Thermodynamics of information

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1. A bit of history.
2. Basic concepts of information theory.
3. Basic concepts of stochastic thermodynamics.
4. Information and the second law.
5. Fluctuation theorems.
6. Feedback reversibility and optimal Maxwell demons.
7. Thermodynamic cost of measurement and erasure.
8. Creating information: symmetry breaking.
9. Maxwell demons in the phase space and microcanonical Szilard engines.
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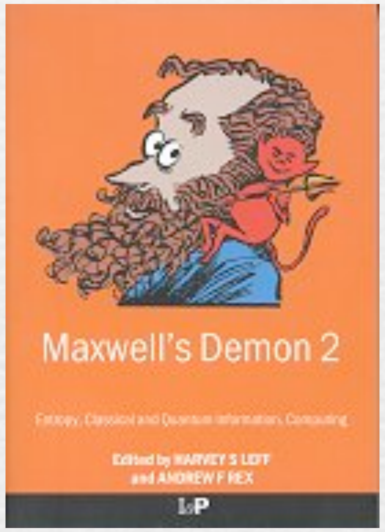
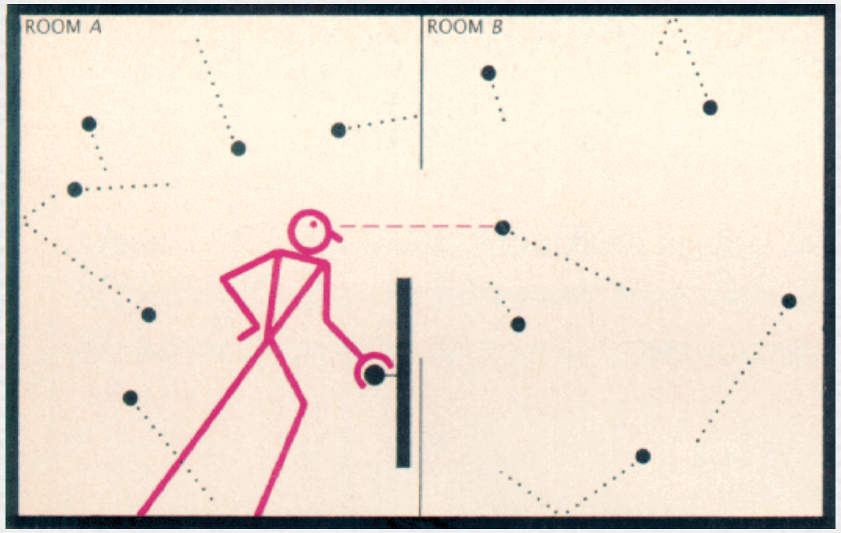
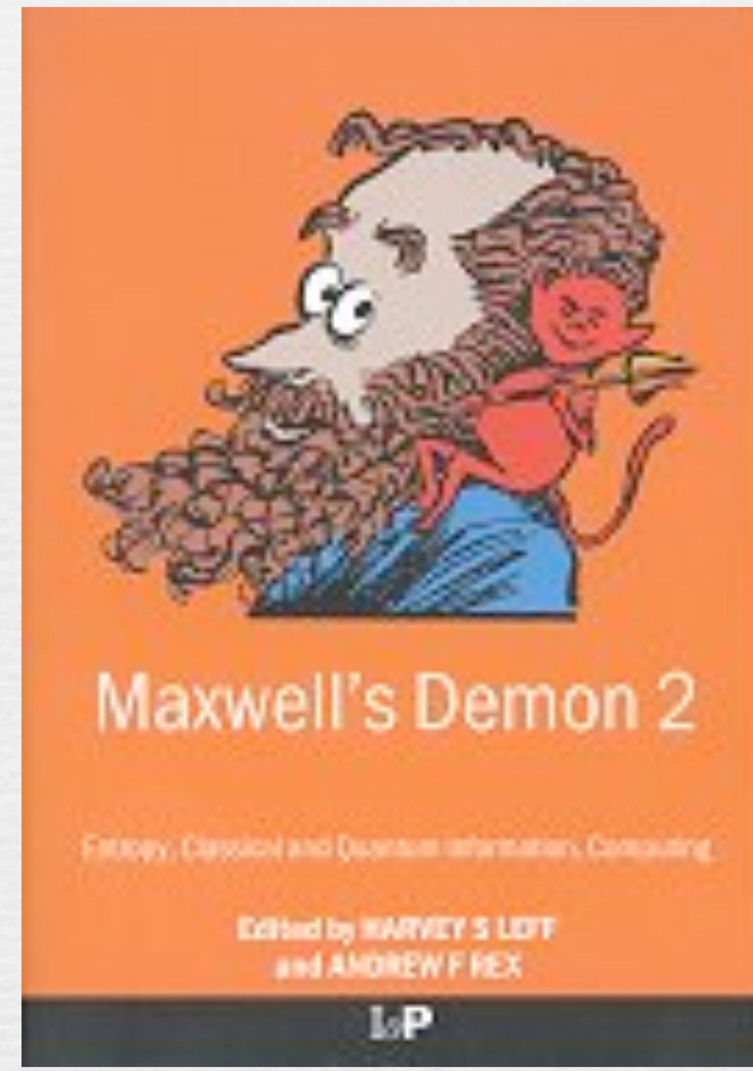




1. A bit of history

- Maxwell demons
- The Szilard engine.
- Landauer's principle.
- Bennett's solution.
- Experimental realizations.
- The two main problems.

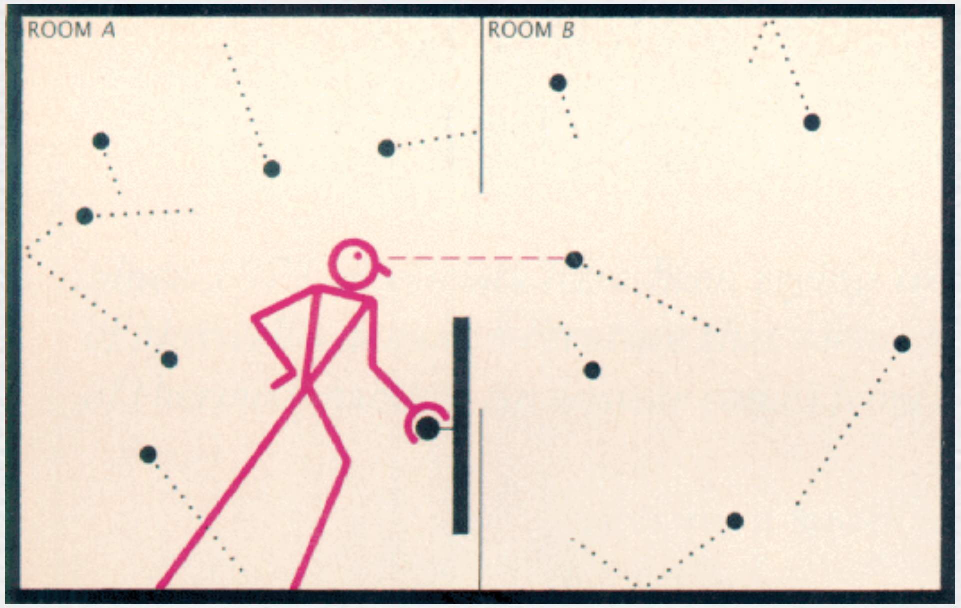


Maxwell demon (letter to Tait, 1867)

To pick a hole — say in the **2nd law of thermodynamics**, that **if two things are in contact the hotter cannot take heat from the colder without external agency.** Now let A and B be two vessels divided by a diaphragm and let them contain elastic molecules in a state of agitation which strike each other and the sides. [...] I have shown that **there will be velocities of all magnitudes in A and the same in B, only the sum of the squares of the velocities is greater in A than in B.**[...]

Now conceive **a finite being who knows the paths and velocities of all the molecules** by simple inspection but who **can do no work except open and close a hole** in the diaphragm by means of a slide without mass. Let him first observe the molecules in A and when he sees one coming the square of whose velocity is less than the mean sq. vel. of the molecules in B let him open the hole and let it go into [...]

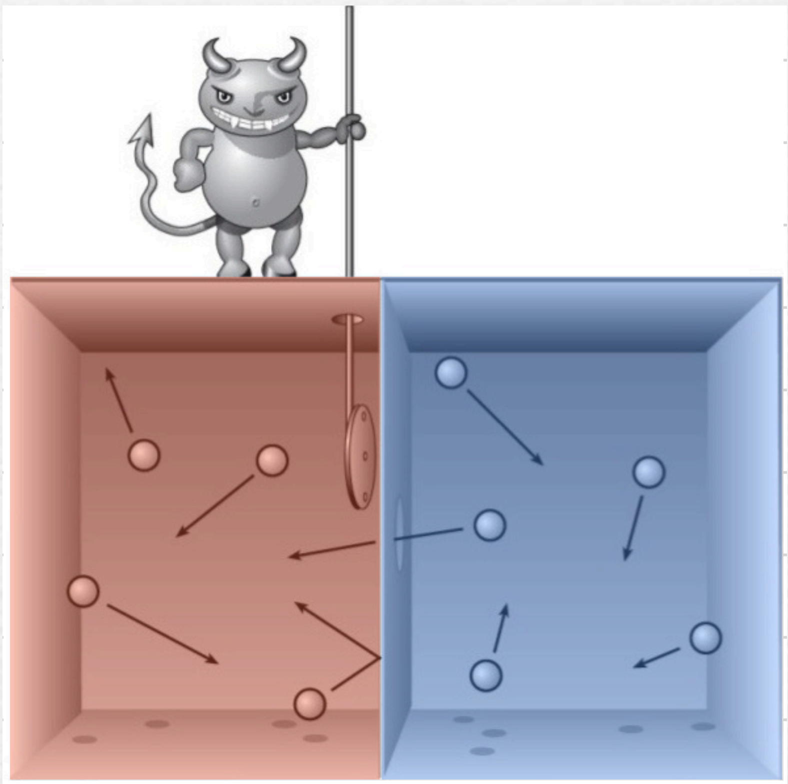
Then the number of molecules in A and B are the same as at first, but the energy in A is increased and that in B is diminished, that is, **the hot system has got hotter and the cold colder and yet no work has been done, only the intelligence of a very observant and neat-fingered being has been employed.**



Temperature Maxwell demon

Heat from cold to hot

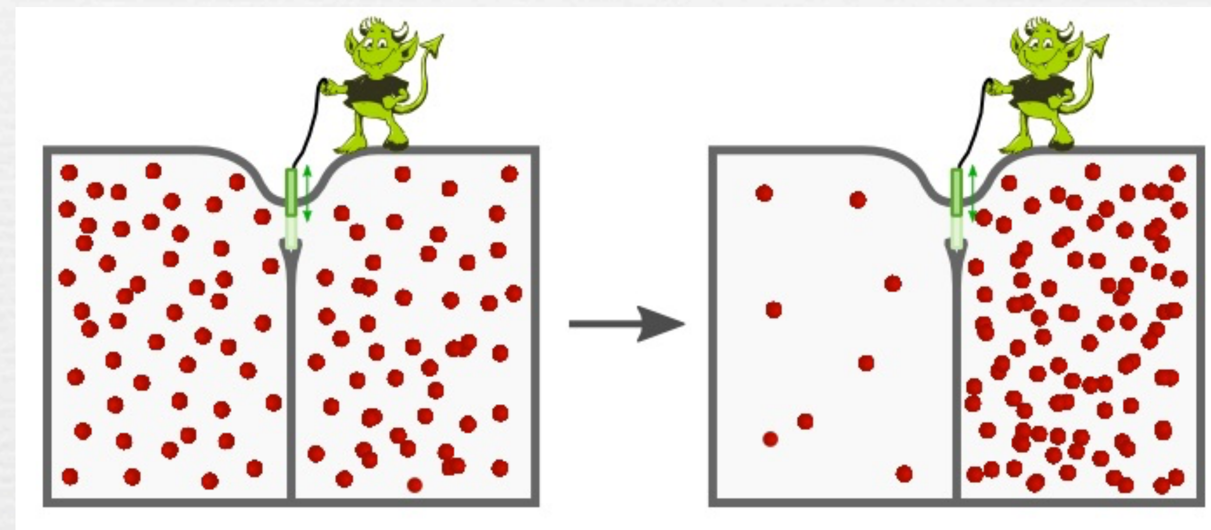
Info: position and speed

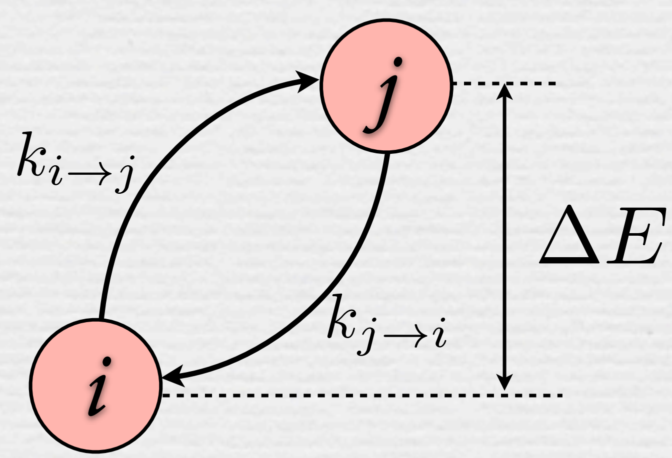


Pressure Maxwell demon

Particles from dense to rare

Info: position



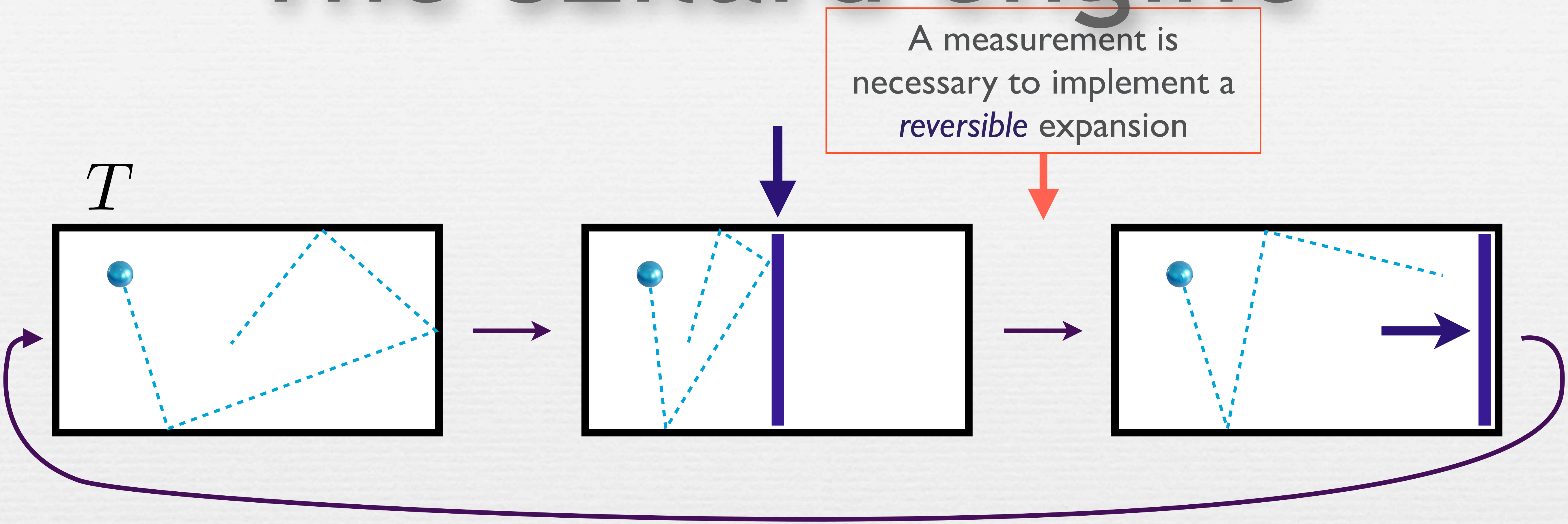


$$\frac{k_{i\to j}}{k_{j\to i}} \neq e^{-\beta \Delta E}$$

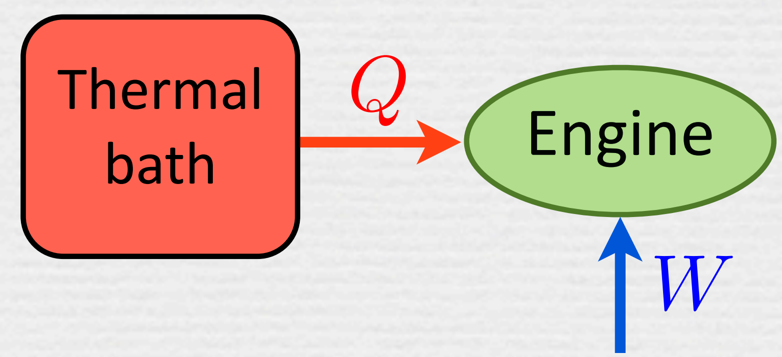
Is any breaking of detailed balance a Maxwell demon?

Ratchet and pawl (*Feynman Lectures*). Serreli, Nature (2007). Esposito, Schaller, EPL (2012)

The Szilard engine



$$W_{\text{extract}} = \int_{V_{\text{init}}}^{V_{\text{fin}}} P dV = \int_{V_{\text{init}}}^{V_{\text{fin}}} \frac{kT}{V} dV = kT \log \frac{V_{\text{fin}}}{V_{\text{init}}}$$



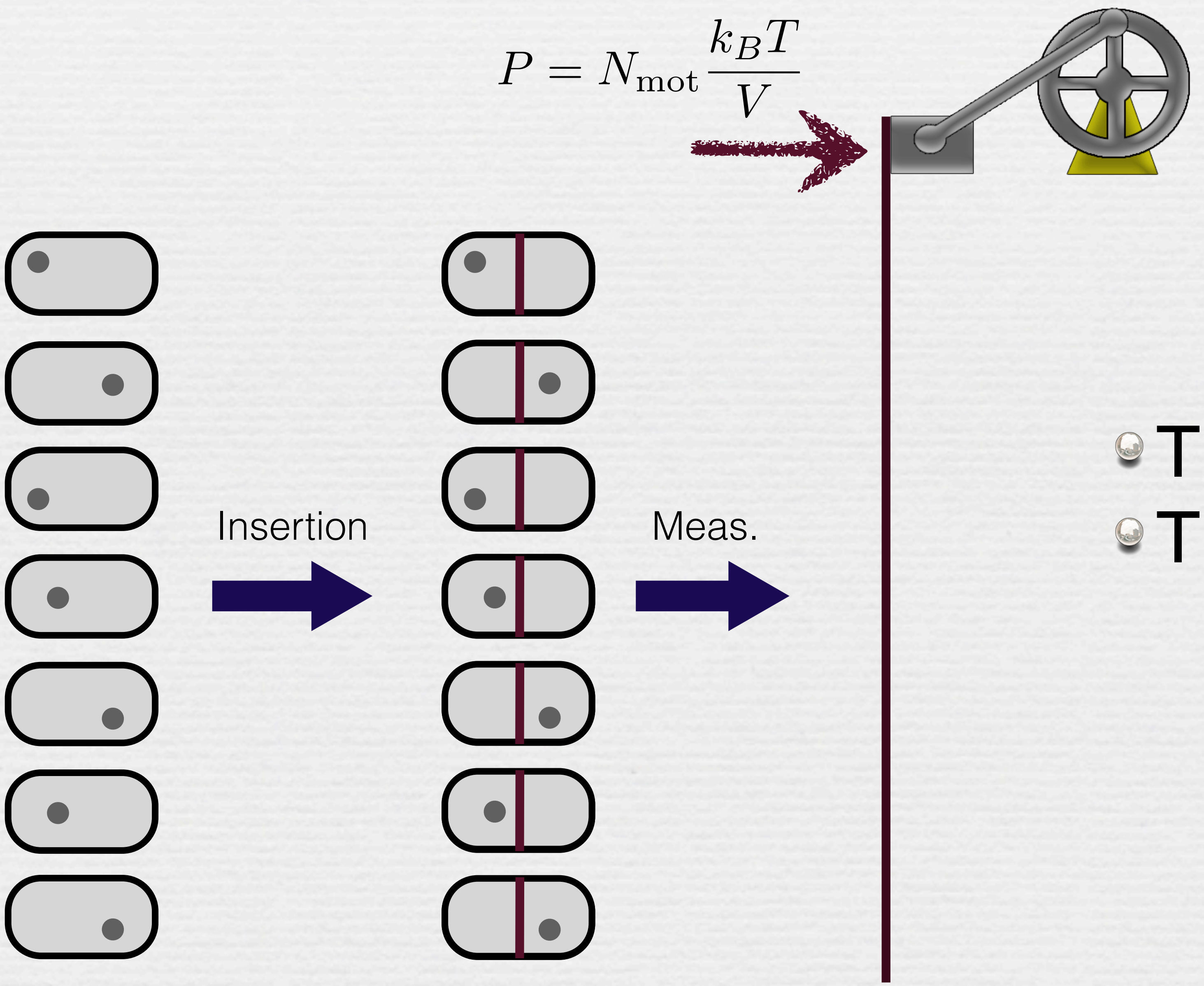
$$W_{\text{extract}} = kT \log \frac{1}{1/2} = kT \log 2 > 0$$

Work done on the system: $$\boxed{W = -W_{\text{extract}} = -kT \log 2}$$

The Szilard engine

N_{mot} Szilard engines

$$P = N_{\text{mot}} \frac{k_B T}{V}$$

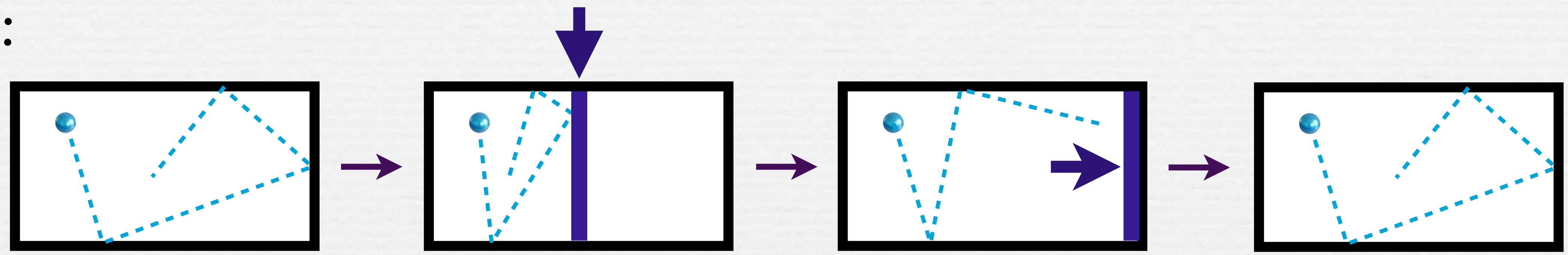


$$W = -N_{\text{mot}}\, k_B T \log 2$$

- The ideal gas equation is valid.
- The measurement is necessary.

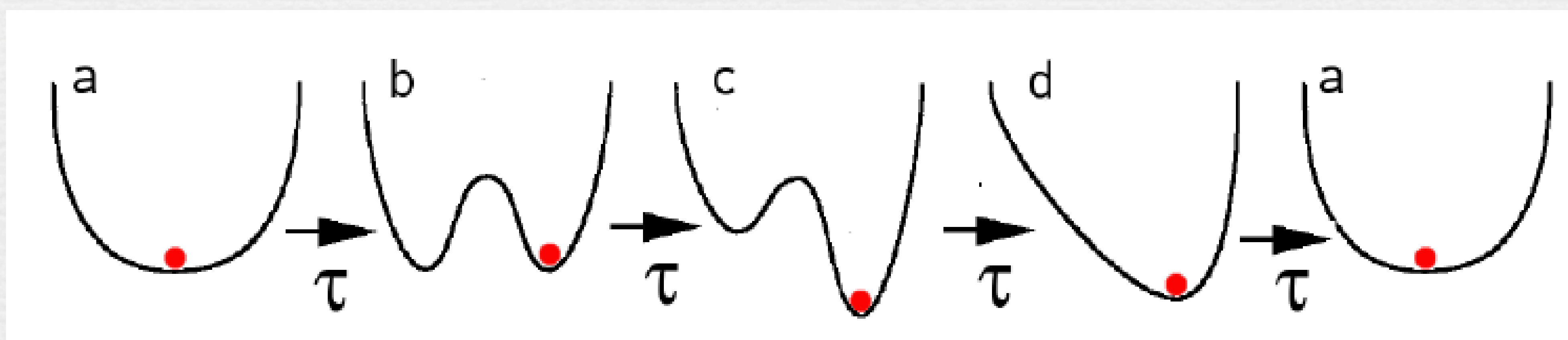
The Szilard engine

- One particle gas:



- Brownian particle:

Kawai, JMRP, van den Broeck. PRL **98**, 080602 (2007).

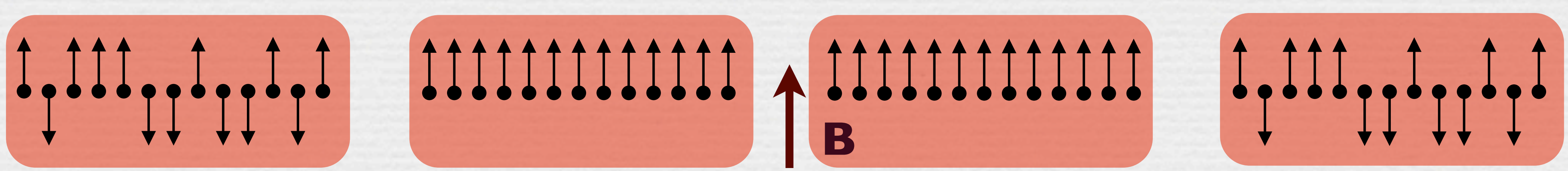


- Ising model:

JMRP. Chaos **11**, 725 (2001)

Coupling Field

$$(J, B)$$

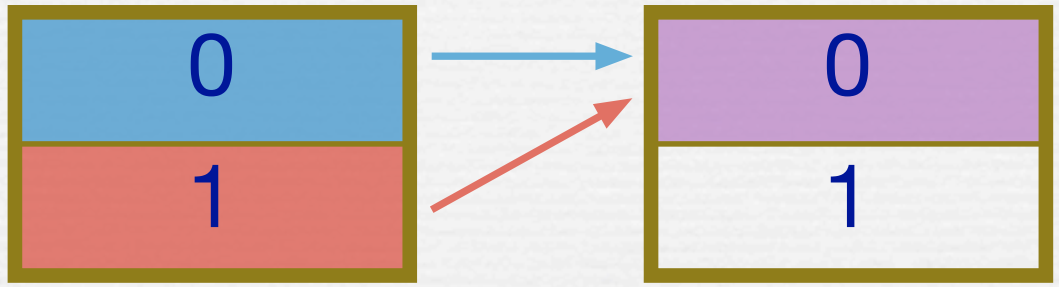


$$(0,0) \rightarrow (J,0) \rightarrow (J,\pm B) \rightarrow (0,\pm B) \rightarrow (0,0)$$

Measurement

Landauer's principle

RESTORE-TO-ZERO process (erasure):



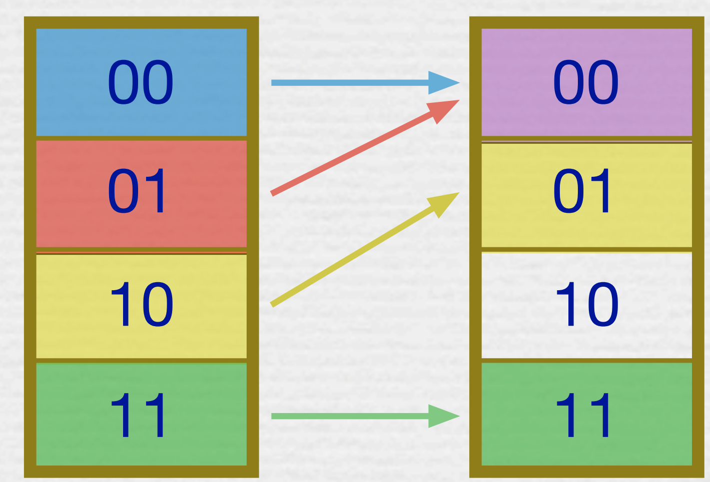
- The available phase space volume shrinks by a factor two.
- This must be compensated by an increase of phase space volume (i.e., entropy) in the surroundings (⇐ Liouville theorem/2nd Law).
- A heat $kT\log 2$ must be dissipated.
- Restore-to-Zero needs a work $kT\log 2$ (if the two states have the same volume in phase space) **but** it can be done **reversibly**

First misconception:

logical irreversibility $\not\Rightarrow$ thermodynamic irreversibililty

⇘

Heat



Bennett's solution

Demon dissipates heat to restore his memory to the initial state.

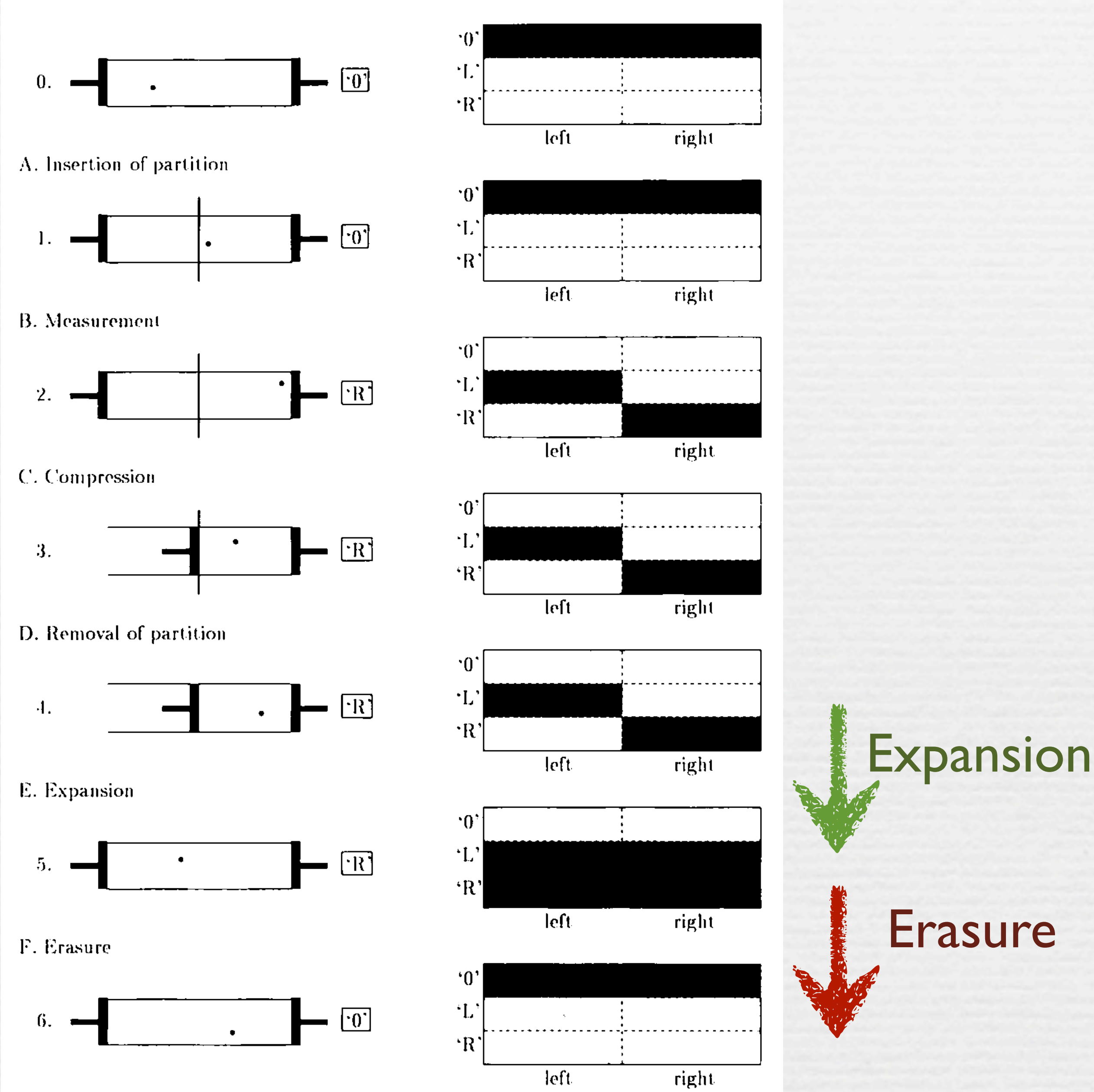


Fig. 2. The Szilard engine. The left column shows one example of the process. The right column shows the phase space after each step.

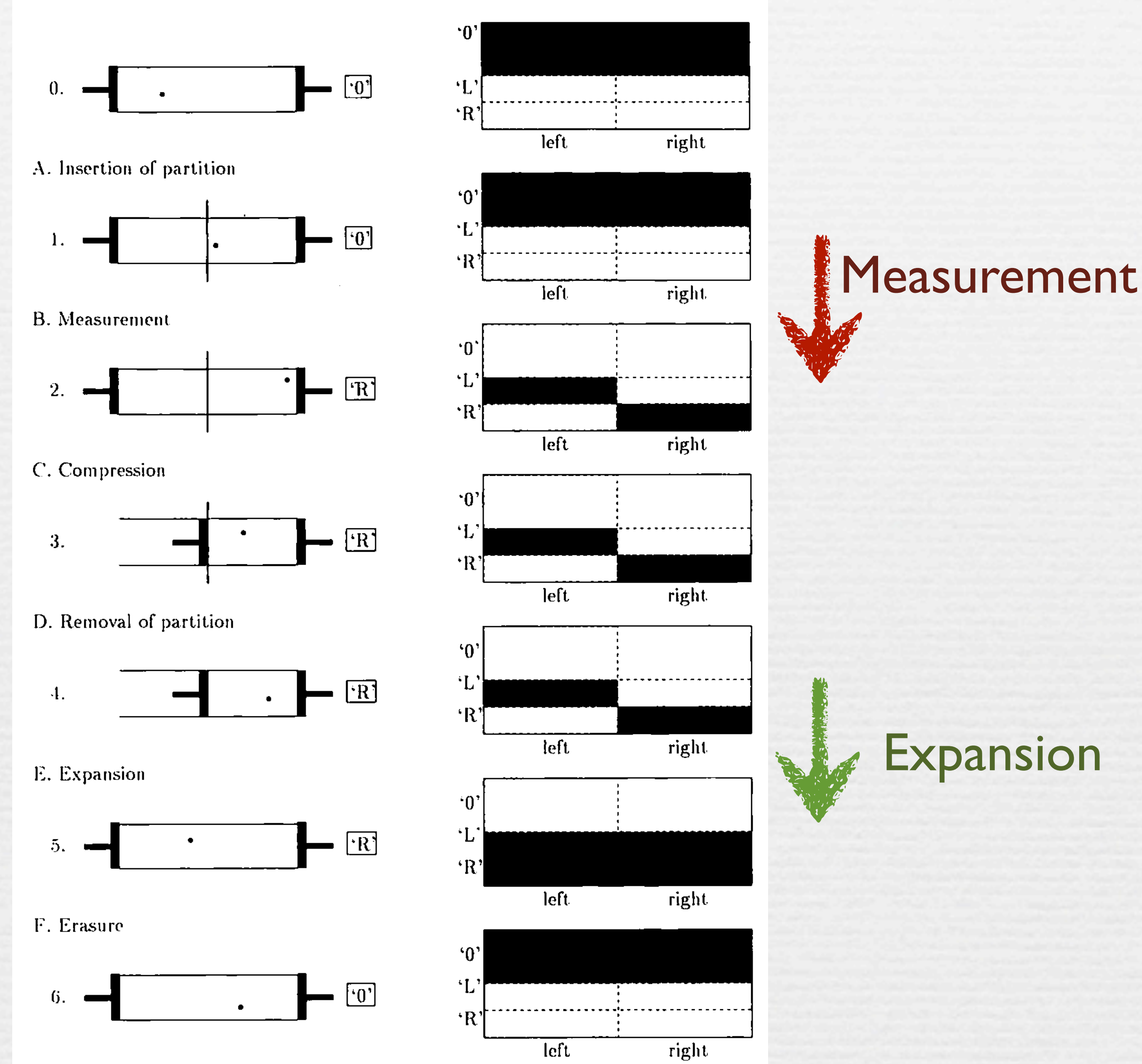


Fig. 4. Szilard engine with memory implemented using two elementary bits, such as two spin-$\frac{1}{2}$ particles.

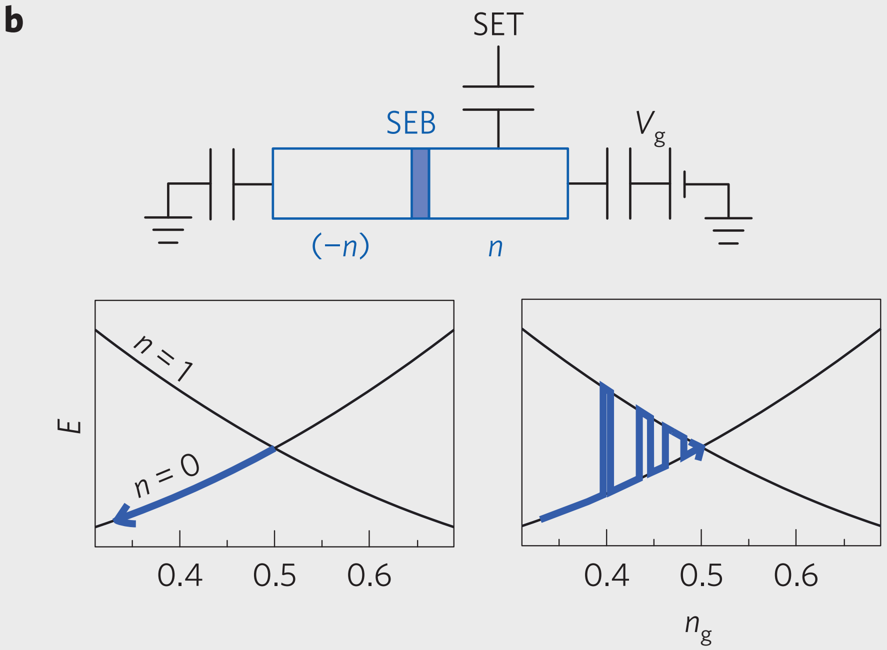
Second misconception: the thermodynamic cost (work) could be partly due to erasure and partly due to measurement.

Fahn, Found. of Phys. (1996). Sagawa&Ueda, PRL (2009)

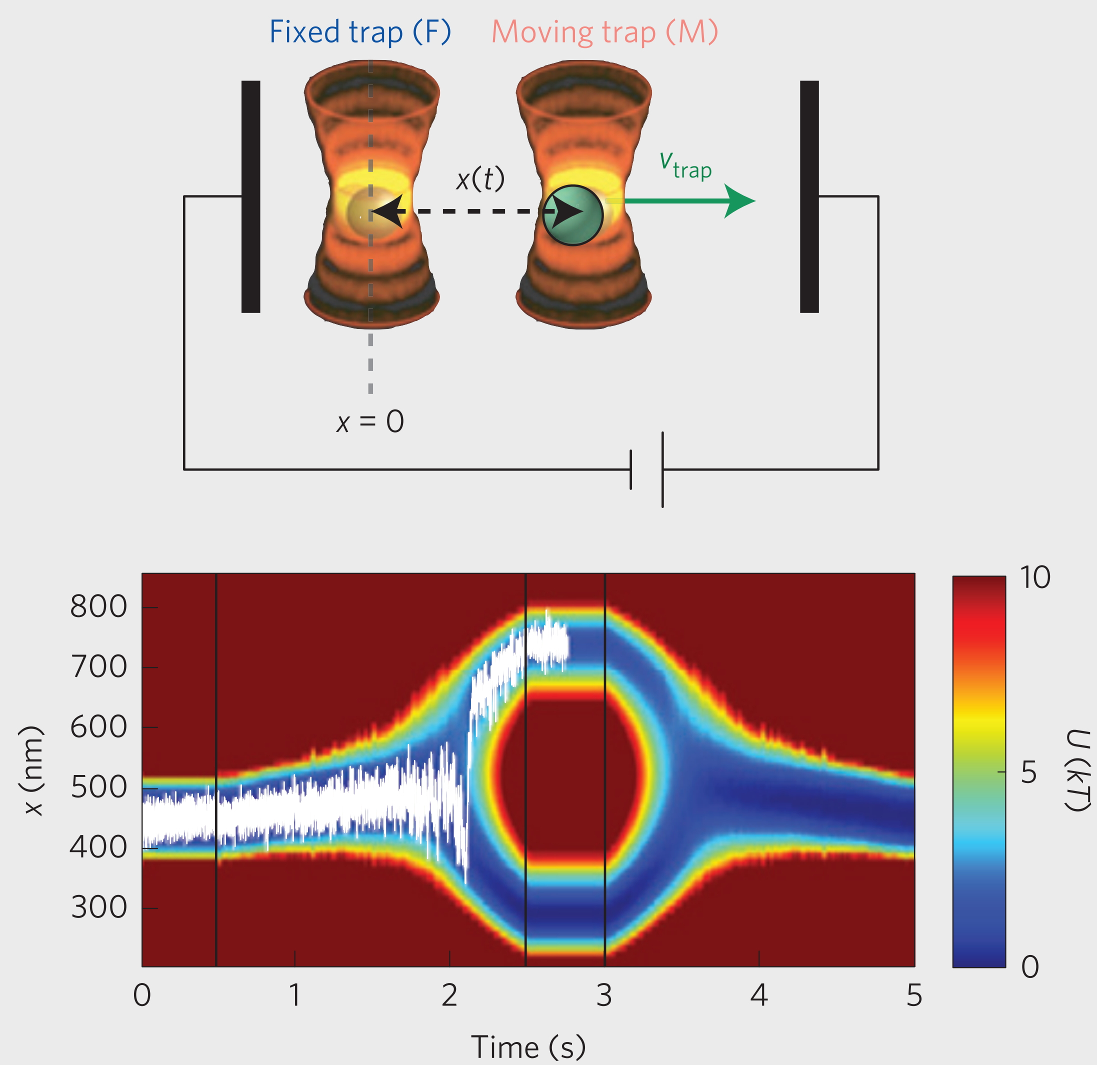
Experimental realizations

Figure 1 | The Szilárd engine and recent experimental realizations. a, In the original Szilárd engine, a partition is inserted into a box containing a single molecule and surrounded by a thermal reservoir at temperature T. The half of the box containing the molecule is measured and the partition is moved performing an isothermal expansion extracting work. **b**, Experimental realization with a single-electron box (SEB) controlled by a gate voltage V_g and monitored by a single-electron transistor (SET; ref. 14). The experimental set-up is shown in the top figure. The lower plots show the energy levels of the box, depending on the electron number ($n = 0,1$), as a function of the normalized gate voltage n_g. The electron number n is measured, and n_g is rapidly changed, decreasing the energy (left plot), and then slowly moved back to the initial value (right plot). There is a net extraction of work in the process due to the thermal excitations of n occurring only when n_g changes slowly. **c**, Experimental realization using a colloidal particle and two optical traps[16]. The top figure shows the experimental set-up: one trap is kept fixed at position $x = 0$ and the other is shifted horizontally at a speed v_{trap}. A controllable electrostatic field created by two electrodes biases the particle towards one of the traps. The bottom figure shows a contour plot of the potential affecting the particle during a process where the moving trap is shifted and then moved back to its initial position. A realization of the particle's trajectory is visualized as a fluctuating white line. The Szilárd cycle is achieved by measuring the trap where the particle lies in the middle of the process and biasing that trap in the second half of the process. **d**, Experimental realization using a rotating colloidal particle[13]. The top figure shows the experimental set-up, where two particles are attached to a cover glass. One of them is regarded as a rotating Brownian particle and is controlled by the electric field induced by four electrodes. The bottom figure shows two shapes of the effective potential, which is a superposition of a sinusoidal potential and a constant torque. The position of the particle is measured and the potential is switched from one shape to the other when the particle crosses the potential minima in the uphill direction.

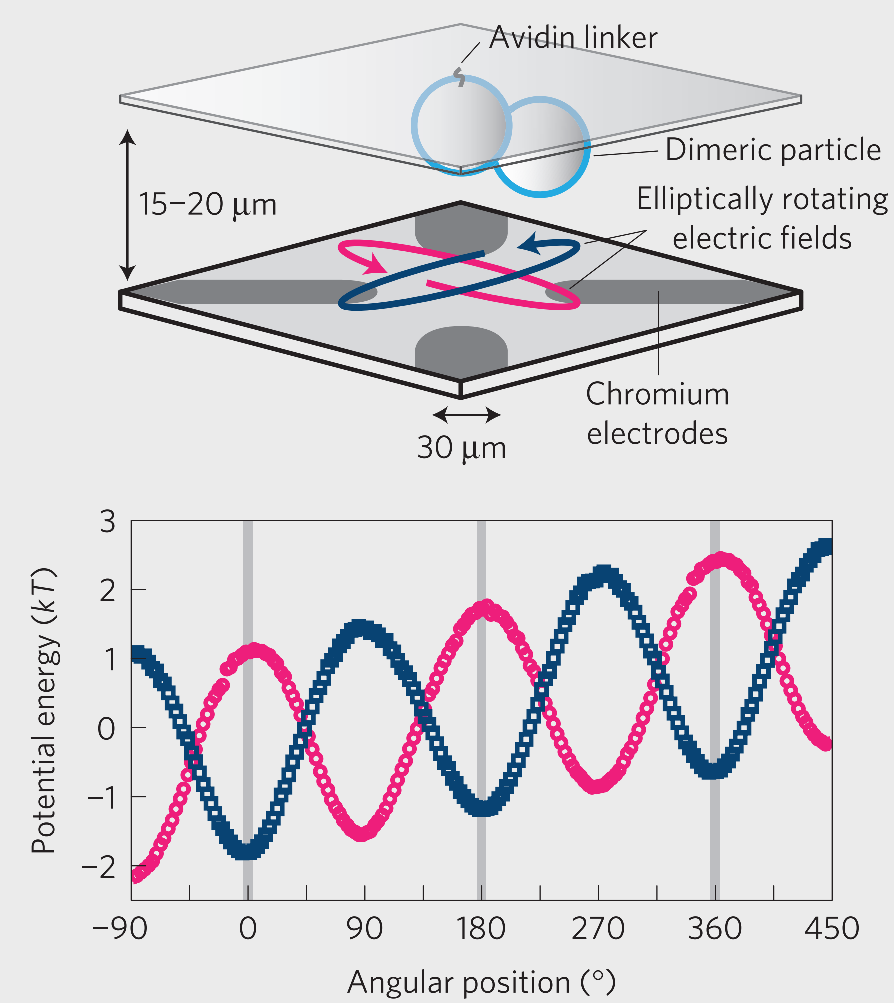
b



c



d



Two main problems

1. Incorporate information in the second law, but do not care about thermodynamic cost of measurement/erasure (sections 4-6).

2. Restore the validity of the second law by taking into account the physical nature of the demon/memory and the thermodynamic cost of information processing (sections 7-10).