"Fractional Hall states from large N fermions" - ICTP

I. context
II. large N model & G/E
III. results

based on work w/ Amir Raz (Austin)

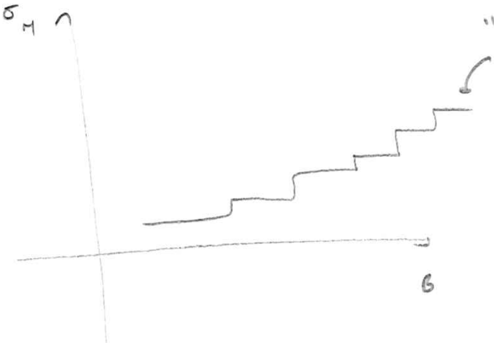
* This is a talk about a subject (FQH) that is quite far from the rest of this workshop.

~~gravity~~ holo~~graphy~~ ~~strings~~ rela~~ti~~vity ~~black holes~~

→ However, the FQH is a simpler instance of strong correlation physics and so of intrinsic interest to anyone who cares about non-perturbative phenomena in QFT.

* What is the FQH? The most simple observation/characterization is, for certain planar materials in a large applied magnetic field,

$\rho_H = \frac{1}{\sigma_H}$




$\frac{2\pi\hbar}{e^2 \nu}$

$= \mathbb{Z}$ (integer quantum Hall)

$\nu = \frac{p}{q} = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots$ (FQH)

each plateau corresponds to a gapped phase of matter w/ a TQFT description at low energies, a CS theory, and, for some $\nu$, fractionally charged massive quasiparticles (anyons)

$S \sim$ "$\frac{\nu}{4\pi}\int A\,dA$"

→ Lore: these states arise from interacting 2d electrons in an applied field.

This has not been demonstrated directly, despite 40 years of FQH physics.

There is a well-developed theory of the FQH, beginning w/ Laughlin and continuing w/ Haldane, Read, Moore, Jain, Fradkin, and many others. But this theory doesn't directly solve this. It focuses instead on identifying ansatze for N-particle wavefunctions hoped for being in the same "universality class" of the actual ground state of this strongly correlated H.

UV = Interacting fermions

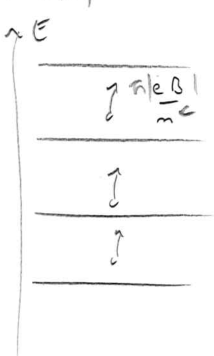
?↓

IR CS EFT

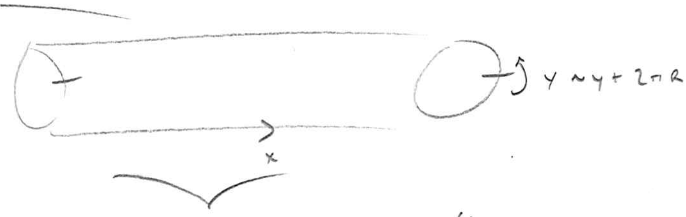
To begin let me introduce you to some of the players in this game.

Non-interacting electrons in B: let's work on a cylinder of (long) radius R.

$\vec{A} = Bx\hat{y}$




spectrum arranges into Landau levels.




L units of magnetic flux

← L-fold degenerate single-particle states

↗ consequence of symmetry

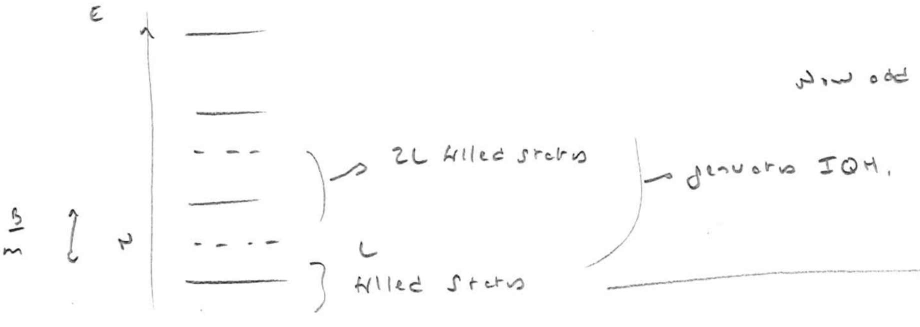
"Fractional Hall from large N" - ICTP

What are symmetries of problem?

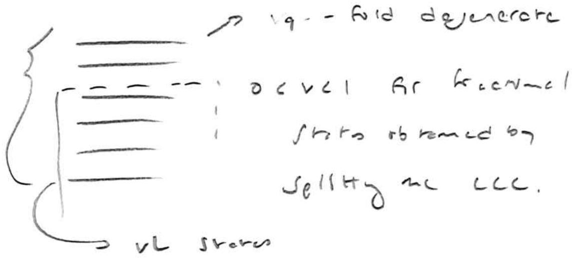
- $U(1)$ → electron number
- time translations → $E$ $\mathbb{R}$
- $y$-translations → $p_y$ $U(1)$
- discrete $x$-translations → $x \to x + \frac{1}{BR}$ ($e = 1$ units) $\mathbb{Z}_L$
- "magnetic translations"

$\mathbb{Z}_L$ governs $L$-fold degeneracy of LLL

In grand canonical ensemble, where you control $N$,




Now add interactions that split the LLs and evidently generate states of "fractional filling"




Take $m \to 0$ so that higher LLs decouple.
(second-quantized)
Many-body problem is described by a NR QFT

$$S = \int dt\, d^2x \left( i\psi^\dagger(\partial_t - i\mu)\psi + \chi^\dagger \bar{D}\psi + \chi D \psi^\dagger - (\text{interactions}) \right)$$

$$\bar{D} = \frac{1}{2}(\partial_x - i\partial_y + Bx) \qquad \chi = \text{Lagrange multiplier enforcing LLL.}$$

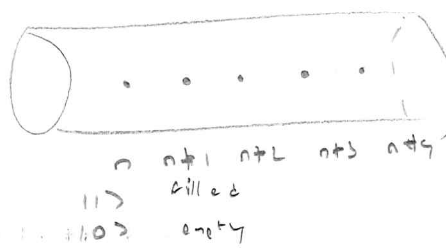
$$\mathcal{L}_{int} = \int d^2x\, d^2y\, dt\; |\psi(x,t)|^2\, \underbrace{V(\vec{x} - \vec{x}')}_{\text{translation-invariant two-body interaction } V(\vec{x}-\vec{x}')}\, |\psi(x',t)|^2$$

expected FQH in Nature arises from Coulomb interactions, but expected that short-range interactions of many types do the job too, i.e., interactions → robustly generate FQH.

$$\bar{D}\psi = 0 \;\to\; \psi(t,x,y) = \sum_n c_n(t)\, f_n(x,y), \qquad f_n(x,y) \propto \exp\left(-\frac{iny}{R} - \frac{B}{2}\left(x - \frac{n}{BR}\right)^2\right)$$

$$S \to \int dt \left( i c_n^\dagger(\partial_t - \mu) c^n - A^{n_1 n_3}_{n_2 n_4}\, c^\dagger_{n_1} c^{n_2} c^\dagger_{n_3} c^{n_4} \right)$$

$$A = \int d^2x\, d^2x'\; f^\dagger_{n_1}(x) f_{n_2}(x) f^\dagger_{n_3}(x') f_{n_4}(x')\, V(\vec{x} - \vec{x}')$$




$H = A c^\dagger c c^\dagger c$ = "all interaction"

Can rewrite the problem as a 1d fermion chain.

This many-body problem is strongly coupled and so intractable.

$\longrightarrow$ Change question so we can answer it. (The usual tradition)

$\psi \to \psi^{\alpha = 1, 2, \dots, N}$ w/ $N \gg 1$.

$$S = \int dt\, d^2x \left( i \psi^+_\alpha (\partial_t - i\mu) \psi^\alpha + \chi^+_\alpha \bar{D} \psi^\alpha + \chi^\alpha D \psi^+_\alpha - (\text{interaction}) \right)$$

$$\longrightarrow \int dt \left( i c^+_{\alpha n} (\partial_t - i\mu) c^\alpha_n - \frac{A^{n_1 n_3}{}_{n_2 n_4}}{N} \left( c^+_{\alpha n_1} c^\alpha_{n_2} \right)\left( c^+_{\beta n_3} c^\beta_{n_4} \right) \right)$$

$\to$ choose to preserve $u(N)$

keep $A$ finite w/ $N \gg 1$ / $A =$ "'t Hooft coupling"

large $N$ $\to$ weakly coupled in the right variables, access finite interaction at the cost of large $N$, i.e., $\frac{1}{N}$ is our small parameter.

For $N=4$ those variables are strings in $AdS_5 \times S^5$.

Here they are bilocal dof "$G/\Sigma$". Same strategy as in CSM, SYK.

$\int$ in $G_{n,m}(t)$, $\Sigma_{n,m}(t)$ w/ $G_{n,m}(t,t') = c^+_{n\alpha}(t)\, c^\alpha_m(t')$, $\Sigma$ enforces it.

$$S \to \int d\tau \left( c^+_{\alpha n} \partial_\tau c^\alpha_n - \mu G_{n,n} + \frac{A^{n_1 n_3}{}_{n_2 n_4}}{N} G_{n_1, n_2} G_{n_3 n_4} \right) - \int d\tau d\tau' \Sigma_{n,m} \left( G_{n,m} - c^+_{n\alpha} c^\alpha_m \right)$$

$t = -i\tau$

Fermion appear quadratically so we can $\int$ them out

$$\hookrightarrow -N \ln \det(\partial_\tau + \Sigma) + \int d\tau d\tau' \Sigma_{m,m} G_{n,m} + \int d\tau \left( -\mu + \frac{A^{n_1 n_3}{}_{n_2 n_4}}{N} G_{n_1 n_2} G_{n_3 n_4} \right)$$

why are $G/\Sigma$ weakly coupled? what do we do?

Ansatz a time-translation-invariant, $\psi$-invariant ansatz $= \langle c^+_{\alpha n}(\omega) c^\alpha_n(\omega) \rangle \left(1 + O\left(\frac{1}{N}\right)\right)$

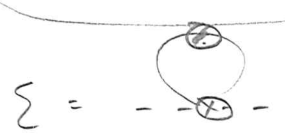
$$G_{n,m}(\omega, \omega') = N \beta\, \delta_{\omega + \omega', 0}\, \delta_{n,m}\, G_n(\omega') \qquad \omega' = \frac{\pi}{\beta}(2k+1)$$

$$\Sigma_{n,m}(\omega, \omega') = \beta\, \delta_{\omega + \omega', 0}\, \delta_{n,m}\, \Sigma_n(\omega') \qquad \frac{S}{N} = \text{finite} \Rightarrow N \sim \frac{1}{\hbar}$$

Solving the thy. to leading order in large $N$ amounts to solving eoms of $(G, \Sigma)$.

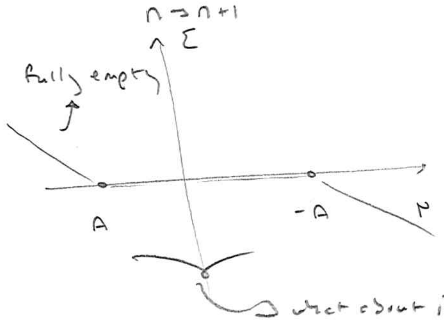
$$\begin{cases} G_n(\omega) = \dfrac{1}{i\omega + \Sigma_n(\omega)} \\[2ex] \Sigma_n(\omega) = -\mu + 2 \displaystyle\sum_{m,n} A_{m,n} \sum_{\omega'} G_m(\omega') \end{cases} \longrightarrow \boxed{\Sigma_n = -\mu + \sum_m A_{m,n} \tanh\left(\frac{\beta \Sigma_m}{2}\right)}$$

$A^{mn}{}_{mn}$



"Fractional Hall from large N" (cont) $(\omega_i)$

Translation invariant sol'ns?




fully gapped one for $|\mu| > |A|$ $\nearrow = \sum_m A_{n,m}$ $< 0$ $\quad A<0$

$\Sigma_n = -\mu \pm A \qquad Q_n = \dfrac{-\tanh\left(\frac{\beta \Sigma_n}{2}\right)+1}{2}$

gapless $\quad \Sigma_n = \dfrac{2}{\beta} \operatorname{arctanh} \dfrac{\mu}{A} \qquad |\mu| < A$

$G_n = \dfrac{1}{i\omega_n + \Sigma_n}$ $\nearrow$ $\Sigma_n > 0$ : $c_n$ empty; $\Sigma_n < 0$ : $c_n$ filled

**NEW SADDLES** : $\mathbb{Z}_L \to \mathbb{Z}_L/\mathbb{Z}_q$ $\quad$ $q$-fold degenerate $\quad \Sigma_{n+q} = \Sigma_n$

$$\Sigma_n = -\mu + \sum_{m=0}^{q-1} A^{(q)}_{m,n} \tanh\left(\frac{\beta \Sigma_m}{2}\right) \to \operatorname{sgn}(\Sigma_m)$$

$A^{(q)}_{m,n} = \sum_r A_{m,n+rq}$

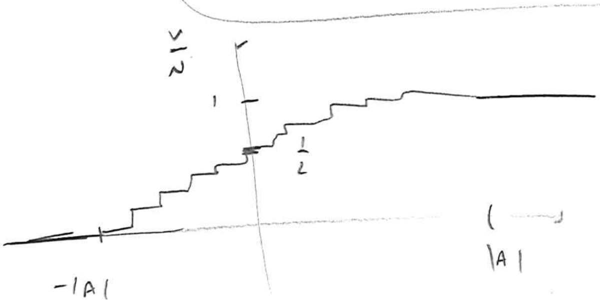
sol'ns determined numerically. depends on $\mu$.

| ↑ | ↓ | ↓ | | ↓ | | | | |
|---|---|---|---|---|---|---|---|---|
| • | • | • | ... | • | • | • | ... | • |
| $n$ | $n+1$ | $n+2$ | | $n+q-1$ | $n+q$ | $n+q+1$ | ... | $n+2q-1$ |

$\Sigma_n \lessgtr 0$, $\Sigma_{n+m} \gtrless 0$, $m < q$ $\Big)$ → fractional filling $\nu = N \dfrac{p}{q}$ ← $p$ = # of $\Sigma$'s $<0$ in unit cell; $q$ = # of $\Sigma$'s in a unit cell

If this is a sol'n, $q$ such sol'ns as determined by broken $\mathbb{Z}_q$.

$\boxed{\text{* We thereby find gapped states } \nu = N\frac{p}{q}, \ q\text{-fold degenerate.}}$




These gapped states have a CS effective description, which computes $\sigma_H$ at $T=0$. get a series of plateaux in the Hall conductivity together w/ transitions between them. Success.

Two remaining points:

- fractionally charged QPs: no time. $\boxed{N\frac{1}{q}}$
- finite N? Remarkably a subset of these results continue to hold away from $N\to\infty$.

The rewriting in terms of $G/\Sigma$ is exact.
Our sol'n are sol'ns to the field eq'ns even at $N=1$
However fluctuations are not small, so phase diagram modified at finite N.

$-\mu + A > 0 \Rightarrow \mu < A$ $\quad A > \mu$

$-\mu - A < 0 \Rightarrow \mu > -A$ $\quad A < \mu$ $\quad -\mu$

$A < 0$

$A > 0$