Ultra-Intense Laser I

Basics – rel plasma physics

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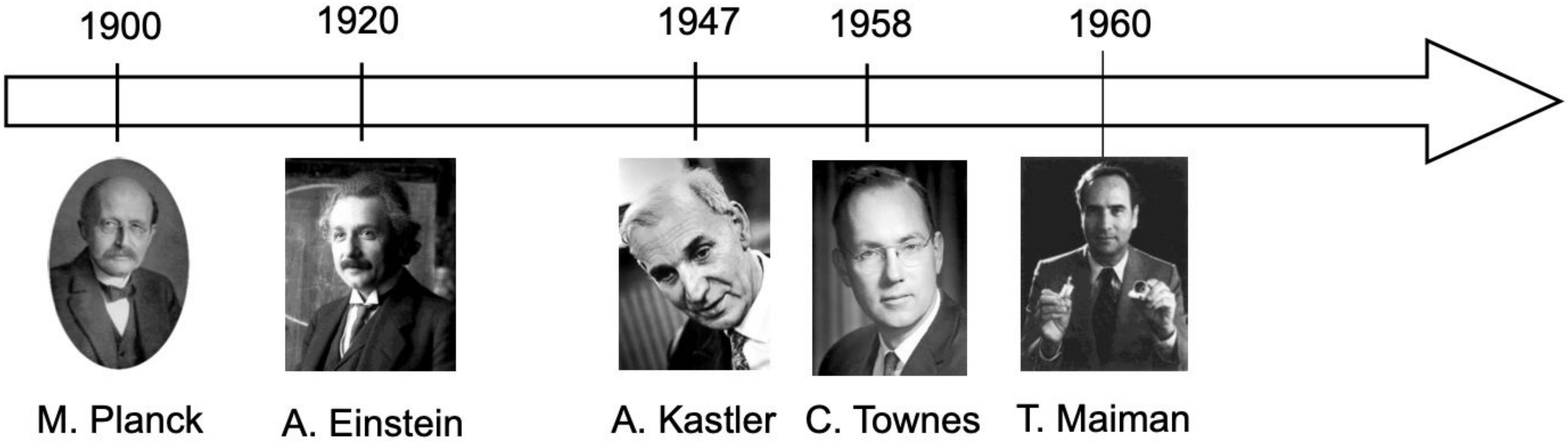
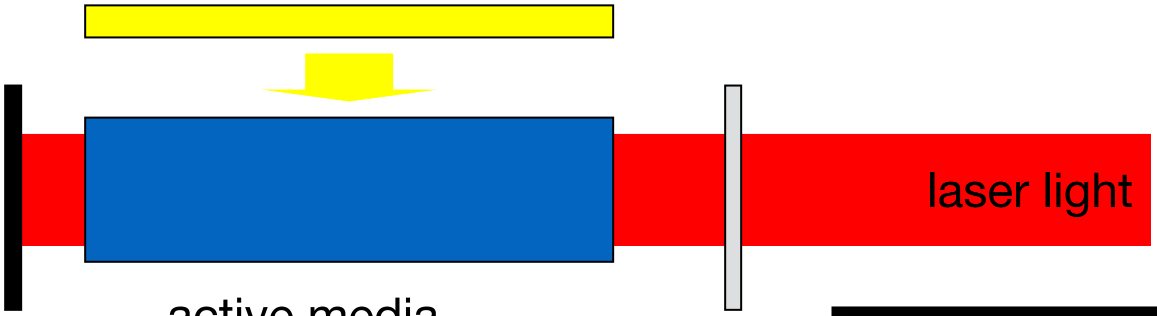
Intro: From photon to laser



What does it need to build a laser?



Pump

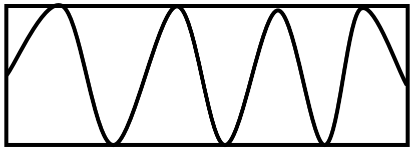
laser light

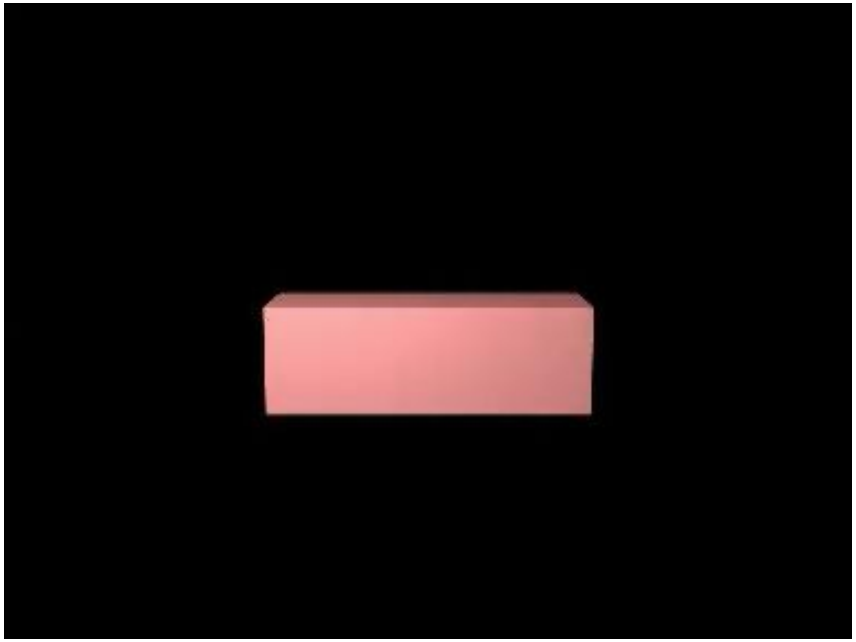
active media

Mirror 1

Mirror 2

Resonator
(Gitarre)





Interaction of matter and radiation (1)

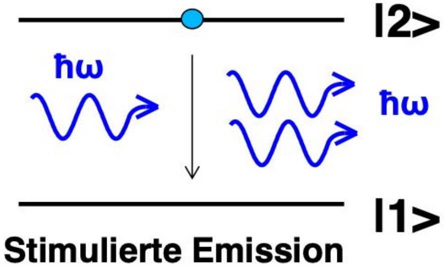
Basics:

An electron in an excited state in an atom decays into a lower level by sptontaneous or stimulated emission

In a thermal equilibrium be n_1 and n_2 the number of atoms in ground level 1 or excited level 2 with the energy E_1 and E_2.

$$\hbar\omega = E_2 - E_1$$

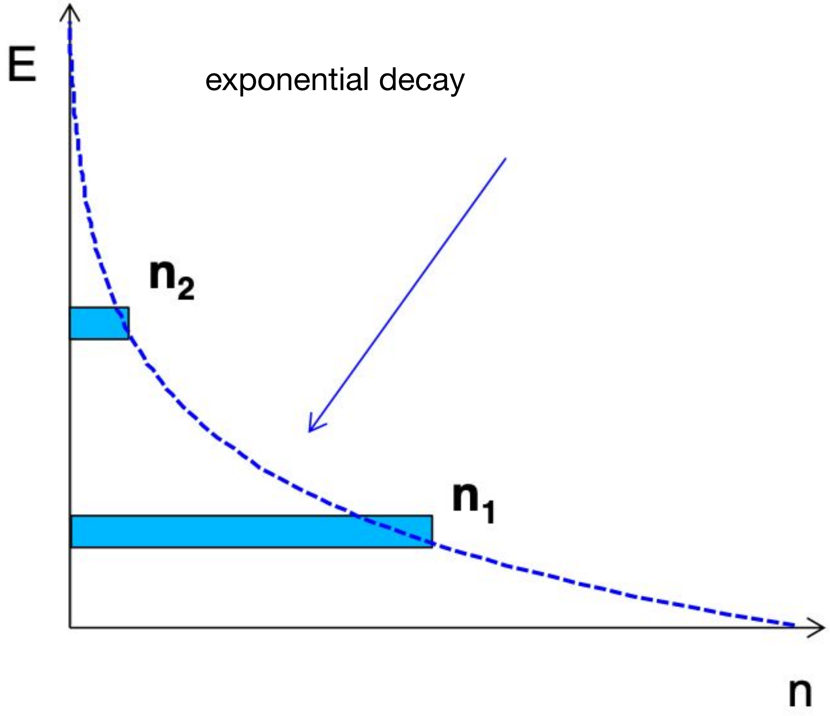
LASER transition: Q.M. emission no different from inducing field. (direction, energy, polarization, phase)



Interaction of matter and radiation (2)

Population of levels in thermal equilibrium

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} \exp\left(- \frac{E_2 - E_1}{k_B T} \right)$$

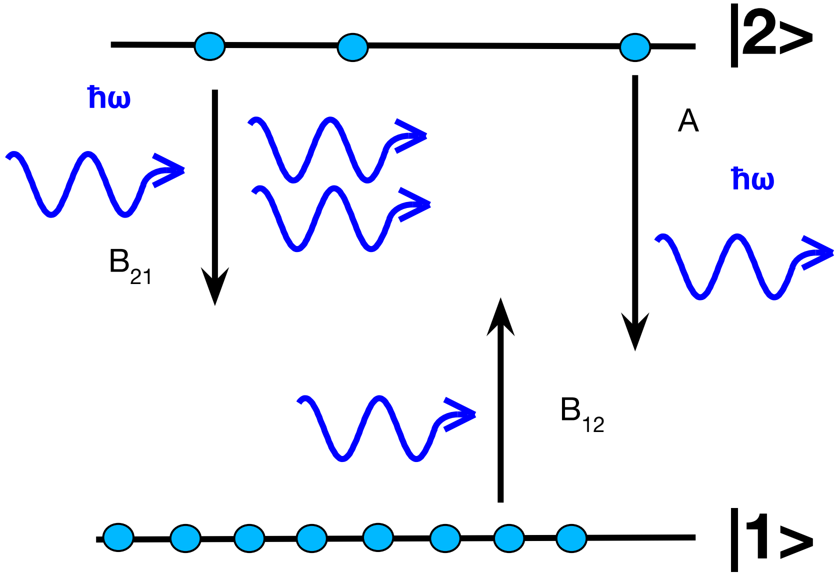


Interaction of matter and radiation (3)

The population follows the rate equations:

$$\frac{dn_2}{dt} = -An_2 - B_{21}U_p(\omega)n_2 + B_{12}U_p(\omega)n_1$$

$$\frac{dn_1}{dt} = +An_2 + B_{21}U_p(\omega)n_2 - B_{12}U_p(\omega)n_1$$



with A, B_{21} and B_{12} as the Einstein-coefficients.

$B_{21}n_2U_p(\omega)$ is the rate of stimulated emission, and
$B_{12}n_1U_p(\omega)$ the rate of stimuliated absorption, and
An_2 is the rate of spontaneous emission

Interaction of matter and radiation (4)

In thermal equilibrium we have:

An_2 +	$B_{21}U_p(\omega)n_2$ =	$B_{12}U(\omega)n_1$
spontaneous emission	**induced emission**	**absorption**

So for the Einstein coefficient it leads to:

$$B_{12}(\frac{g_1}{g_2}) = B_{21} = B \qquad \frac{A}{B} = \frac{\hbar\omega^3}{\pi^2 c^3} \qquad \frac{A}{BU_p(\omega)} = exp(\frac{\hbar\omega}{k_B T}) - 1$$

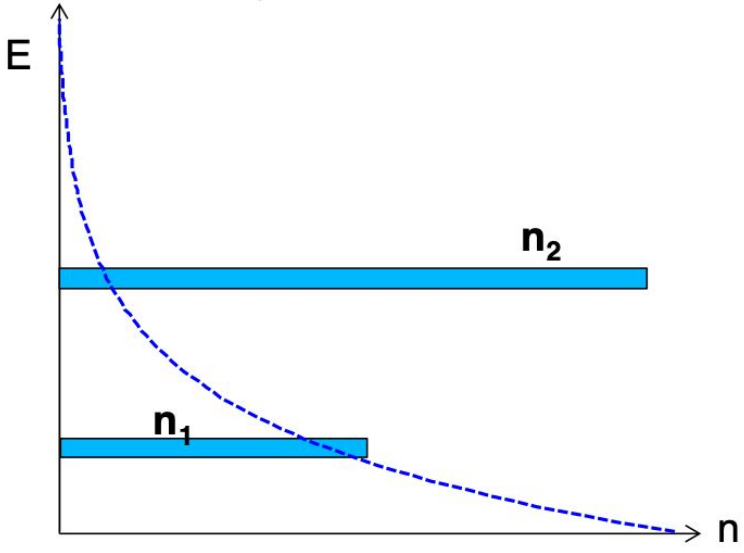
This makes it so much harder to pump lasers in the short wavelength range

Interaction of matter and radiation (5)

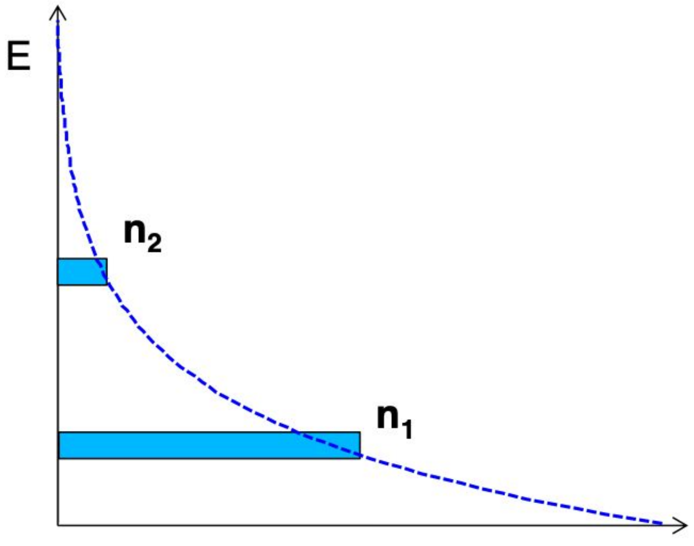
As line profiles we have to take into acoount: nat. lifetime, doppler broadening, stark-broadening.

To achieve a population inversion in a medium, we have to pump energy into the system.

Population inversion and thermal equilibrium exclude each other



Besetzungsinversion



System in thermischen Gleichgewicht

Interaction of matter and radiation (6)

For absorption and stimulated emission we have to take line profiles into account. We separate homogeneous and inhonogeneous broadened line profiles.

Homogeneous broadened lines:

- Each atom has the same line shape and frequency response.
- Each atom has the same probability for a transition.
- Lorentz-line shape.

Important difference to inhomogeneous broadened line shapes:

Saturation

Natural LB, pressure broadening.

Interaction of matter and radiation (7)

Inhomogeneous broadened line shape:

- Each central wavelength of an atom is shifted.
- Different atoms have different resonance frequencies
- Signal pulse interacts only with a fraction of the atoms
- Gauss-Line shape

Spectral Hole Burning

Doppler broadening, Crystal inhomogenities

Interaction of matter and radiation (8)

Example for spectral hole burning zum spectral hole burning:

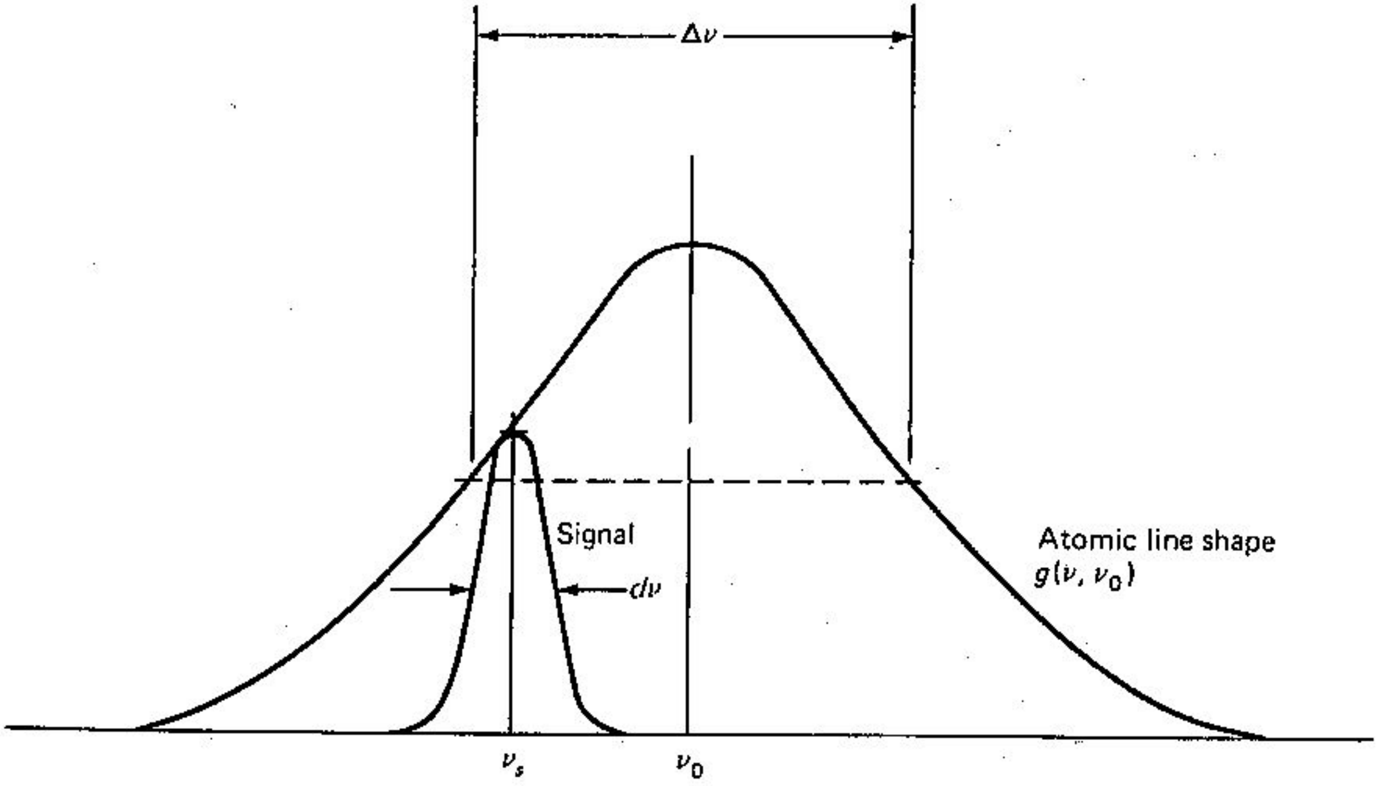
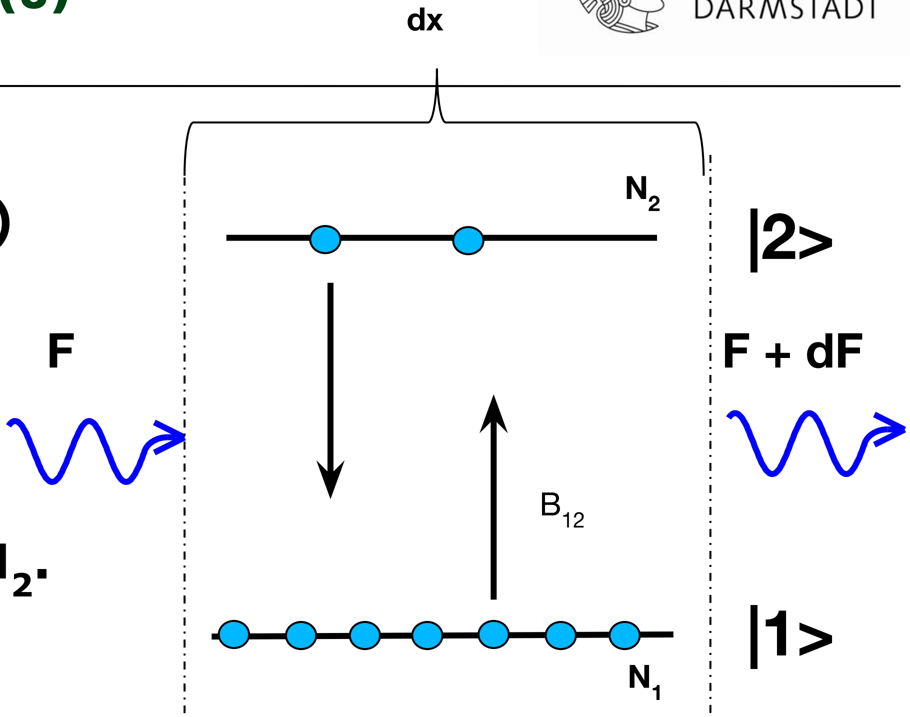


Bild2.2.2 . **Linewidth-broadened atomic transition line centered at ν_0 and narrow band signal centered at ν_s**

Interaction of matter and radiation (9)

Almost parallel beam of energy Density F = $F(\nu)\partial\nu$ (Fluence (J/m^2)) in an absorbing medium of thickness dx.



<u>Assuming:</u>

Only 2 levels, populated N_1 and N_2. Regard only absorption and stimulated emission.

$$dF = -h\nu . dN_2 = h\nu . dN_1 \qquad (2.1)$$

$$dN_2 = -\left(\frac{F(\nu)}{\partial x}\right) B_{12} . dt . \left(N_2 - N_1 \frac{g_2}{g_1}\right) = \frac{F(\nu) B_{12}}{c} \left(N_2 - N_1 \frac{g_2}{g_1}\right) \qquad (2.2)$$

n

Interaction of matter and radiation (10)

For the change of fluence in the medium we get

$$F = F_0 \exp\left[\frac{h\nu_s g(\nu_s, \nu_0) B_{21}}{c}\left(n_2 - n_1 \frac{g_2}{g_1}\right)x\right]$$

With the absorption coefficient

$$\alpha(\nu_s) = \left(\frac{g_2}{g_1} n_1 - n_2\right)\sigma_{21}(\nu_s)$$

(function g is the line shape signal and levels)

And the cross section for stimulated emission

$$\sigma_{21}(\nu_s) = \frac{h\nu_s g(\nu_s, \nu_0) B_{21}}{c}$$

Interaction of matter and radiation (11)

$$F = F_0 \exp\left[-\alpha\left(\nu_s\right)x\right]$$

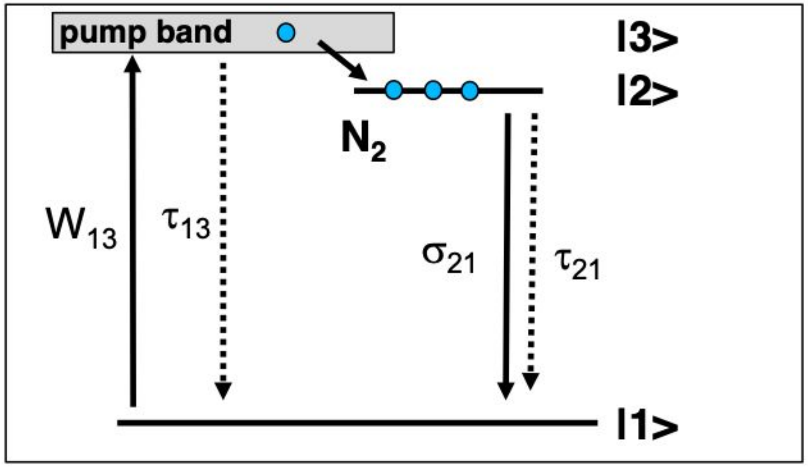
- **Well known absorption behaviour in a medium.**

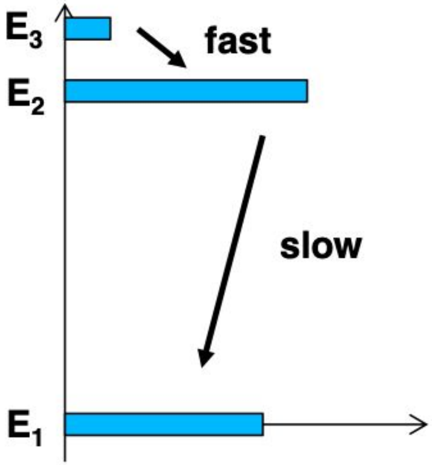
Population inversion: α gets negative -> $\partial F\left(\nu\right)/\partial x > 0$

- **Population inversion is a non-natural state**
- **Threshold, where upper and lower levels are equally populated are called inversion threshold**
- **Dependend on laser material this can be reach only temporary (pulsed lasers) or stable (cw lasers)**
- **2 level systems are no laser medium as the lower level will be populated too fast**
- **Real lasers are 3 level and 4 level lasers.**

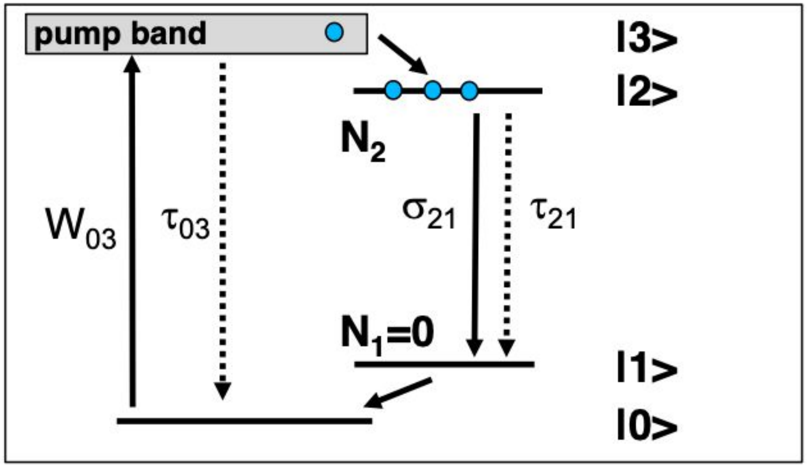
Interaction of matter and radiation (12)

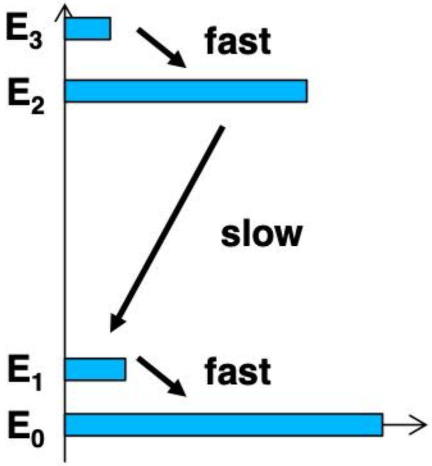
3 level system





4 level system

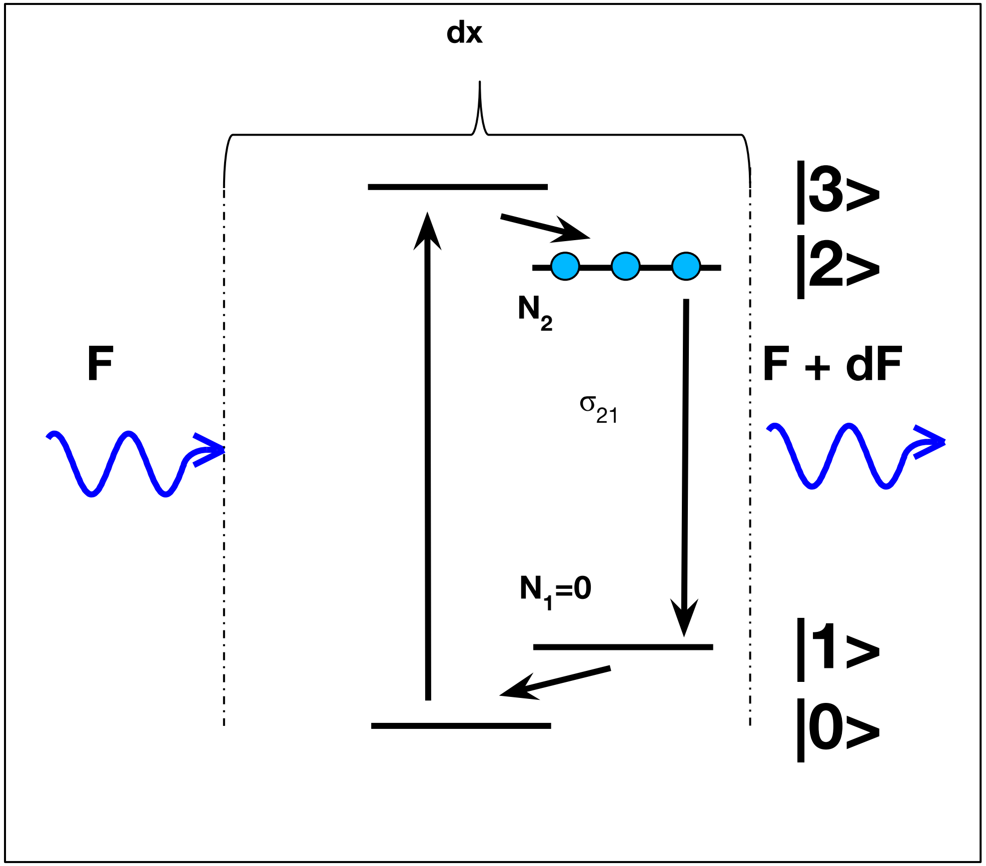




Interaction of matter and radiation (13)

Pulse amplification:

- **Neglect pump rate and flourescency within pulse duration**
- **4 level system, length l**



Stored energy density:

$$F_s = h\nu n_2 l$$

σ_{12}: **cross section**

$$dF = -h\nu dN_2 = \sigma_{21} F n_2 dx$$

Interaction of matter and radiation (14)

if $dn_2/n_2 << 1$, we get by simple integration:

$$F_{aus} = F_{ein} \exp\left(\frac{h\nu n_2 l}{\frac{h\nu}{\sigma_{21}}} \right) = F_{ein} \exp\left(\frac{F_s}{F_{sat}} \right)$$

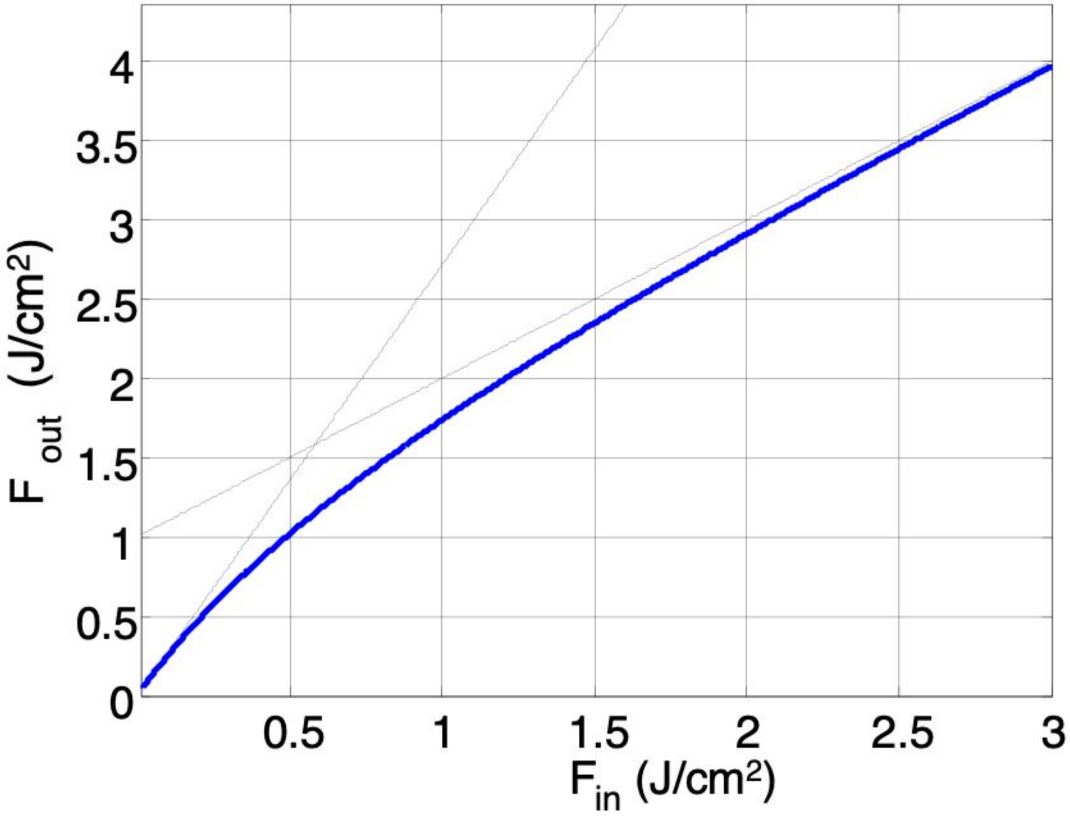
With F_{sat} as saturation fluence and G_0 the small signal gain

$$\boxed{F_{sat} = \frac{h\nu}{\sigma_{21}}}$$

$$\boxed{G_0 = \frac{F_{aus}}{F_{ein}} = \exp\left(\frac{F_s}{F_{sat}} \right)}$$

Interaction of matter and radiation (15)

In real life there is a deviation between small signal gain and gain saturation effects



Simulationsparameter:
F_{sat} = 1 J/cm²
(Nd:YLF @ 1053 nm)
F_{stored} = 1 J/cm²

Interaction of matter and radiation (16)

Taking saturation effects into account we get the time integrated Frantz-Nodwik equation:

$$\boxed{F_{aus} = F_{sat} \log\left(1 + G_0\left(\exp\left(\frac{F_{ein}}{F_{sat}}\right) - 1\right)\right)}$$

High intensity lasers

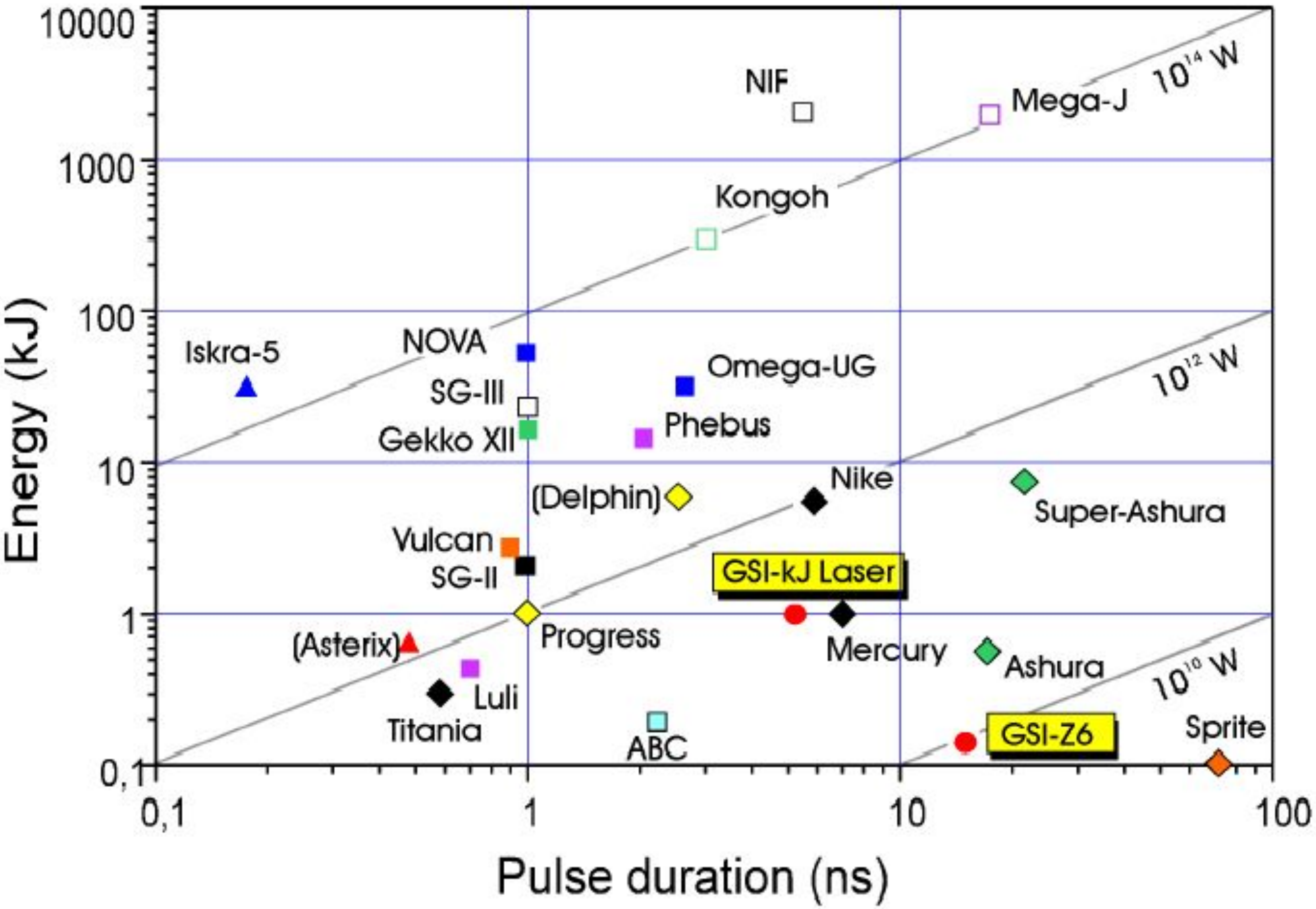
Energy

Time

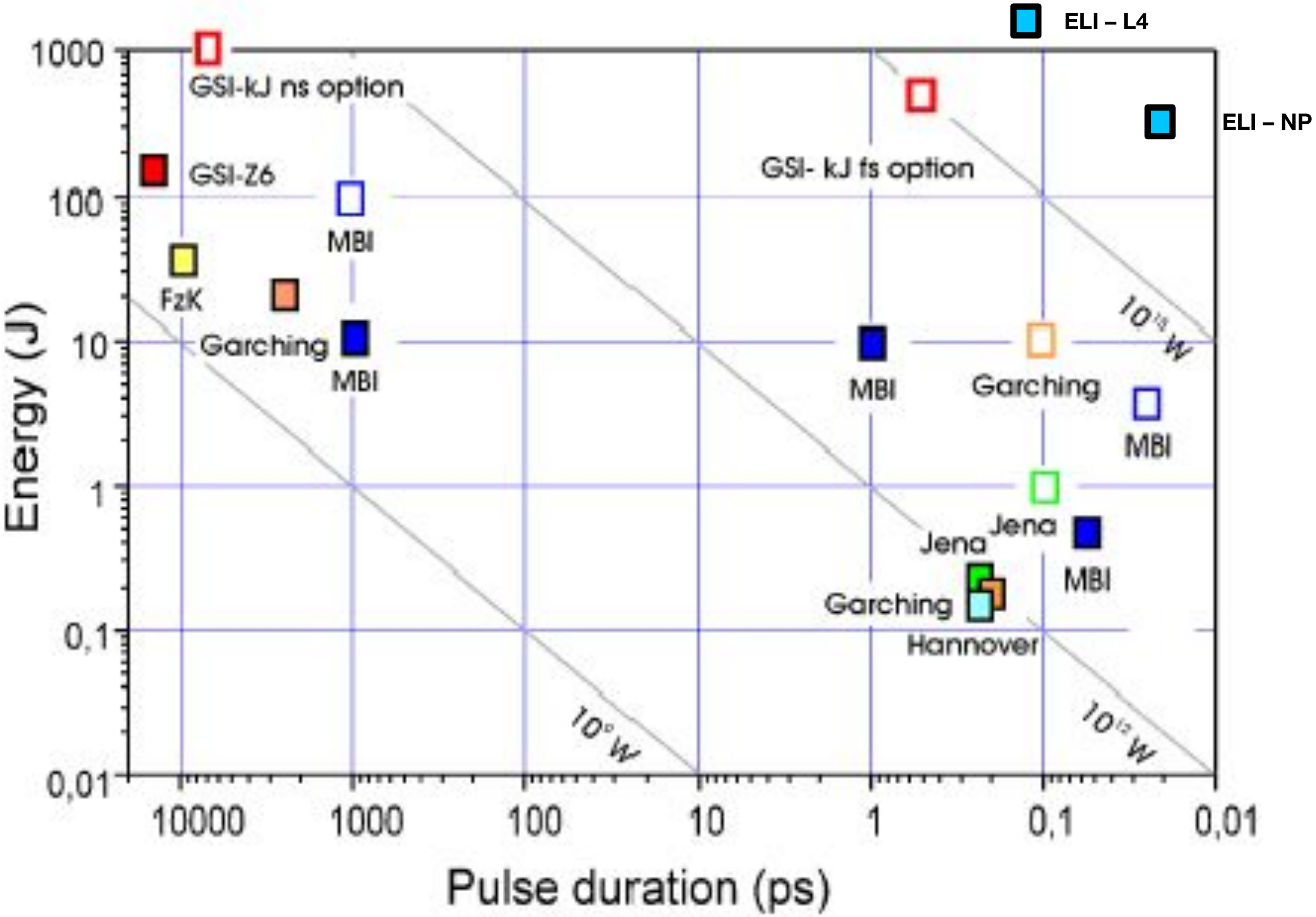
Space

High power lasers

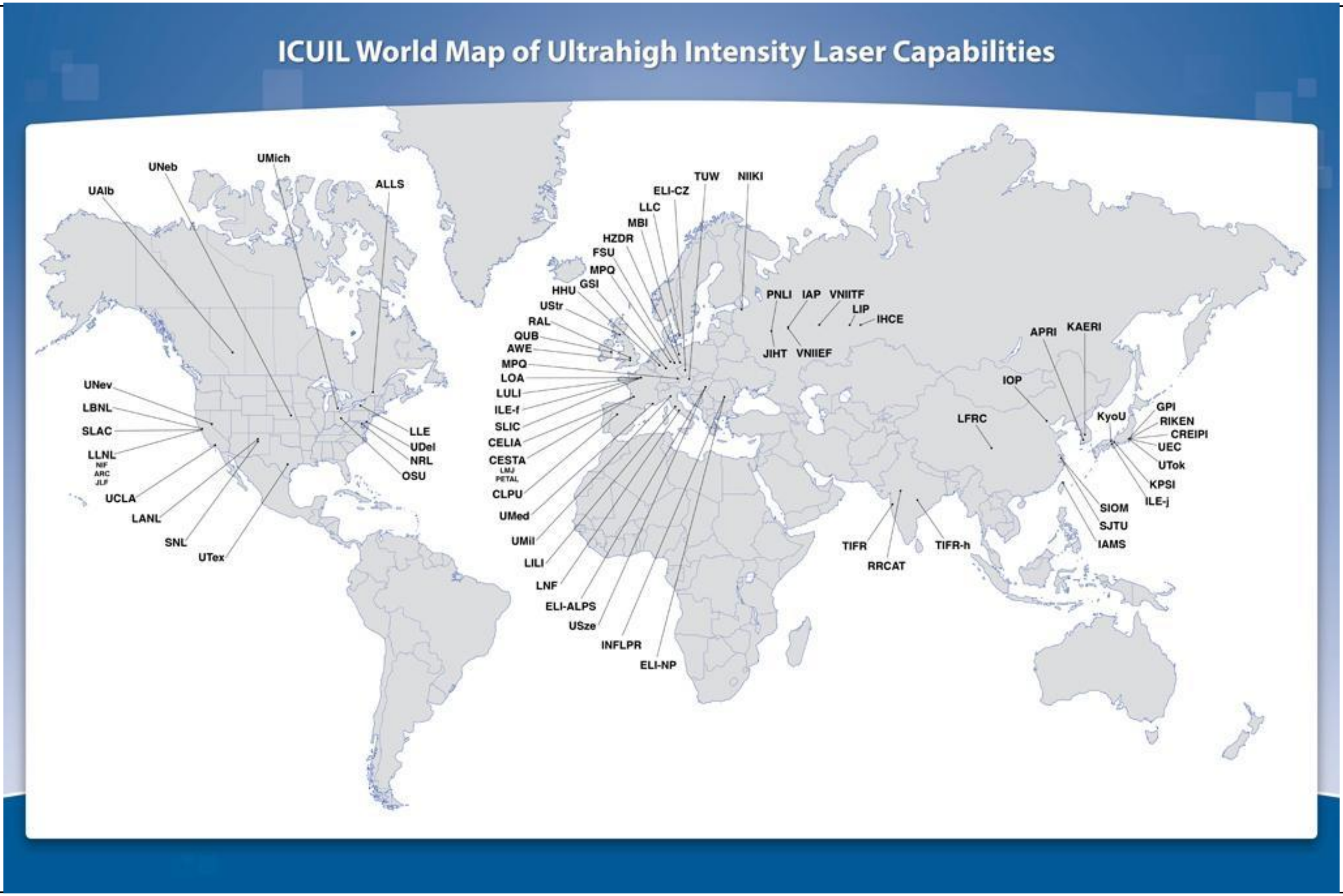
Overview:



High power lasers



UHI lasers world-wide



ICUIL World Map of Ultrahigh Intensity Laser Capabilities

a look back on 50 years of high peak power lasers

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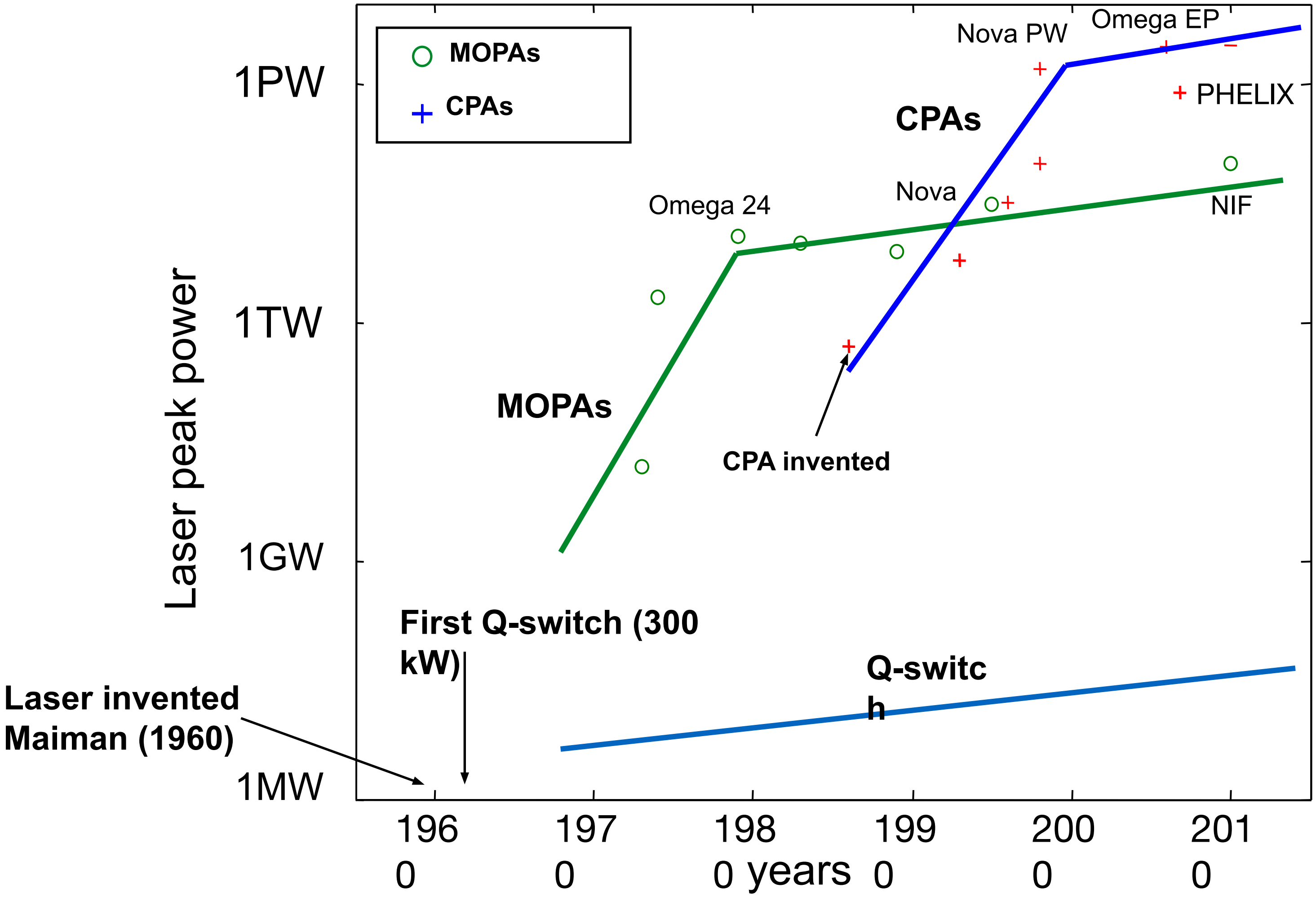


2 TW compares to the 2008 world average electrical power production

Energy, Power, Intensity

Energy [J]

Power = Energy /time W= J/s

Intensity = power / area W / cm^2

High power is crucial for experiments in basic research

Therefore the goal is to quench the energy in time and to focus the energy to a very small spot:

todays lasers:

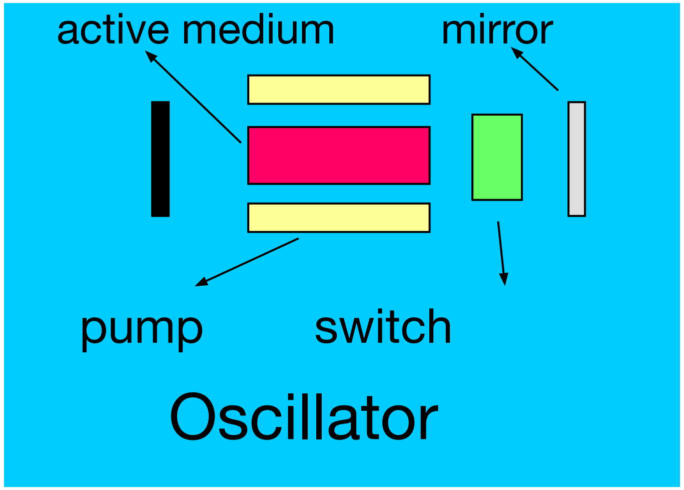
- Energy (from wall plug) MJ -> stored in capacitors in minutes
- Flashlamps pump the medium within microseconds (efficiency ~ 0.1 %)
- Laser light is compressed to 25 fs (0.000 000 000 000 025 s) / factor 10^{15}
  and focused from 50 cm beam diameter down to 5 micrometer / factor 10^9
- Energy is compressed by factor of 10^{24} in space and time!

most powerful laser in the world (so far) (ELI-NP, Romania, 25fs)

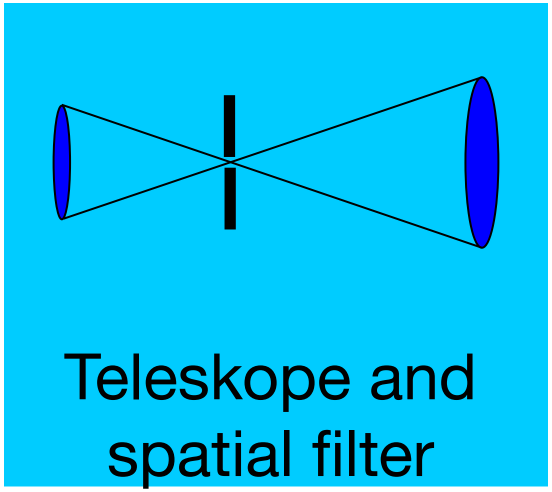
Energy: 2x250 J Power: 20 PW Intensity: 10^{23} W/cm^2

How to do A LOT of laser light

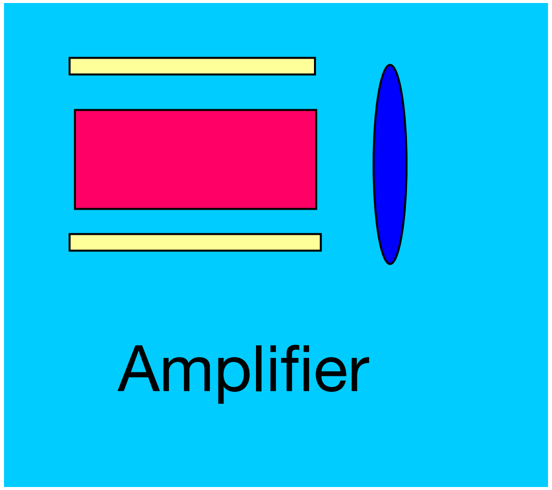
MOPA- scheme:



Oscillator



Teleskope and spatial filter



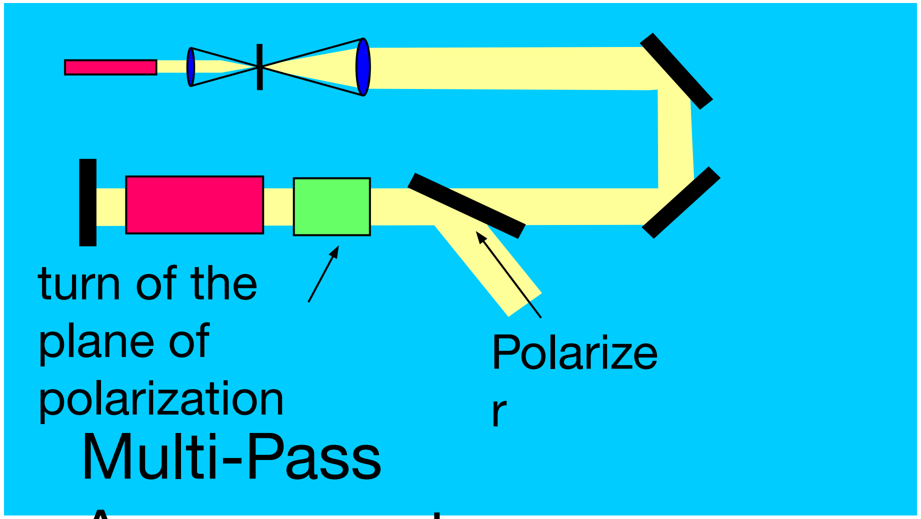
Amplifier

Limits of the amplification:

- Damage threshold of the medium
- non-linear refraction index

$$B = \Delta\Phi = \frac{2\pi}{\lambda}\int I(l)\, n_{nl}\, dl$$

self- filamentation
beam quality



Multi-Pass Arrangement

Time/frequency equivalence with short pulse lasers

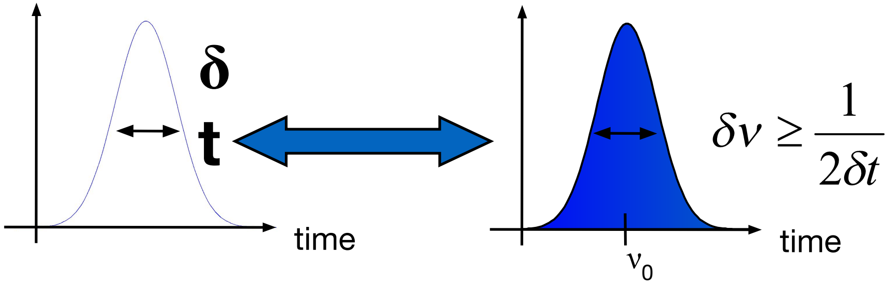
A description of short laser pulses can be made advantageously in the spectral domain

Heisenberg principle

$$\delta\nu.\delta t \geq \frac{1}{2}$$

frequency
~ spectrum width

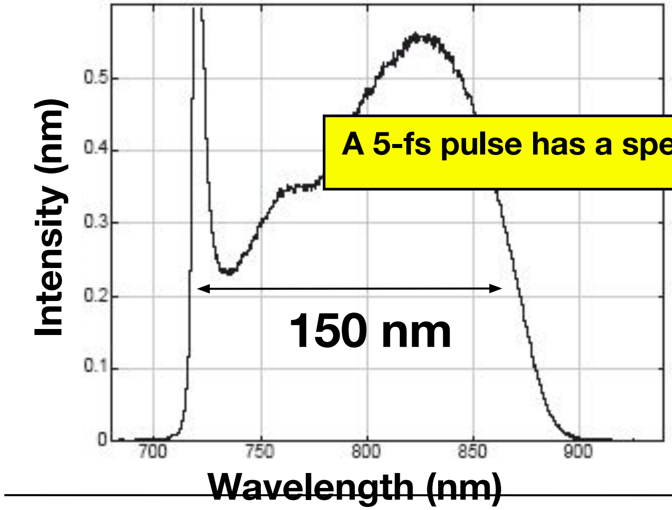
time
~ pulse width



Laser pulse

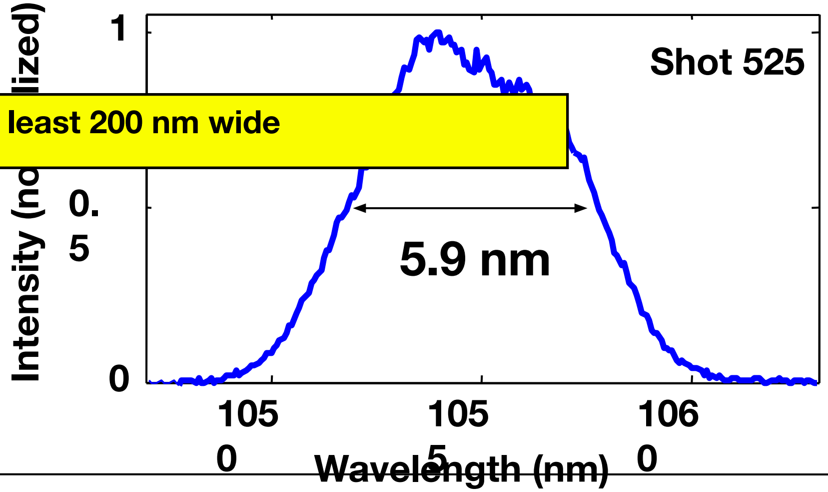
Laser spectrum

Spectrum of a 10 fs oscillator



A 5-fs pulse has a spectrum at least 200 nm wide

Spectrum of 500 fs PHELIX pulse



4.1.5 Oszillator - longitudinale Moden (3)

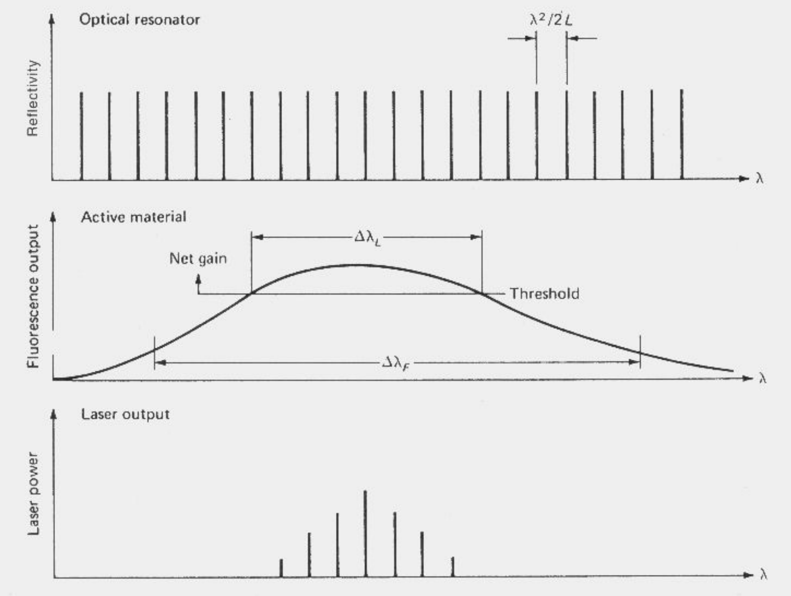
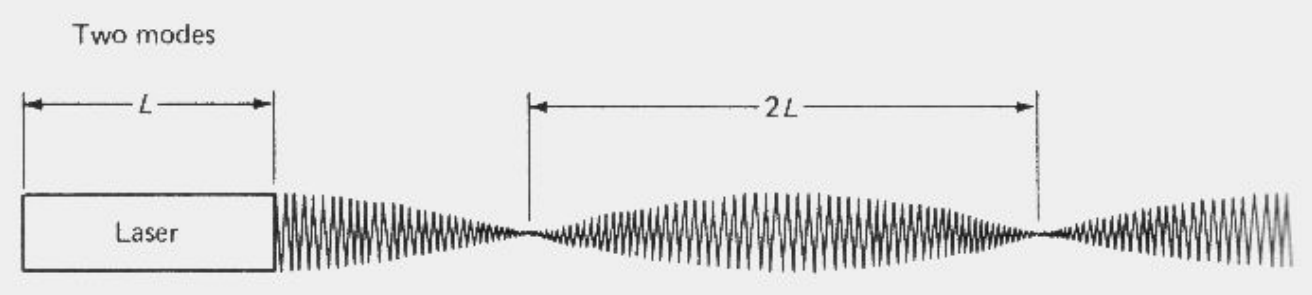


FIGURE 5.18. Schematic diagram of spectral output of a laser without mode selection.

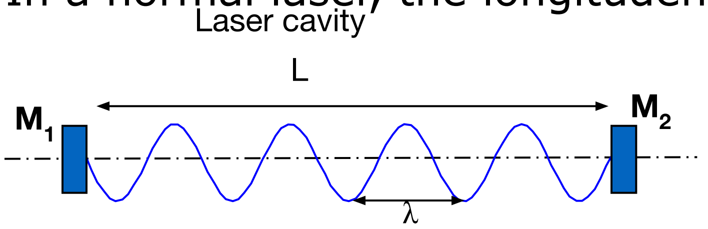
A beam with multiple longitudinal modes is highly modulated.

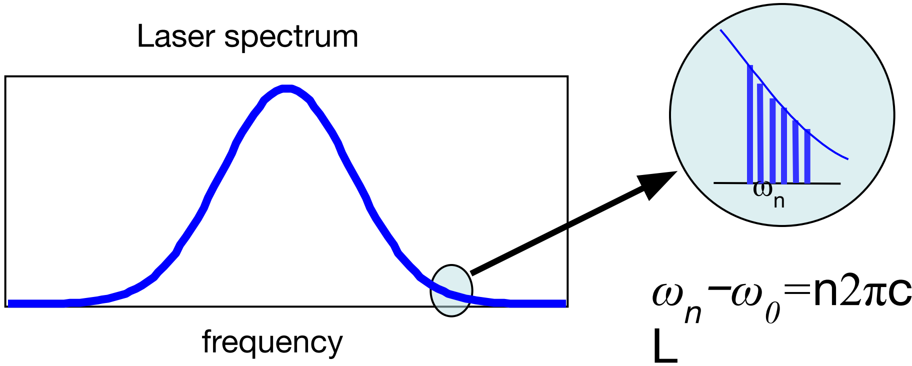


Short pulse generation: the mode-locked laser

The effect of mode locking explained in the spectral domain

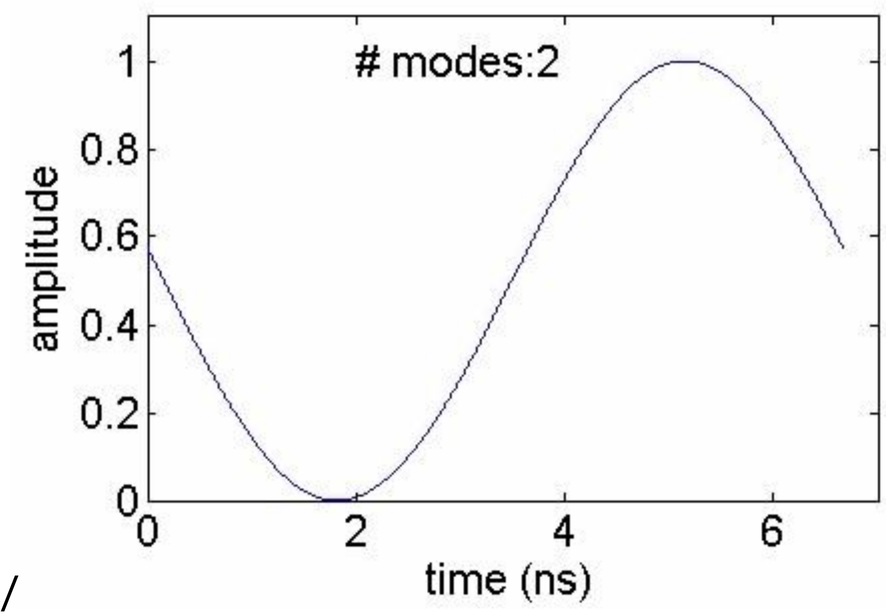
In a normal laser, the longitudenal modes are not in phase





$\omega_n - \omega_0 = n2\pi c/L$

Time intensity of a laser with unlocked modes



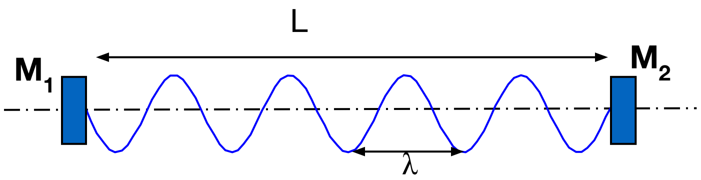
$$E(t) = e^{i\omega_0 t} \sum_n e^{i(\omega_n - \omega_0)t} e^{i\varphi_n}$$

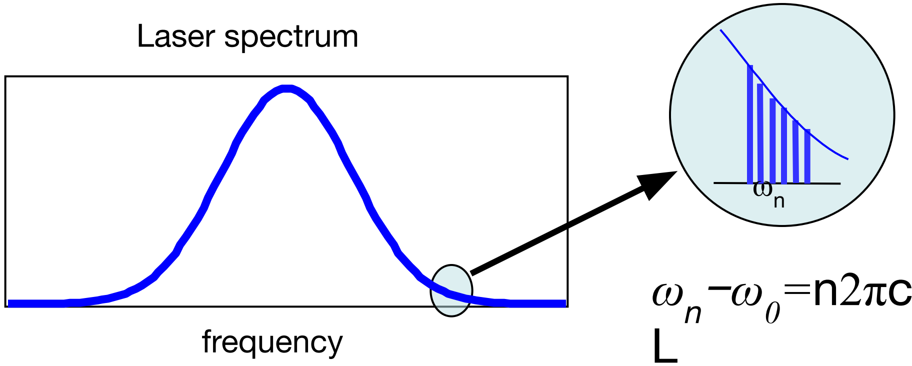
$$\varphi_n = 2\pi rand(\ )$$

Effect of mode locking on the temporal behavior of the laser

In a mode-locked cavity, the phase between modes follows deterministic laws

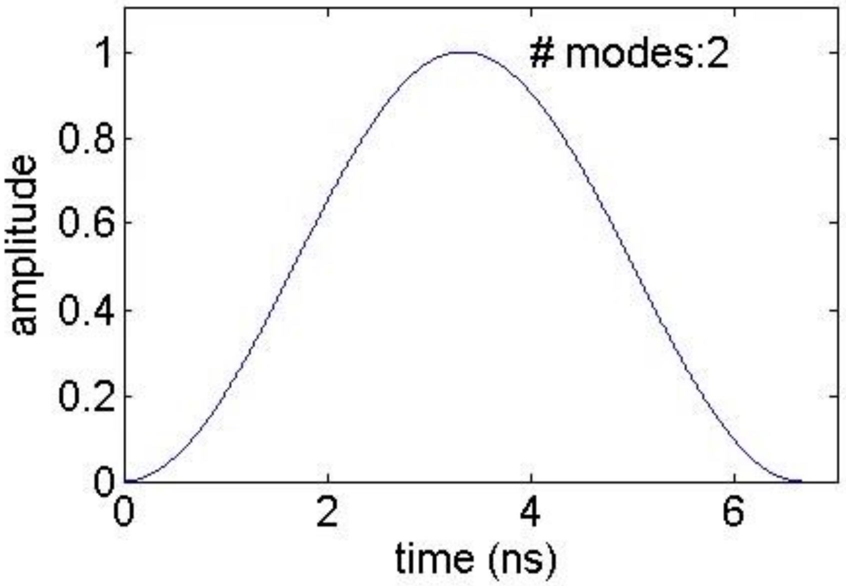
Laser cavity





$\omega_n - \omega_0 = n2\pi c/L$

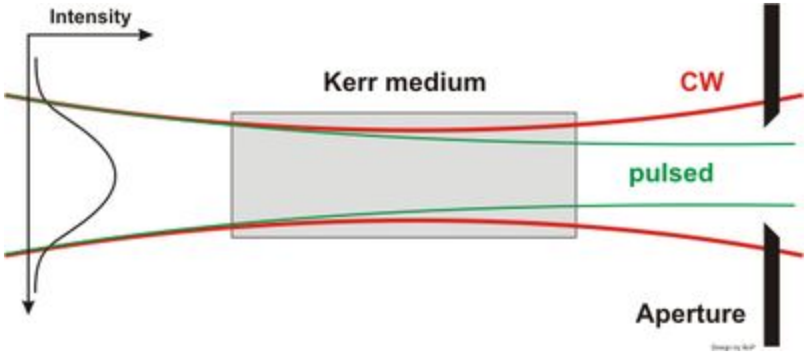
Time intensity of a laser with locked modes



$$E(t) = e^{i\omega_0 t}\sum_n e^{i(\omega_n - \omega_0)t} e^{i\varphi_n}$$

$$\boxed{\varphi_n = CST}$$

Techniques for locking modes together

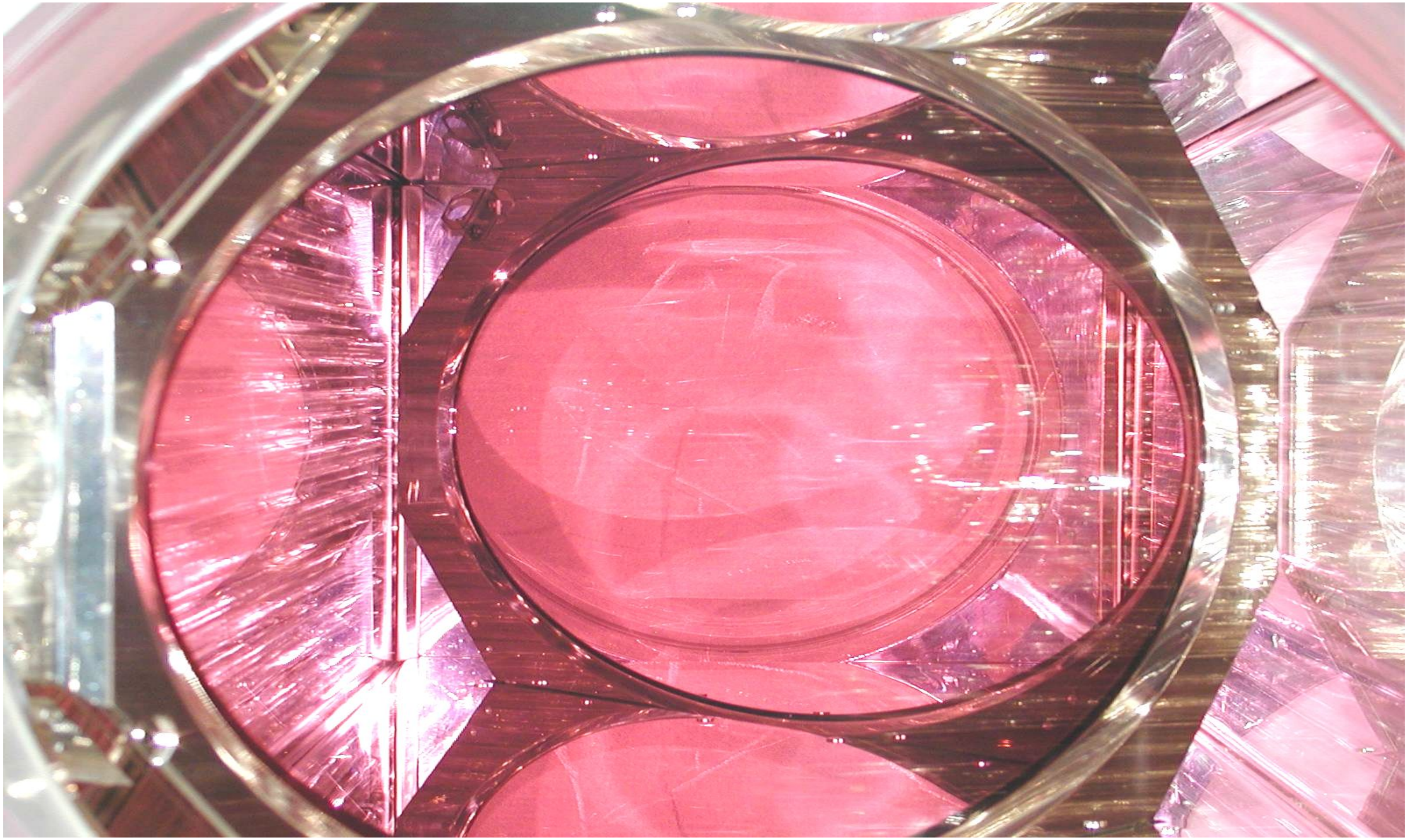


Kerr Lens Mode locking
(operation mode
dependent losses)

- **SESAM (Saturable absorber) that introduces a few % losses in CW mode**

- **AOM that modulates the losses in the cavity (active)**

Part 2: Amplification of short pulses



A view of the PHELIX 31.5 cm amplifier section

Non-linear refraction index

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Strong laser beam in medium: Change in refractive index to first order:

$$n = n_0 + n_2\langle E^2\rangle = n_0 + \gamma I$$

With n_0 as linear and n_2 as nonlinear refractive index and the intensity I.

The intensity I results from the temporal average $\langle E^2\rangle$ in electrostat. units (esu)

$$I = (n_0/4\pi c)\,\langle E^2\rangle$$

Example: (Ti:Saphhire) $n_0 = 1.76$
$n_2 = 3 \times 10^{-20}\ \mathrm{m^2/W}$

In literature the refractive index is shown as n_2 or γ.

$$\gamma\ [\mathrm{cm^2/W}] = (4\pi n_2)/(n_0 c)\ (\times 10^7) = 4{,}19 \times 10^{-3}\ n_2/n_0\ [\mathrm{esu}]$$

Effect on the E-Field:

$$\widetilde{E}(x,t) = \int_{\infty}^{-\infty} E_0(\Delta\omega)\cdot e^{-i(\omega t - kx)}\,d(\Delta\omega) = \int_{\infty}^{-\infty} E_0(\Delta\omega)\cdot e^{-i\omega\left(t - \frac{x}{c}(n_0 + \Delta n(I))\right)}\,d(\Delta\omega)$$

Non-linear refractive index

Example for non-linear refractive:

Nd:Glass: in 10^{-13} esu

Q-246 (Silicate)	LG-670 (Silicate)	Q-88 (Phosphate)	LHG-8 (Phosphate)	LG-760 (Phosphate)
1.4	1.41	1.1	1.13	1.04

	ED-2 (Silicate)	LHG-5 (Phosphate)	EV-1(Phosphate)	Fluorphosphate	
	1.41	1.16	0.91	0.71	
γ=	3.77	3.15	2.53	2.0	[10^{-16} cm^2/W]

The non-linear refractive index has two components acting on different time scales :

Electronic part: Distortion of the electron cloud around the nucleus. Relaxation times are typically a few optical cycles.

Nuclear part: nucleus attempts to minimize local field-matter interaction. Relaxation times: rotation or vibration cycles.

In glass: electronic part 70-85%.

Nonlinear refractive index in glass is smaller for circularly polarized light compared to linear polarized light by about 2/3.

B-Integral

Phase difference of a wave in vacuum and in a medium with index n:

$$\Delta\phi = \frac{2\pi}{\lambda}\int_0^L n(z)dz \qquad (5.3)$$

With λ as vacuum-wavelength and optical path L

$$\Delta\phi = \frac{2\pi}{\lambda}n_0 L + \frac{2\pi}{\lambda}\int \gamma I(z)dz \qquad (5.4)$$

Also constant term Φ_0 and intensity dependent part with phase B:

$$B = \frac{2\pi}{\lambda}\int_0^L \gamma I(z)dz = \frac{8\pi^2\times 10^7}{\lambda c}\int_0^L \frac{n_2}{n_0} I(z)dz \qquad (5.5)$$

B-Integral (Breakup Integral)

B-Integral

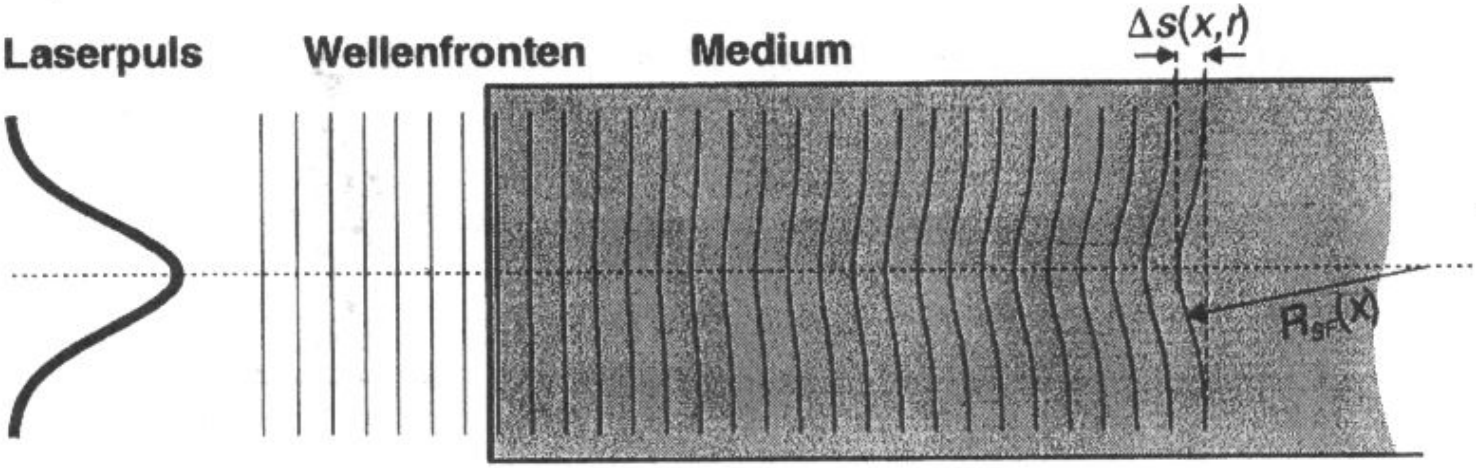
The B-Integral is main parameter describing the non-linear effects.

B-integral creates self-focussing on small and large scales.

For Fusion lasers values of B-Integral exceeding 9 (1.5 wavelength) are not to be tolerated

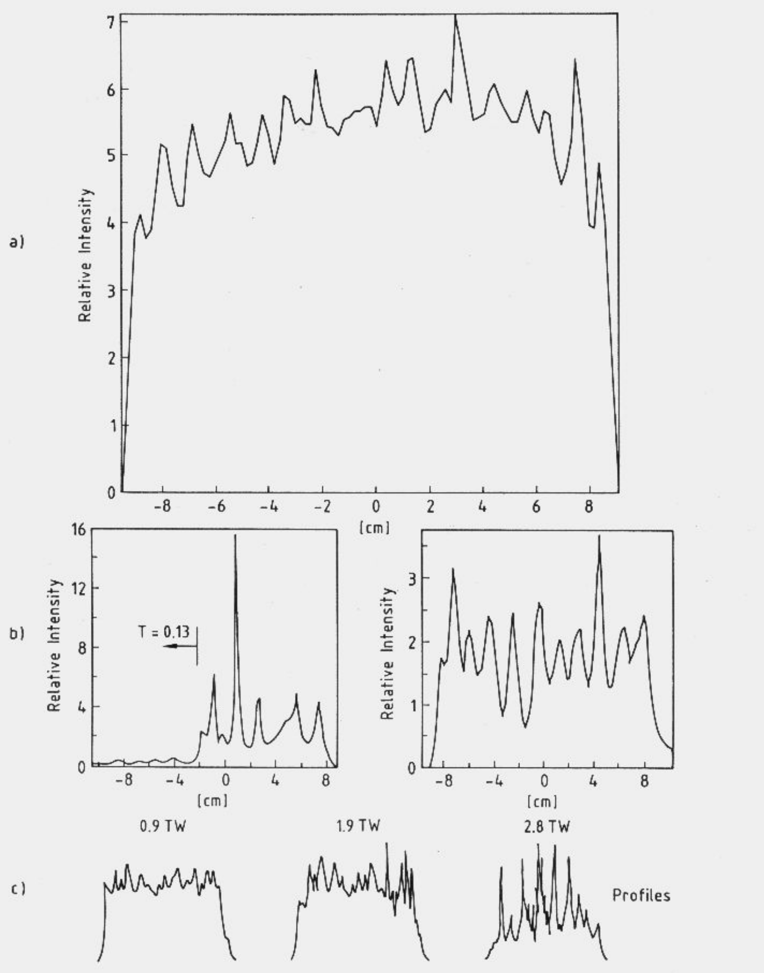
For UHI laser we need to minimize the B-integral

For ultra-short pulses: B-Integral below 1



So much harder to amplify ultra-short pulses because of the B-integral requirement

Small-Scale Self Focusing

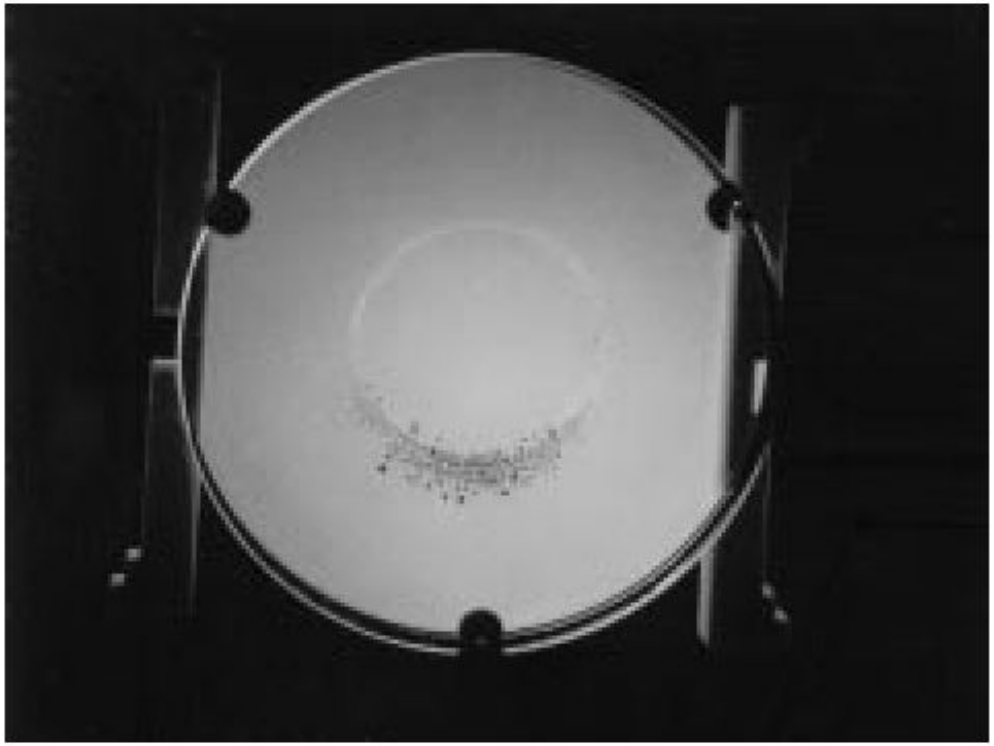


One arm of the SHIVA Laser,
700 J in 600 ps

Relative Intensity of a
Beamline of the SHIVA Laser
at 1.5 TW, before and after the
Installation of spatial filters

Beam profile of a large Nd:glass
laser with beam breakup by
SSSF with increasing beam power

Results of SSSF



G3887

Figure 70.2

A malfunctioning spatial filter pinhole assembly caused the accidental damage to this liquid crystal polarizer component (135-mm diameter).

Damage of laser material

Damage to laser material result in deterioration of the beam profile

Laser systems try to operate at highest possible intensities

Limit: Damage Threshold of the material

Costs proporational to optics aperture (non-linear)

Cause of damage not fully understood so far

Many possible reasons

Lots of experimental data

Transparent media
Idea: Free electrons by multi-photon ionization and avalanche-like grow of electrons

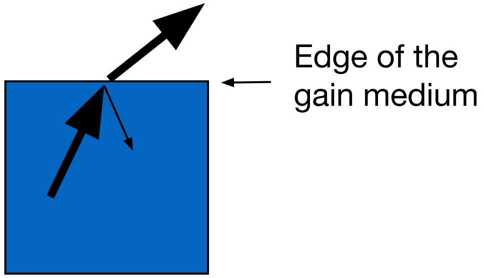
Plasma transports absorbed energy in surrounding material.

Damage in bulk, at the surface or at dielectric coatings

How far can one go with amplification?

Single-pass amplification is limited by fluorescence

- → Longitudinally, amplification is creating an ASE pedestal
- → Transversally, the fluorescence is depleting the gain medium
  - → Transverse gain parameters (F_{stored}/F_{sat} or $g_0 l$) of 3 to 4 are maximum (G_0 of 20 to 50)
  - → High gain materials are not favorable to energy storage
  - → Large width/length ratio are prohibited



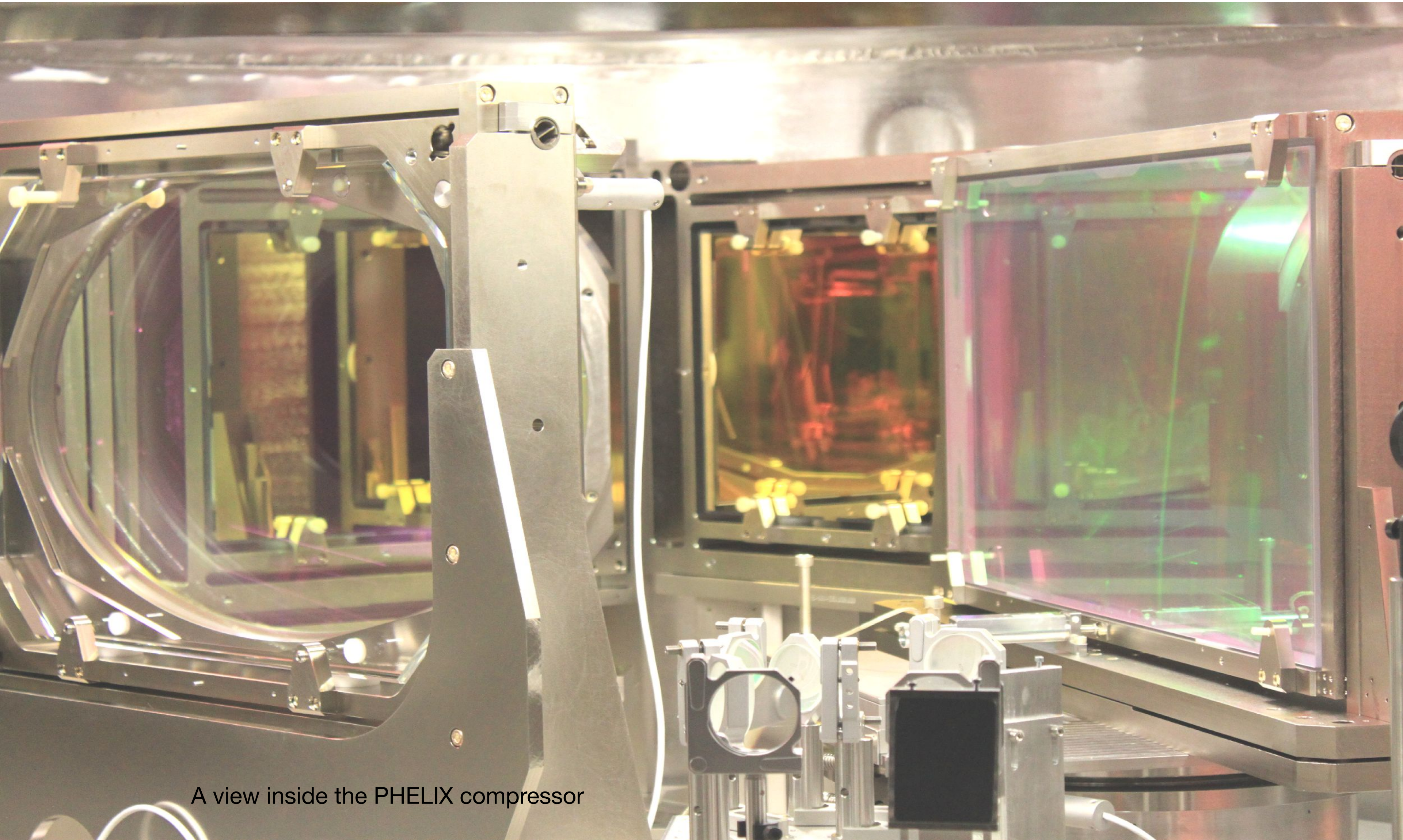
PHELIX
Single pass gain:10,
Aspect ratio : 0.7

Amplification in multi-pass is a solution to mitigate problems

In the end $E_{\max} \approx F_{saturation} .S$

A ride on the beamline of the National Ignition Facility

Part 3: Manipulation of short Pulses



A view inside the PHELIX compressor

Introduction to the spectral phase

In the spectral domain, the electric field may be defined by its complex amplitude and a phase around the central frequency ω_0

$$\phi(\omega)=\phi(\omega_0)+(\omega-\omega_0)\frac{\partial\phi}{\partial\omega}(\omega_0)+\frac{(\omega-\omega_0)^2}{2}\frac{\partial^2\phi}{\partial\omega^2}(\omega_0)+O(\omega^3)$$

Characterizes the phase velocity

Characterizes the group velocity

CEP

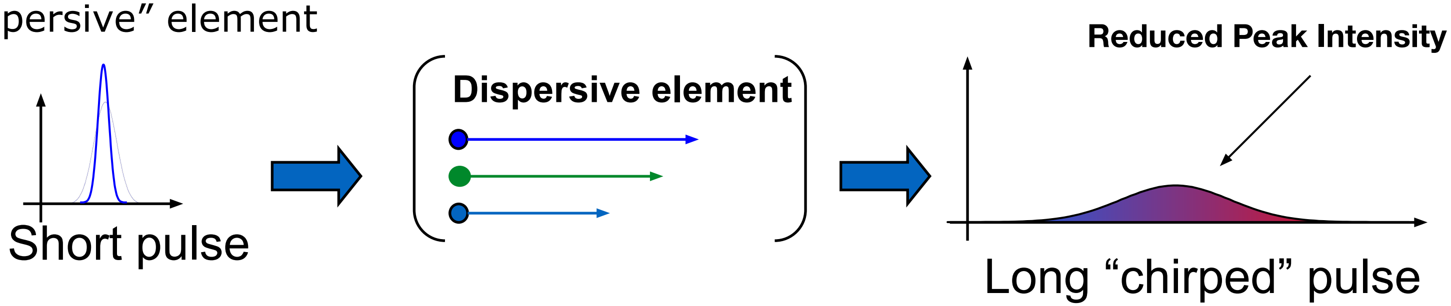
Linear chirp (change) of pulse duration

Higher order terms (pulse distortions)

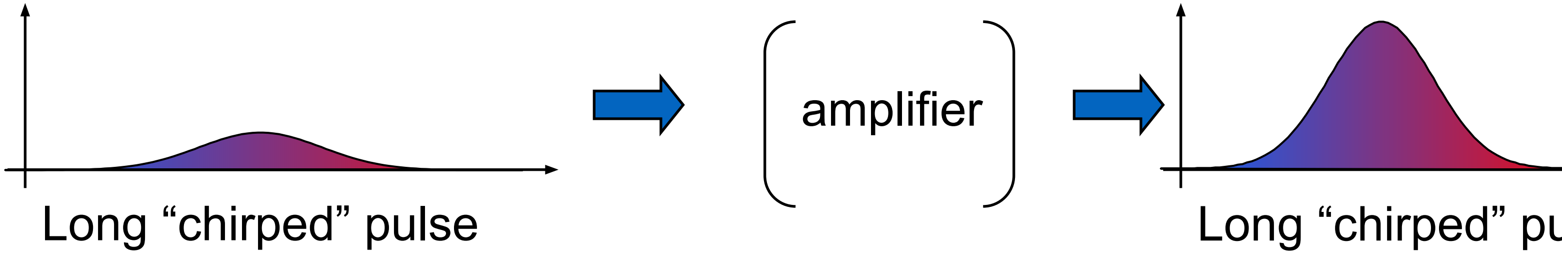
The natural or controlled dispersion of laser components is used to modify the shape of laser pulses

The best example of spectral phase modification is the CPA

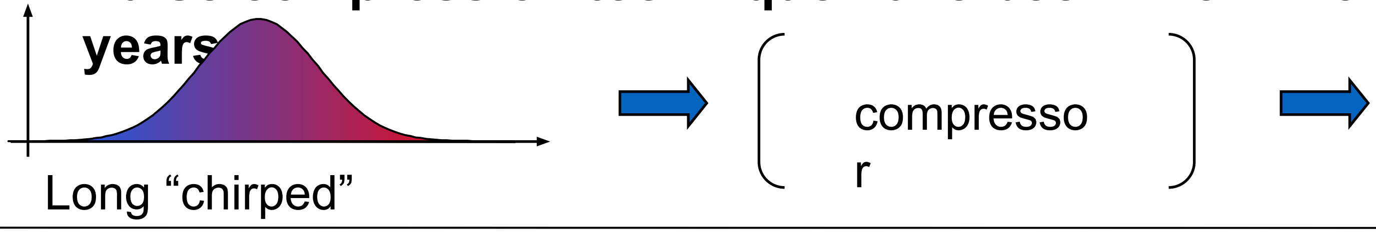
A short laser pulse can be chirped (temporally stretched) into a longer pulse using a "dispersive" element

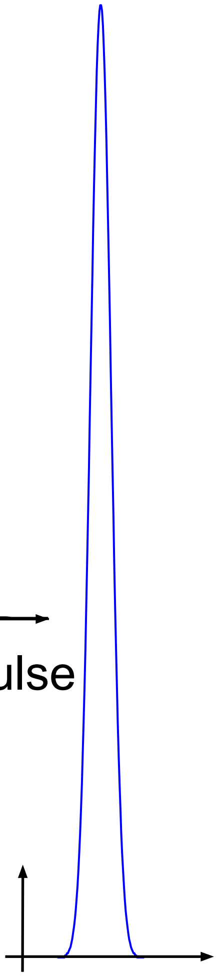


- **A long pulse can be amplified without the risk of damage**

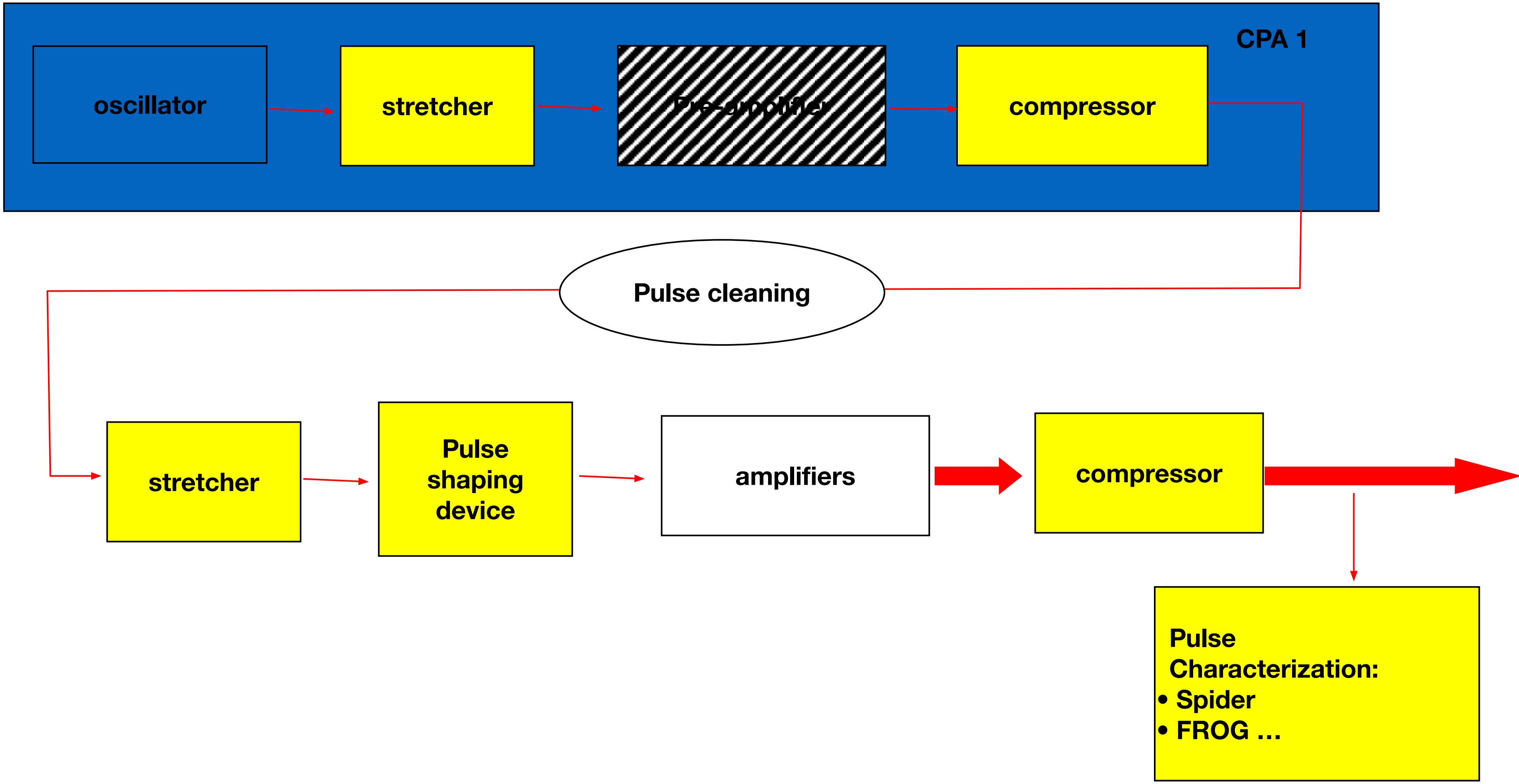


- **Pulse compression technique have been known for 35 years**





Modern PW short pulse lasers incorporate mostly pulse shaping devices



CPA 1

oscillator

stretcher

Pre-amplifier

compressor

Pulse cleaning

stretcher

Pulse shaping device

amplifiers

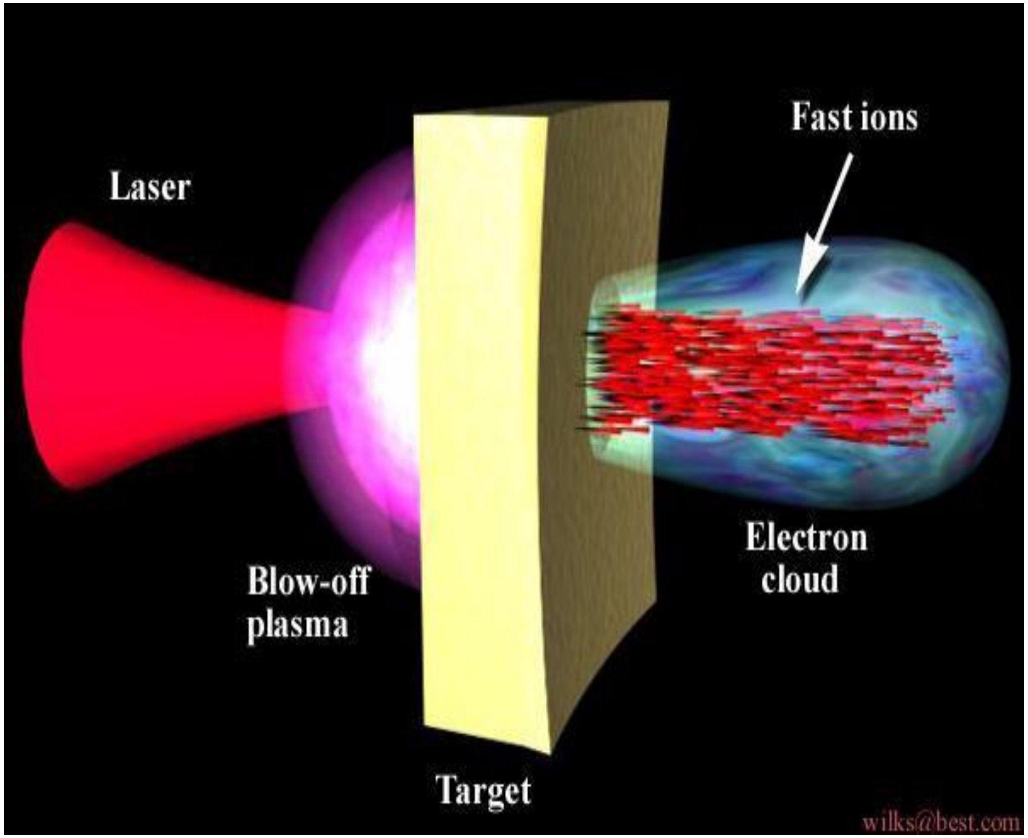
compressor

Pulse Characterization:
- **Spider**
- **FROG …**

Example of a big CPA system

Relativistic Plasma physics

Basics



The Petawatt opens a new regime of ultra-relativistic laser-matter interactions

Enormous EM fields

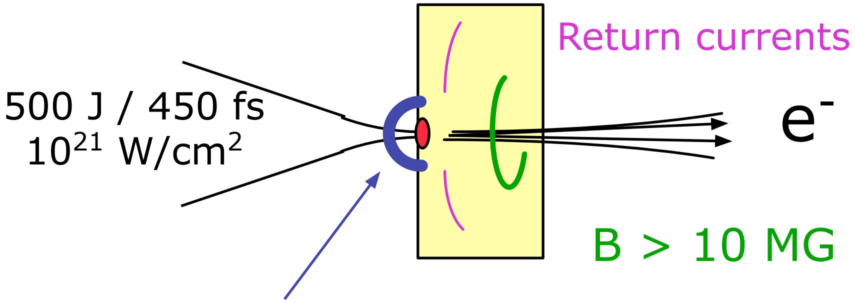
at $I = 10^{21}$ W/cm^2 :

$E \sim I^{1/2} \lambda = 10^{14}$ V/m

$B = E/c = 3\mathrm{x}10^{5}$ Tesla

$P_{rad} = I/c = 3\mathrm{x}10^{10}$ J/cm^3

$= 300$ GBar



500 J / 450 fs
10^{21} W/cm^2

Return currents

e^-

B > 10 MG

Pre-formed plasma

$\Phi_{sc} \sim$ MV

Laser-plasma effects

filamentation
self-focusing
plasma acceleration
» dependence on pre-pulse

Relativistic Electron Motion

Cycle-averaged oscillation energy:

$$E_{avg} = mc^2[1+ a_o^2/2]^{1/2}$$

$$a_o = eA/mc^2 = [I/(1.37\mathrm{x}10^{18})]^{1/2}\lambda(\mu m)$$

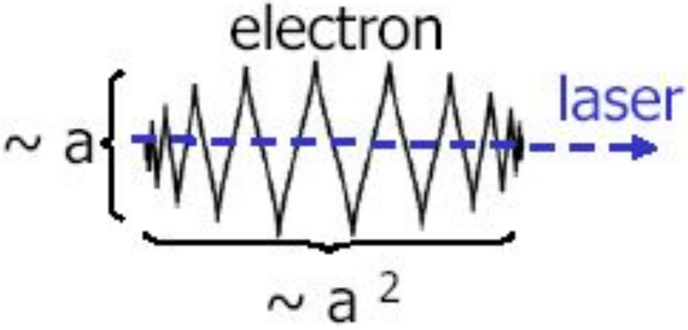
at 10^{21} W/cm^2, $a_o \sim 27$, $E_{avg} > 10$ MeV

Electron & bremsstrahlung energies can greatly exceed nuclear excitation & pair-creation thresholds

Electrons in a laser field

Electron trajectory in plane laser pulse

$$\vec{F} = -e\Big[\underbrace{\vec{E}}_{\sim a} + \underbrace{(\vec{v} / c)\times \vec{B}}_{\sim a^{2}}\Big]$$

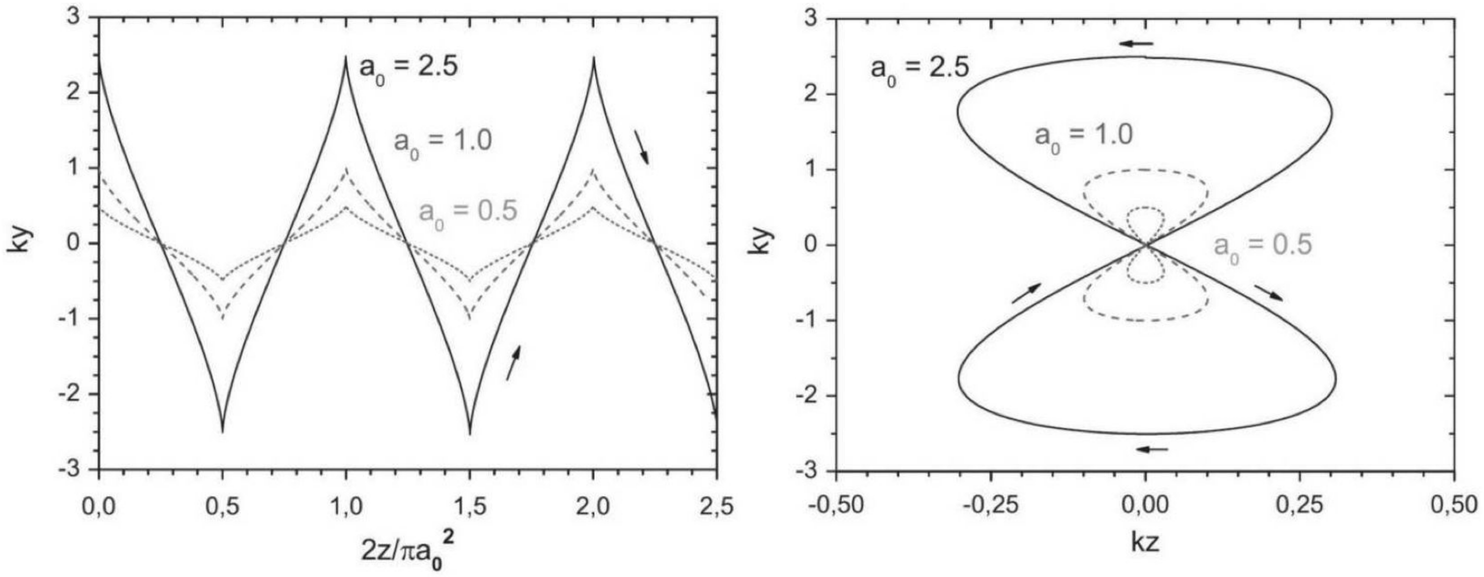


$a = eA/mc^{2}$

laser amplitude

Relativistic plasmas

drift in rel. laser field: compare laboratory system and co-moving system



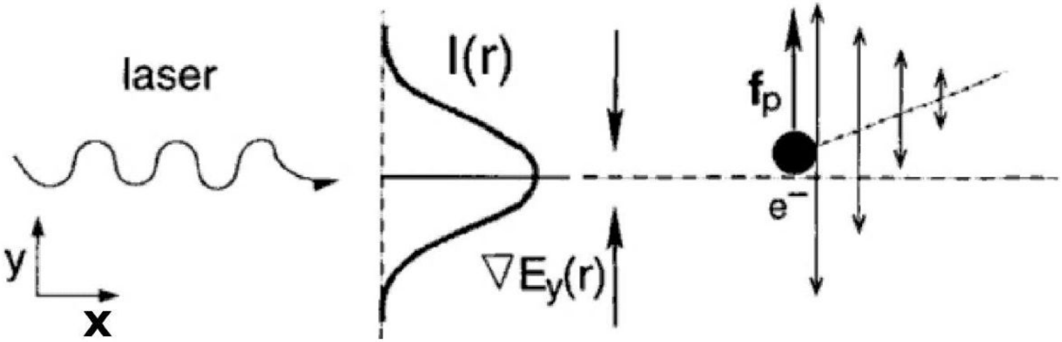
Lawson-Woodward Theorem!

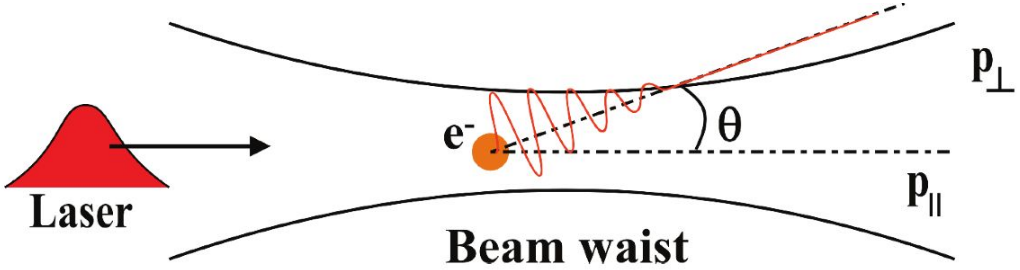
Relativistic plasmas

ponderomotive acceleration of a free electron

$$\boxed{\vec{f}_p = -\frac{e^2}{4m_e\omega_L^2}\nabla\langle\vec{E}\cdot\vec{E}^*\rangle}$$

$$\boxed{\phi_p \propto \langle\vec{E}\cdot\vec{E}^*\rangle = (\gamma-1)m_e c^2}$$





$$p_\parallel = \frac{E_p}{c} = (\gamma-1)m_e c$$

angular distribution:
parallel, longitudinal momentum component is conserved

$$p_\parallel = \frac{p_\perp^2}{2m_e c}$$

relation to perpendicular

angular distribution: $\tan(\theta) = \frac{p_\perp}{p_\parallel} = \sqrt{\frac{2}{\gamma-1}}$

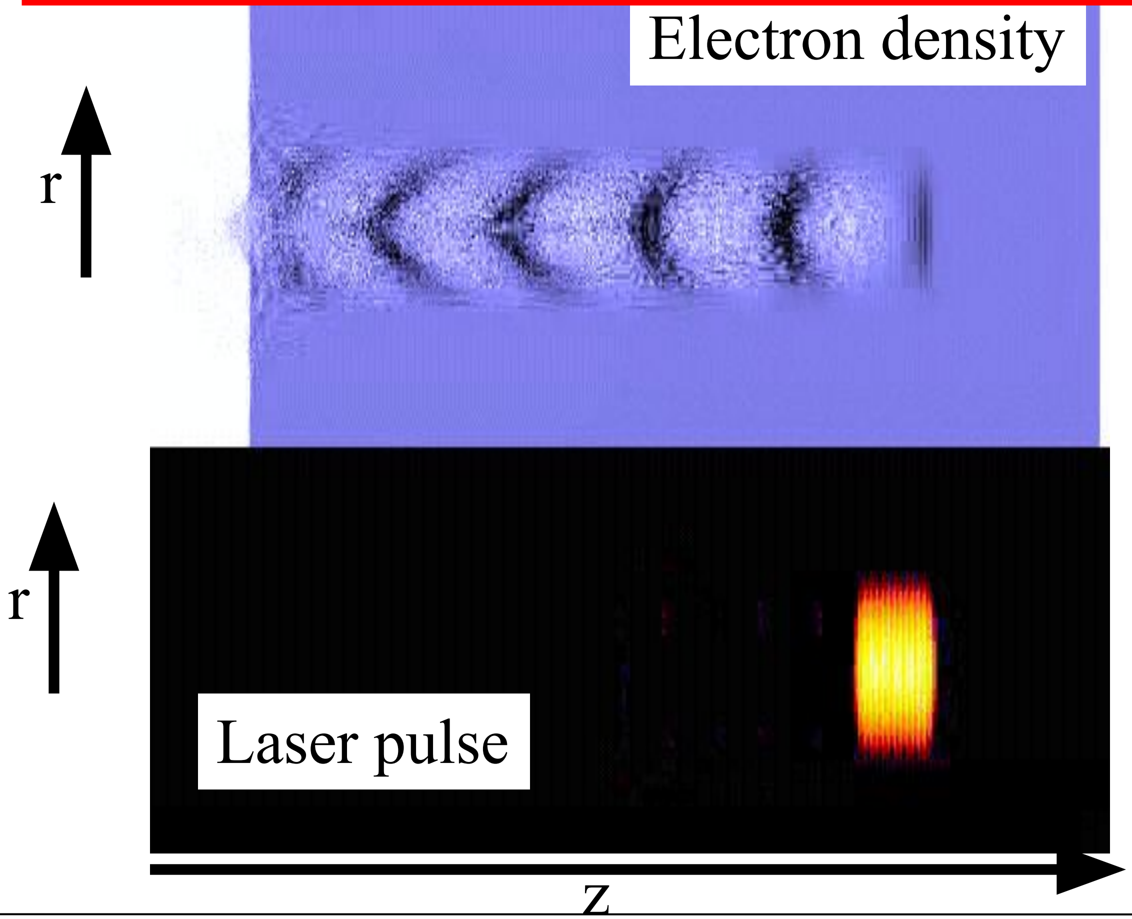
the higher the gamma-factor, the smaller the emission angle

„Surfing“ on plasma waves – Wake Field Acceleration

2D PIC Resonant LWFA Simulation

A. Pukov et al, MPI - Quantenoptic, Germany

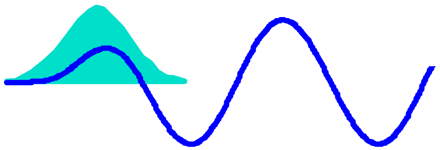
$I=4 \times 10^{18}$ W/cm^2, t=70 fs, ne=3×10^{17} cm^{-3}



Electron acceleration

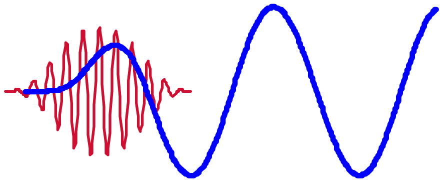
- Plasma Wake Field Accelerator(PWFA)

  A high energy electron bunch



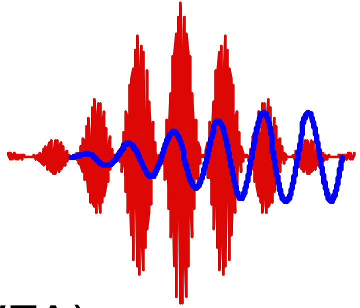
- Laser Wake Field Accelerator(LWFA)

  A single short-pulse of photons



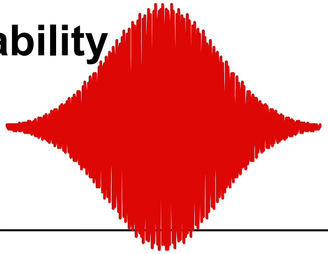
- Plasma Beat Wave Accelerator(PBWA)

  Two-frequencies, i.e., a train of pulses

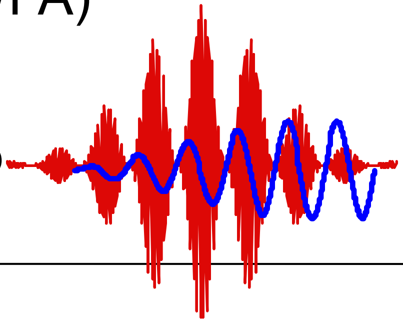


- Self Modulated Laser Wake Field Accelerator(SMLWFA)

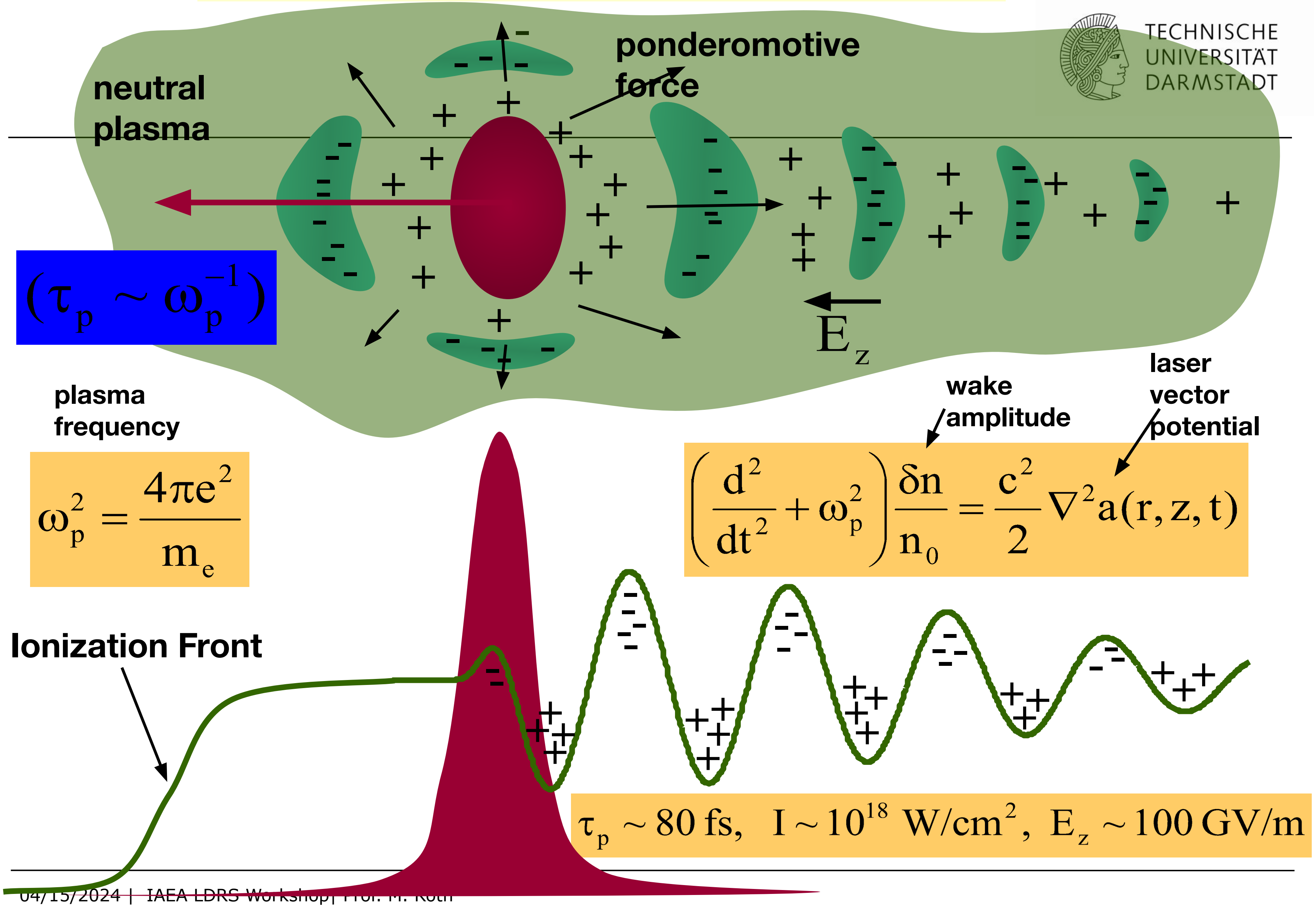
  Raman forward scattering instability



evolves to



Resonant Laser Wakefield Generation:



$$(\tau_p \sim \omega_p^{-1})$$

plasma frequency

$$\omega_p^2 = \frac{4\pi e^2}{m_e}$$

$$\left(\frac{d^2}{dt^2} + \omega_p^2\right)\frac{\delta n}{n_0} = \frac{c^2}{2}\nabla^2 a(r,z,t)$$

$$\tau_p \sim 80\text{ fs},\quad I \sim 10^{18}\text{ W/cm}^2,\quad E_z \sim 100\text{ GV/m}$$

For the optimum plasma condition, $\lambda_p = \pi\sigma_z$

Diffraction limitation:

$$L_{diff} = \pi Z_R = \frac{\pi^2 r_0^2}{\lambda_0}$$

$$\Delta W_{dif}[\mathrm{GeV}] \cong 0.85 P[\mathrm{TW}]\lambda_0[\mu\mathrm{m}]/\left(\gamma_0 \tau_L[\mathrm{fs}]\right)$$

Dephasing limitation:

$$L_d[\mathrm{cm}] = 0.18\times 10^{-4}\,\tau_L^3[\mathrm{fs}]\gamma_0/\lambda_0^2[\mu\mathrm{m}]$$

$$\Delta W_d[\mathrm{GeV}] \cong 0.01 P[\mathrm{TW}]\tau_L^2[\mathrm{fs}]/r_0^2[\mu\mathrm{m}]$$

Pump depletion:

$$L_{pd}[\mathrm{cm}] = 1.06\times 10^{-6}\,\tau_L^3[\mathrm{fs}]r_0^2[\mu\mathrm{m}]\gamma_0^3/\left(\lambda_0^4[\mu\mathrm{m}]P[\mathrm{TW}]\right)$$

$$\Delta W_{pd}[\mathrm{GeV}] = 0.91\times 10^{-3}\,\tau_L^2[\mathrm{fs}]\gamma_0^2/\lambda_0^2[\mu\mathrm{m}]$$

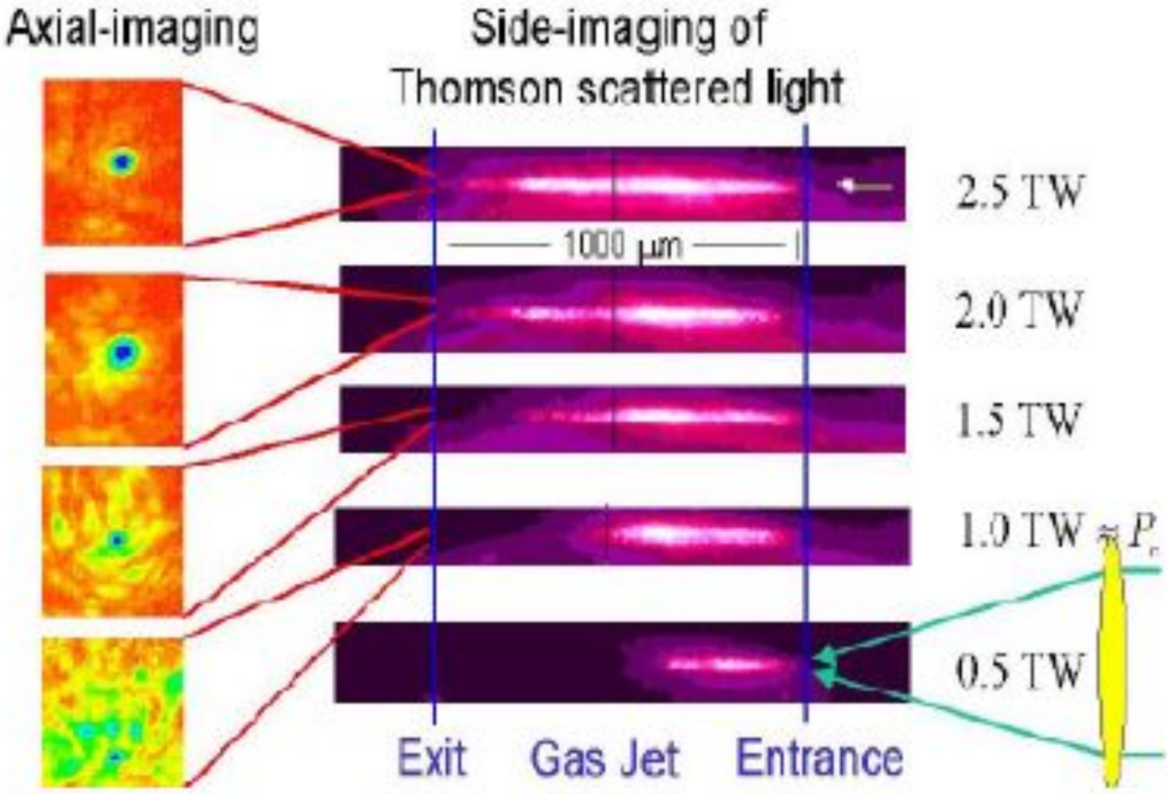
e.g.
$\lambda_0 = 0.8\mu\mathrm{m}$,
$P = 2\mathrm{TW}$,
$\tau = 100\mathrm{fs}$,
$r_0 = 10\mu\mathrm{m}$
$Z_R \cong 0.4\mathrm{mm}$
$a_0 = 0.77(\gamma_0 = 1.14)$
$n_0 \cong 3.5\times 10^{17}\mathrm{cm}^{-3}$
$\lambda_p \cong 60\mu\mathrm{m}$

Diffraction limit:
$L_{diff} \cong 1.2\mathrm{mm}$
$\Delta W_{dif} \cong 12\,\mathrm{MeV}$

Dephasing limit:
$L_d \cong 32\mathrm{cm}$
$\Delta W_d \cong 2\,\mathrm{GeV}$

Pump depletion limit:
$L_{pd} \cong 192\mathrm{cm}$
$\Delta W_{pd} \cong 18.5\,\mathrm{GeV}$

Scaling


Axial-imaging
Side-imaging of
Thomson scattered light
2.5 TW
1000 µm
2.0 TW
1.5 TW
1.0 TW ≈ P_c
0.5 TW
Exit
Gas Jet
Entrance

Limit: Wave breaking

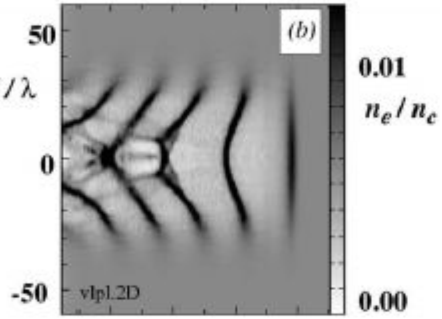
Wave breaking E-field

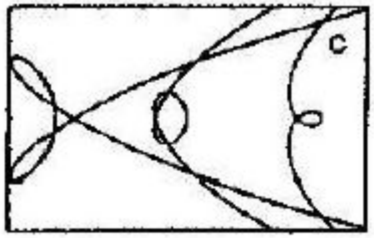
$$E_{wb} / E_0 = \sqrt{2(\gamma_{ph} - 1)} \propto \sqrt{\omega_L / \omega_p}$$

Akhieser, Polovin (1956)

2D wave breaking

Bulanov et al.
PRL 78 (1997)

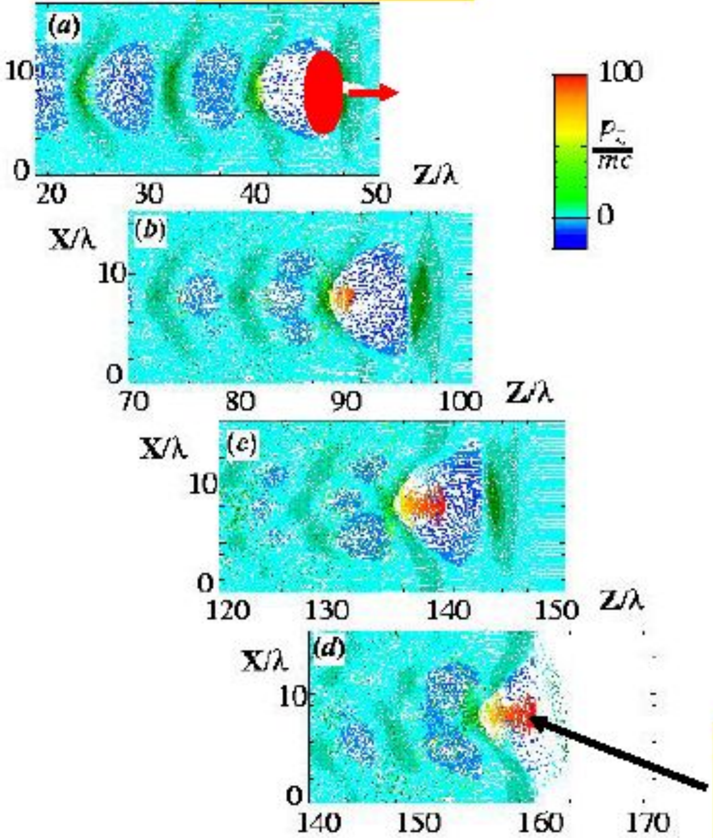




Generating monenergetic bunches

Pukhov, MTV Appl.Phys. B (March 2002)

Laser pulse



Trapped electrons: colour ~ p_z/mc

Plasma density:

3.5×10^{19} /cm^3

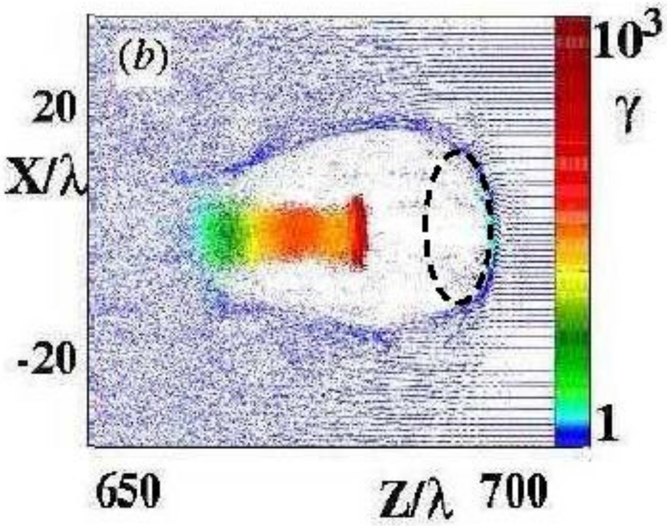
Laser pulse :

6.6 fs
20 mJ
3 TW
$a_0=1.7$

„Bubble Regime“

Wakefield Bubbles

Pukhov, MtV, Appl.Phys. B (March 2002)



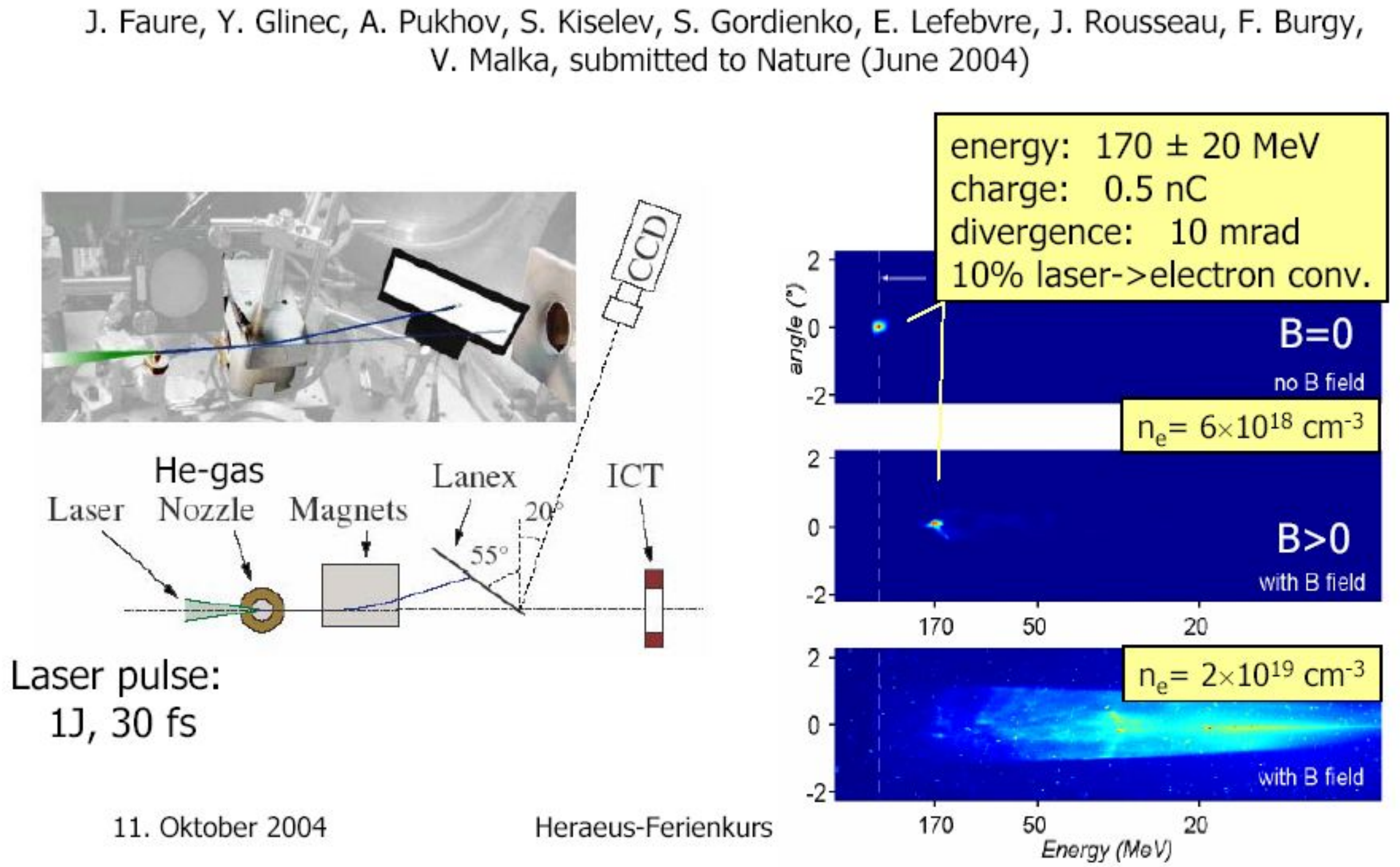
Plasma density:

1.0×10^{19} /cm^3

Laser pulse :

33 fs
12 J
350 TW
a=10

Current record: around 10 GeV



J. Faure, Y. Glinec, A. Pukhov, S. Kiselev, S. Gordienko, E. Lefebvre, J. Rousseau, F. Burgy, V. Malka, submitted to Nature (June 2004)

energy: 170 ± 20 MeV
charge: 0.5 nC
divergence: 10 mrad
10% laser->electron conv.

$n_e = 6\times10^{18}\ \text{cm}^{-3}$

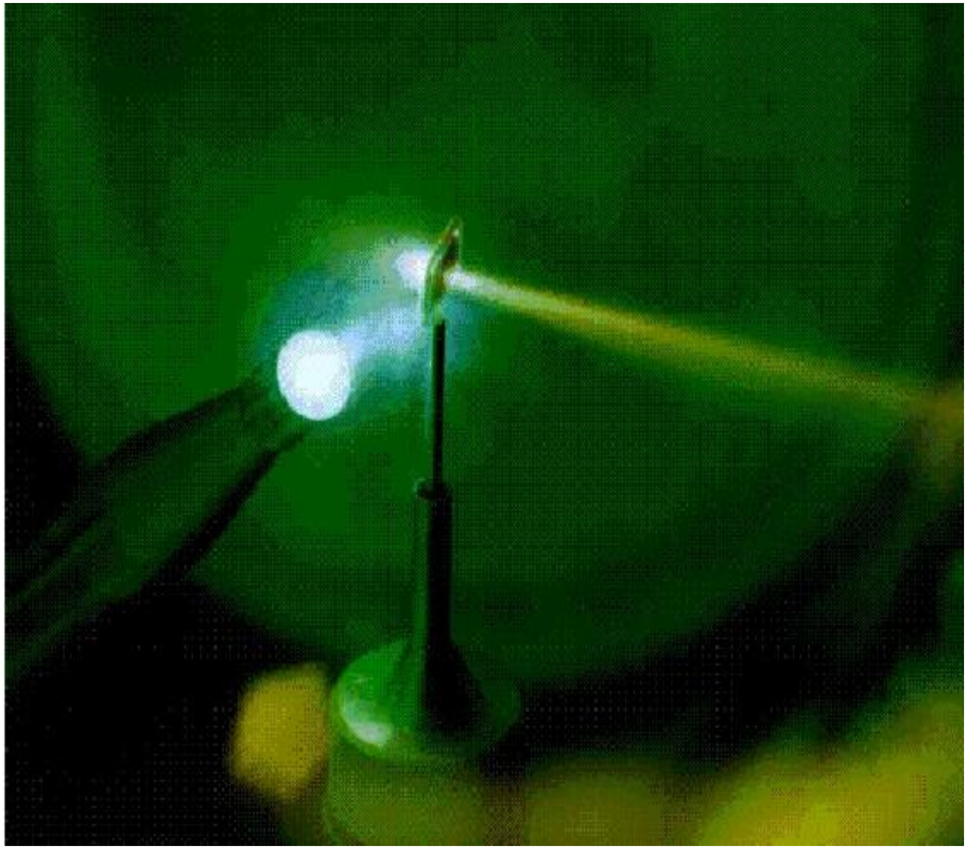
$n_e = 2\times10^{19}\ \text{cm}^{-3}$

Laser pulse:
1J, 30 fs

11. Oktober 2004

Heraeus-Ferienkurs

Ion Beams



Petawatt experiments
3/99

Number of accelerated protons:
10^{13} (LLNL- Petawatt)

Pulse duration:
several Picoseconds

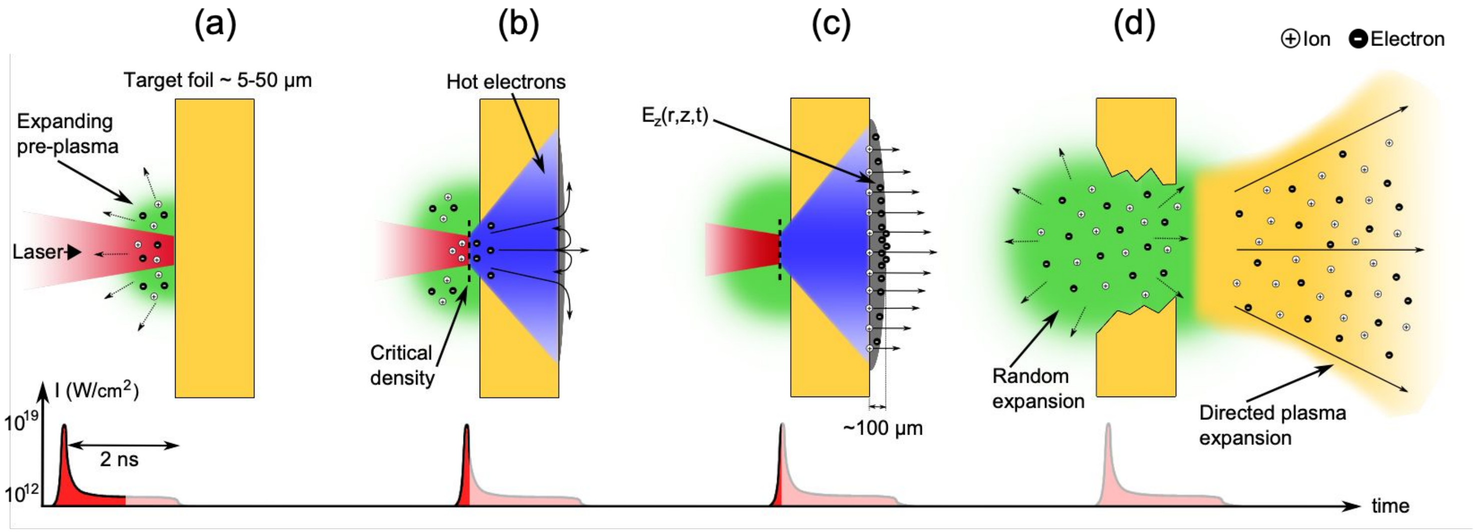
Maximum energy:
60 MeV (LLNL Petawatt)

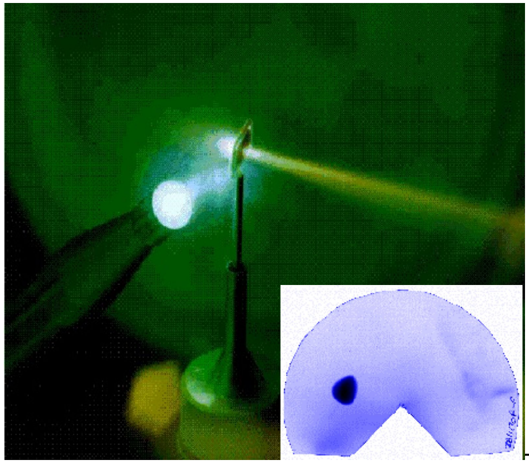
Divergence:
<10° for high energy part

always normal to the rear surface

origin of protons: surface contaminants

Proton acceleration with lasers : Static electric fields

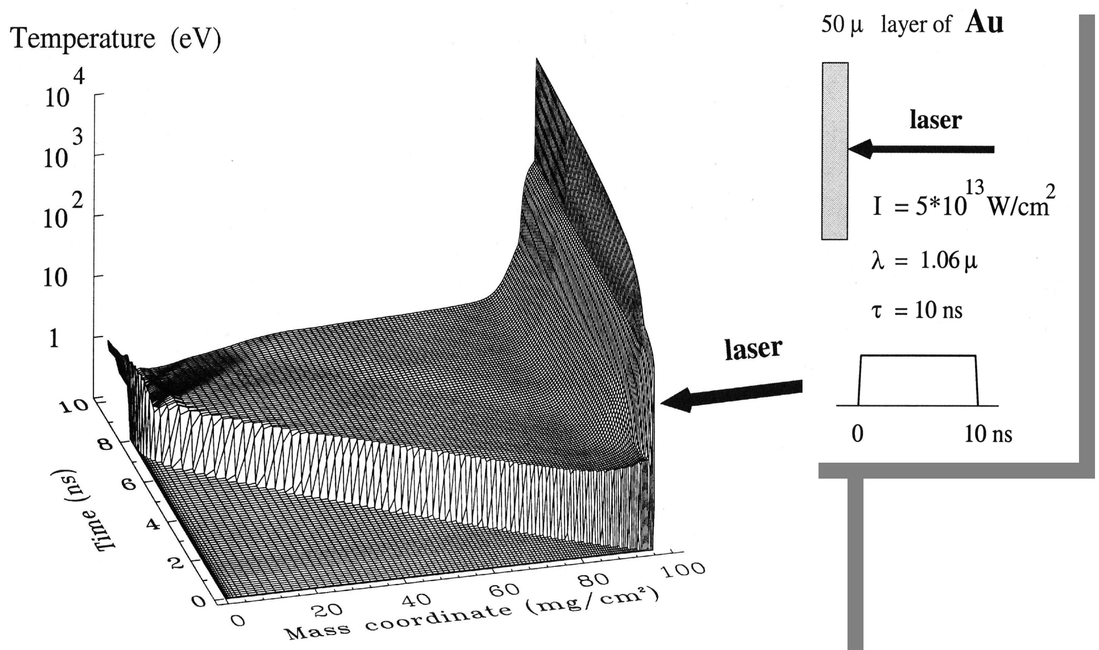


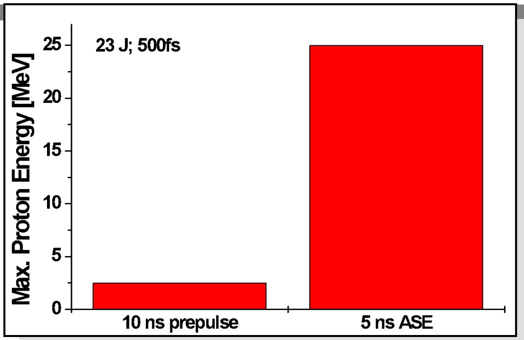


TV/m fields lead to:

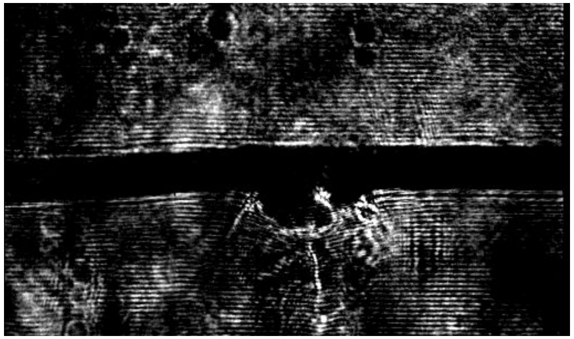
ps pulse duration
excellent beam quality
high particle numbers
no space charge

Initial Target Conditions may be altered by pre-pulse



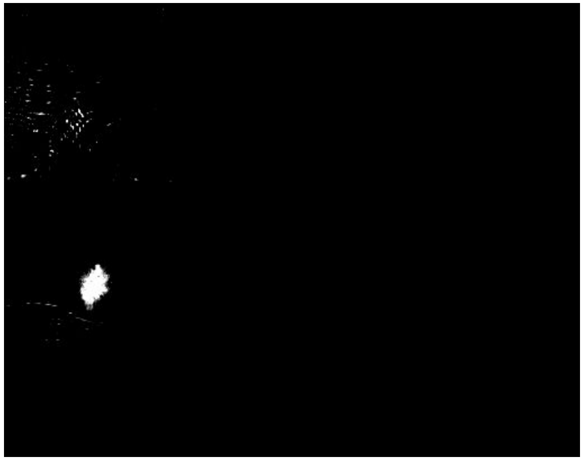


Pre-pulse induced shock wave must not disturb the rear surface



48 mum Au, 21 J, t= +2ps

Protons



125 mum CH, 22 J, t=+4ps

No Protons

High contrast Lasers (PHELIX)

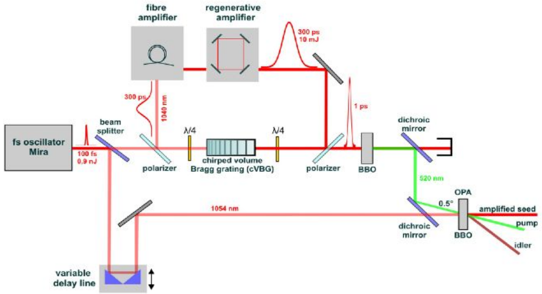
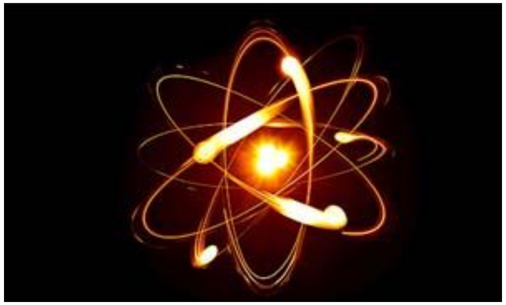
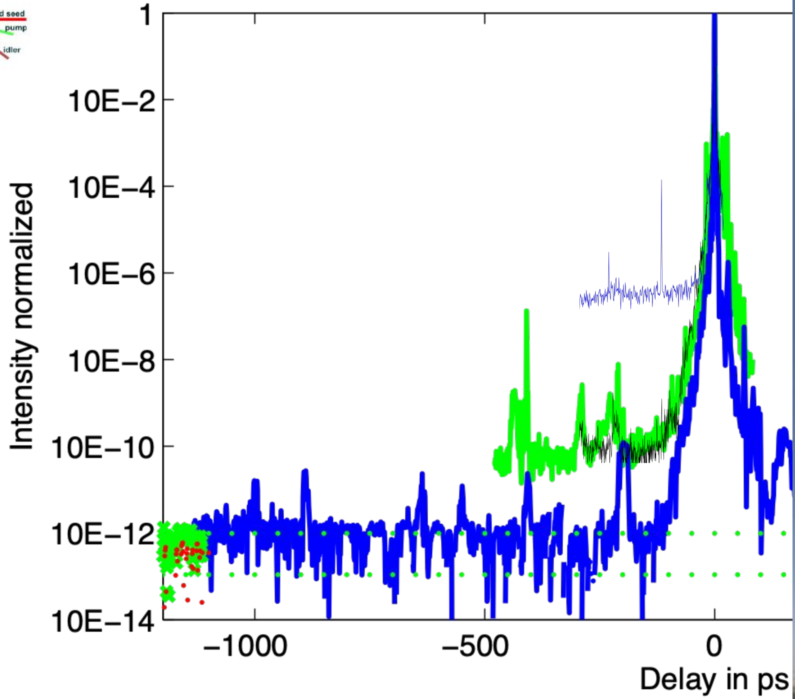


Fig. 2: Setup of the contrast-boosting module



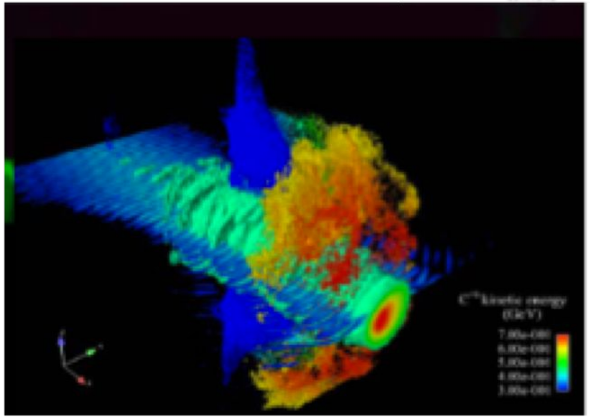
Over an area of 50 km!

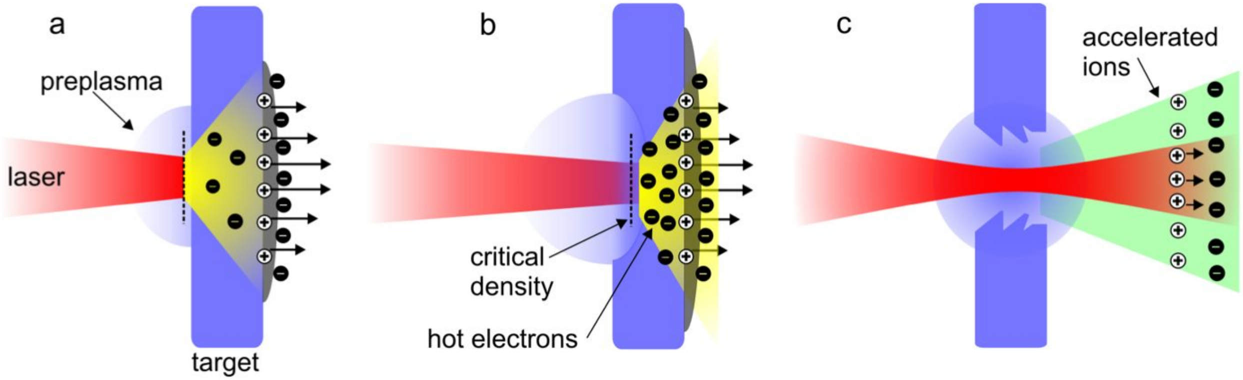


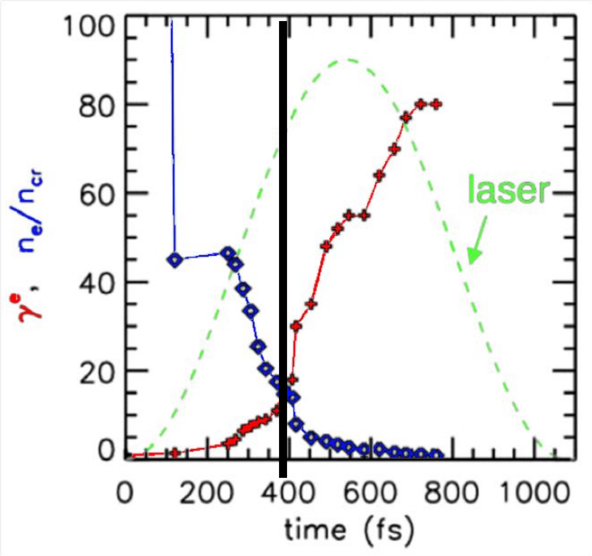
V. A. Schanz, F. Wagner, M. Roth, and V. Bagnoud
Optics Express Vol. 25, Issue 8, pp. 9252-9261
(2017)

Burj Khalifa

Proton acceleration with lasers :
Static electric fields







$$\omega_p^2 = \frac{e^2 n_e}{\varepsilon_0 \bar{\gamma} m_e}$$