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May/13/2024

Magnetic Confinement

Force on a Charged Particle:

$$\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$$

Simplest case:

$$\vec{B} = B\hat{z} \quad \text{constant} \qquad \vec{E} = \emptyset$$

$$m\dot{\vec{v}} = q(\vec{v} \times B\hat{z}) \qquad \vec{v} \equiv v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$$

$$\dot{v}_z\hat{z} = \frac{q}{m}(\vec{v} \times B\hat{z}) \cdot \hat{z}\,\hat{z} = \emptyset$$

$$\dot{v}_z = \emptyset \rightarrow \boxed{v_z = ct}$$

$$\dot{v}_x = \frac{q}{m}(\vec{v} \times B\hat{z})_{in\ x} = \frac{qB}{m} v_y$$

coupled

$$\dot{v}_y = q/m\,(\vec{v} \times B\hat{z})_{in\ y} = -qB/m\, v_x$$

$$\ddot{v}_x = \omega_c \dot{v}_y = -\omega_c^2 v_x$$

$$\ddot{v}_y = -\omega_c \dot{v}_x = -\omega_c^2 v_y$$

Where

$$\boxed{\omega_c \equiv \frac{qB}{m}}$$

The general solution is

$$v_x = v_\perp \cos(\omega_c t + \phi)$$
$$\dot{v}_x = -\omega_c v_\perp \sin(\omega_c t + \phi) = \omega_c v_y$$

$$v_y = -\, {}^{q}/_{|q|}\, v_\perp \sin(\omega_c t + \phi)$$

${}^{q}/_{|q|}$ is just the sign of the charge, so:

$$\left.\begin{aligned} v_y &= \mp v_\perp \sin(\omega_c t + \phi) \\ \text{for } v_x &= v_\perp \cos(\omega_c t + \phi) \end{aligned}\right\}$$

integrating:

$$\left.\begin{aligned} x &= x_0 + {}^{v_\perp}/_{\omega_c} \sin(\omega_c t + \phi) \\ y &= y_0 \pm {}^{v_\perp}/_{\omega_c} \cos(\omega_c t + \phi) \end{aligned}\right\}$$

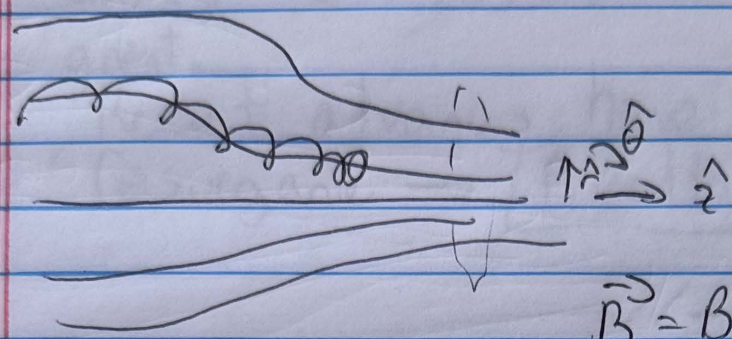
So particles gyrate around a point (x_0, y_0) with a frequency $\boxed{\omega_c = \frac{|q| B}{m}}$

and a radius: $\boxed{\rho \equiv \frac{v_\perp}{\omega_c} = \frac{m v_\perp}{|q| B}}$

For ITER, i = Deuterium, $T_i \approx 10$ KeV, $B \simeq 5T$

$\boxed{v_{T_i} = \sqrt{\frac{kT_i}{m_i}} \simeq 700 \text{ km/s}}$; $\boxed{\rho_i \simeq 3 \text{ mm} \quad \text{for } T_e = T_i \rightarrow \rho_e = 0.05 \text{ mm}}$

First Magnetic Configuration: Mirror



$$\vec{B} = B_r \hat{r} + B_z \hat{z}$$

B_r causes $m \frac{d v_z}{\partial t} = -q v_\theta B_r$

results in a net force in $-\hat{z}$

$$F_z = -\frac{m v_\perp^2}{2B} \frac{\partial B_z}{\partial z} = -\mu \frac{\partial B_z}{\partial z}$$

where $\mu \equiv \frac{m v_\perp^2}{2B}$ = magnetic moment adiabatic invariant

$\Downarrow$

$\boxed{\text{as } B \text{ increases } v_\perp \text{ increases}}$

Power = $\vec{v} \cdot \vec{F} \propto \vec{v} \cdot (v \times B) = \emptyset \Rightarrow$ Conservation of energy

so $\frac{1}{2} m v_\perp^2 + \frac{1}{2} m v_z^2 = ct$

As particles flow towards $\hat{z}$, B increases, $v_\perp$ increases, v_z decreases. Sometimes $v_z = \emptyset$ and reflects $\Rightarrow$ mirror

While this approach is simple to build, the losses have been too great.

Recent advances have led to a resurgance → $\boxed{\text{Realta}}$

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Lets add an $\vec{E} = \vec{E}_\perp + \vec{E}_\parallel \qquad \vec{B} = \vec{B}_\parallel$

$$F = m\frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B} + \vec{E})$$

$$\dot{v}_\parallel = \frac{q}{m}\vec{E}_\parallel \rightarrow \boxed{v_\parallel = v_{\parallel 0} + \frac{q}{m}E_\parallel t \quad \text{constant acceleration}}$$

$$\dot{v}_\perp = q/m(\vec{v} \times \vec{B} + \vec{E}_\perp)$$

For simplicity, assume $\vec{B} = B\hat{z}$, $\vec{E} = E\hat{x}$

$$\dot{v}_x = q/m(v_y B + E)$$
$$\dot{v}_y = -q/m(v_x B)$$

$$v_y' \equiv v_y + E/B \rightarrow \dot{v}_y' = \dot{v}_y$$

$$\dot{v}_x = q/m(v_y' B) \rightarrow \ddot{v}_x = -\omega_c^2 v_x$$

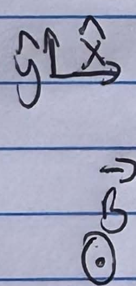
$$\ddot{v}_y' = -\omega_c^2(v_y')$$

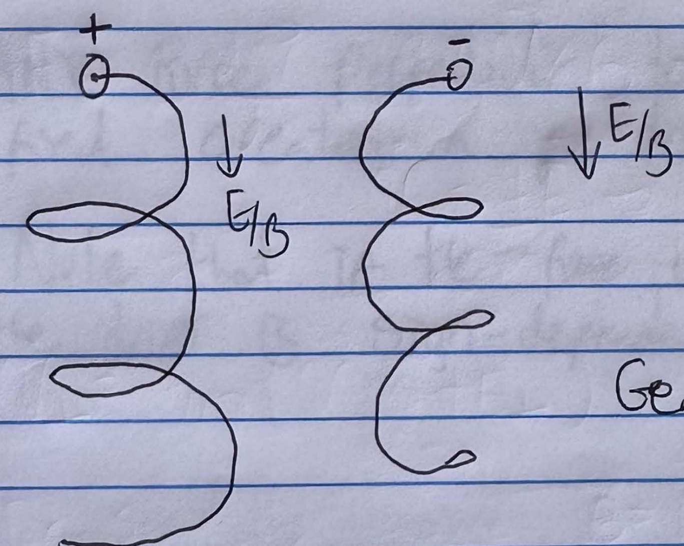
the solutions are similar to the $E=0$ solutions:

$$v_x = v_\perp \cos(\omega_c t + \phi)$$

$$v_y' = \mp v_\perp \sin(\omega_c t + \phi)$$

$$v_y = \mp v_\perp \sin(\omega_c t + \phi) - \underbrace{\frac{E}{B}}_{v_{E\times B}}$$





Same direction for both charges

Generalized $\boxed{\dfrac{\vec{E}\times\vec{B}}{B^2} = v_{E\times B}}$

We can generalize to any external force perpendicular to $\vec{B}$: $\vec{F}_{E\perp}$

$$m\frac{d\vec{v}}{dt} = q\vec{v}\times\vec{B} + \vec{F}_{E\perp}$$

$$\dot{v}_\perp = \frac{q}{m}\vec{v}_\perp\times\vec{B} + \frac{\vec{F}_{E\perp}}{m} \Rightarrow v_\perp' \equiv v_\perp - \frac{\vec{F}_{E\perp}\times\vec{B}}{qB^2}$$

$$+ \left[-\vec{F}_{E\perp}\underbrace{(\vec{B}\cdot\vec{B})}_{B^2} + \vec{B}\cancel{(\vec{F}_{E\perp}\cdot\vec{B})}\right)$$

$$\vec{v}_\perp' \times \vec{B} = \vec{V}_\perp \times B - \frac{(\vec{F}_{E\perp} \times B) \times B}{q B^2}$$

$$\vec{v}_\perp' \times \vec{B} = \vec{V}_\perp \times \vec{B} + \frac{\vec{F}_{E\perp}}{q}$$

So $\dot{\vec{V}}_\perp' = \frac{q}{m} \vec{V}_\perp' \times B \Rightarrow$ Gyro motion

So $\vec{V}_\perp$ has a drift $=$ $\boxed{V_D = \frac{\vec{F}_{E\perp} \times \vec{B}}{q B^2}}$

ANY force perpendicular to the magnetic field creates a drift perp. to both

Note that if the force is indep. of q, the drift is sign-dependent.

We move to the obvious Question:
How about a magnetic field that bites its own tail?

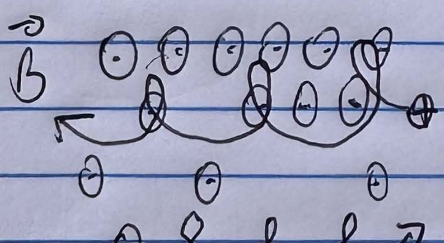
Curvature Drift arises as an effect of the centrifugal force in the plasma frame:

$$F_{CF} = \frac{m v_{\parallel}^2}{R_c} \hat{r}$$

$$\Downarrow$$

$$\boxed{V_R = \frac{m v_{\parallel}^2}{q B^2} \frac{\vec{R}_C \times \vec{B}}{R_C^2}}$$

∇B Drift arises due to varying mag. fields perp. to $\vec{B}$:



$$\boxed{V_{\nabla B} = \frac{m v_{\perp}^2}{2 q B} \frac{\vec{B} \times \nabla B}{B^2}}$$

These add to make purely toroidal fields untenable (refer to slides)