What we don't know about holography

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Outline

Some history

Historically, holography and black hole physics were intimately related, sharing a 50 year long history (if one counts from the Phys Rev. D paper of Jacob Bekenstein in 1973).

Though originally viewed as a a mysterious property of black holes, during the past 30 years holography was generalized and extended in numerous directions and even proposed as a general principle of gravitational physics (t'Hooft, Beckenstein and Bousso bounds etc). Under the name of "gauge-gravity duality", holographic ideas and models were used to make predictions regarding various field theories and even models in condensed matter physics. More recently, ideas inspired by the Ryu-Takayanagi entropy formula and the bulk reconstruction program led to "it from qubit" type proposals aiming to derive models of bulk matter and geometry directly from quantum information theory considerations.

This led to an impressive amount of scientific effort, papers and even blogging, newspaper articles, youtube, radio & TV appearances.

Despite all this, the subject still abounds in open problems, conflicting proposals, conjectures and speculation. We still lack precise answers to numerous questions – and especially answers that might one day satisfy mathematical physicists and mathematicians.

- We still lack a convincing non-perturbative definition of quantum gravity in spacetime dimensions ≥ 4. In string theory, we lack a definition of M-theory.
- As a result, we cannot yet transform holography proposals into actual conjectures – since formulating a proper duality conjecture requires that we have independent definitions of *both sides* of the duality as well as a precise proposal for the the full equivalence map between them. Even in various simplifying limits, we lack precise expansions in the limiting small parameter which could be tested for asymptotic bounds or resummed.
- Most tests and explicit proposals concern the large N limit, usually also at large t'Hooft coupling (strong boundary theory coupling, when bulk gravity becomes classical). "Stringy corrections" (at weak string coupling but finite α') were not considered or computed systematically in most cases. In these restrictive regimes, most precision tests concern BPS quantities in supersymmeric theories – and there are very few results which could be viewed as precision tests in non-supersymmetric or condensed matter cases. The first orders of effective field theories dominate the literature.
- For these and other reasons, we do not understand the status of speculations regarding the "holographic principle".
- We don't know how states of the bulk theory should map to the boundary far away from the background approximation of the bulk: Wheeler's "bags of gold", fully accounting for boundary entropy, the correct relation between bulk and boundary state spaces.

- We still lack a convincing resolution of the black hole information paradox. The most detailed program in that direction (the blackhole microstate program) has not yet been successful.
- We do not understand the issue of black hole horizon structure.
- Approaches to thermal boundary theories tend to rely on Euclidean Quantum Gravity, which is known to be problematic. In its naive form, EQG is *not* a well-defined theory and its predictions cannot be trusted beyond leading orders in the saddle point approximation.
- The *general* formulation of "holographic dualities" remains obscure. What are the precise geometries and classes of theories which can support such dualities and what is their general formulation for strongly interacting bulk theories ? When does such a duality makes sense when one includes the non-perturbative regime of quantum gravity ? What is the precise and general duality map for interaction terms between bulk and boundary theories ?
- What is the full "quantum RT formula", which correctly includes *all* quantum corrections ? Does the notion of "quantum minimal volume" makes sense non-perturbatively in the bulk ?
- How can we fully reconstruct the bulk theory ? How does this work for all pure and mixed states of the boundary theory ? Can we fully and unambiguously reconstruct the bulk geometry ? If so, for what class of boundary theories and what class of boundary geometries ?

On "It from qubit" ideology

- The idea of somehow "deriving" matter models from information models is reminiscent of ancient philsophical ideas of the idealist current (Plato, Descartes, German idealism etc.)
- Information theory uses discrete models – this includes quantum information theory, which uses finite-dimensional Hilbert spaces (it assumes a finite number of qubits). Applying these to gravity requires a discrete model of spacetime. Applying these to bosonic fields requires adaptation (think of Bose-Einstein condensates).
- Quantum models of information *are* models of quantum systems. In this sense, one is not deriving matter from information but translating the description of a quantum system into that of another. This can be viewed as a re-encoding or learning process (the latter via RG flow).
- Most applications to date involved entropy, error correcting codes and various notions of complexity. One's notion of complexity depends markedly on the problem under consideration; there seems to exist no notion of complexity which is "canonical" or "universal".
- Most applications only reconstruct discrete/combinatorial approximations to bulk geometry, such as quantum tensor networks or other kinds of graphs. This largely stems from the fact that the von Neumann and Renyi entropies are far from sufficient to uniquely characterize mixed states.
- A deeper approach might attempt to model radial RG flow as a (singular) classical or quantum learning machine in the spirit of Watanabe's SLT.

Some ideas and problems

- The algebraic structure of EQG. A proper formulation of Euclidian quantum gravity may be expected to obey the rules of the monoidal functor formalism, where one sums over smooth cobordisms of different topologies. This should reflect in algebraic operations on the corresponding state space, leading to a rich structure. Identifying this algebraic structure may provide a guide to fully reconstructing the bulk theory from the boundary theory. This program should be easier in dimensions ≥ 5 but much harder in dimension 4.
- Give a mathematical characterization of the most general BPS microstate geometries in Bena's program and use it for entropy counting. This requires describing all solutions with the required asymptotics and global/local supersymmetry for each dimension of interest, *without computing them explicitly*, relying on the theory of stratified G-structures and other techniques connected to spin geometry. There exist mathematical results on G-spaces which seem to be relevant to this problem.
  - A characterization of certain generalizations of Bena-Warner solutions exists in terms of spinors on Kleinian surfaces with prescribed asymptotics at certain points.
  - A nontrivial example of this involves the $\mathrm{AdS}_3 \times X_7/\mathrm{CFT}_2$ correspondence with $\mathcal{N} = (1, 0)$ susy, where results of CIL and E. Babalic could be used to extract a characterization of the M-theory lifts of the most general relevant solutions.

- Connections to Bayesian learning theory. Describe the holographic RG flow as a Bayesian learning machine. Via the Wegner-Morris interpretation of Polchinski's RG, this would likely connect it with an infinite-dimensional difusion-convection process and a corresponding Langevin type process. This would require a very broad extension of the ideas of Wegner and Morris but may lead to better understanding of various issues in the microstate and bulk reconstruction programs and might flesh out some ideas put forth by "it from qubit" proposals. The theory of stochastic processes seems to be relevant to this circle of ideas.
- The Bayesian approach to increasingly refined and sequential measurement of complex systems seems well-suited to describe numerous questions in information-theoretic approaches to holography but its applications to quantum systems are insufficiently developed. The "it from qubit" ideology might be viewed as a particular case of a theory of "quantum Bayesian learning", which seems to underline the process of sequential measurement of quantum systems.