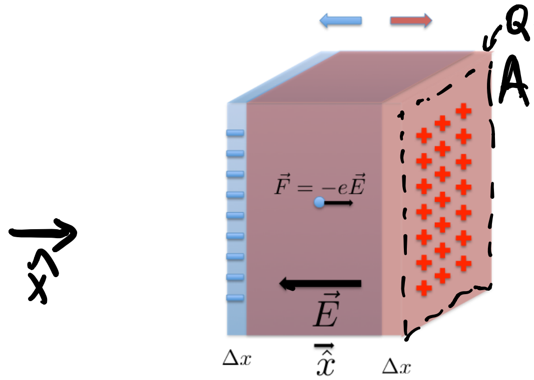
Plasma Frequency



$$Q = \rho_e V = e n_e V = e n_e \Delta x A$$

Figure 1: Moving the center of mass of the electrons with respect to the ions creates a restoring force

$$\vec{E} = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0} = \frac{(e n_e \Delta x A)/A}{\varepsilon_0} = \frac{e n_e \Delta x}{\varepsilon_0}(-\hat{x})$$

The force on one of the electrons displaced a distance Δx.

$$\vec{F} = -e\vec{E} = -\frac{e^2 n_e \Delta x}{\varepsilon_0} = m_e \vec{a}$$

$$\vec{a} = \boxed{\ddot{\Delta x} = -\frac{e^2 n_e}{\varepsilon_0 m_e}\Delta x} \quad \text{S.H.O.}$$

$$\omega^2 = \frac{e^2 n_e}{\varepsilon_0 m_e} \Rightarrow \boxed{\omega_{pe} = \left(\frac{e^2 n_e}{\varepsilon_0 m_e}\right)^{1/2}}$$

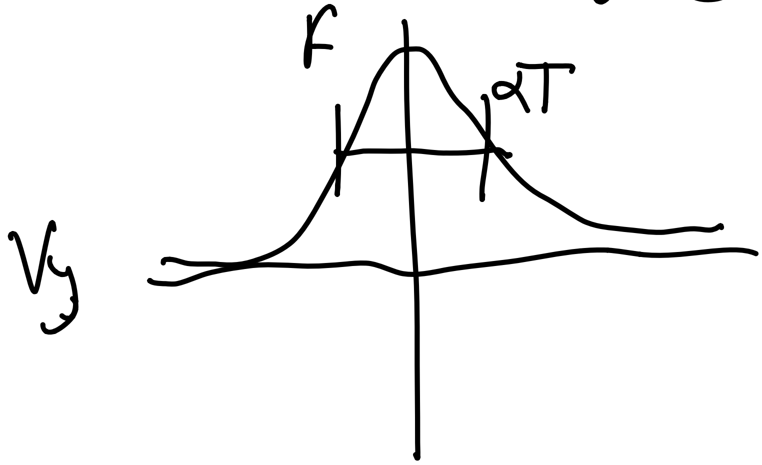
$$m_i \gg m_e$$

Temperature

As particles heat up, they collide and thermalize. They develop a Maxwell Boltzmann distribution:

$$f(x,y,z,V_x,V_y,V_z) = \underbrace{n(x,y,z)}_{\text{Position Dependent}} \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\left[\frac{m(V_x^2+V_y^2+V_z^2)}{2KT}\right]}$$

$$f(V_x,V_y,V_z) = \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\left[\frac{m(V_x^2+V_y^2+V_z^2)}{2KT}\right]}$$



$f(V)$? $V = (V_x^2 + V_y^2 + V_z^2)^{1/2}$

treat it like Spherical Coordinates

$$dv_x\,dv_y\,dv_z \rightarrow \boxed{}\, dv\, d\phi\, d\theta$$

$$v^2 \sin\phi\, dv\, d\phi\, d\theta$$

Isotropic $\rightarrow 4\pi v^2 dv$

$$\boxed{f(v) = 4\pi n_0 \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{\frac{-mv^2}{2kT}}}$$

Assuming Spatial Isotropy

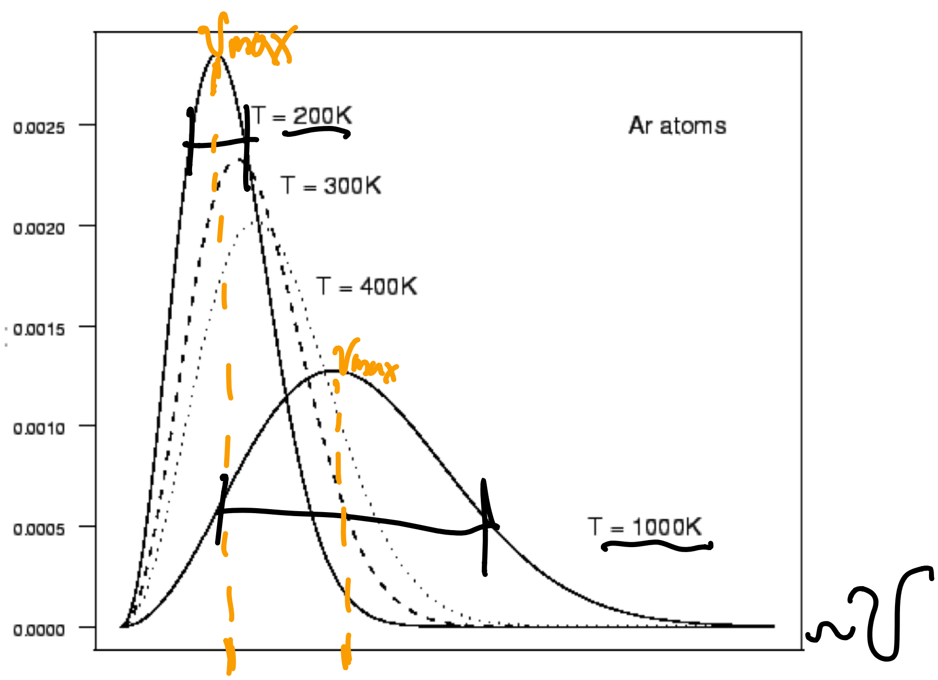


Figure 4: Maxwell-Boltzmann distribution function of Argon gas at different temperatures. The $\hat{x}$-axis is proportional to the *speed* of the atoms. Note that the area under the curve must be 1.

Temperature defines a <u>characteristic</u> <u>speed</u> $v_{max} = \sqrt{\frac{2kT}{m}}$, $v_{mean} = \sqrt{\frac{3kT}{m}}$

$$v_{th} \equiv \sqrt{\frac{kT}{m}}$$

$$\boxed{v_{the} = \sqrt{\frac{kT_e}{m_e}}} \qquad v_{thi} = \sqrt{\frac{kT_i}{m_i}}$$

If $T_e = T_i \Rightarrow \frac{v_{the}}{v_{thi}} = \sqrt{\frac{m_i}{m_e}} \gg 1$

$kT \sim mv^2 \sim$ Energy $\Rightarrow$ Plasma Physicists talk about Temperature in eV

$$\boxed{\omega_{pe} \equiv \left(\frac{e^2 n_e}{\varepsilon_0 m_e}\right)^{1/2}}$$

Frequency

$$\boxed{v_{the} = \sqrt{\frac{KT_e}{m_e}}}$$

Velocity

What Unit can we derive?

LENGTH!

$$v \sim \frac{L}{T} \rightarrow L \sim v \cdot T \sim \frac{v}{\omega} \rightarrow \lambda_{De} \equiv \frac{v_{the}}{\omega_{pe}}$$

$$\lambda_{De} = \sqrt{\frac{KT_e}{m_e}} \bigg/ \sqrt{\frac{e^2 n_e}{\varepsilon_0 m_e}} \rightarrow \boxed{\lambda_{De} = \sqrt{\frac{KT_e \varepsilon_0}{e^2 n_e}}}$$

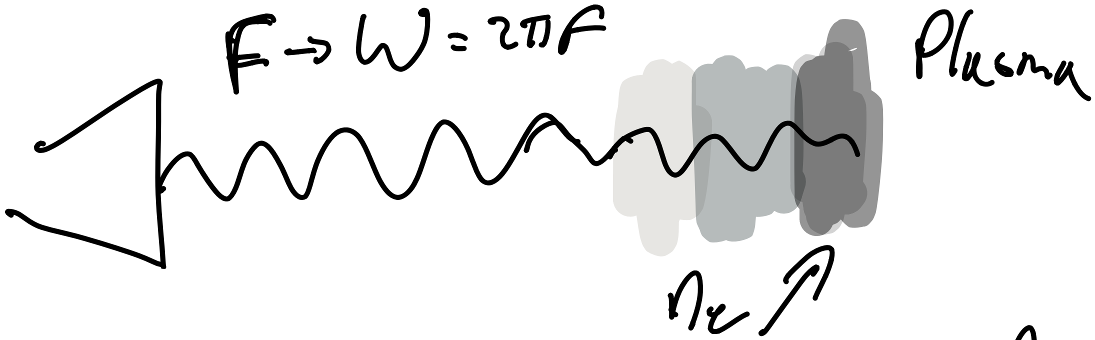
Electron Oscillates as:

$$\Delta \ddot{x} = -\omega_{pe}^2 \Delta x$$

$$\Delta x = A \sin(\omega_{pe} t + \cancel{\phi})$$

$$\Delta \dot{x} = \underbrace{\omega_{pe} A}_{v} \cos(\omega_{pe} t) \rightarrow \lambda_{De} = v_{the}/\omega$$

Quick diagnostic detour



$\omega > \omega_{pe} \to$ travel

$\omega < \omega_{pe} \to$ evanescent

So waves will travel into the plasma until they reach $\omega = \omega_{pe}$ then they will bounce back.

Basis of reflectometry!

System	$n_e[m^{-3}]$	$T_e[eV]$	$\omega_{pe}[s^{-1}]$	$\lambda_D[m]$
Interstellar gas	10^6	1	10^5	10
Solar Wind	10^7	10	10^5	10
Van Allen belts	10^9	10^2	10^6	1
Ionosphere	10^{11}	10^{-1}	10^7	10^{-2}
Solar Corona	10^{13}	10^2	10^8	10^{-3}
Candle flame	10^{14}	10^{-1}	10^9	10^{-4}
Neon lights	10^{15}	1	10^9	10^{-4}
Gas Discharge	10^{18}	2	10^{11}	10^{-5}
Process Plasma	10^{18}	10^2	10^{11}	10^{-4}
Fusion Experiment	10^{19}	10^3	10^{11}	10^{-4}
Fusion Reactor	10^{20}	10^4	10^{12}	10^{-4}
Lightning	10^{24}	3	10^{14}	10^{-8}
Electrons in metal	10^{29}	10^{-2}	10^{16}	10^{-12}

Table 1: Plasma Frequency and Debye length for various systems

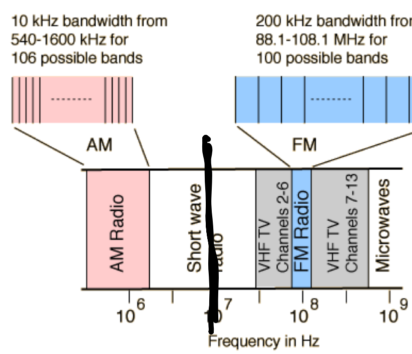
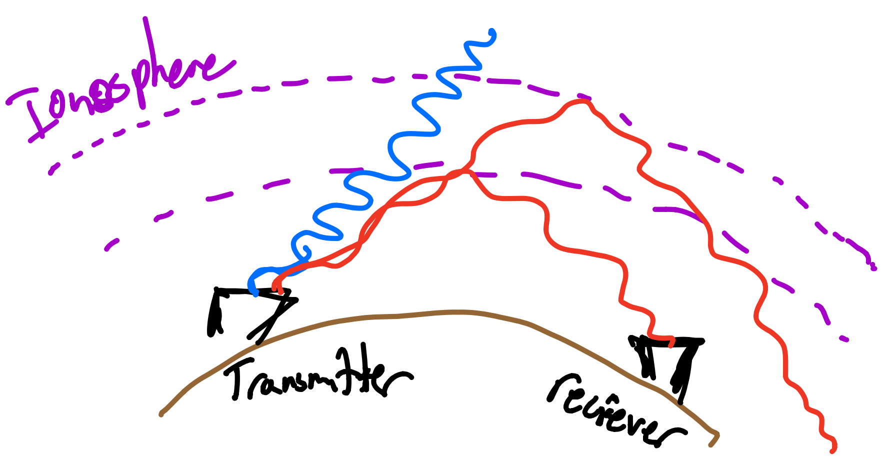
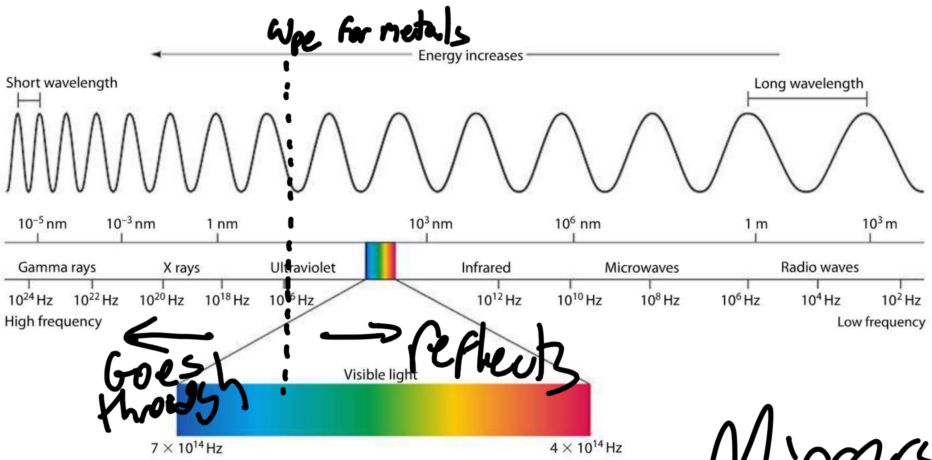


Figure 7: The AM spectrum is well below the $\approx 10MHz$ ω_{pe} of the ionosphere, leading to their reflection. FM waves, at higher frequency, penetrate it.



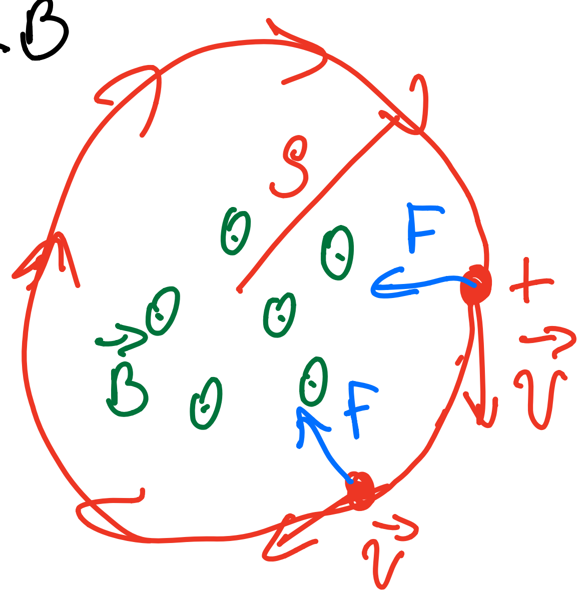


Mirrors!

Final topic: motion in a magnetic field

$$\vec{F} = q(\cancel{\vec{E}} + \vec{v} \times \vec{B}) \Rightarrow \vec{F} = q\,\vec{v} \times \vec{B}$$

when $\vec{v} \perp \vec{B}$



$$F = q\vec{V} \times B = qVB = ma$$

Centripetal acceleration!

$$a_c = \frac{V^2}{\rho} \Rightarrow \frac{mV^{\cancel{2}}}{\rho} = q\cancel{V}B$$

$$\rho = \frac{mV}{qB}$$

$$\boxed{\rho_e = \frac{m_e V_{the}}{eB}}$$

for circular motion, $V = \omega \rho$

in this case $V_{the} = \omega\, \rho$

$$\omega_{ce} \equiv \frac{V_{the}}{\rho} = \frac{V_{the}}{\left(\frac{m_e V_{the}}{eB}\right)}$$

$$\boxed{\omega_{ce} \equiv \frac{eB}{m_e}}$$

$$\omega_{ci} = \frac{ZeB}{m_i}$$

Final Point: direction of rotation creates a $\vec{B}$ counter the original $\vec{B}$. Makes sense or else we'd have a positive feedback!

RGDX