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FOR THE SCIENCE OF LIGHT

Kerr squeezing in fibers and sub-shot-noise interferometry

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Outline

- Light modes
- Coherent and squeezed states
- Kerr squeezing
- Stokes parameters and polarization squeezing
- Polarization squeezing generation in fibers
- Interferometer sensitivity
- Kerr-squeezed interferometer

Classic mode of light

- Wave equation in an isotropic medium $$\nabla^2 \mathbf{E}(\mathbf{r},t) - \frac{1}{c}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t) = 0$$
- General solution $$\mathbf{E}(\mathbf{r},t) = E_0\left[\alpha(\mathbf{r},t)\exp(i2\pi\nu t) + \alpha^*(\mathbf{r},t)\exp(-i2\pi\nu t)\right] \quad \mathbf{p}(\mathbf{r},t)$$
- For α, the fundamental solution is Gaussian mode $$\alpha(\mathbf{r},t) = \frac{\alpha_0}{q(z)}\exp\left(\frac{-ik\rho^2}{2q(z)}\right)$$
- There are higher modes $$q(z) = z + iz_0$$

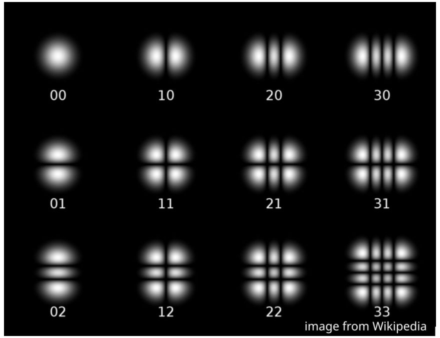
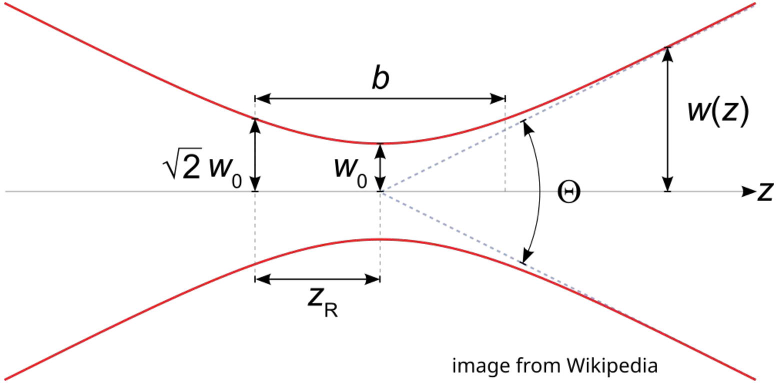


image from Wikipedia



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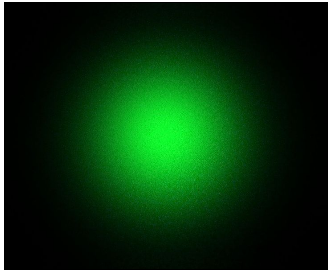


Classic mode of light

- To talk about a single Gaussian mode, you have to specify:
  - Propagation direction z
  - Waist position in space z_w, x_w, y_w
  - Waist radius w_0
  - Polarization $\mathbf{p}$
  - Frequency ν
- Properties of the mode:
  - Intensity I_0 or amplitude $|\alpha_0|$
  - Phase φ_0

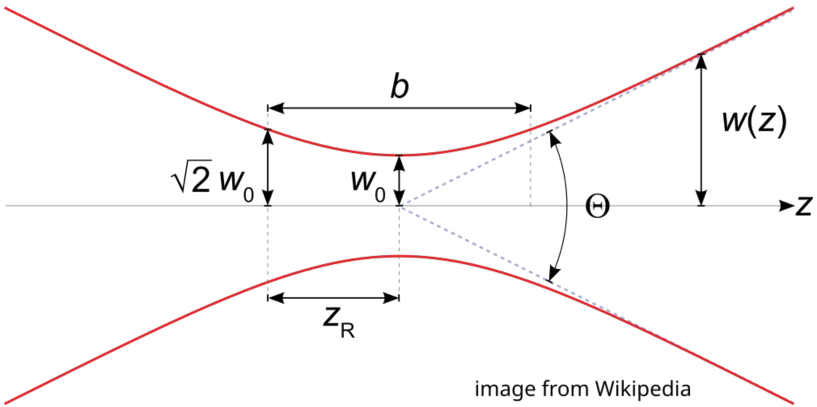
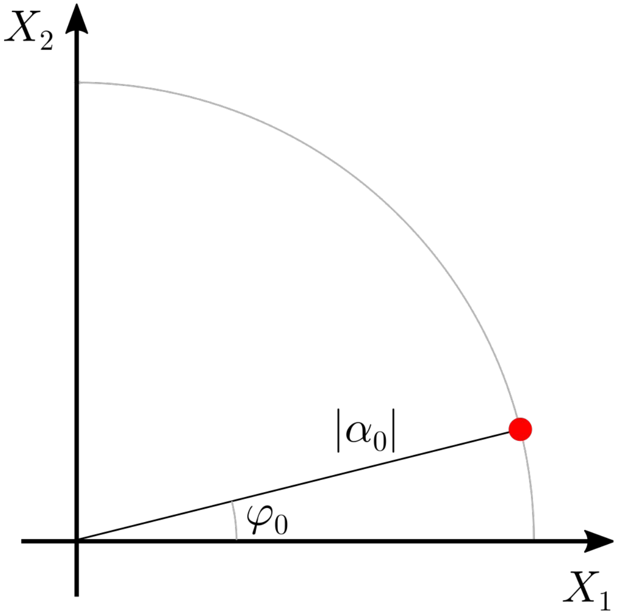


image from Wikipedia

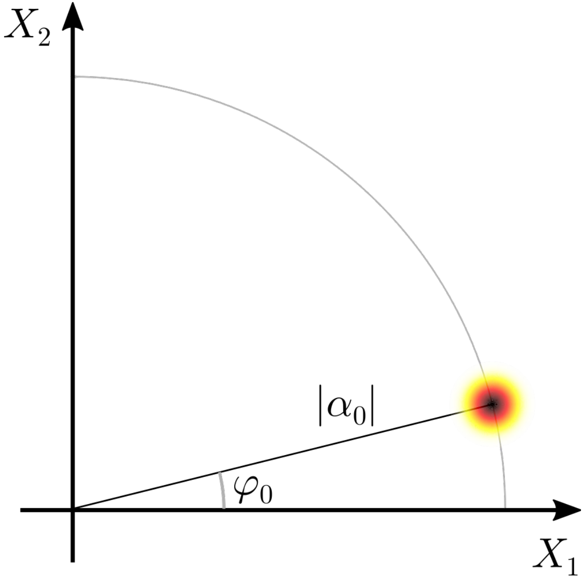
Quadrature space

- Intensity and phase are conveniently plotted on the quadrature (phase) space
- X_1 and X_2 are called the quadratures of the field
- Classic light is described with a single point (X_1, X_2)
- Passing through various optical elements changes the quadratures



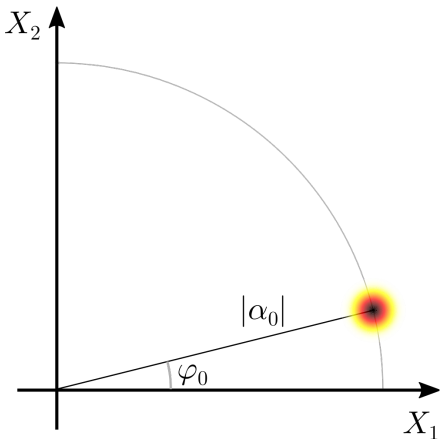
Quantum mode of light

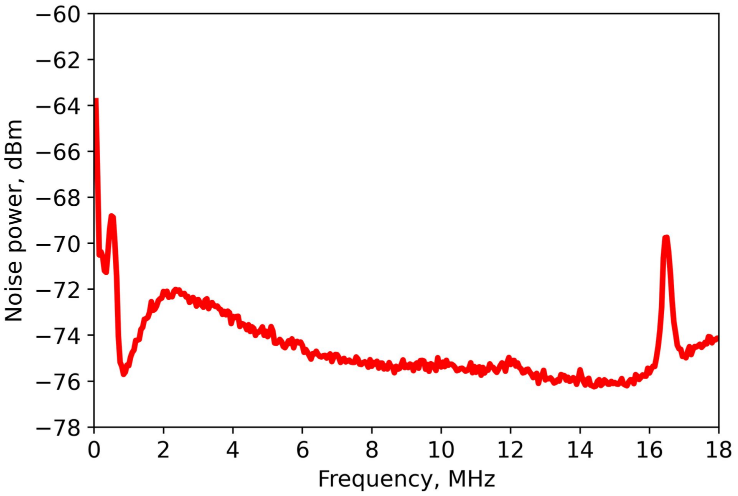
- Heisenberg uncertainty principle:
  conjugate variables can’t be precisely defined simultaneously
- The principle applies to the quadrature variables of a light mode
- The mode is no longer represented with a point, but must have some uncertainty area*
- This uncertainty is the source of noise in measurements
- * There are states with a more complicated representation



Quantum uncertainty and noise

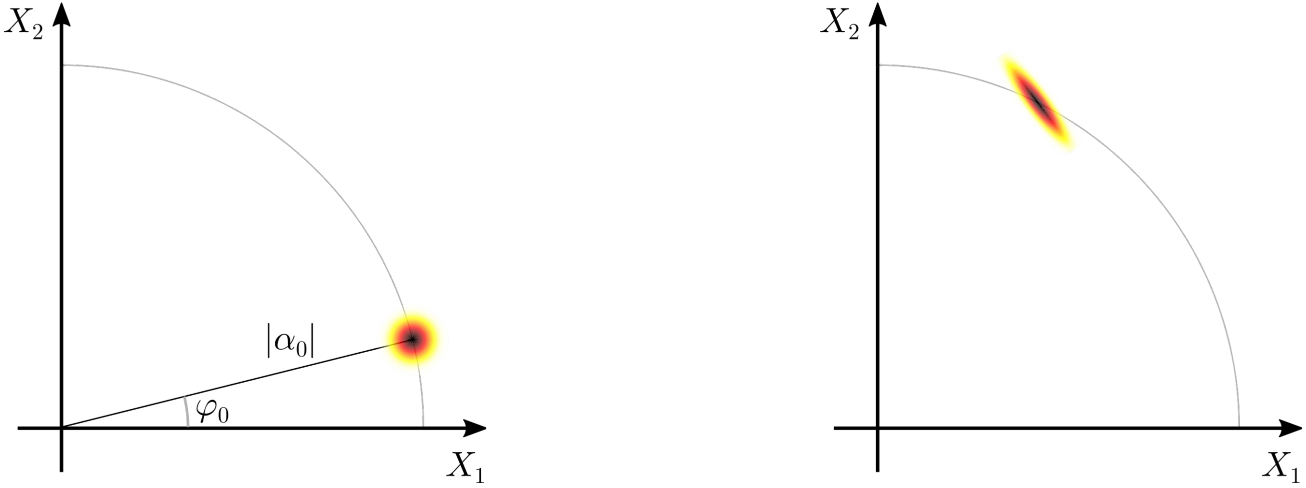
- When a variable such as intensity is measured, the result is taken from the uncertainty distribution
- Classical noise has a certain timescale: when measured fast enough, consecutive measurements give similar results
- Quantum noise has no timescale, result is random at every measurement
- Spectral power of quantum noise is flat



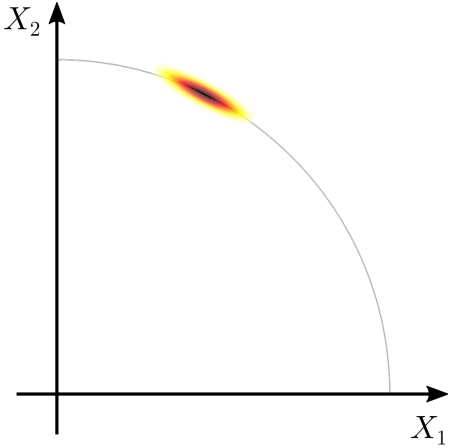


Coherent and squeezed states

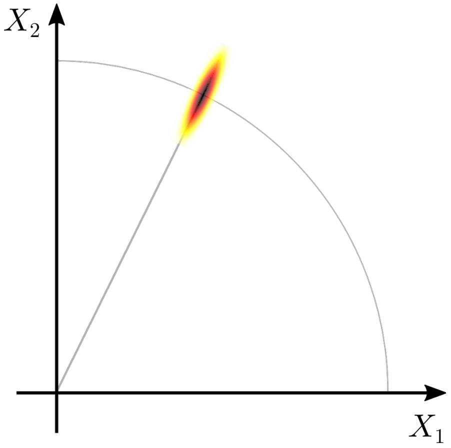
- Coherent states have a symmetric uncertainty in phase space
  - Closest description of the laser radiation
- Squeezed state's uncertainty reminds an ellipse with the minor axis smaller than the diameter of a coherent state



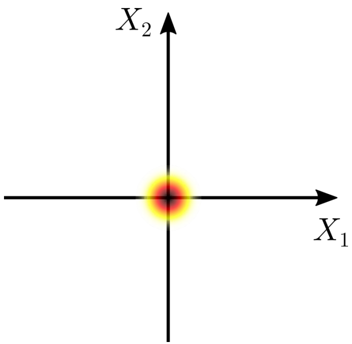
Special squeezed states



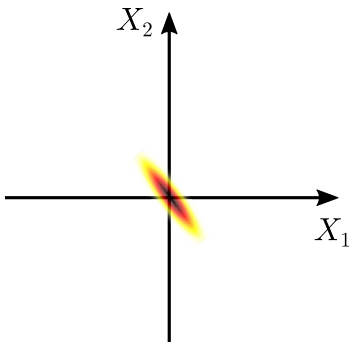
Amplitude squeezed state



Phase squeezed state



(Coherent) vacuum
Exactly 0 photons
Lowest energy



Squeezed vacuum
>0 photons

Kerr effect

- When the electric field is strong enough, the induced polarization in the medium is no longer proportional to the electric field
- In isotropic media such as glasses, the third-order nonlinearity is the lowest
- The change can be induced both by external electric field and by self-action (optical Kerr effect)
- Effectively, the refraction index is changed proportionally to the intensity
- Kerr effect leads to a number of effects in time and space

$$\mathbf{P}(t) = \varepsilon_0 \left(\chi^{(1)} \mathbf{E}(t) + \chi^{(2)} \mathbf{E}^2(t) + \chi^{(3)} \mathbf{E}^3(t) + \ldots \right),$$

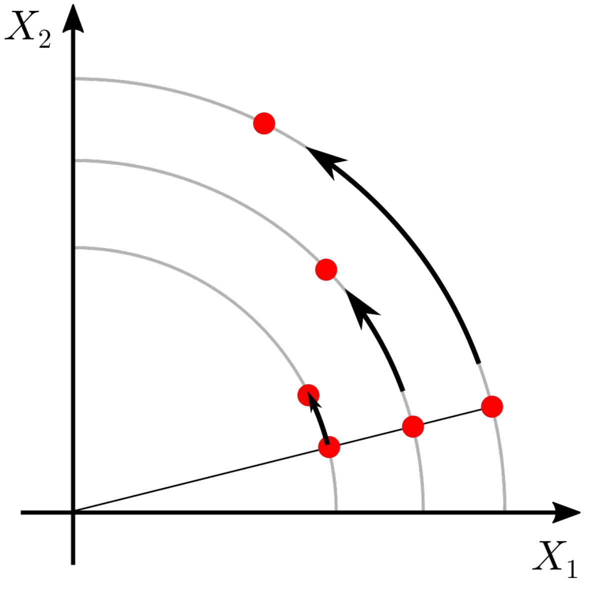
$$\tilde{n} = n + \bar{n}_2 |E|^2$$

Self-phase modulation

- In fibers, Kerr nonlinearity is responsible for a number of classical effects, most notably the self-phase modulation (SPM)
- One of the important terms in the nonlinear Schrödinger equation (NLSE) is SPM

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + i\gamma|A|^2 A$$

- SPM is intensity-dependent phase shift
- Most important in intrapulse effects
- Also important in quantum description!

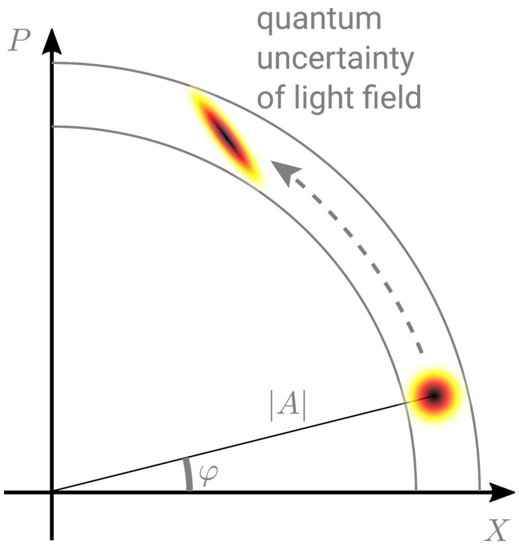


Kerr squeezing

- Kerr nonlinearity produces squeezed state in phase space with the Kerr Hamiltonian

$$\hat{H}_K = \hbar\chi\hat{a}^{\dagger 2}\hat{a}^2$$

- In fibers we can confine pulses in time and space to create high effective nonlinearity
- Challenges:
  - Raman and guided acoustic wave Brillouin scattering reduce
  - Uncertainty ellipse is tilted, no intensity noise reduction



Quadrature space

Solitons

- In fibers, dispersion tends to make pulse longer, while SPM compresses the pulse

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + i\gamma|A|^2 A$$

- When the pulse is of correct shape, duration and energy, these effect compensate each other
- Such pulses don't change the shape during propagation, and called solitary waves, or solitons

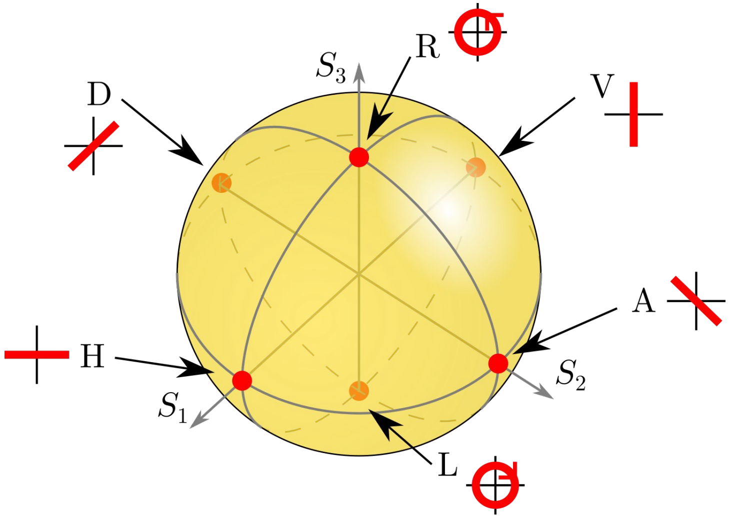
Stokes parameters and Poincaré sphere

- Stokes parameters

$$S_0 = |E_x|^2 + |E_y|^2 = I$$
$$S_1 = |E_x|^2 - |E_y|^2 = |E_x|^2 - |E_y|^2$$
$$S_2 = 2\mathrm{Re}(E_x E_y^*) \quad = |E_a|^2 - |E_d|^2$$
$$S_3 = -2\mathrm{Im}(E_x E_y^*) = |E_r|^2 - |E_l|^2$$

- Classical polarization state is a point on the sphere of radius ${S_0}^2 = {S_1}^2 + {S_2}^2 + {S_3}^2$ – the Poincaré sphere

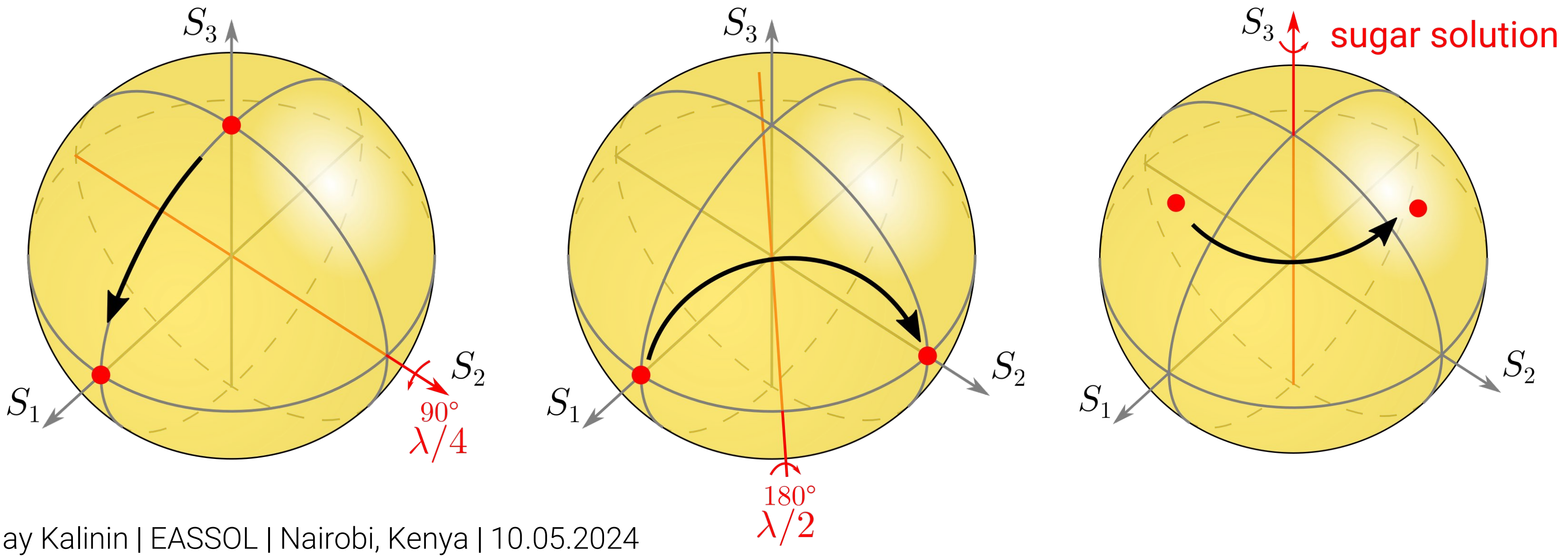
- Note: in other fields, the sphere is also known as Bloch and Riemann spheres



Polarization states on the Poincaré sphere

Poincaré sphere and waveplates

- Waveplates rotates the Poincaré sphere
- The rotation axis connects the fast and slow axes of the waveplate
- The rotation angle is the retardance of the waveplate



Polarization squeezing

- Quantum Stokes parameters are similar to classical

$$\hat{S}_0 = \hat{a}_x^\dagger \hat{a}_x + \hat{a}_y^\dagger \hat{a}_y, \quad \hat{S}_1 = \hat{a}_x^\dagger \hat{a}_x - \hat{a}_y^\dagger \hat{a}_y,$$

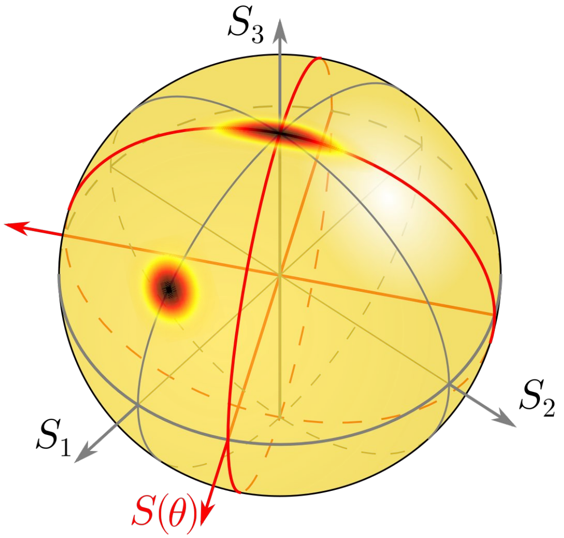
$$\hat{S}_2 = \hat{a}_x^\dagger \hat{a}_y + \hat{a}_y^\dagger \hat{a}_x, \quad \hat{S}_3 = i(\hat{a}_y^\dagger \hat{a}_x - \hat{a}_x^\dagger \hat{a}_y),$$

- Arbitrary linear combination of Stokes parameters can be easily measured in experiment

- Uncertainty relation has to be satisfied:

$$\mathrm{Var}\left(\hat{S}_i\right) \mathrm{Var}\left(\hat{S}_j\right) \geq \left|\epsilon_{ijk} \left\langle \hat{S}_k \right\rangle\right|^2$$

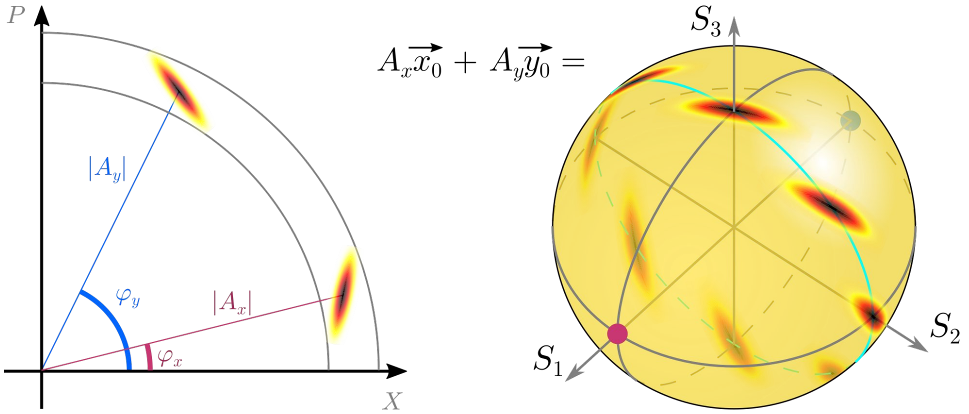
- A state is not a point, but an uncertainty area on the sphere



A coherent and a squeezed states on the Poincaré sphere

Polarization Kerr squeezing

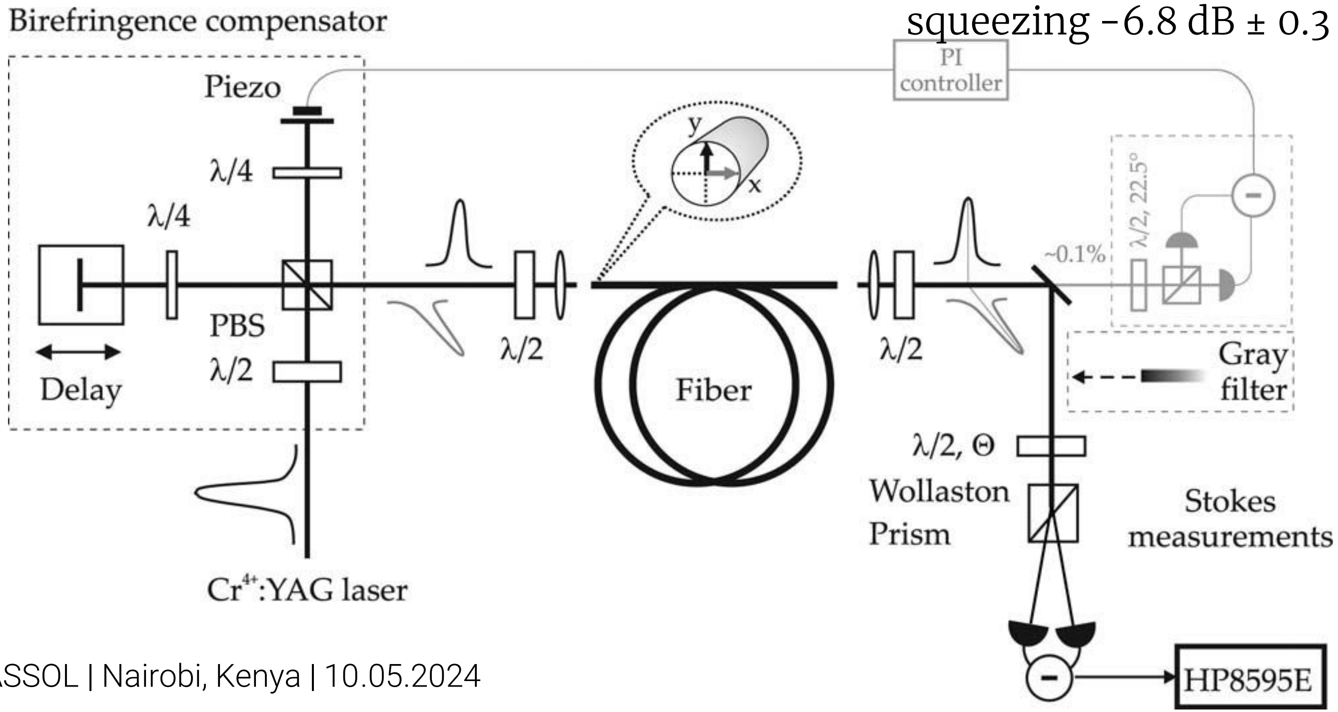
- Combining two equal Kerr squeezed states in two orthogonal polarization modes produces polarization-squeezed state
- The value of polarization squeezing is equal to the quadrature squeezing in each mode
- A convenient instrument to observe Kerr squeezing



Polarization Kerr squeezing in fibers

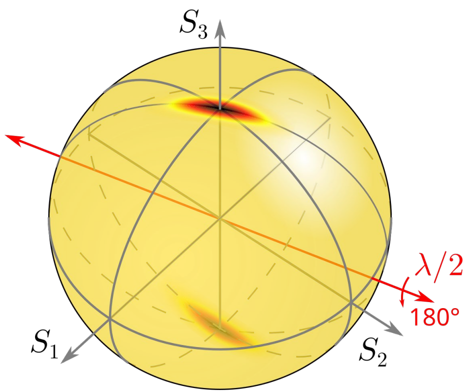
- Using two modes of a polarization-maintaining (PM) fiber we generate a squeezed state

R. Dong et al., Opt. Lett. 33, 116 (2008)
squeezing -6.8 dB ± 0.3 dB

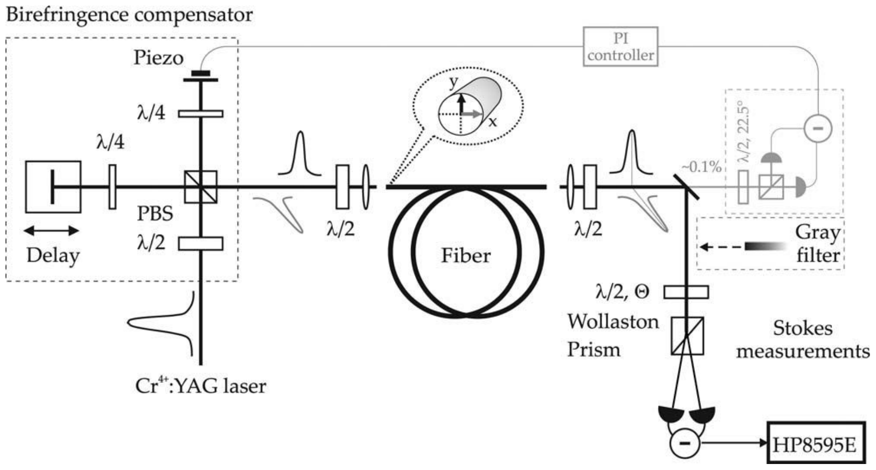


Polarization Kerr squeezing

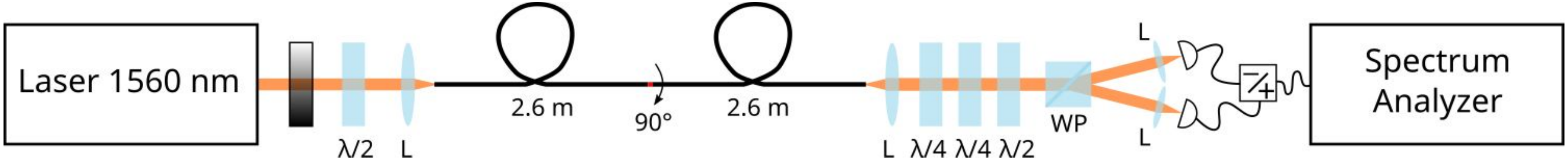
- Advantages:
  - PM fiber is useful
  - Almost insensitive to thermal noise
- Challenges:
  - Group delay should be matched
  - Phase should be controlled



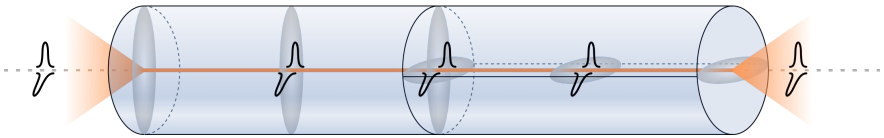
Action of a halfwave plate



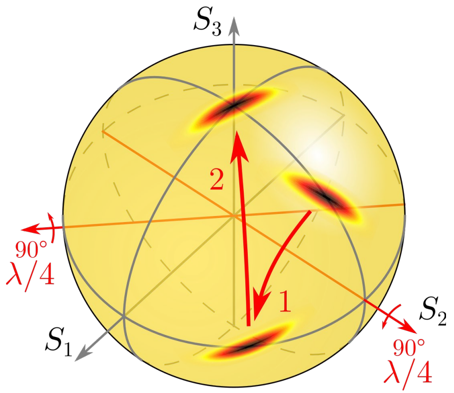
Polarization Kerr squeezing – new more robust setup



- Group delay can be compensated by a 90° splice at the middle of the fiber
- All-fiber setup, no need for active phase matching, phase fine-tuning is done with waveplates

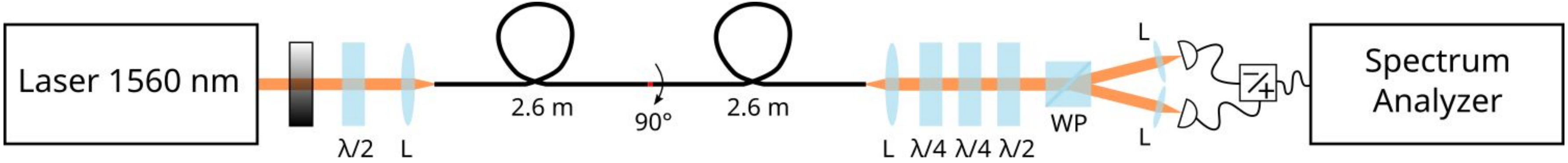


PM fiber with a 90° splice

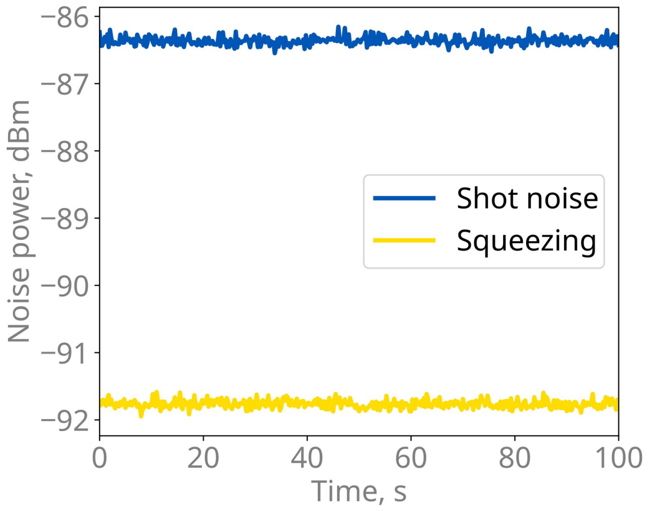


Waveplates are used to set circular polarization

Polarization Kerr squeezing – new more robust setup

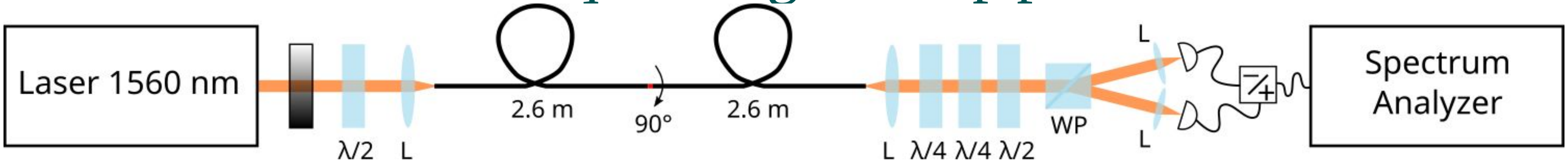


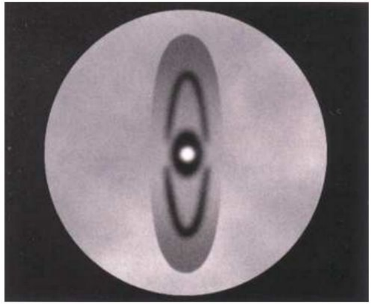
- Results:
  - >7 dB squeezing
  - Very stable (days without adjustment)
- Parameters:
  - 200 fs pulse duration
  - 175 pJ per pulse
  - 80 MHz repetition rate



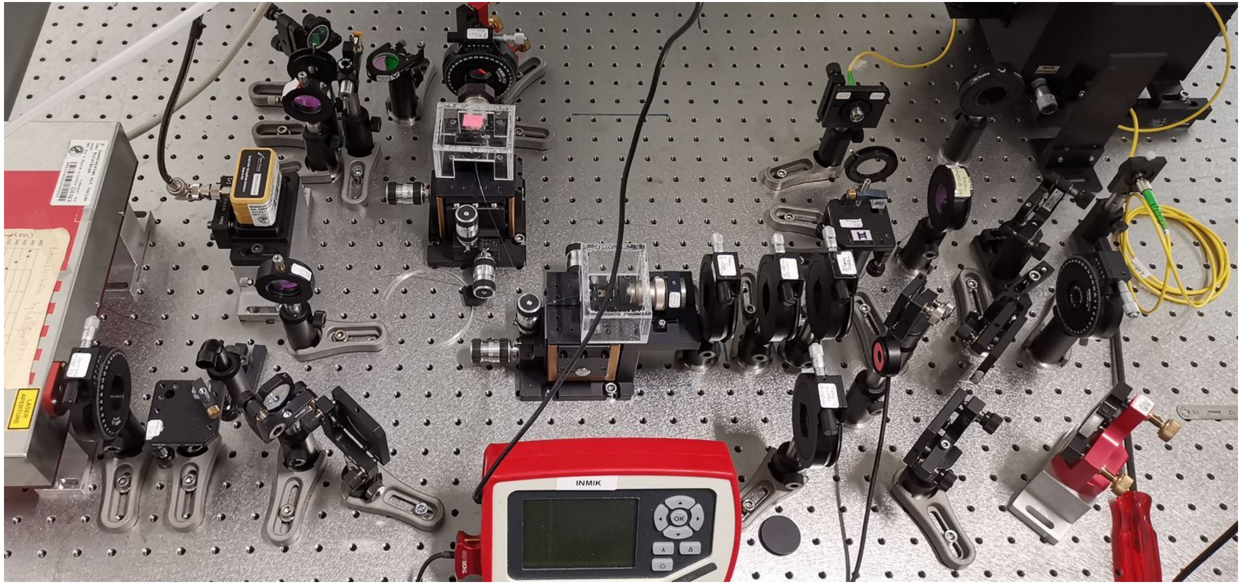
N. Kalinin et al., Adv. Quantum Technol. 6, 2200143 (2023)

Polarization Kerr squeezing – setup photo



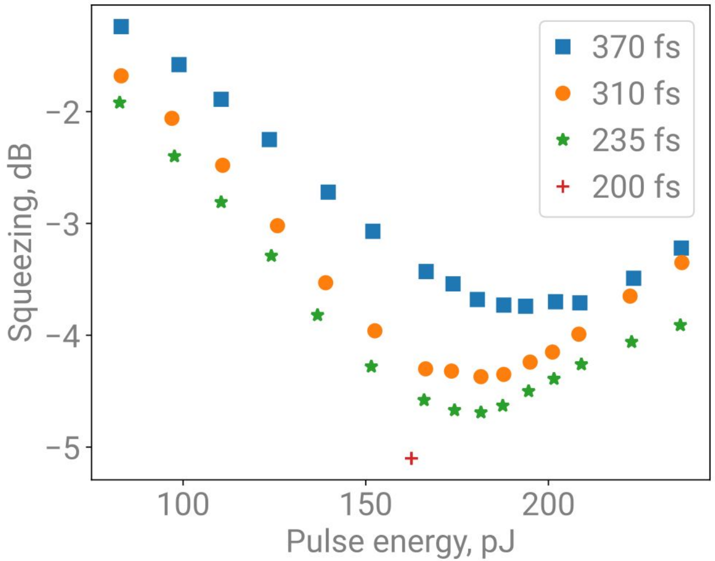


Used PM fiber
pic. from Sheu et al.,
Proc. SPIE 9665

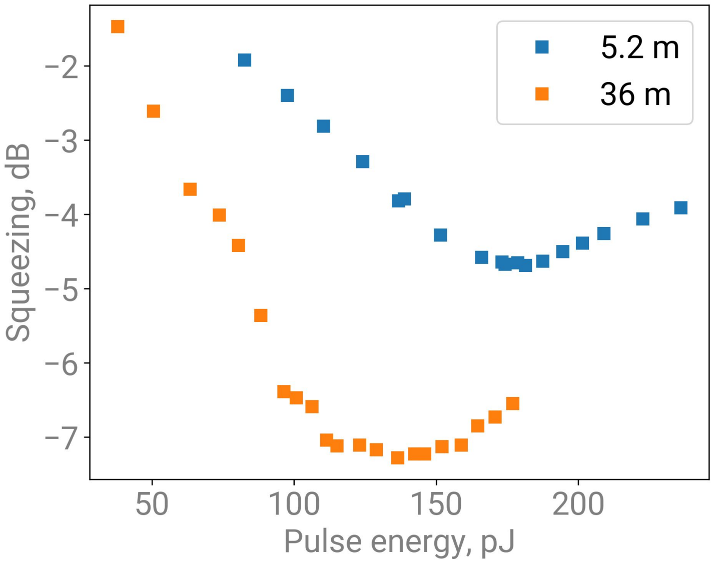


Parameters fine-tuning

- 3 variables: power, pulse duration, fiber length
- Best squeezing so far 7.3 dB



5.2 m fiber length



235 fs pulse duration

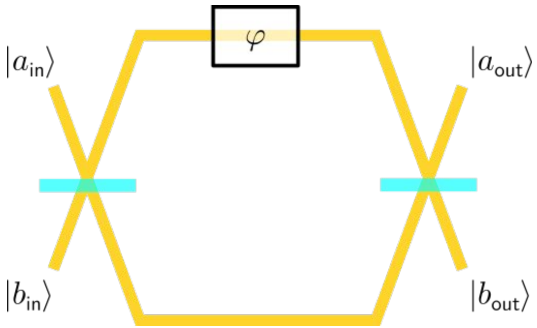
Interferometric sensitivity

- A general interferometer has two inputs and two outputs
- Measuring a function P of both output states, the phase shift φ is investigated
- The smallest observable shift $\Delta\varphi$ is defined by

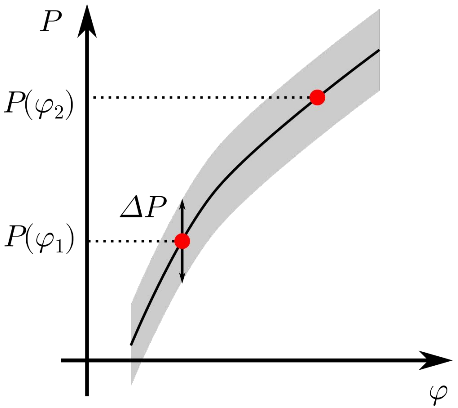
$$\Delta\varphi = \frac{\Delta P}{\left|\frac{\partial\langle\hat{P}\rangle}{\partial\varphi}\right|_{\varphi_0}} \qquad \Delta P = \left(\langle\hat{P}^2\rangle - \langle\hat{P}\rangle^2\right)^{1/2}$$

- Using only coherent inputs, the best achievable value is

$$\Delta\varphi_{\min}^{\mathrm{coh}} = \frac{1}{\sqrt{N}} \qquad N = \langle\hat{n}\rangle_a + \langle\hat{n}\rangle_b$$



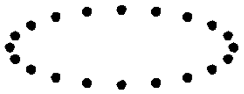
Mach–Zehnder (unfolded Michelson) interferometer



Gravitational waves detection

- Gravitational waves are generated by accelerating masses, they shrink and elongate orthogonal space in orthogonal directions

- Hypothetical GW of strain $h{=}0.5$:

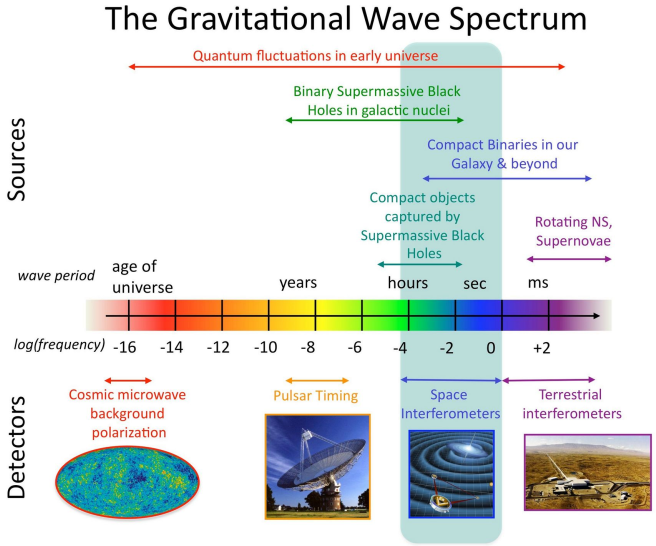


- First detected GW had $h = 0.9 \times 10^{-21}$

- On the scale of 4 kilometers, the length change

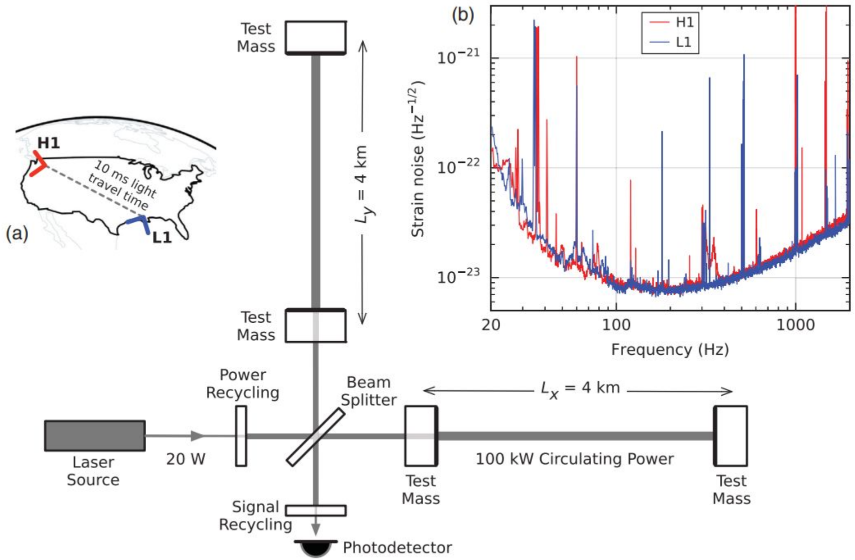
$$\Delta L = hL = 4 \times 10^{-18}\text{m}$$

- How to detect such small signal?

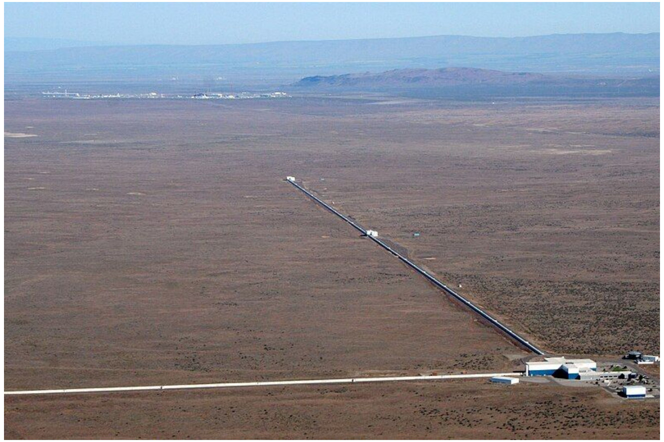


Laser Interferometer Gravitational-Wave Observatory

- Huge Michelson interferometers to observe the changes



Simplified diagram of an Advanced LIGO detector

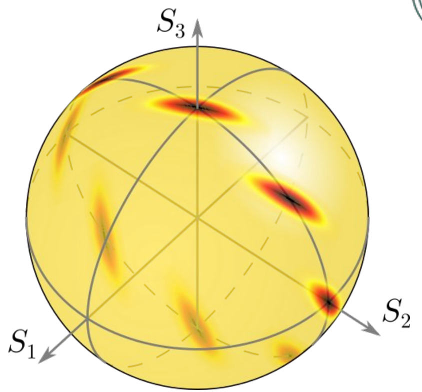


LIGO Hanford Observatory

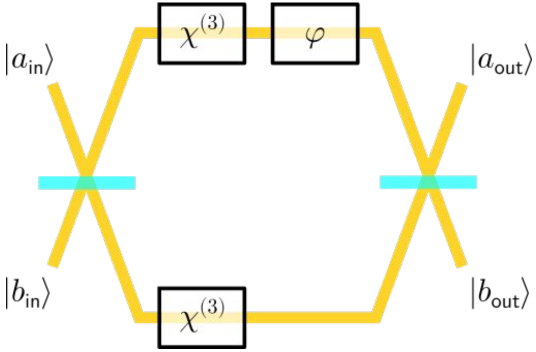
B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration)
Phys. Rev. Lett. 116, 061102, 2016

Interferometric sensitivity and Kerr tilting

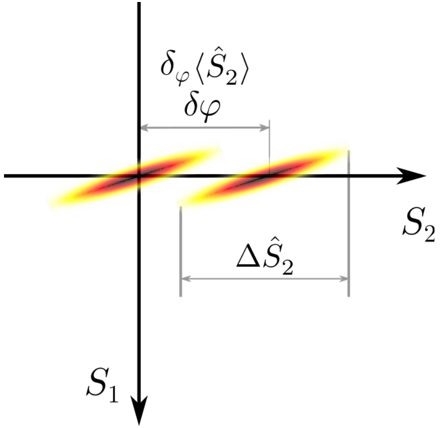
- Two fiber modes as interferometer arms?
  - No sensitivity enhancement
  - Depending on the measurement P, either the measured Stokes parameter has little dependency on the phase shift, or has a large uncertainty



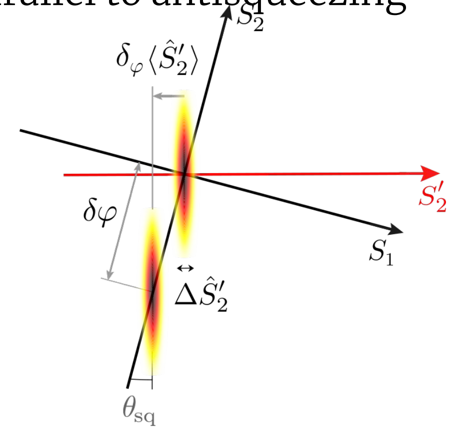
Changes in φ are almost parallel to antisqueezing



Kerr interferometer
no sensitivity improvement



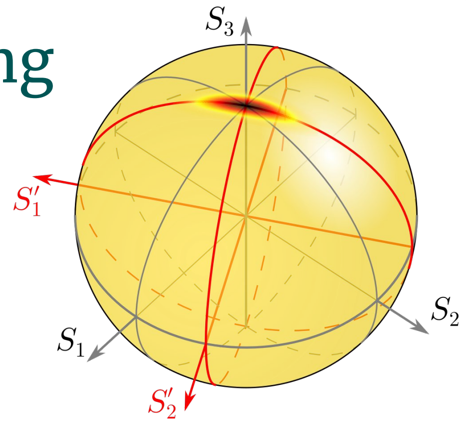
$P{=}S_2$: large noise



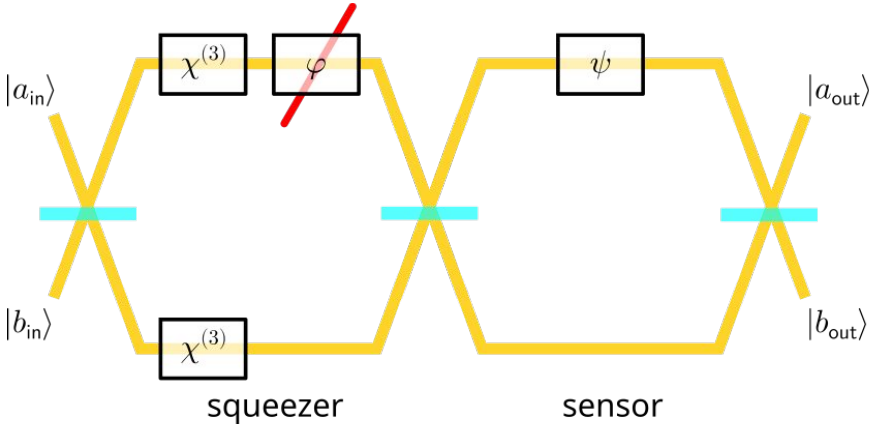
$P{=}S(\theta)$: small signal

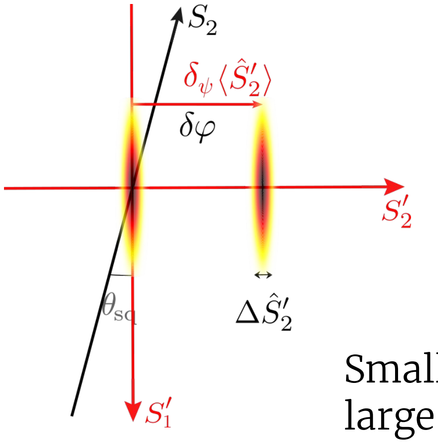
Interferometric sensitivity and Kerr tilting

- The same variable must be squeezed, measured, and depend on the changing phase
- Need to realign the uncertainty ellipse before the phase difference is introduced
- Two interferometers: squeezer and sensor



Basis change on the Poincaré sphere

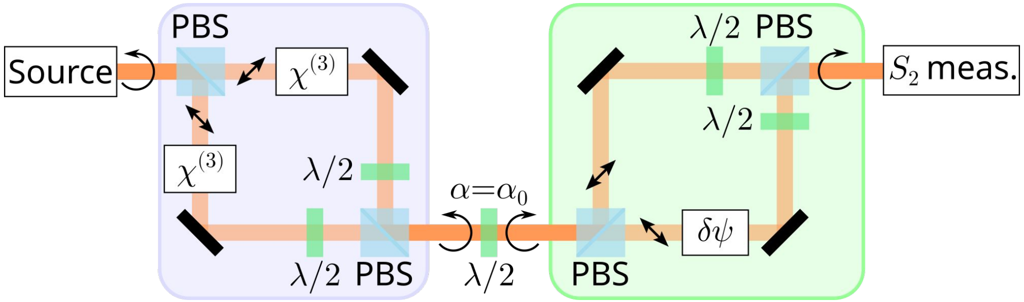




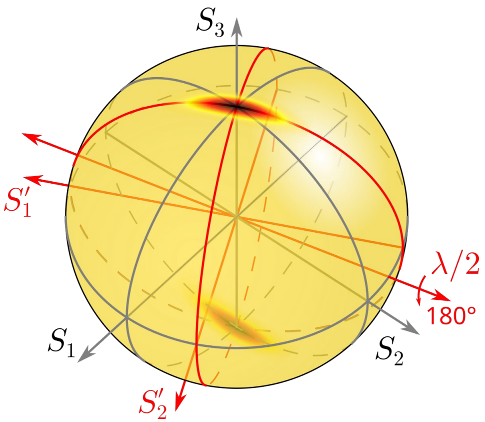
Small uncertainty, large signal

Interferometric sensitivity enhancement

- With polarization modes, the correct basis change can be implemented with a single halfwave plate.



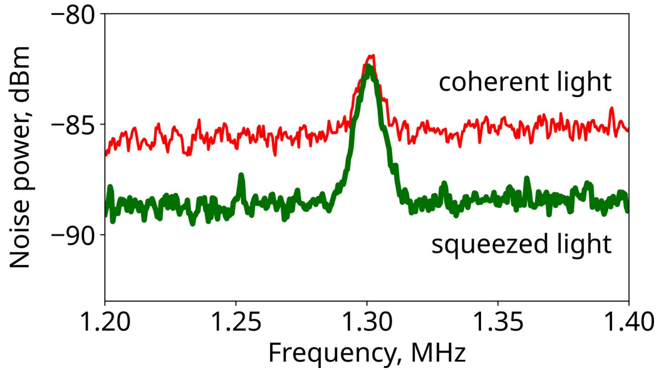
General experimental setup



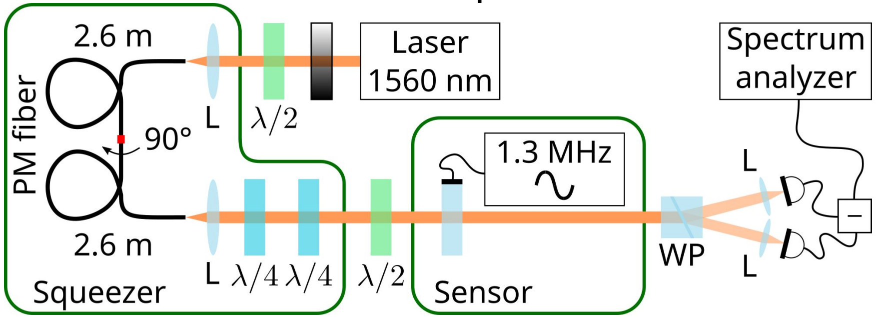
A halfwave plate rotates the basis around S_3

Interferometric sensitivity enhancement

- Implementation:
  - Halfwave plate aligns the uncertainty ellipse
  - Glass plate with small birefringence serves as a sensor polarization interferometer
- Results:
  - First Kerr sub-SNL phase interferometer
  - 4 dB SNR improvement at 1.3 MHz
  - Method suitable for free-space interferometers



SNR improvement



N. Kalinin et al., Nanophotonics 12 (14), 2945 (2023)

Thank you!

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