Data Analysis and Filter Optimization for Online Pulse-Amplitude Measurement

A Case Study on High-Resolution X-ray Spectroscopy

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ICTP - Multidisciplinary Laboratory

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The Abdus Salam
International Centre
for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

Trieste

Main Areas of Expertise

- Read-out electronics and high performance **Digital Signal Processing**.
- Advanced **FPGA Design** and **Programmable Systems-on-Chip**.
- Reconfigurable virtual instrumentation for **Particle Detectors**.
- Novel architectures for **Supercomputing Based on FPGA**.
- Instruments and methods for **X-Ray Imaging and Analytical Techniques**.

Outline

- Introduction
- Pulsed signals: Description levels
- Processing chain: Detector/Sensor, Preamplification, Pulse shaping, Data acquisition, transmission, . . .
- Digital Pulse Processor (DPP): Main functional blocks, Features extraction, Dead times, Pattern recognition, . . .
- DPP Optimization
    - Data analysis
    - Pulse modeling
    - Digital Penalized Least Mean Squares (DPLMS) method for filtering optimization
- Discussion and Conclusions

More technical details in

sensors MDPI

Article

Data Analysis and Filter Optimization for Pulse-Amplitude Measurement: A Case Study on High-Resolution X-ray Spectroscopy

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Digital Penalized LMS method for filter synthesis with arbitrary constraints and noise

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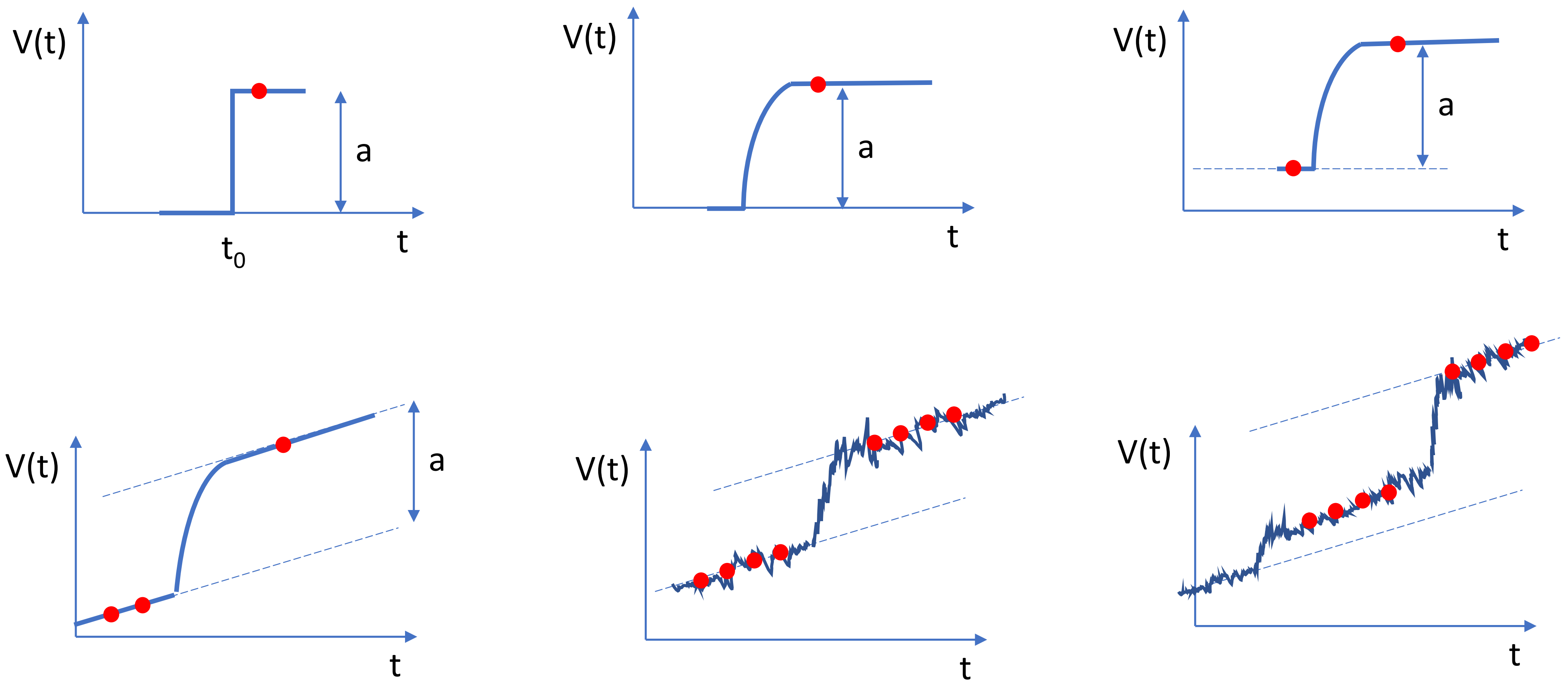
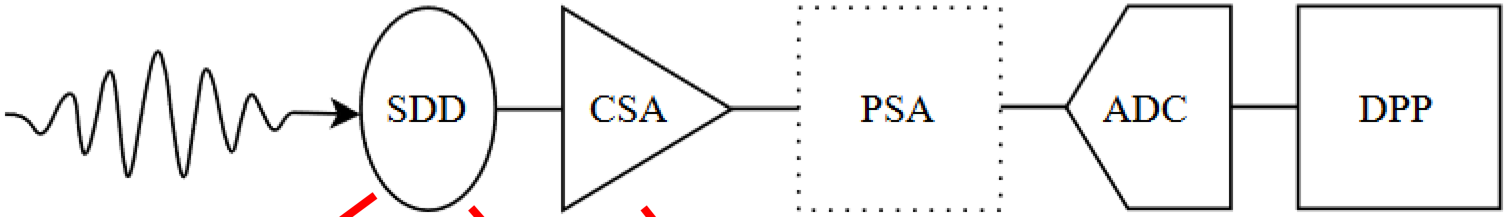
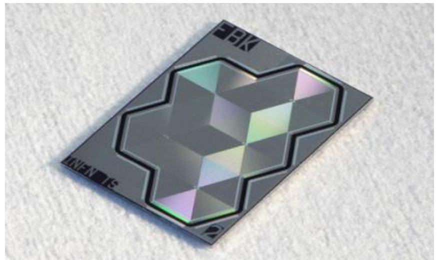
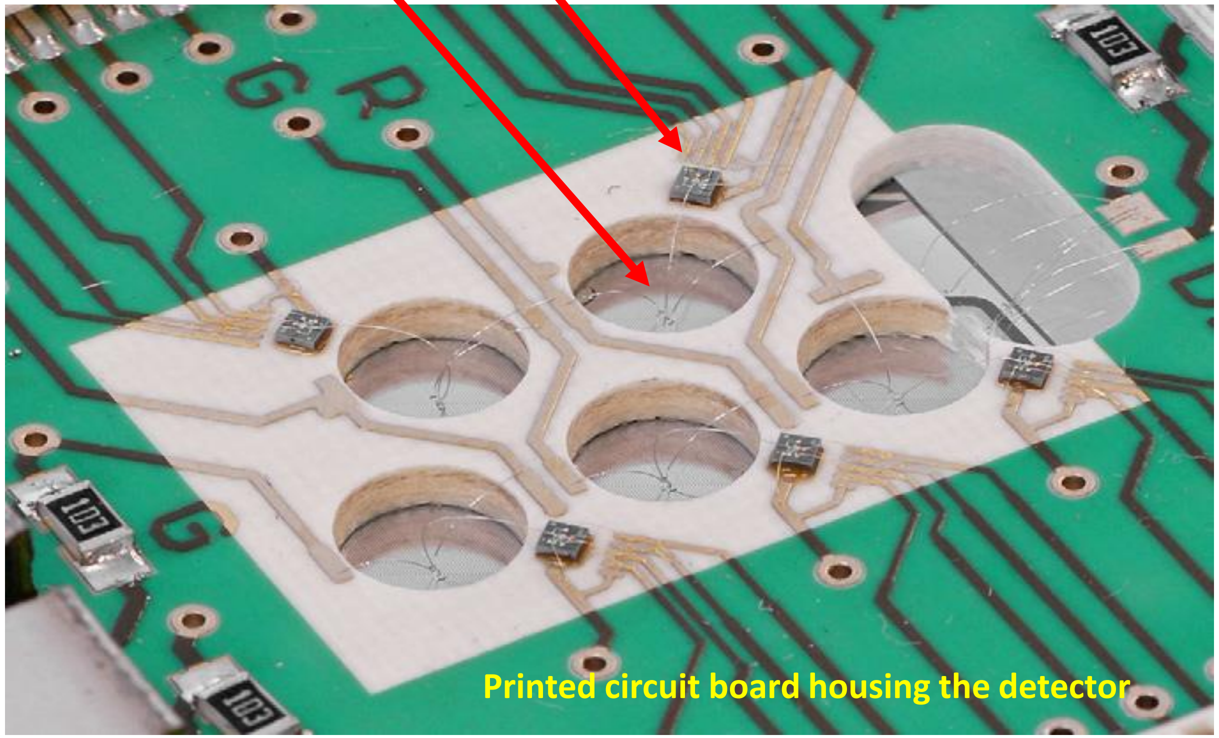
Pulsed signals: Description levels



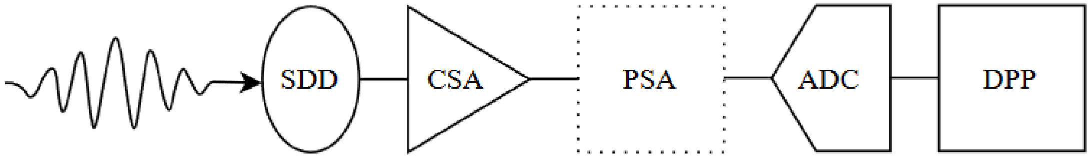
Processing chain

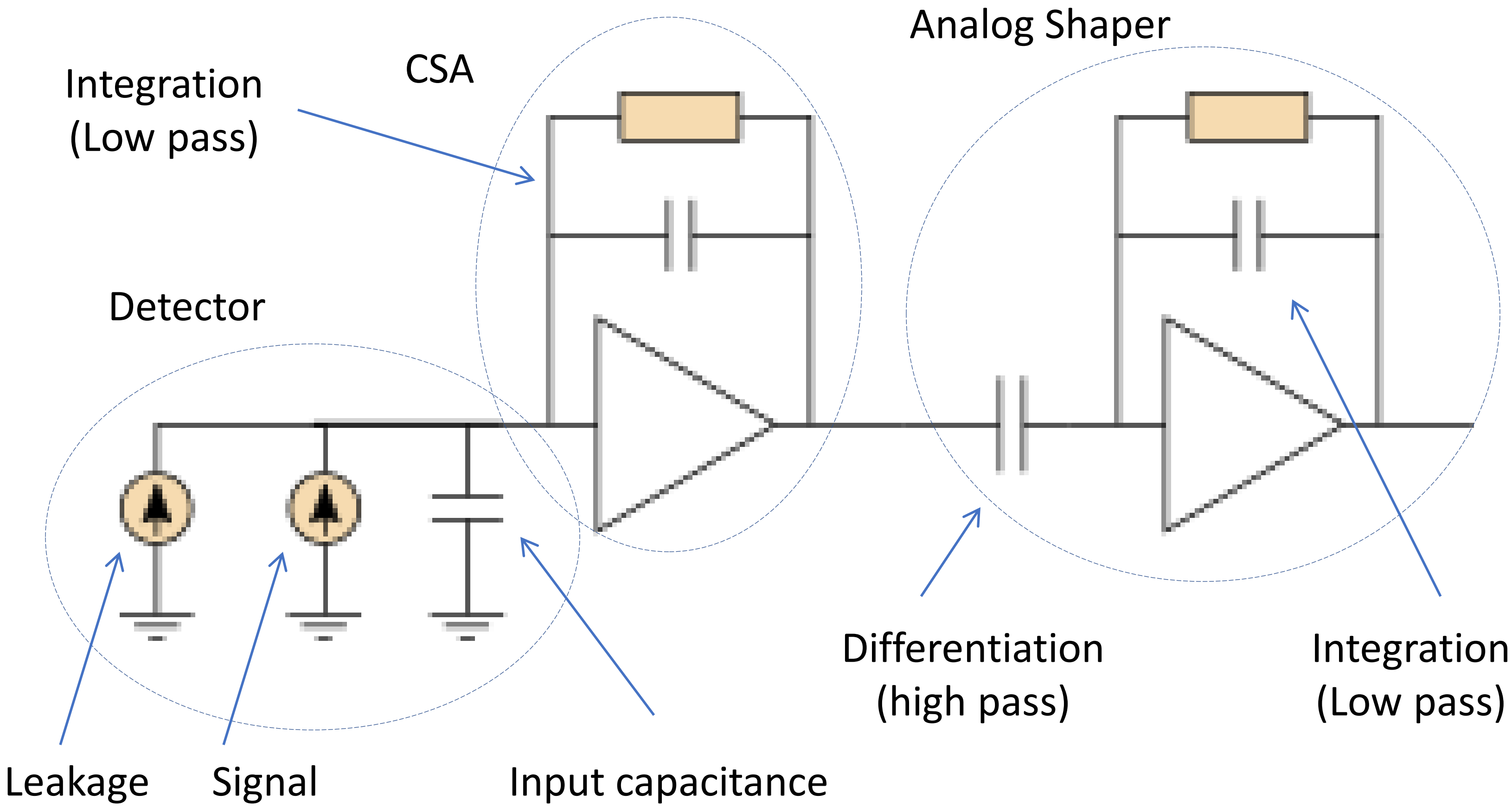




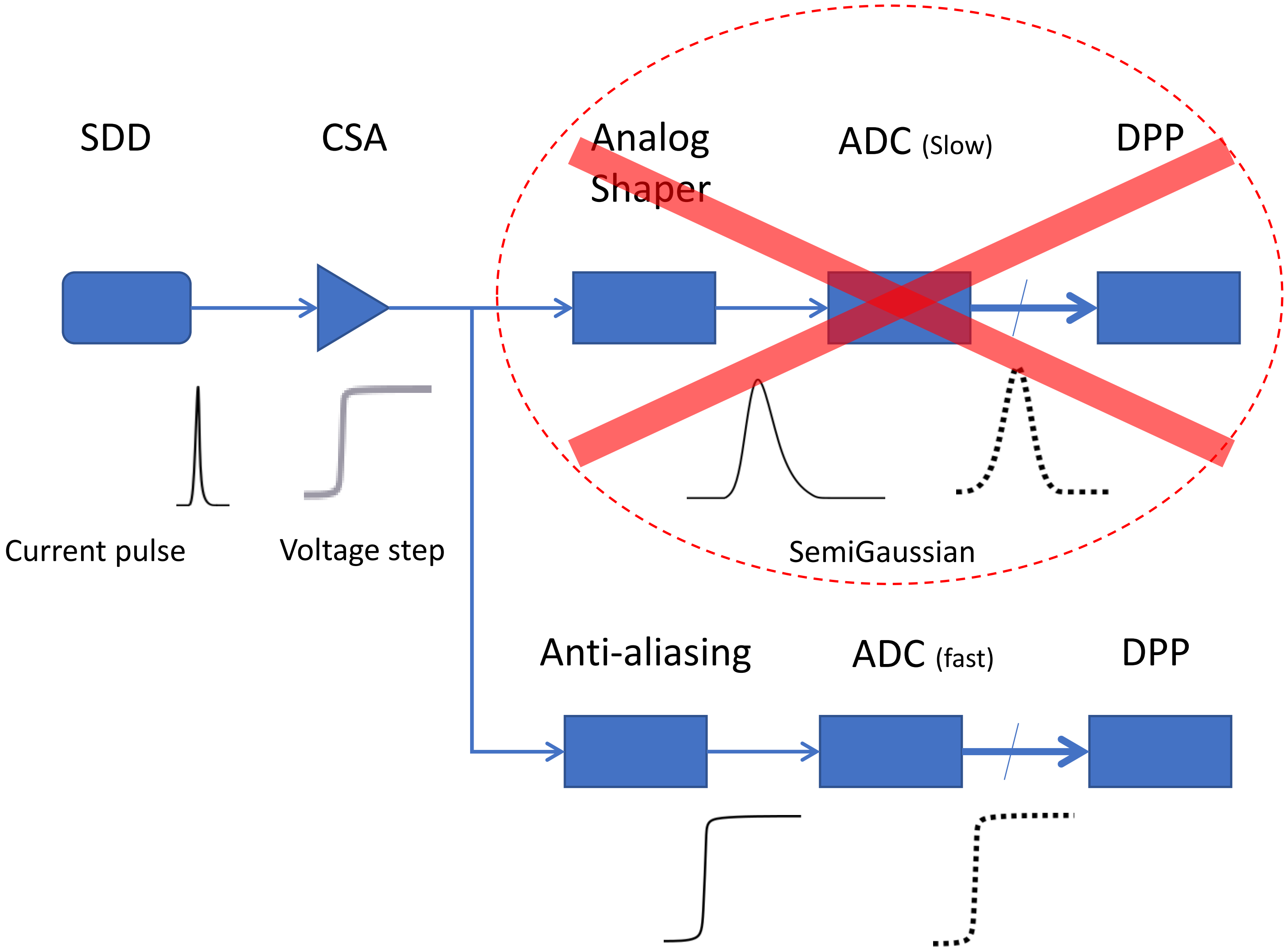


Detector, CSA, Pulse Shaper





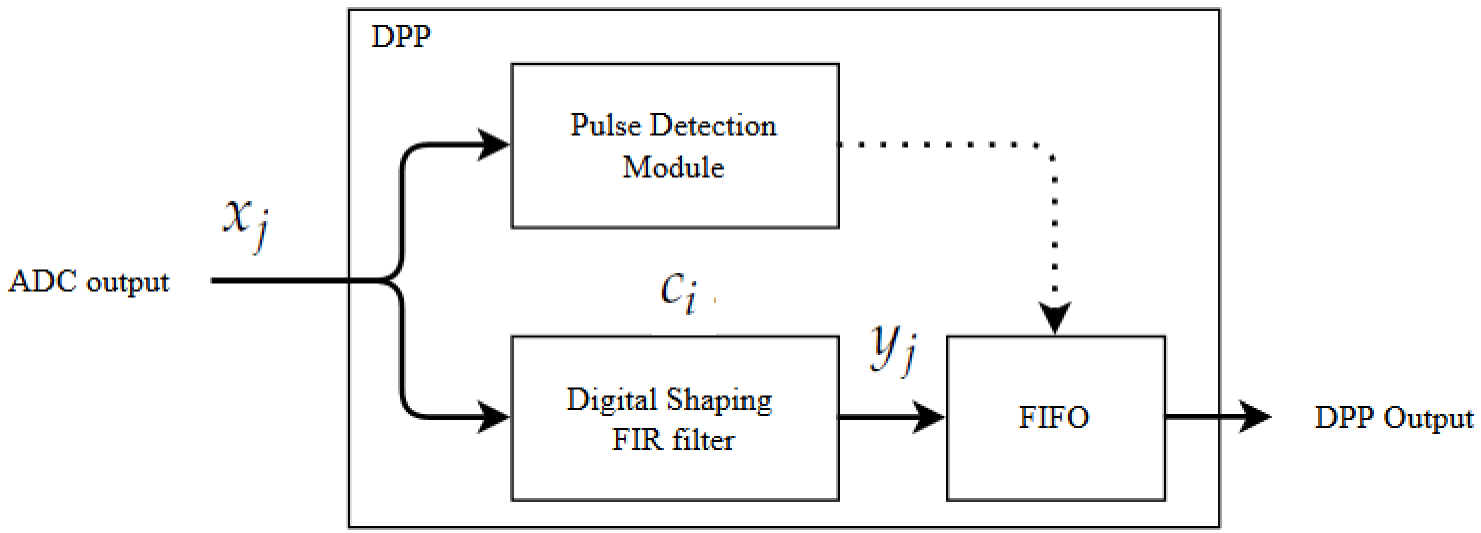
Pulse Processing Chain



Digital Pulse Processor (DPP)

Main functional blocks, Features extraction, Dead times, Pattern recognition, . . .

Minimal version

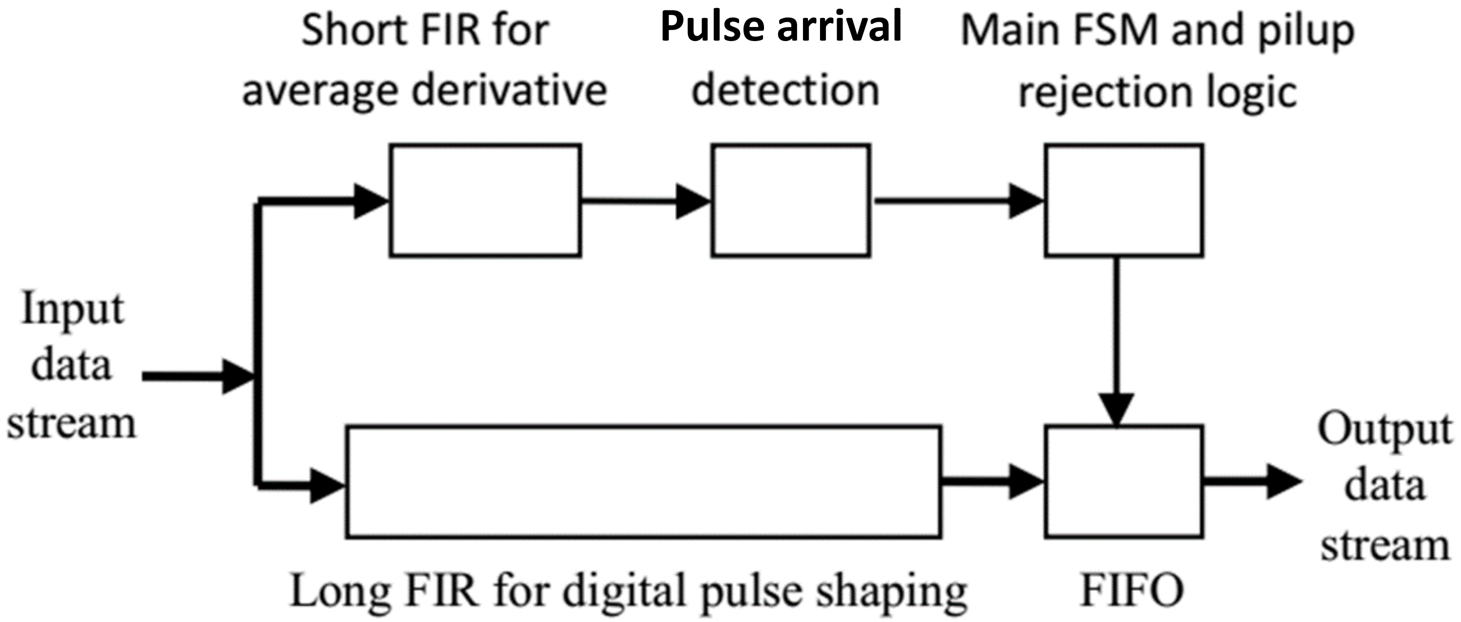


convolution

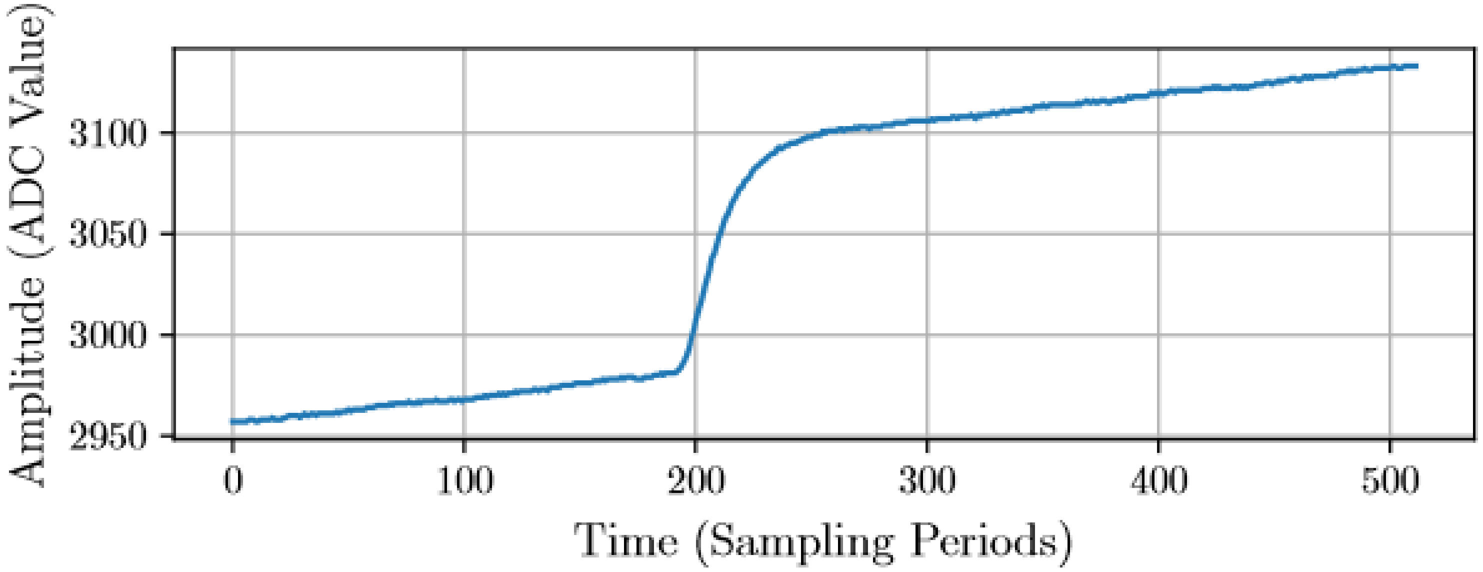
$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i}$$

Digital Pulse Processing Strategy for High-Resolution and High-Performance Amplitude Measurement

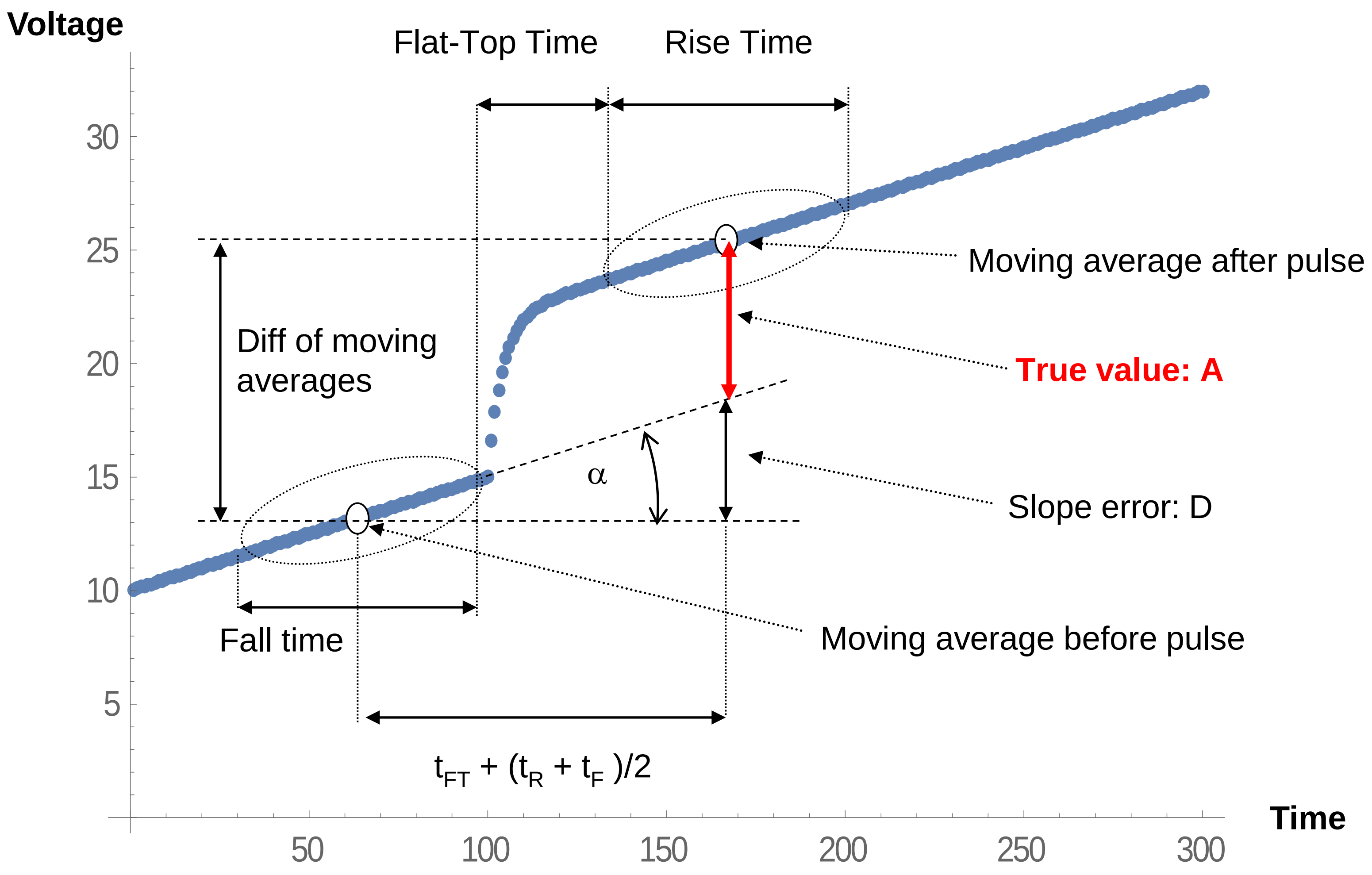
$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i}$$



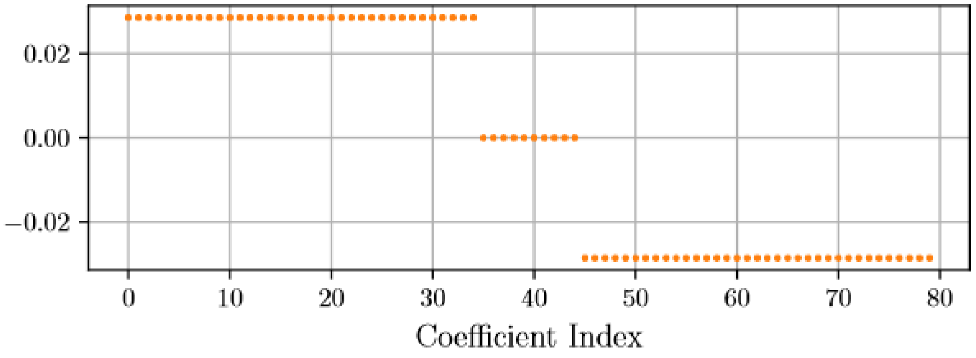
A typical experimental pulse

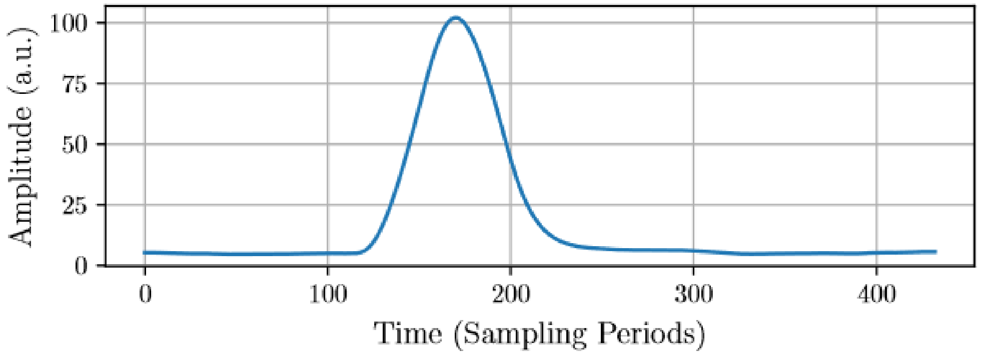


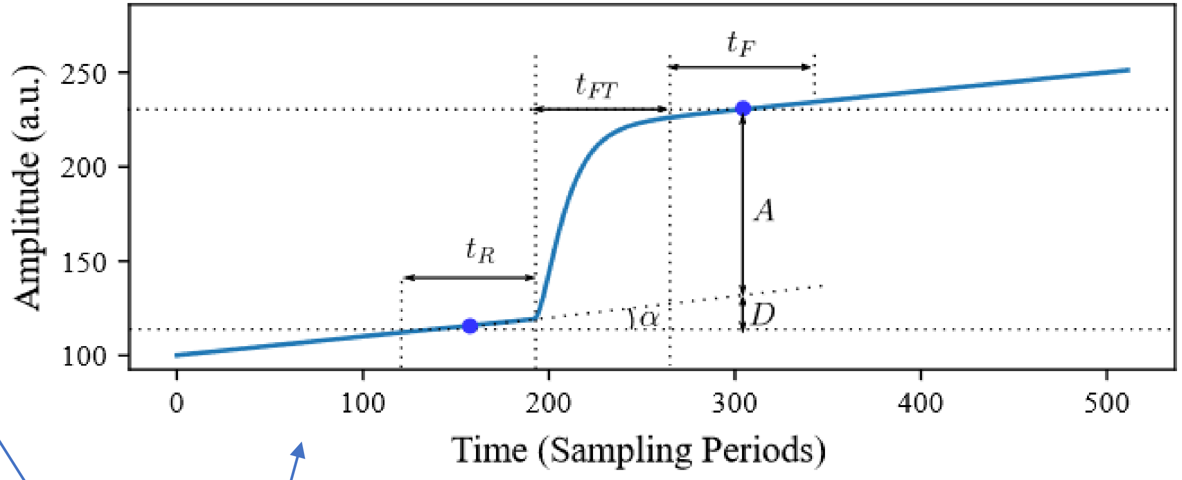
Pulse amplitude measurement



A simple trapezoidal shaper







$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i}$$

convolution

$$tan\ \alpha = \frac{D}{\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F}$$

$$tan\ \alpha \approx \sum_{i=0}^{t_R-1} -6\left(\frac{1+t_R-2i}{t_R^3 - t_R}\right)x_i$$

Errata corrige: t_R ⇄ t_F

After some algebra . . .

$$D = \sum_{i=0}^{t_R-1} -6\left(\frac{1+t_R-2i}{t_R^3-t_R}\right)\left(\tfrac{1}{2}t_R + t_{FT} + \tfrac{1}{2}t_F\right)x_i$$

$$A = \frac{1}{t_F}\sum_{i=t_R+t_{FT}}^{t_R+t_{FT}+t_F-1} x_i - \frac{1}{t_R}\sum_{i=0}^{t_R-1} x_i - \sum_{i=0}^{t_R-1} -6\left(\frac{1+t_R-2i}{t_R^3-t_R}\right)\left(\tfrac{1}{2}t_R + t_{FT} + \tfrac{1}{2}t_F\right)x_i$$

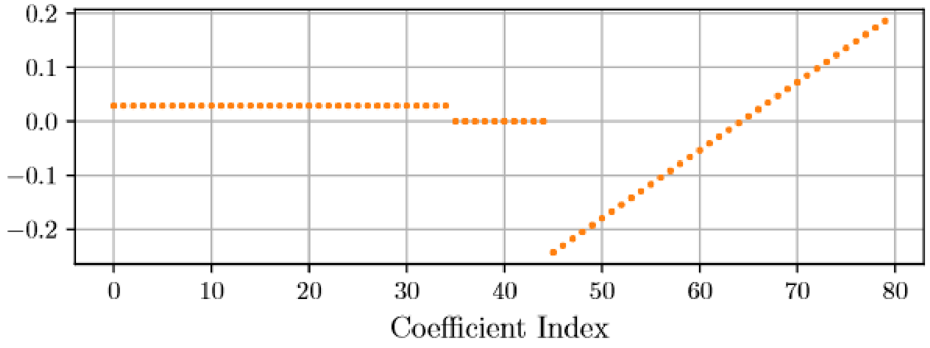
$$A = \sum_{i=t_R+t_{FT}}^{t_R+t_{FT}+t_F-1} \frac{1}{t_F}x_i + \sum_{i=0}^{t_R-1}\left[-\frac{1}{t_R} + 6\left(\frac{1+t_R-2i}{t_R^3-t_R}\right)\left(\tfrac{1}{2}t_R + t_{FT} + \tfrac{1}{2}t_F\right)\right]x_i$$

← Linear combination

$$c_i = \begin{cases} \frac{1}{t_F}, & 0 \leqslant i < t_F; \\ 0, & t_F \leqslant i < t_F + t_{FT}; \\ -\frac{1}{t_R} + 6\left(\frac{1+t_R-2i}{t_R^3-t_R}\right)\left(\tfrac{1}{2}t_R + t_{FT} + \tfrac{1}{2}t_F\right), & t_F + t_{FT} \leqslant i < t_F + t_{FT} + t_R; \end{cases}$$

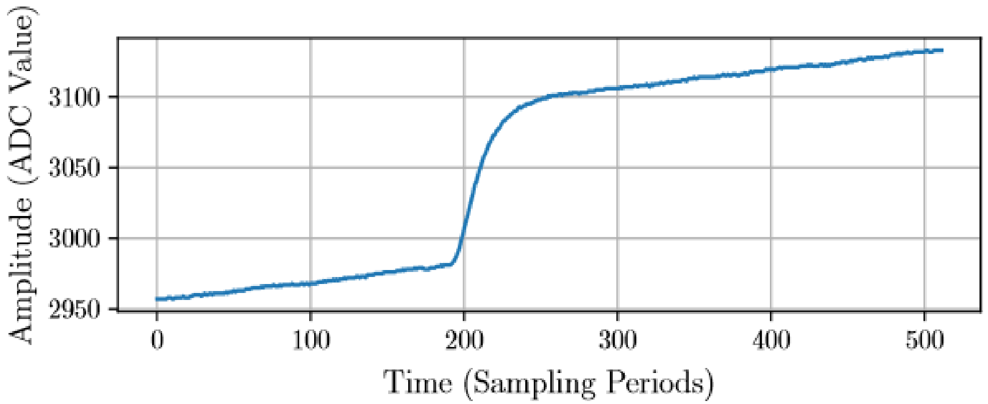
Errata corrige: t_R ⇄ t_F

Geometrically Derived FIR Filter

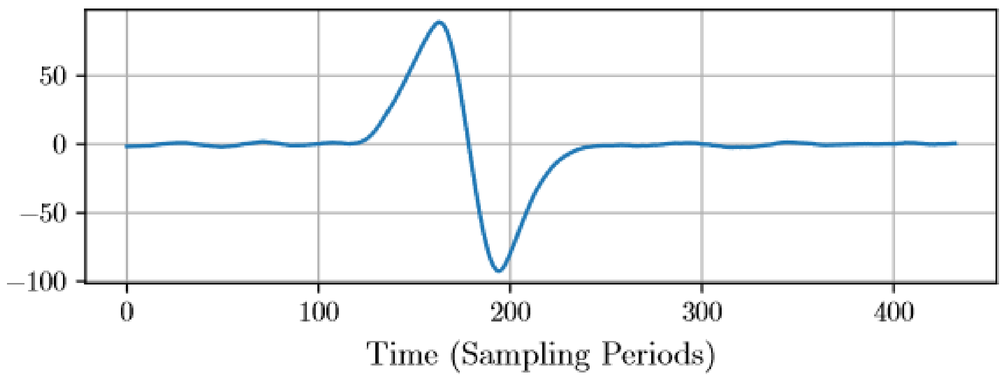


$$c_i = \begin{cases} \frac{1}{t_F}, & 0 \leqslant i < t_F; \\ 0, & t_F \leqslant i < t_F + t_{FT}; \\ -\frac{1}{t_R} + 6\left(\frac{1+t_R-2i}{t_R^3 - t_R}\right)\left(\frac{1}{2}t_R + t_{FT} + \frac{1}{2}t_F\right), & t_F + t_{FT} \leqslant i < t_F + t_{FT} + t_R; \end{cases}$$

Input pulse



Output pulse



DPP Optimization

Pulse modeling

$$V(t) = \begin{cases} 0, & t \leqslant t_0; \\ A(1 - e^{\frac{-(t-t_0)}{\tau}}), & t > t_0; \end{cases}$$

$$V(t) = \begin{cases} B_0 + B_1 t + n(t), & t \leqslant t_0; \\ A(1 - e^{\frac{-(t-t_0)}{\tau}}) + B_0 + B_1 t + n(t), & t > t_0; \end{cases}$$

$$x_i = \begin{cases} B_0 + B_1 i + n_i, & i \leqslant t_0; \\ A(1 - e^{\frac{-(i-t_0)}{\tau}}) + B_0 + B_1 i + n_i, & i > t_0; \end{cases}$$

DPP Optimization

Pulse modeling

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

Deterministic component
(ideal pulse)

$$S_i = \begin{cases} B_0 + iB_1, & i \leq t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1, & i > t_0 \end{cases}$$

Increasing information

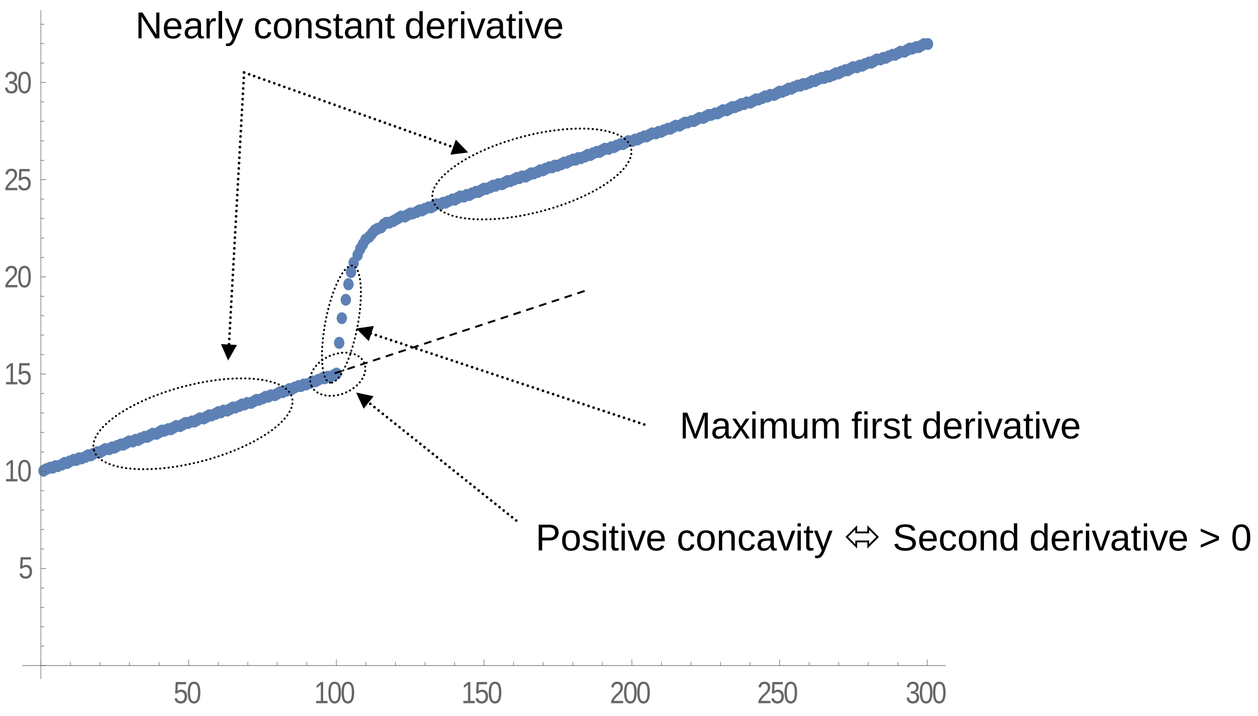
Stochastic component

$$S_i = \begin{cases} 0, & i \leq t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right), & i > t_0 \end{cases}$$

$$S_i = \begin{cases} 0, & i \leq t_0 \\ A, & i > t_0 \end{cases}$$

Deterministic component

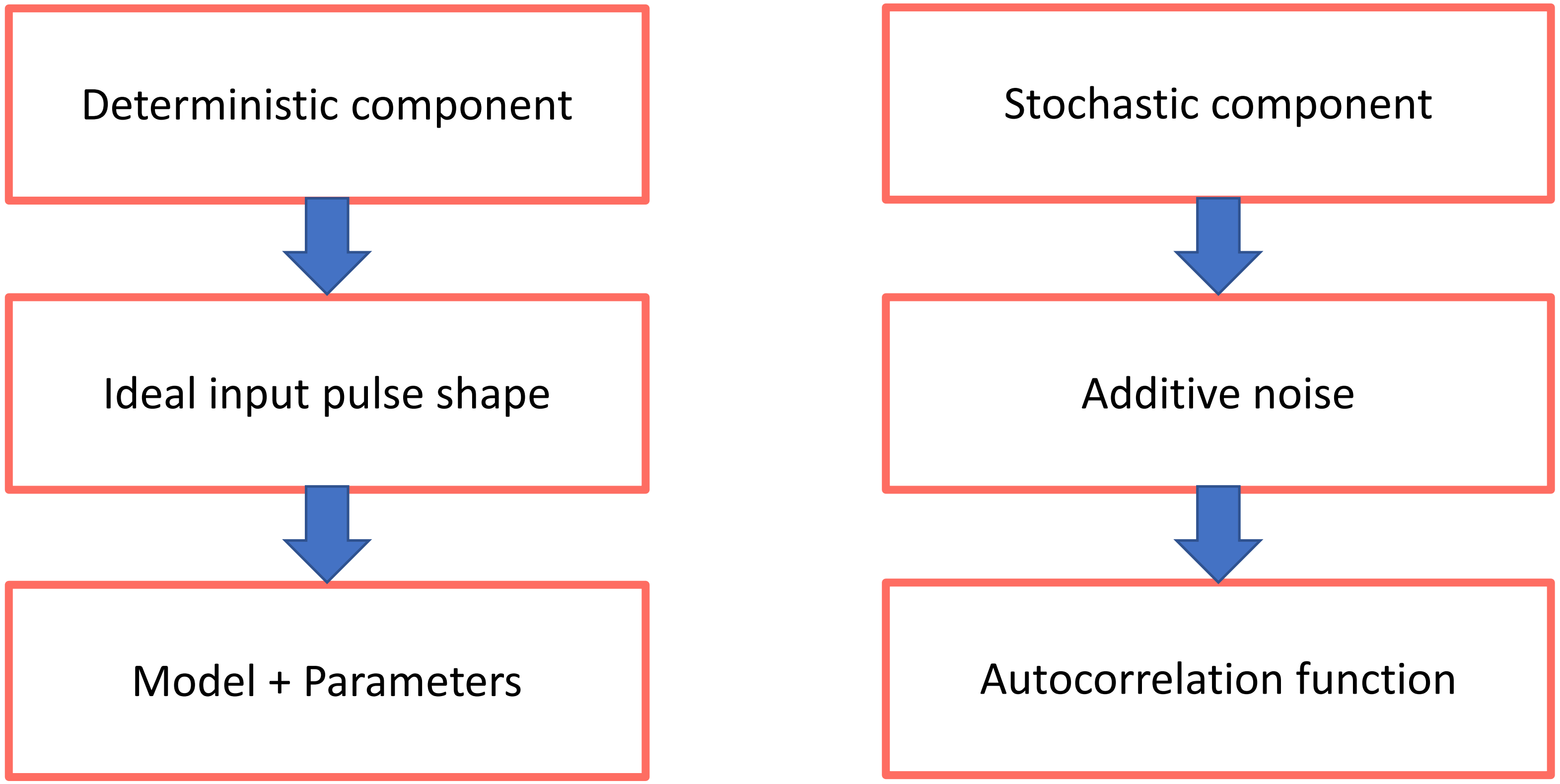
Digital Pulse Processing: Detecting Arrival Time



A short FIR can compute different discrete derivatives

FIR Design and Optimization

Input signal analysis



FIR Design and Optimization

Input pulse modeling I

The ideal case corresponding to a single photon detection is represented by the step function S_i

$$S_i = \begin{cases} 0, & i \le t_0 \\ A, & i > t_0 \end{cases}$$

The finite frequency response of the CSA determines a limited rise time that could be modeled (1st aprox) as an exponential growth

$$S_i = \begin{cases} 0, & i \le t_0 \\ A(1 - e^{-(i-t_0)/\tau}), & i > t_0 \end{cases}$$

A constant detector leakage current determines a baseline with a steady slope and a variable offset on top of which the signal segment must be processed

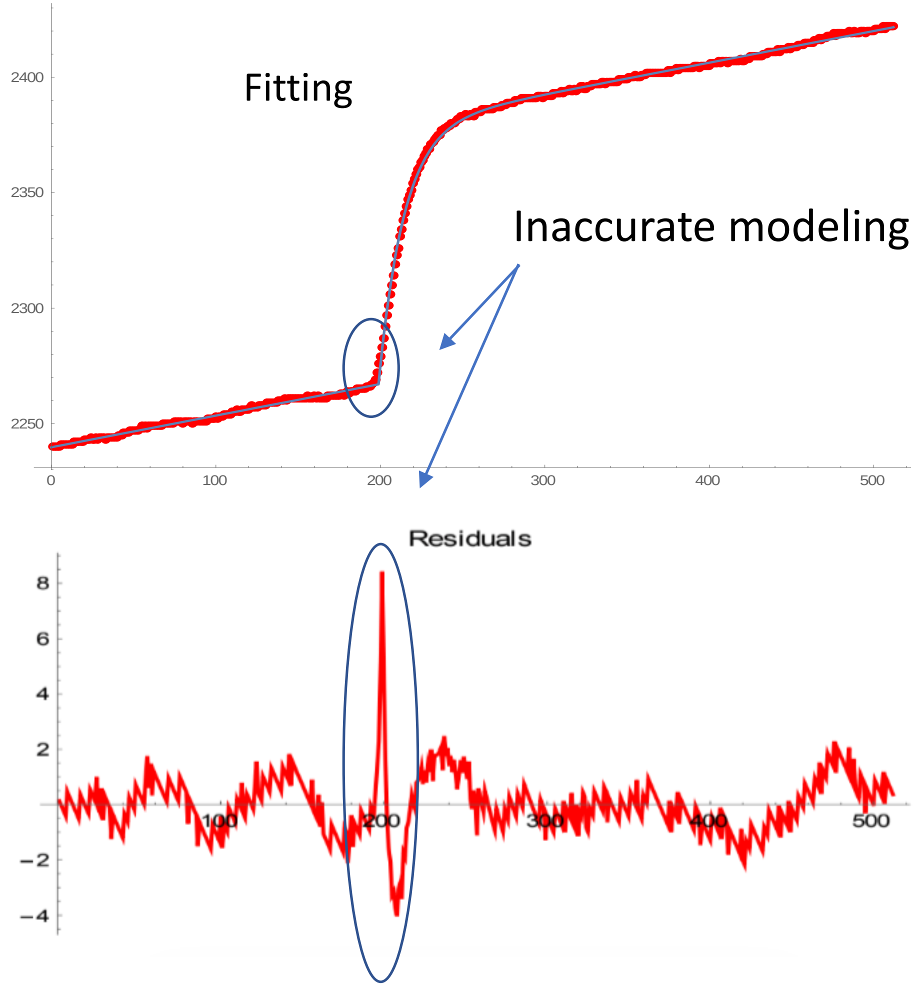
$$S_i = \begin{cases} B_0 + iB_1, & i \le t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1, & i > t_0 \end{cases}$$

Several sources of noise will contribute with an additive spurious signal n_i that degrades the voltage step measurement

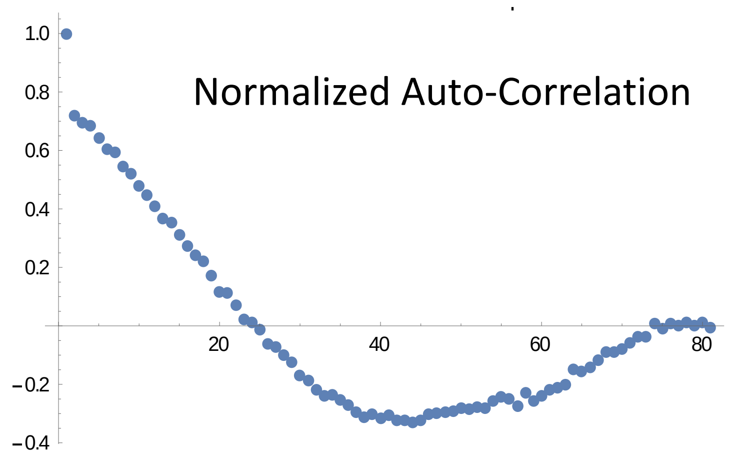
$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \le t_0 \\ A\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

FIR Design and Optimization

Input noise characterization



$$ACF(j) = \frac{\sum_{i=1}^{N-j} x_i \, x_{i+j}}{\sum_{i=1}^{N-j} x_i^2}$$



Some statistic results from the extracted parameters after fitting 1447 segments with the special bi-exponential function.

The proposed signal model has five (quite independent) parameters

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A_0\left(2\left(1 - e^{-(i-t_0)/\tau}\right) - \left(1 - e^{-2(i-t_0)/\tau}\right)\right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

(1) Amplitude,

(2) Arrival Time,

(3) Exponential Time,

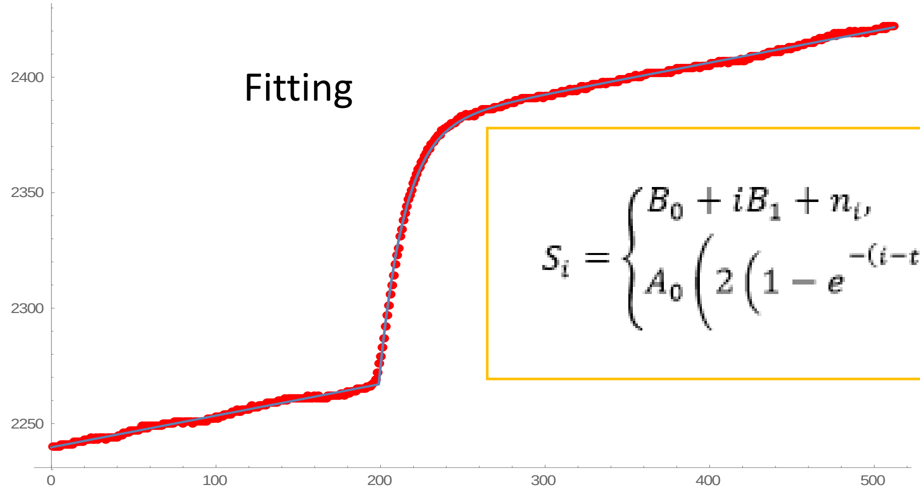
(4) Offset,

(5) Slope Coefficient

FIR Design and Optimization

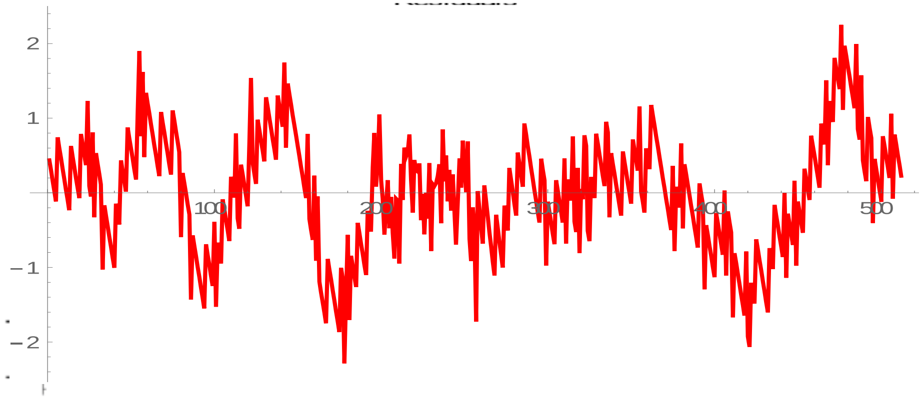
Input noise characterization

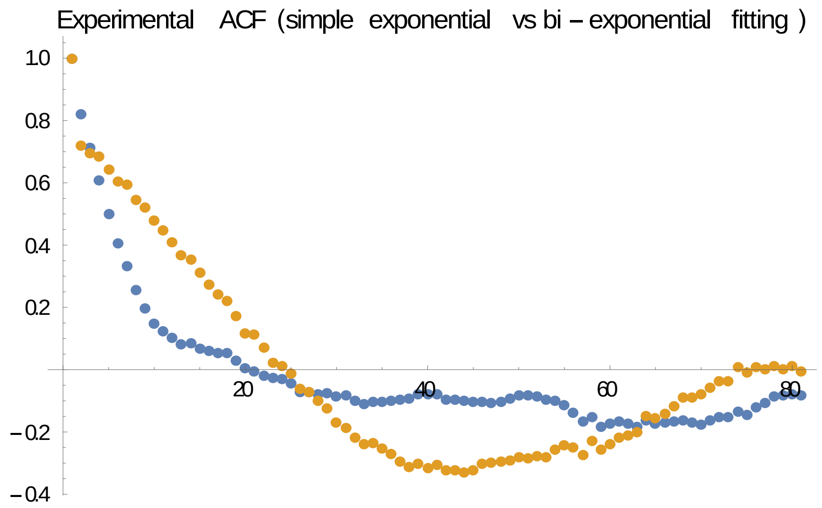
Fitting



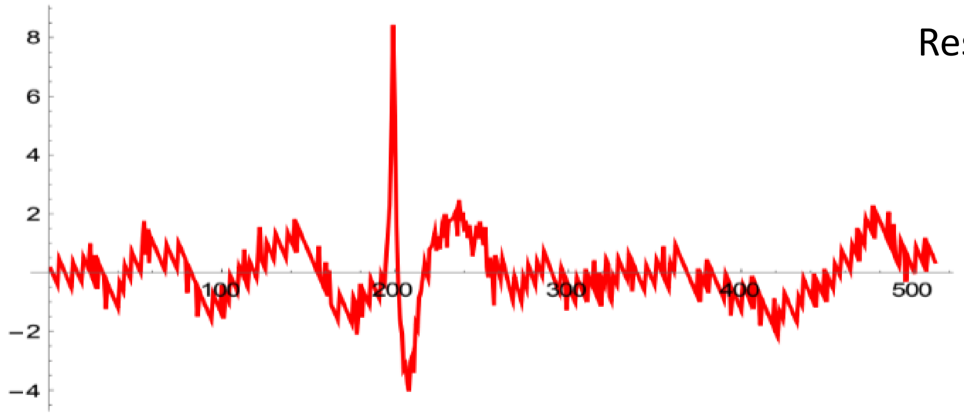
$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \leq t_0 \\ A_0\left(2\left(1 - e^{-(i-t_0)/\tau}\right) - \left(1 - e^{-2(i-t_0)/\tau}\right)\right) + B_0 + iB_1 + n_i, & i > t_0 \end{cases}$$

Residuals (noise)

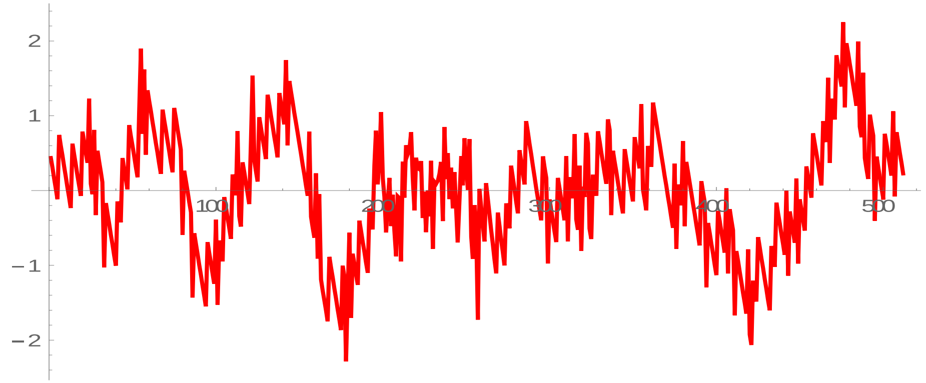




Pulse models comparison

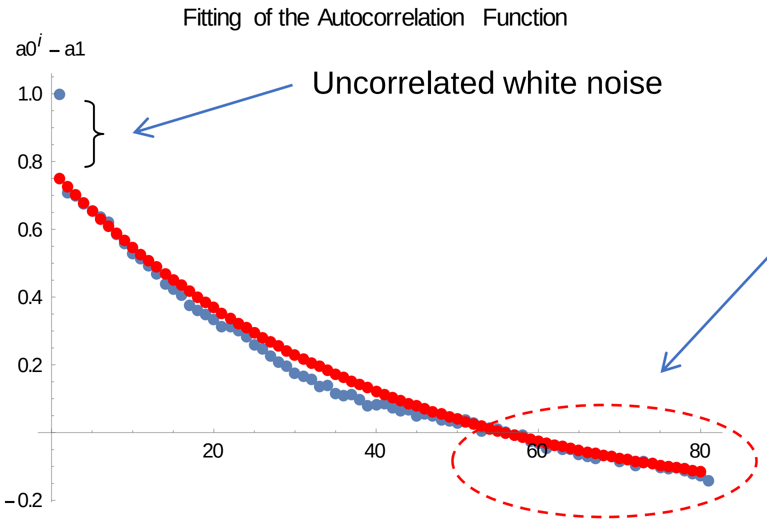


Residuals (noise)

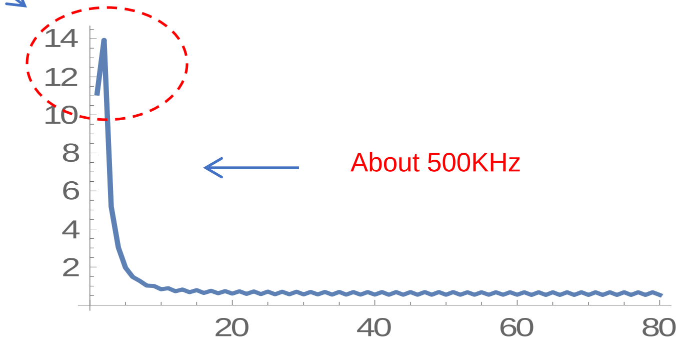


	Exponential Model	Bi-Exponential Model
Mean quadratic residuals	6201	5914
Mean peak-to-peak residuals	13.7	6.6
Mean Akaike information criterion	1720	1397

Autocorrelation model



Negative values determine a peak in the PSD
(via the Winner-Khintchine theorem)



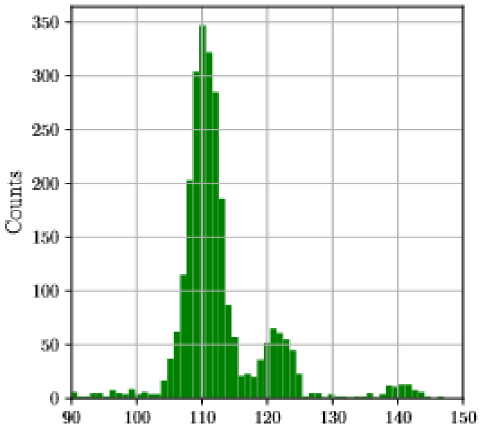
$$ACF(i, a_0, a_1) = \begin{cases} 1 & , \quad i = 0 \\ {a_0}^i + a_1 & , \quad i \geq 1 \end{cases}$$

In this case the average normalized ACF can be approximated with $\boldsymbol{a_0}$***=0.965*** and $\boldsymbol{a_1}$***=-0.2***

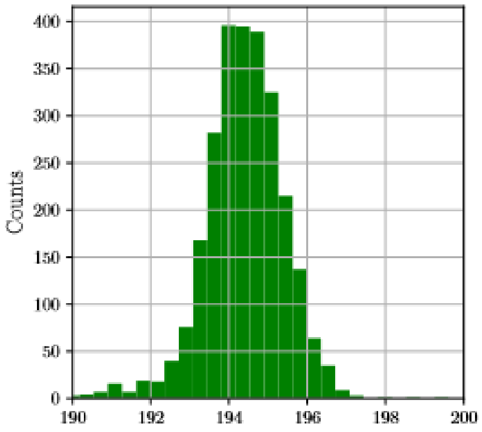
Histograms of fitting parameters corresponding to the bi-exponential model.

They looks nice but . . .

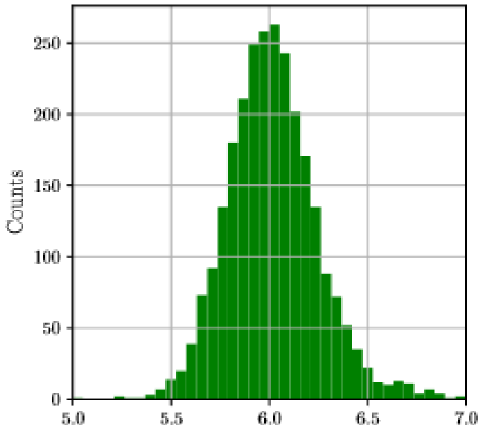
. . . scattered plots and correlation distances between pair of parameters may reveal non idealities of the DAQ system.



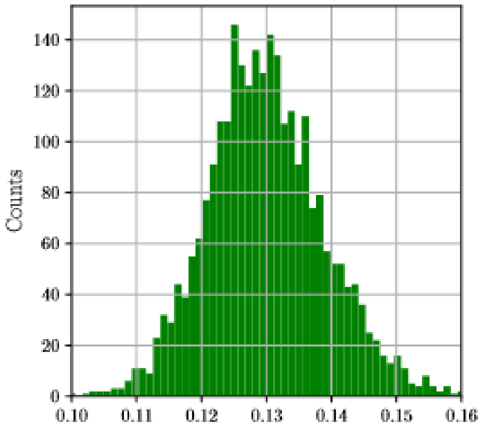
(a) Amplitude (A)



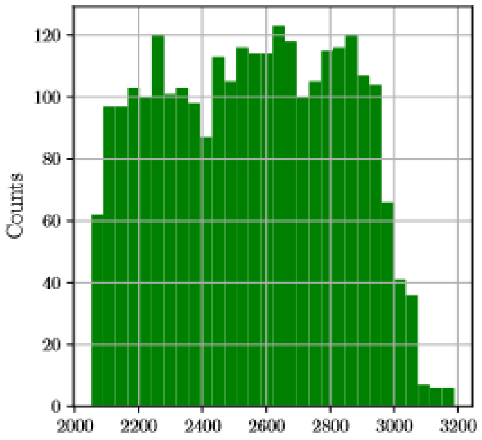
(b) Arrival time (t_0)



(c) Exponential time (τ)

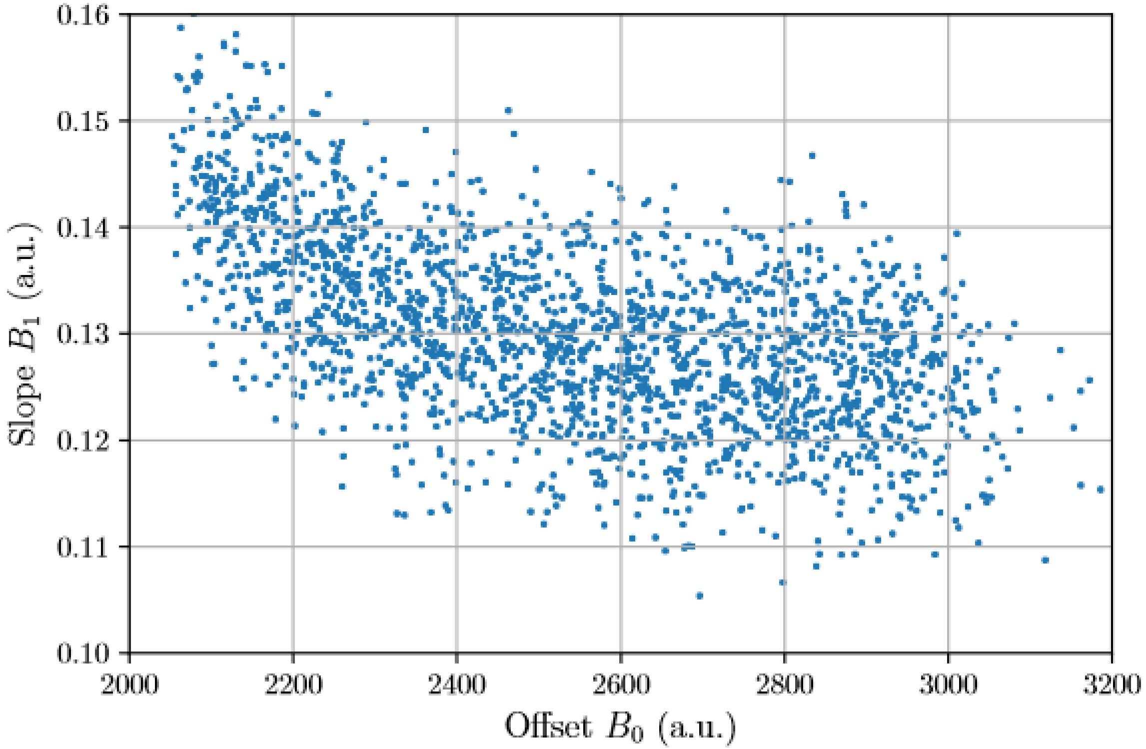


(d) Slope (B_1)

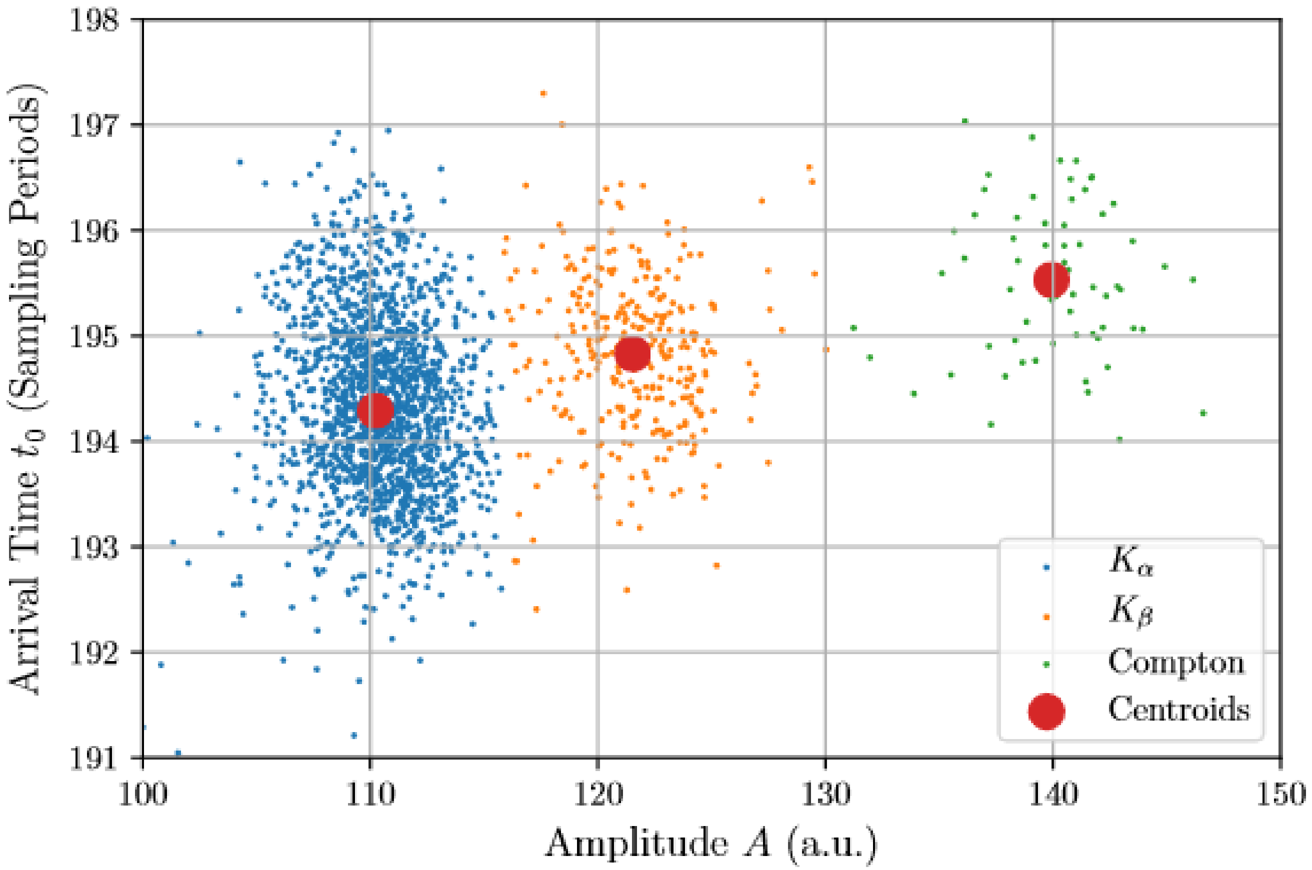


(e) Offset (B_0)

Non linearity: Amplification gain depends on offset (!)



Detection arrival time depends on pulse amplitude (!)



DPP: Digital Penalized LMS Method for filtering optimization

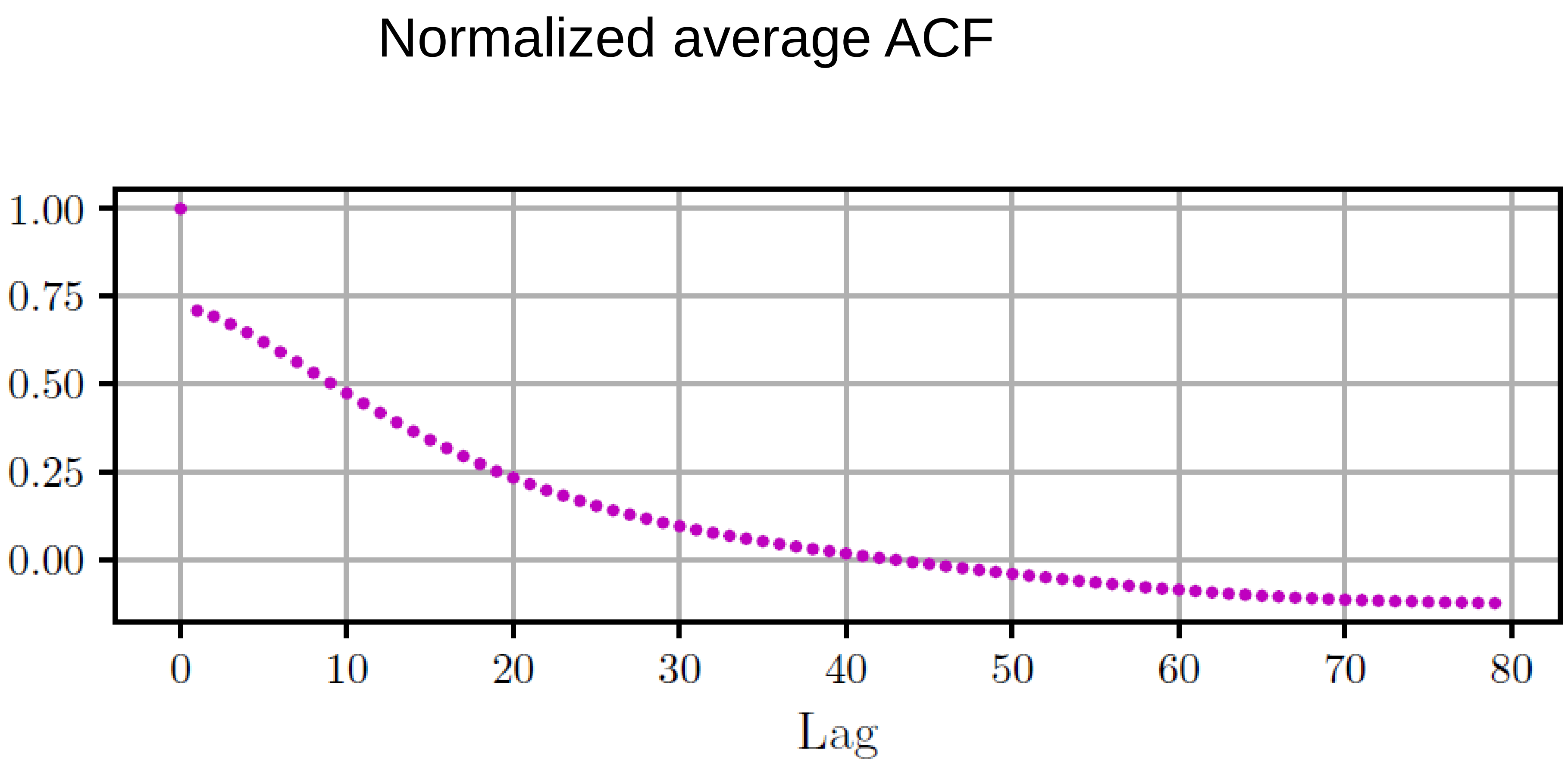
$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i}$$

$$\sigma_y^2 = \left\langle (y - \langle y \rangle)^2 \right\rangle = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} c_i \, c_j \, \underbrace{\langle x_i - \langle x_i \rangle \rangle \, \langle x_j - \langle x_j \rangle \rangle}_{\text{Covariance Matrix } V_{i,j}}$$

$$\sigma_y^2 = \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} c_i \, c_j \, ACF(|i-j|)$$

$$ACF(j) = \frac{\sum_{i=1}^{N-j} x_i \, x_{i+j}}{\sum_{i=1}^{N-j} x_i^2}$$

DPP: Digital Penalized LMS Method for filtering optimization



DPP: Digital Penalized LMS Method for filtering optimization

$$\sum_{i=0}^{k-1} c_i = 0$$

Offset rejection

$$\sum_{i=0}^{k-1} c_i \, i = 0$$

Slope rejection

Ideal requirements

$$\sigma_y^2 = \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} c_i \, c_j \, ACF(|i-j|)$$

Output noise

$$y_j = \sum_{i=0}^{k-1} c_i \, x_{j-i} = A, \qquad j \in [t_R, t_R + t_{FT} - 1]$$

Output flat top

DPP: Digital Penalized LMS Method for filtering optimization

$$\Psi(c_0, c_1, \ldots, c_{k-1}) = \alpha_1 \left(\sum_{i=0}^{k-1} c_i \right)^2 + \alpha_2 \left(\sum_{i=0}^{k-1} c_i\, i \right)^2 + \alpha_3 \sum_{j=t_R}^{t_R+t_{FT}-1} \left(\sum_{i=0}^{k-1} c_i\, x_{k+j-i} - A \right)^2 +$$

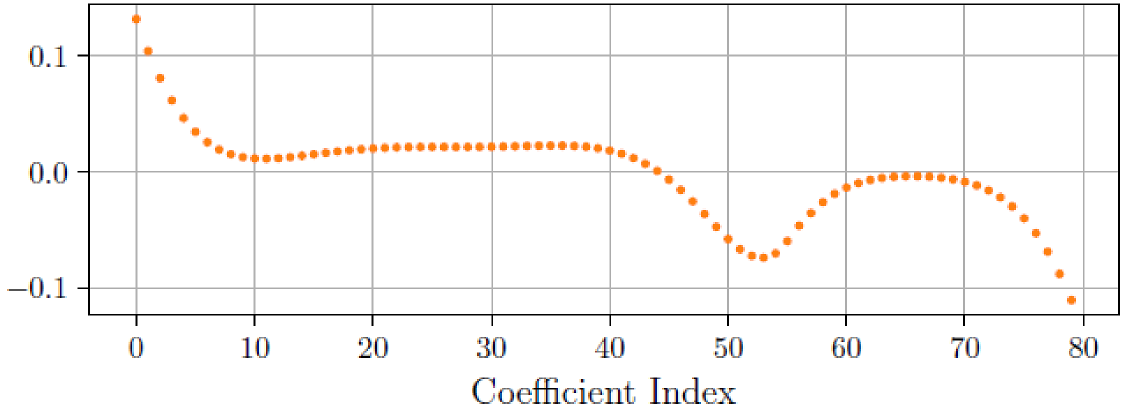
$$\alpha_4 \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} c_i\, c_j\, ACF_{|i-j|}$$

$$y_j = \sum_{i=0}^{k-1} c_i\, x_{j-i} = A, \qquad j \in [t_R, t_R + t_{FT} - 1]$$

$$\{c_0, c_1, \ldots, c_{k-1}\}_{opt} = \underset{\{c_0, c_1, \ldots, c_{k-1}\}}{argmin}\ \Psi(c_0, c_1, \ldots, c_{k-1})$$

DPP: Digital Penalized LMS Method for filtering optimization

$$\{c_0, c_1, \ldots, c_{k-1}\}_{opt} = \underset{\{c_0, c_1, \ldots, c_{k-1}\}}{argmin} \Psi(c_0, c_1, \ldots, c_{k-1})$$



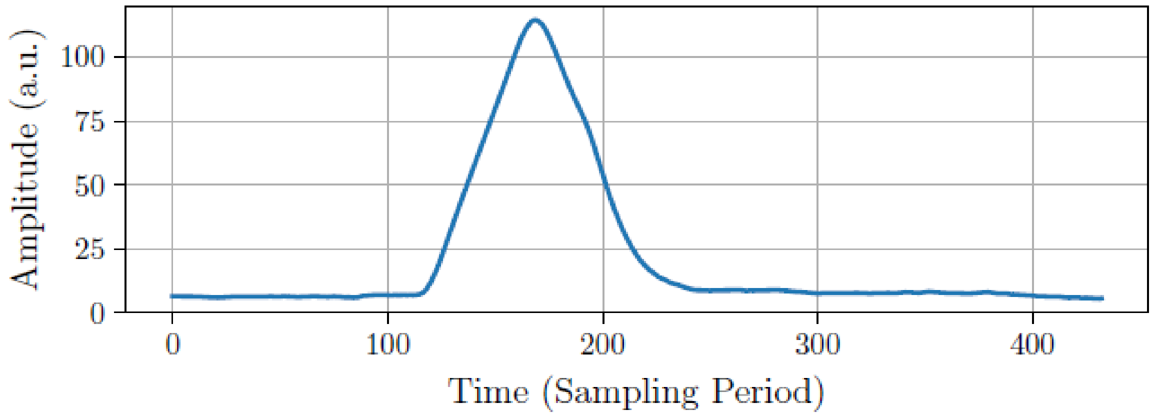


Table 3. Comparison of energy resolutions with different methods to estimate the energy spectrum.

Method	**FWHM K_α [eV]**	**FWHM K_β [eV]**	**Slope-Error Correction**
GD FIR	286 ± 4	316 ± 16	yes
Fitting †	267 ± 4	288 ± 17	yes
Trapezoidal FIR	207 ± 3	247 ± 17	no
DPLMS FIR	202 ± 2	233 ± 12	no

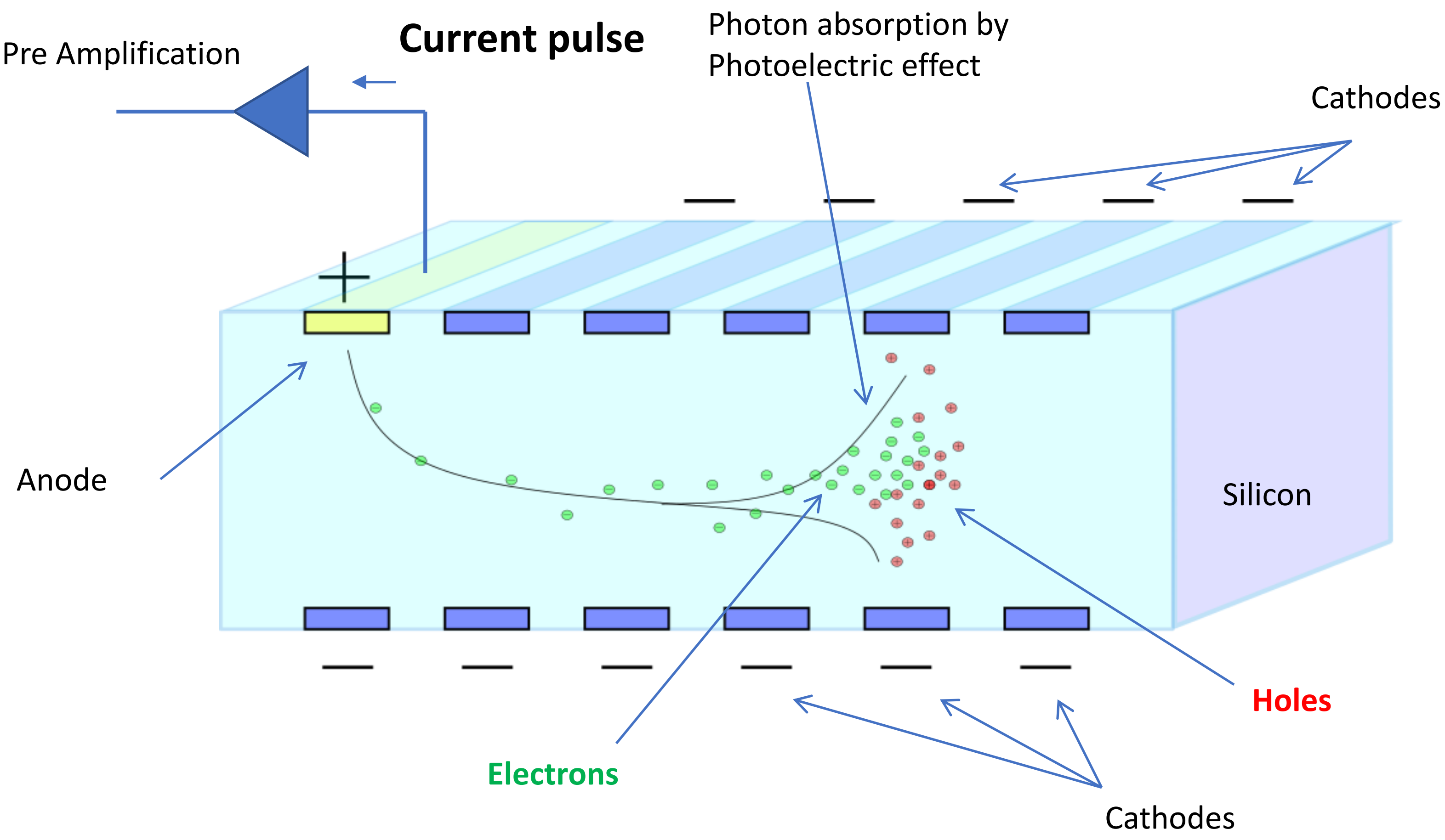
† These results correspond to the histogram of the amplitudes obtained by fitting all available photon traces.

Conclusions

- High-resolution pulse amplitude measurement can be achieved by considering concrete experimental noise and accurate pulse modeling.
- DPP can be optimized through DPLMS method allowing satisfactory trade-off among competing requirements that cannot be all simultaneously satisfied.
- An appropriate data analysis provides the necessary information to apply the DPLMS method, and it may also provide information about the quality the frontend electronics and data acquisition system.

Tank you !

Backup slides: X-Ray Photon detection with Silicon Drift Detectors (SDD)



Backup slides: Pile up (1)

Pile up: Being a Poissonian process, two or more photons could be absorbed in the SDD within any arbitrary small time window. The superposition of two photons absorbed at times t_0 and t_1 and respectively with amplitudes A_0 and A_1 is then given by

$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \le t_0 \\ A_0\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1 + n_i, & t_0 < i \le t_1 \\ A_0\left(1 - e^{-(i-t_0)/\tau}\right) + A_1\left(1 - e^{-(i-t_1)/\tau}\right) + B_0 + iB_1 + n_i, & i > t_1 \end{cases}$$

Backup slides: Pile up (2)

. . . and in general for *m+1* photons

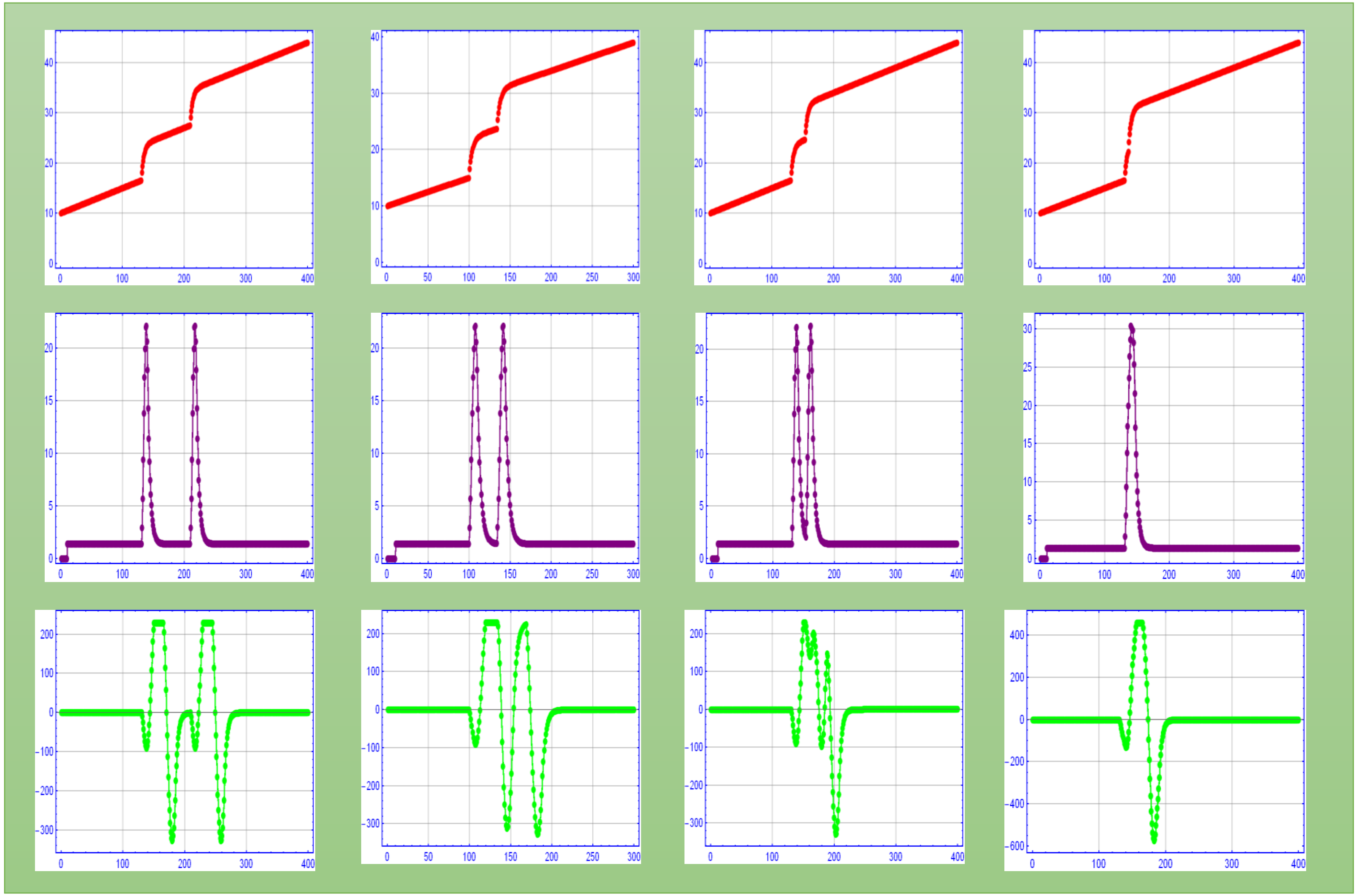
$$S_i = \begin{cases} B_0 + iB_1 + n_i, & i \le t_0 \\ A_0\left(1 - e^{-(i-t_0)/\tau}\right) + B_0 + iB_1 + n_i, & t_0 < i \le t_1 \\ A_0\left(1 - e^{-(i-t_0)/\tau}\right) + A_1\left(1 - e^{-(i-t_1)/\tau}\right) + B_0 + iB_1 + n_i, & t_1 < i \le t_2 \\ \vdots & \\ \sum_{j=0}^{m} A_j\left(1 - e^{-(i-t_j)/\tau}\right) + B_0 + iB_1 + n_i, & i > t_m \end{cases}$$

Backup slides: Pileup rejection

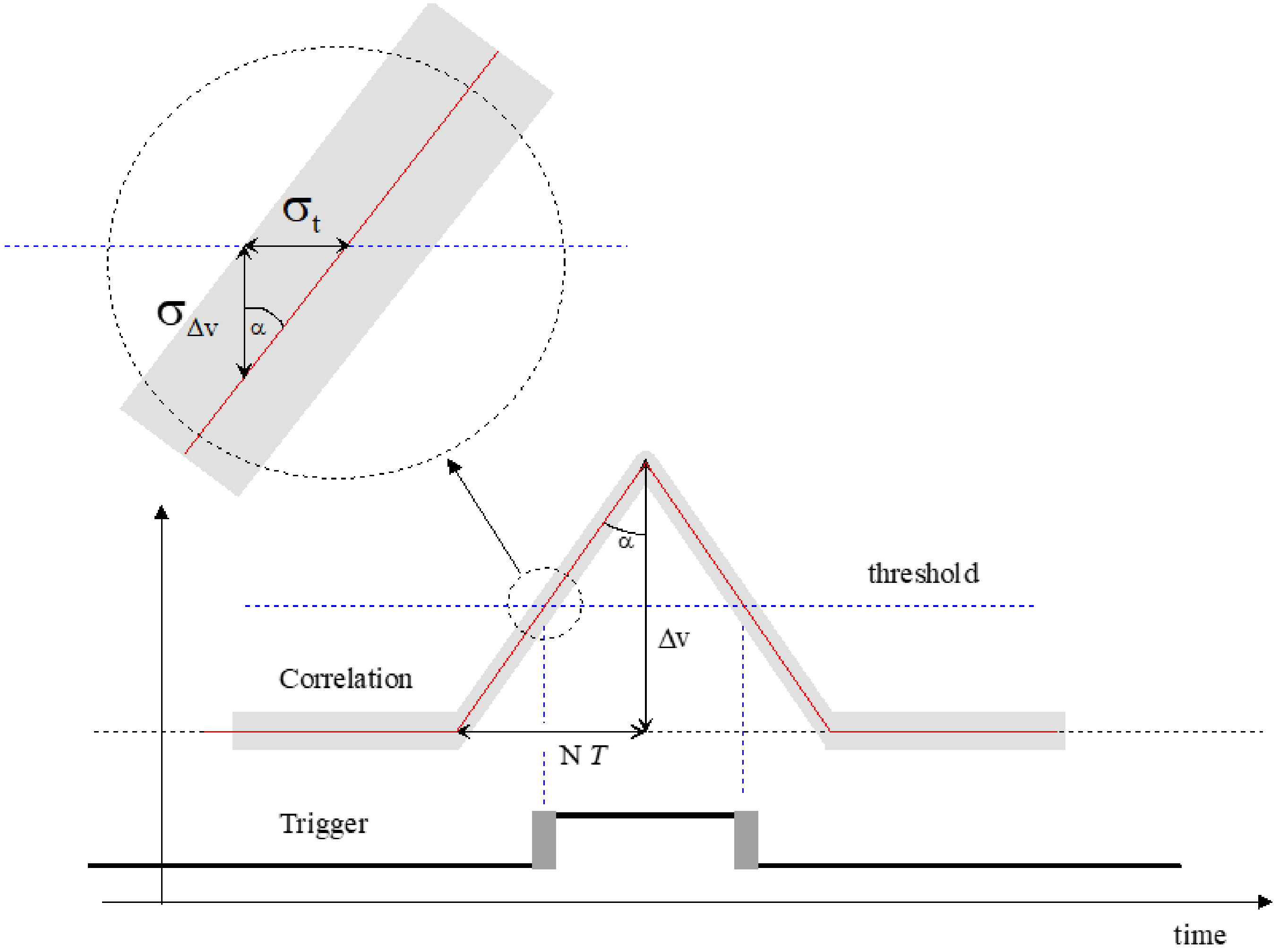
Input Pulses

First Derivs

Shaped Pulses



Backup slides: Uncertainty relation between **Energy** and **Time**



$$\sigma_t \sigma_{\Delta V} = \frac{2T}{\Delta V} {\sigma_x}^2$$