Optics and quantum information
– introduction to field quantization

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Optics and quantum information

- field quantization
  - wavization
    - optics
    - mechanics
  - application to fields
- importance of measurement in quantum physics
- quantum uncertainty and correlations
- applications
  - interferometry
    - sensing
    - quantum computing
  - sensing
  - communication

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Quantum Statistical Properties of Radiation

WILLIAM H. LOUISELL
Professor of Physics and Electrical Engineering
University of Southern California

Wiley Classics Library Edition Published 1990

1.10 QUANTIZATION; EXAMPLE OF CONTINUOUS SPECTRUM

In this section we solve an eigenvalue problem in which the eigenvalue spectrum is continuous. This simple example demonstrates how to treat quantum-mechanically a system that has a classical analog.

…

the operators must obey. This requires an additional *postulate* for the theory; it is given in terms of the commutation relations for p and q, namely,

$$[q, q] = 0 \qquad [p, p] = 0$$
$$[q, p] \equiv (qp - pq) = i\hbar, \tag{1.10.3}$$

q and p satisfy (1.10.3). The justification for the quantum postulate is the remarkable agreement between theory and experiment. It is possibly the most profound and fundamental postulate in the theory.

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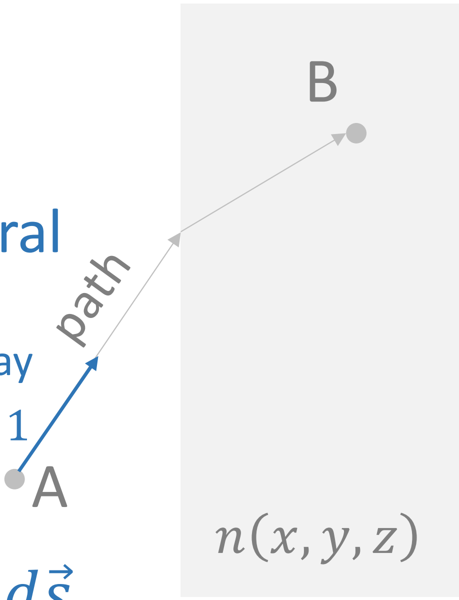
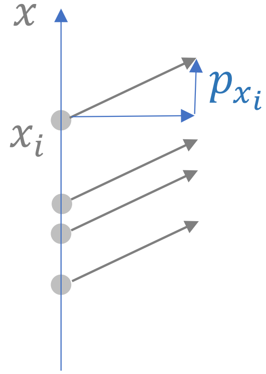
Fermat‘s principle

- minimize path integral



paraxial regime …



path $= \int_A^B \vec{p}(x, y, z) \cdot d\vec{s}$

free space

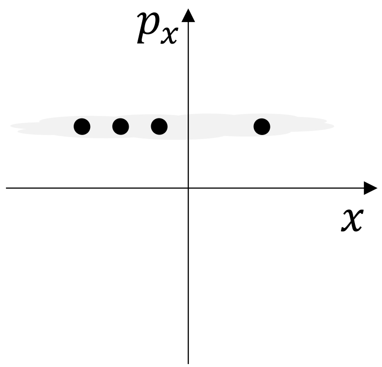
$$1 - p_x^2 - p_y^2 - p_z^2 = 0$$

varying refractive index

$$n^2(x, y, z) - p_x^2 - p_y^2 - p_z^2 = 0$$

→ ray optics

phase space

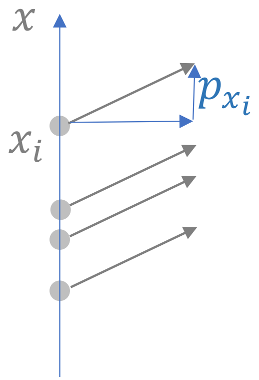


Fermat‘s principle → ray optics

$\vec{p}$ = direction of ray

$|\vec{p}|^2 = 1$

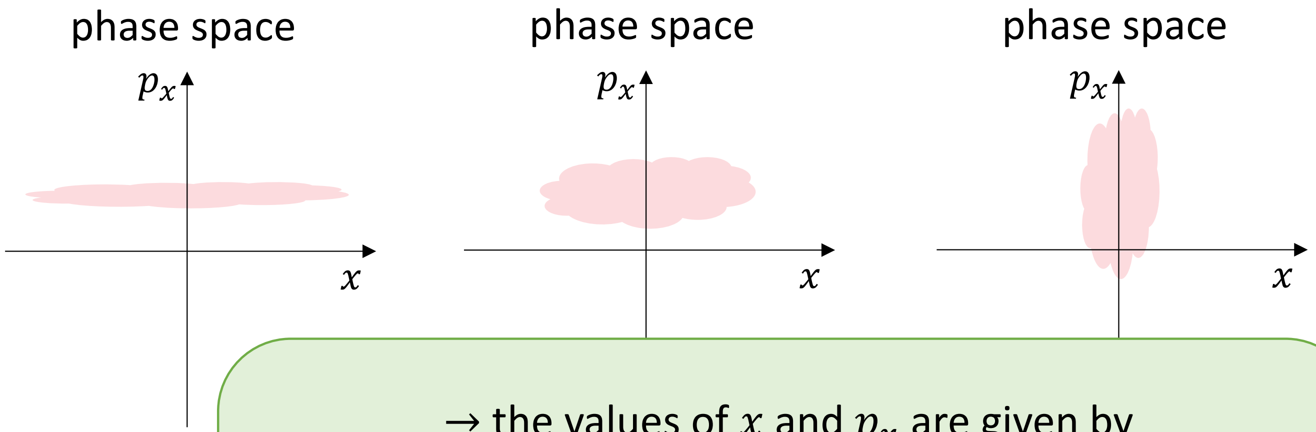
paraxial regime …



varying refractive index

$$n^2(x,y,z) - p_x^2 - p_y^2 - p_z^2 = 0$$

now: experimental observation of diffraction at a slit



→ the values of x and p_x are given by
phase space distribution functions,
which are related by Fourier transformation!

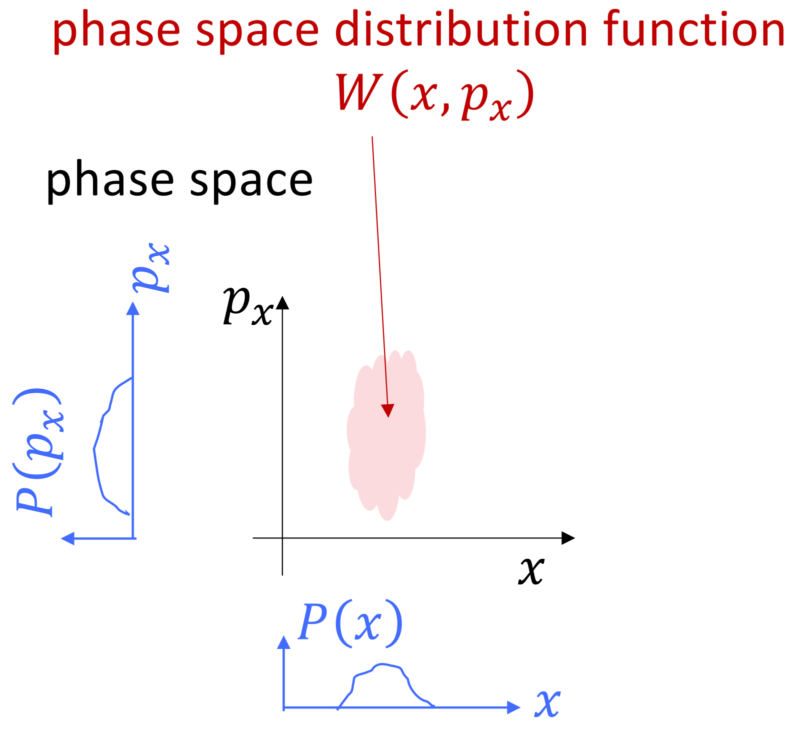
typical for wave phenomena !!!

→ x and p_x are called conjugate variables

It is clear from the experiment, that there is a minimum area in phase space. The product of the variances of x and p_x has a lower bound, so there must be a **Fourier transform relationship** !!!!

dynamics of rays → dynamics of distributions

Are the marginal distributions $P(x)$ and $\tilde{P}(p_x)$ the functions related by Fourier transformation?



marginal probability distributions:

$$P(x) = \int_{-\infty}^{\infty} W(x, p_x)\, dp_x\ , \quad P(p_x) = \int_{-\infty}^{\infty} W(x, p_x) dx$$

conjugate variables have inverse dimensions…
so p has not the right dimension to be
conjugate to x: $k_i \equiv (2\pi/\lambda)p_i$

Are the marginal distributions $P(x)$ and $P(k_x)$
the functions related by Fourier transformation?

?
$$\tilde{P}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx P(x) e^{-ikx} \qquad P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dp \tilde{P}(k) e^{ikx}$$
?

k (and p) is related to $\frac{-i\partial}{\partial x}$, because $k\,P(x) = \frac{-i\partial}{\partial x} P(x)$

test:

$$\int dx \frac{-i\partial}{\partial x} P(x) = \int dx \frac{-i\partial}{\partial x} \frac{1}{\sqrt{2\pi}} \int dk \tilde{P}(k) e^{ikx} = \frac{1}{\sqrt{2\pi}} \int dx \int dk\, k\, \tilde{P}(p) e^{ikx} = \int dk\, k\, \tilde{P}(k)\, \delta(0) = 0$$

No!

how about: $P(x) = \Psi^*(x)\Psi(x)$? → $\Psi(x) = \frac{1}{\sqrt{2\pi}} \int dk \tilde{\Psi}(k) e^{ikx}$, $\tilde{\Psi}^*(k) = \frac{1}{\sqrt{2\pi}} \int dx \tilde{\Psi}^*(x) e^{ikx}$

2nd test:

$$\int dx \Psi^*(x) \frac{-i\partial}{\partial x} \Psi(x) = \frac{1}{\sqrt{2\pi}} \int dx\, \Psi^*(x) \int dk\, k\, \tilde{\Psi}(k) e^{ikx} = \int dk\, \tilde{\Psi}^*(k) k\, \tilde{\Psi}(k) = \int dk\, k\, P(k) = \langle k \rangle$$

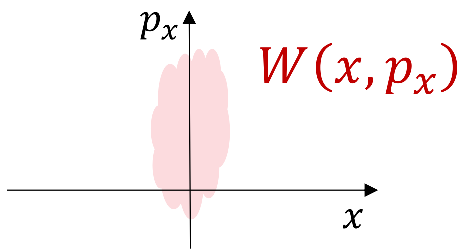
Yes!

experiment: diffraction at a slit

Fermat's principle → ray optics

varying refractive index

$$n^2(x,y,z) - p_x^2 - p_y^2 - p_z^2 = 0$$



Now we have everything

to turn this into a differential equation: $p_x \to \frac{\lambda}{2\pi}\frac{-i\partial}{\partial x}$, $p_y \to \frac{\lambda}{2\pi}\frac{-i\partial}{\partial y}$, $p_z \to \frac{\lambda}{2\pi}\frac{-i\partial}{\partial z}$

$$\left(\frac{2\pi}{\lambda}\right)^2 n^2(x,y,z)\Psi + \left(\frac{\partial}{\partial x}\right)^2 \Psi + \left(\frac{\partial}{\partial y}\right)^2 \Psi + \left(\frac{\partial}{\partial z}\right)^2 \Psi = 0$$

→ $$\vec{\nabla}^2\Psi + \left(\frac{2\pi n}{\lambda}\right)^2 \Psi = 0$$ Helmholtz equation

Wigner function and diffraction

- examples -

$$W(x,k) = \int_{-\infty}^{\infty} dy\, \psi^*\left(x+\frac{y}{2}\right)\psi\left(x-\frac{y}{2}\right)e^{iky}$$

$$W_1(x,k) = \int_{-\infty}^{\infty} dy\, \psi_1^*\left(x+\frac{y}{2}\right)\psi_1\left(x-\frac{y}{2}\right)e^{iky}$$

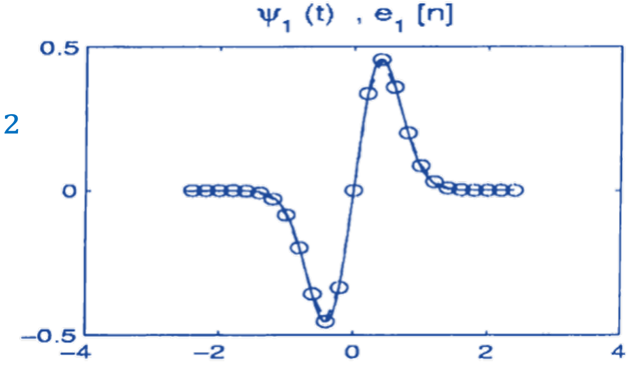
$$= \int_{-\infty}^{\infty} dy\, \left(x+\frac{y}{2}\right)\left(x-\frac{y}{2}\right)e^{-a\left\{\left(x+\frac{y}{2}\right)^2+\left(x-\frac{y}{2}\right)^2\right\}}e^{iky}$$

$$= e^{-2ax^2}\int_{-\infty}^{\infty} dy\, \left(x^2-\frac{y^2}{4}\right)e^{-\frac{a}{2}\left(y^2-\frac{i2k}{a}y\right)}$$

...

$$= \sqrt{\frac{\pi}{2a^3}}\; e^{-2ax^2-\frac{k^2}{2a}}\left(2ax^2+\frac{k^2}{2a}-\frac{1}{2}\right)$$

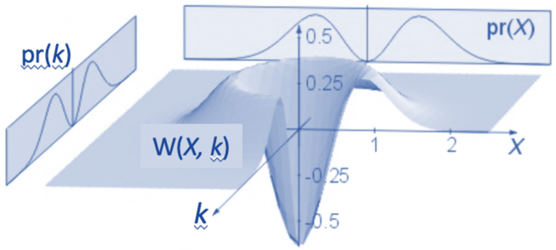
$$\psi_1(x) = x\, e^{-ax^2}$$



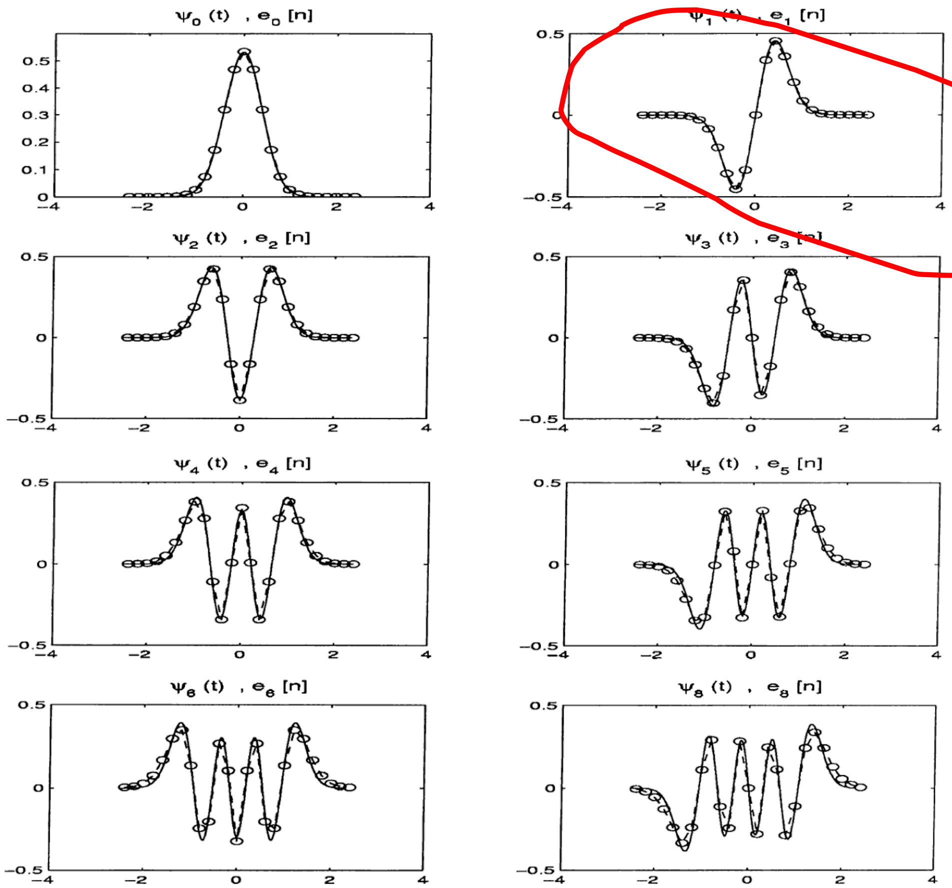
a laser mode

hint:

$$\int_{-\infty}^{\infty} dy\, e^{-a(y+ib)^2} = \sqrt{\frac{\pi}{a}}$$



Hermite Gaussian functions

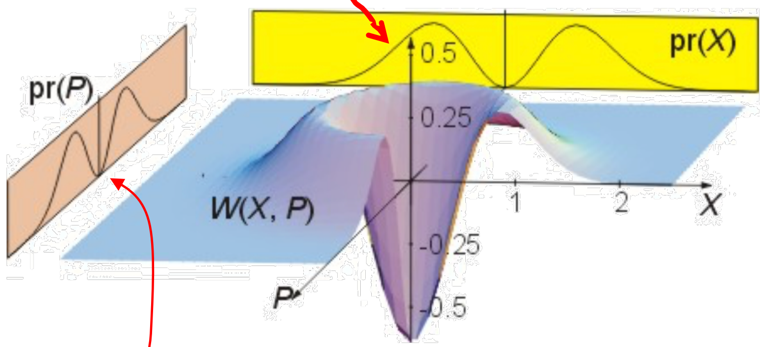


Transverse mode pattern in lasers

→ invariant under propagation

→ invariant under Fourier transformation

→ rotationally symmetric Wigner function



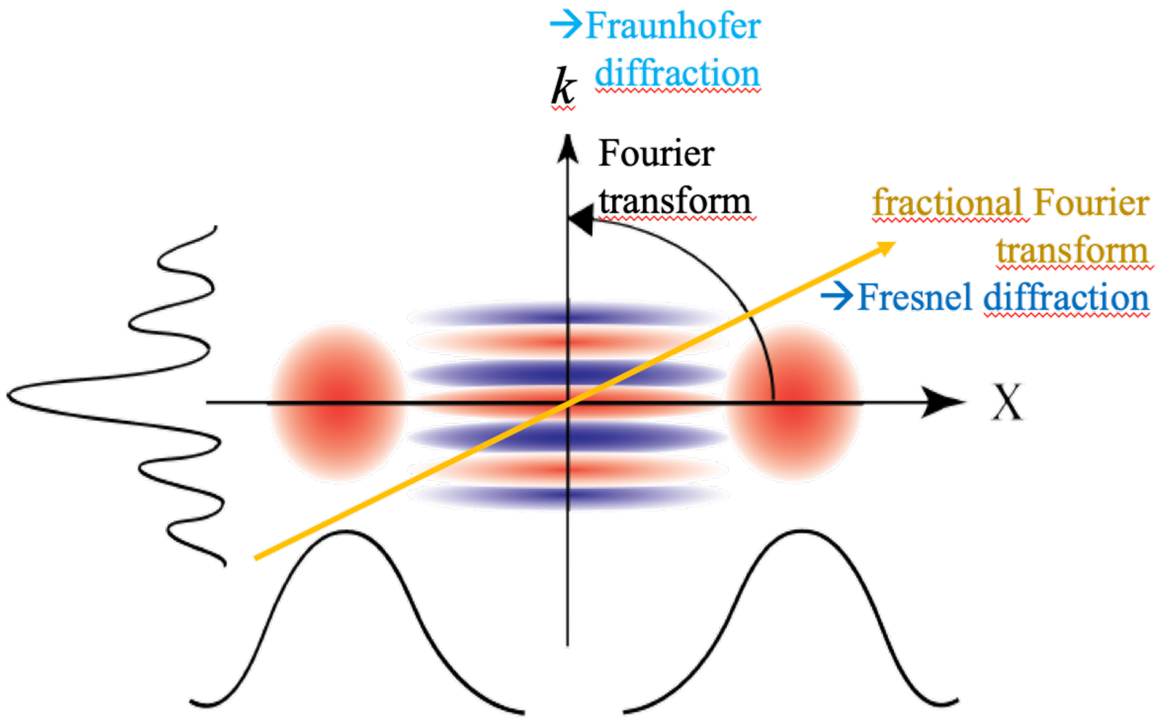
zero of pr(P) requires negative values of W(X,P)

Phase space description of diffraction:

$$W(x,k) = \int_{-\infty}^{\infty} dy\, \psi^*\left(x + \frac{y}{2}\right) \psi\left(x - \frac{y}{2}\right) e^{iky}$$

→ two slit interference

W(x,k) → Wigner function



see: "Wigner distribution function and its application to first order optics",
M J Bastiaans, J.Opt.Soc.Am. **69**, 1710 (1979)

summary of first part: **wavization**

"classical" phase space description

experiments give hints for

- *lower bound for occupied volume in phase space*
- *FT relationship between phase space variables*
- *typical interference pattern*

→ "wave" phase space description

- non-commuting conjugate variables

➔ works also for 1st quantization, we will use it for 2nd quantization

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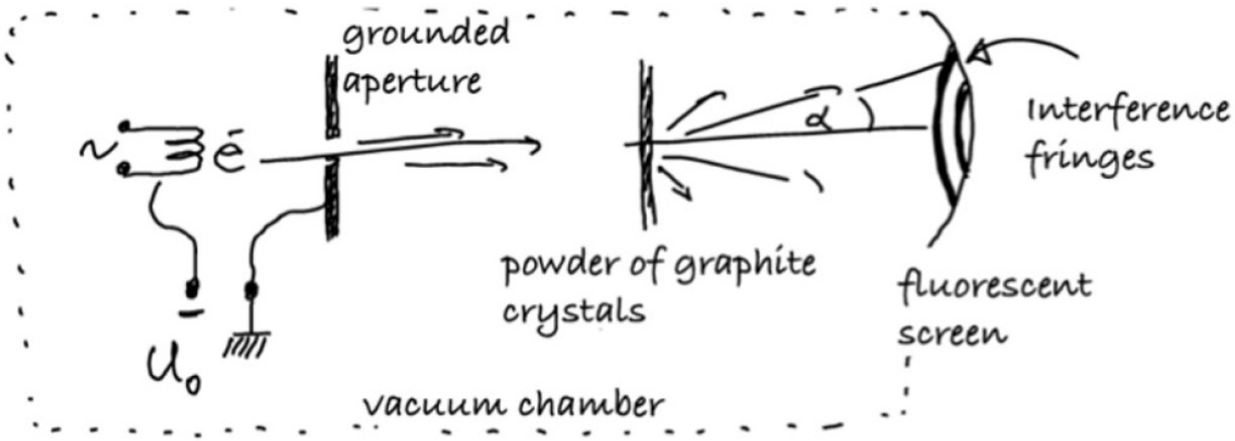
other example of "wavization"

Newtonian mechanics → quantum mechanics



→$p = \hbar k = m \frac{\partial \omega}{\partial k}$

→dispersion relation

→Schrödinger equation

… and others

G Leuchs **Wave phenomena and wave equations**

Lecture at the Enrico Fermi Summer School at Varenna, Course 197 'Foundations of Quantum Theory', 2016, Organizers: Ernst M Rasel, Wolfgang P Schleich and Sabine Wölk (published in the proceedings)

phase space description

Other names in other fields ...

• Wigner function	quantum physics	space / angle versus momentum
Moyal:	evolution of Wigner function	
• Ville function		time versus frequency
• ambiguity function	RADAR	
• Woodward's ambiguity fct.	Electrical engineering	

J. Ville, "Théorie et Applications de la Notion de Signal Analytique." Câbles et Transmissions **2**, 61 (1948)

J. E. Moyal, "Quantum mechanics as a statistical theory", Math. Proc. Cambr. Phil. Soc. 45, 99 (1949)

P.M. Woodward, "Probability and Information Theory with Applications to Radar", Norwood, MA: Artech House, (1980)

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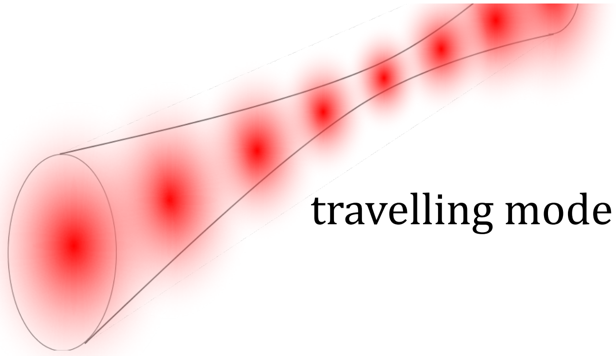
Fig. 1: optical resonator



Maxwell's equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}(\vec{r},t) = 0$$

separation of variables

$$\vec{E}(\vec{r},t) = \vec{u}(\vec{r})q(t) \quad \longrightarrow \quad \int_V \varepsilon_0 \vec{u}^2(\vec{r})\,dV = 1$$

→ q^2 has dimension **energy**

$$\begin{cases} (a) \quad \vec{\nabla}^2\vec{u}(\vec{r}) + k^2\vec{u}(\vec{r}) = 0 \\ (b) \quad \frac{1}{c^2}\frac{\partial^2}{\partial t^2}q(t) + k^2 q(t) = 0 \end{cases}$$

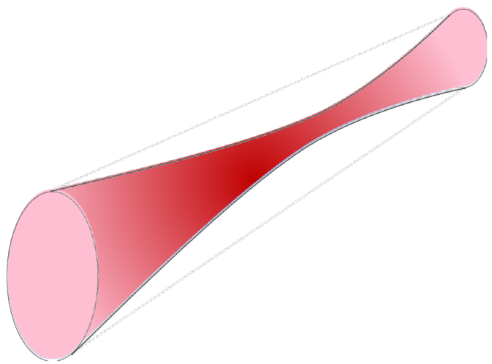
← Helmholtz equation

Equations describing spatio-temporal evolution

$$\begin{cases} (a) \quad \vec{\nabla}^2 \vec{u}(\vec{r}) + k^2 \vec{u}(\vec{r}) \; = \; 0 \\ (b) \, \dfrac{1}{c^2} \dfrac{\partial^2}{\partial t^2} q(t) + k^2 q(t) = 0 \end{cases}$$

→ Harmonic oscillator

excitation of mode



$$q(t) = q(0)\cos(\omega t) + \frac{\dot{q}(0)}{\omega}\sin(\omega t)$$

(b) → $$\frac{1}{\omega^2}\left(\dot{q}(0)\right)^2 + \left(q(0)\right)^2 = S$$

abstract phase space
where excitation lives

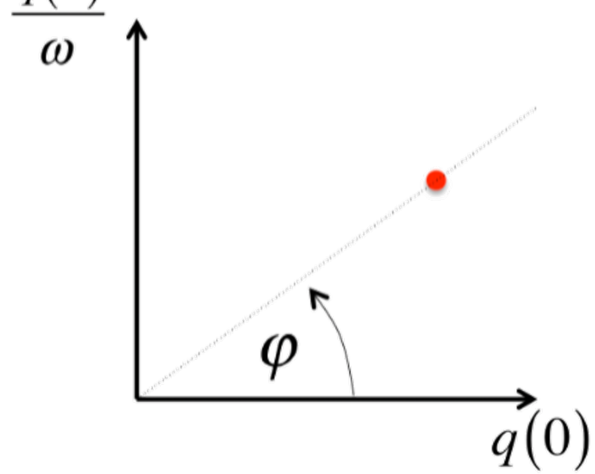
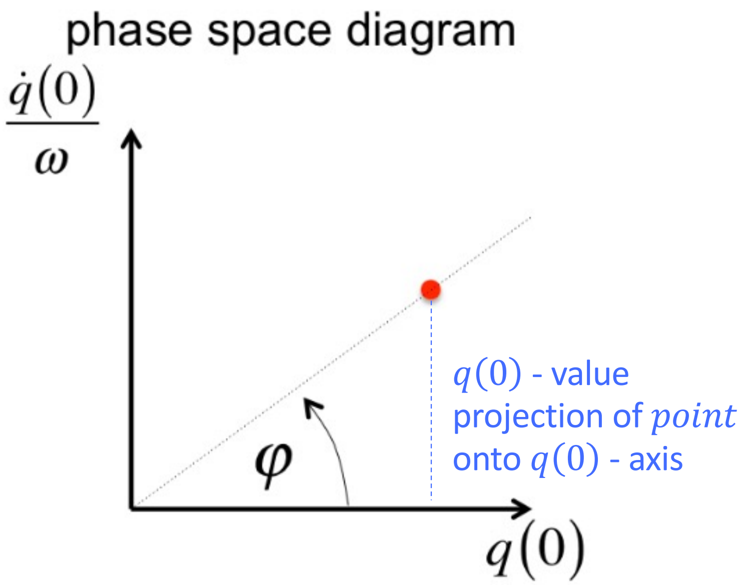
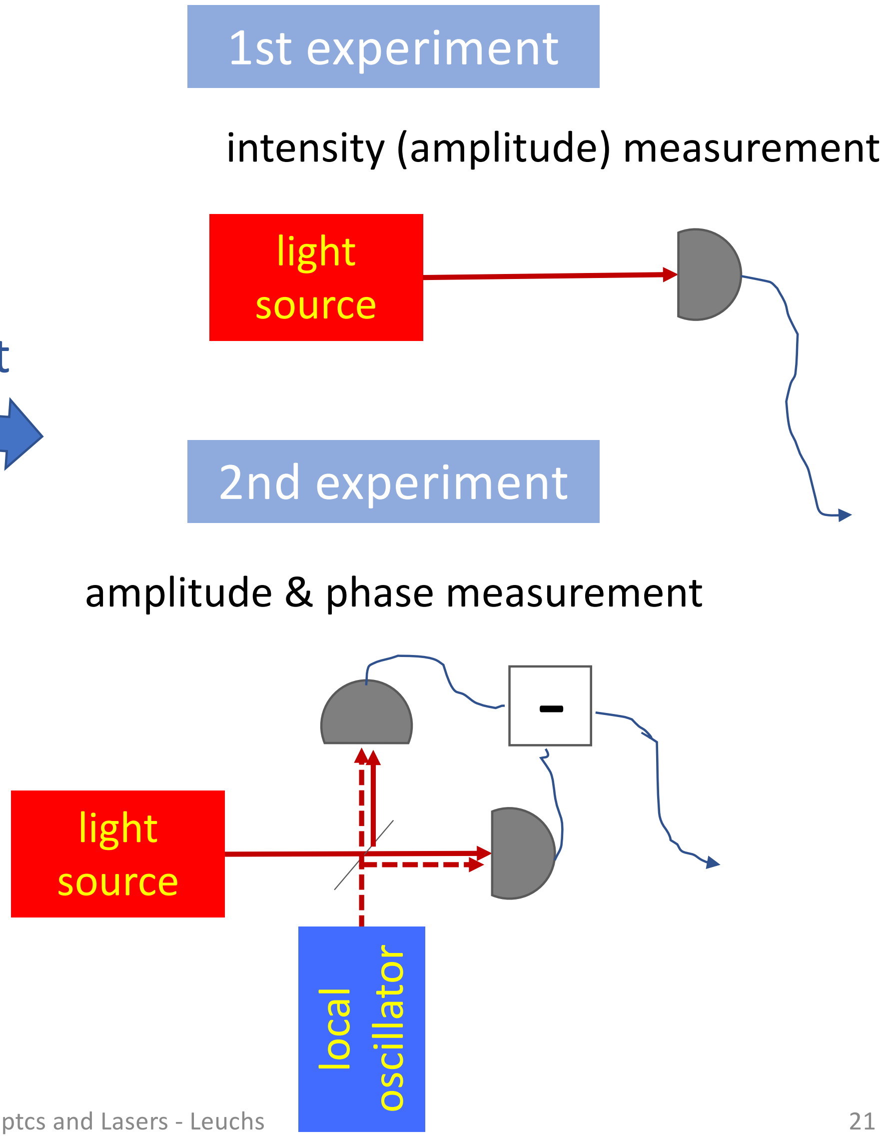


Fig. 2: phase space representation of a classical field: a single point.

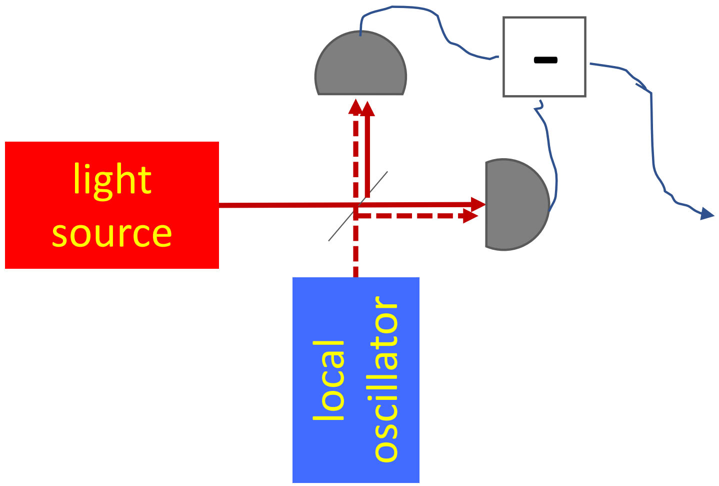
→ q^2 has dimension energy


phase space diagram
$\frac{\dot{q}(0)}{\omega}$
φ
$q(0)$ - value
projection of *point*
onto $q(0)$ - axis
$q(0)$


experiment

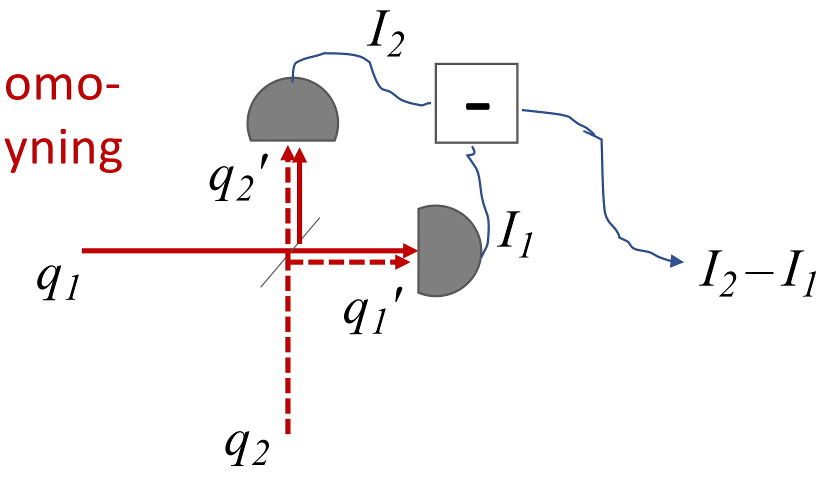

1st experiment
intensity (amplitude) measurement
light source
2nd experiment
amplitude & phase measurement
−
light source
local oscillator

amplitude & phase measurement



amplitude & phase measurement

homo-
dyning



$$q_1' = (q_1 - q_2)/\sqrt{2}$$
$$q_2' = (q_1 + q_2)/\sqrt{2}$$

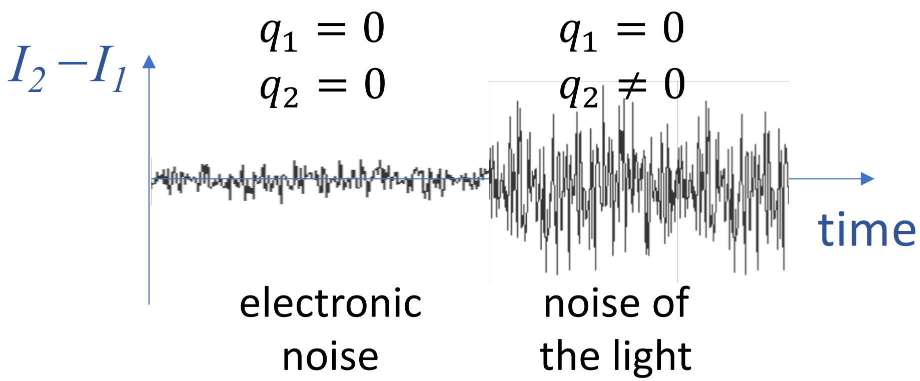
$$I_1 = |q_1'|^2 = (|q_1|^2 + |q_2|^2 - q_1 q_2^* - q_2 q_1^*)/2$$

$$I_2 = |q_2'|^2 = (|q_1|^2 + |q_2|^2 + q_1 q_2^* + q_2 q_1^*)/2$$

$$I_2 - I_1 = 2\,\Re\mathrm{e}\{q_1 q_2^*\}$$

If $q_1 = 0$ → $I_2 - I_1 = 0$

But experiment shows noise:

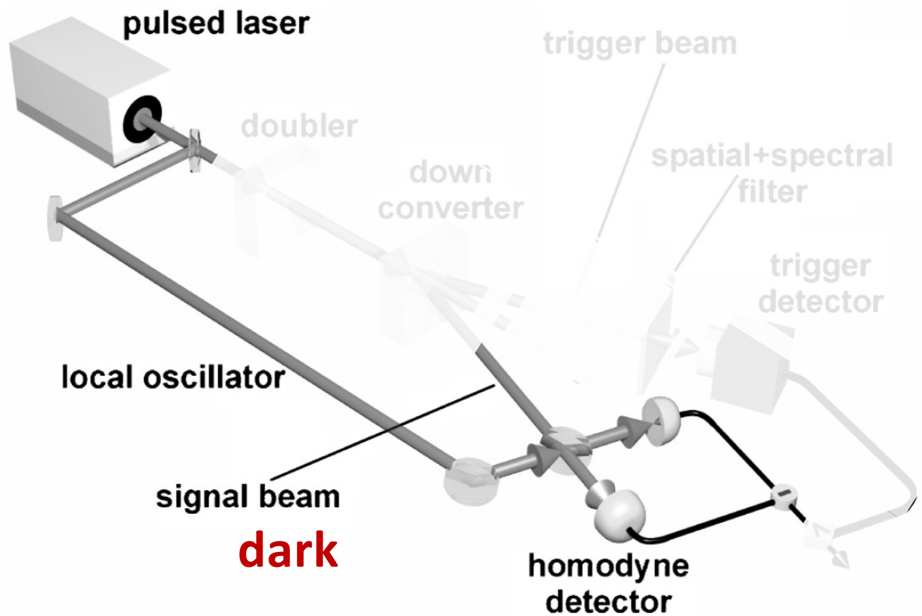


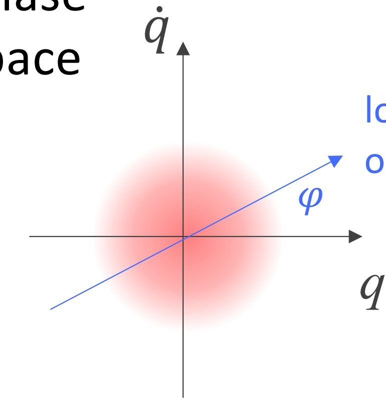
→obviously:
$\langle q_1 \rangle = 0$ and $\langle q_1^2 \rangle \neq 0$

quantum state reconstruction of the vacuum state (zero-photon Fock state)

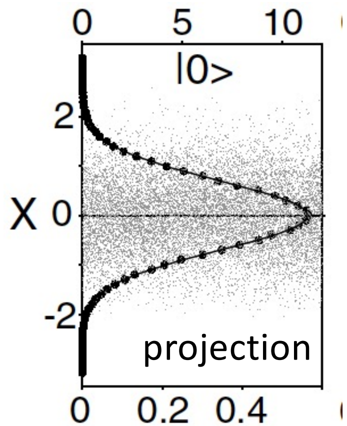
A I Lvovsky et al., Phys. Rev. Lett. 87, 050402 (2001)

Homodyning → projection of phase space distribution onto "φ"





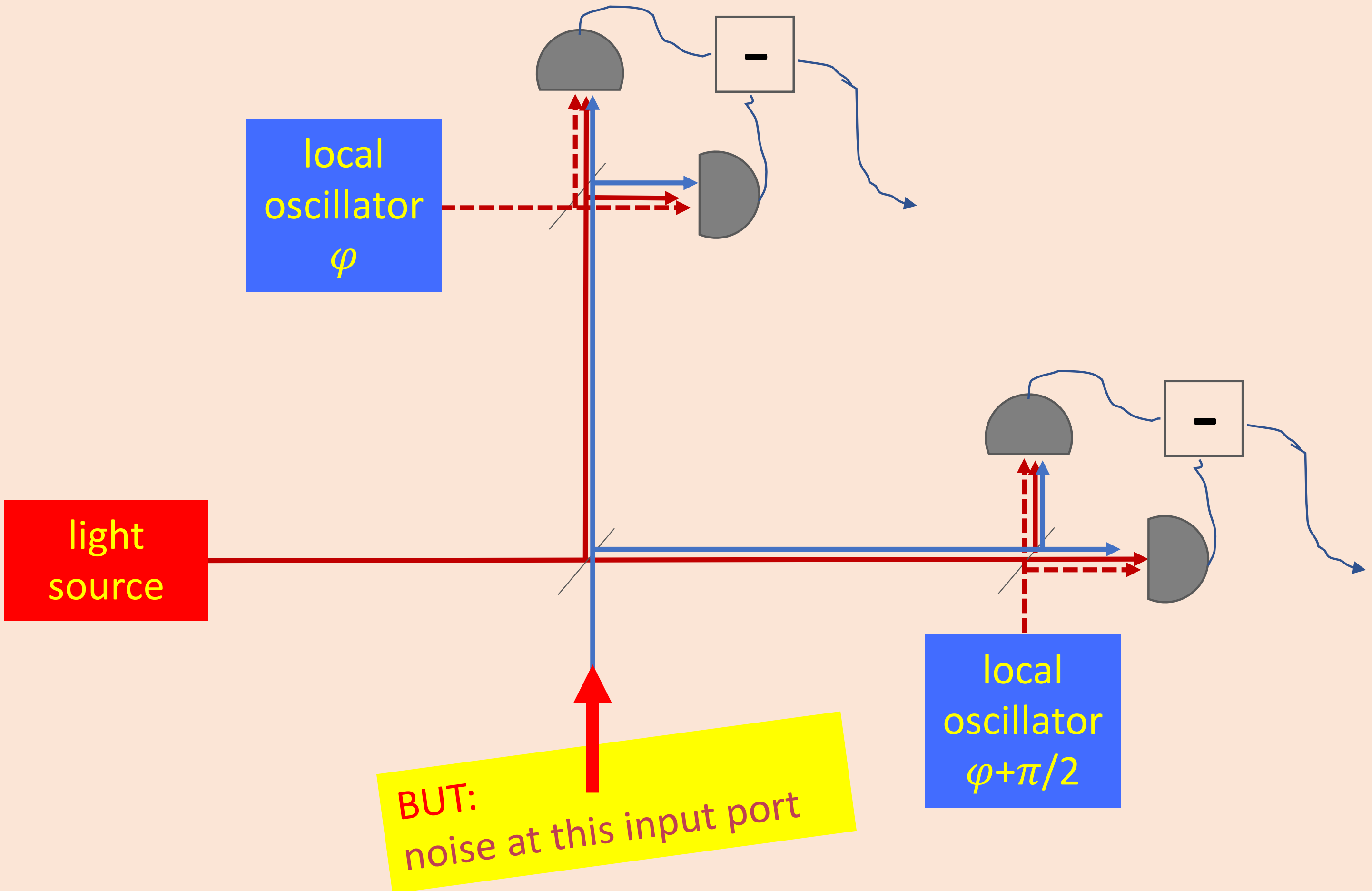
Measure for different φ and reconstruction, e.g. Radon transformation

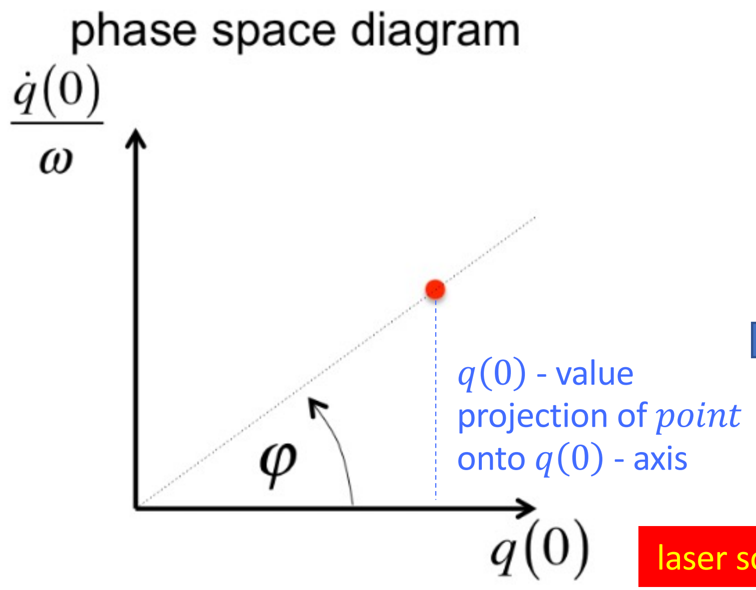


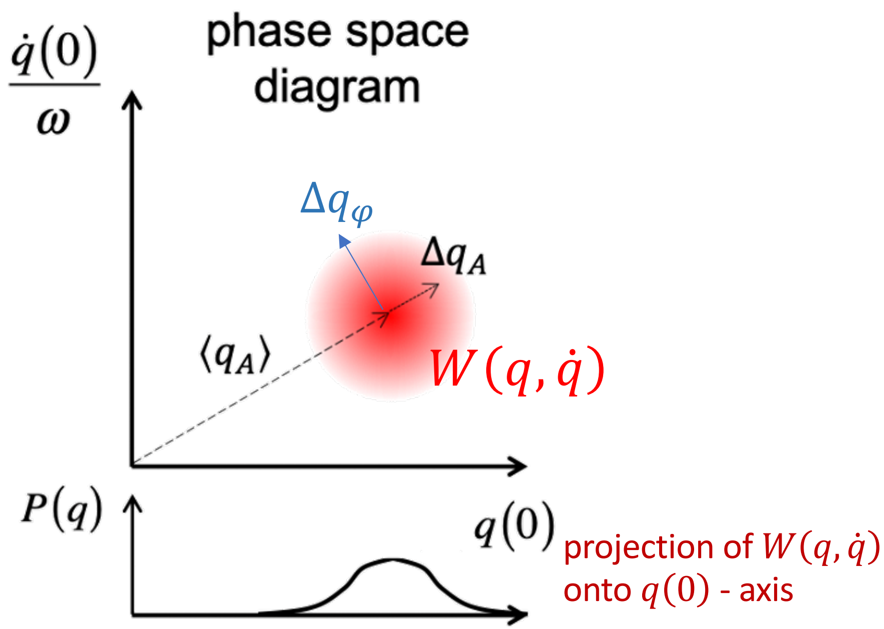
$\langle q_1 \rangle = 0$
and
$\langle q_1^2 \rangle \neq 0$

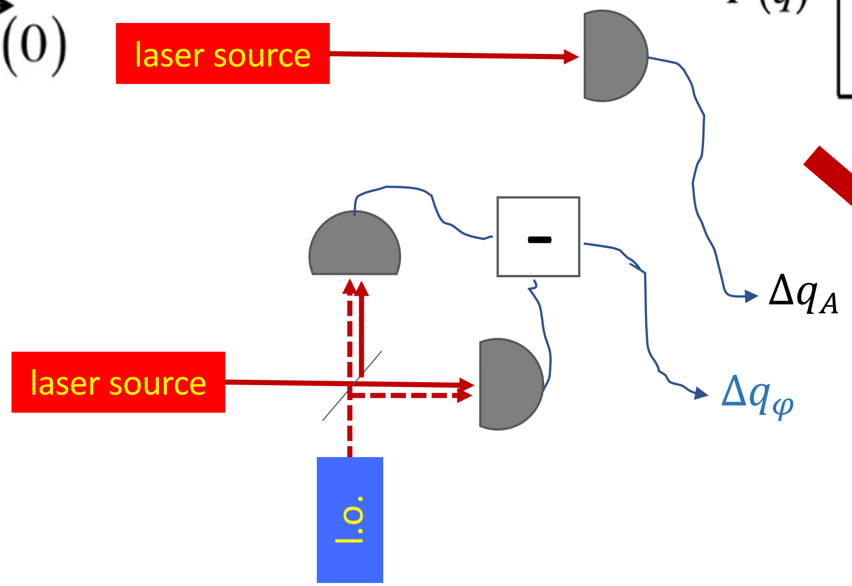
simultaneous amplitude & phase measurement

3rd experiment



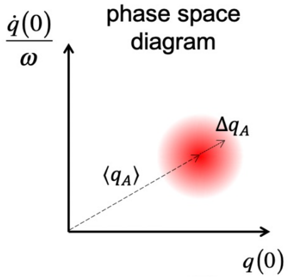






- distributions $P(q)$, $P(\dot{q})$
- minimum area in phase space
- for laser: symmetric

➢ $P(q)$ dependent on $P(\dot{q})$



p is conjugate to variable q

→ $p \sim \dot{q}$

$$P(q) = \Psi^*(q)\Psi(q)$$

$$\Psi(q) = \int dp \tilde{\Psi}(p) e^{iqp}$$

$$\int dq \Psi^*(q)(-i)\frac{\partial}{\partial q}\Psi(q) = \int dp \tilde{\Psi}^*(p) p \tilde{\Psi}(p)$$

variable	conjugate variable			inertia
	Fourier	action complement	time derivative	
x	$k = i\,\partial/\partial x$	p	$\dot{x} = p/m$	m
t	$\omega = i\,\partial/\partial t$	E	-	-
φ	$i\,\partial/\partial\varphi$	L	$\dot{\varphi} = L/\Theta$	$\Theta = \mathrm{mr}^2$
	property of waves or oscillations	particle property		
$M^\dagger$	$i\,\partial/\partial M$		$\dot{M}$	

† arbitrary variable

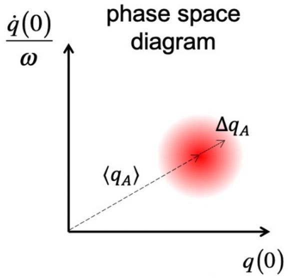
$$p \equiv -i\frac{\partial}{\partial q}$$

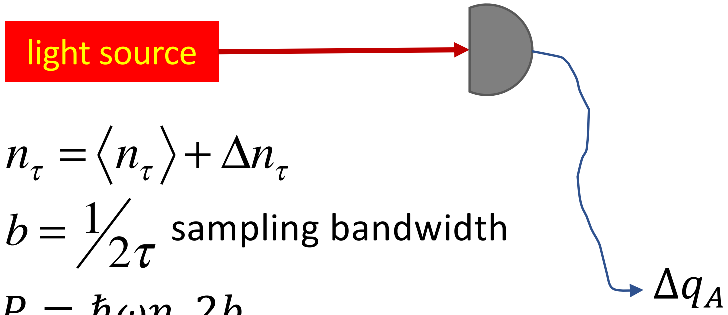
still missing :

width of $\Psi(q)$

and

$$p \equiv -i\frac{\partial}{\partial q} \qquad \leftrightarrow \qquad \dot{q}$$





$n_\tau = \langle n_\tau \rangle + \Delta n_\tau$

$b = {}^{1}\!/_{2\tau}$ sampling bandwidth

$P = \hbar\omega n_\tau 2b$

shot noise $\langle (\Delta P)^2 \rangle = \hbar\omega 2b \langle P \rangle$

area in phase space

$\langle (\Delta q)^2 \rangle = \left\langle \left(\Delta \frac{\dot{q}}{\omega} \right)^2 \right\rangle \approx 1.5 \cdot 10^{-19} J$

with Fourier conjugate variable $\langle (\Delta p)^2 \rangle = {}^{1}\!/_{\langle (\Delta q)^2 \rangle}$

$\frac{\dot{q}}{\omega} \approx 1.5 \cdot 10^{-19} [J]\, p$

$\approx 0.5 \cdot 10^{-34} [Js]\, \omega\, p$

$\hbar$

$\rightarrow \frac{\dot{q}}{\omega} \approx \frac{\hbar\omega}{2} p$

<u>quantitative result:</u>

He Ne laser of 1mW

root mean square power fluctuations in radio frequency band of 1 MHz:

$\langle (\Delta P)^2 \rangle = \mathbf{2.5 \cdot 10^{-8}}$ **Watt**

(same for phase .....)

$\langle (\Delta P)^2 \rangle /_{4b^2} = 4 \langle q_A \rangle^2 \langle (\Delta q_A)^2 \rangle$

→ q^2 has dimension energy

$$\hat{a} = \frac{1}{\sqrt{\hbar\omega}} q + \frac{\sqrt{\hbar\omega}}{2} \frac{\partial}{\partial q} = \frac{1}{\sqrt{\hbar\omega}} q + i \frac{\sqrt{\hbar\omega}}{2} p$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{\hbar\omega}} q - \frac{\sqrt{\hbar\omega}}{2} \frac{\partial}{\partial q} = \frac{1}{\sqrt{\hbar\omega}} q - i \frac{\sqrt{\hbar\omega}}{2} p$$

interestingly enough:
eigen functions the same as for
laser modes !!!

$$\frac{1}{\omega^2}(\dot{q}(0))^2 + (q(0))^2 = S$$

$$-\left(\frac{\hbar\omega}{2}\frac{\partial}{\partial q}\right)^2 \Psi(q) + q^2\Psi(q) = S\Psi(q)$$

$$\frac{\hbar\omega}{2}(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger)\Psi(q) = S\Psi(q)$$

eigen functions

$$\Psi_0(q) = N_0 e^{-\frac{q^2}{\hbar\omega}}$$

$$\Psi_1(q) = N_1 q e^{-\frac{q^2}{\hbar\omega}}$$

$\vdots$

$\Big\} \; |n\rangle$

eigen values

$$S_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

Field operators

$$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

Field operators

$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$

$\hat{a}^{\dagger}|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$\hat{a}^{\dagger}\hat{a}|n\rangle = n\,|n\rangle$

$\hat{a}\hat{a}^{\dagger}|n\rangle = (n+1)\,|n\rangle$

$[\hat{a}, \hat{a}^{\dagger}] = \hat{a}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a} = \hat{1}$

$\{|n\rangle, n = 0, 1, 2, \ldots\}$
ortho-normal
basis of Hilbert space

eigen functions of $\hat{a}$?

$$\hat{a}\sum_{n=0}^{\infty} c_n|n\rangle \overset{?}{=} \alpha \sum_{n=0}^{\infty} c_n|n\rangle \equiv \alpha|\alpha\rangle$$

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$ ✓

eigen functions of $\hat{a}^{\dagger}$?

$$\hat{a}^{\dagger}\sum_{n=0}^{\infty} c_n|n\rangle \overset{?}{=} \beta \sum_{n=0}^{\infty} c_n|n\rangle \equiv \beta|\beta\rangle$$

No!

Continuous versus disrete variables

discrete
dichotomic variables

$\Psi = \alpha\,|0\rangle + \beta\,|1\rangle =$

$$= \sum_{i=1}^{2} \alpha_i |i\rangle$$

many photons

$$\Psi = \sum_{i=1}^{\infty} \alpha_i |i\rangle$$

→ ∞ dim Hilbert space

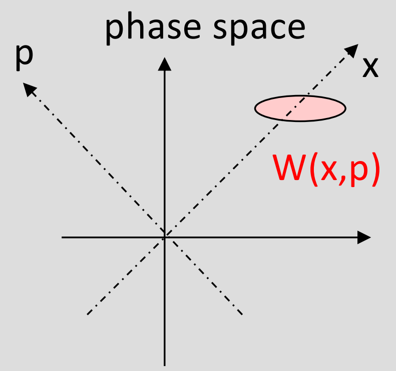
'click'-detection

alternatively :

continuous variables

x , p

→ Wigner-function

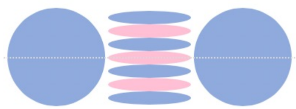


$$W(x,p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} d\xi \, \exp(-\frac{ip\xi}{\hbar}) \; \Psi^*(x - \frac{1}{2}\xi) \; \Psi(x + \frac{1}{2}\xi)$$

types of continuous quantum variables

- field quadratures
- Stokes variables (polarization)

field quadrature detection (amplitude, phase, …)

	wave function in q	wave function Fock/coherent basis	Wigner function $\alpha = \frac{q}{\sqrt{\hbar\omega}} + ip\sqrt{\hbar\omega}$
vacuum state (n=0)	$N_0 e^{-\frac{q^2}{\hbar\omega}}$	$\|0\rangle$	$\frac{2}{\pi} e^{-2\|\alpha\|^2}$
Fock state, n=1	$N_1 q e^{-\frac{q^2}{\hbar\omega}}$	$\|1\rangle$	$\frac{1}{\pi}(4\|\alpha\|^2 - 1)e^{-2\|\alpha\|^2}$
coherent state $\|\beta\rangle$	…	$\|\beta\rangle = e^{-\|\beta\|^2/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}\|n\rangle$	$\frac{2}{\pi} e^{-2\|\alpha-\beta\|^2}$
thermal state	N.A.	N.A.	$\frac{w}{\pi} e^{-\|\alpha\|^2 w}$, $w = \frac{1}{\langle n\rangle + \frac{1}{2}}$
squeezed state	$N_{w_0} e^{-\frac{(q-q_0)^2}{2w_0^2\hbar\omega}+ip_0 q}$	$\frac{1}{\sqrt{\cosh r}}\sum_{n=0}^{\infty}(-e^{i\phi}\tanh r)^n \frac{\sqrt{(2n)!}}{2^n n!}\|2n\rangle$	$\frac{1}{\pi} e^{-rq^2/(\hbar\omega)-p^2\hbar\omega/r}$
cat state	…	$N(\|\alpha\rangle \pm \|-\alpha\rangle)$	

note: phase space distribution function is unique in claical physics, but in quantum physics it is not

Optics and quantum information

- field quantization
  - wavization
    - optics
    - mechanics
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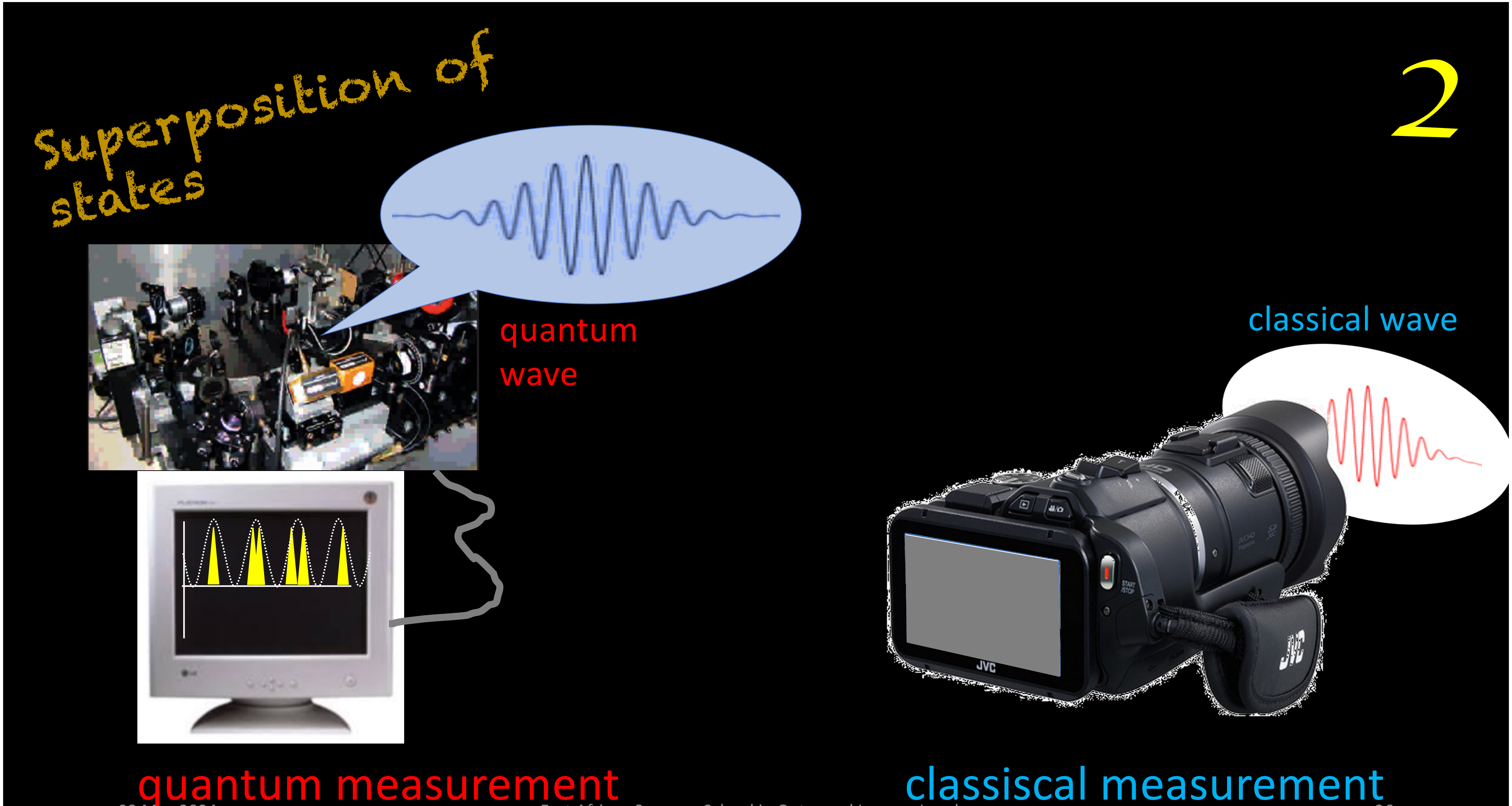
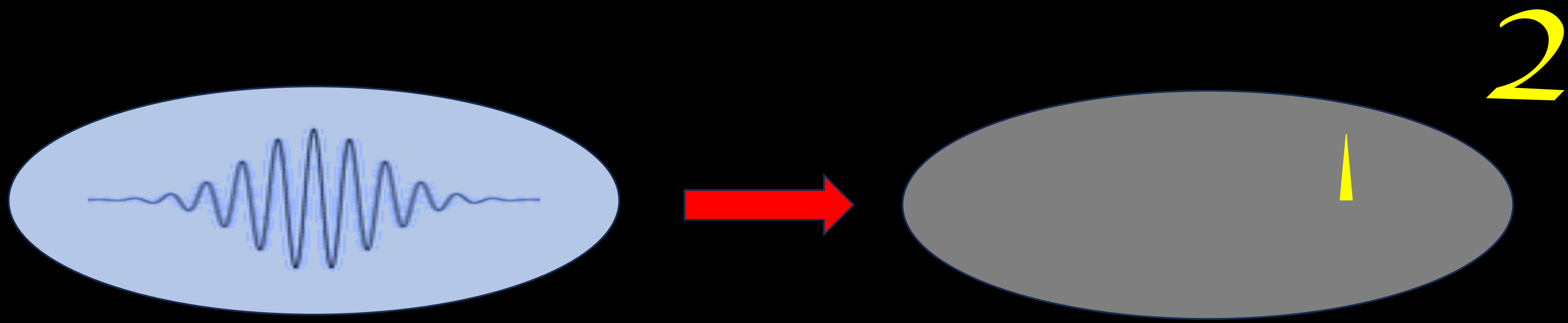
  - interferometry
    - sensing
    - quantum computing
  - sensing
  - communication



Superposition of states
2
quantum wave
classical wave
quantum measurement
classiscal measurement



"collapse of the wave function"

or

"projection"

measurement process in quantum physics

2

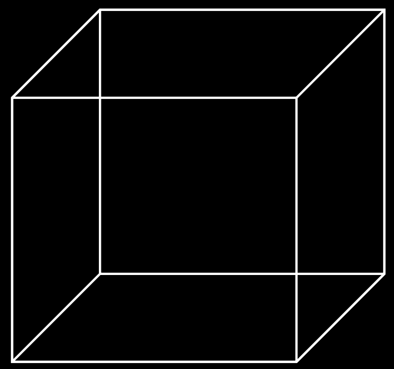
- collapse of the wave function ... *or*
- projection on to one of the superposed states

... it takes time to get used to it... visualization??

a **superposition state** contains information about two or more states so different that they seem to be mutually exclusive for us

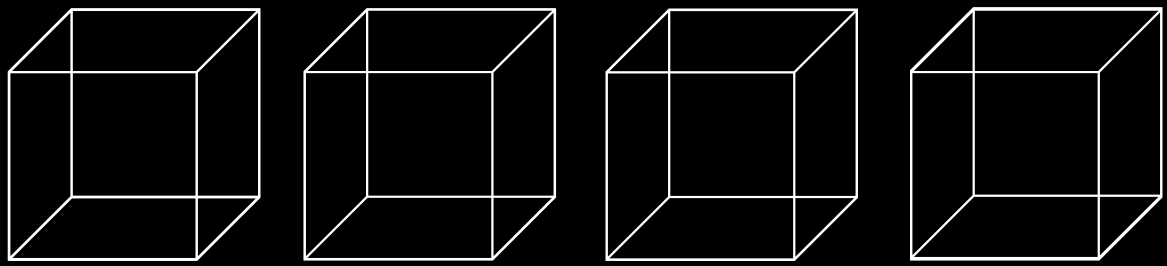
another more abstract example:

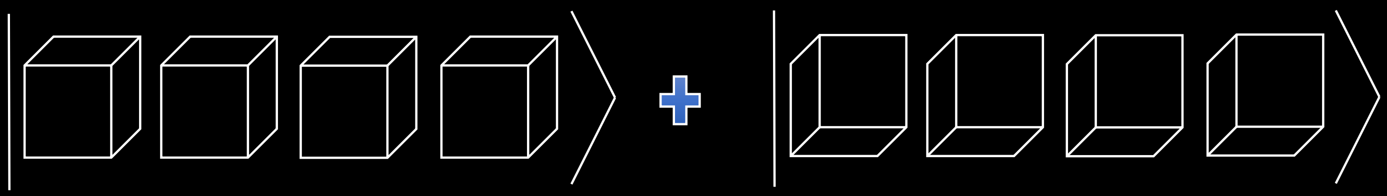
2



state 1 state 2

quantum - entanglement ... and measurement 2

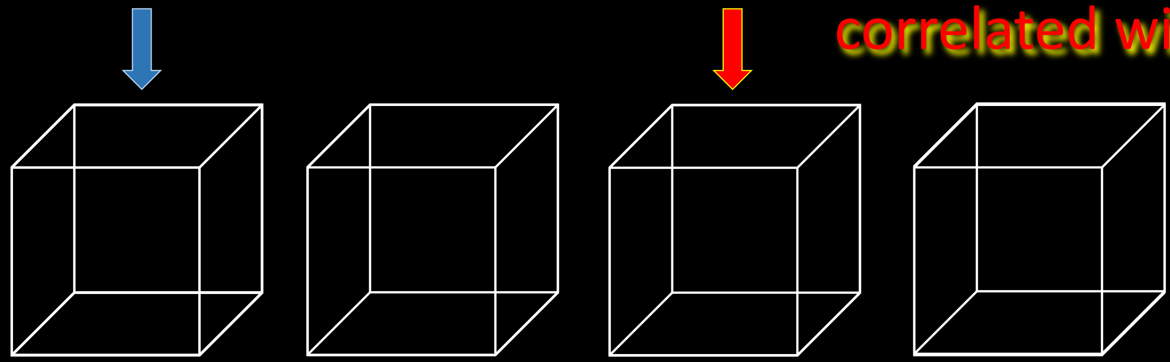




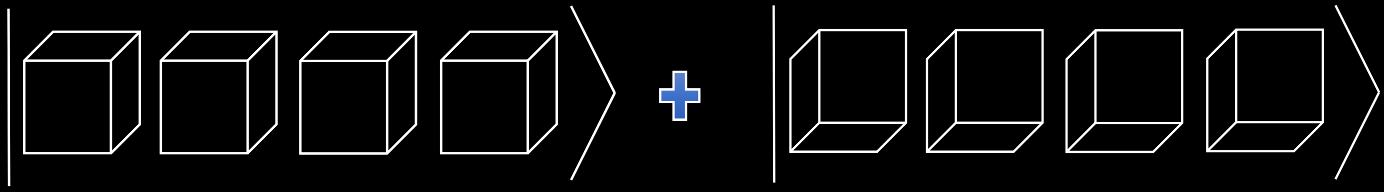
quantum - entanglement … and measurement 2

1. measurement

2. measurement … is then strictly correlated with the first one



… → projection , e. g.



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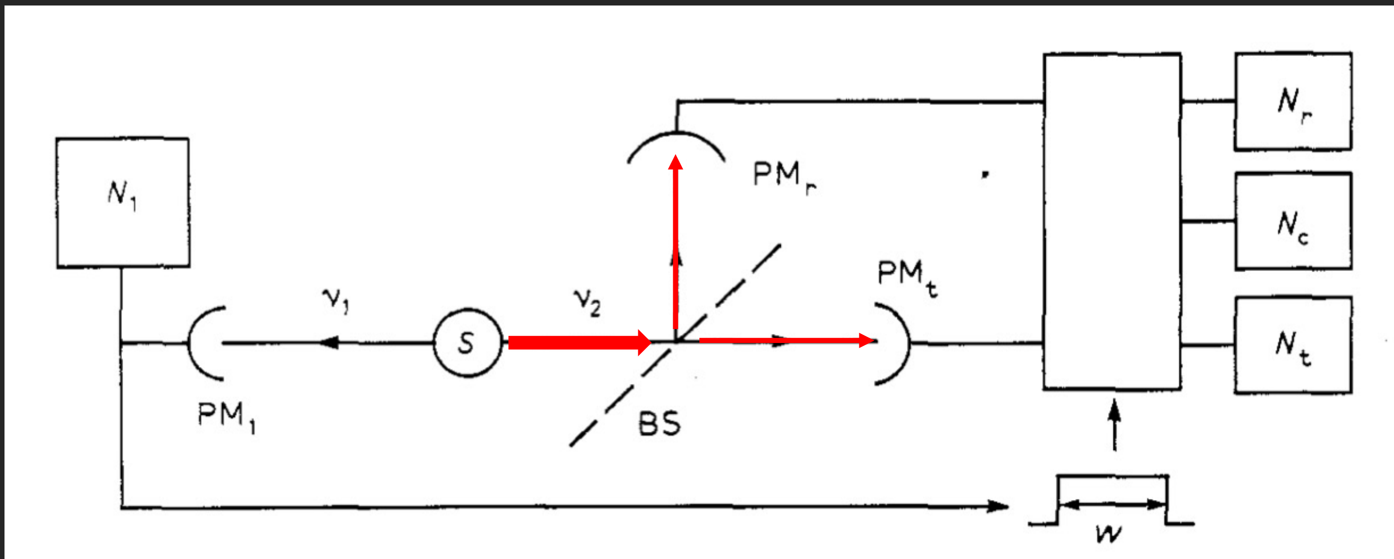
EUROPHYSICS LETTERS 15 February 1986

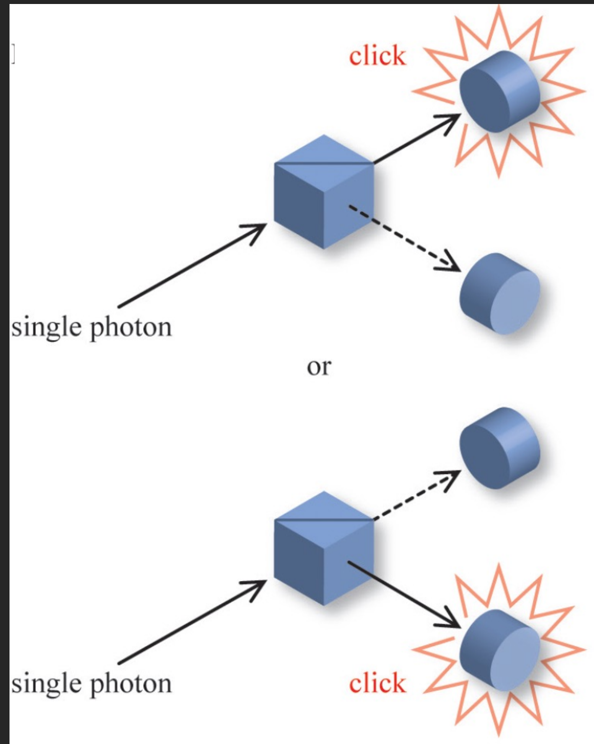
Europhys. Lett., **1** (4), pp. 173-179 (1986)

Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A New Light on Single-Photon Interferences.

P. GRANGIER, G. ROGER and A. ASPECT (*)

Institut d'Optique Théorique et Appliquée, B.P. 43 - F 91406 *Orsay, France*





a single photon is either reflected or transmitted but not both at the same time

experiment

VOLUME 92, NUMBER 18 PHYSICAL REVIEW LETTERS week ending 7 MAY 2004

Experimental Demonstration of Single Photon Nonlocality

Björn Hessmo,[1,*] Pavel Usachev,[2] Hoshang Heydari,[1] and Gunnar Björk[1]
...electronics and Information Technology, Royal Institute of Technology (KTH), S-16440 Kista, Sweden
...Academy of Sciences, Politekhnicheskaya ul. 26, St. Petersburg, 194021 Russia

theory

PHYSICAL REVIEW A, VOLUME 64, 042106

Single-particle nonlocality and entanglement with the vacuum

Gunnar Björk* and Per Jonsson
Department of Microelectronics and Information Technology, Royal Institute ...
SE-164 40 Kista, Sweden

Luis L. Sánchez-Soto
Departamento de Óptica, Facultad de Ciencias Físicas, Universidad ...

(Received 15 March 2001; published 13 Septe...

January 2016

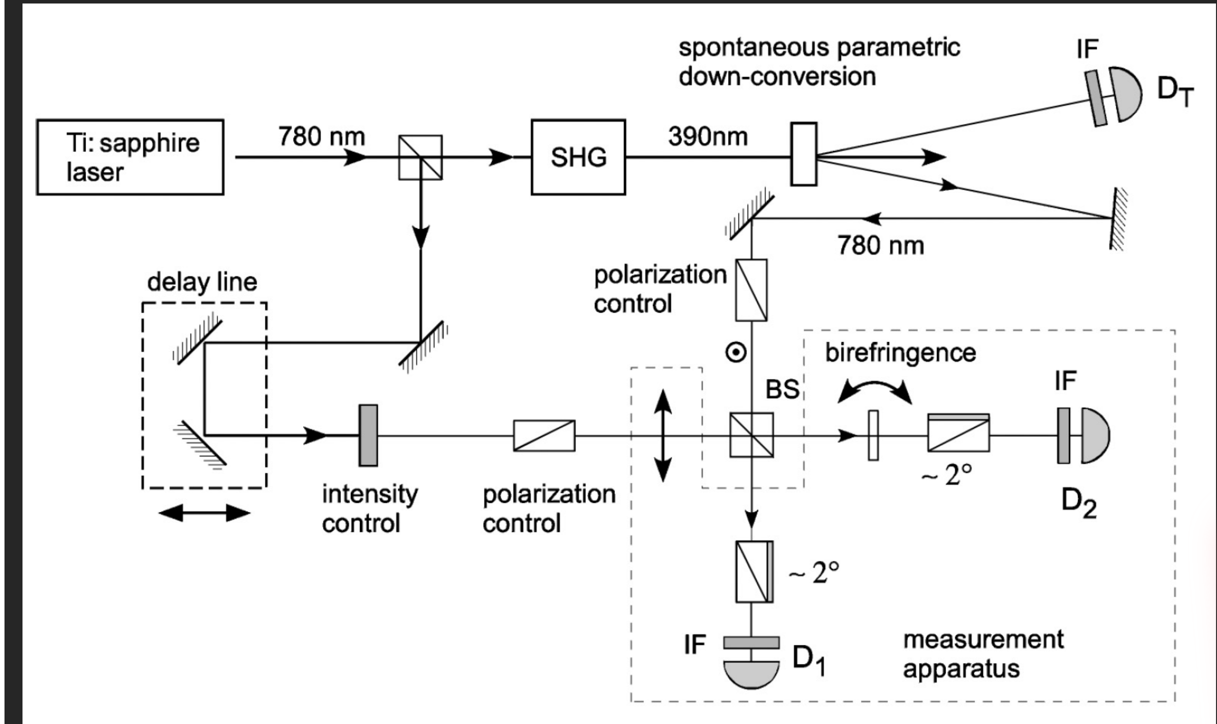
NEWSLETTER

Commission Internationale d'Optique • International Commission for Optics

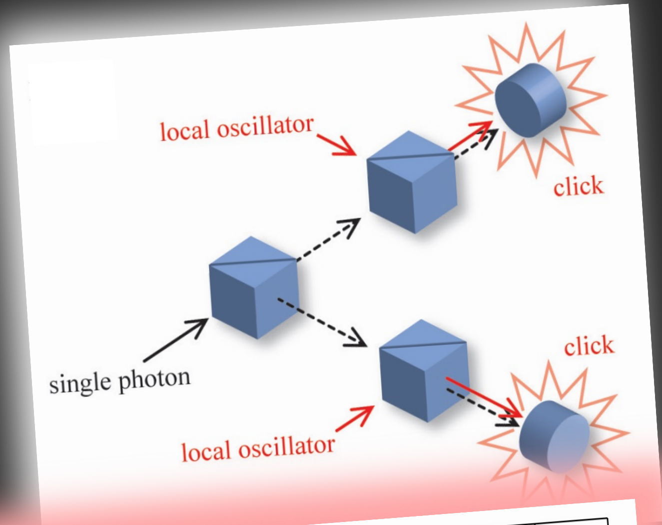
Getting used to quantum optics ...

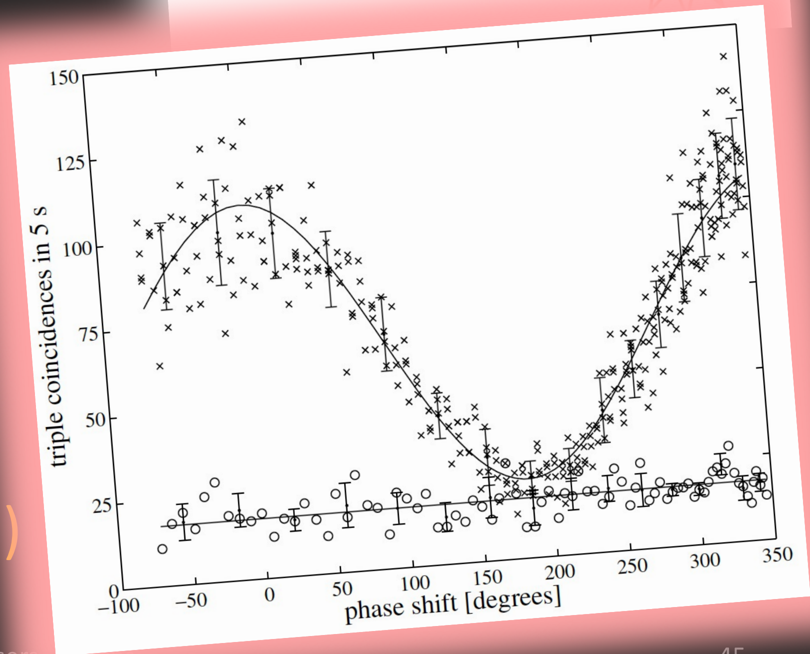
... and measuring one photon at both output ports of a beam splitter.

INTERNATIONAL YEAR OF LIGHT 2015



B Hessmo, P Usachev, H Heydari, G Björk, Phys Rev Lett 92, 180401 (2004)





measurement of energy

versus

measurement of field amplitudes (quadratures)

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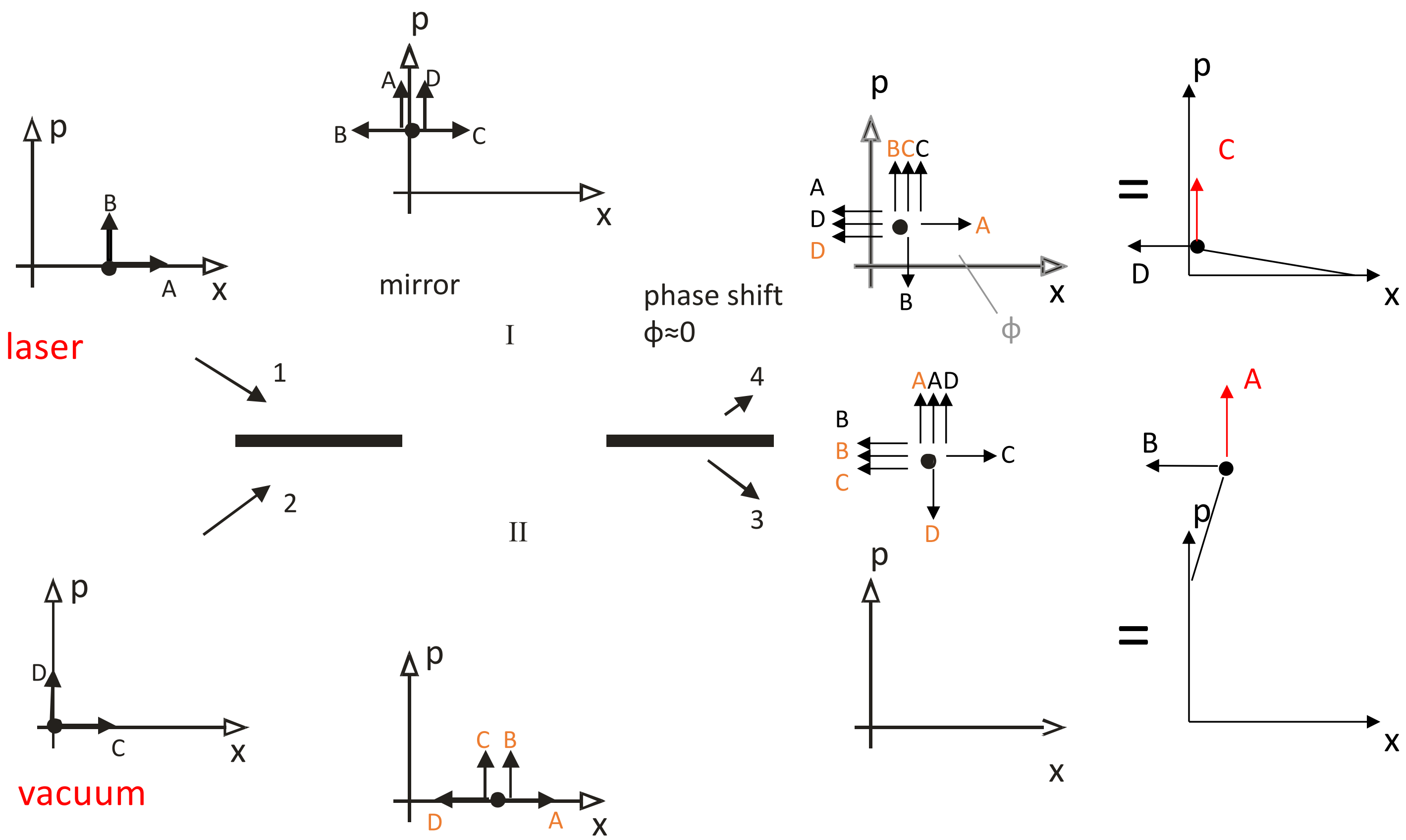
interferometer



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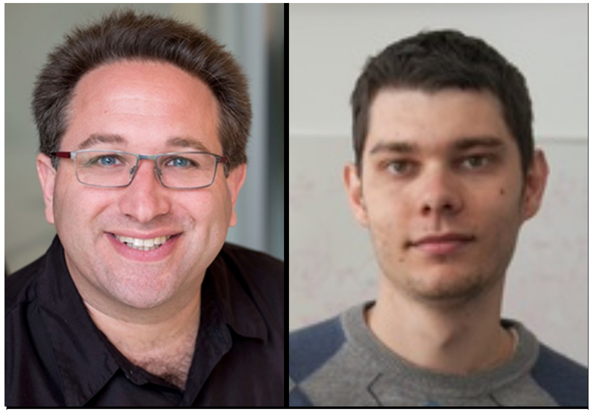
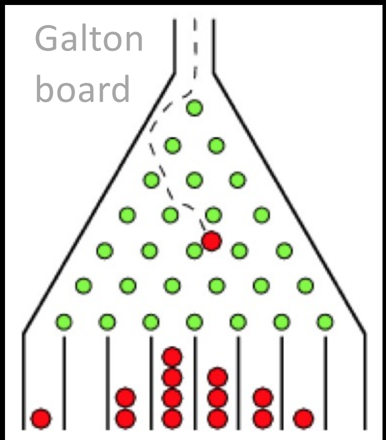
THEORY OF COMPUTING, Volume 9 (4), 2013, pp. 143–252
www.theoryofcomputing.org

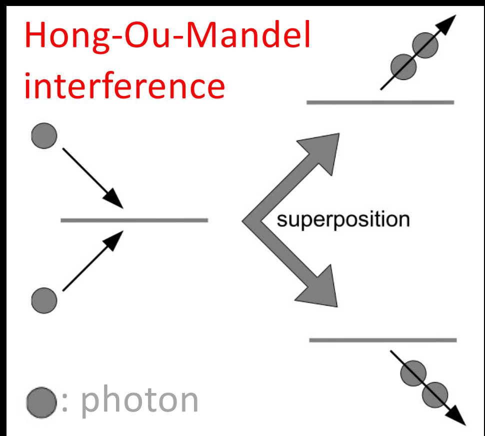
The Computational Complexity of Linear Optics*

Scott Aaronson[†] Alex Arkhipov[‡]

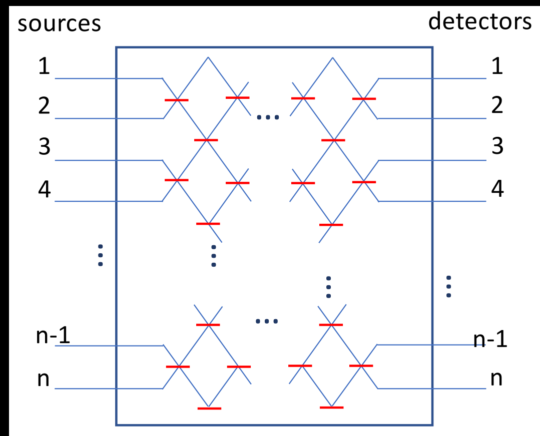


Scott Aaronson Alex Arkhipov





single photon states at the input



photon number resolving detectors at the output: $P(n)$

M. A. Broome et al., Science 339, 794 (2013).
M. Tillmann et al., Science 339, 798 (2013).
A. Crespi et al., Nat. Photonics 7, 545 (2013).

H. Wang et al., Nat. Photonics 11, 361 (2017).
Y. He et al., Phys. Rev. Lett. 118, 190501 (2017).
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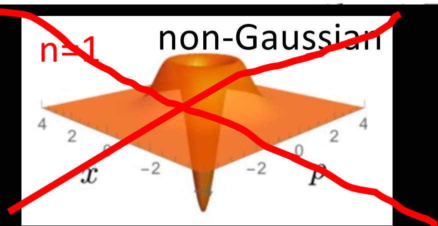
PRL 119, 170501 (2017) PHYSICAL REVIEW LETTERS week ending 27 OCTOBER 2017

Gaussian Boson Sampling

Craig S. Hamilton,[1,*] Regina Kruse,[2] Linda Sansoni,[2] Sonja Barkhofen,[2] Christine Silberhorn,[2] and Igor Jex[1]

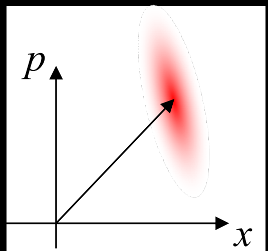
Christine Silberhorn

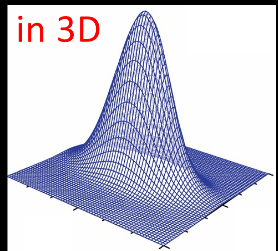


deterministic

Gaussian light source:

squeezed light





beam splitters

phase shifters

photon number resolving detector

non linearity,
non Gaussian operation

Igor Jex

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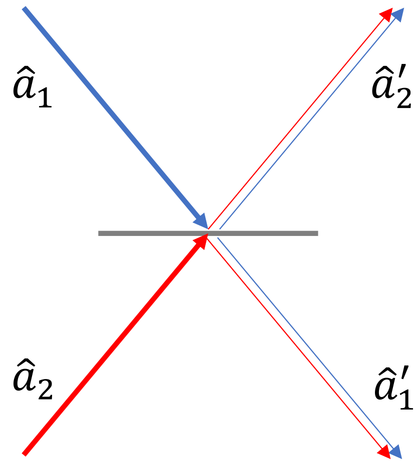
quantum communication

- secure communication of classical information (quantum key distribution QKD)
    - security through loss of information by measurement
    - fibre technology
    - free space / satellite technology

- exchange of quantum information e. g. between quantum computers thank you

state at input

$f(\hat{a}_1, \hat{a}_2)|0,0\rangle$



state at output

$f(\hat{a}_1', \hat{a}_2')|0,0\rangle$

$$\hat{a}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1' + \hat{a}_2')$$

$$\hat{a}_2 = \frac{1}{\sqrt{2}}(-\hat{a}_1' + \hat{a}_2')$$

examples:

$\hat{a}_1^\dagger|0,0\rangle = |1,0\rangle$ → $\frac{1}{\sqrt{2}}\left(\hat{a}_1'^\dagger + \hat{a}_2'^\dagger\right)|0,0\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle + |0,1\rangle)$

$\hat{a}_1^\dagger \hat{a}_2^\dagger|0,0\rangle = |1,1\rangle$ → $\frac{1}{2}\left(\hat{a}_1'^\dagger + \hat{a}_2'^\dagger\right)\left(-\hat{a}_1'^\dagger + \hat{a}_2'^\dagger\right)|0,0\rangle = \frac{1}{\sqrt{2}}(-|2,0\rangle + |0,2\rangle)$

product states → entangled states

$\frac{1}{\sqrt{2}}\hat{a}_1^{\dagger 2}|0,0\rangle = |2,0\rangle$ → $\frac{1}{2\sqrt{2}}\left(\hat{a}_1'^\dagger + \hat{a}_2'^\dagger\right)^2|0,0\rangle = \frac{1}{2}\left(|2,0\rangle + \sqrt{2}|1,1\rangle + |0,2\rangle\right)$

parametric down conversion:

Hamiltonian $\hat{H}_{PCD} = i\gamma\left(\hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{a}_1 \hat{a}_2\right)$

or

$$\hat{H}_{PCD} = i\gamma\left(\hat{b}\hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{b}^\dagger \hat{a}_1 \hat{a}_2\right)$$

$\hat{a}_1 = \hat{a}_2$

amplification:

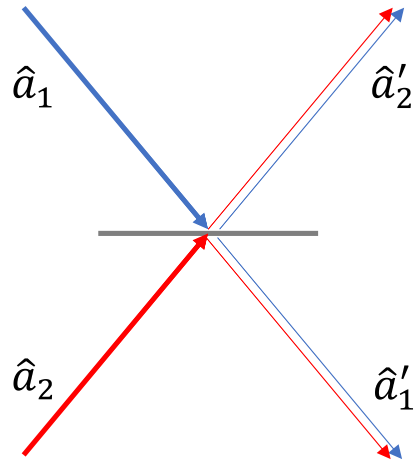
$$\hat{c} = \sqrt{G}\,\hat{a} + \sqrt{G-1}\,\hat{b}^\dagger$$

phase sensitive amplification:

$$\hat{c} = \sqrt{G}\,\hat{a} + \sqrt{G-1}\,\hat{a}^\dagger$$

state at input

$f(\hat{a}_1, \hat{a}_2)|0,0\rangle$



state at output

$f(\hat{a}_1', \hat{a}_2')|0,0\rangle$

$$\hat{a}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1' + \hat{a}_2')$$

$$\hat{a}_2 = \frac{1}{\sqrt{2}}(-\hat{a}_1' + \hat{a}_2')$$

examples:

$$|\alpha, 0\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \hat{a}_1^{\dagger n} |0,0\rangle$$

$$= e^{-\frac{|\alpha|^2}{2}} e^{\alpha \hat{a}_1^\dagger} |0,0\rangle$$

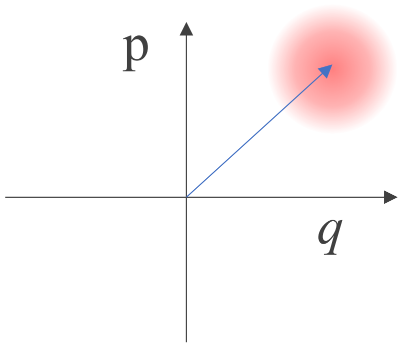
$$\rightarrow \quad e^{-\frac{|\alpha|^2}{2}} e^{\alpha \frac{\hat{a}_1'^\dagger + \hat{a}_2'^\dagger}{\sqrt{2}}} |0,0\rangle = e^{-\frac{\left|\frac{\alpha}{\sqrt{2}}\right|^2 + \left|\frac{\alpha}{\sqrt{2}}\right|^2}{2}} e^{\frac{\alpha}{\sqrt{2}} \hat{a}_1'^\dagger + \frac{\alpha}{\sqrt{2}} \hat{a}_2^\dagger} |0,0\rangle$$

$$= \left| \frac{\alpha}{\sqrt{2}}, \frac{\alpha}{\sqrt{2}} \right\rangle$$

product states $\rightarrow$ product states

Continuous variables q, p

their distribution described by phase space distribution



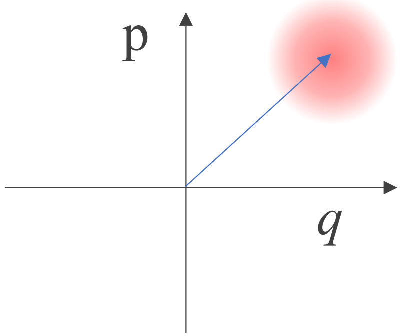
$$\langle n\rangle = \langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle = |\alpha|^2$$

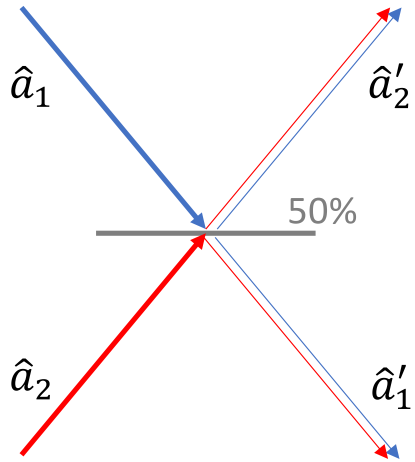
$$\sqrt{\langle\Delta n^2\rangle} = \sqrt{\langle n^2\rangle - \langle n\rangle^2} = \left[\langle\alpha|\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}|\alpha\rangle - \langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle^2\right]^{\frac{1}{2}}$$

$$= \sqrt{|\alpha|^4 + |\alpha|^2 - |\alpha|^4} = \sqrt{\langle n\rangle}$$

$\langle n \rangle = |\alpha|^2$

$\sqrt{\langle \Delta n^2 \rangle} = |\alpha|$





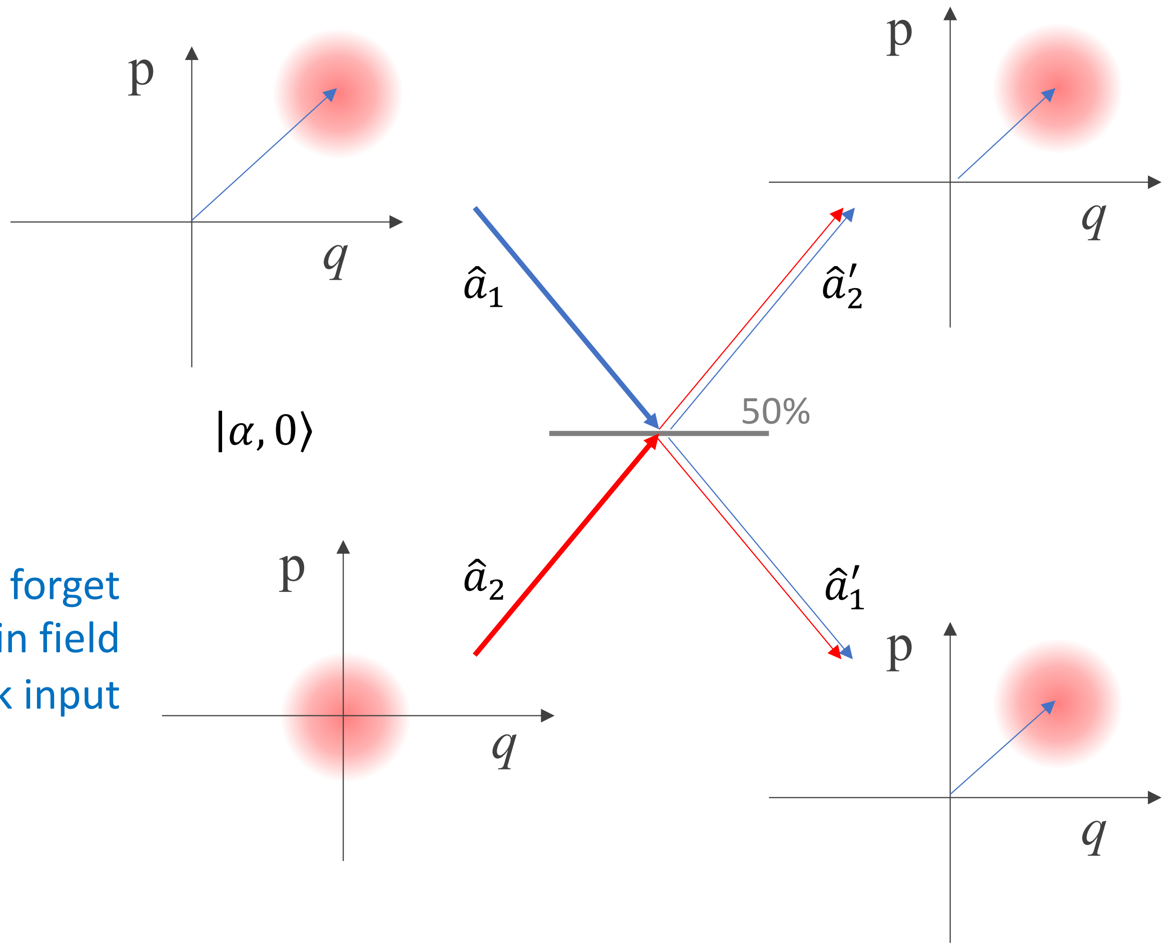
$\langle n \rangle = \frac{1}{2}|\alpha|^2$

~~$\sqrt{\langle \Delta n^2 \rangle} = \frac{1}{2}|\alpha|$~~ $\sqrt{\langle \Delta n^2 \rangle} = \frac{1}{\sqrt{2}}|\alpha|$

$\langle n \rangle = \frac{1}{2}|\alpha|^2$

~~$\sqrt{\langle \Delta n^2 \rangle} = \frac{1}{2}|\alpha|$~~ $\sqrt{\langle \Delta n^2 \rangle} = \frac{1}{\sqrt{2}}|\alpha|$

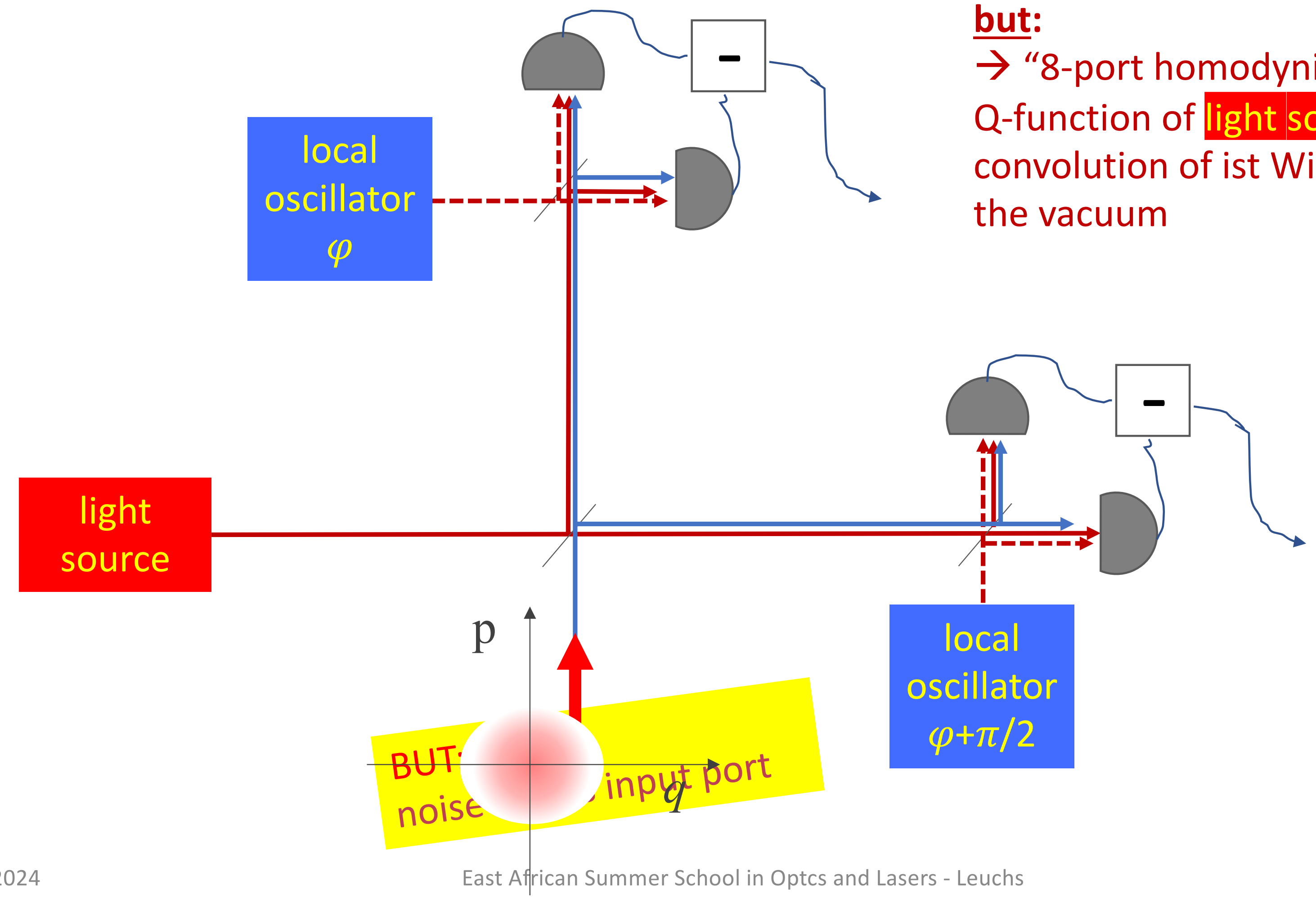
noise higher than expected!



do not forget
uncertain field
at dark input

convolution of
the two inputs
(but careful – in general their
can be correlations)

amplitude & phase measurement



homodyne:
measures Wigner function projection

but:
→ "8-port homodyning" measures Q-function of light source, which is the convolution of ist Wigner function with the vacuum

Table 4.1 Overview of quasi-probability distributions

Ordering	Normal	Symmetric	Antinormal
Energy	$\langle n \vert \hat{a}^\dagger \hat{a} \vert n \rangle = n$	$\langle n \vert \frac{1}{2}(\hat{a}^\dagger \hat{a} + \hat{a}\hat{a}^\dagger) \vert n \rangle = n + \frac{1}{2}$	$\langle n \vert \hat{a}\hat{a}^\dagger \vert n \rangle = n + 1$
Detection scheme	Direct detection: click detector, photon number resolving	Homodyne: 4-port detection	Double-homodyne: 8-port detection
Determining phase-space distribution	Reconstruction by deconvoluting the Wigner function	Tomographic reconstruction from homodyne data	Phase-space distribution directly measured with heterodyne detection
Corresponding representation	P-distribution	Wigner function	Q-function

Quantum Interference between a Single-Photon Fock State and a Coherent State

A Windhager et al., arXiv:1009.1844v2

the case

$$\hat{a}_1^{\dagger}|0{,}0\rangle = |1{,}0\rangle \quad \rightarrow \quad \frac{1}{\sqrt{2}}\left(\hat{a}_1'^{\dagger} + \hat{a}_2'^{\dagger}\right)|0{,}0\rangle = \frac{1}{\sqrt{2}}\left(|1{,}0\rangle + |0{,}1\rangle\right)$$

becomes much more involved in the continuous variable picture

$$W_{\hat{\rho}}(q,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} \langle q - y/2|\,\hat{\rho}\,|q + y/2\rangle \exp(iyp/\hbar)dy$$

$$\hat{\rho}_3 = |t'|^2\,|t\alpha\rangle_3\,\langle t\alpha|_3 + |r'|^2\,\hat{D}_3(t\alpha)\,|1\rangle_3\,\langle 1|_3\,\hat{D}_3^{\dagger}(t\alpha).$$

$$\hat{\rho}_2 = |r|^2\,|r\alpha\rangle_2\,\langle r\alpha|_2 + |t|^2\,\hat{D}_2(r\alpha)\,|1\rangle_2\,\langle 1|_2\,\hat{D}_2^{\dagger}(r\alpha)$$

$$W_{\hat{\rho}_3} = |t'|^2\,W_{\hat{\rho}(\hat{D}(t\alpha)|0\rangle)} + |r'|^2\,W_{\hat{\rho}(\hat{D}(t\alpha)|1\rangle)}$$

$$W_{\hat{\rho}(\hat{D}(t\alpha)|0\rangle)}(q,p) = W_{\hat{\rho}(|0\rangle)}(q',p') = \frac{1}{\pi\hbar}\exp\left[-\left(\frac{q'}{q_0}\right)^2 - \left(\frac{p' q_0}{\hbar}\right)^2\right]$$

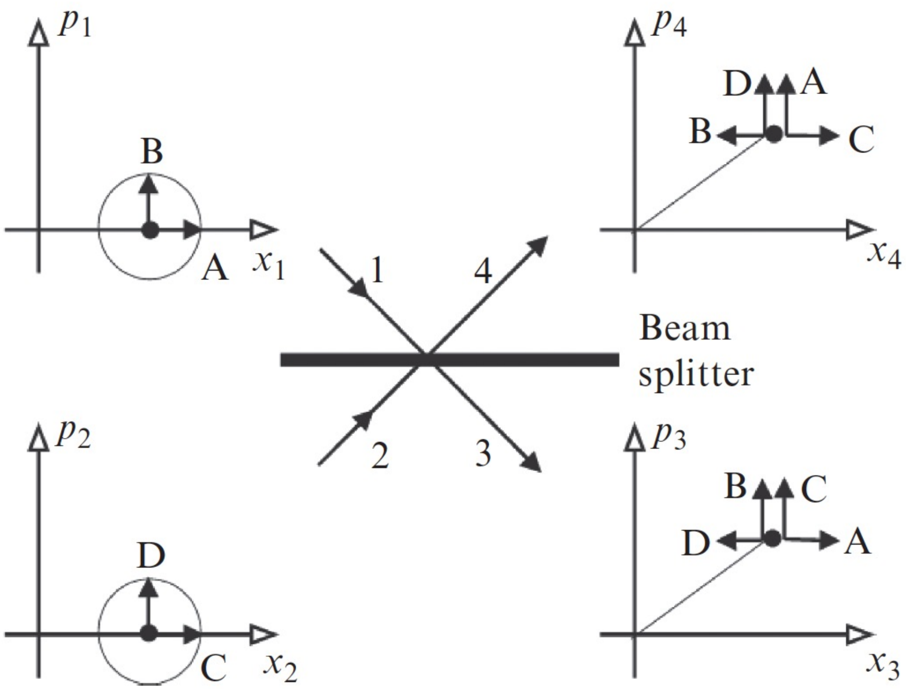
$$W_{\hat{\rho}(\hat{D}(t\alpha)|1\rangle)}(q,p) = W_{\hat{\rho}(|1\rangle)}(q',p')$$

$$= -\frac{1}{\pi\hbar}\exp\left[-\left(\frac{q'}{q_0}\right)^2 - \left(\frac{p' q_0}{\hbar}\right)^2\right]\left[1 + 2\left(-\left(\frac{q'}{q_0}\right)^2 - \left(\frac{p' q_0}{\hbar}\right)^2\right)\right]$$

the case

Gaussian states → (Gaussian states)‘

is, however, much simpler in the continuous variable picture



thank you

merci

спасибо

خیلی ممنون

çox sağ ol

非常感谢你

الشكر لكم

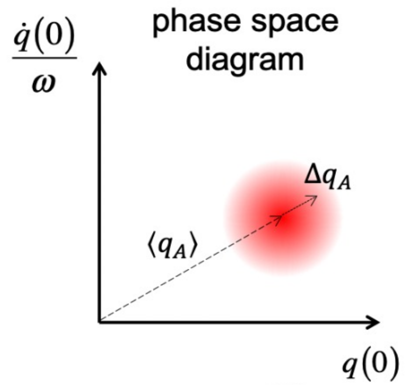
धन्यवाद आप

gracias

molto grazie

ehara koe i a ia!

abstract phase space where excitation “lives“



$q(t)\,,\ \dot{q}(t)$

$q = q(0), \quad \dot{q} = \dot{q}(0)$

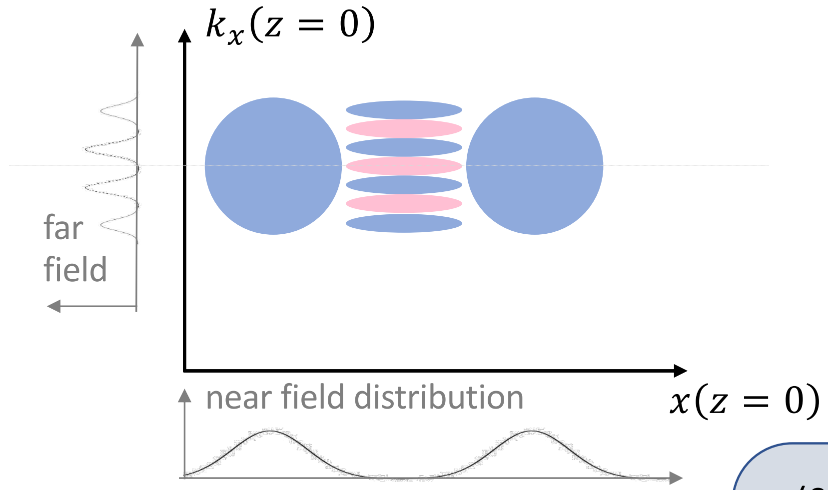
$P(q)\ , \qquad P(\dot{q})$

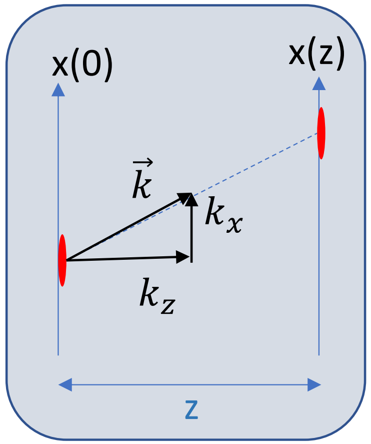
$\psi(q), \quad ! \quad \psi(\dot{q})$

(fractional) Fourier transform

$t \Leftrightarrow z$

“lab“ phase space of classical optics





$x(0)\,,\ k_x(0)$

$x\,, \qquad \frac{\partial x}{\partial z} = \frac{k_x}{k_z} \approx \frac{k_x}{k}$

$I(x)\,, \qquad I(k_x)$

$u(x)\,, \qquad u(k_x)$

(fractional) Fourier transform

Coherent state wave function

$$\psi_\alpha(x,t) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left[-\frac{m\omega}{2\hbar}\left(x - \sqrt{\frac{2\hbar}{m\omega}}\mathrm{Re}\,\alpha\right)^2 + i\sqrt{\frac{2m\omega}{\hbar}}\mathrm{Im}\,\alpha\right]$$

Two-component cat state wave function (identical components, located at \pm x_0)

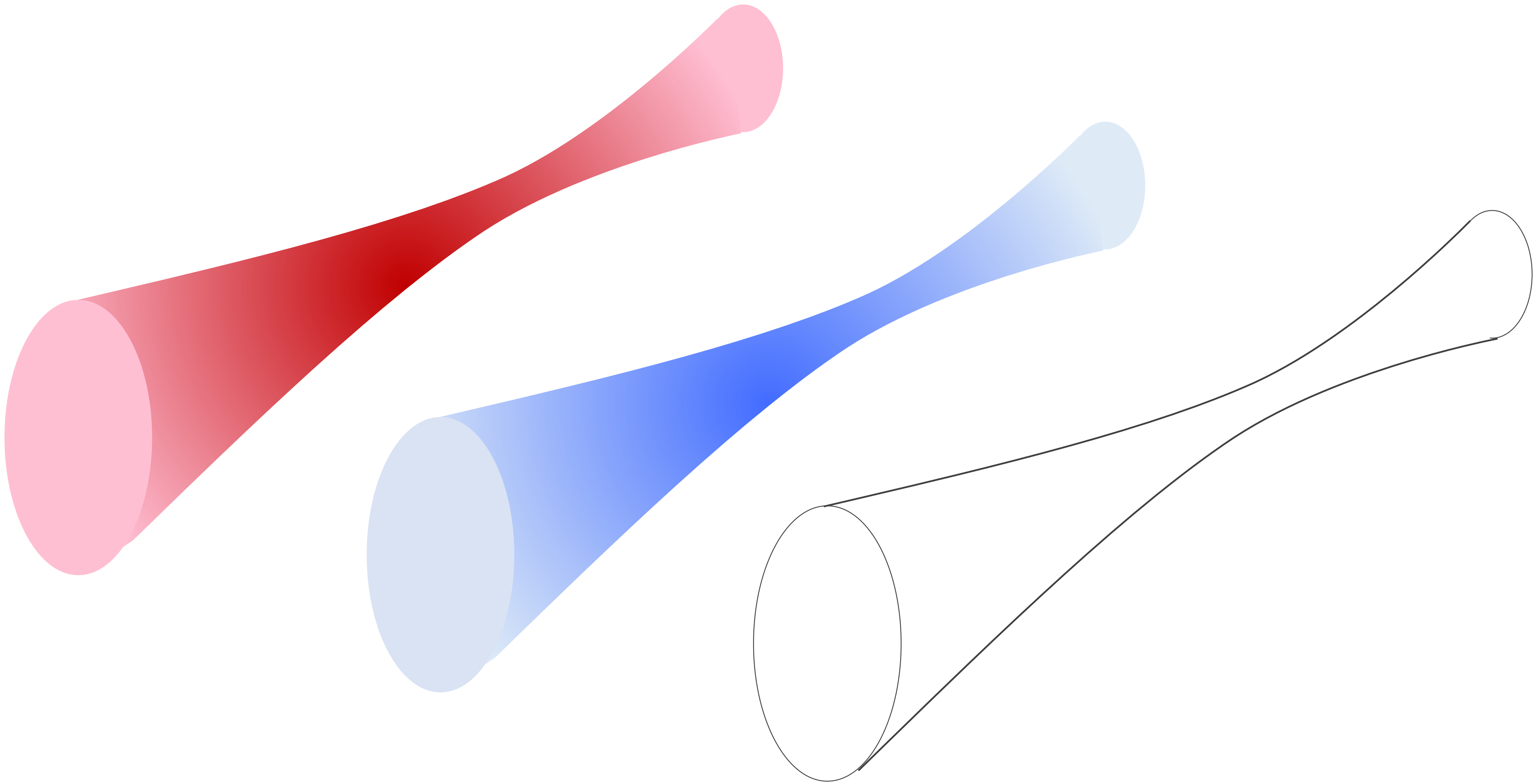
$$\psi_{\text{cat}}(x) = N_3(A_+ \exp[-\alpha(x-x_0)^2] + A_- \exp[-\alpha(x+x_0)^2]$$

$$|N_3|^2 = \sqrt{\frac{2\alpha}{\pi}}[|A_+|^2 + |A_-|^2 \ + \exp[-2\alpha x_0^2(A_+^* A_- + A_-^* A_+)]$$

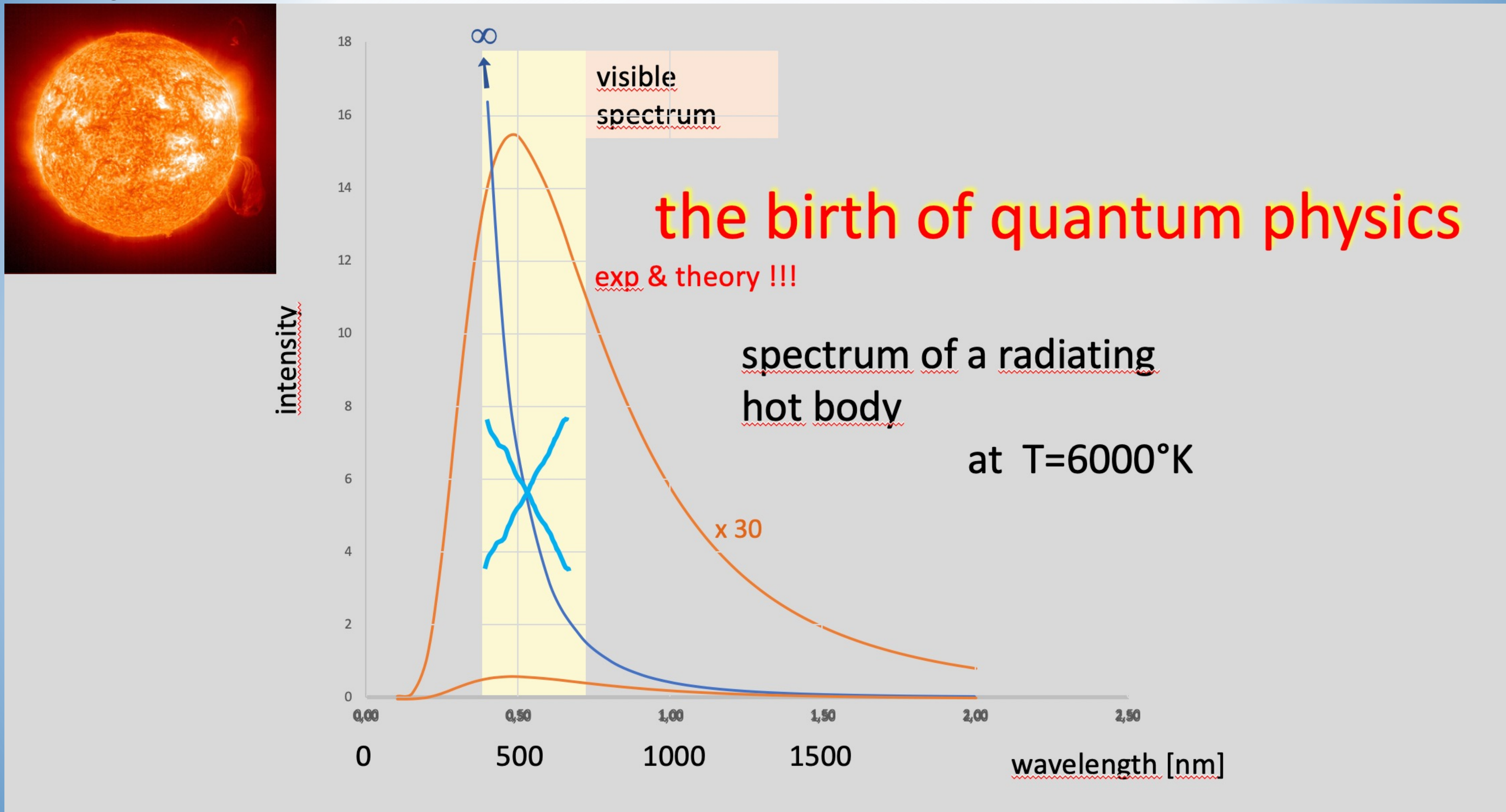
Wigner function

This was calculated in

https://www.sciencedirect.com/science/article/abs/pii/0031891474902158?via%3Dihub



First experimental evidence



thermodynamical equilibrium

- population of states of different energy: Boltzmann distribution $e^{-\frac{\varepsilon}{kT}}$
  - ... mean energy

$$\langle\varepsilon\rangle = \int d\varepsilon\, P(\varepsilon)\, \varepsilon = kT$$

$$P(\varepsilon) = N\ e^{-\frac{\varepsilon}{kT}}$$

Max Planck 1900

$$\langle\varepsilon\rangle = \sum_{n=0}^{\infty} P(\varepsilon)\, \varepsilon = \frac{hc/\lambda}{e^{\frac{hc}{\lambda\, kT}}-1} \xrightarrow{T\to\infty} kT$$

$$P(\varepsilon) = N'\ e^{-\frac{n\Delta}{kT}}$$

$$\varepsilon = n\,\Delta$$

$\Delta = hc/\lambda$ → quantum physics

photons per mode

$$\varepsilon = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

$T \rightarrow \infty$

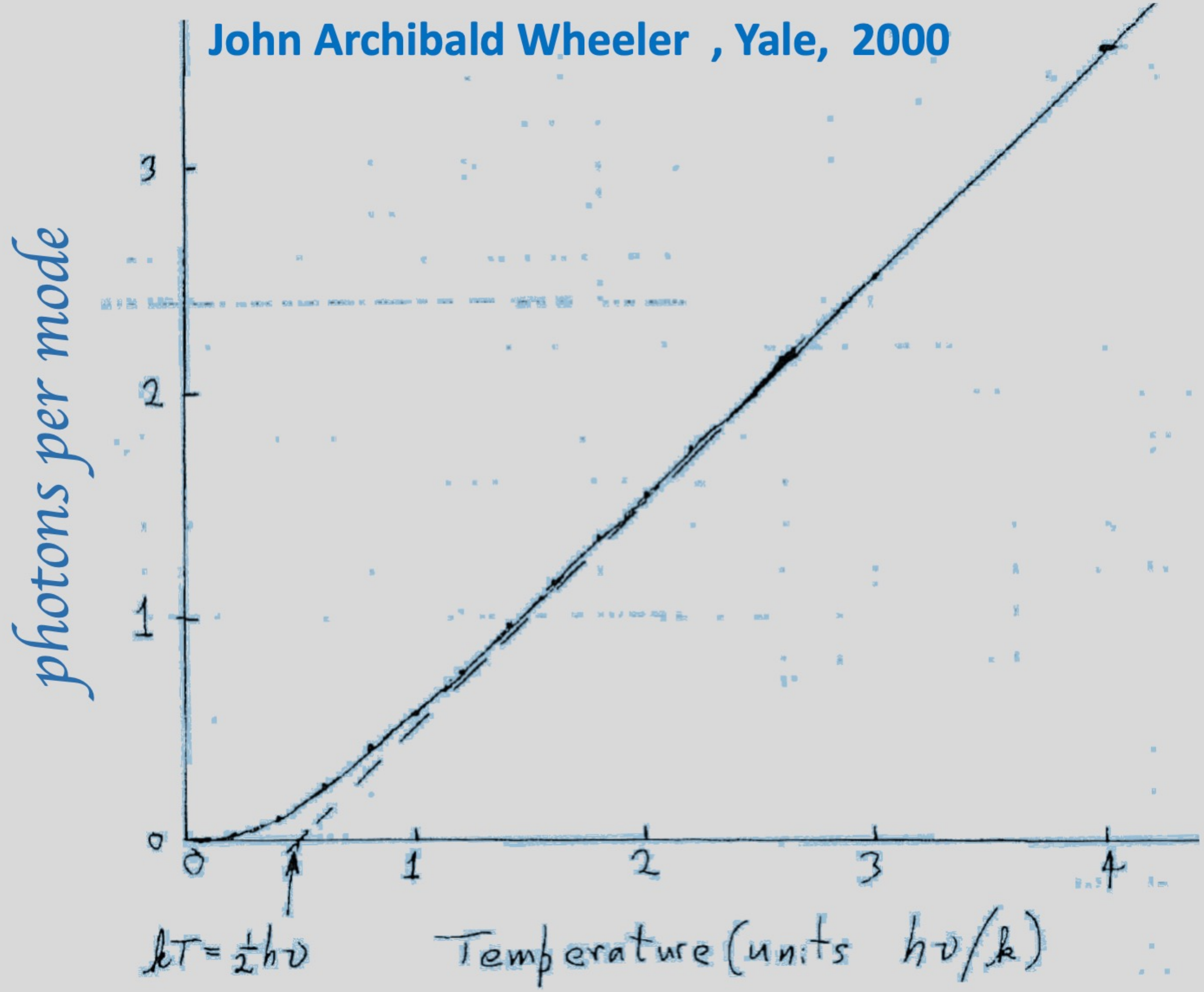
$\varepsilon \rightarrow kT$ **?**

$\varepsilon \rightarrow kT - \frac{1}{2} h\nu$

ground state energy of
a mode ≡ a harmonic oscillator

$$\rightarrow \varepsilon = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} + \frac{1}{2} h\nu$$

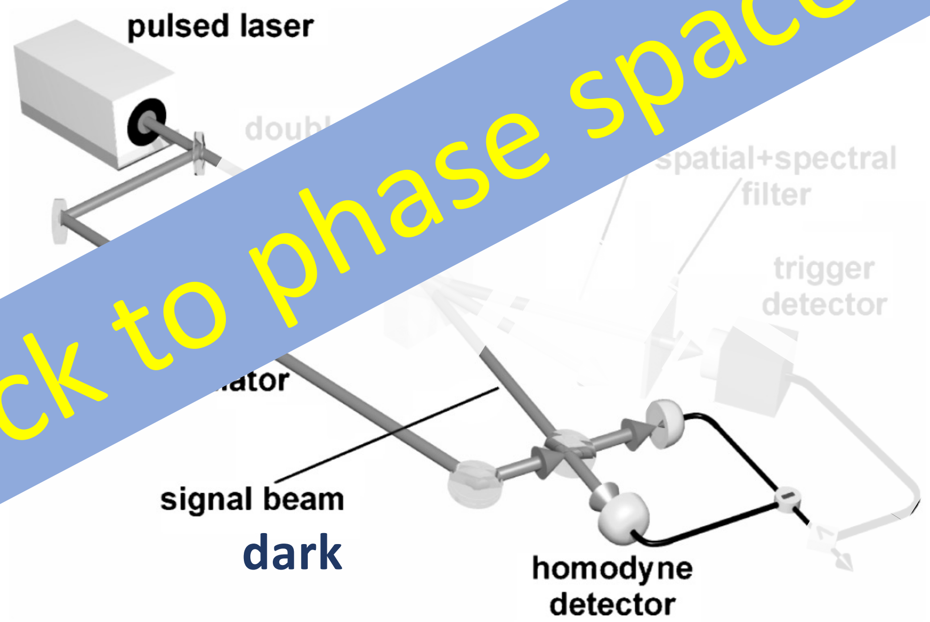
John Archibald Wheeler , Yale, 2000



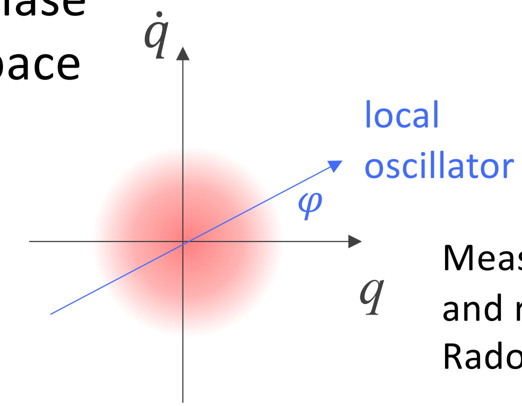
quantum state reconstruction of the vacuu ... photon Fock state)

A I Lvovsky et al., Phys. Rev. Lett. 87, 0504

back to phase space distribution



Homodyning → projection of phase space distribution onto "φ"



Measure for different φ and reconstruction, e.g. Radon transformation