Lecture

Fourier optics

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Contents

- Apodization
- Wavefront sensing: Shack Hartmann, Shear interferometers, curvature sensing

References: Introduction to Fourier Optics, J. Goodman
Adaptive Optics for Astronomical Telescopes, J. W. Hardy

Apodization

Apodize means to introduce attenuation in the exit pupil of an imaging system

We will consider two examples:

- softening of the edges of the pupil, with the objective to suppress ringing effects caused by the edge waves
- "inverse" apodization: give more weight to the field at the edge of the pupil

Apodization

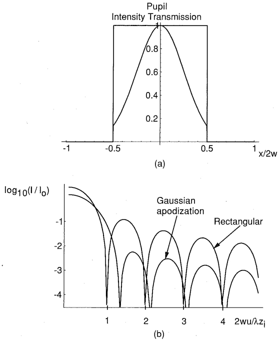

FIGURE 6.14
Apodization of a rectangular aperture by a Gaussian function.
(a) Intensity transmissions with and without apodization.
(b) Point-spread functions with and without apodization.

Reference: Goodman

Apodization

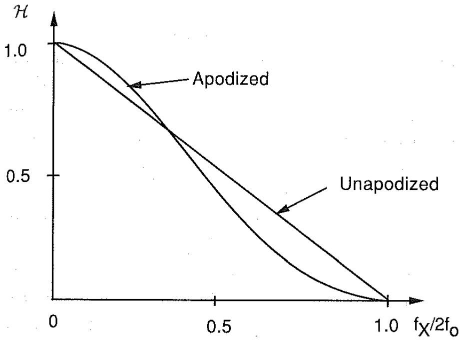

FIGURE 6.15
Optical transfer functions with and without a Gaussian apodization.

Reference: Goodman

Apodization

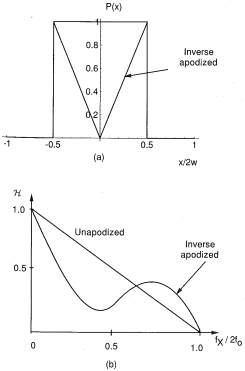

FIGURE 6.16
Pupil amplitude transmittance and the corresponding OTF with and without a particular "inverse" apodization.

Reference: Goodman

Wavefront sensing

- Direct measurement of the wavefront: Shack Hartmann, lateral shear
- Indirect measurement of the wavefront: Image sharpening

Wavefront sensing

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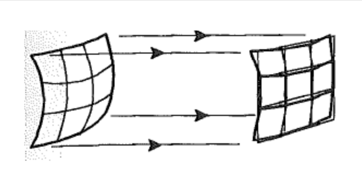
Direct wavefront sensing

Zonal or Modal:

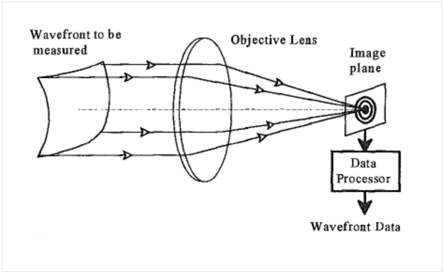
Zonal: wavefront slope is measured within a number of zones in the pupil

Modal: Decompose the wavefront into different surface shapes and modes



Indirect wavefront sensing

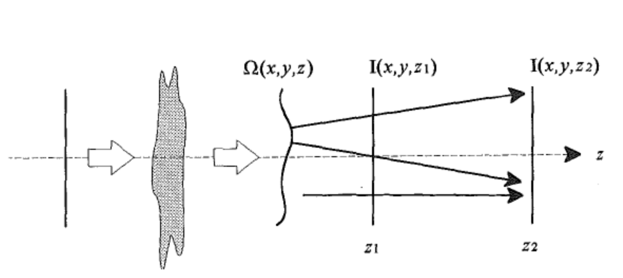
Wavefront errors are *deduced* from their effect on a related parameter, for example the intensity distribution at or near the image plane



Deducing the wavefront from measurement at the image plane require deconvolution: computer intensive, ambiquities, does not work well for large aberrations. Data at other than the image plane (out-of-focus plane) are sometimes used to resolve ambiquities

Wavefront slope sensing

Relate angular deviations of rays with intensity variations



Wavefront slope sensing

Consider a field $U(x, y, z)$ propagating along the z axis

$$U(x, y, z) = (I(x, y, z))^{1/2} e^{ikW(x,y,z)} \tag{1}$$

where $W(x, y, z)$ is the wavefront surface at distance z. The change in irradiance is given by:

$$\frac{\partial I}{\partial z} = -(\nabla I . \nabla W + I \nabla^2 W) \tag{2}$$

where $\nabla = \partial/\partial x + \partial/\partial y$
The first term is the irradiance variation caused by transverse shift of the beam due to the local tilt of the wavefront
The second term is the irradiance variation caused by convergence (divergence) of the beam, local curvature proportional to $\nabla^2 W$

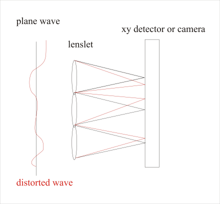
Wavefront slope sensing

Problem: measurement of ∇W and $\nabla^2 W$

In principle, one can measure the intensity distribution at two planes (z_1 and z_2)

If variations are small, the distance needs to be large to produce measurable variations

Wavefront slope sensing: Shack Hartmann



Array of lenslets is placed in the pupil of the optical beam to be measured
An array of spots are produced in the image plane.

- For plane wave, each spot will be located on the optical axis of the corresponding lenslet
- Distorted wavefront produces a local gradient $(\alpha(x, y)$ over the lenslets, displacing the spot a distance $s(x, y) = \alpha(x, y)Z$, where Z if the focal length of the lenslet

Wavefront slope sensing: Shack Hartmann

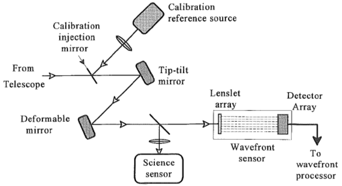
Measurement errors:

- random errors in determining the positions of the spots: photon noise, electrical noise, thermal noise, etc. $\rightarrow$ unavoidable
- bias errors due to misalignment of the optics and variations in the responsivity of the detectors $\rightarrow$ can be reduced or calibrated

Wavefront slope sensing: Shack Hartmann

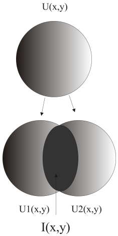
Calibration issues

- calibration of the null point of each subaperture with a plane wave input
- dark frame



Shearing interferometers

Combines original phase front with a replica that is displaced with respect to the original one so that interference occurs
Optical phase differences are converted into intensity variations



The field to be measured is given

$$U(x,y) = A(x,y)e^{ikW(x,y)} \tag{3}$$

Shearing interferometers

The two copies of the wavefront that are generated are given by

$$U_1(x,y) = A(x+s/2,y)e^{ikW(x+s/2,y)} \tag{4}$$

$$U_2(x,y) = A(x-s/2,y)e^{ikW(x-s/2,y)} \tag{5}$$

and the corresponding intensity:

$$I(x,y) = 2(1 + |A(x+s/2,y)||A(x-s/2,y)| \tag{6}$$

$$\cos(kW(x+s/2,y) - kW(x-s/2))) \tag{7}$$

when the shear distance is small compared to the spatial period of $W(x,y)$

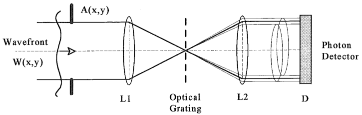
$$I(x,y) = 2(1 + |A(x,y)|^2 cos(ksdW/dx)) \tag{8}$$

By measuring the intensity of the interference pattern and knowing the shear distance, one can determine the wavefront slope in the shear direction

Two measurements are needed to find the slope in two dimensions: lateral shear in x and y directions

Shearing interferometers

Example; shearing interferometer using moving grating



Input field at aperture

$$U(x,y) = A(x,y)e^{ikW(x,y)} \tag{9}$$

Shearing interferometers

Consider grating with amplitude transmittance $M(x_0, t)$
The field after the grating is:

$$U(x_0, y_0, t) = \tilde{U}(x_0, y_0)M(x_0, t) \tag{10}$$

where $\tilde{U}(x_0, y_0)$ is the Fourier transform of $U(x, y)$
The second lens produces an image of the pupil containing the wavefront distortions at plane D

$$I(x, y, t) = |U'(x, y, t)|^2 \tag{11}$$

where $U'(x, y, t)$ is the convolution of $\tilde{U}(x_0, y_0)$ with the Fourier transform of $M(x_0, y)$

Shearing interferometers

Example: Sine-wave amplitude grating in the x direction:

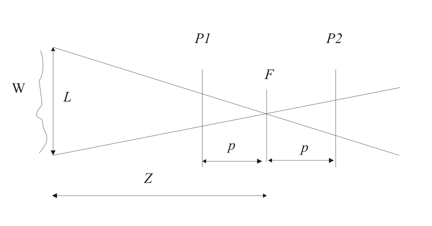
$$I(x,y,t) = \frac{1}{2} + \frac{1}{2}\cos(k(W(x-s,y) - W(x+s,y)) + 2\omega t) \quad (12)$$

One shear pattern: interference of the $+1$ and -1 orders
The phase difference between the two beams is:

$$\phi(x,y) = k(W(x-s,y) - W(x+s,y)) \quad (13)$$

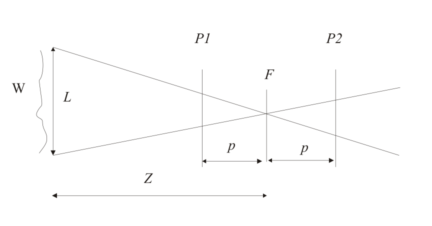
with $s = \lambda Z/g$ is the shear distance between orders.
measurement of the phase ϕ at x, y gives the wavefront difference between points $x+s$ and $x-s$. For small values of s, this is equivalent to the wavefront slope

Curvature sensing



- wavefront curvature determined from 2 displaced focal planes
- local wavefront curvature error makes the light converge closer to planes $P1$ or $P2$ $\rightarrow$ excess of illumination in one plane and lack of illumination in the other.

Curvature sensing



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Curvature sensing

Consider wavefront fluctuations r_0 (diffracts over an angle λ/r_0
Plane $P1$ is a blurred pupil image with a blur size $\approx \lambda(f - l)/r_0$.
This should be small compared to the fluctuations to be measured (r_0 scaled down by a factor l/f. This gives the condition:

$$\frac{\lambda(Z-p)}{r_0} << \frac{r_0 p}{Z} \tag{14}$$

Curvature sensing

Suppose a incoming local wavefront curvature $C_w = 1/r_w$, with r_w being the local radius of curvature over a small area a_0.
The curved wavefront will focus at a distance

$$z_c = \frac{Zr_w}{Z + r_w} \tag{15}$$

The focal shift is

$$\Delta z = Z - z_c = \frac{Z^2}{Z + r_w} \tag{16}$$

Curvature sensing

For a beam of area a_0, at the lens, the area at planes $P1$ and $P2$ are

$$a_{1,2} = a_0 (\frac{p \mp \Delta z}{Z + \Delta z})^2 \tag{17}$$

Consider H the irradiance in W/m^2 at the aperture, the irradiances at $P1$ and$P2$ are:

$$H_{1,2} = H(a_0/a_{1,2})^2 \tag{18}$$

Curvature sensing

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$$H_{1,2} = H(a_0/a_{1,2})^2 \tag{18}$$

Curvature sensing

The normalized difference in irradiance at planes $P1$ and $P2$ are:

$$\Delta I = \frac{H_1 - H_2}{H_1 + H_2} = \frac{a_2^2 - a_1^2}{a_2^2 + a_1^2} = \frac{2p\Delta z}{p^2 + \Delta z^2} \tag{19}$$

Using the approximation that $r_w >> Z$ and $p^2 >> \delta z^2$, we can simplify ΔI as:

$$\Delta I = \frac{2Z^2}{pr_w} = \frac{2Z^2 C_w}{p} \tag{20}$$

Curvature sensing

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