Progress in Flat Space Holography

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Motivation

1. Gauge-gravity correspondence:

- AdS/CFT: low energy gravity observables ~ correlators in strongly coupled CFT

  bulk geometry ~ entanglement in CFT

  low energy states in gravity ~ CFT code subspace

  black hole microstate counting

  tensionless strings: string worldsheet ~ boundary CFT

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Top-down: string theory $\Longrightarrow$ AdS/CFT $\longleftarrow$

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Top-down: string theory $\Longrightarrow$ AdS/CFT $\longleftarrow$ $\begin{cases} \text{black hole microstate counting} \\ \text{tensionless strings: string worldsheet} \sim \text{boundary CFT} \end{cases}$

Bottom-up: $\begin{cases} \bullet \text{ Asymptotic symmetries in gravity} \sim \text{symmetries of CFT in lower dim.} \\ \bullet \text{ Black hole entropy} \quad S = \dfrac{A}{4G} + \cdots \end{cases}$

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Top-down: string theory $\Longrightarrow$ AdS/CFT

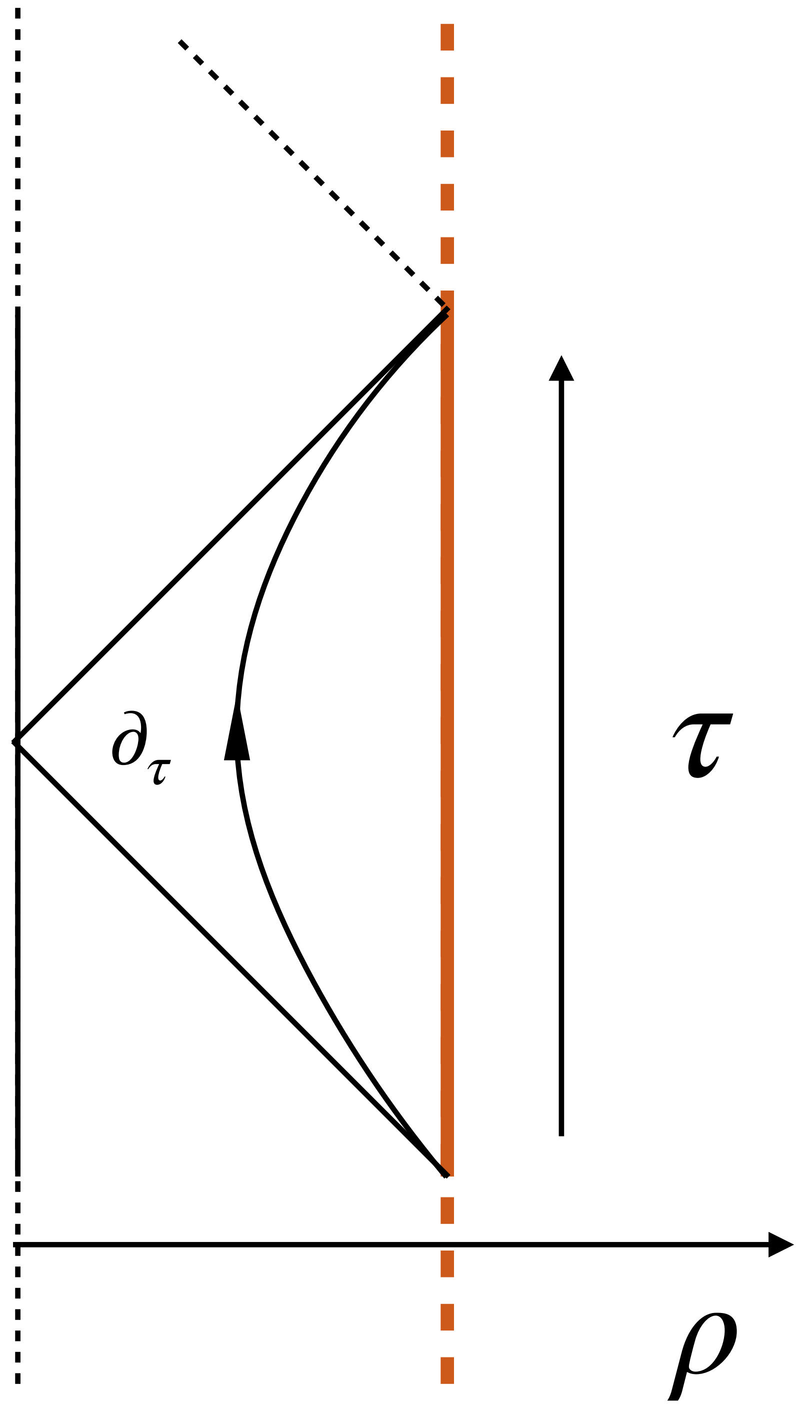
Bottom-up:

- Asymptotic symmetries in gravity ~ symmetries of CFT in lower dim.
- Black hole entropy $S = \dfrac{A}{4G} + \cdots$

Which aspects of gravity are captured by conformal field theories?

Motivation

2. Towards "all-Λ holography" ?



AdS: $\Lambda < 0$

- (Usually) no flux through boundary
- Bulk time evolution ~ boundary time evolution $\Longrightarrow$

Plausible to postulate that: $\begin{cases} \text{Bulk } \mathscr{H} \sim \text{boundary } \mathscr{H} \\ \text{Bulk unitarity} \sim \text{boundary unitarity} \end{cases}$

- Can explicitly check match at weak coupling/large N

Motivation

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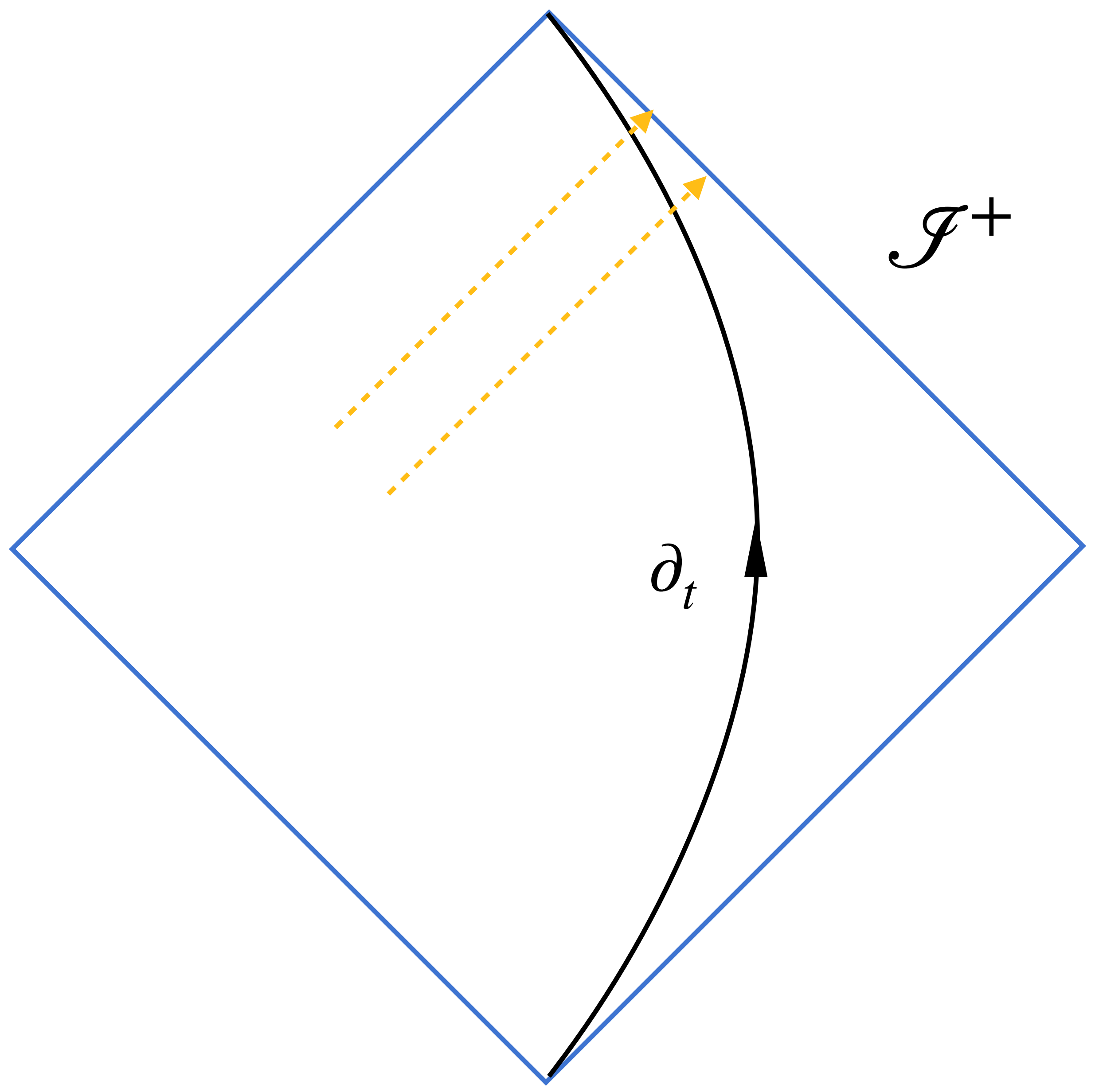
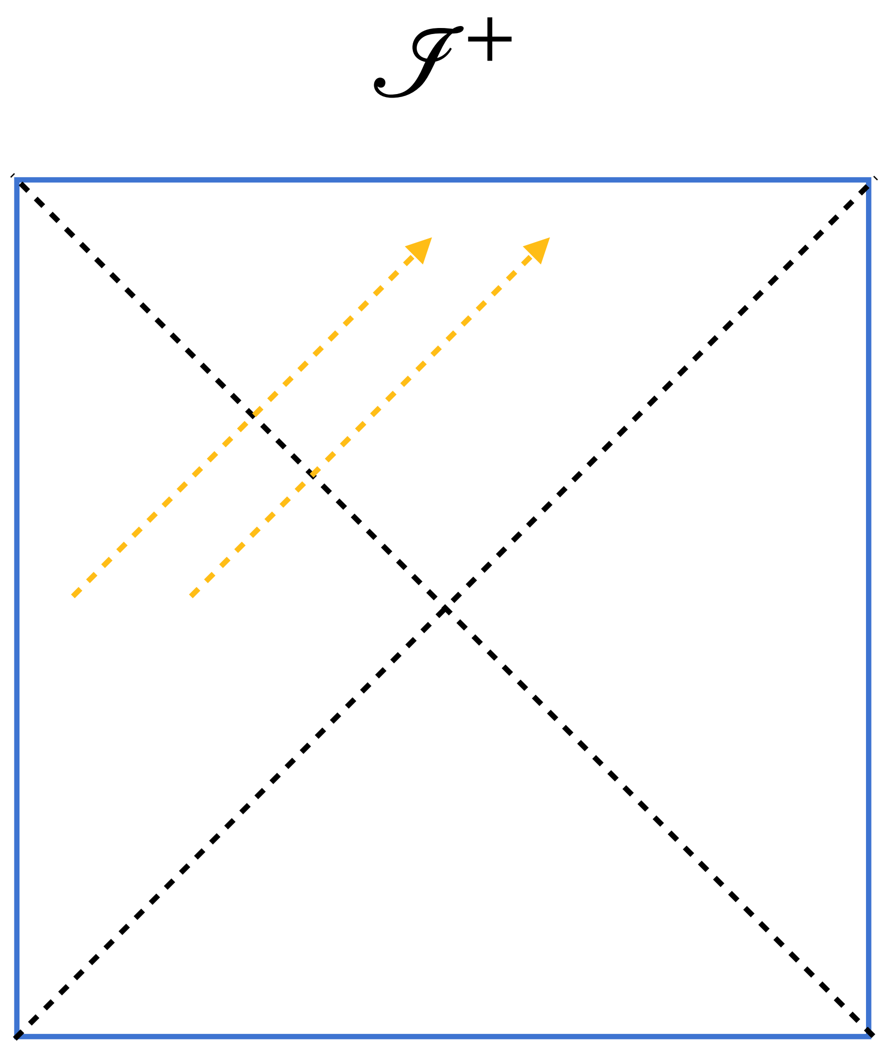


AFS: $\Lambda = 0$



dS: $\Lambda > 0$

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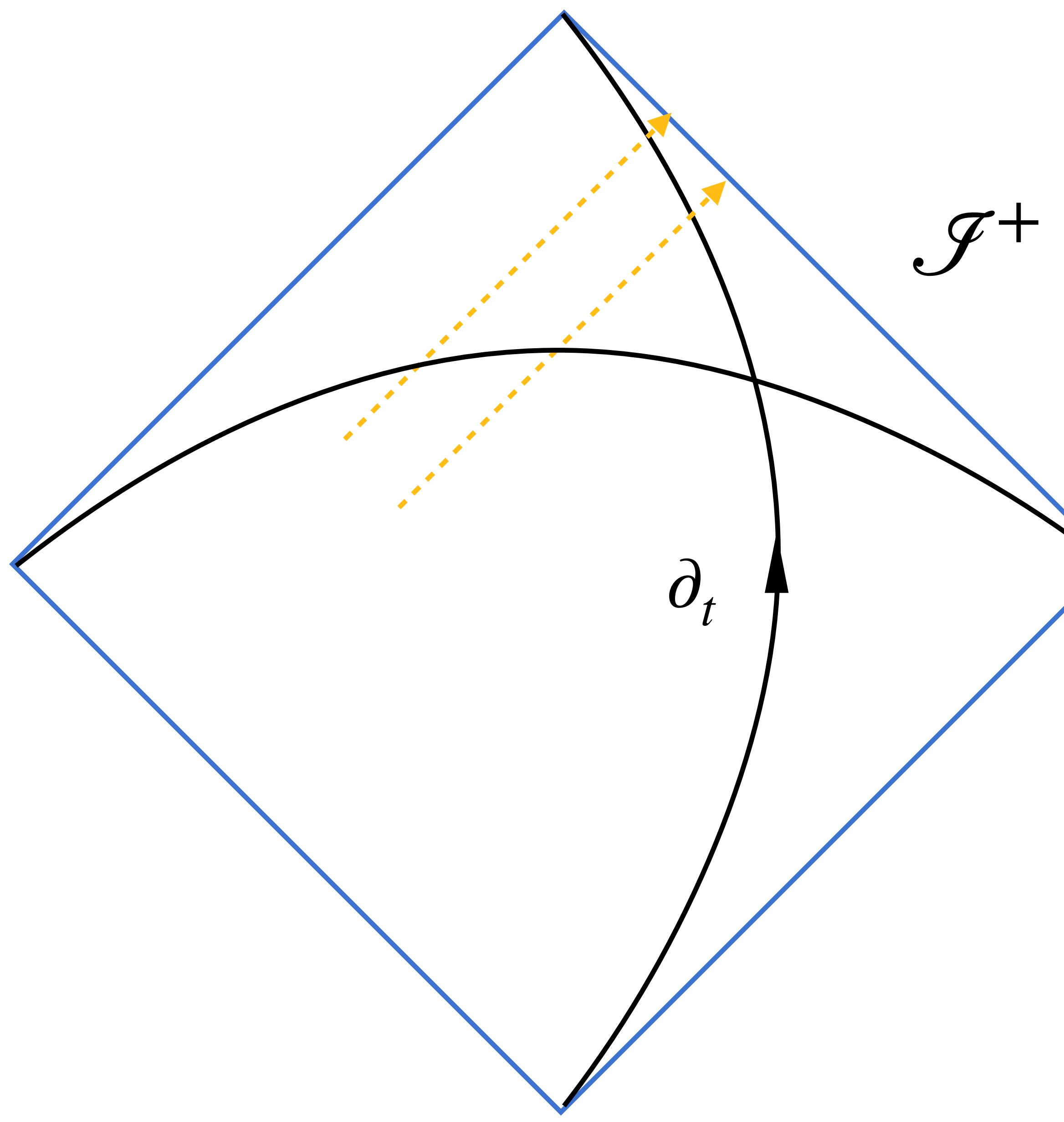
2. Towards "all-Λ holography" ?

- Flux through boundary $\Longrightarrow$

  Boundary conditions?

  No obvious place for QFT/CFT



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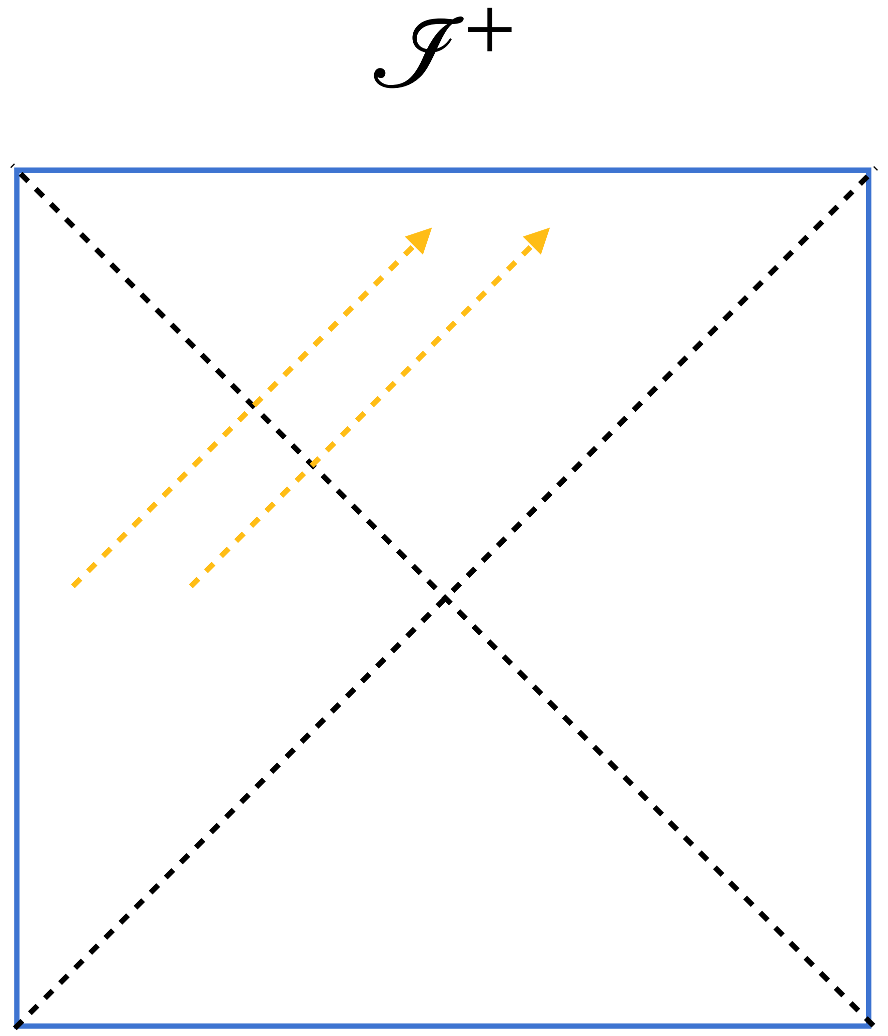

dS: $\Lambda > 0$

[Dio's talk]

Motivation

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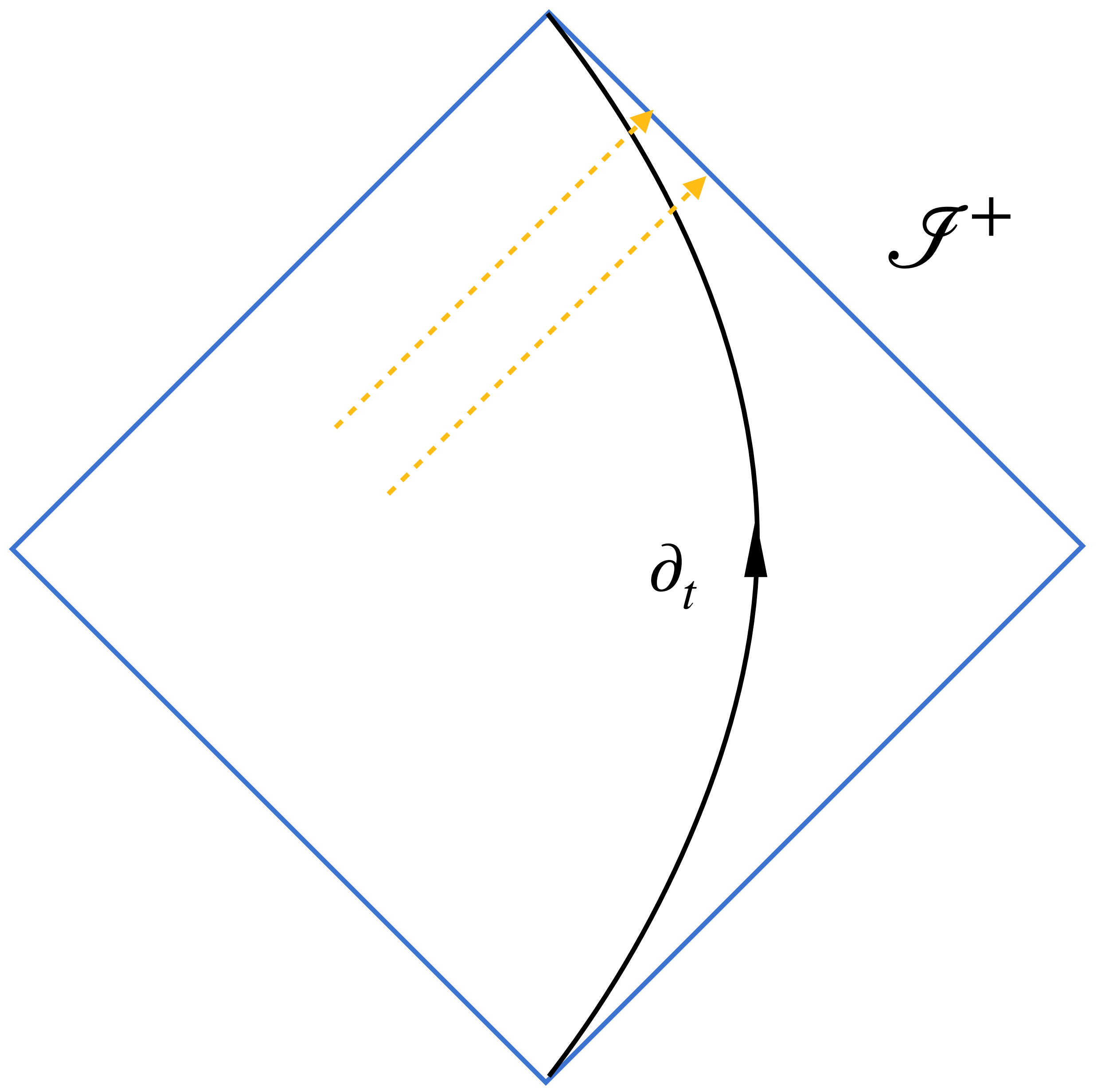
  Boundary conditions?

  No obvious place for QFT/CFT

Focus on $\Lambda = 0$

- Hints of CFT structure in sector of GR



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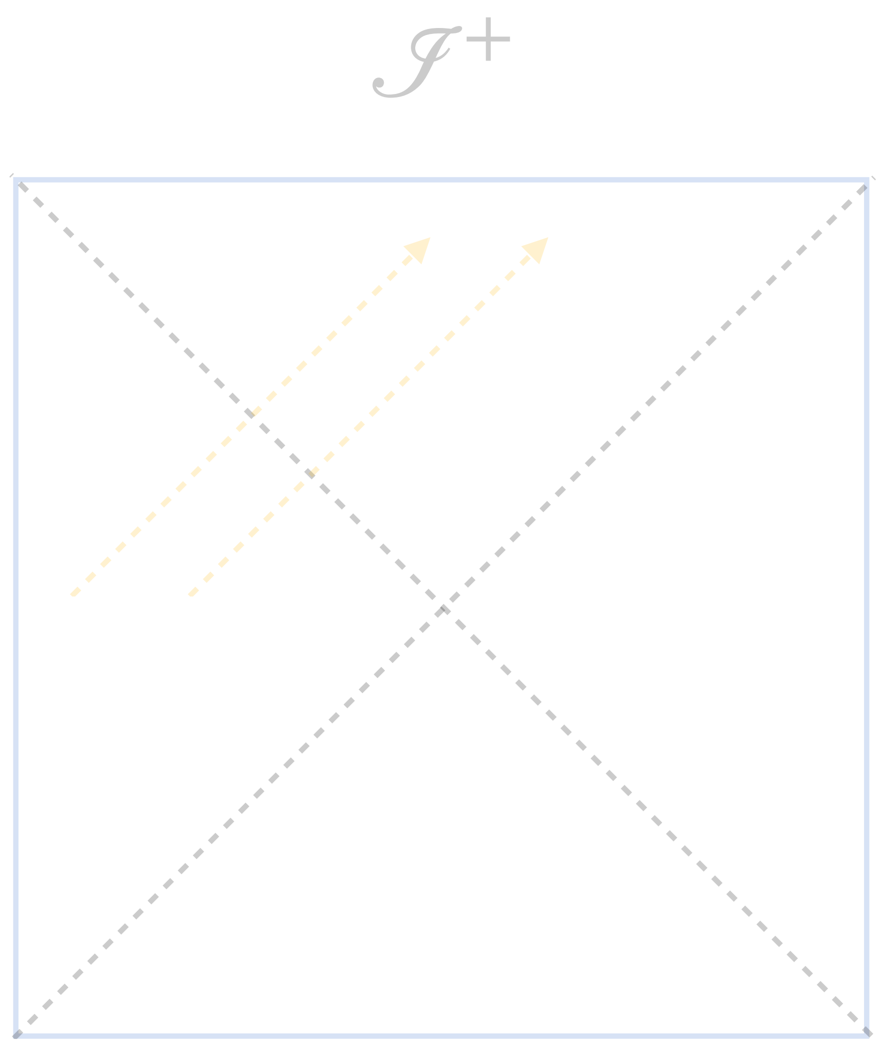

dS: $\Lambda > 0$

Motivation and outline

3. New structures in gravity in 3+1 dimensional AFS:

   A. Asymptotic symmetries and gravitational memory

   B. Celestial amplitudes

   C. Towards bulk reconstruction in general relativity

   D. Flat space limit of AdS/CFT

   E. Twistors, self-dual sector and top-down holography in AFS

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Asymptotic, semiclassical Virasoro symmetry

Infinite towers of symmetries

Infinity of charges ~ multipoles (discrete basis)

Emergent symmetries in flat limit

?

A. Asymptotic symmetries and gravitational memory

General relativity in Bondi gauge

Asymptotic observables:

- gravitational waves: GR, scattering amplitudes, numerics, …

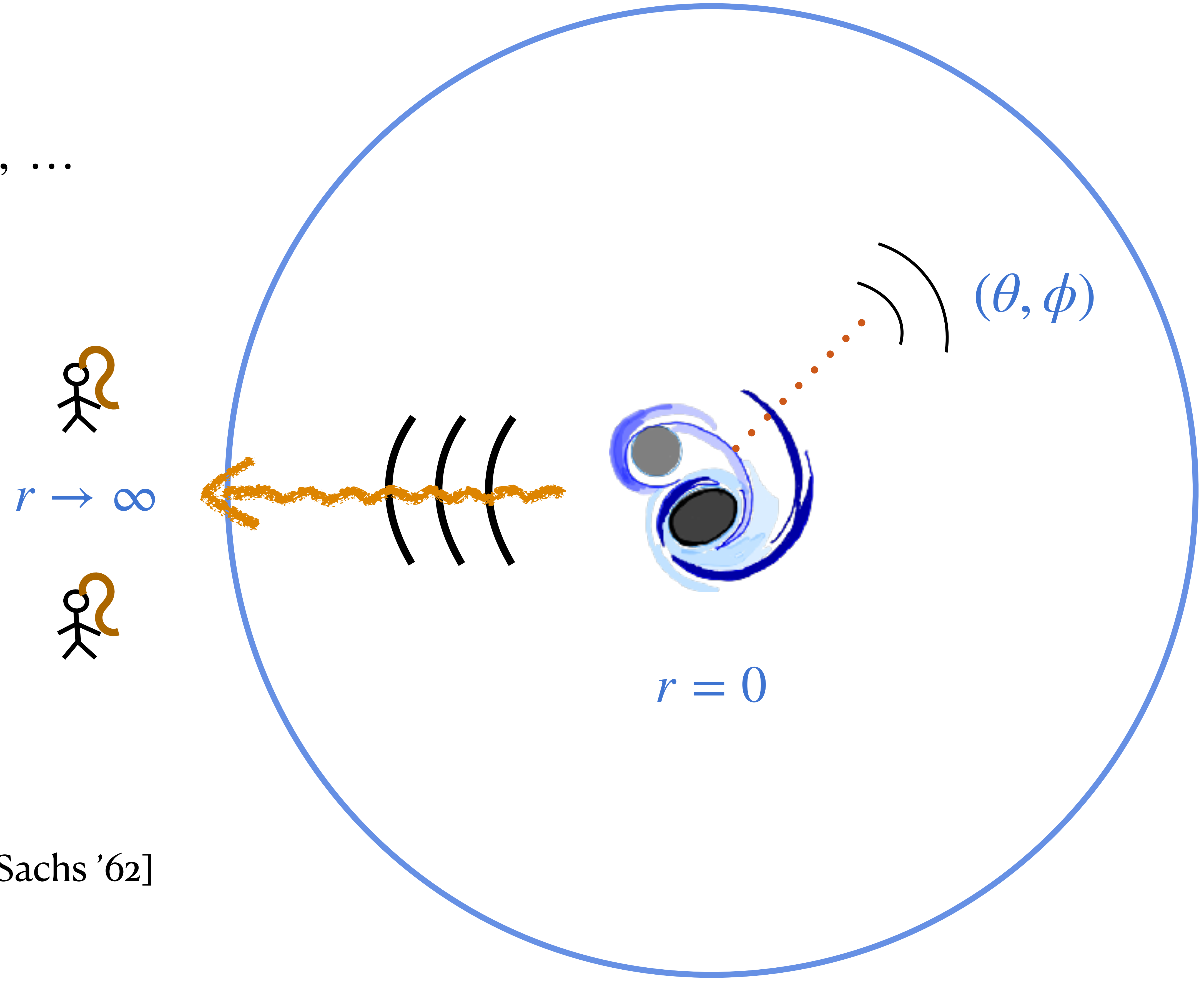
Perturbations around flat (Minkowski) background:

- spherical coordinate system centered at source
- Bondi gauge:

$$g_{rr} = g_{r\theta} = g_{r\phi} = 0 \text{ (radial propagation)}$$

$$\partial_r \det\left(r^{-2} g_{AB}\right) = 0 \quad \text{(spherical wavefronts)}$$

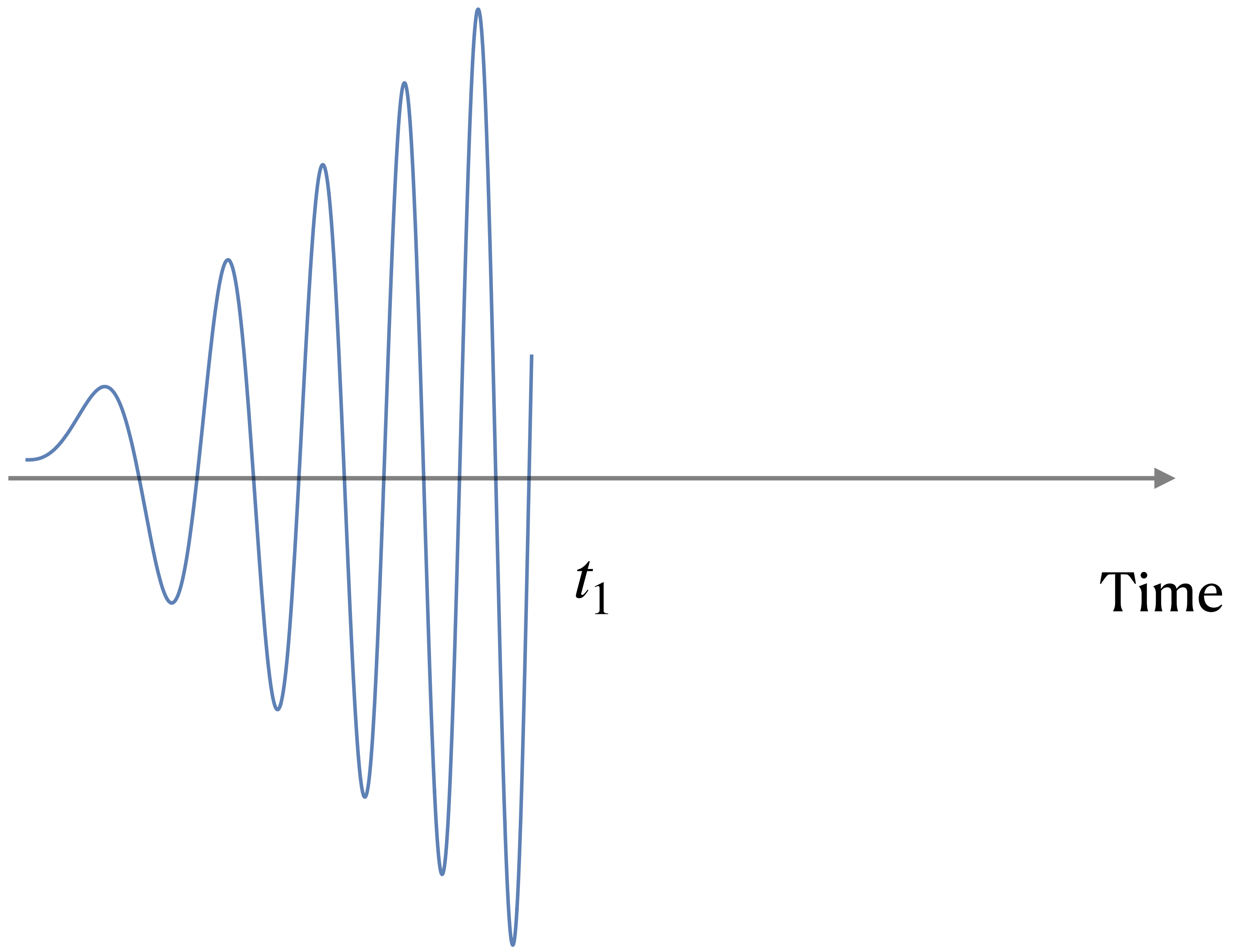
[Bond, van der Burg, Metzner, Sachs '62]

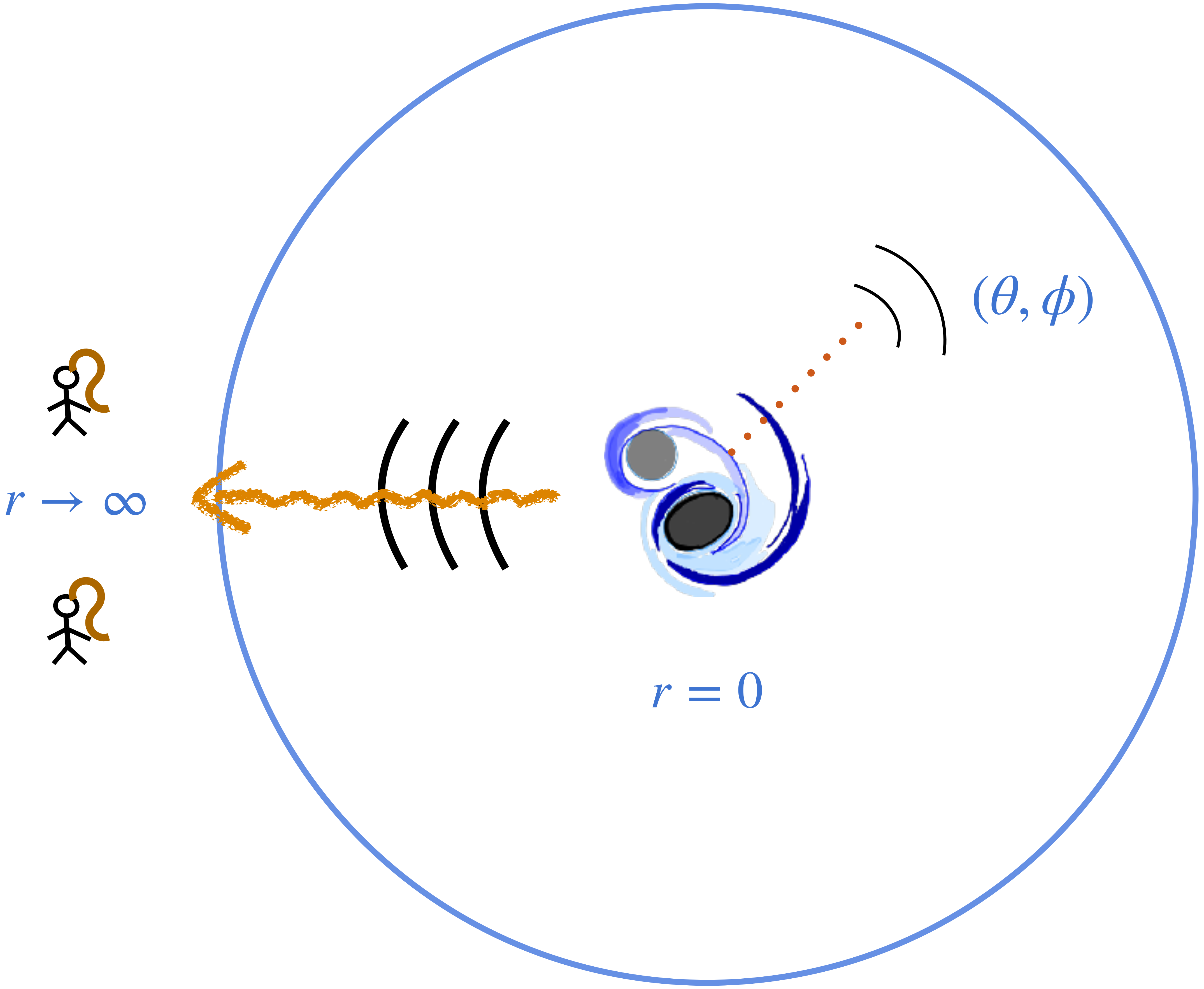


General relativity in Bondi gauge

- Metric near null infinity ($\mathscr{I}^+$):

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$
$$+rC_{zz}(u,z,\bar{z})dz^2 + rC_{\bar{z}\bar{z}}(u,z,\bar{z})d\bar{z}^2 + \frac{m_B}{r}du^2 + \cdots$$





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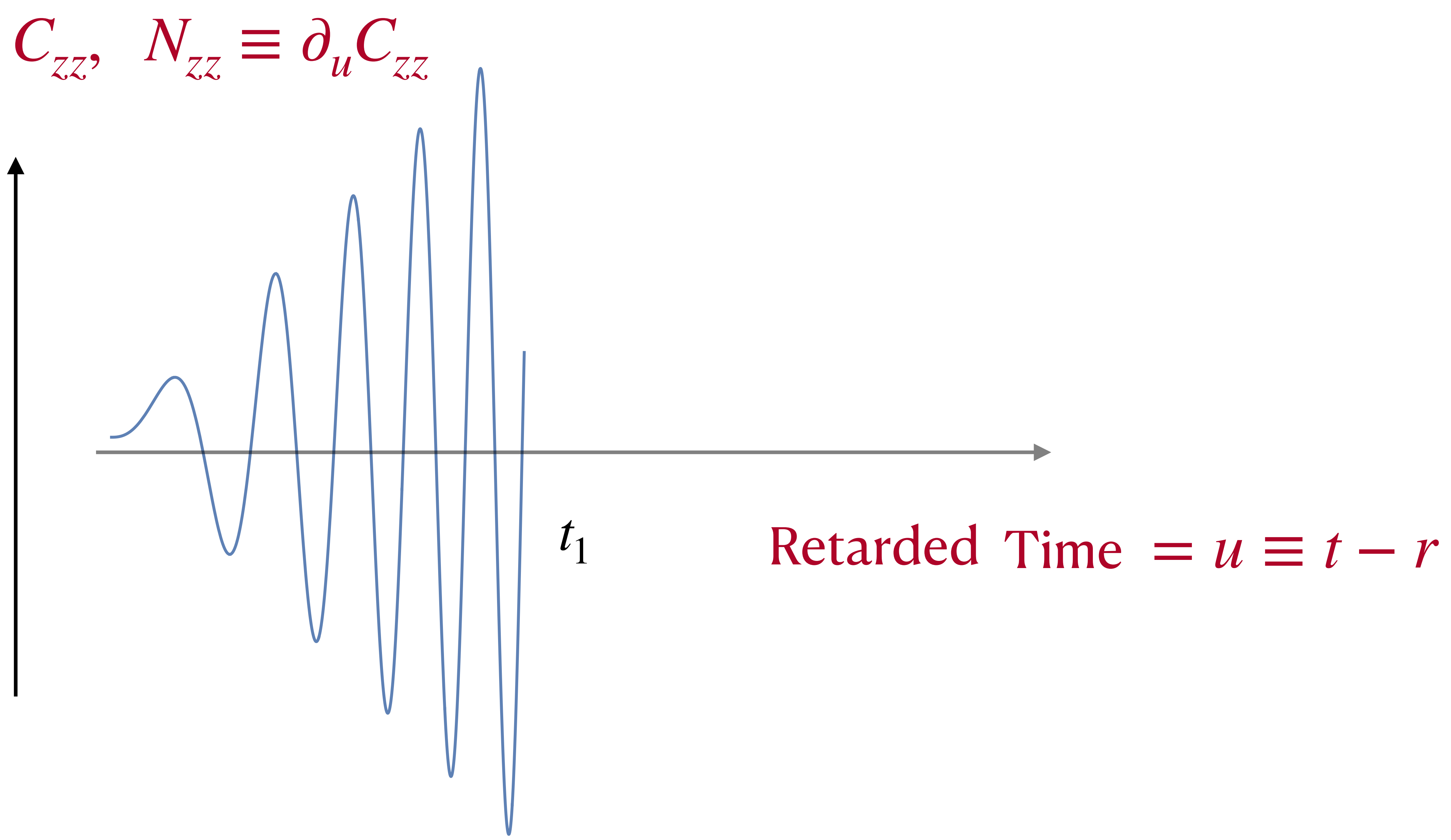
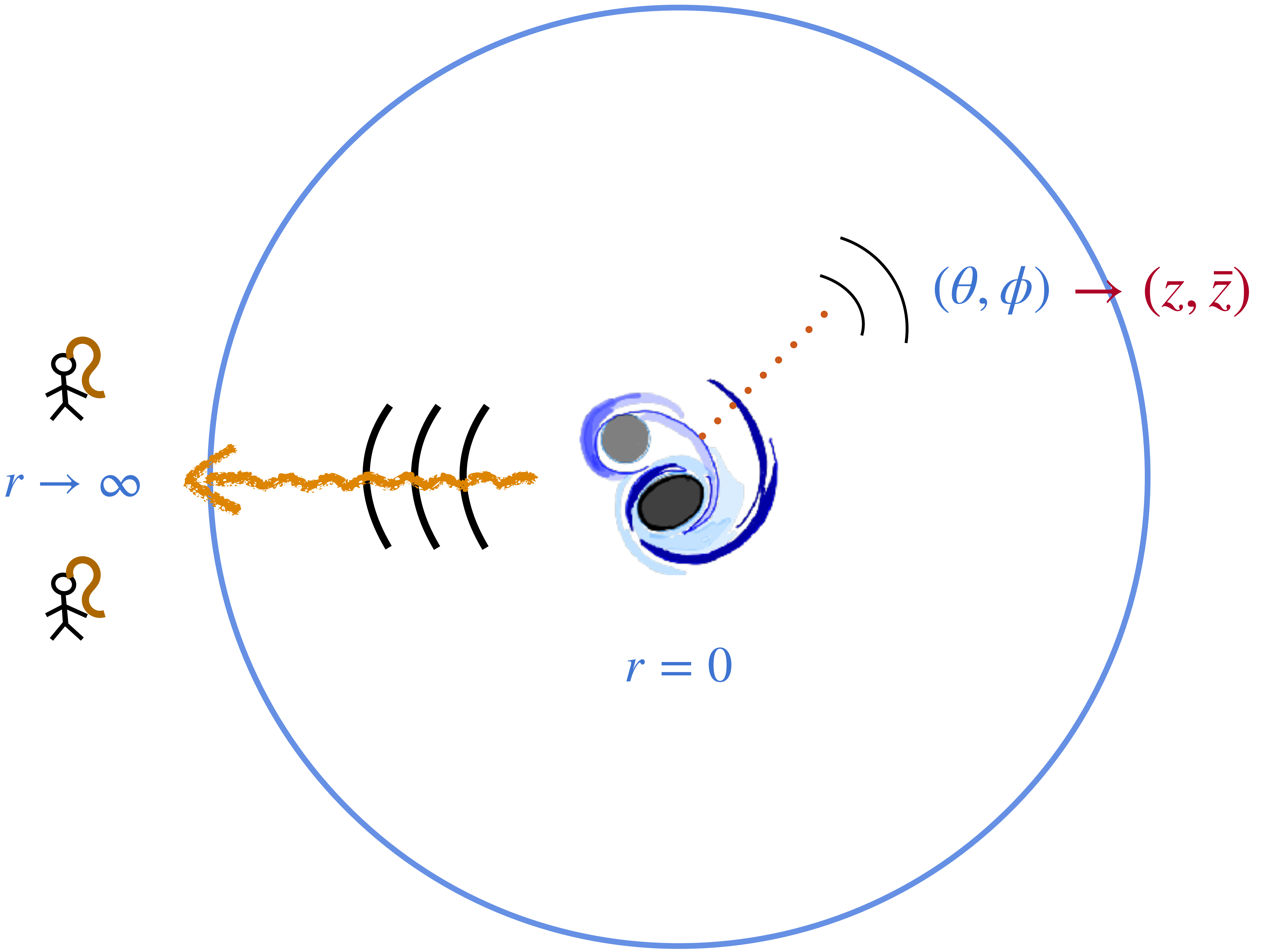


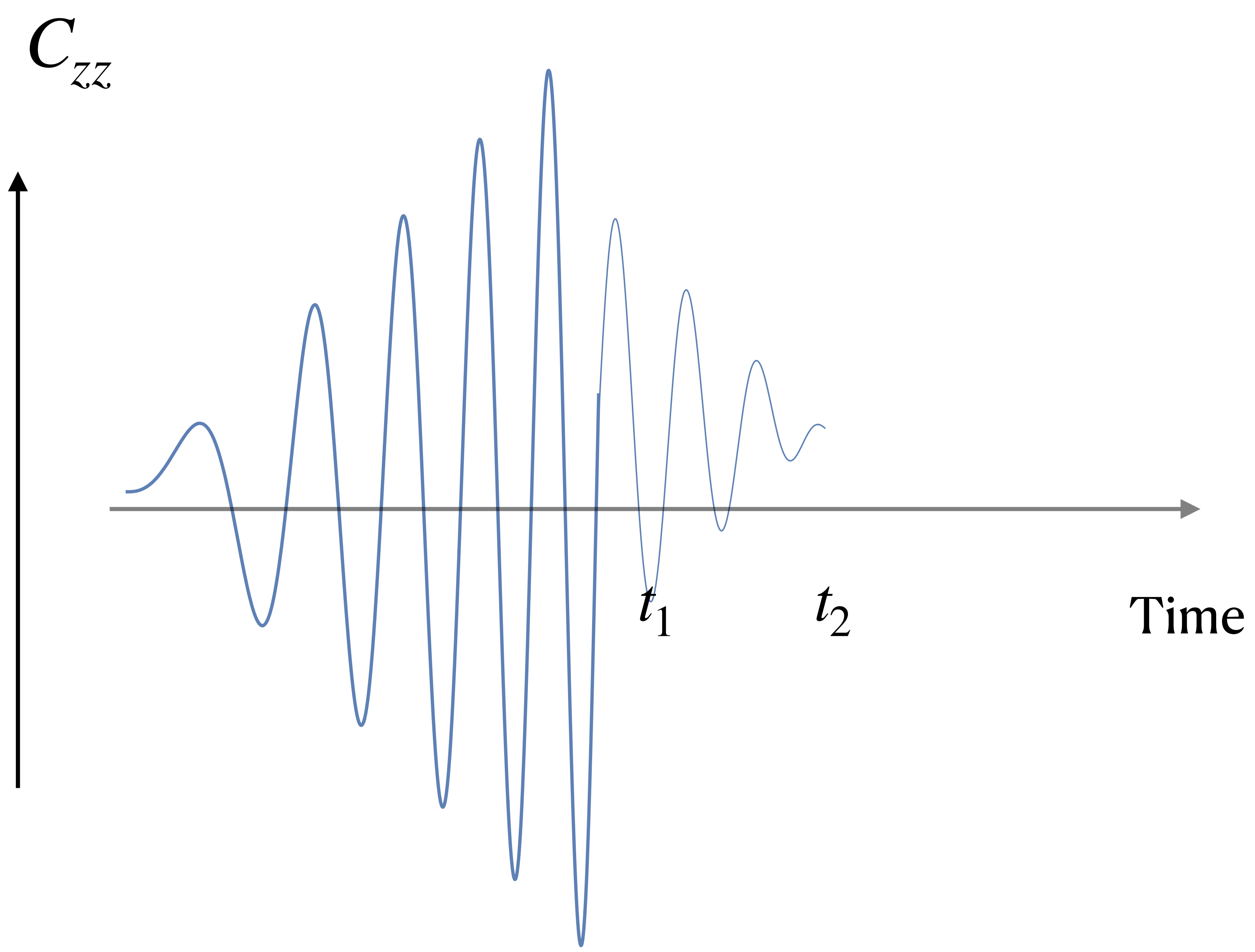


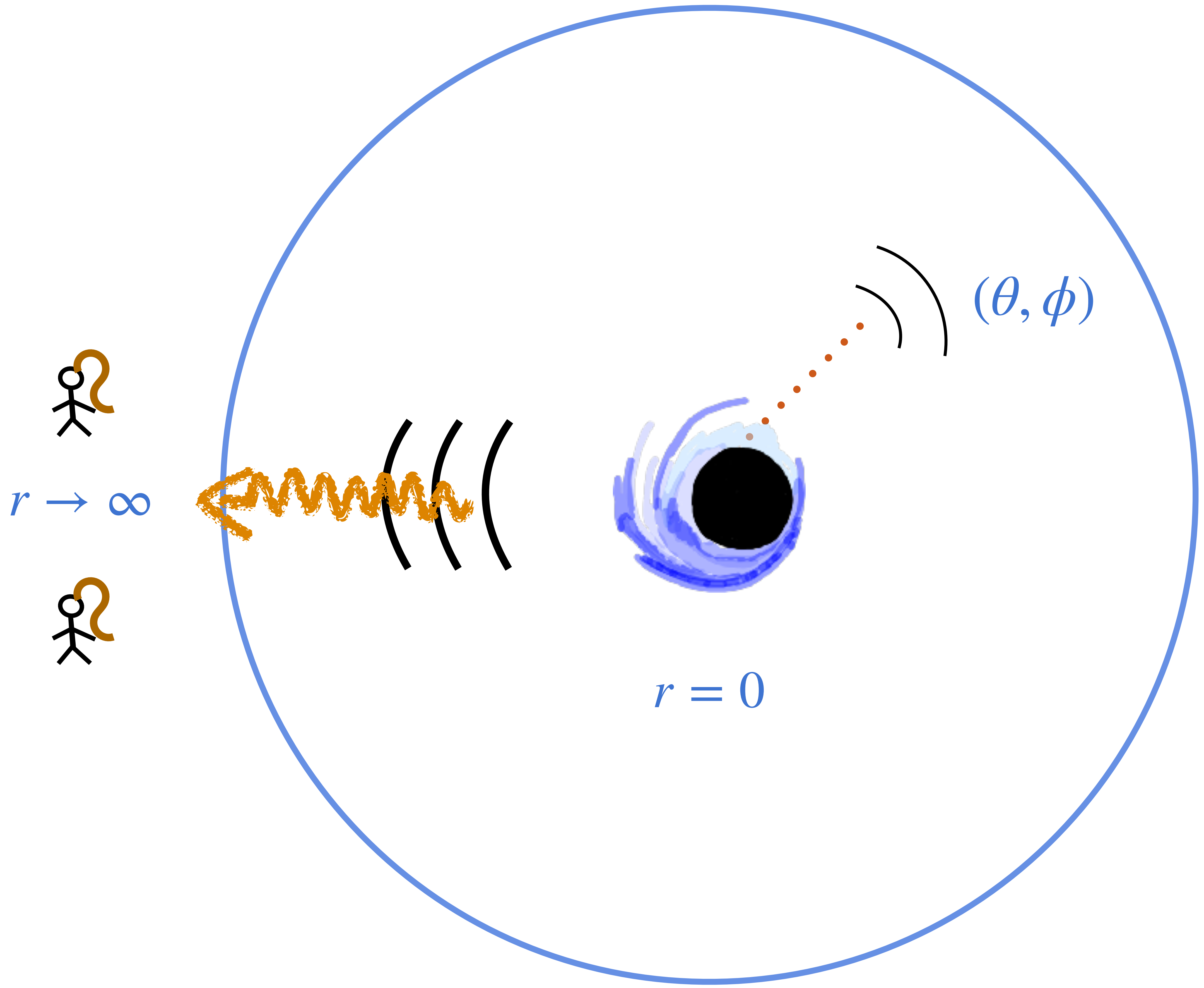
General relativity in Bondi gauge

- Solve Einstein equations perturbatively at large $r \Longrightarrow$

eg. Bondi mass loss formula (from G_{uu} @ $\mathcal{O}(r^{-2})$)

$$\partial_u m_B = \frac{1}{4}\left(D_z^2 N^{zz} + D_{\bar{z}}^2 N^{\bar{z}\bar{z}}\right) - T_{uu}$$

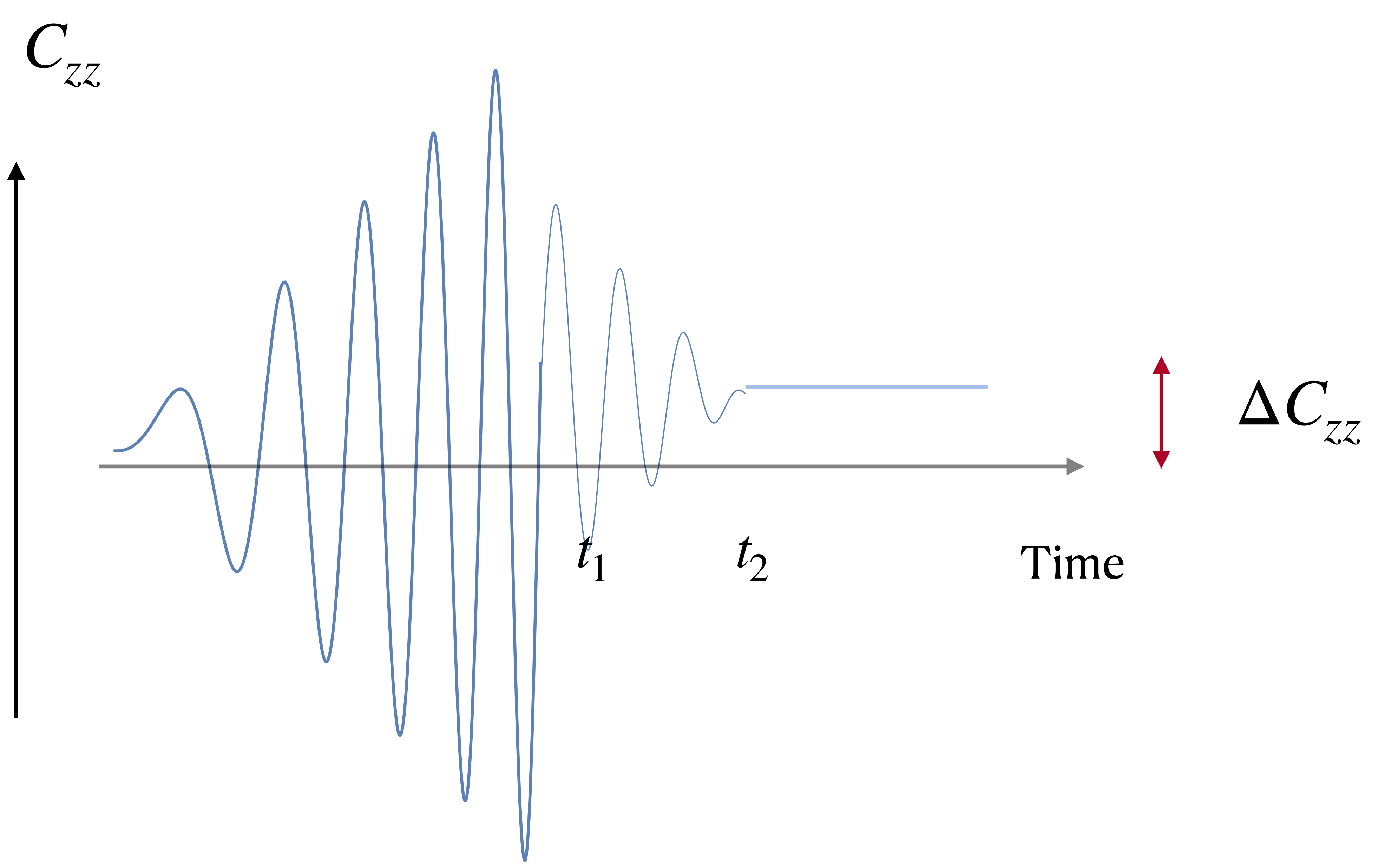


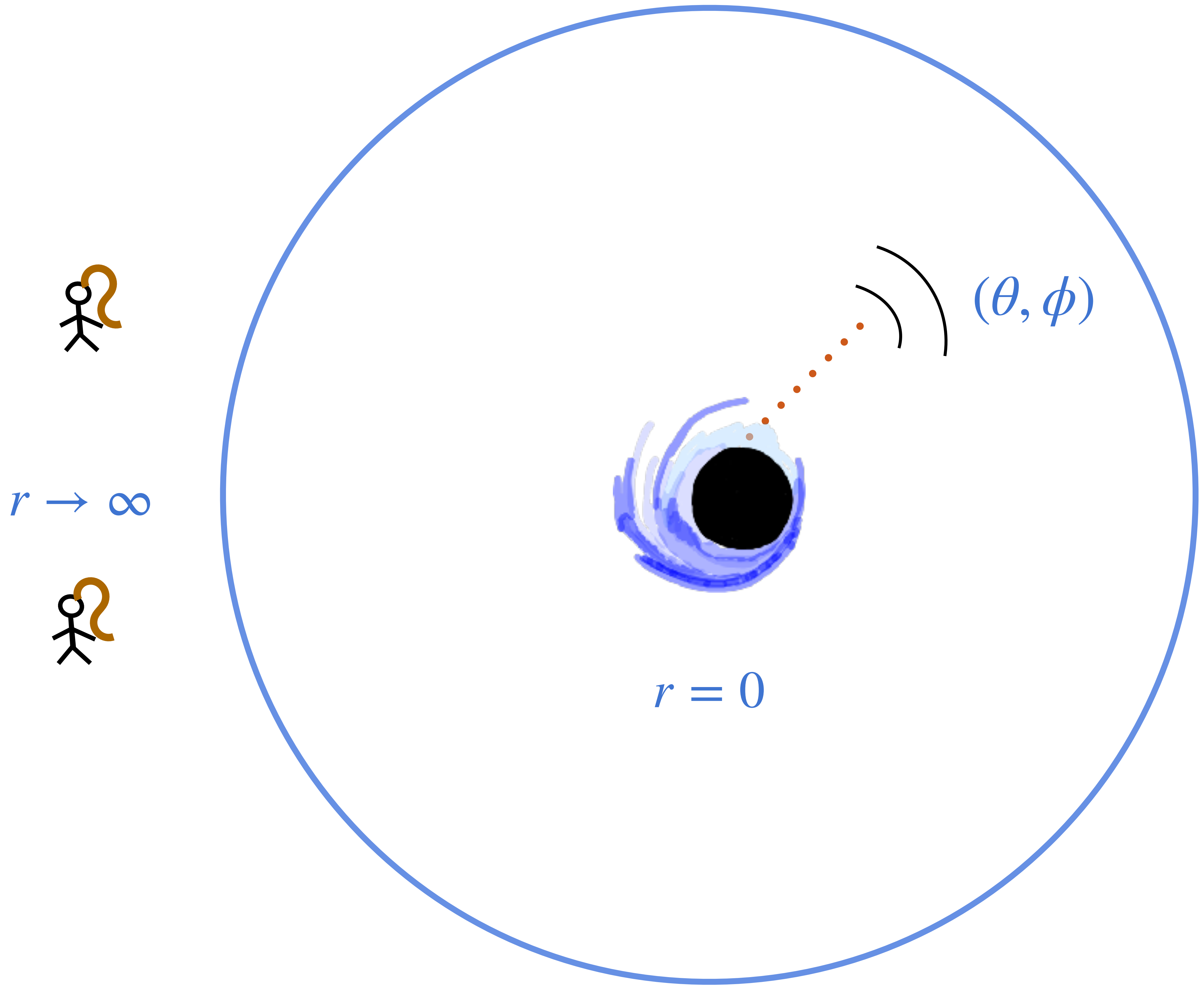


Memory effects

Gravitational memory:

$$\frac{1}{4}\left(D_z^2 \Delta C^{zz} + D_{\bar{z}}^2 \Delta C^{\bar{z}\bar{z}}\right) = \Delta m_B - \int du T_{uu}$$





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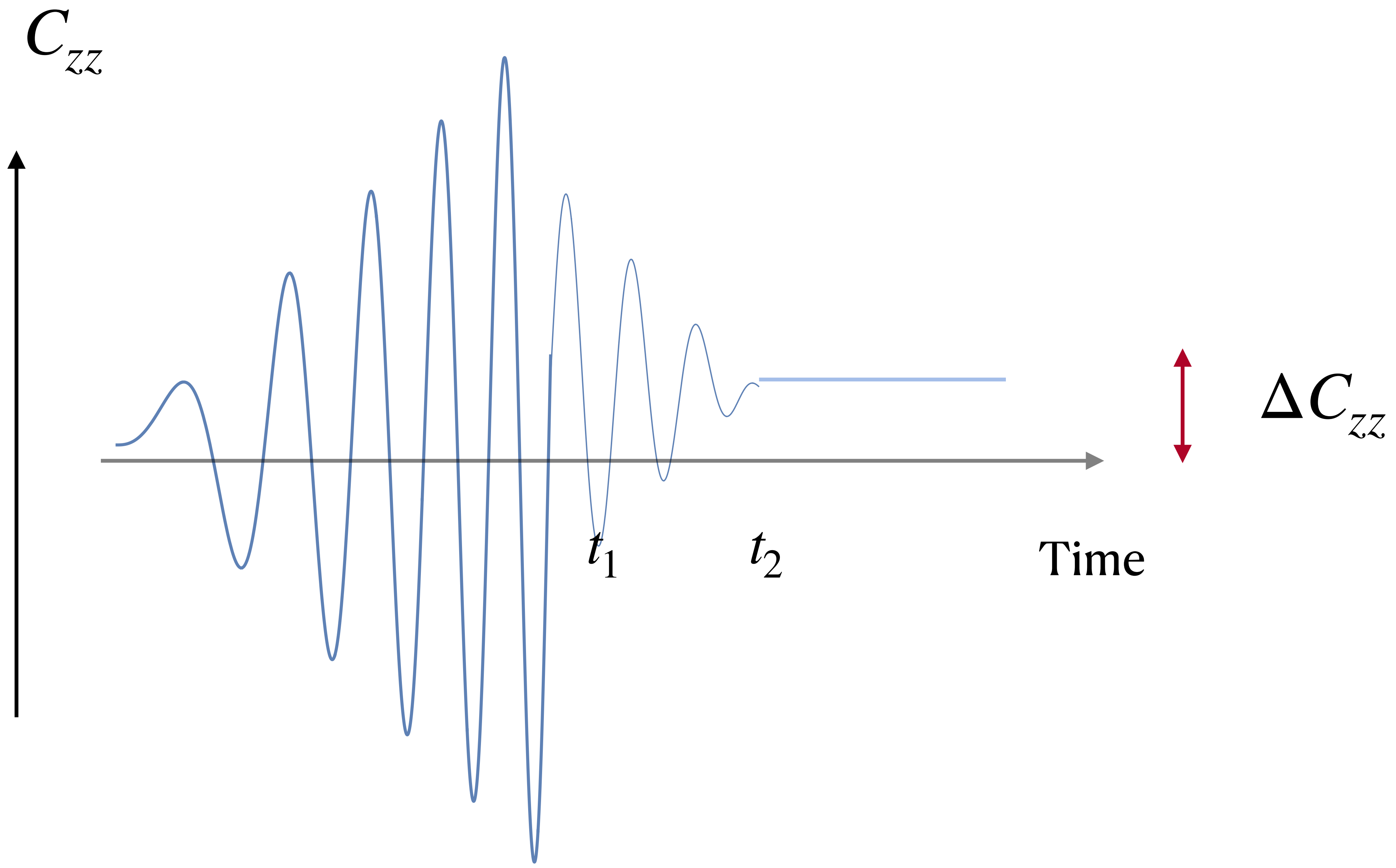
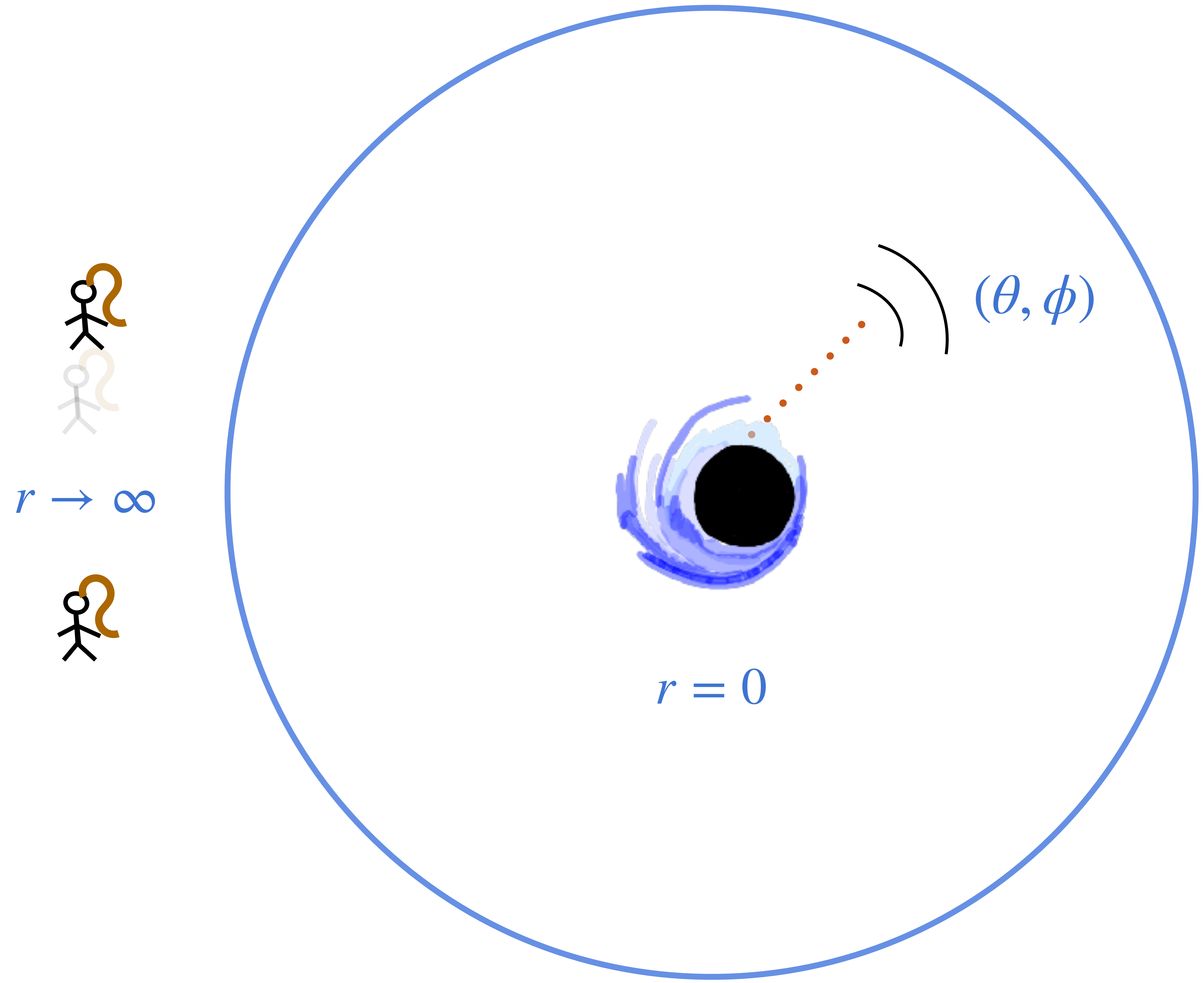
$\Longrightarrow$ Net relative displacement between observers

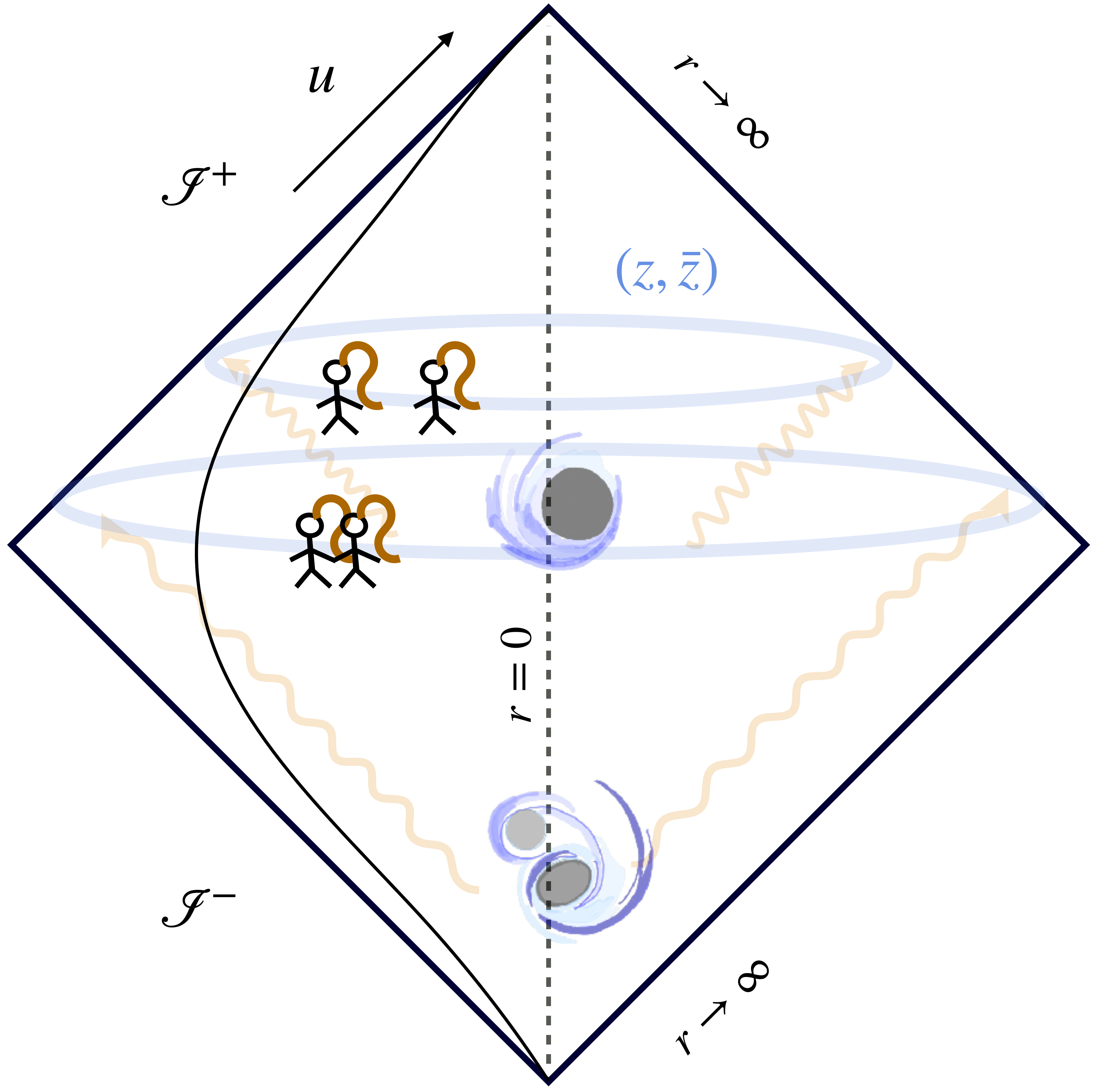




Conformal compactification

Compact representation of spacetime preserving causal structure

Boundary consists of: - future and past null infinities $(\mathscr{I}^+, \mathscr{I}^-)$



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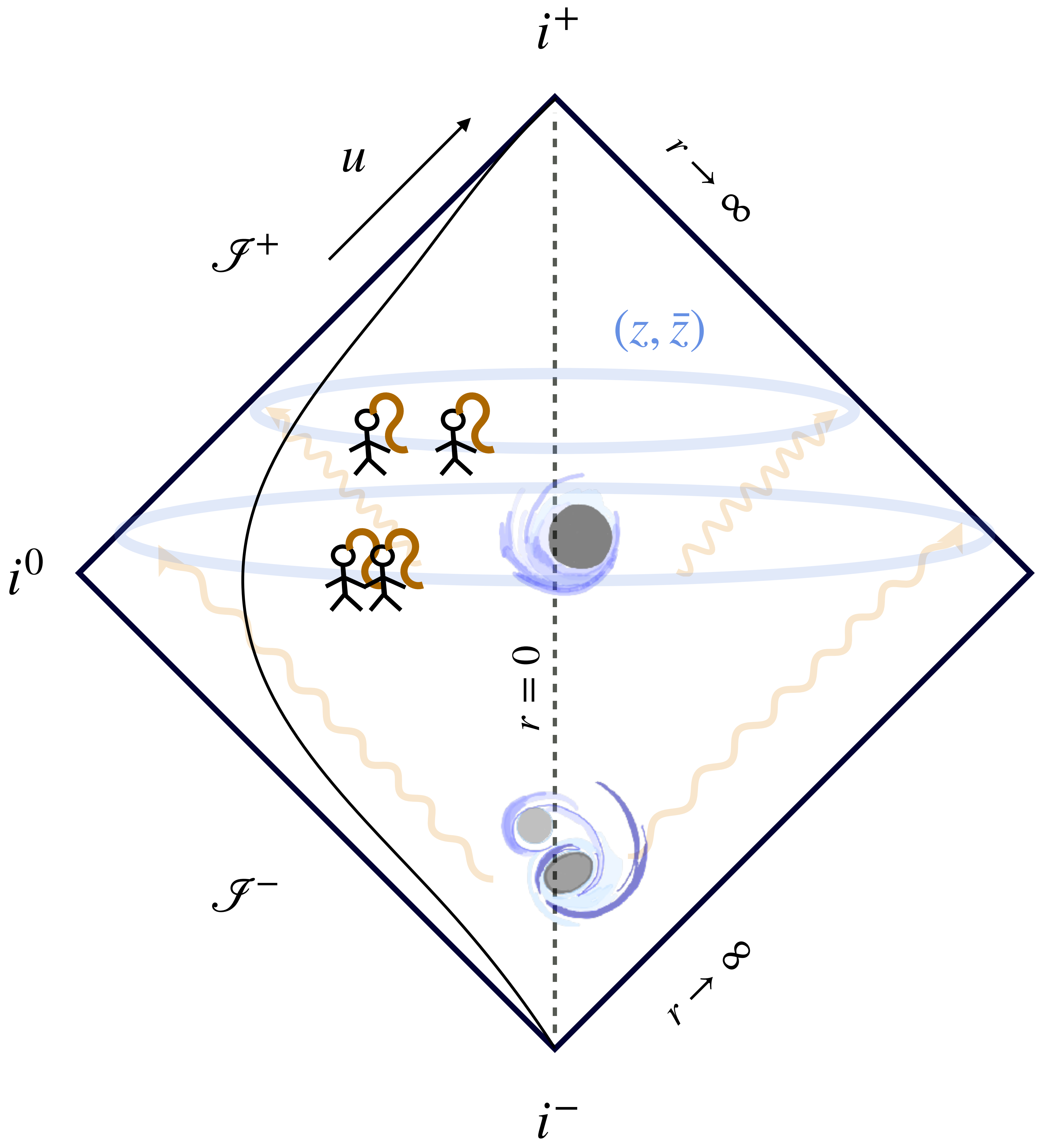

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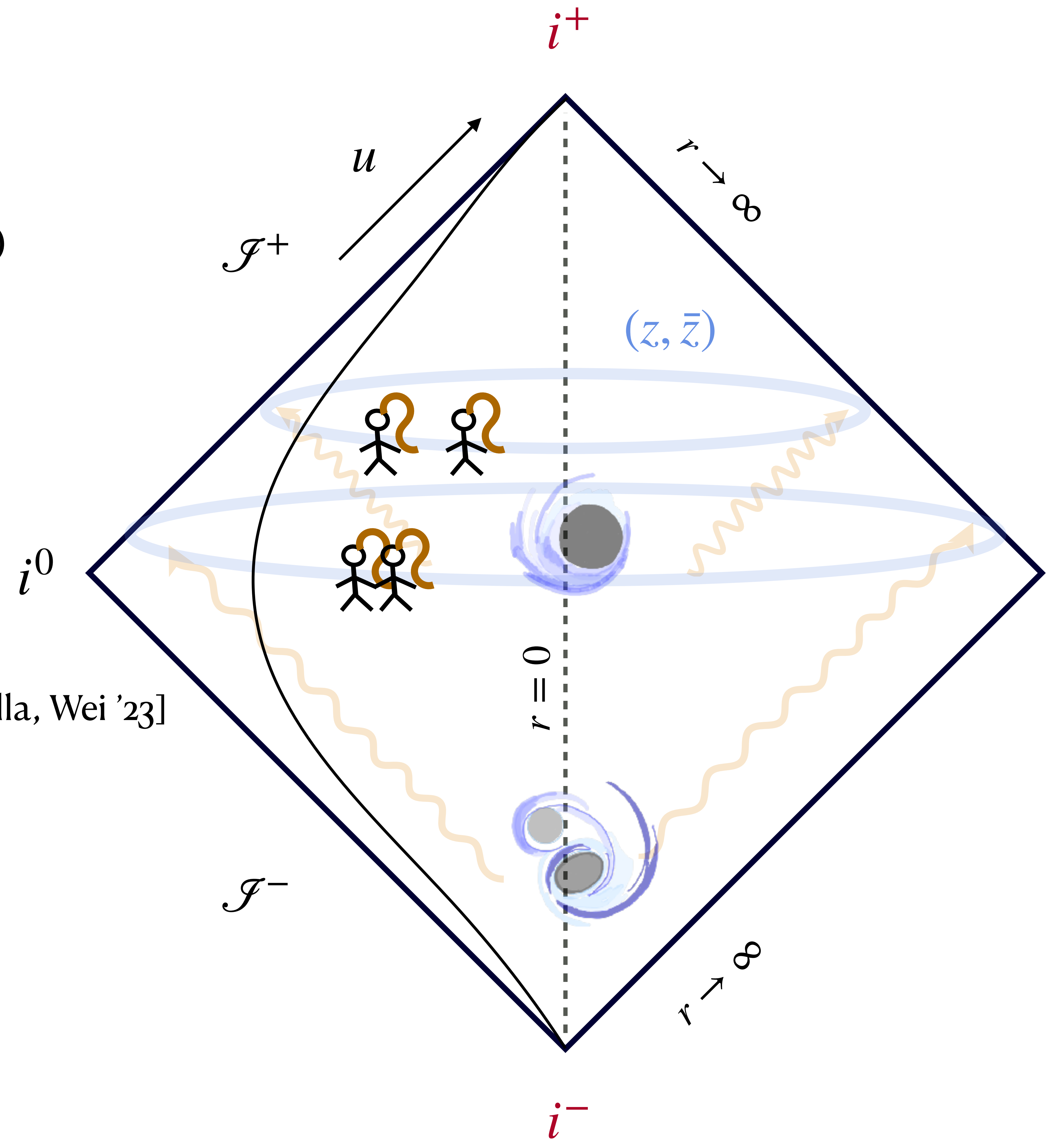
Can be blown up to 3d surfaces

- Timelike infinity important for massive particles [Compere, Gralla, Wei '23]
- Spacelike infinity hosts asymptotic charges



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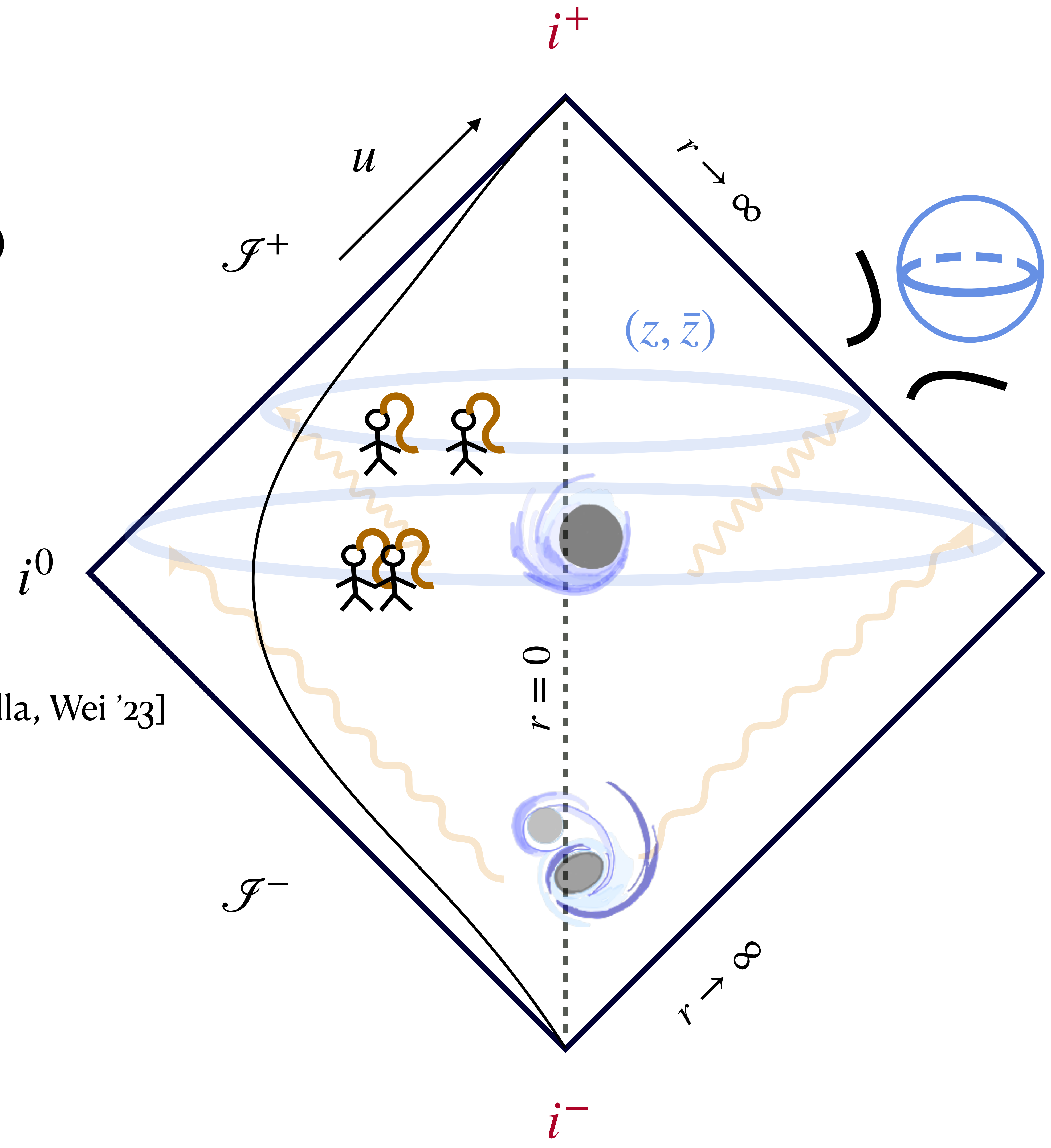
→ at any cut from Einstein equations (constraints)



Asymptotic symmetries

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Diffeomorphism invariance left after imposing Bondi gauge:

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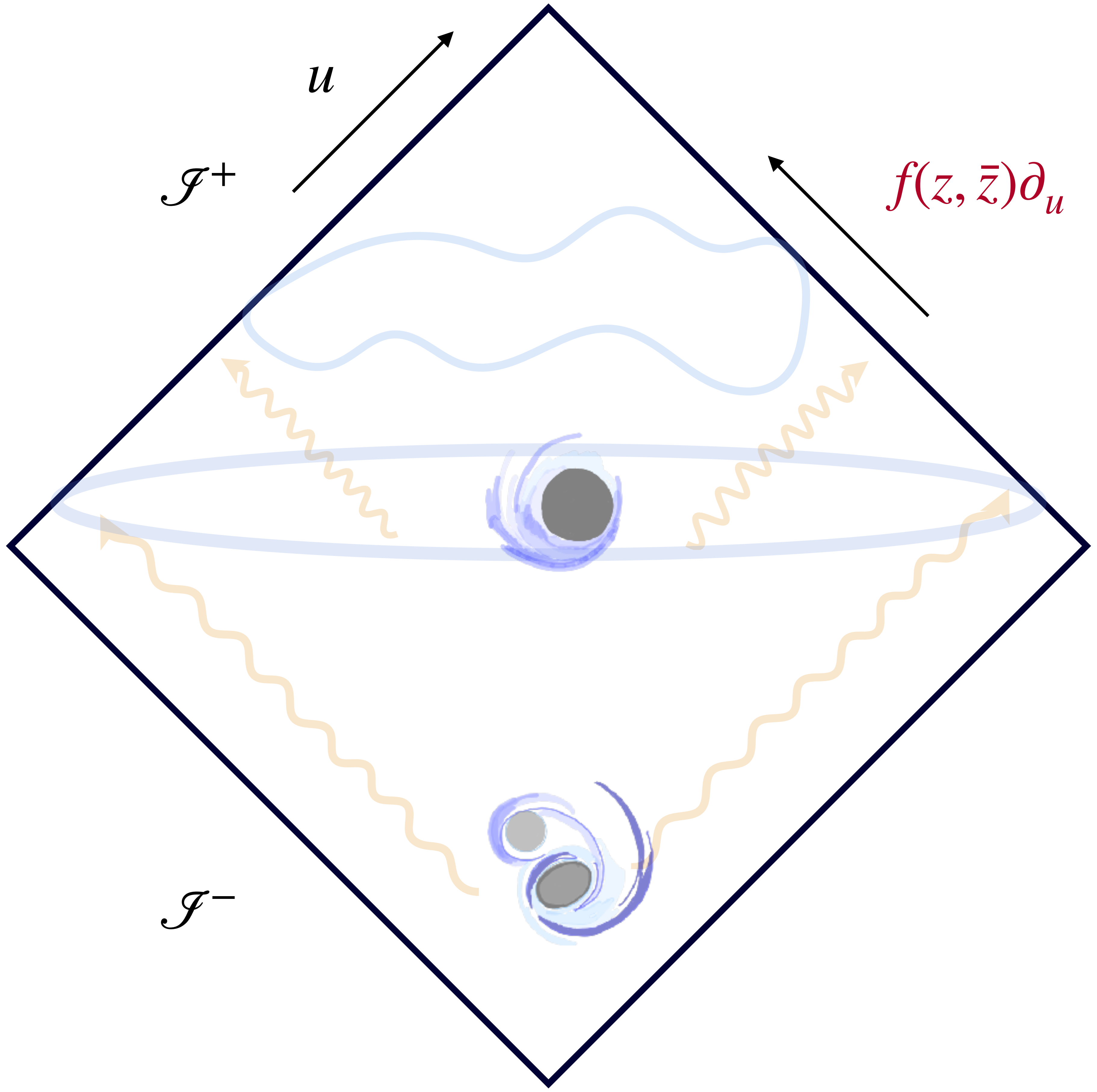
Look for vector fields ξ that preserve the metric at large r

$$\mathcal{L}_\xi g_{\mu\nu} \equiv \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

Asymptotic symmetries

Poincare symmetries of Minkowski spacetime enhanced:

- Translations $\longrightarrow$ Supertranslations $\xi_F \equiv f(z,\bar{z})\partial_u + \cdots$



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- Boosts

$\Big\} \longrightarrow$ Superrotations $\xi_Y \equiv Y^A(z, \bar{z})\partial_A + \cdots$

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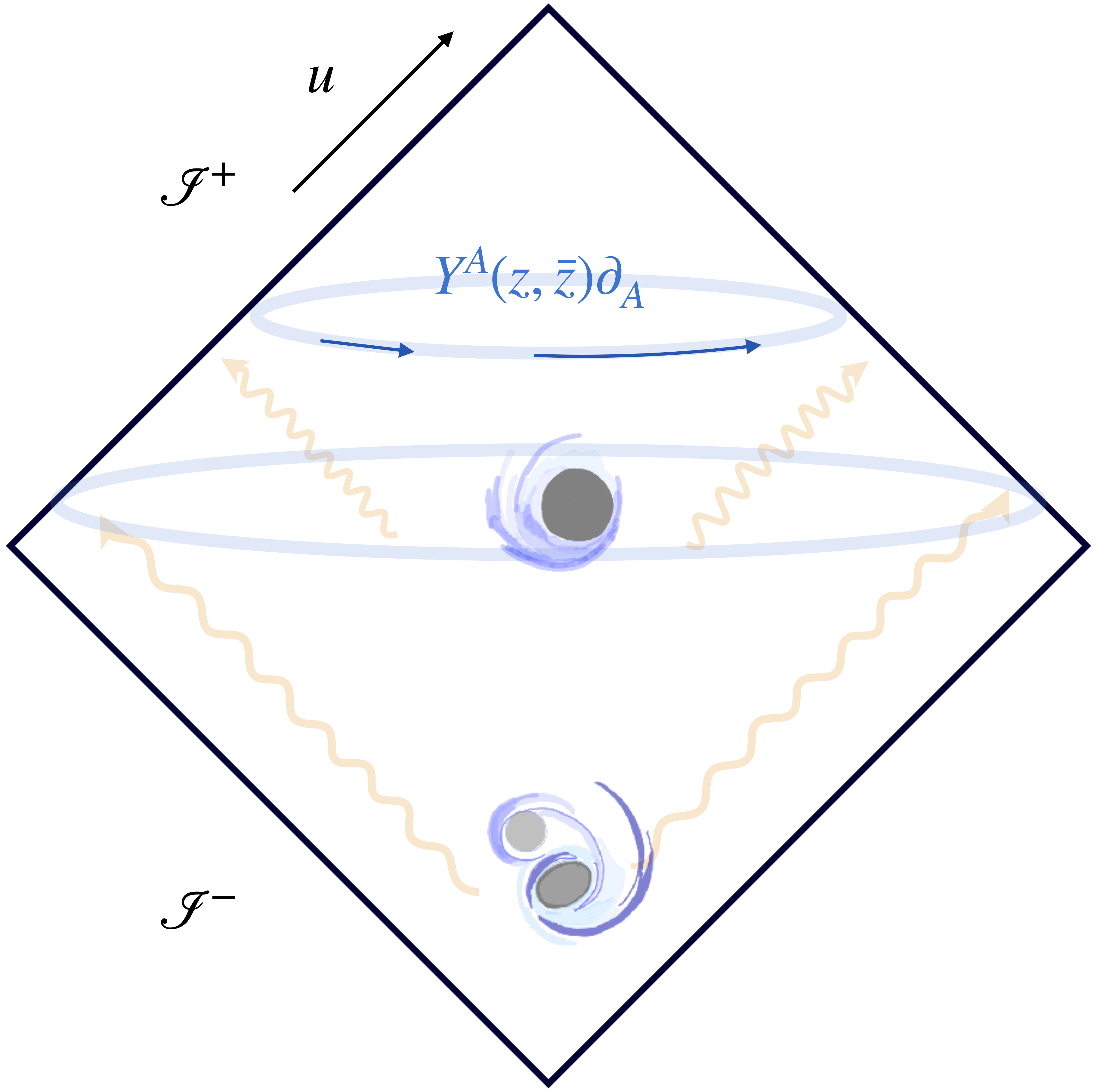
[Bondi, van der Burg, Metzner, Sachs ’62, Barnich, Troessaert ’09]



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[bulk signature of 2D Virasoro symmetry]

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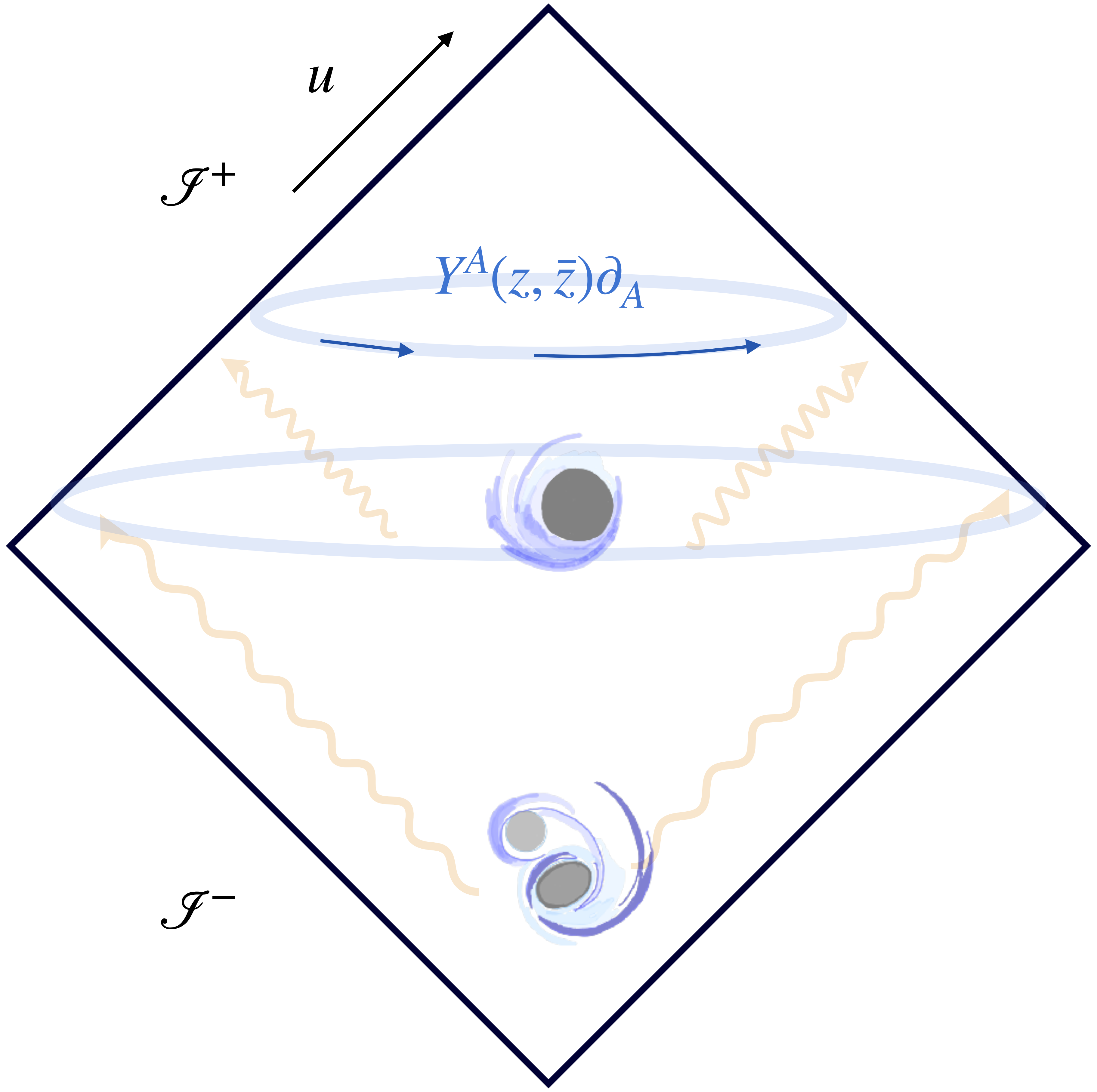
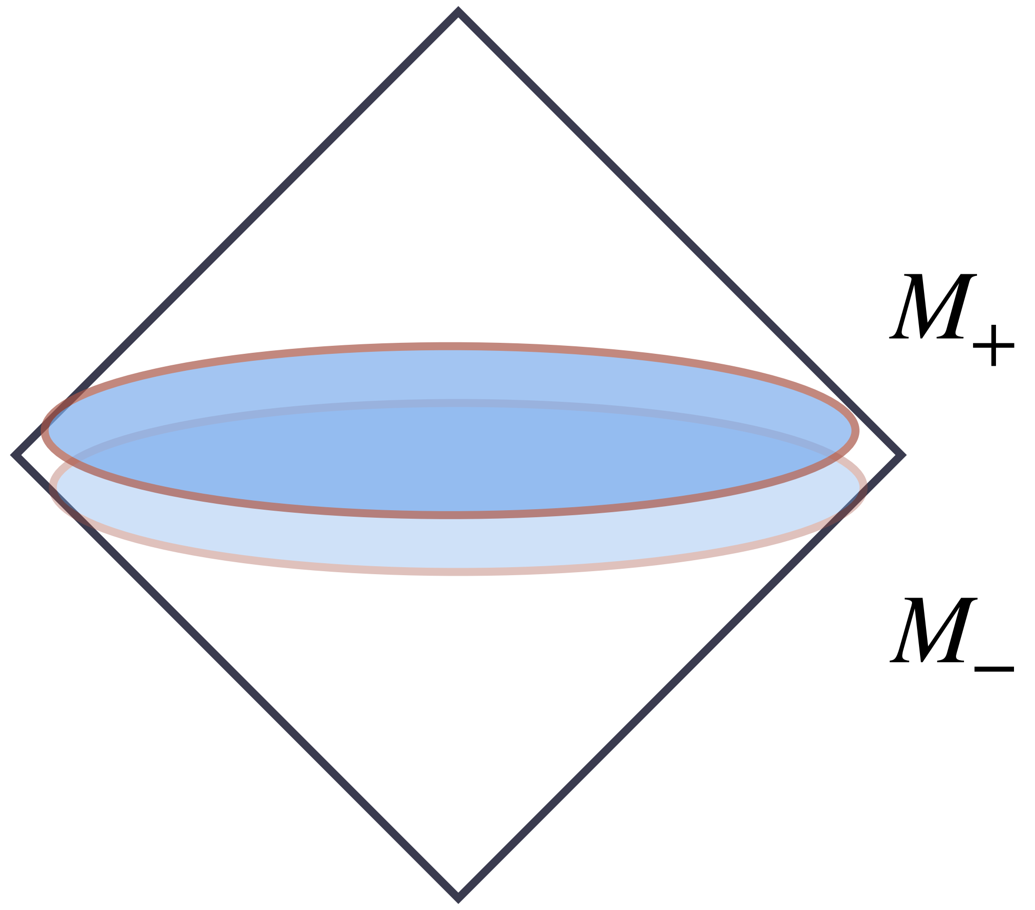


Matching condition

Einstein equations at infinity can be recast as evolution equations for (generalized) BMS covariant quantities

$$G_{uu}=0: \quad \partial_u m_B = \frac{1}{4}\left(D_z^2 N^{zz} + D_{\bar z}^2 N^{\bar z\bar z}\right) - T_{uu} \qquad \longrightarrow \qquad \partial_u M_{\mathbb{C}} = \frac{1}{2} D^2 N + \frac{1}{4} C \partial_u N$$



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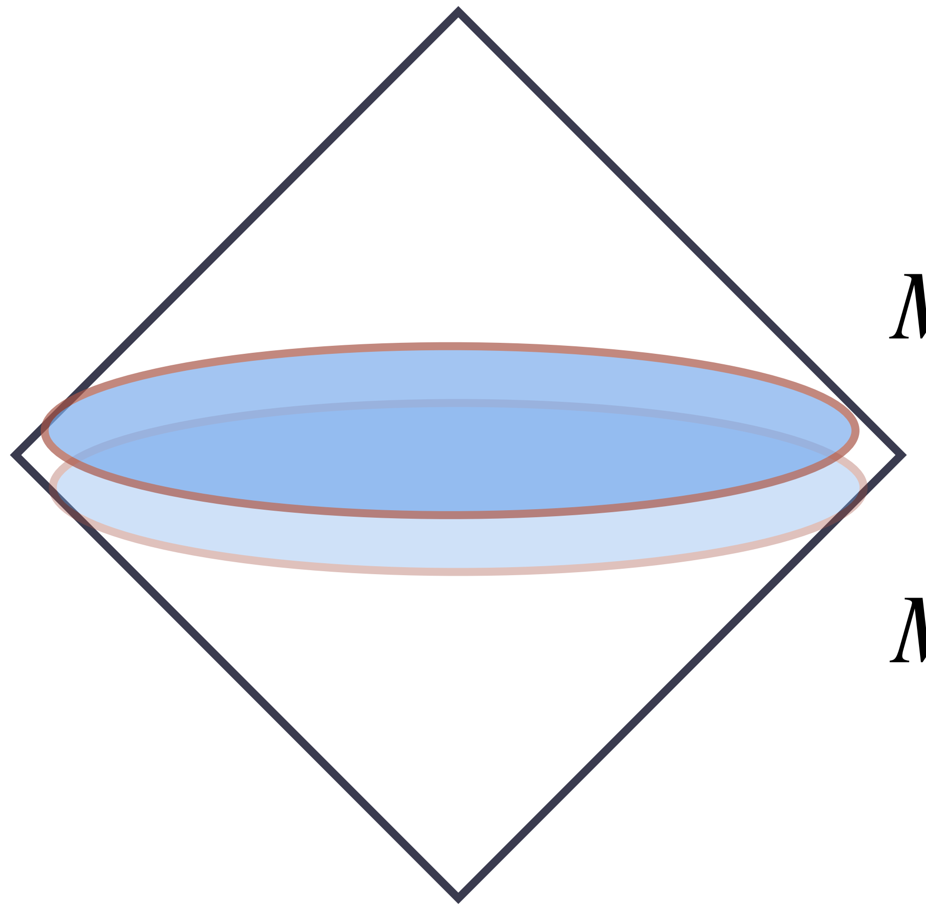

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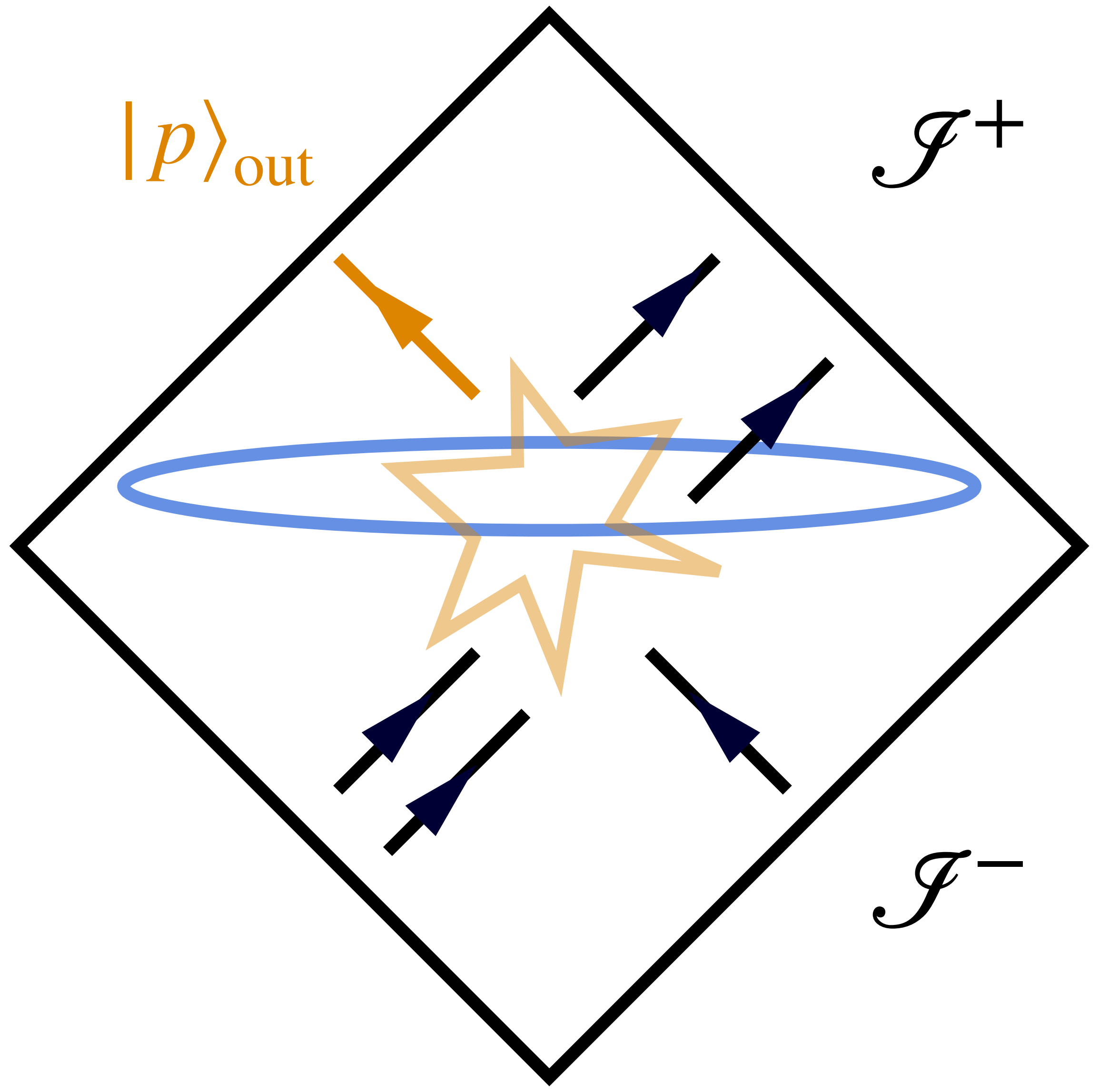
- Imposing matching on supertranslations at $\mathcal{I}^{\pm}$ allows for pairing:

$$\int_{\mathcal{I}^{\pm}_{-}} d^2 z f(z,\bar{z}) M_+(z,\bar{z}) = \int_{\mathcal{I}^{-}_{+}} d^2 z f(z,\bar{z}) M_-(z,\bar{z}) \equiv Q_f$$

[Strominger '13]

Implications for scattering



- Particles near infinity weakly interacting ~ free
- Incoming and outgoing Fock spaces: $a^\dagger_{\text{in/out}}(p_1)\cdots a^\dagger_{\text{in/out}}(p_n)|0\rangle$

$$a_{\text{out}} = S^\dagger a_{\text{in}} S \implies {}_{\text{out}}\langle p_{n+m}\cdots p_{m+1}|p_1\cdots p_m\rangle_{\text{in}} = \langle p_{n+m}\cdots p_{m+1}|S|p_1\cdots p_m\rangle$$

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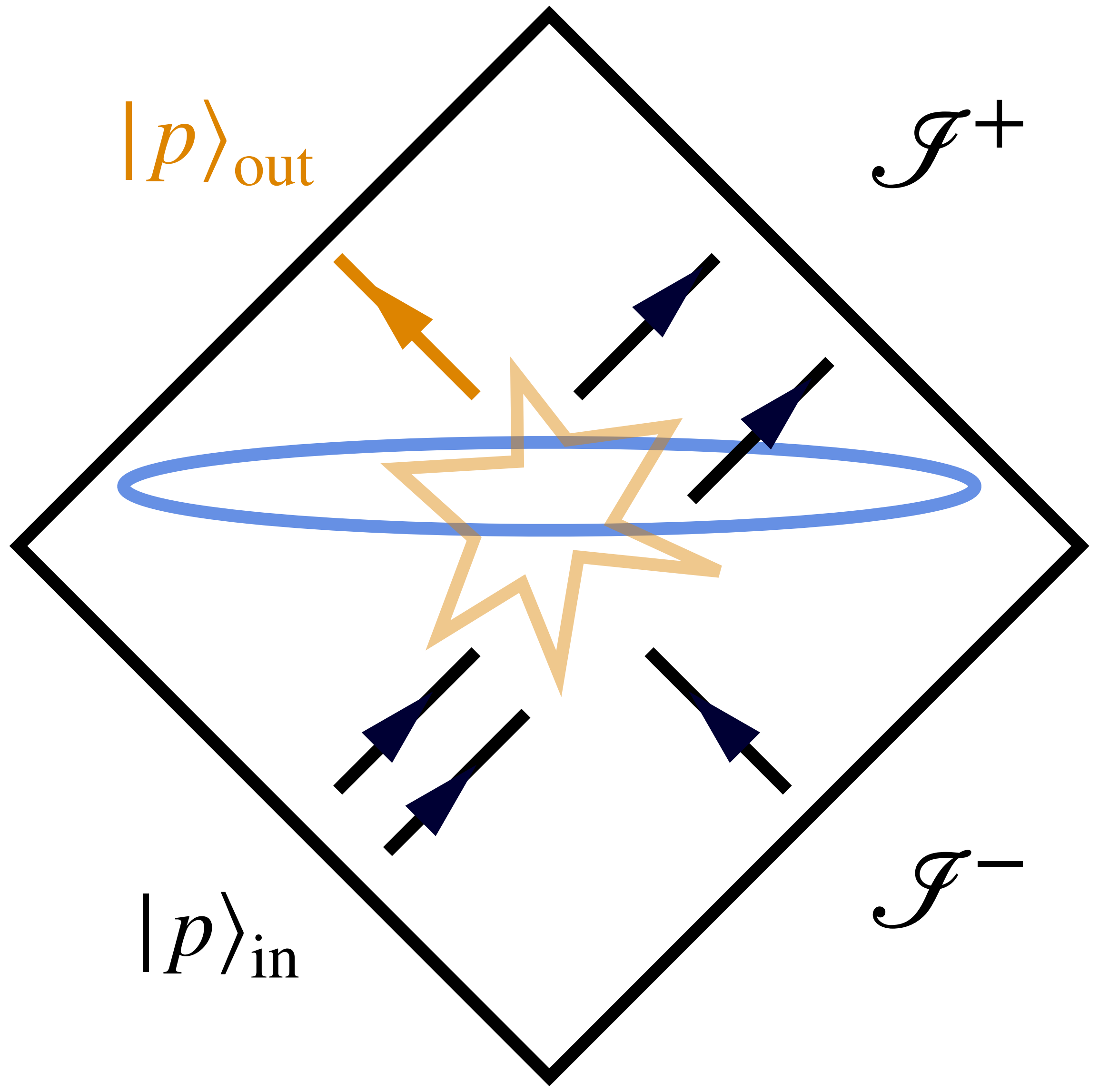

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S-matrix

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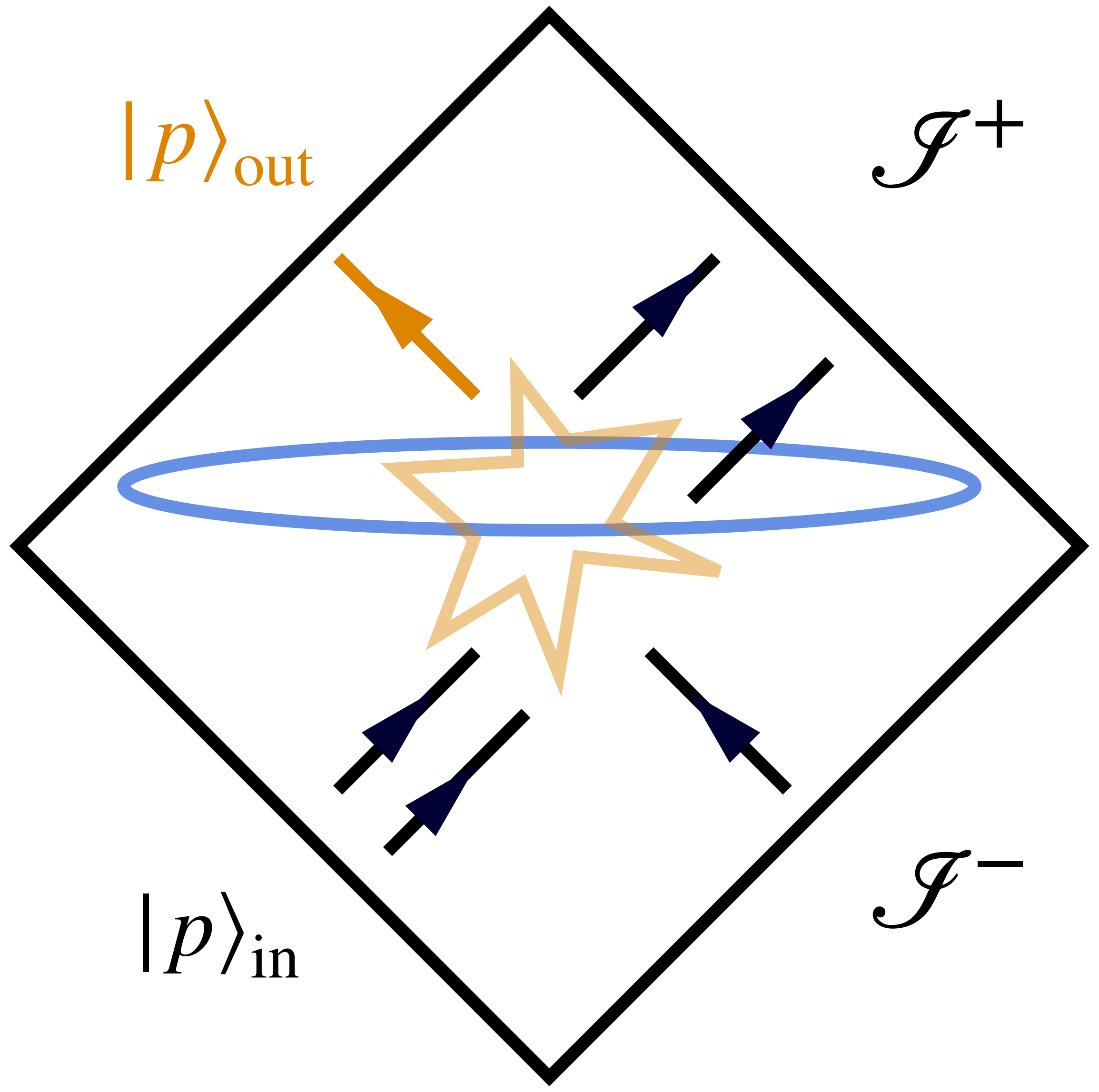

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S-matrix

- Scattering states are acted upon by the asymptotic charges:

$$Q_f \equiv \int_{\mathscr{I}^\pm_-} d^2z f(z,\bar{z}) M_{\mathbb{C}}(z,\bar{z}) = \int_{\mathscr{I}^+} du d^2z f(z,\bar{z}) \partial_u M_{\mathbb{C}}(z,\bar{z})$$

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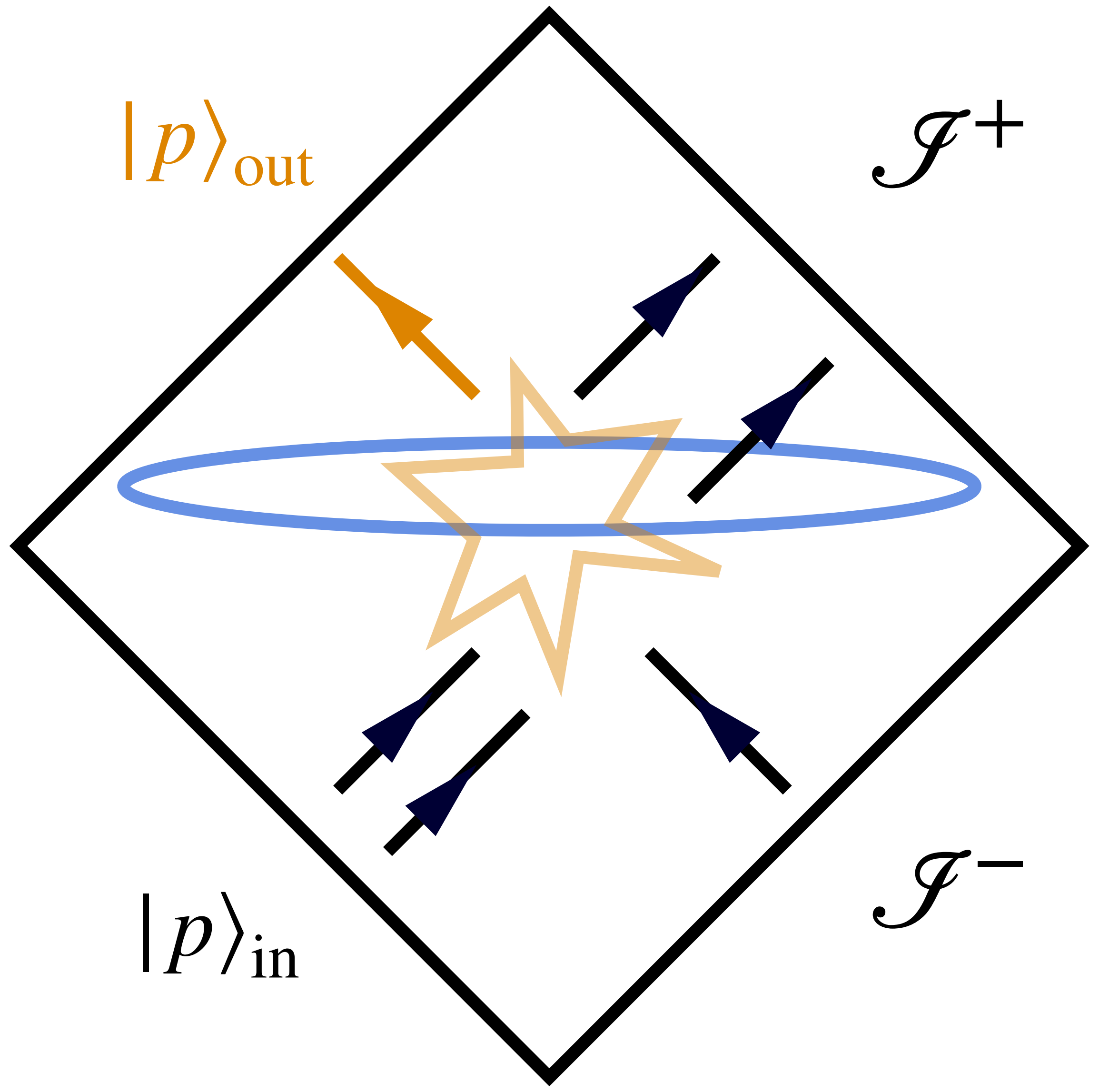

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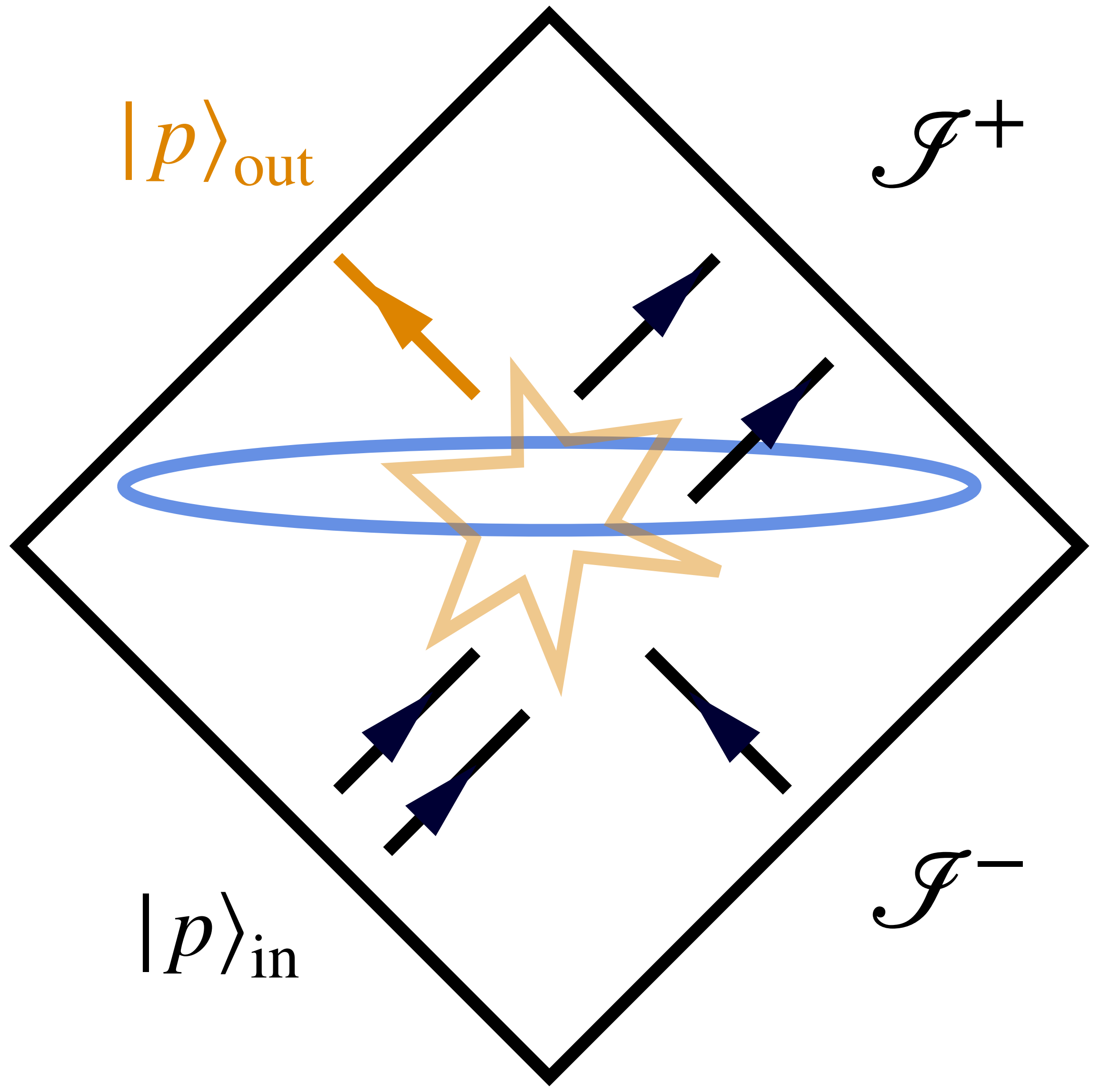

- Gravitons = quantized modes of the news:

$$N(u,z,\bar{z}) = -\frac{\kappa}{8\pi^2}\int_0^\infty d\omega\omega\left(a_+^\dagger e^{i\omega u} + a\_e^{-i\omega u}\right)$$

[He, Mitra, Strominger '14]

- Charge action on Fock space computed via canonical brackets [Ashtekar '82]

$$[N(u), C(u')] = -i\frac{\kappa^2}{2}\delta(u-u')\delta^2(z,z')$$

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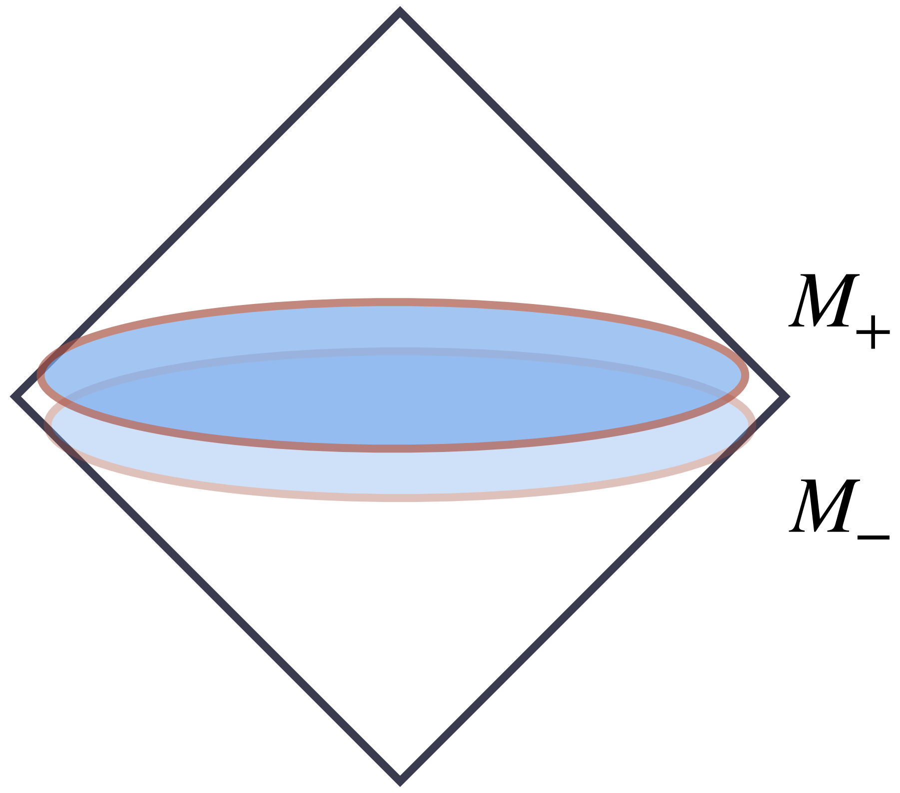
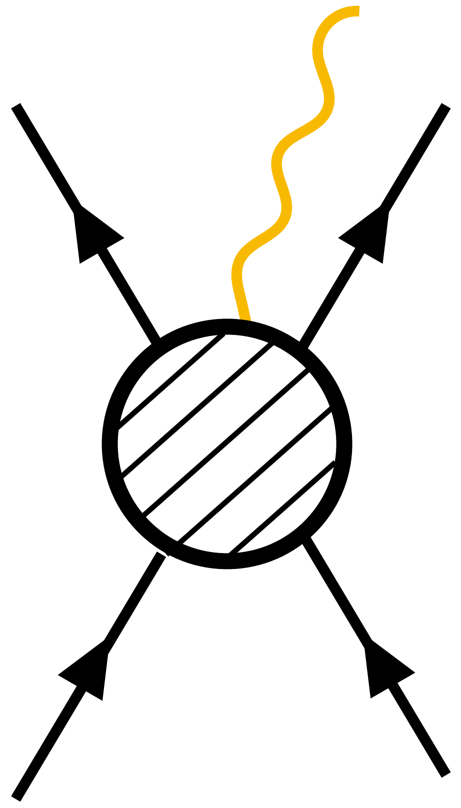
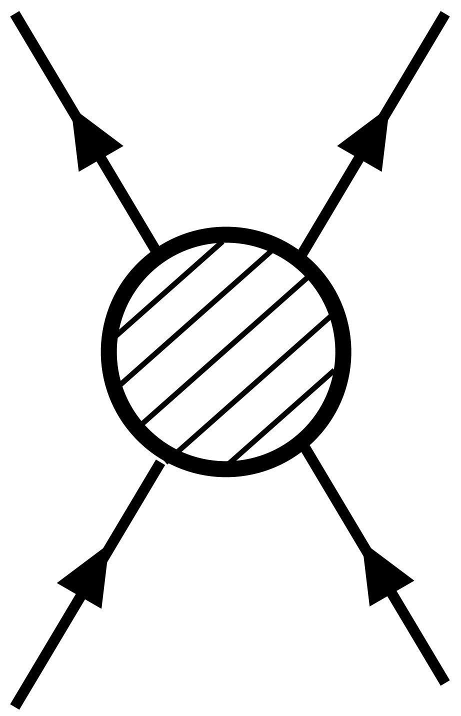
Implications for scattering

Matching condition $\Longrightarrow$ charge conservation: $\langle \text{out} | Q_f^+ \mathcal{S} - \mathcal{S} Q_f^- | \text{in} \rangle = 0$ $\quad\Big|\quad$ $Q_f \equiv \int_{\mathcal{I}^+} du d^2 z f(z,\bar{z}) \Big(\underbrace{\frac{1}{2} D^2 N}_{Q_{\text{soft}}} + \underbrace{\frac{1}{4} C \partial_u N}_{Q_{\text{hard}}} \Big)$

$$\Longrightarrow \quad \langle \text{out} | [Q_{\text{soft}}, \mathcal{S}] | \text{in} = - \langle \text{out} | [Q_{\text{hard}}, \mathcal{S}] | \text{in} \rangle$$



$\Longrightarrow \quad \lim_{q \to 0}$  $= \mathbf{S} \times$  $+\mathcal{O}(q^0)$

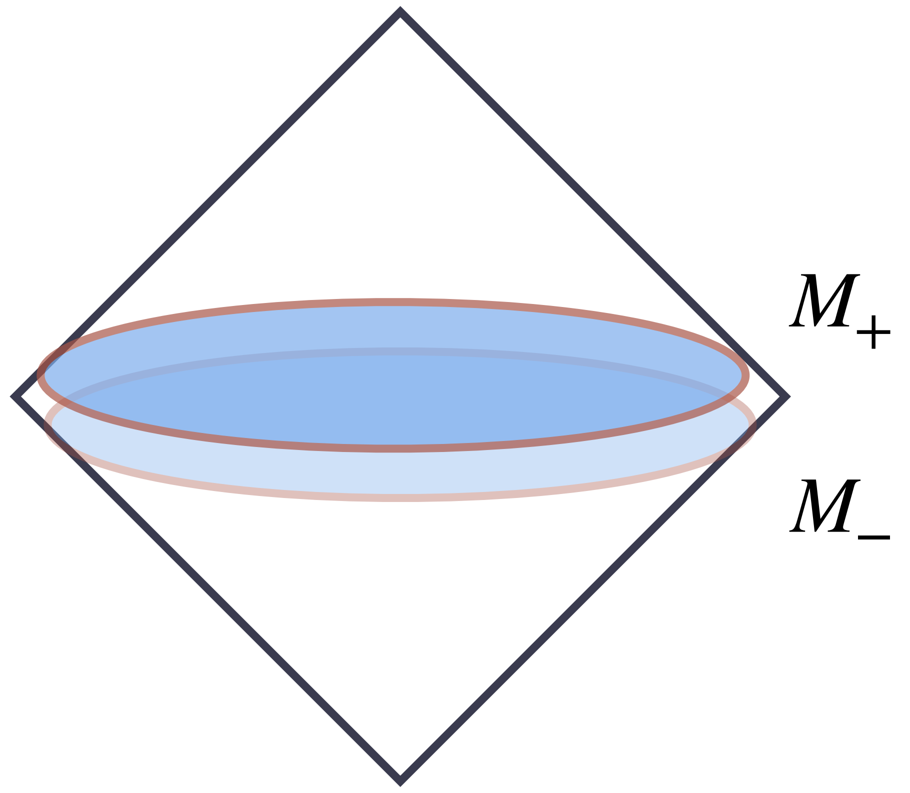
Implications for scattering

Matching condition $\Longrightarrow$ charge conservation: $\langle \text{out} | Q_f^+ \mathcal{S} - \mathcal{S} Q_f^- | \text{in} \rangle = 0$

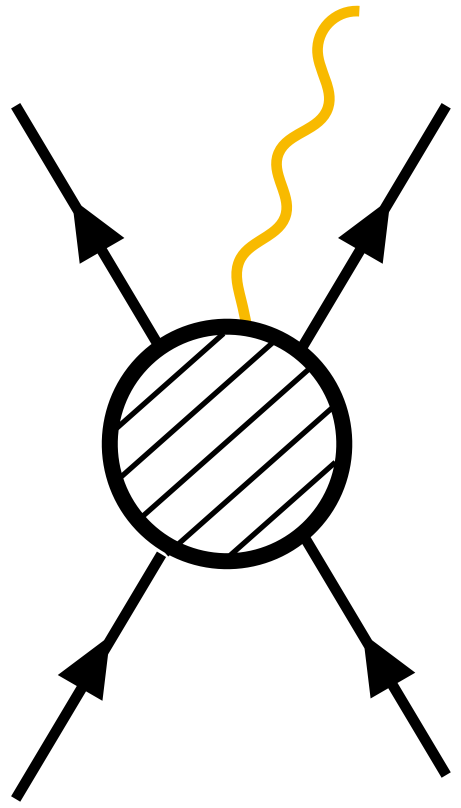
$$Q_f \equiv \int_{\mathcal{J}^+} du d^2z f(z,\bar{z}) \Big(\underbrace{\frac{1}{2} D^2 N}_{Q_{\text{soft}}} + \underbrace{\frac{1}{4} C \partial_u N}_{Q_{\text{hard}}} \Big)$$

$$\Longrightarrow \quad \langle \text{out} | [Q_{\text{soft}}, \mathcal{S}] | \text{in} = - \langle \text{out} | [Q_{\text{hard}}, \mathcal{S}] | \text{in} \rangle$$

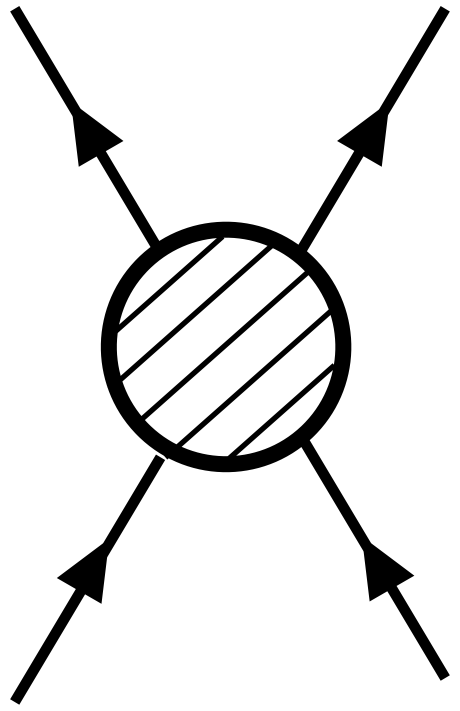
$$Q_{\text{soft}} \propto \int d^2z D^2 f \int_{-\infty}^{\infty} du N(u, z, \bar{z})$$ ~ zero mode of news ~ soft graviton



$\Longrightarrow \quad \lim_{q \to 0}$



$= \underbrace{\mathbf{S}}_{\text{SOFT FACTOR}} \times$



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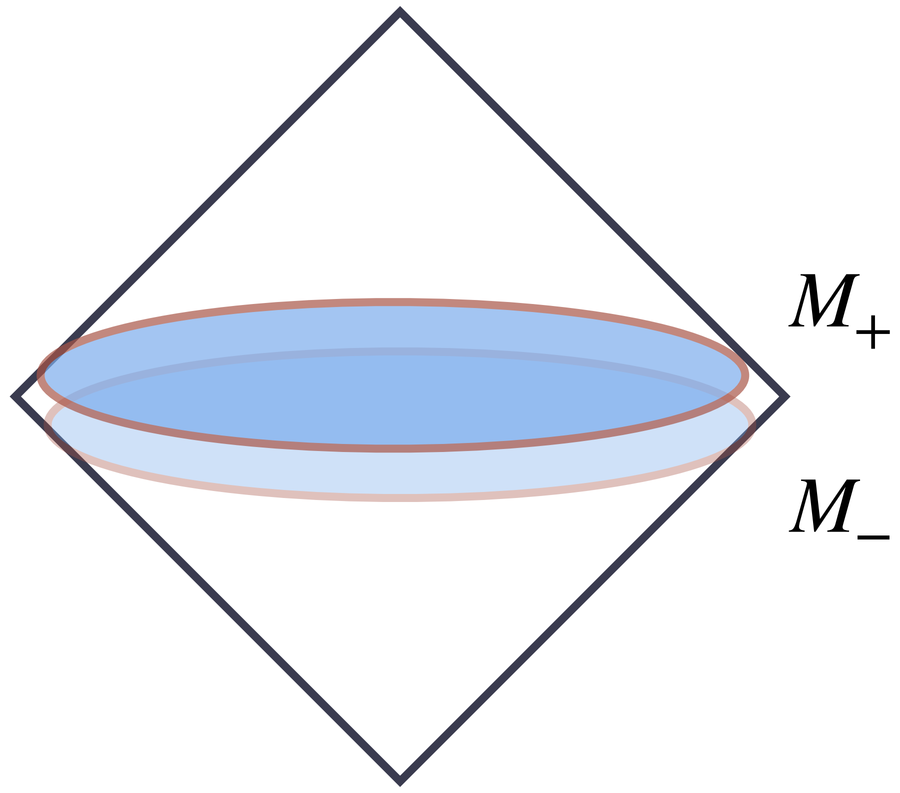
Q_{hard} generates asymptotic symmetry transformation on scattering states ~ <u>leading</u> soft factor in momentum space



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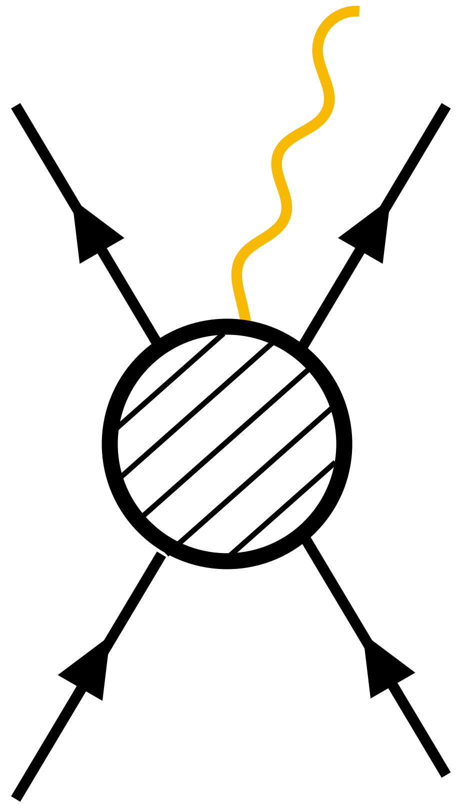

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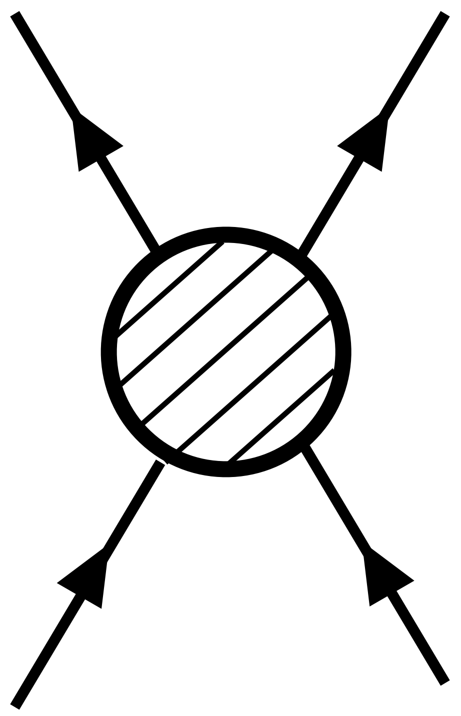

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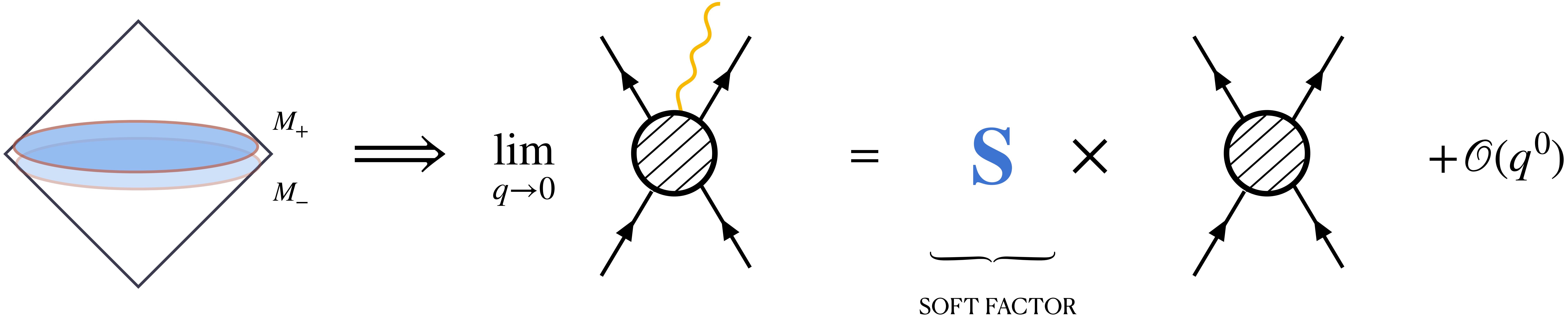
Implications for scattering

Repeat for superrotations $\Longrightarrow$ subleading soft graviton theorem [Cachazo, Strominger '14; Kapec, Lysov, Pasterski, Strominger '14]

$$\mathbf{S} = \frac{1}{\omega} S^{(0)} + \boxed{S^{(1)}} + \omega S^{(2)} + \cdots$$



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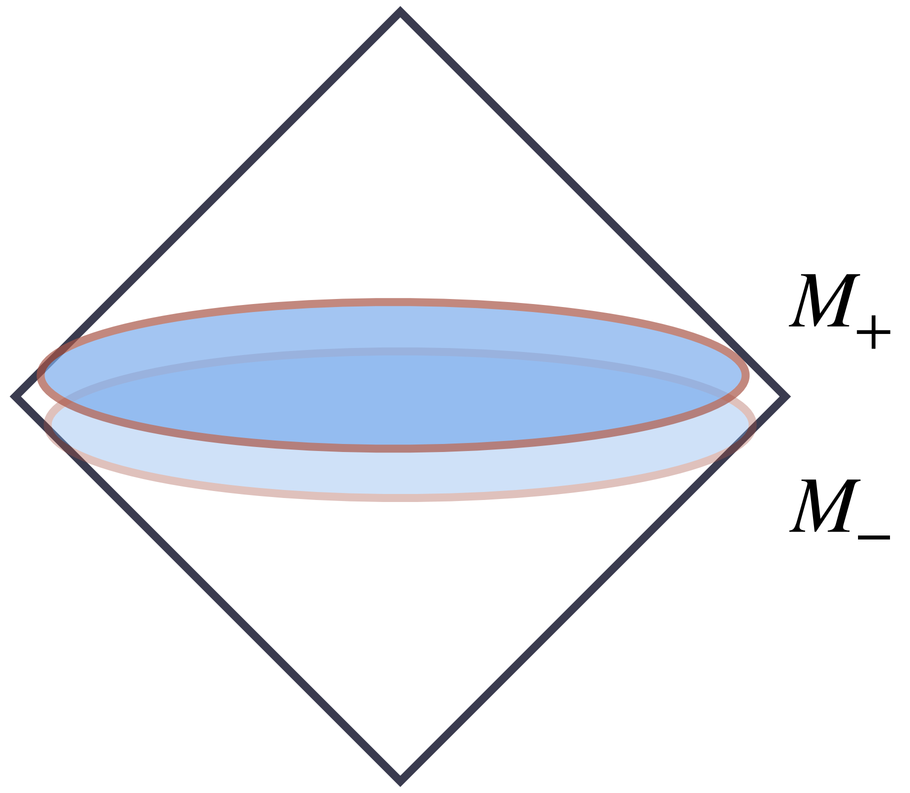
Tower of corrections $\Longrightarrow$ tower of conservation laws?



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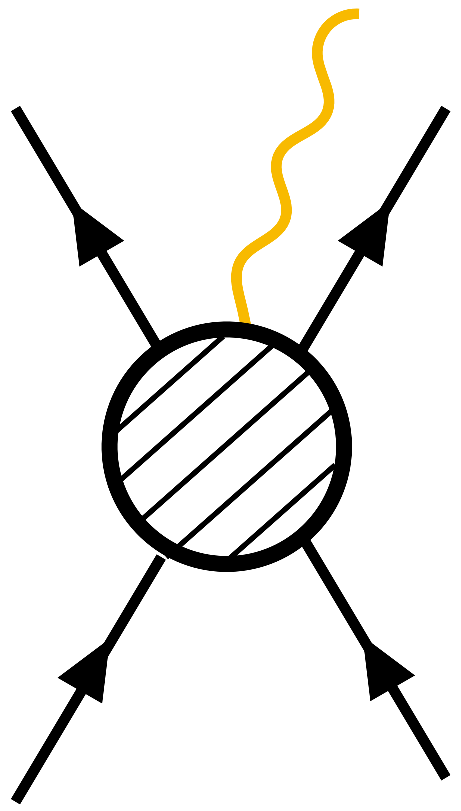

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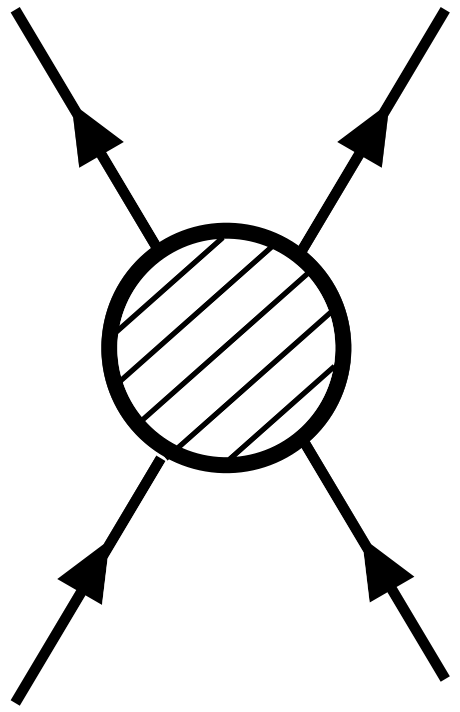

$+\mathcal{O}(q^0)$

B. Celestial amplitudes

Conformal primary basis

Lorentz algebra in 4D ~ global conformal algebra in 2D

- Subleading soft graviton ↔ generator of 2D conformal (Virasoro) symmetry
- 4D Lorentz boost/2D dilation action on momentum eigenstates is non-diagonal: $\delta_{Y^z=z}\mathcal{O}_s(\omega,0) \propto (-\omega\partial_\omega + s)\mathcal{O}(\omega,0)$
- Diagonalize $\delta_Y \implies$ reorganize asymptotic data in representations of 2D conformal algebra

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For massless scalars achieved by: $\widetilde{\mathcal{O}}^{\pm}_{\Delta}(z,\bar{z}) \equiv \int_0^\infty d\omega\omega^{\Delta-1}\mathcal{O}^{\pm}(\omega,z,\bar{z})$ Massless momenta $q = \pm\,\omega\left(1,\hat{n}(z,\bar{z})\right)$

Equivalently, solve eom with conformal primary wavepackets: $\Psi^{\pm}_{\Delta}(\hat{q};X) \propto \dfrac{1}{(-\hat{q}\cdot X_{\pm})^{\Delta}}$

$$\widetilde{O}^{\pm}_{\Delta}(z,\bar{z}) \equiv -\,i\langle\Psi^{\pm}_{\Delta}(\hat{q};X),\mathcal{F}(X)\rangle$$

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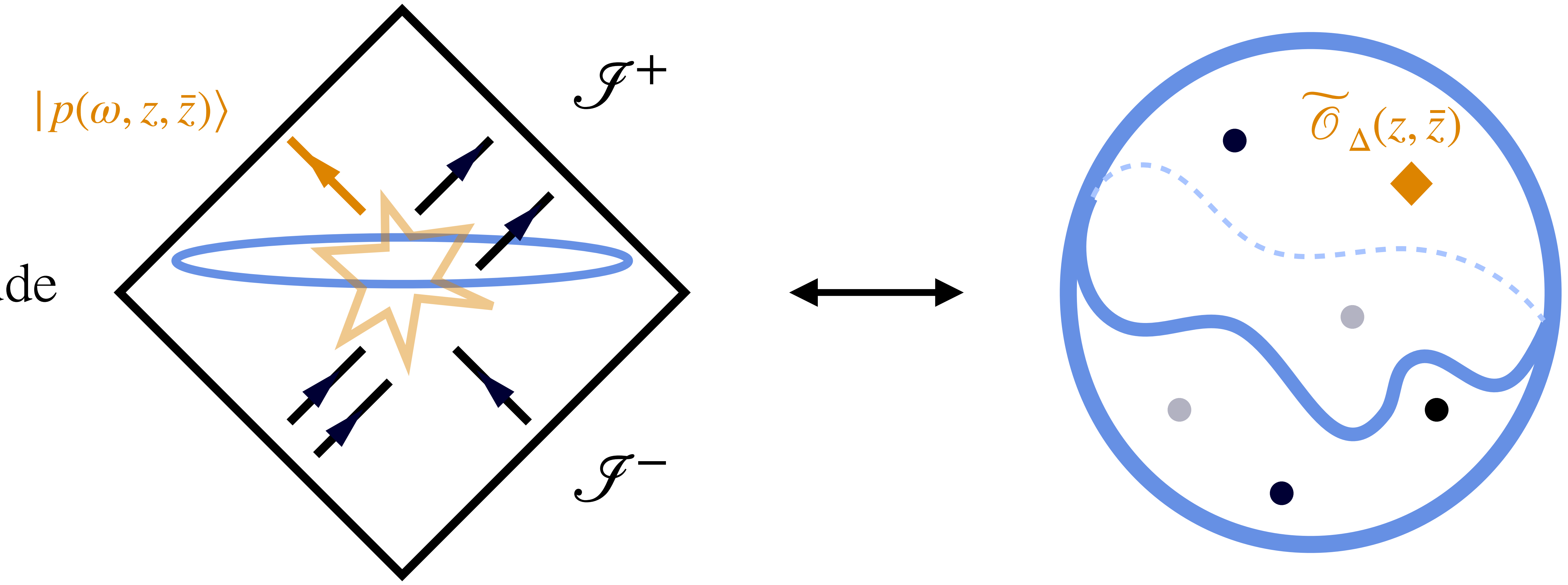
Conformal primary basis

Massless
scattering amplitude
$\langle \text{out} | \mathcal{S} | \text{in} \rangle$



Celestial amplitude
$\langle \widetilde{\mathcal{O}}_{\Delta_1}(z_1, \bar{z}_1) \; \cdots \; \widetilde{\mathcal{O}}_{\Delta_n}(z_n, \bar{z}_n) \rangle$

$$\langle \widetilde{\mathcal{O}}_{\Delta_1}(z_1, \bar{z}_1) \; \cdots \; \widetilde{\mathcal{O}}_{\Delta_n}(z_n, \bar{z}_n) \rangle = \langle \text{out} | \mathcal{S} | \text{in} \rangle \rangle |_{\text{boost}}$$

- transforms like primary correlator in 2D CFT
- enjoys additional symmetries from 4D bulk
- other constraints $\Longleftrightarrow$ 4D bulk crossing symmetry, unitarity, causality, …

Celestial amplitudes

Generalization for spinning (massless) particles: $$\widetilde{\mathscr{M}}(\Delta_a, z_a, \bar{z}_a) = \prod_{i=1}^{n}\left(\int_0^\infty d\omega_i \omega_i^{\Delta_i - 1}\right) \underbrace{M(p_a, \sigma_a)\delta^4\left(\sum_{a=1}^{n} p_a\right)}$$

Momentum space amplitude

(contracted with polarization tensors)

- Poincare symmetry $\Longrightarrow \widetilde{\mathscr{M}}(\Delta_i, z_i) = \widetilde{M}(\Delta_i, z_i)\delta(r - \bar{r}), \quad r \equiv \dfrac{z_{13}z_{24}}{z_{12}z_{34}} \quad (n = 4)$

$$\widetilde{M}(\Delta_1 + 1, \Delta_2, \cdots, \Delta_n) + \cdots + \widetilde{M}(\Delta_1, \Delta_2, \cdots, \Delta_n + 1) = 0$$

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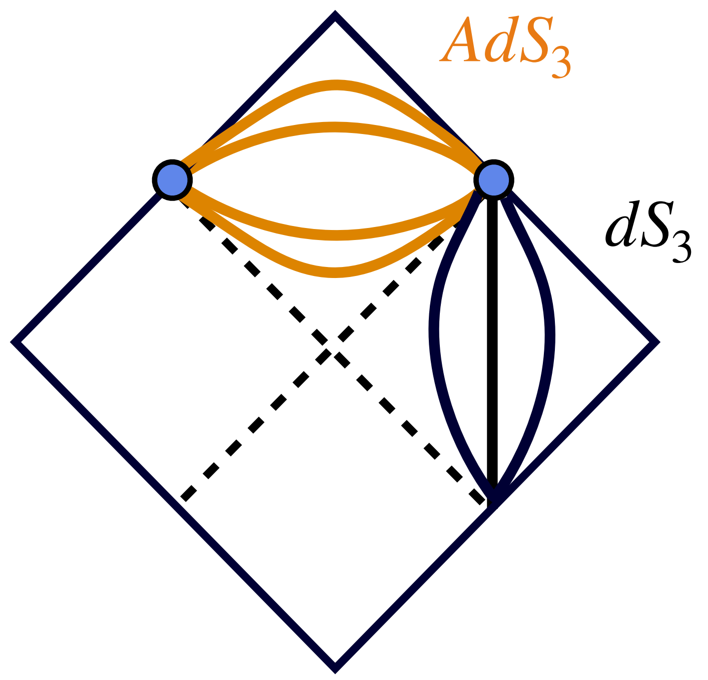
Generalization for (spinning) massive particles:

- Mellin integral replaced by integral against 3D AdS/dS bulk-to-boundary propagators
- Harder to compute, less studied so far - focus on massless case



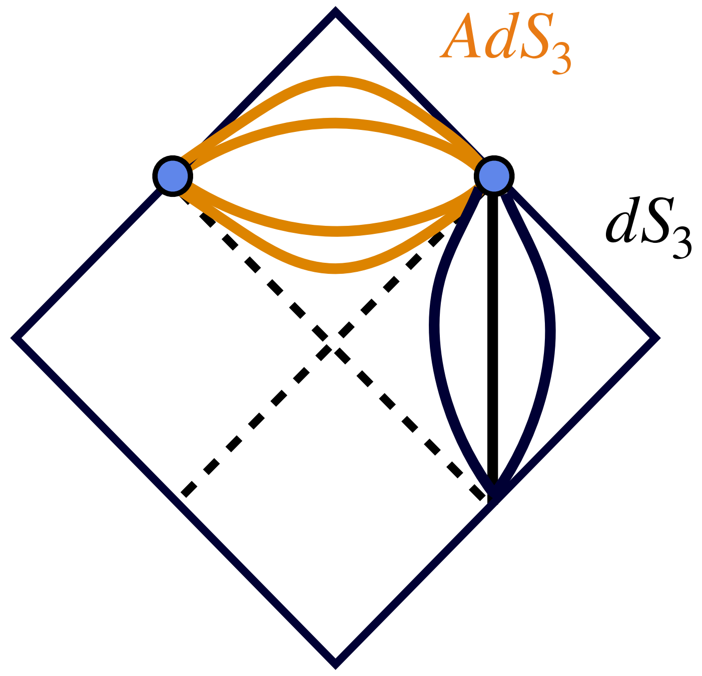
[de Boer, Solodukhin ’03, Taronna, Sleight ’22, ’23]

Examples: Non-perturbative backgrounds

Translation breaking backgrounds appear to smoothen out singularities

$$\widetilde{\mathcal{M}}_B(1^-,2^-,3^+,\cdots n^+) \sim \frac{z_{12}^3}{z_{23}z_{34}\cdots z_{n1}} \int \widetilde{d^3Q}\, g(Q) \int d\omega_1 \omega_1^{\Delta_1} \int d\omega_2 \omega_2^{\Delta_2} \int \prod_{j=3}^{n} d\omega_j \omega_j^{\Delta_j-2} \delta^{(4)}\left(Q + \sum_i \eta_i \omega_i \hat{q}_i\right)$$

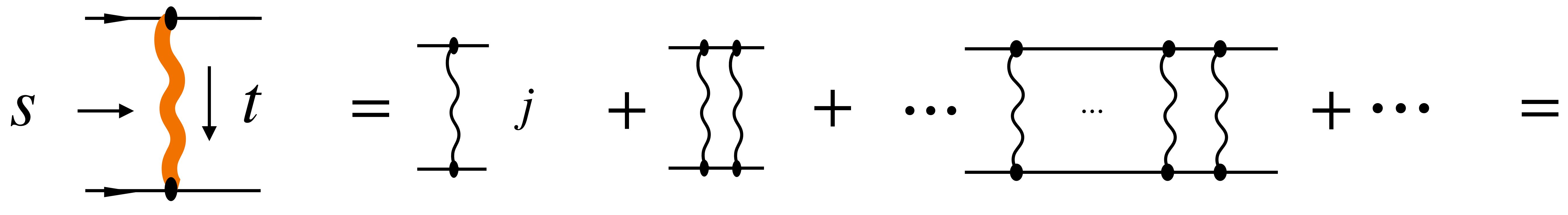
conformal primary massive scalar ($\Delta = 2$)



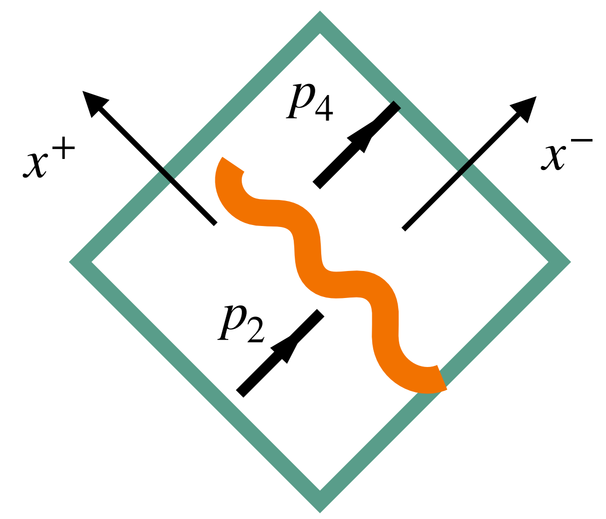
- 3-point function $\propto$ standard CFT 3-point function [Casali, Melton, Strominger '22; Stieberger, Taylor, Zhu '22; Sleight, Taronna '23]
- Celestial two-point functions in various different backgrounds recently computed to leading order in the coupling

[Gonzo, McLaughlin, Puhm '22]

Examples: Non-perturbative backgrounds



Propagation through shockwave ($j = 2$)



- Celestial 2-point function $$\widetilde{A}_{\text{shock}}(\Delta_2, z_2, \bar{z}_2; \Delta_4, z_4, \bar{z}_4) = 4\pi \int d^2x_\perp \frac{i^{\Delta_2+\Delta_4}\Gamma(\Delta_2+\Delta_4)}{\left[-q_{24,\perp}\cdot x_\perp - h(x_\perp) + i\epsilon\right]^{\Delta_2+\Delta_4}}$$

- Can be directly obtained from flat space limit of holographic correlator in AdS

[de Gioia, A.R. ’22]

2D conformal representations

For $\Delta = 1 + i\lambda$, $\lambda \in \mathbb{R}$, conformal primary wavefunctions $\Psi^{\pm}_{\Delta}(\hat{q}; X) \propto \dfrac{1}{(-\hat{q} \cdot X_{\pm})^{\Delta}}$ form a basis:

$$\mathcal{F}(X) = \int_{\mathbb{R}} d\lambda \int d^2 z \left(O^{\dagger}_{1+i\lambda}(z) \Psi_{1+i\lambda}(z; X) + h.c. \right)$$

Bulk soft modes instead fall into integer $\Delta \leq 1$ representations eg.: $\lim\limits_{\omega \to 0} \omega \mathcal{O}(\omega, z, \bar{z}) = \lim\limits_{\Delta \to 1} (\Delta - 1) \underbrace{\int_0^{\infty} d\omega \omega^{\Delta - 1} \mathcal{O}(\omega, z, \bar{z})}_{\widetilde{\mathcal{O}}_{\Delta}(z, \bar{z})}$

- $\Delta = -n$: (sub)$^{n+1}$ - leading soft modes organize into finite-dimensional representations

[A.R., Pate, Strominger '21; Adamo, Mason, Sharma '19; Puhm '19]

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- $\Delta = -n$: $(\text{sub})^{n+1}$-leading soft mode
- Leading soft mode $(n = -1)$ has a primary descendant

$$\begin{tikzcd} & \Phi^0_{\Delta=-1} \equiv \mathcal{S}Q^0_{\text{soft}} \arrow[dl, "\partial_z^2"'] \arrow[dr, "\partial_{\bar{z}}^2"] & \\ O^{++}_{\Delta=1} \arrow[dr, "\partial_{\bar{z}}^2"'] & & (\mathcal{S}O)^{--}_{\Delta=1} \arrow[dl, "\partial_z^2"] \\ & Q^0_{\text{soft}} & \end{tikzcd}$$

Q^0_{soft} = soft component of supertranslation charge

[Gelfand '65; Donnay, Pasterski, Puhm, Strominger, Trevisano, ...]

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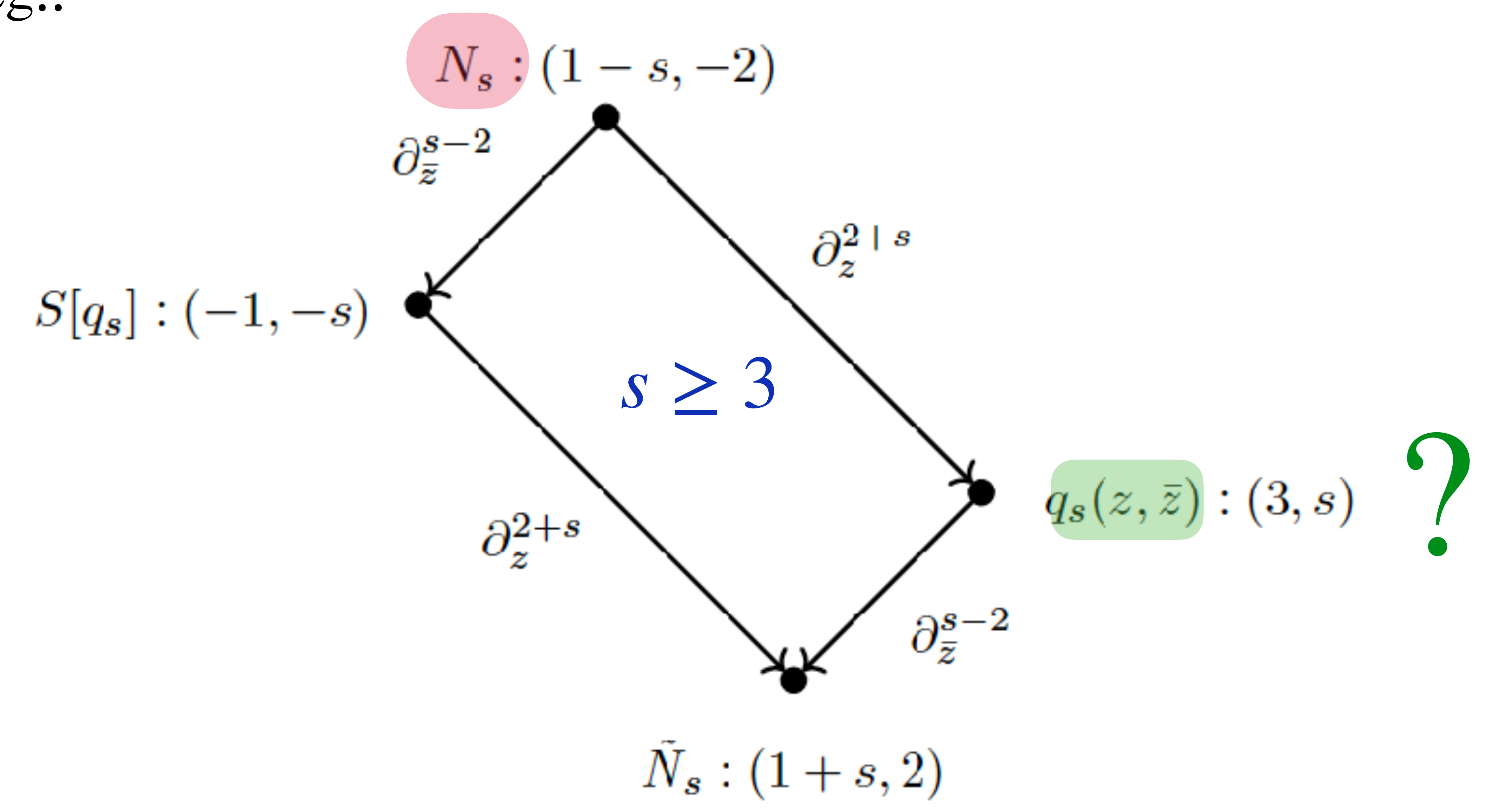
- $\Delta = -n$: (sub)$^{n+1}$-leading soft mode N_s

- Set $\Delta = 1 - s$



[Gelfand '65; Donnay, Pasterski, Puhm, Strominger, Trevisano, ...]

Celestial operator products

Lorentzian Celestial (C)CFT with global conformal group $SL_L(2,\mathbb{R}) \times SL_R(2,\mathbb{R})$

Upon resuming contributions from $SL_L(2,\mathbb{R})$ descendants, the OPE ($\bar{z}_{12} \to 0$, z_1, z_2 fixed) of two gravitons takes the form:

$$O^{--}_{\Delta_1}(z_1) O^{\pm\pm}_{\Delta_2}(z_2) \sim -\frac{\kappa}{2}\frac{1}{\bar{z}_{12}} \sum_{n=0}^{\infty} B(2h_1 + 1 + n, 2h_{2\pm} + 1)\frac{z_{12}^{n+1}}{n!}\partial^n O^{\pm\pm}_{\Delta_1+\Delta_2}(z_2) + \mathcal{O}(\bar{z}^0_{12}) \qquad h \equiv \frac{\Delta + J}{2}$$

[Pate, A.R., Strominger, Yuan '19; Guevara, Himwich, Pate, Strominger '21]

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$$\boxed{h \equiv \frac{\Delta + J}{2}}$$

- $n = 0$ from (bulk) collinear factorization (also fixed by symmetry)
- $n > 0$ from resuming $SL_L(2,\mathbb{R})$ descendants
- $\Delta_1 = 1 - s, \quad \Delta_2 = 1 - s'$ & integral-transform (light-transform) $\Longrightarrow$

$$[w^p_m, w^q_n] = \big(m(q-1) - n(p-1)\big)\, w^{p+q-2}_{m+n}, \qquad p = \frac{s+3}{2}, \quad q = \frac{s'+3}{2}$$

[Guevara, Himwich, Pate, Strominger ’21; Strominger ’21]

C. Towards bulk reconstruction in general relativity

Infinite tower of bulk charges

Recall $G_{uu} = 0$ constraint: $\boxed{\partial_u M}_{\mathbb{C}} = \frac{1}{2} D^2 N + \frac{1}{4} C \partial_u N$ Supertranslation charge conservation $\Longrightarrow$ leading soft theorem

Remaining components of Einstein equations:

$$@\mathcal{O}(r^{-2}): \begin{cases} G_{uz} = 0 \implies \boxed{\partial_u J} = DM + \frac{1}{2} CDN \\ G_{zz} = 0 \implies \boxed{\partial_u T} = DJ + \frac{3}{2} CM \end{cases}$$

Superrotation charge conservation $\Longrightarrow$ sub-leading soft theorem

``Spin-2'' charge conservation $\Longrightarrow$ sub-sub-leading soft theorem

[Freidel, Pranzetti, A.R. '21]

$G_{zz} = 0 \quad @\mathcal{O}(r^{-(s+1)}) \implies$ tower of evolution equations that truncate to:

$$\frac{d\mathcal{Q}_s}{du} = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2} C\mathcal{Q}_{s-2} + \cdots, \quad s \geq 0, \ \ s \in \mathbb{Z}$$

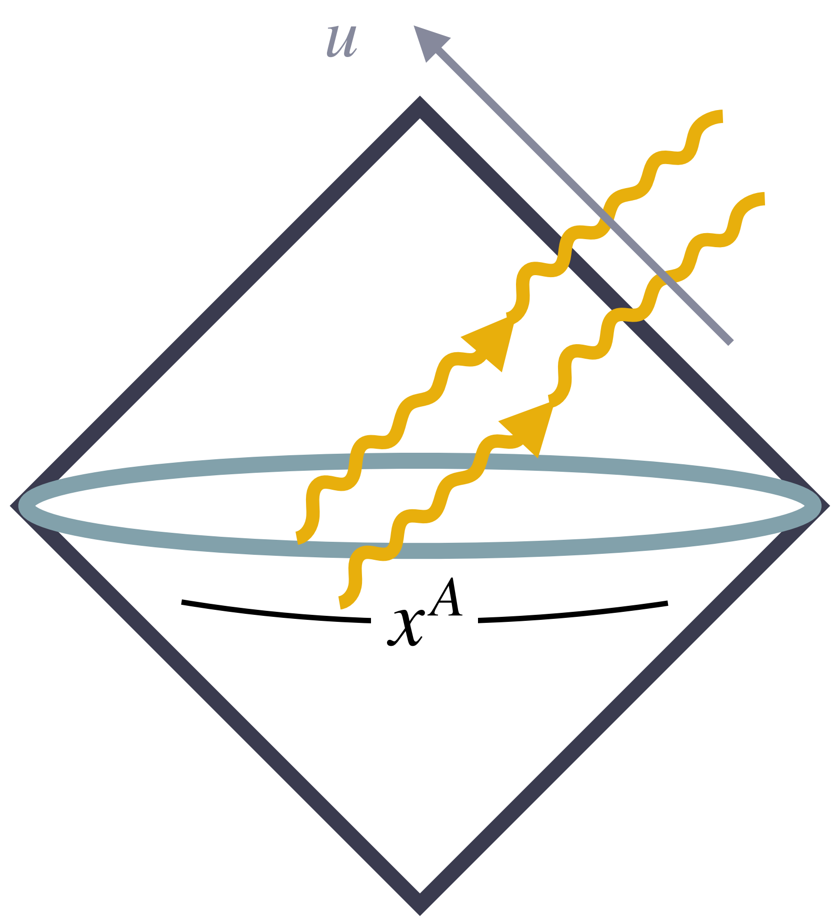
Infinite tower of bulk charges

Asymptotic Einstein equations reorganized into a hierarchy of recursive, non-linear differential equations:

$$\frac{d\mathcal{Q}_s}{du} = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2}C\mathcal{Q}_{s-2} + \cdots, \quad s \geq 0, \quad s \in \mathbb{Z}$$

[Freidel, Pranzetti, A.R. '21]

$$ds^2 = -2e^{2\beta}du(dr + \Phi du) + r^2\gamma_{AB}\left(dx^A - \frac{\Upsilon^A}{r^2}du\right)\left(dx^B - \frac{\Upsilon^B}{r^2}du\right)$$



$$\Phi = \frac{1}{2} - \frac{m}{r} + \mathcal{O}(r^{-2}), \quad \gamma_{AB} = q_{AB} + \frac{1}{r}C_{AB} + \mathcal{O}(r^{-2}).$$

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[Freidel, Pranzetti, A.R. '21]

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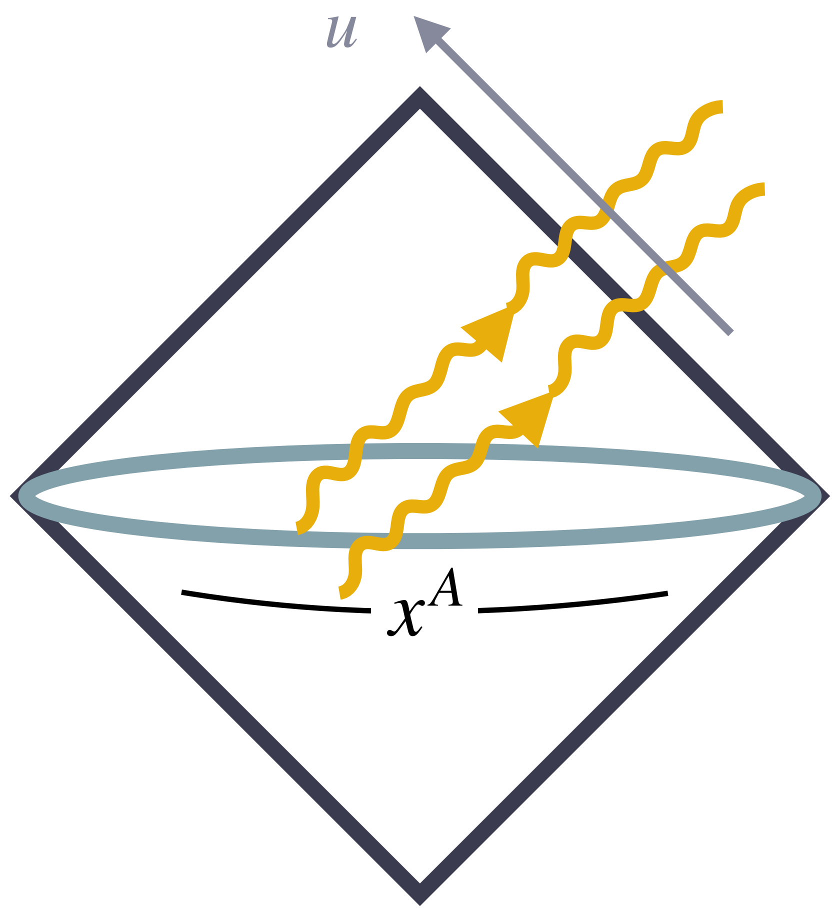

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$\mathcal{Q}_0$

mass/energy flux

Infinite tower of bulk charges

Asymptotic Einstein equations reorganized into a hierarchy of recursive, non-linear differential equations:

$$\frac{d\mathcal{Q}_s}{du} = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2}C\mathcal{Q}_{s-2} + \cdots\,, \quad s \geq 0, \quad s \in \mathbb{Z}$$

[Freidel, Pranzetti, A.R. '21]

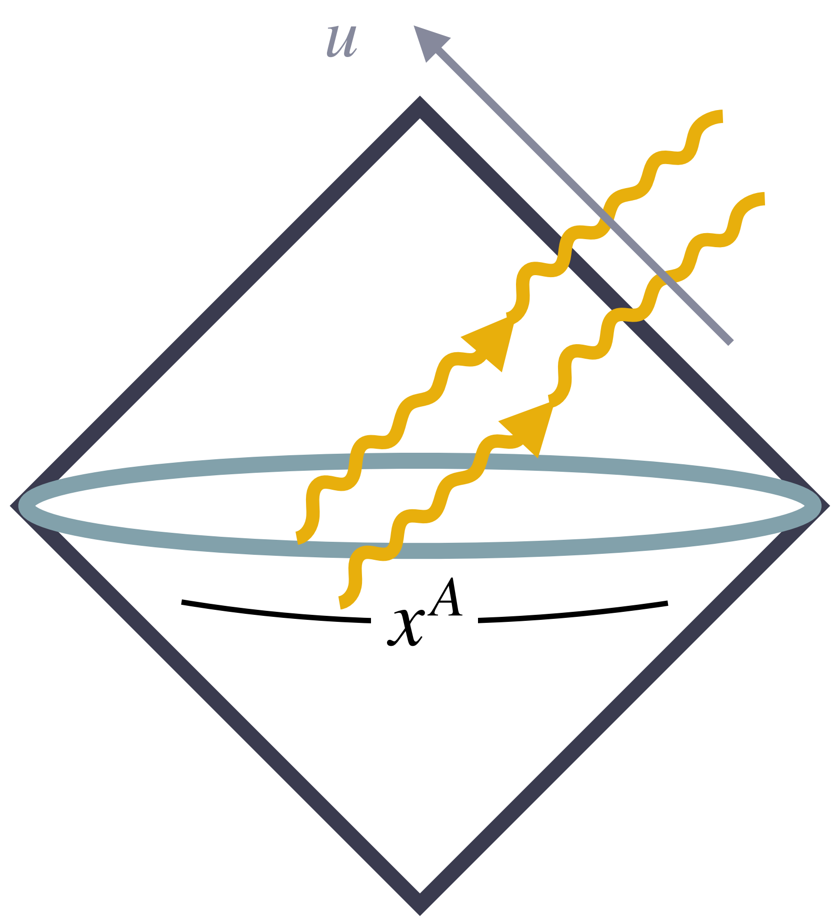
$$ds^2 = -2e^{2\beta}du(dr + \Phi du) + r^2\gamma_{AB}\left(dx^A - \frac{\Upsilon^A}{r^2}du\right)\left(dx^B - \frac{\Upsilon^B}{r^2}du\right)$$

Q_1 angular momentum

$$\Phi = \frac{1}{2} - \frac{m}{r} + \mathcal{O}(r^{-2}), \quad \gamma_{AB} = q_{AB} + \frac{1}{r}C_{AB} + \mathcal{O}(r^{-2})\,.$$

Q_0

mass/energy flux



Infinite tower of bulk charges

Asymptotic Einstein equations reorganized into a hierarchy of recursive, non-linear differential equations:

$$\frac{d\mathcal{Q}_s}{du} = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2}C\mathcal{Q}_{s-2} + \cdots\,, \quad s \geq 0, \;\; s \in \mathbb{Z} \;\; \leftrightarrow \;\; \text{asymptotic 4D metric} \leftrightarrow \text{Ward identities in 2D CCFT}$$

[Freidel, Pranzetti, A.R. '21]

$\mathcal{Q}_1$ angular momentum

$$ds^2 = -2e^{2\beta}du(dr + \Phi du) + r^2\gamma_{AB}\left(dx^A - \frac{\Upsilon^A}{r^2}du\right)\left(dx^B - \frac{\Upsilon^B}{r^2}du\right)$$

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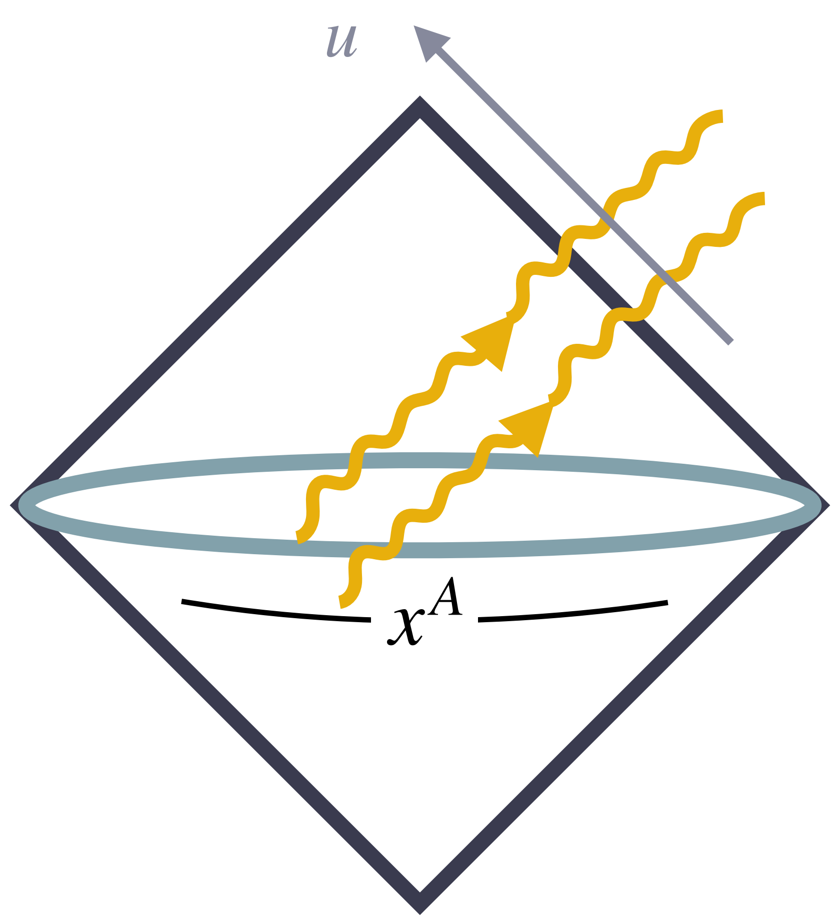
$\mathcal{Q}_0$

mass/energy flux

$\mathcal{Q}_2, \mathcal{Q}_3, \cdots$ higher multipole moments of the gravitational field!



[Compere, Oliveri, Seraj '22]

Discrete basis

The news can be reconstructed from the tower of soft charges provided it decreases rapidly as $\omega \to \infty$:

$$\widetilde{N}_{\pm}(\omega) = \sum_{n=0}^{\infty} \omega^n \mathscr{M}_{\pm}(n),\ n \in \mathbb{N} \qquad \text{subject to UV condition}$$

- if also $C(u)\Big|_{u\to\infty} \propto |u|^{-1-n},\ \forall n > 0 \implies C_{\pm}(u) = \dfrac{i}{2\pi}\displaystyle\sum_{n=0}^{\infty}\frac{(-iu)^n}{n!}\mathscr{S}_{\pm}(n)$, where

$$\mathscr{S}_{\pm}(n) = \lim_{\Delta\to n} \widehat{N}_{\pm}(\Delta),\ n \in \mathbb{N} \qquad \text{subject to IR condition}$$

- UV & IR conditions $\implies N(u) \in$ space of Schwartz functions

$\mathscr{M}_{\pm}(n)$ (alternatively $\mathscr{S}_{\pm}(n)$) form a basis on this function space!

[Freidel, Pranzetti, A.R.'22]

Phase space symmetry and infrared divergences

- Renormalized Q_s realize a higher spin symmetry algebra on the gravitational phase space at linearized order

  Evidence from YM theory that this continues to hold in the non-linear theory [Freidel, Pranzetti, A.R. '23]

  Discrete basis $\Longrightarrow$ (tree level) gravity amplitudes determined by higher spin symmetry Ward identities?

  [Cotler, Miller, Strominger, '23]

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  [Cotler, Miller, Strominger, '23]

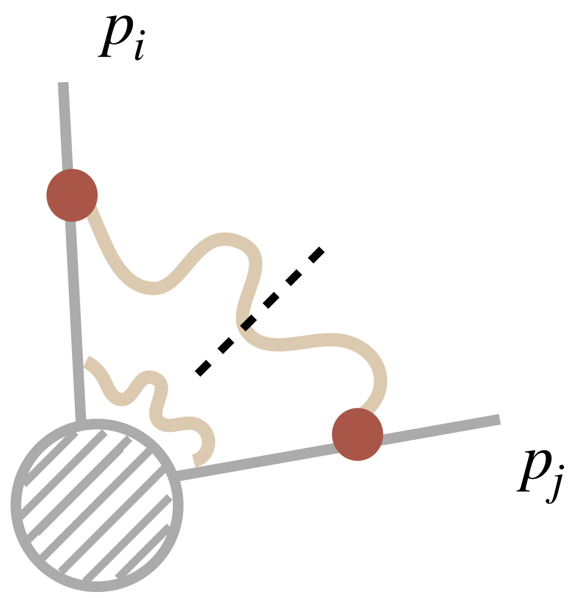
  Caveats: full Einstein equations, mixed helicity sectors, loop corrections, infrared divergences…

Phase space symmetry and infrared divergences

- $\mathcal{S}(n)$ allows to construct all order generalization of Dirac-Faddeev-Kulish dressings diagonalizing $\mathcal{M}(n)$:

[Dirac '31; Weinberg '65; Chung; Kibble; Faddeev, Kulish...]

$$\langle p | \mathcal{D}_\pm \equiv \langle p | \exp\left\{ \sum_{s=0}^{\infty} \frac{(-1)^s}{\pi\kappa^2} \int d^2 z Q_\mp(s,z;p) \left[\mathcal{G}_\pm(s,z) - \mathcal{G}^*_\mp(s,z) \right] \right\}, \quad \mathcal{S}_+(s) = D_+^{s+2} \mathcal{G}_+(s)$$



$\langle p_{\rm out} | \mathcal{D}(p_1) \cdots \mathcal{D}(p_n) | p_{\rm in} \rangle$ reproduces amplitude of finite energy graviton exchanges

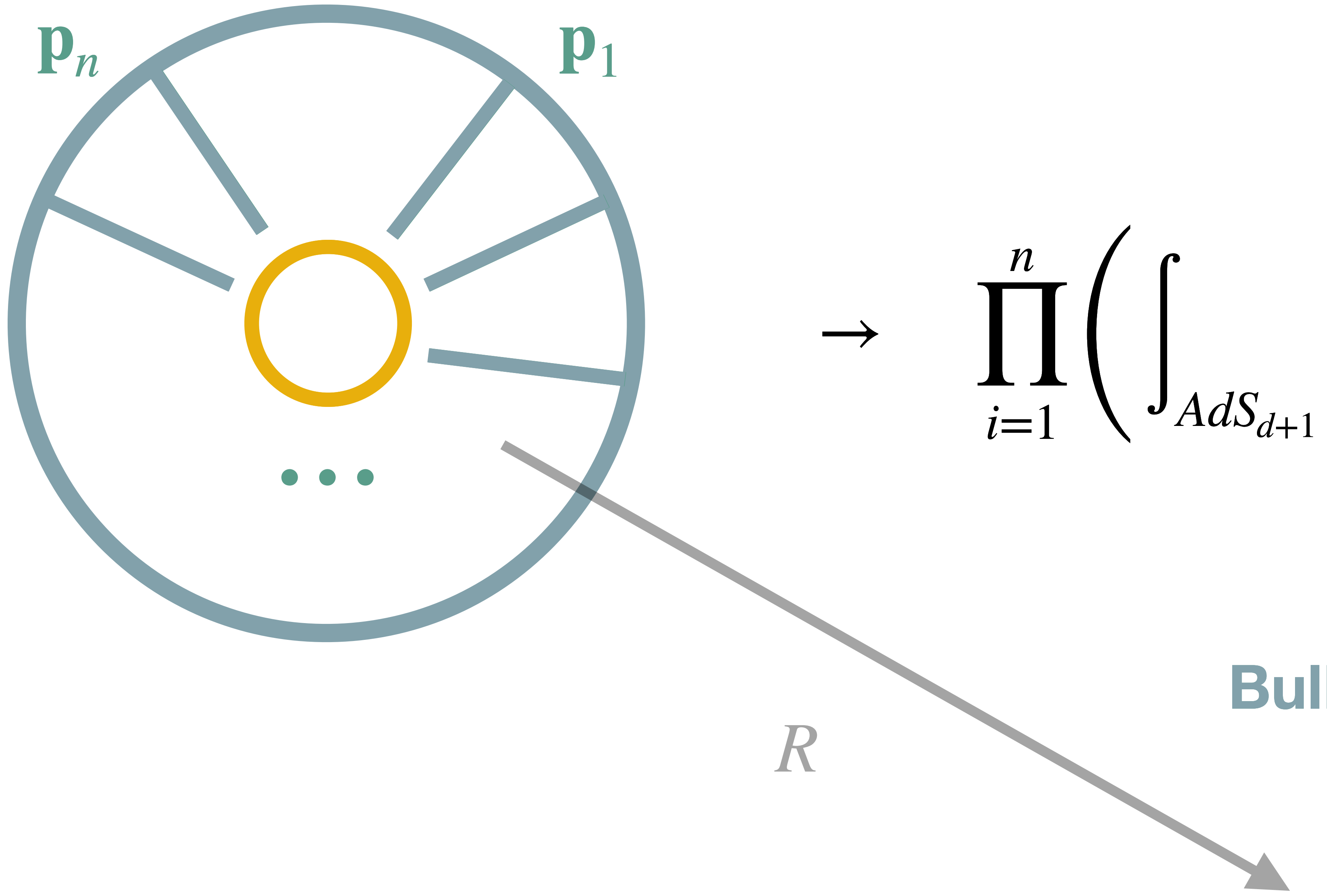
D. Flat space limit of AdS/CFT

AdS/CFT in the flat space limit

$\sum$ Amplitudes in AdS_{d+1} (Witten diagrams) $\leftrightarrow$ Correlation functions in CFT_d

- CFT (Mellin) correlators related to flat space scattering amplitudes at large AdS radius

[Polchinski '99; Susskind '99; Giddings '99; Penedones '10;…; Hijano, Neuenfeld '20]

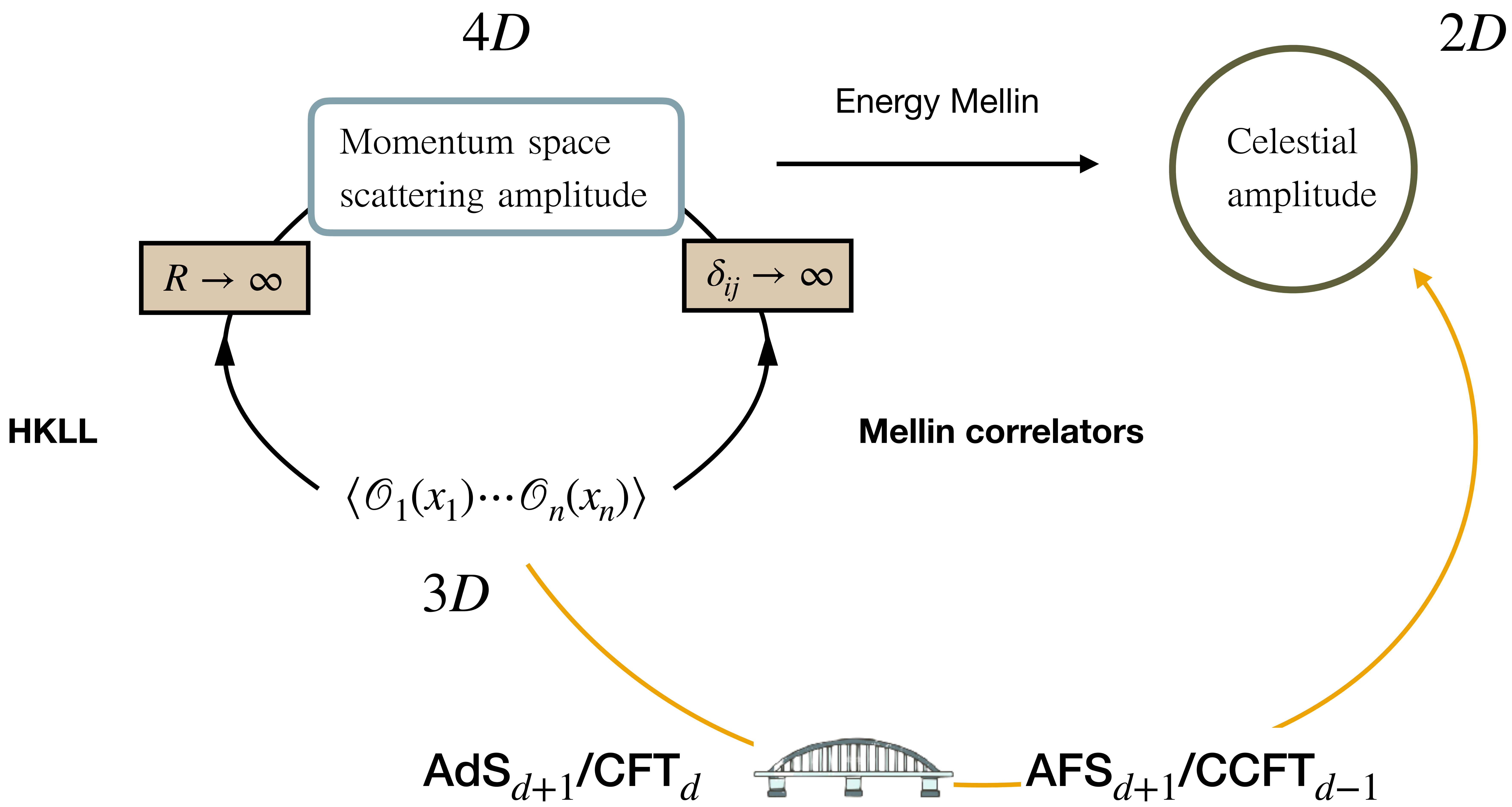


$$\rightarrow \prod_{i=1}^{n}\left(\int_{AdS_{d+1}} d^{d+1}\mathbf{x}_i K_{\Delta_i}(\mathbf{p}_i,\mathbf{x}_i)\right)\mathbf{\Pi}_{\Delta}(\mathbf{x}_i) \xrightarrow{R\to\infty} \langle p_{\text{out}}|\mathcal{S}|p_{\text{in}}\rangle$$

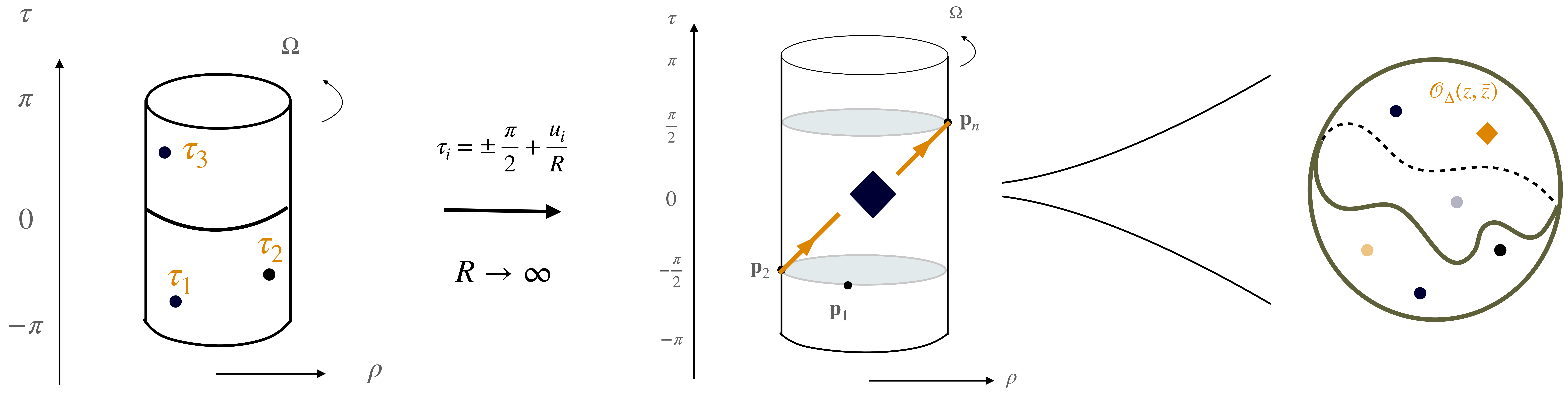
Bulk-to-boundary propagator

Bulk-to-bulk propagators

Celestial amplitudes from AdS/CFT



Celestial amplitudes from AdS/CFT

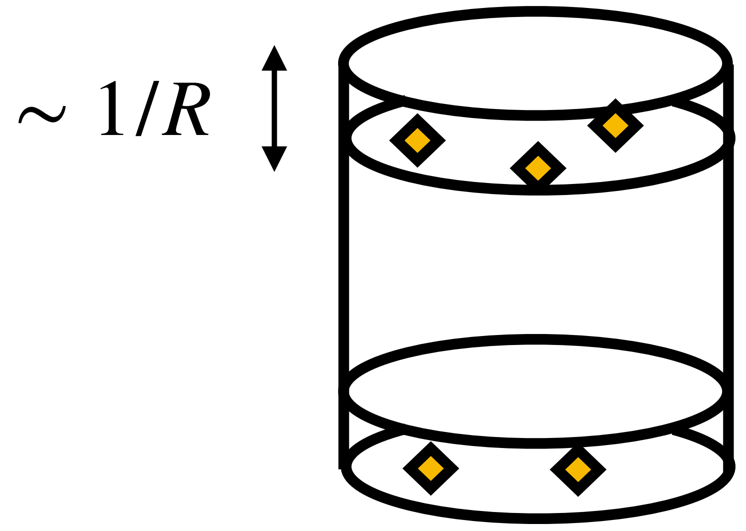


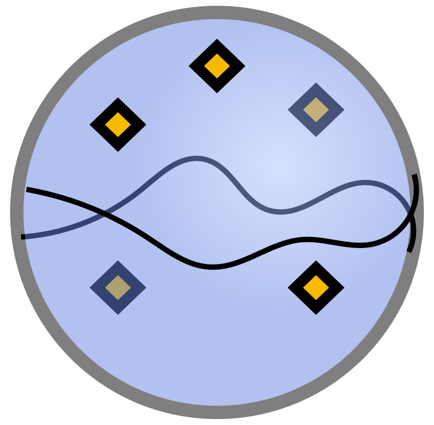
AdS$_4$ boundary observables $\longrightarrow$ 3+1D flat space celestial observables

[de Gioia, A.R. '22]

Celestial sector in CFT

Infinity of symmetries from CFT_d



$\mathrm{CFT}_3 \xrightarrow{R \to \infty}$  CCFT_2

$\cup$ $\qquad\qquad$ $\cup$

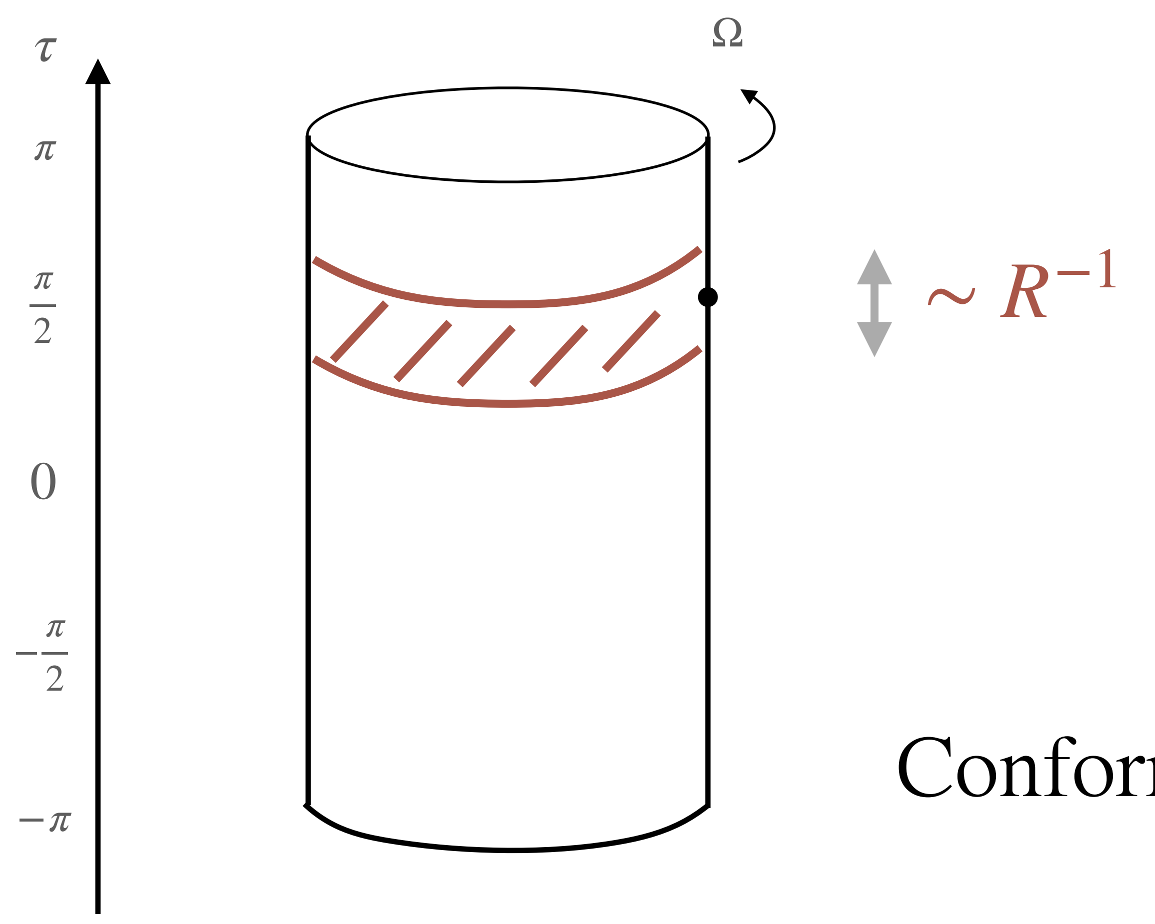
$$so(3,2) \longrightarrow \; ? \; \longrightarrow \begin{cases} \text{Supertranslations} \\ \text{Virasoro} \times \text{Virasoro} \\ \ldots\, w_\infty\,? \end{cases}$$

from [shadow of] T_{CFT_3}!

[de Gioia,. A.R. '23]

Symmetries of infinitesimal time intervals in 3D CFT

Analyze conformal symmetries in infinitesimal interval on the Lorentzian cylinder:



$$ds^2 = -d\tau^2 + 2\gamma_{z\bar{z}}dzd\bar{z}, \quad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2} \xrightarrow{\tau = \tau_0 + \frac{u}{R}} ds^2 = -R^{-2}du^2 + 2\gamma_{z\bar{z}}dzd\bar{z}$$

Conformal Killing vectors in the interval: $\nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu = \frac{2}{d}\nabla \cdot \epsilon(x) g_{\mu\nu}$

[de Gioia, A.R. '23]

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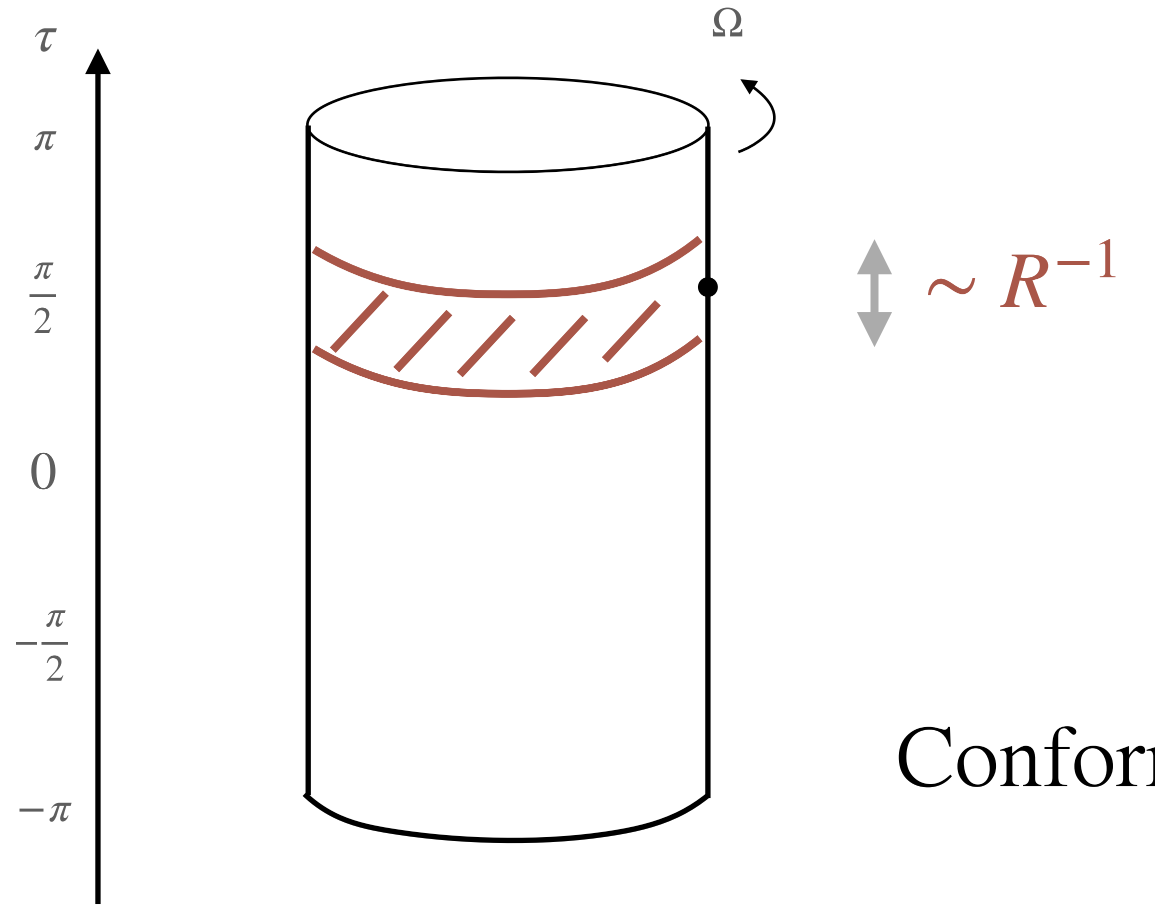

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$\rightarrow 0$ as $R \rightarrow \infty$

Conformal Killing vectors in the interval: $$\nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu = \frac{2}{d}\nabla \cdot \epsilon(x) g_{\mu\nu}$$

As $R \rightarrow \infty$, solutions parameterized by a function f and a vector field Y^A on the sphere:

$$\epsilon^\pm = \left[\mp \frac{iR}{2} F_\pm(u) D \cdot Y(z,\bar{z}) + f(z,\bar{z})\right]\partial_u + F_\pm(u) Y^A(z,\bar{z})\partial_A$$

[de Gioia, A.R. '23]

BMS_4 algebra in the strip

- For constant f and Y global CKV, $\epsilon^{\pm}$ reorganize into generators of $so(3,2)$ - Lorentz generators $M^{\mu\nu}$ in 5d embedding space

- Inonu-Wigner contraction $\mathscr{P}^{\mu} = \frac{1}{R} M^{4\mu}, \quad \mu = 0, \cdots, 3$ with $\mathscr{P}^{\mu}$, $M_{\mu\nu}$ fixed as $R \to \infty$ yields 4D Poincare algebra

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- For $Y^A(z,\bar{z})$ arbitrary CKV, contraction yields $$\begin{cases} L_Y = iY^A\partial_A + i\frac{u}{2}D\cdot Y\partial_u + O(R^{-2}) \\ T_f \equiv i\epsilon_f = if(z,\bar{z})\partial_u + O(R^{-2}) \end{cases}$$

which generate $\mathfrak{ebms}_4$ $\quad [T_{f_1}, T_{f_2}] = O(R^{-2}), \quad [L_{Y_1}, L_{Y_2}] = iL_{[Y_1,Y_2]} + O(R^{-2}), \quad [T_f, L_Y] = iT_{f'=\frac{1}{2}(D\cdot Y)f - Y(f)} + O(R^{-2})$

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→ asymptotic symmetry algebra of 4D AFS from kinematic limit of 3D CFT!

CCFT operators from 3D CFT operators

Conformal transformations in the strip ~ celestial symmetries in the flat space limit

$$\delta_\epsilon \mathcal{O}_\Delta(x) = -\left[(\nabla\cdot\epsilon)\frac{\Delta}{3} + \epsilon^\mu\nabla_\mu + \frac{i}{2}\nabla_\mu\epsilon_\nu S^{\mu\nu}\right]\mathcal{O}_\Delta(x) \longrightarrow$$

transformation of CCFT_2 primary operator

$$\mathfrak{h} \equiv \frac{\hat{\Delta}+s}{2}, \quad \bar{\mathfrak{h}} \equiv \frac{\hat{\Delta}-s}{2}, \quad \hat{\Delta} \equiv \Delta + u\partial_u$$

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Diagonalize weights via $\widehat{\mathcal{O}}_\Delta(z,\bar{z};\Delta_0) \equiv N(\Delta,\Delta_0)\int_{-\infty}^{\infty} du\; u^{-\Delta_0}\mathcal{O}_\Delta(u,z,\bar{z})$

- Same as transform relating Carrollian and celestial conformal field theories [Donnay, Fiorucci, Herfray, Ruzziconi ‘22]
- Shadow stress tensor Ward identity in 3D CFT lead to leading and subleading conformally soft graviton theorems in 2D CCFT

[Kapec, Mitra ’18; de Gioia, A.R. ’23]

E. Twistors, self-dual sector and top-down holography in AFS

Twistor space of Minkowski spacetime

Dynamics of sectors of gauge/gravity theories in 3+1-dimensional (complexified) Minkowski space M, $x^\mu \to x_{\alpha\dot{\alpha}} \equiv x_\mu \sigma^\mu_{\alpha\dot{\alpha}}$

encoded in geometry of $\mathbb{CP}^3$, $Z^A = (Z_1, Z_2, Z_3, Z_4) \sim \lambda Z^A$, $\lambda \in \mathbb{C}^*$; $Z^A = (\mu^{\dot{\alpha}}, \lambda_\alpha)$

Twistor space = open subset of $\mathbb{CP}^3$

Twistor correspondence: $\mu^{\dot{\alpha}} = x^{\alpha\dot{\alpha}}\lambda_\alpha$ (incidence relations)

[Adamo '18]

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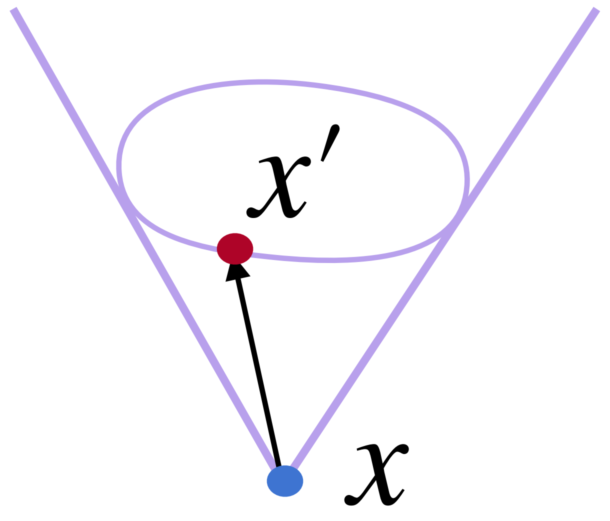

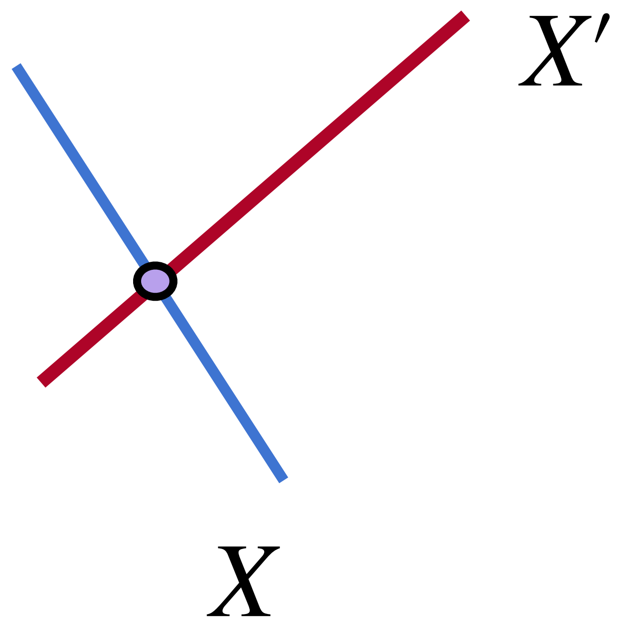

- Points in Minkowski $\longleftrightarrow$ $\mathbb{CP}^1$ (Riemann sphere) in twistor space
- Null cone in Minkowski $\longleftrightarrow$ Point (intersection of two lines) in twistor space

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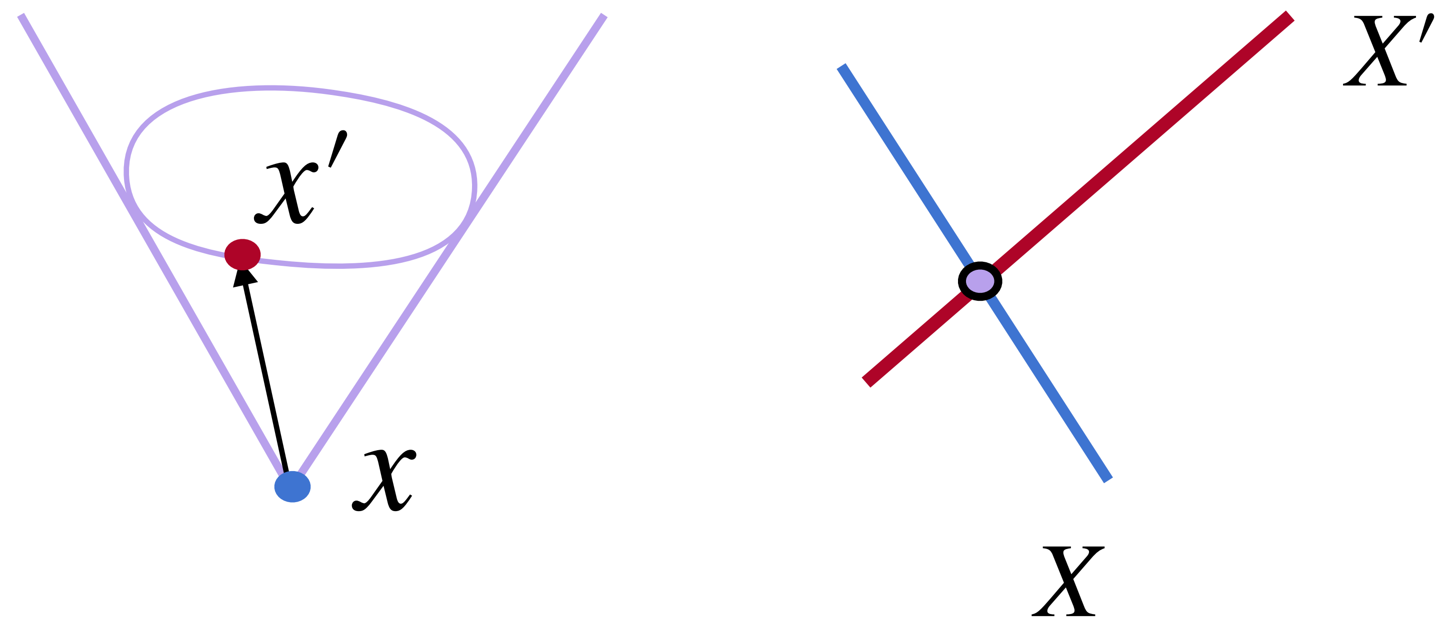

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Penrose transform: solutions to massless eq. of motion in Mink. ~ cohomology classes in twistor space

[Adamo '18]

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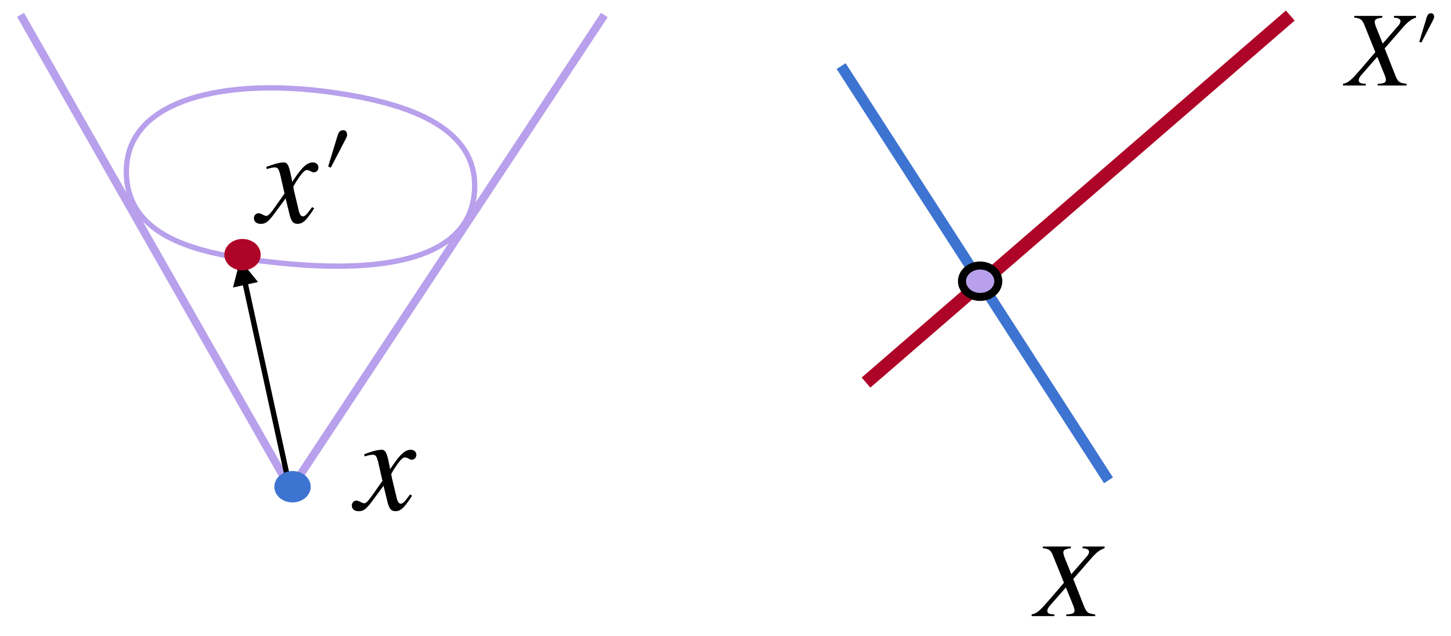

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Ward transform: solutions to self dual YM eqn. ~ holomorphic vector bundle with flat connection $\bar{D}$ on $\mathbb{PT}$

[Adamo '18]

Chiral algebras

- 2D QFT with $SL(2,\mathbb{C})$ symmetry generated by $\left\{ \begin{array}{lll} L_{-1} = -\partial_z, & L_0 = -z\partial_z, & L_{-1} = -z^2\partial_z \\ \bar{L}_{-1} = -\partial_{\bar{z}}, & \bar{L}_0 = -\bar{z}\partial_{\bar{z}}, & \bar{L}_{-1} = -\bar{z}^2\partial_{\bar{z}} \end{array} \right\}$

$sl(2)_L \times sl(2)_R$ commutation relations $[L_m, L_n] = (m-n)L_{m+n}, \quad [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n}$

- Meromorphicity condition $\bar{\partial}\mathcal{O}^s_\Delta(z,\bar{z}) = 0 \implies \mathcal{O}^s_\Delta(z,\bar{z}) = \mathcal{O}_h(z)$ of dimension/weight $\Delta = h = s \in \mathbb{N}/2$

$\implies$ infinity of conserved charges $O_n \equiv \oint dz z^{n+h-1}\mathcal{O}(z)$ $\qquad \mathcal{O}_h(z) = \sum_n \frac{O_n}{z^{n+h}}$

- Global subalgebra = modes that annihilate the vacuum at both 0 and $\infty \implies 1-h \leq n \leq h-1$

Chiral algebras from higher dimensions

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees '13]

- Chiral algebras are special, rigid structures; theories that possess them are vastly constrained
- Unexpected to encounter beyond 2D CFT
- Nevertheless they may appear as sectors of higher-dimensional SCFT, eg. $4D\ \mathcal{N} = 2$ SYM!

Consider $\mathbb{R}^2 \subset \mathbb{R}^4$ preserving $sl(2)_L \times sl(2)_R \subset so(6)$ & look for operators that transform trivially under an $sl(2)$ copy

Naive obstruction: Trivial under $sl(2) \implies$ trivial under full $so(6)$

Bypass by looking for $\widehat{sl}(2)$ that is exact with respect to some operator $\mathbb{Q}$ such that $\mathbb{Q}^2 = 0$ and take cohomology wrt. $\mathbb{Q}$

Chiral algebras from higher dimensions

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees '13]

Taking cohomology wrt. $\mathbb{Q}$ = twisting

- Schematically $\mathbb{Q} \sim \mathcal{Q} + \mathcal{S} \implies [\mathcal{O}]_{\mathbb{Q}}(z)$ where $[\mathcal{O}]_{\mathbb{Q}} = \left\{ \mathcal{O} \,\middle|\, \{\mathbb{Q}, \mathcal{O}] = 0,\ \ \mathcal{O} \neq \{\mathbb{Q}, \mathcal{O}'] \right\}$

Examples of chiral algebras for $\mathcal{N} = 2$ SCFT:

Free hypermultiplet → free symplectic boson algebra: $q_I(z) q_J(w) \sim \dfrac{\varepsilon_{IJ}}{z - w}$

Free vector multiplet → (b, c) system: $b(z)\partial c(w) \sim \dfrac{1}{(z-w)^2}, \quad \partial c(z) b(w) \sim -\dfrac{1}{(z-w)^2}$

Top down holography in AFS

Celestial gluon and graviton algebras related to 2D chiral algebras arising from SCFT/twisted holography/twistor theory

[Gaiotto, Costello '18; Adamo, Bu, Costello, Mason, Paquette, Sharma '21, '22, '23]

Look for 4D asymptotically flat bulk theory dual to celestial 2D chiral algebras

Top down toy model

[Costello, Paquette, Sharma '22, '23]

Consider $\mathbb{R}^4 \simeq \mathbb{C}^2$ with coordinates $x^\mu \to x = x_\mu \sigma^\mu$

$u^{\dot\alpha} \equiv x^{1\dot\alpha}, \quad \hat{u}^{\dot\alpha} \equiv x^{2\dot\alpha}$

Equip $\mathbb{C}^2 \backslash \{0\}$ with metric associated to Kähler form $\omega = \partial\bar\partial K$,

$$K = \|u\|^2 + \log\|u\|^2$$

$\partial \equiv du^{\dot\alpha}\partial_{u^{\dot\alpha}}, \quad \bar\partial \equiv d\hat{u}^{\dot\alpha}\partial_{\hat{u}^{\dot\alpha}}, \quad \|u\|^2 = u^{\dot\alpha}\hat{u}_{\dot\alpha}$

- also known as the Burns metric
- self-dual ($\Psi = 0$), $R = 0$, $R_{\mu\nu} \neq 0$, asymptotically flat:

$$g_{\mu\nu} = \delta_{\mu\nu} + \mathcal{O}(\|u\|^{-2}), \quad \|u\|^2 \propto \delta_{\mu\nu}x^\mu x^\nu \to \infty$$

- bulk theory: WZW_4 on Burns space

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- bulk theory: WZW_4 on Burns space

$\widetilde{\mathbb{C}}^2 = \mathbb{C}^2$ with origin replaced by $\mathbb{CP}^1$

$$\mathcal{S} = \frac{N}{8\pi^2}\int_{\widetilde{\mathbb{C}}^2} \partial\bar\partial K \wedge tr\left(g\partial g^{-1} \wedge g\bar\partial g^{-1}\right) - \frac{N}{24\pi^2}\int_{\widetilde{\mathbb{C}}^2\times[0,1]} \partial\bar\partial K \wedge tr\left(\widetilde{g}d\widetilde{g}^{-1}\right)^3$$

$$g : \widetilde{\mathbb{C}}^2 \to SO(8), \quad \frac{iN}{2\pi}\int_{\mathbb{CP}^1} \partial\bar\partial K = N \in \mathbb{Z}_+$$

Top down toy model

[Costello, Paquette, Sharma '22]

$$\mathcal{S} = \frac{N}{8\pi^2}\int_{\widetilde{\mathbb{C}}^2} \partial\bar{\partial}K \wedge tr\left(g\partial g^{-1} \wedge g\bar{\partial}g^{-1}\right)$$

$$-\frac{N}{24\pi^2}\int_{\widetilde{\mathbb{C}}^2\times[0,1]} \partial\bar{\partial}K \wedge tr\left(\widetilde{g}d\widetilde{g}^{-1}\right)^3$$

$$\to \frac{N}{8\pi^2}\int_{\mathbb{C}^2} \partial\bar{\partial}K \wedge tr\left(\partial\phi \wedge \bar{\partial}\phi - \frac{1}{3}\phi[\partial\phi, \bar{\partial}\phi]\right) + \mathcal{O}(\phi^4)$$

- look for perturbative solutions by setting $g = e^{\phi}$, $\widetilde{g} = e^{t\phi}$
- asymptotic states obey the wave equation on Burns space admitting a family of solutions:

$$\phi_a(z, \widetilde{\lambda}) = \sum_{k,\ell} \frac{1}{k!\ell!}\widetilde{\lambda}_1^k\widetilde{\lambda}_2^\ell \phi_a[k,\ell](z)$$

Top down toy model

[Costello, Paquette, Sharma '22]

$$\mathcal{S} = \frac{N}{8\pi^2}\int_{\widetilde{\mathbb{C}}^2} \partial\bar{\partial}K \wedge tr\left(g\partial g^{-1} \wedge g\bar{\partial}g^{-1}\right)$$

$$-\frac{N}{24\pi^2}\int_{\widetilde{\mathbb{C}}^2\times[0,1]} \partial\bar{\partial}K \wedge tr\left(\tilde{g}d\tilde{g}^{-1}\right)^3$$

$$\to \frac{N}{8\pi^2}\int_{\mathbb{C}^2} \partial\bar{\partial}K \wedge tr\left(\partial\phi \wedge \bar{\partial}\phi - \frac{1}{3}\phi[\partial\phi, \bar{\partial}\phi]\right) + \mathcal{O}(\phi^4)$$

- look for perturbative solutions by setting $g = e^{\phi}$, $\quad \tilde{g} = e^{t\phi}$
- asymptotic states obey the wave equation on Burns space admitting a family of solutions:

$$\phi_a(z, \tilde{\lambda}) = \sum_{k,\ell} \frac{1}{k!\ell!}\tilde{\lambda}_1^k\tilde{\lambda}_2^\ell \phi_a[k,\ell](z)$$

Holographic dictionary

$$\phi_a[k,\ell](z) \leftrightarrow J_a[k,\ell](z)$$

$J_a[k,\ell](z)$ 2D chiral algebra generators

Top down toy model

[Costello, Paquette, Sharma '22]

$$\mathcal{S} = \frac{N}{8\pi^2}\int_{\widetilde{\mathbb{C}}^2} \partial\bar{\partial}K \wedge tr\left(g\partial g^{-1} \wedge g\bar{\partial} g^{-1}\right) - \frac{N}{24\pi^2}\int_{\widetilde{\mathbb{C}}^2\times[0,1]} \partial\bar{\partial}K \wedge tr\left(\widetilde{g}d\widetilde{g}^{-1}\right)^3$$

$$\rightarrow \frac{N}{8\pi^2}\int_{\mathbb{C}^2} \partial\bar{\partial}K \wedge tr\left(\partial\phi \wedge \bar{\partial}\phi - \frac{1}{3}\phi[\partial\phi, \bar{\partial}\phi]\right) + \mathcal{O}(\phi^4)$$

- look for perturbative solutions by setting $g = e^{\phi}$, $\widetilde{g} = e^{t\phi}$
- asymptotic states obey the wave equation on Burns space admitting a family of solutions:

$$\phi_a(z, \widetilde{\lambda}) = \sum_{k,\ell} \frac{1}{k!\ell!}\widetilde{\lambda}_1^k \widetilde{\lambda}_2^\ell \phi_a[k,\ell](z)$$

$ij \rightarrow a$

4D perturbative gluon amplitudes on Burns space $\longleftrightarrow$ 2D OPE of $J_{ij}[\widetilde{\lambda}](z)$

- two-point $\quad A(1,2) = -\frac{N}{z_{12}^2}J_0\left(2\sqrt{\frac{[12]}{z_{12}}}\right) tr\left(T_{a_1}T_{a_2}\right)$ $\quad$ matches identity contribution to OPE
- Matching also established for the three-point amplitudes/correlators

[`gravity` in Costello, Paquette, Sharma '23]

Summary

- Gravity at $\Lambda = 0$ in (3+1)D has a rich symmetry structure in the IR — tower of soft theorems — gravitational memory
- Subleading soft theorem (tree level) — Virasoro symmetry
- S-matrix in boost 'basis' — celestial amplitude
- Truncation of Einstein equations reorganize into hierarchy of recursive differential equations — w_∞ on phase space
- Celestial symmetries (eg. leading and subleading soft symmetries) emerge in flat limit of CFT_3
- Connections to chiral algebras and twistors — underlying mathematical structures??

Outlook

- Bulk/geometric interpretation of flat space limit
- AdS boundary conditions - TTbar deformations?
- Central extensions
- Entanglement entropy; black holes?
- Spectrum of CCFT; conformal block decompositions
- Connections to all Λ via AdS/dS slicing
- Top down constructions

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Connections to scattering amplitudes

- OPE from EFT
- Higher-derivative & loop corrections to OPE

[He, Jiang, Ren, Spradlin, Taylor, Volovich, Zhu….]

- Double copy constructions [Casali, Puhm, Sharma,…]
- IR divergences
- Self-dual amplitudes and black holes
- Discrete basis

Asymptotic symmetries and Carrollian FT

String theory, BFFS, …

[Adamo, Ball, Cotler, Crawley, Donnay, Fan, Fiorucci, Guevara, He, Kapec, Mason, Mitra, Narayana, Ruzziconi, Salzer, Storminger, Sharma, Tropper, Wang…]

Thank you!