Joint ICTP-IAEA Fusion Energy School, 2024

International Centre for Theoretical Physics, Trieste, Italy

INTERACTING NONLINEAR WAVES

MAY 7TH, 2024

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SUMMARY

Monday 6th

- New nonlinear solutions of the old wave equation
- Accelerating self-modulated nonlinear waves in magnetized plasmas

Tuesday 7th

- Can we surf a gravitational wave?
- Amplification of electromagnetic plasma waves due to gravitational wave
- Amplification of electromagnetic plasma waves due to cosmological gravitational waves

Wednesday 8th

- Interacting quantum and classical waves: Resonant and non-resonant energy transfer to electrons immersed in an intense electromagnetic wave.
- Statistical model for relativistic quantum fluids interacting with an intense electromagnetic wave.

CAN WE SURF A GRAVITATIONAL WAVE?

YES, WE CAN. BUT ONLY A SPECIAL KIND OF GRAVITATIONAL WAVE

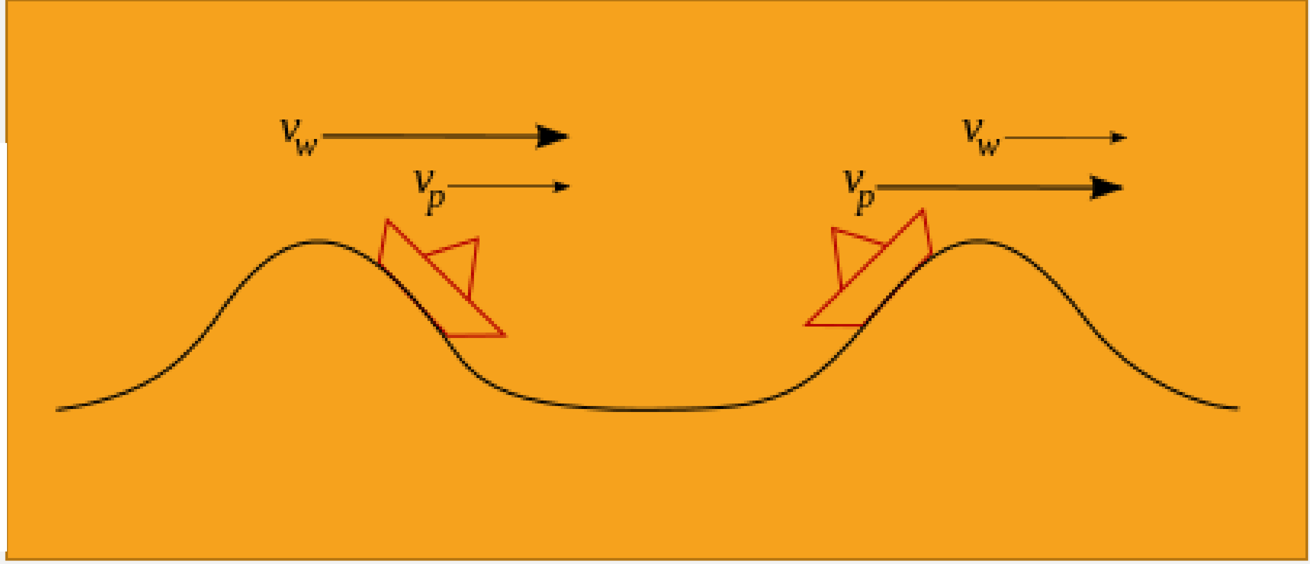
Asenjo & Mahajan, Phys. Rev. D 101, 063010 (2020)

PRELIMINAR: LANDAU DAMPING

$$\mathbf{E} = -\nabla\phi.$$

$$\frac{\partial f_e}{\partial t} + \mathbf{v}\cdot\nabla f_e - \frac{e}{m_e}\mathbf{E}\cdot\nabla_v f_e = 0,$$

$$\nabla^2\phi = -\frac{e}{\epsilon_0}\left(n - \int f_e\, d^3\mathbf{v}\right).$$



$$1 + \frac{e^2}{\epsilon_0\, m_e\, k^2}\int \frac{\mathbf{k}\cdot\nabla_v f_0}{\omega - \mathbf{k}\cdot\mathbf{v}}\, d^3\mathbf{v} = 0.$$

$$\omega = \omega_0 + \delta\omega$$

$$\delta\omega \simeq -\mathrm{i}\sqrt{\frac{\pi}{8}}\frac{\Pi_e}{(k\,\lambda_D)^3}\exp\left[-\frac{1}{2\,(k\,\lambda_D)^2}\right].$$

Resonance mechanism between longitudinal EM waves and particles, it prevents instabilities.

Only it occurs if the wave and the particles move with similar velocities.

The EM wave can transfer energy to the slower particles, accelerating them.The wave damps.

¿Can exist a similar effect for gravitational waves?

We need:

- Massive particles must interact with gravitational waves moving slower than the speed of light.
- Phase velocity of the wave must be similar to velocity of the massive particles.
- Thus, gravitational waves must be dispersive
- If this occurs, we get resonance effects!

PHYSICAL REVIEW D **101**, 063010 (2020)

Resonant interaction between dispersive gravitational waves and scalar massive particles

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- We model massive particles with the Klein-Gordon equation

- The massive particle is in a dispersive gravitational wave background

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\Phi\right)=m^2\Phi$$

Dispersive GW

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu};$$

$$\eta_{\mu\nu}=(-1,1,1,1)$$

$$h_{\mu\nu}<<\eta_{\mu\nu}$$

$$h_{22}=-h_{33}=h_+(\chi)$$

$$h_{23}=h_{32}=h_\times(\chi)$$

$$\chi=\omega t-kz$$

$$\omega^2-k^2=\omega_G{}^2\neq 0$$

$$0=-\frac{\partial^2\Phi}{\partial t^2}+\frac{\partial^2\Phi}{\partial z^2}-\omega f(\chi)\frac{\partial\Phi}{\partial t}-kf(\chi)\frac{\partial\Phi}{\partial z}-m^2\Phi,$$

$$f(\chi)=-h_+h'_+-h_\times h'_\times$$

$$\Phi(t,z)=\Phi(\chi)$$

$$0=\Phi''+f\Phi'+\frac{m^2}{\omega_G^2}\Phi.$$

By defining $\Phi(\chi)=(-g)^{-1/4}\varphi(\chi)$, Eq. (3) transforms to

$$0=\varphi''+\zeta^2\varphi,$$

where

$$\zeta(\chi)=\sqrt{\frac{m^2}{\omega_G^2}-\frac{f'}{2}-\frac{f^2}{4}}.$$

resonance with the gravitational wave

WKB SOLUTION

$$\varphi(\chi) = \frac{1}{\sqrt{2W(\chi)}} \exp\left(-i \int W(\chi) d\chi\right),$$

$$W^2 = \zeta^2 - \frac{W''}{2W} + \frac{3(W')^2}{4W^2}.$$

Let us now assume a slowly varying spacetime, such that:

- the derivatives of W are small compared to m/ω_G.
- consider a regime in which ω_G remains essentially constant such that $m \gg \omega_G$
- and m/ω_G is larger than any possible spacetime variation of the gravitational wave

In this case $W \approx m/\omega_G$

$$E \approx \frac{m\omega}{\omega_G} \gg m, \qquad P \approx \frac{mk}{\omega_G}.$$

$$\Phi(\chi) \approx \sqrt{\frac{\omega_G}{2m}} \exp\left(-i \frac{m}{\omega_G} \chi\right)$$

Energization!

resonance

$$v \sim \frac{E}{P} \sim \frac{\omega}{k}.$$

CAN OCCUR SOMETHING SIMILAR TO EM WAVES?

AMPLIFICATION OF ELECTROMAGNETIC PLASMA WAVES DUE TO GRAVITATIONAL WAVE

Mahajan & Asenjo, Phys. Rev. E 107, 035205 (2023)

¿Can exist a similar effect for EM waves induced by gravitational waves?

We need:

- Plasma interacting with gravitational waves moving slower than the speed of light.
- Gravitational waves must be dispersive
- Phase velocity of the waves must be similar
- Again, we get resonance effects!

PHYSICAL REVIEW E **107**, 035205 (2023)

Parametric amplification of electromagnetic plasma waves in resonance with a dispersive background gravitational wave

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EM PLASMA WAVE EQUATION IN CURVED SPACETIME

$$\nabla_\nu F^{\mu\nu} + \Omega_p{}^2 A^\mu = 0$$

$$\frac{1}{\sqrt{-g}}\partial_\nu[\sqrt{-g}g^{\mu\alpha}g^{\nu\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)] + \Omega_p^2 g^{\mu\alpha}A_\alpha = 0.$$

Dispersive GW

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu};$$
$$\eta_{\mu\nu} = (-1,1,1,1)$$
$$h_{\mu\nu} << \eta_{\mu\nu}$$
$$h_{11} = -h_{22} = h(\chi)$$
$$\chi = \omega t - kz$$
$$\omega^2 - k^2 = \omega_G{}^2 \neq 0$$

$$\frac{\partial}{\partial t}\left(f_\pm \frac{\partial A_\pm}{\partial t}\right) - \frac{\partial}{\partial z}\left(f_\pm \frac{\partial A_\pm}{\partial z}\right) + \Omega_p^2 f_\pm A_\pm = 0,$$

where $f_+ = \sqrt{-g}g^{11} \approx 1 - h$, and $f_- = \sqrt{-g}g^{22} \approx 1 + h$.

Polarizations + and -

$$A_\pm(t,z) = A_\pm(\chi)$$

Resonance

$$\frac{d^2 A_\pm}{d\chi^2} \mp \frac{dh}{d\chi}\frac{dA_\pm}{d\chi} + \left(\frac{\Omega_p}{\omega_G}\right)^2 A_\pm = 0,$$

CHOOSING A PROFILE $h(\chi) = h_0 \cos\chi$

$$\frac{d^2 A_\pm}{d\chi^2} \pm h_0 \sin\chi \frac{dA_\pm}{d\chi} + \left(\frac{\Omega_p}{\omega_G}\right)^2 A_\pm = 0. \quad (9)$$

The oscillator is clearly subject to a periodic potential; this has immensely interesting consequences. Noting that $A_-(\chi) = A_+(\chi - \pi)$, it is enough to solve for only one polarization. For the A_- polarization, a change of variable

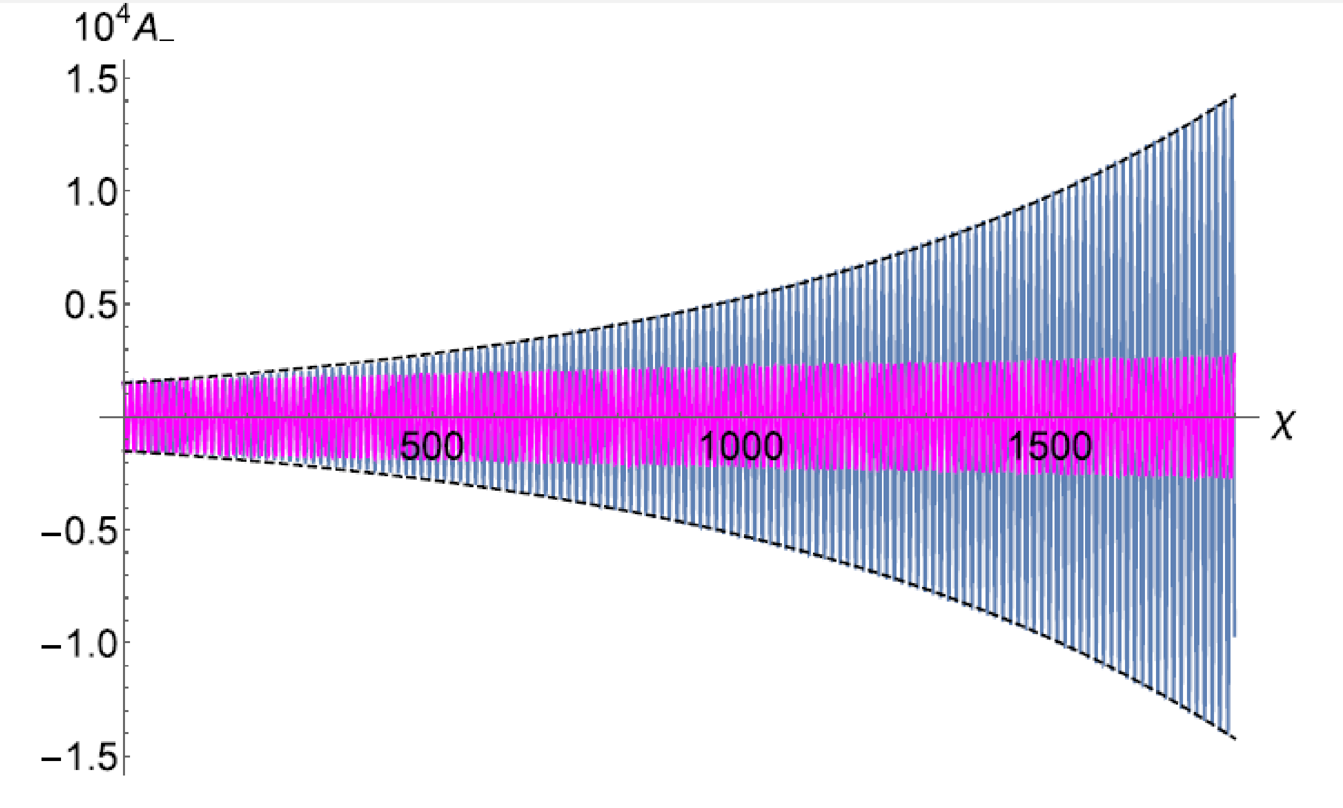
$$\mathcal{A}_- = \exp\left(\frac{h_0}{2}\cos\chi\right) A_- \quad (10)$$

converts Eq. (9) into the more standard form of a Whittaker-Hill equation,

$$\frac{d^2 \mathcal{A}_-}{d\chi^2} + \left[\left(\frac{\Omega_p}{\omega_G}\right)^2 + \frac{h_0}{2}\cos\chi - \frac{h_0^2}{4}\sin^2\chi\right]\mathcal{A}_- = 0. \quad (11)$$

Neglecting higher orders of h_0^2 the equation approximates to the Mathieu equation, which has an unstable solution for

$$\left(\frac{\Omega_p}{\omega_G}\right)^2 = \frac{1 \pm h_0}{4} \Rightarrow |A_-| \propto \exp\left(\frac{h_0}{4}\chi\right).$$



Amplification EM plasma wave

Numerical solution
Blue line: above unstable solution
Magenta line: other arbitrary stable solution

CAN OCCUR SOMETHING SIMILAR TO EM WAVES IN COSMOLOGY?

AMPLIFICATION OF ELECTROMAGNETIC PLASMA WAVES DUE TO COSMOLOGICAL GRAVITATIONAL WAVES

Asenjo & Mahajan, in preparation (2024)

EM PLASMA WAVE EQUATION IN COSMOLOGY

$$\nabla_\nu F^{\mu\nu} + \Omega_p{}^2 A^\mu = 0 \quad \longrightarrow \quad \frac{1}{\sqrt{-g}}\partial_\nu[\sqrt{-g} g^{\mu\alpha} g^{\nu\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)] + \Omega_p^2 g^{\mu\alpha} A_\alpha = 0.$$

No need for dispersive GW

$$g_{00} = -1; \quad g_{11} = g_{33} = a^2$$

$$g_{22} = a^2 + h_{22}$$

$$\frac{\partial^2 A_2}{\partial \tau^2} - \frac{\partial \xi}{\partial \tau}\frac{\partial A_2}{\partial \tau} + \left(k^2 + \omega_{p0}^2 V\right) A_2 = 0$$

$$\frac{\partial^2 h_{22}}{\partial \tau^2} + \frac{2}{a}\frac{\partial a}{\partial \tau}\frac{\partial h_{22}}{\partial \tau} + K^2 h_{22} = 0$$

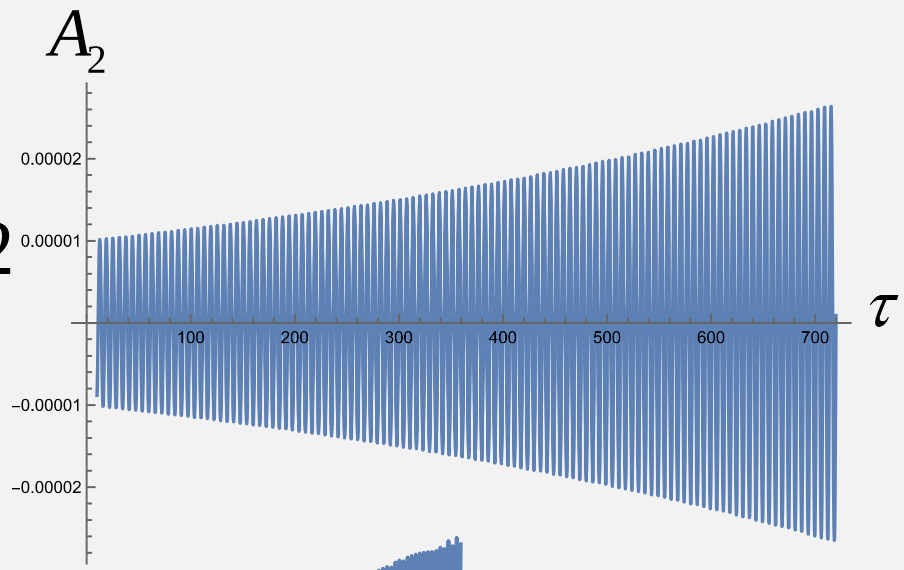
$$\tau = \int dt / a; \quad \xi = h_{22} / \left(2a^2\right)$$

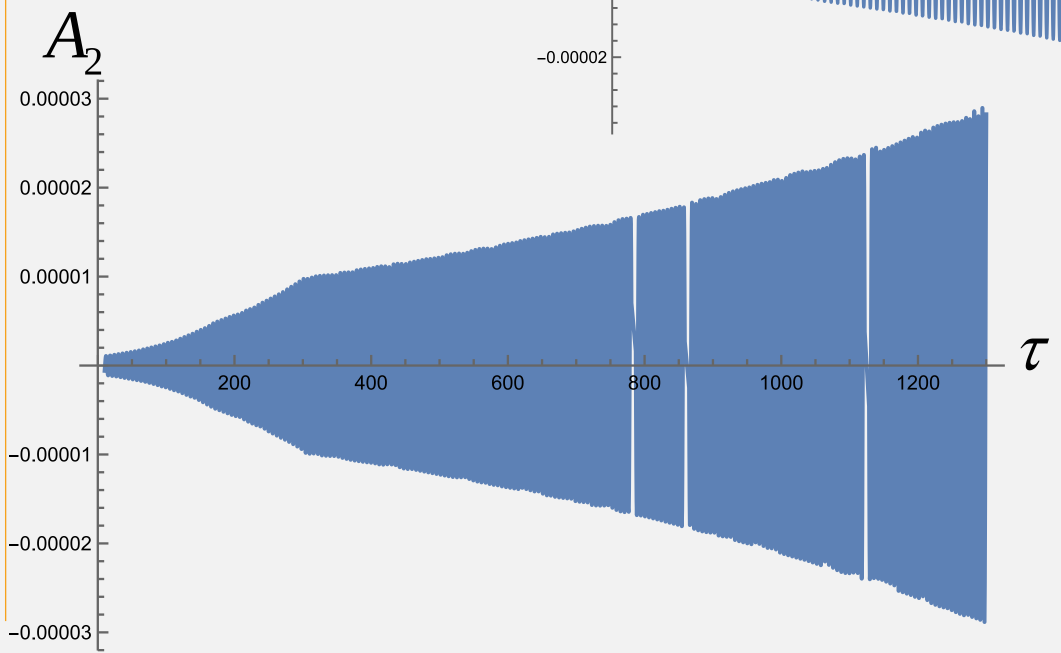
$$V(a) = f_0 / (a f(a))$$

For a perfect fluid

$$p = w\varepsilon$$

$$w = -1/2$$





$$w = 0.7$$

CONCLUSIONS

- Background "waves" can interact in a nonlinear fashion with particles and waves.
- They can be in resonance when the pase velocities are similar.
- In these cases, the background gives energy to EM waves, it occurs amplification.