\right]$$

For $M_{\text{bh}} \to \infty$ (i.e., for $\mu \to \infty$) the radius $\mu^{1/d}\ell$ of $\mathbf{S}^{d-1}$ diverges and hence, $\mathbf{S}^{d-1}$ becomes flat and looks locally as $\mathbf{R}^{d-1}$.

Now we introduce coordinates x^i near a point $P \in \mathbf{S}^{d-1}$ such that at P, $d\Omega_{d-1}^2 = \ell^{-2}\mu^{-2/d} d\boldsymbol{x}^2$. Then the metric becomes

$$ds^2 = \frac{\ell^2}{\rho^2}\left[\left(1-\frac{\rho^d}{\ell^d}\right)d\tau^2 - \left(1-\frac{\rho^d}{\ell^d}\right)^{-1} d\rho^2 - d\boldsymbol{x}^2\right]$$

where $$d\boldsymbol{x}^2 = \sum_{i=1}^{d-1} dx^i dx^i$$

The metric describes an asymptotically AdS_{d+1} geometry with a planar BH horizon located at $\rho = \ell$. Note that dependence on Newton's constant G_{d+1} has disappeared, and the metric involves only one scale: the AdS curvature ℓ. It is easy to show by simultaneously rescaling the coordinates that this metric is equivalent to the whole class of planar BHs represented by

$$ds^2 = \frac{\ell^2}{\rho^2}\left[\left(1-\frac{\rho^d}{\rho_{\mathrm{pl}}^d}\right)d\tau^2 - \left(1-\frac{\rho^d}{\rho_{\mathrm{pl}}^d}\right)^{-1} d\rho^2 - dx^2\right]$$

with the horizon located at an **arbitrary** $\rho_{\mathrm{p}l}$.

As before, one can calculate the horizon temperature by demanding that the periodicity β_{pl} is such that the conical singularity is removed. One finds

$$T_{\text{pl}} = \frac{1}{\beta_{\text{pl}}} = \frac{d}{4\pi\rho_{\text{pl}}}$$

Hence, the temperature of the planar BH increases when the horizon shifts towards the AdS boundary at $\rho = 0$. However, since ρ_{pl} is not fixed, the temperature T_{pl} is ambiguous, and local observations are independent of ρ_{pl} . In other words, since the AdS curvature ℓ is the only scale in the problem, local observers see no excitations beyond this scale.

Since small ρ corresponds to large r, one would naively conclude that the thermodynamic behavior of the AdS planar BH is consistent with that of the large AdS Schwarzschild BH. However, this conclusion could be wrong as it is not obvious that the planar BH temperature is in any way related to the temperature of the AdS Schwarzschild BH.

The entropy of AdS planar BHs

The AdS Schwarzschild BH entropy is derived by E. Witten (Adv. Theor. Math. Phys. (1998))

$$S_{\rm Sch} = \frac{r_+^{d-1}\Omega_{d-1}}{4G_{d+1}}$$

which demonstrates the area law in agreement with Bekenstein and Hawking .

To calculate the horizon entropy of the AdS planar BH, we transform the coordinate ρ back to $r = \rho_{\rm pl}^2/\rho$ so that r goes from $r = \rho_{\rm pl}$ to $r = \infty$, and change the Minkowski signature to Euclidean

$$ds_{\rm E}^2 = \frac{r^2}{\ell^2}\left[\left(1 - \frac{\rho_{\rm pl}^d}{r^d}\right)d\tau^2 + \left(1 - \frac{\rho_{\rm pl}^d}{r^d}\right)^{-1}\frac{\ell^4}{r^4}dr^2 + dx^2\right]$$

Following Hawking and Page (Commun. Math. Phys. (1983)) and Witten, we describe the BH free energy as a renormalized **on-shell** Euclidean bulk action. The Euclidean bulk action is given by

$$S_{\mathrm{E}} = -\ln Z = \frac{1}{16\pi G_{d+1}} \int_0^{\beta_{\mathrm{pl}}} d\,\tau \int d^d x \sqrt{g} \left(-R - \frac{d(d-1)}{\ell^2} \right)$$

The on-shell action is obtained by plugging in the solution $R = -d(d+1)/\ell^2$. Then, the free energy is

$$F_{\mathrm{pl}} = -\frac{1}{\beta_{\mathrm{pl}}} \ln Z = \frac{d}{8\pi G_{d+1} \beta_{\mathrm{pl}}} \underbrace{\int_0^{\beta_{\mathrm{pl}}} d\,\tau \int d^d x \sqrt{g}}_{\text{d+1 volume}}$$

Since the free energy is proportional to an infinite d+1-volume of the AdS BH spacetime, it must be regularized and renormalized.
The regularization is achieved by integrating the space volume up to a large radius R. Then, if we subtract the pure AdS volume from the AdS BH volume, we obtain the renormalized action. More explicitly

$$F_{\mathrm{pl}} = \frac{d\ell^{-2}}{8\pi G_{d+1}\beta_{\mathrm{pl}}} \lim_{R\to\infty} \left(V_{\mathrm{pl}} - V_{\mathrm{AdS}}\right)$$

where

$$V_{\mathrm{pl}} = \int_0^{\beta_{\mathrm{pl}}} d\,\tau \int_0^R d\,r\sqrt{g}\left(\int_{-L/2}^{L/2} d\,x\right)^{d-1}$$

$$V_{\mathrm{AdS}} = \int_0^{\beta_{\mathrm{AdS}}} d\,\tau \int_0^R d\,r\sqrt{g}\left(\int_{-L/2}^{L/2} d\,x\right)^{d-1}$$

β_{AdS} is determined by identifying the local AdS periodicity with the planar BH periodicity at the hypersurface $r = R$, i.e.,

$$\beta_{\mathrm{AdS}}\sqrt{g_{00}^{\mathrm{AdS}}} = \beta_{\mathrm{pl}}\sqrt{g_{00}^{\mathrm{pl}}} \qquad \text{yielding} \qquad \beta_{\mathrm{AdS}} = \beta_{\mathrm{pl}}\left(1 - \frac{\rho_{\mathrm{pl}}^d}{R^d}\right)^{1/2}$$

We obtain the free energy expressed as a function of ρ_{pl}

$$F_{\mathrm{pl}} = \frac{L^{d-1}}{16\pi G_{d+1}\ell}\left(\frac{\rho_{\mathrm{pl}}}{\ell}\right)^{d} = \frac{L^{d-1}}{16\pi G_{d+1}\ell}\left(\frac{4\pi\ell T_{\mathrm{pl}}}{d}\right)^{-d}$$

where L^{d-1} is the area of the hyperplane $r = \mathrm{const.}$ From this we derive the planar BH entropy

$$S_{\mathrm{pl}} = -\frac{\partial F_{\mathrm{pl}}}{\partial T_{\mathrm{pl}}} = \frac{L^{d-1}}{4G_{d+1}}\left(\frac{\rho_{\mathrm{pl}}}{\ell}\right)^{d+1}$$

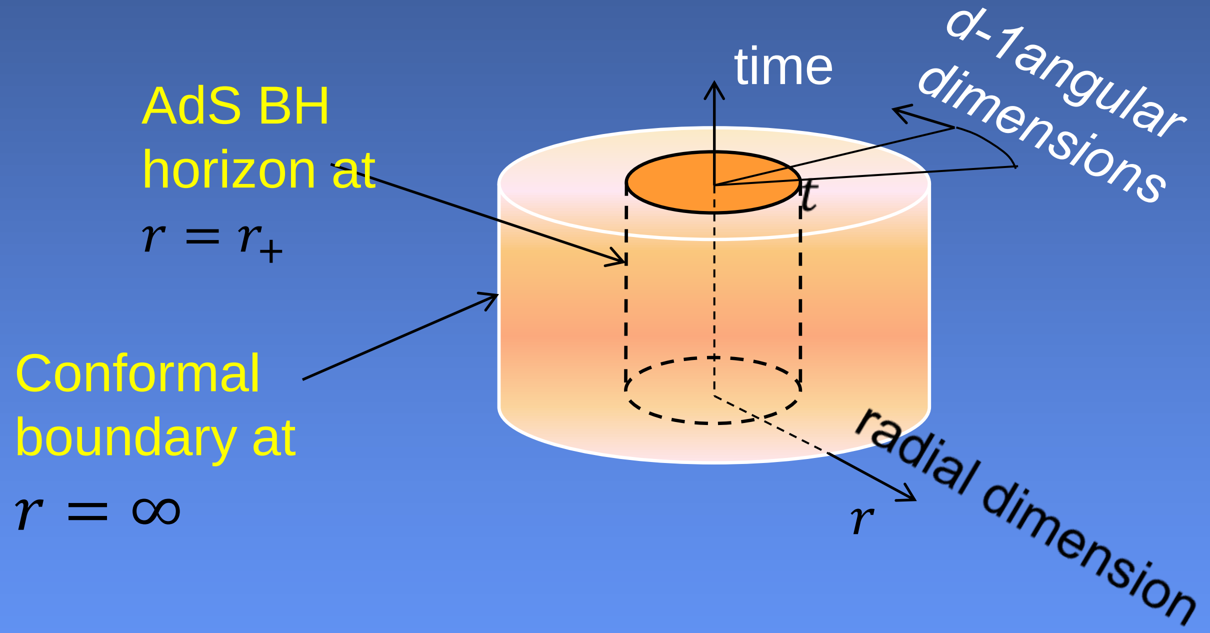
We recover the standard area law for the planar BH entropy if $\rho_{\mathrm{pl}} = \ell$. Hence, if we demand the validity of the area law, the line element

$$ds^2 = \frac{\ell^2}{\rho^2}\left[\left(1 - \frac{\rho^d}{\ell^d}\right)d\tau^2 - \left(1 - \frac{\rho^d}{\ell^d}\right)^{-1} d\rho^2 - dx^2\right]$$

will be a natural representative of all AdS planar BHs belonging to the general class with $0 < \rho_{\mathrm{pl}} < \infty$.

AdS/CFT holography

AdS/CFT correspondence is a holographic duality between gravity in d+1-dim space-time and quantum conformal field theory (CFT) on the d-dim boundary.



The vacuum expectation value of the CFT stress tensor describes matter on the **holographic brane** at the conformal boundary of the asymptotic AdS_{d+1}

To study the CFT at the boundary, it is convenient to transform both the AdS Schwarzschild metric and the AdS planar BH metric to the Fefferman-Graham coordinates. In Fefferman-Graham coordinates the general asymptotically AdS metric can be expressed as

$$ds^2 = \frac{\ell^2}{z^2}\left(g_{\mu\nu}(z)dx^\mu dx^\nu - dz^2\right)$$

The d-dimensional metric $g_{\mu\nu}$ near the boundary at $z = 0$ can be expanded as, (S. de Haro, S. N. Solodukhin, and K. Skenderis, Commun. Math. Phys. (2001),

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + \cdots . + z^d g_{\mu\nu}^{(d)} + h_{\mu\nu} z^d \ln z^2 + \mathcal{O}\left(z^{d+1}\right)$$

The logarithmic term appears only for d even. Using this expansion one can express the vacuum expectation value of the boundary CFT stress tensor as

$$\langle T_{\mu\nu}\rangle = -\frac{d\ell^{d-1}}{16\pi G_{d+1}} g_{\mu\nu}^{(d)} + X_{\mu\nu}\left[g^{(n)}\right]$$

The term $X_{\mu\nu}\left[g^{(n)}\right]$ is a function of $g_{\mu\nu}^{(n)}$ with $n < d$.

The exact form of $X_{\mu\nu}$ depends on the spacetime dimension and it reflects the **conformal anomalies** of the boundary CFT
M. Henningsson and K. Skenderis, JHEP 07, 023 (1998)

For d odd, there are no gravitational conformal anomalies and $X_{\mu\nu} = 0$.
For d even, we may expect that the conformal anomaly related to the $X_{\mu\nu}$ term will correspond to the logarithmic correction to the BH entropy.
D.V. Fursaev, PRD (1995); R.G. Cai, L.M. Cao and N. Ohta, JHEP (2010);

In the following, we will ignore $X_{\mu\nu}$ and focus on the traceless part of
$\langle T_{\mu\nu}\rangle \sim g^{(d)}_{\mu\nu}$

AdS Schwarzschild and planar BH geometries in Fefferman-Graham coordinates

We will focus on the leading term in the BH entropy for which we expect to correspond to the traceless part of the universal term proportional to $g_{\mu\nu}^{(d)}$. Hence, we need to identify this term for both the AdS Schwarzschild and planar BH metrics.

We will first transform the AdS Schwarzschild and AdS planar BH metrics to the Fefferman-Graham coordinates. Then, by making a comparison with the expansion of $g_{\mu\nu}$ near $z = 0$, we will extract the coefficients $g_{\mu\nu}^{(n)}$.

AdS Schwarzschild BH geometry

By transforming the radial coordinate $r = \ell^2/\rho$, we rewrite the AdS Schwarzschild BH metric as

$$s^2 = \frac{\ell^2}{\rho^2}\left[\left(1+\frac{\rho^2}{\ell^2}-\mu\frac{\rho^d}{\ell^d}\right)d\tau^2-\left(1+\frac{\rho^2}{\ell^2}-\mu\frac{\rho^d}{\ell^d}\right)^{-1}d\rho^2-\ell^2 d\Omega_{d-1}^2\right].$$

Then, we use the transformation

$$\frac{dz}{z}=\frac{d\rho}{\rho}\left(1+\frac{\rho^2}{\ell^2}-\mu\frac{\rho^d}{\ell^d}\right)^{-1/2}$$

The integration of this cannot be carried out in terms of elementary functions, with the exception of $d = 2$ and $d = 4$. However, as we are interested in the expansion near $z = 0$ up to the order z^d, we can apply an ansatz

$$ds^2=\frac{\ell^2}{z^2}\left[H(z)d\tau^2-F(z)\ell^2 d\Omega_{d-1}^2-dz^2\right]$$

where

$$F(z)=1+\sum_{n=2}^{\infty}f_n\frac{z^n}{\ell^n},\quad H(z)=1+\sum_{n=2}^{\infty}h_n\frac{z^n}{\ell^n},$$

Keeping only the terms up to z^d we find

$$F(z) = 1 - \frac{1}{2}\frac{z^2}{\ell^2} + \frac{1}{16}\frac{z^4}{\ell^4} + \frac{1}{d}\mu\frac{z^d}{\ell^d} + \cdots, \qquad H(z) = 1 + \frac{1}{2}\frac{z^2}{\ell^2} + \frac{1}{16}\frac{z^4}{\ell^4} - \frac{d-1}{d}\mu\frac{z^d}{\ell^d} + \cdots,$$

so the background is the static Einstein universe $\mathbf{R} \times \mathbf{S}^{d-1}$ with line element

$$g^{(d)}_{\mu\nu}dx^\mu dx^\nu = -\frac{\mu}{d\ell^d}\left[(d-1)d\tau^2 + \ell^2 d\Omega^2_{d-1}\right].$$

For $d \neq 4$ we have

$$g^{(d)}_{\mu\nu}dx^\mu dx^\nu = -\frac{\mu}{d\ell^d}\left[(d-1)d\tau^2 + \ell^2 d\Omega^2_{d-1}\right],$$

and a similar expression for $d = 4$.

Then, according to the prescription, we obtain the boundary CFT stress tensor

$$\left\langle T^{(d)\mu}{}_{\nu}\right\rangle = \frac{\ell^{-1}\mu}{16\pi G_{d+1}} \; \mathrm{diag}(d-1, -1, -1, \ldots) = \mathrm{diag}(\varepsilon_{\mathrm{CF}}, -p_{\mathrm{CF}}, -p_{\mathrm{CF}}, \ldots)$$

in the form of a d-dimensional conformal fluid with the equation of state

$$p_{\mathrm{CF}} = \varepsilon_{\mathrm{CF}}/(d-1),$$

where p_{CF} is the pressure and $\varepsilon_{\mathrm{CF}}$ the energy density.

By writing the BH mass parameter μ in terms of r_+ , we have

$$\varepsilon_{\mathrm{CF}} = \frac{(d-1)\ell^{-1}}{16\pi G_{d+1}}\left(\frac{r_+}{\ell}\right)^d\left(1+\frac{\ell^2}{r_+^2}\right),$$

which for large r_+ scales as T_{Sch}^d

The corresponding entropy density is

$$s_{\mathrm{CF}} = \frac{1}{T}(p_{CF} + \varepsilon_{\mathrm{CF}})$$

So far the temperature T is unknown. However, the energy density $\varepsilon_{\mathrm{CF}}$ of the conformal fluid should scale as the energy density of the black-body radiation, i.e., as $\sim T^d$, so we can assume that T is proportional to the temperature $T_{\mathrm{Sch}} = dr_+/(4\pi\ell^2)$ of the large BH. So, if $T = \gamma T_{\mathrm{Sch}}$, where γ is a dimensionless constant, find

$$s_{\mathrm{CF}} = \frac{1}{4G_{d+1}}\frac{1}{\gamma}\left(\frac{r_+}{\ell}\right)^{d-1}$$

which scales as $\sim T_{\mathrm{Sch}}^{d-1}$.

If we set $\gamma = 1$ and multiply $s_{\rm CF}$ by the volume $\ell^{d-1}\Omega_{d-1}$ of the background space, the obtained entropy coincides with AdS Schwarzschild BH entropy derived by Witten,

$$S_{\rm Sch} = \frac{r_+^{d-1}\Omega_{d-1}}{4G_{d+1}}$$

and confirms the area law.

Hence, the temperature T of the conformal fluid at the holographic boundary can be identified with the horizon temperature $T_{\rm Sch}$ of the large Schwarzschild BH.

AdS Planar BH geometry

Next, we apply the above procedure to a planar BH metric. Starting from the metric written in terms of the coordinate ρ we transform ρ into z by

$$\frac{dz}{z} = \frac{d\rho}{\rho}\left(1 - \frac{\rho^d}{\rho_{\rm pl}^d}\right)^{-1/2}$$

Integrating this equation and substituting z for ρ in the metric we find

$$ds^2 = \frac{\ell^2}{z^2}\left[\left(1 - \frac{1}{4}\frac{z^d}{\rho_{\rm pl}^d}\right)^2\left(1 + \frac{1}{4}\frac{z^d}{\rho_{\rm pl}^d}\right)^{4/d-2} d\tau^2 - \left(1 + \frac{1}{4}\frac{z^d}{\rho_{\rm pl}^d}\right)^{4/d} dx^2 - dz^2\right]$$

Expanding in z^n up to $n = d$ and comparing with the general expression for the asymptotically AdS metric we find the coefficients

$$g_{\mu\nu}^{(0)} = \eta_{\mu\nu}, \qquad g_{\mu\nu}^{(d)} = -\frac{1}{d\rho_{\rm pl}^d}\ \mathrm{diag}(d-1,1,1,\ldots)$$

so the traceless part of the CFT stress tensor is

$$\left\langle T^{(d)\mu}{}_{\nu}\right\rangle = \frac{\ell^{d-1}\rho_{\rm pl}^{-d}}{16\pi G_{d+1}}\ \mathrm{diag}(d-1,-1,-1,\ldots) = \mathrm{diag}(\varepsilon_{\rm CF},-p_{\rm CF},-p_{\rm CF},\ldots)$$

As in the case of the AdS Schwarzschild BH, this form is identical to the stress tensor of the d-dimensional conformal fluid with the equation of state $p_{\rm CF} = \varepsilon_{\rm CF}/(d-1)$, where the energy density is given by

$$\varepsilon_{\rm CF} = \left\langle T^{(d)0}{}_{0}\right\rangle = \frac{(d-1)\ell^{d-1}\rho_{\rm pl}^{-d}}{16\pi G_{d+1}}.$$

Assuming $T = T_{\rm pl} = d/\left(4\pi\rho_{\rm pl}\right)$ the obtained the entropy density

$$s_{\rm CF} = \frac{1}{4G_{d+1}}\left(\frac{\ell}{\rho_{\rm pl}}\right)^{d-1};$$

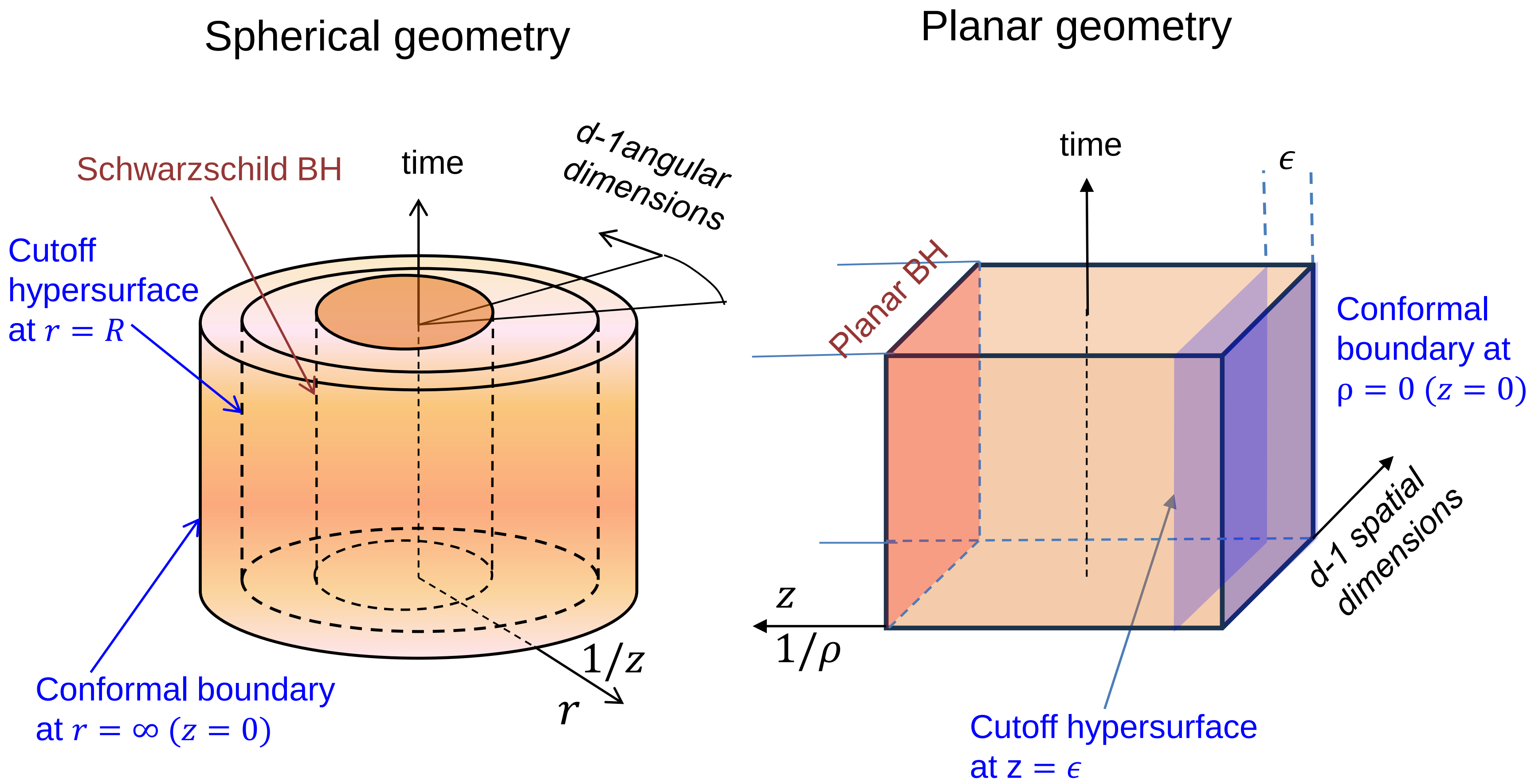
scales as $\sim T_{\rm pl}^{d-1}$. Multiplied by the background volume L^{d-1}, this quantity coincides with the planar BH entropy $S_{\rm pl}$ derived previously for $\rho_{\rm pl} = \ell$

$$S_{\rm pl} = \frac{L^{d-1}}{4G_{d+1}}$$

Next, we assume equality of the total entropies of the conformal fluid on the AdS boundary for AdS Schwarzschild and planar BHs. This assumption is quite natural since we have explicitly demonstrated that the entropy density of the conformal fluid at the boundary scales as $T_{\mathrm{Sch}}^{d-1}$and as T_{pl}^{d-1}, exactly as the entropy of a large AdS Schwarzschild and planar BHs.

To make a comparison, we need the total entropy contained in the space volume of the boundary. Since the boundary space volume is infinite (owing to the conformal factor $1/z^2$ in the AdS metric), the total CF entropy is infinite. To make it finite, we will choose a cutoff near the AdS boundary.

For the spherical geometry, the cutoff hypersurface is a large hypersphere of radius $R \to \infty$. For the planar geometry, the corresponding cutoff hypersurface is a hyperplane at $\rho = \epsilon$.



Then, the total entropy of the conformal fluid in the spherical geometry is

$$S_{\mathrm{CF}}|_{\mathrm{Sch}} = \frac{1}{4G_{d+1}}\left(\frac{4\pi\ell T_{\mathrm{Sch,loc}}}{d}\right)^{d-1} \lim_{R\to\infty} R^{d-1}\Omega_{d-1}$$

where $R^{d-1}\Omega_{d-1}$ is the area of the cutoff hypersphere which approaches the boundary as $R \to \infty$.

Similarly, for the planar geometry

$$S_{\mathrm{CF}}|_{\mathrm{pl}} = \frac{1}{4G_{d+1}}\left(\frac{4\pi\ell T_{\mathrm{pl,loc}}}{d}\right)^{d-1} \lim_{\epsilon\to 0} \frac{\ell^{d-1}}{\epsilon^{d-1}} L^{d-1}$$

where $\left(\ell^{d-1}/\epsilon^{d-1}\right)L^{d-1}$ is the area of the cutoff hyperplane which approaches the boundary as $\epsilon \to 0$.

The quantities $T_{\mathrm{Sch,loc}}$ and $T_{\mathrm{pl,loc}}$ are the local temperatures at the cutoff hypersurfaces.

Now, if we equate the cutoff areas, i.e.,

$$\frac{\ell^{d-1}}{\epsilon^{d-1}} L^{d-1} = R^{d-1} \Omega_{d-1}$$

As a consequence, from the equation $S_{\mathrm{CF}}|_{pl} = S_{\mathrm{CF}}|_{\mathrm{Sch}}$, we obtain a relation between the local temperatures of the planar BH and the large Schwarzschild BH measured near the AdS boundary

$$T_{\mathrm{Sch,loc}} = T_{\mathrm{pl,loc}}$$

This is our main result: **for a local observer, the large AdS Schwarzschild BH appears as hot as the corresponding planar BH**.

In addition, equating $S_{\mathrm{CF}}|_{pl}$ and $S_{\mathrm{CF}}|_{\mathrm{Sch}}$ with previously derived entropies at the boundary, we recover the Tolman law for both spherical and planar geometries

$$T_{\mathrm{Sch,loc}} \equiv \frac{T_{\mathrm{Sch}}}{\sqrt{g_{00}}} = \frac{T_{\mathrm{Sch}}}{R/\ell}, \qquad T_{\mathrm{pl,loc}} \equiv \frac{T_{\mathrm{pl}}}{\sqrt{g_{00}}} = \frac{T_{\mathrm{pl}}}{\ell/\epsilon}.$$

Nota bene: The **local** observers near the AdS boundary measure the redshifted local temperatures $T_{\mathrm{Sch,loc}}$ and $T_{\mathrm{pl,loc}}$ at the cutoff hypersurfaces. These temperatures diverge as one approaches the boundary, so the horizon temperatures T_{Sch} and T_{pl} do not have the meaning of asymptotic temperatures as is the case of the usual Schwarzschild spacetime.

The quantity R is a cutoff radius near the AdS boundary with spherical geometry, and ϵ is the corresponding cutoff with planar geometry. We have introduced these cutoffs to regularize the entropy of the conformal fluid at the AdS boundary, so it is natural to choose $R/\ell \sim \ell/\epsilon$ of the same order of magnitude.

As a consequence, we obtain a relation between the horizon temperatures of the planar BH and the large Schwarzschild BH

$$\frac{T_{\mathrm{Sch}}}{R/\ell} = \frac{T_{\mathrm{pl}}}{\ell/\epsilon} \quad \Longrightarrow \quad T_{\mathrm{Sch}} \sim T_{\mathrm{pl}}$$

Making use of the previously derived expression $T_{\mathrm{pl}} = d/(4\pi\ell)$, we find

$$T_{\mathrm{Sch}} \sim \frac{d}{4\pi\ell}$$

Thus, although the local observers measure a large local temperature, the actual horizon temperature is not large. Our result confirms the argument of Hubeny, Marolf, and Rangamani based on the translational invariance of the planar BH horizon.

Summary

The temperature relationships are obtained invoking three interrelated principles:

1) the AdS/CFT conjecture,
2) the area law for the entropy of AdS planar BHs,
3) the total entropies of the CFT on the holographic boundary for planar and spherical geometries are identical and scale as the black-body radiation at the boundary.

Combining the expressions for entropies we have obtained

$$\frac{T_{\rm Sch}}{R/\ell} = \frac{T_{\rm pl}}{\ell/\epsilon} \qquad \text{and} \qquad T_{\rm Sch} \sim T_{\rm pl} = \frac{d}{4\pi\ell}$$

The physical meaning of these two equations is clear: the temperature of a large Schwarzschild BH and that of the corresponding planar BH, as seen by an observer near the AdS boundary, are of the same order of magnitude. This result resolves the horizon temperature ambiguity of the large AdS Schwarzschild BH and its relation to the AdS planar BH.

Outlook

We have neglected the $X_{\mu\nu}$ term related to the conformal anomaly and the log corrections to the BH entropy.

In this context, it would be of interest to apply the presented formalism to study the departure of the boundary CFT fluid from ideal. In particular, one could use the AdS/CFT correspondence to calculate the bulk and shear viscosities of that fluid.

Thank you

Conclusions

1. We predict that the temperatures of large AdS Schwarzschild and planar BHs are of the same order of magnitude as seen by an observer near the AdS boundary.

2. Because of the diverging redshift near the boundary, local observers measure a large redshifted temperature of the AdS Schwarzschild BH. This contrasts with the case of ordinary Schwarzschild BH where asymptotic observers see the actual horizon temperature.

3. Our result confirms the argument of Hubeny, Marolf, and Rangamani, based on the translational invariance of the planar BH horizon

Black body radiation at the holographic boundary

We now assume that the conformal fluid at the boundary is equivalent to the **black-body radiation** at a temperature T . For a theory with n_{B} massless bosons and n_{F} massless fermions in $d-1$ spatial dimensions, the energy density and pressure of a gas of massless particles at a temperature T is given by

$$\varepsilon = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left(\frac{n_B k}{e^{k/T}-1} + \frac{g n_F k}{e^{k/T}+1} \right) = a_d T^d, \qquad p = \frac{\varepsilon}{d-1}$$

and the entropy density by

$$s = T^{-1}(p+\varepsilon) = \frac{d a_d}{d-1} T^{d-1}$$

where

$$a_d = \frac{\Omega_{d-2}\Gamma(d)\zeta(d)}{(2\pi)^{d-1}} \left(n_{\mathrm{B}} + \left(1-2^{1-d}\right) g n_{\mathrm{F}}\right).$$

The factor g is included to account for the degeneracy due to the internal degrees of freedom of the fermions. For example, for Weyl fermions in $3+1$ dimensional spacetime, $g=2$.

As in the Schwarzschild case, the total CF entropy on the cutoff hypersurface is the entropy density s_{CF} multiplied by the area of the hypersurface. For $\rho_{\mathrm{pl}} = \ell$ we find

$$S_{\mathrm{CF}} = \frac{1}{4G_{d+1}} \lim_{\epsilon \to 0} \frac{\ell^{d-1}}{\epsilon^{d-1}} L^{d-1}$$

where $\left(\ell^{d-1}/\epsilon^{d-1}\right) L^{d-1}$ is the area of the cutoff hyperplane at $\rho = \epsilon$, which approaches the boundary as $\epsilon \to 0$.

The total black-body radiation entropy on the cutoff hypersurface

$$S = sV = \underbrace{\frac{da_d}{d-1} T^{d-1} \lim_{R\to\infty} R^{d-1} \Omega_{d-1}}_{\text{spherical geometry}} = \underbrace{\frac{da_d}{d-1} T^{d-1} \lim_{\epsilon\to 0} \frac{\ell^{d-1}}{\epsilon^{d-1}} L^{d-1}}_{\text{planar geometry}}$$

Then, from the equation $S = S_{\mathrm{CF}}$ for a large Schwaryschild BH we find a relationship between the temperatures T and T_{Sch}

$$T = \frac{4\pi\ell}{d} \left(\frac{d-1}{4da_d G_{d+1}} \right)^{1/(d-1)} T_{\mathrm{Sch}}$$

for the planar BH we find

$$T = \left(\frac{d-1}{4da_d G_{d+1}} \right)^{1/(d-1)}$$

The total black-body radiation entropy on the cutoff hypersurface

$$S = sV = \underbrace{\frac{da_d}{d-1} T^{d-1} \lim_{R\to\infty} R^{d-1} \Omega_{d-1}}_{\text{spherical geometry}} = \underbrace{\frac{da_d}{d-1} T^{d-1} \lim_{\epsilon\to 0} \frac{\ell^{d-1}}{\epsilon^{d-1}} L^{d-1}}_{\text{planar geometry}}$$

Then, from the equation $S = S_{\mathrm{CF}}$ for a large Schwaryschild BH we find a relationship between the temperatures T and T_{Sch}

$$T = \frac{4\pi\ell}{d} \left(\frac{d-1}{4da_d G_{d+1}} \right)^{1/(d-1)} T_{\mathrm{Sch}}$$

From the equation $S = S_{\mathrm{CF}}$ for the planar BH we find

$$T = \left(\frac{d-1}{4da_d G_{d+1}} \right)^{1/(d-1)}$$

$$T_{\mathrm{Sch}} = \frac{d}{4\pi\ell}$$

From these two equations we obtain

As a consequence, we obtain a relation between the redshifted local temperatures of the planar BH and the large Schwarzschild BH measured near the AdS boundary

$$\frac{T_{\mathrm{Sch}}}{R/\ell} = \frac{T_{\mathrm{pl}}}{\ell/\epsilon}$$

where R is a cutoff radius near the AdS boundary with spherical geometry and ϵ is the corresponding cutoff with planar geometry. We have introduced these cutoffs to regularize the entropy at the AdS boundary, so it is natural to choose $R/\ell \sim \ell/\epsilon$ of the same order of magnitude. Then, we have

$$T_{\mathrm{Sch}} \sim T_{\mathrm{pl}}$$

Making use of the previously derived expression $T_{\mathrm{pl}} = d/(4\pi\ell)$, we find an estimate for the temperature of a large Schwarzscild BH

$$T_{\mathrm{Sch}} \sim \frac{d}{4\pi\ell}$$

Outline

1. **Thermodynamics of AdS Schwarzschild and planar BHs**
   - Entropy of AdS planar BHs
2. **AdS/CFT holography**
   - AdS Schwarzschild BH
   - AdS planar BH
   - Temperature relationships
3. **Conclusions and outlook**

In the spherical case, the total black-body radiation entropy

$$S = sV = \frac{d a_d}{d-1} T^{d-1} \lim_{R\to\infty} R^{d-1} \Omega_{d-1},$$

Then, from the equation $S = S_{\mathrm{CF}}$ for a large BH we find a relationship between the temperatures T and T_{Sch}

$$T = \frac{4\pi\ell}{d} \left(\frac{d-1}{4 d a_d G_{d+1}} \right)^{1/(d-1)} T_{\mathrm{Sch}}$$

Note: According to Tolman's law, the **local** temperature of the AdS Schwarzschild spacetime is

$$T_{\mathrm{Sch,loc}} \equiv \frac{T_{\mathrm{Sch}}}{\sqrt{g_{00}}} = \frac{T_{\mathrm{Sch}}}{R/\ell}.$$

Hence, the above expression for T relates the temperature of the holographic CF to the **local** temperature of the Schwarzschild BH near the AdS boundary at $r = R$.

Similarly, in the planar BH geometry, the total entropy of the black-body radiation on the boundary is given by

$$S = \frac{d a_d}{d-1} T^{d-1} \lim_{\epsilon\to 0} \frac{\ell^{d-1}}{\epsilon^{d-1}} L^{d-1}$$

where $\left(\ell^{d-1}/\epsilon^{d-1}\right) L^{d-1}$ is the area of the cutoff hypersurface at $\rho = \epsilon$, which approaches the boundary as $\epsilon \to 0$. As in the case of AdS Schwarzschild BH, we assume that the total entropy of the CFT at the holographic boundary equals the AdS planar BH entropy Then, from the equation $S = S_{\text{pl}}$ we find

$$T = \left(\frac{d-1}{4 d a_d G_{d+1}}\right)^{1/(d-1)}$$

were we have set $\rho_{pl} = \ell$, as required by the BH entropy area law, and used the previously derived expression for $T_{\text{pl}} = d/(4\pi\ell)$.

$$T = \frac{4\pi\ell}{d}\left(\frac{d-1}{4d a_d G_{d+1}}\right)^{1/(d-1)} \frac{T_{\mathrm{pl}}}{\ell/\epsilon}$$

Here, similar to case of Schwarzschild BH geometry, the quantity $T_{\mathrm{pl}}/(\ell/\epsilon)$ can be regarded as the **local** temperature near the AdS boundary at $\rho = \epsilon$,

$$T_{\mathrm{pl,loc}} \equiv \frac{T_{\mathrm{pl}}}{\sqrt{g_{00}}} = \frac{T_{\mathrm{pl}}}{\ell/\epsilon}.$$

Now we compare the above expression for T with the similar expression derived for the AdS Schwarzschild geometry

$$T = \frac{4\pi\ell}{d}\left(\frac{d-1}{4d a_d G_{d+1}}\right)^{1/(d-1)} \frac{T_{\mathrm{Sch}}}{R/\ell}.$$

$$T = \frac{4\pi\ell}{d}\left(\frac{d-1}{4d a_d G_{d+1}}\right)^{1/(d-1)} \frac{T_{\rm pl}}{\ell/\epsilon}$$

Here, similar to case of Schwarzschild BH geometry, the quantity $T_{\rm pl}/(\ell/\epsilon)$ can be regarded as the **local** temperature near the AdS boundary at $\rho = \epsilon$,

$$T_{\rm pl,loc} \equiv \frac{T_{\rm pl}}{\sqrt{g_{00}}} = \frac{T_{\rm pl}}{\ell/\epsilon}.$$

Now we compare the above expression for T with the similar expression derived for the AdS Schwarzschild geometry

$$T = \frac{4\pi\ell}{d}\left(\frac{d-1}{4d a_d G_{d+1}}\right)^{1/(d-1)} \frac{T_{\rm Sch}}{R/\ell}.$$