Calabi-Yau threefold flops as quiver varieties from monopole deformations

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Work in collaboration with Roberto Valandro, and Andrés Collinucci.

June 13, 2024 String-Math 2024

SISSA

Non-toric flop geometries

Type IIA String Theory on trivially fibered ADE surfaces:

Background geometry: $\mathbf{R}^{1,3} \times \mathrm{X}_2^g \times \mathbb{C}_w, \qquad X_2^g = \mathbb{C}^2/\Gamma^g, \quad g \in \{A, D, E\},$

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X_2^g's complex deformations encoded in the background profile of : $\Phi \in Adj(g)$

$\Phi \quad \leftarrow \quad$ complex scalar in the 6d EFT at the singularity

$$\Phi \neq const: \qquad X_2^g \times \mathbb{C}_{\{w\}} \xrightarrow[\Phi(w)]{} \begin{array}{ccc} \mathrm{X}_2^{g,def} & \longrightarrow & \mathrm{X}_3^g(\Phi) \\ & & \downarrow \\ & & \mathbb{C}_{\{w\}} \end{array}$$

Note: The resolution pattern of $\mathrm{X}_3^g(\Phi)$ is also specified by $\Phi(w)$ [Collinucci, De Marco, Sangiovanni, Valandro]

Quiver Varieties from Monopole Deformations:

Towards a Systematic Construction

- Recover the geometry $X_3^g(\Phi)$ from the moduli space of a D-brane probe.

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The theory on $X_3^g(\Phi)$ is obtained by adding a $N = 2$ preserving deformation:

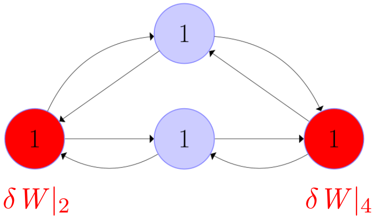
$$W = W_{N=4} + \mathrm{Tr}(\Phi(\{\phi_i\})\mu), \quad \mu \to \text{g-moment map}, \quad \phi_i \to \text{CB chirals}$$

$$W \xrightarrow[IR]{} W_{\mathit{eff}} \Rightarrow X_3^g(\Phi)\text{'s defining equation in } \mathbb{C}^4$$

Analogous results have been obtained through different approaches [Witten, Klebanov; Cachazo, Vafa, Katz; Gubser, Nekrasov, Shatashvili]

1) $A_3 \times \mathbb{C}_w \quad \Rightarrow \quad$ Reid Pagoda

$$x^2 + y^2 = z^4, \quad \forall\, w \quad \Rightarrow \quad x^2 + y^2 = z^4 - w^2 \quad (x, y, z, w) \subset \mathbb{C}^4$$



$$W = W_{N=4} + \delta W(W_2^{\pm}, W_4^{\pm}, \phi_2, \phi_4)$$

Monopole deformations have been studied [Giacomelli, Collinucci, Valandro, Savelli; Benini, Benvenuti, Pasquetti]

Quiver Varieties from Monopole Deformations: Examples

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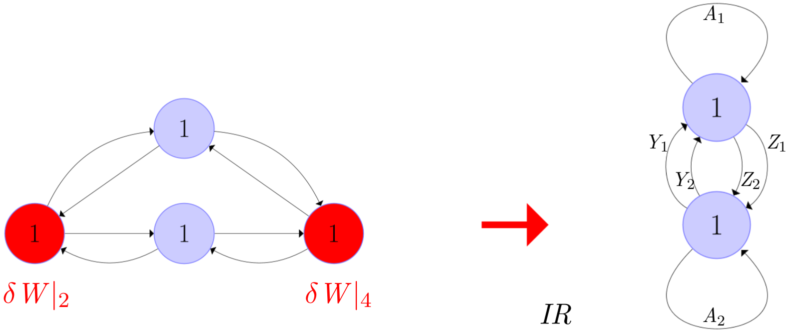
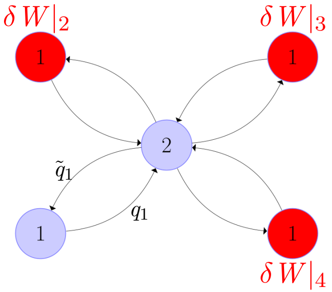


$$W = W_{N=4} + \delta W(W_2^{\pm}, W_4^{\pm}, \phi_2, \phi_4) \qquad W_{eff} = (A_1 - A_2)(Y_1 Z_1 + Y_2 Z_2 + 2 A_1 A_2)$$

Monopole deformations have been studied [Giacomelli, Collinucci, Valandro, Savelli; Benini, Benvenuti, Pasquetti]

2) $D_4 \times \mathbb{C}_w \quad \Rightarrow \quad$ Brown-Wemyss Threefold

$$x^2 + y^2 z = z^3, \quad \forall\, w \quad \Rightarrow \quad x^2 + y^2 z = (z - w)(zw^2 + (z-w)^2) \quad (x, y, z, w) \subset \mathbb{C}^4$$



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Integrating out massive degrees of freedom...

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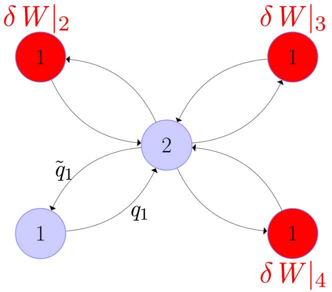
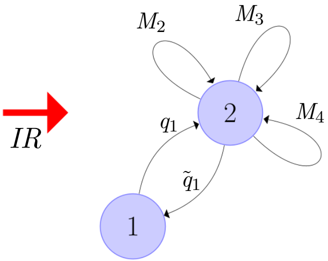




$$W = W_{N=4} + \delta W(W_2^{\pm}, W_3^{\pm}, W_4^{\pm}, \phi_2, \phi_3, \phi_4) \qquad W_{\mathit{eff}} = F(M_2, M_3, M_4, q_1, \tilde{q}_1)$$

Integrating out massive degrees of freedom...

Conclusions and Outlook

- We provide a simple recipe to extract the $N = 2$ superpotential of a large class of non toric CY threefolds.
- We propose a physical explanation for a non-commutative geometry algorithm that derives the quiver and the relations of the threefold from the non-affine [Cachazo, Katz, Vafa] and the affine [Karmazin] Dynkin diagram of the starting ADE algebra.
- We aim at applying our technique to simple flops of any length.

Thanks!

Abelianization of Virasoro conformal blocks at $c = 1$

Qianyu Hao, University of Genève
(joint work in progress with Andrew Neitzke)

June 2024

Introduction

The (chiral) Virasoro conformal blocks are solutions to the 2d CFT Ward identities.

In this talk, by a Virasoro block Ψ we mean a system of correlation functions

$$\langle T(p_1)\cdots T(p_n)\rangle_\Psi$$

, $\forall n \geq 0$, which only have poles at the diagonal $p_i = p_j$, governed by the OPE

$$T(p_i)\,T(p_j) = \frac{c/2}{(p_i-p_j)^4} + \frac{2T(p_j)}{(p_i-p_j)^2} + \frac{\partial_{p_j}T(p_j)}{p_i-p_j} + \text{reg}\,.$$

Under change of coordinates, T transforms as

$$T(p')^w = \left(\frac{\mathrm{d}p}{\mathrm{d}p'}\right)^2\left(T(p) - \frac{c}{12}\{p',p\}\right)\,.$$

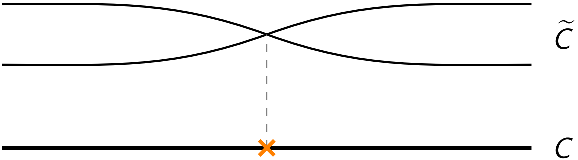
My talk is to describe a new way to construct the $c=1$ Virasoro conformal blocks.

A new way to compute Virasoro blocks at $c = 1$

We construct a nonabelianization map which simplifies the computation to the one of the abelian objects, Heisenberg blocks:

$$\mathcal{F}_{\mathcal{W}}: \quad \mathrm{Conf}(\widetilde{C}, \mathrm{Heis}) \quad \to \quad \mathrm{Conf}(C, \mathrm{Vir}_{c=1})$$

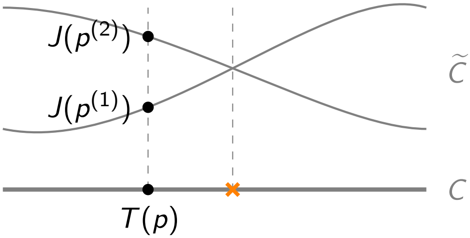
- $\mathrm{Conf}(C, \mathrm{Vir}_{c=1})$ is the space of conformal blocks for the Vertex algebra $\mathrm{Vir}_{c=1}$ over a Riemann surface C
- $\mathcal{W}$ is a spectral network. [Gaiotto-Moore-Neitzke]
  The main new idea of our work is to use the spectral network.
- Heis stands for the Heisenberg vertex algebra ($\hat{\mathfrak{u}}(1)$ vertex algebra).
- $\widetilde{C}$ is a branched double cover of C.



Introduce the dictionary

$$T(p) \rightsquigarrow \frac{1}{4} :(J(p^{(1)}) - J(p^{(2)}))^2:,$$

where J is the generator of the Heisenberg algebra. This gives correct OPE away from the branch point.



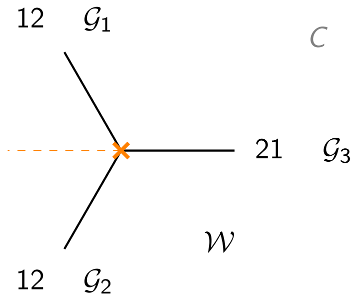
Simply using the dictionary, we get

$$\mathcal{F}_{\varnothing}: \quad \mathrm{Conf}(\widetilde{C}, \mathrm{Heis}) \quad \to \quad \mathrm{Conf}\left(C, \mathrm{Vir}_{c=1}; W_{\frac{1}{16}}(b_1)\cdots W_{\frac{1}{16}}(b_n)\right),$$

where $W_{\frac{1}{16}}(b_i)$ is a primary field at branch point b_i. These image blocks with extra insertions are not what we are after.

[Zamolodchikov, Dixon-Friedan-Martinec-Shenker, Gavrylenko-Marshakov, $\cdots$]

The main new idea in our work is to use the spectral network $\mathcal{W}$ to cancel the unwanted insertions at branch points.



We introduce operator $E(\mathcal{W})$, and the desired nonabelianization map $\mathcal{F}_{\mathcal{W}}$ is

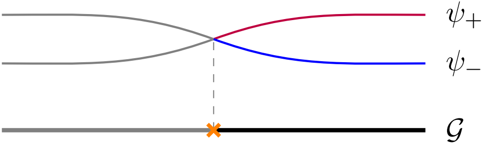
$$\langle 1 \rangle_{\mathcal{F}_{\mathcal{W}}(\Psi)} = \langle E(\mathcal{W}) \rangle_{\Psi} \, ,$$

$$\langle T(p) \rangle_{\mathcal{F}_{\mathcal{W}}(\Psi)} = \left\langle \frac{1}{4} \colon \left(J(p^{(1)}) - J(p^{(2)}) \right)^2 \colon E(\mathcal{W}) \right\rangle_{\Psi} \, ,$$

$$\vdots$$

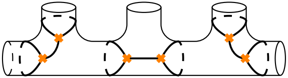
$$E(\mathcal{W}) = \exp\left[\frac{1}{2\pi i}\int_{\mathcal{W}} \psi_+(z^{(1)})\psi_-(z^{(2)})\,dz\right],$$

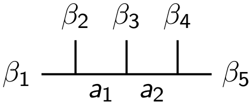
where $\psi_\pm$ are usually called free fermions.



For each spectral network, we get a type of the Virasoro blocks.

In particular, we expect that Virasoro blocks obtained from Fenchel-Nielsen type of spectral networks correspond to Nekrasov partition functions. E.g.





Future directions

- Generalization to W_N-algebras at $c = N - 1$.
- Generalization to other central charges c.
- $\ldots$

Thank you!

Symmetry topological field theories from self-dual fields in string theory

Saghar S. Hosseini

KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

THE UNIVERSITY OF TOKYO

String-Math 2024 - ICTP

based on

[2404.16028 with I. García Etxebarria]

[Work in progress with Y. Tachikawa and H. Zhang]

Symmetry TFT

Symmetries may be defined in terms of **topological operators**, as **categories**.

[Gaiotto, Kapustin, Seiberg, Willett '14]

- Consider a QFT $\mathcal{T}_{\text{relative}}$ with a fixed local structure.
- $\mathcal{T}_{\text{relative}}$ may admit **different choices of symmetries**.

[Aharony, Seiberg, Tachikawa '13]

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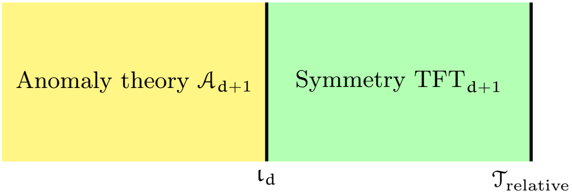
The topological data and choices of symmetries may be characterized by a topological field theory in one higher dimension, known as the **Symmetry TFT**.

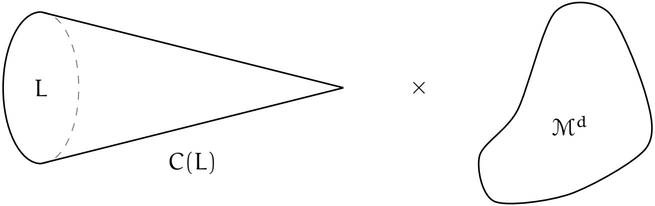
[Witten '98], [Kapustin and Seiberg '14]... [Freed, Moore, Teleman '22]



Fixing the interface ι_d results in an absolute theory $\mathcal{T}_{\text{absolute}}$ and the corresponding anomaly theory.

Symmetry TFT from string theory

Geometric engineering: String theory on $\mathsf{C}(\mathsf{L}) \times \mathcal{M}^d$

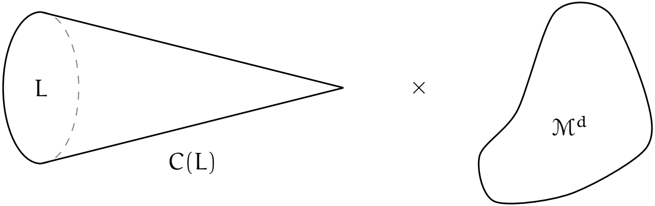


results in a SQFT $\mathcal{T}$ on $\mathcal{M}^d$.

[Katz, Klemm '96], [Katz, Vafa '97], [Katz, Klemm, Vafa '97]

Symmetry TFT from string theory

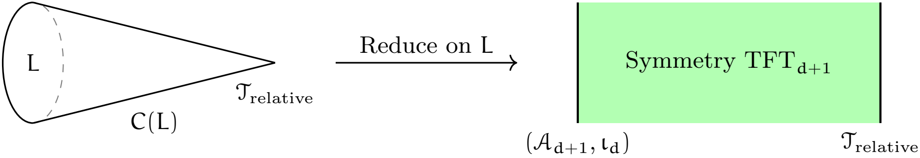
Geometric engineering: String theory on $C(L) \times \mathcal{M}^d$



results in a SQFT $\mathcal{T}$ on $\mathcal{M}^d$.

[Katz, Klemm '96], [Katz, Vafa '97], [Katz, Klemm, Vafa '97]

The SymTFT is determined by the topology of L.



[Del Zotto, Heckman, Park, Rudelius, '15], [García Etxebarria, Heidenreich, Regalado '19] ...
[Apruzzi, Bonetti, García Etxebarria, SH, Schäfer-Nameki '21]

Symmetry descent

In type II string theory, the self-dual RR field is a H-twisted differential K-theory class. Can we write an action for it?

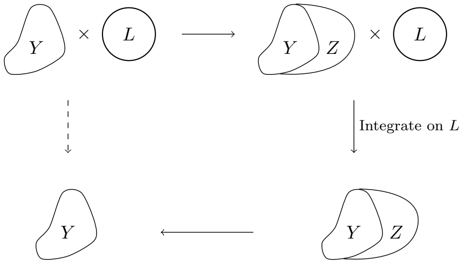
Symmetry descent

In type II string theory, the self-dual RR field is a H-twisted differential K-theory class. Can we write an action for it?

We can construct an 11d Chern-Simons action, given by quadratic refinement in twisted differential K-theory, that describes a self-dual field as a boundary mode.

[Witten '97] [Freed '00], [Hopkins and Singer '02], [Belov, Moore '06] ...
[Hsieh, Tachikawa, Yonekura '20]

The dimensional reduction of this Chern-Simons theory results in the symmetry TFT.



In equation:

$$\Delta \int_{L}^{\hat{K}_H} \mathcal{L}_{CS} = \delta \mathcal{L}_{\mathrm{SymTFT}}$$

Some open questions

- For configurations that require a formulation where the NSNS field is also dynamical, H-twisted K-theory seems to be no longer a good approximation. How do we extend to this case?

- How do we extend to M-theory?

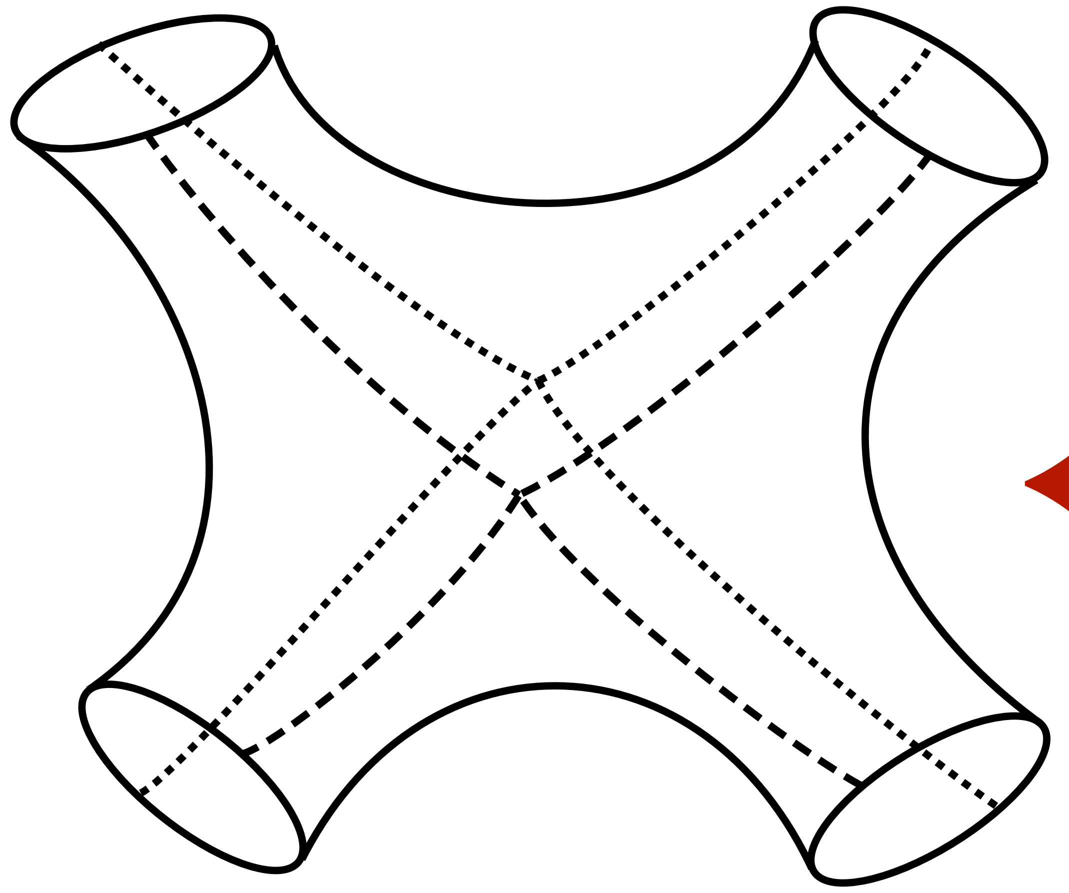
- How can we adopt a more categorical perspective in our analysis? How does the symmetry theory obtained from string theory describe categorical symmetries and their anomalies? Do we need a generalised cohomology theory that works with T-duality?

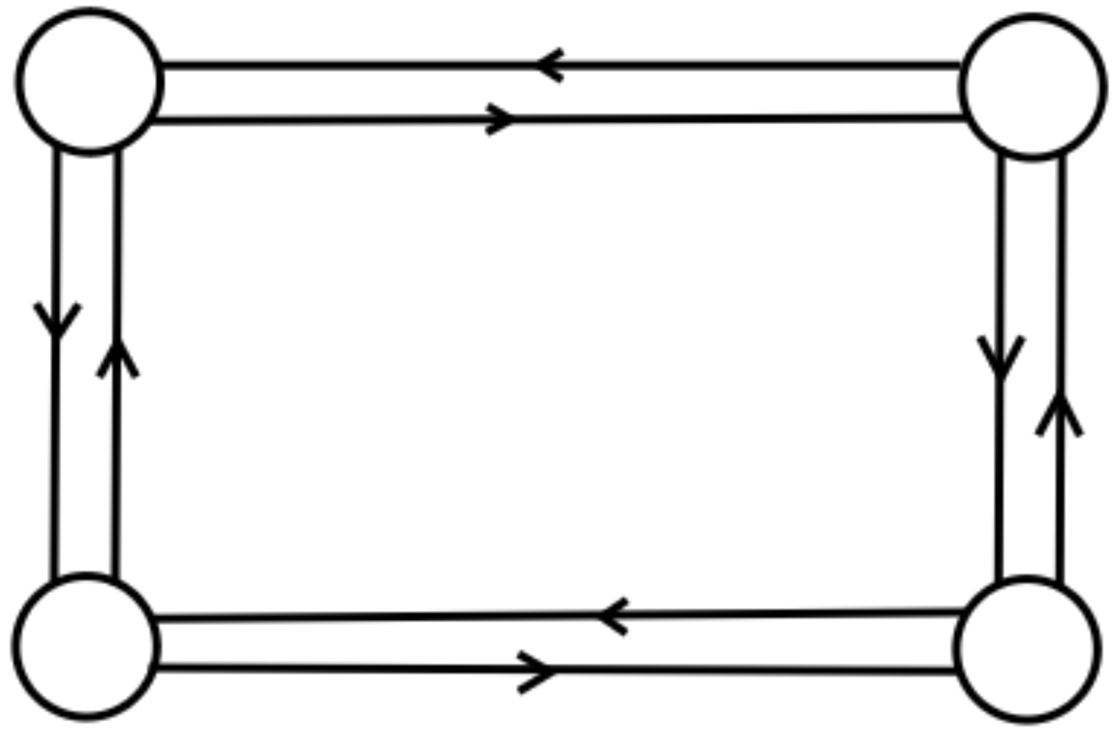
  [Apruzzi, Bah, Bonetti and Schäfer-Nameki '22], [García Etxebarria '22], ...

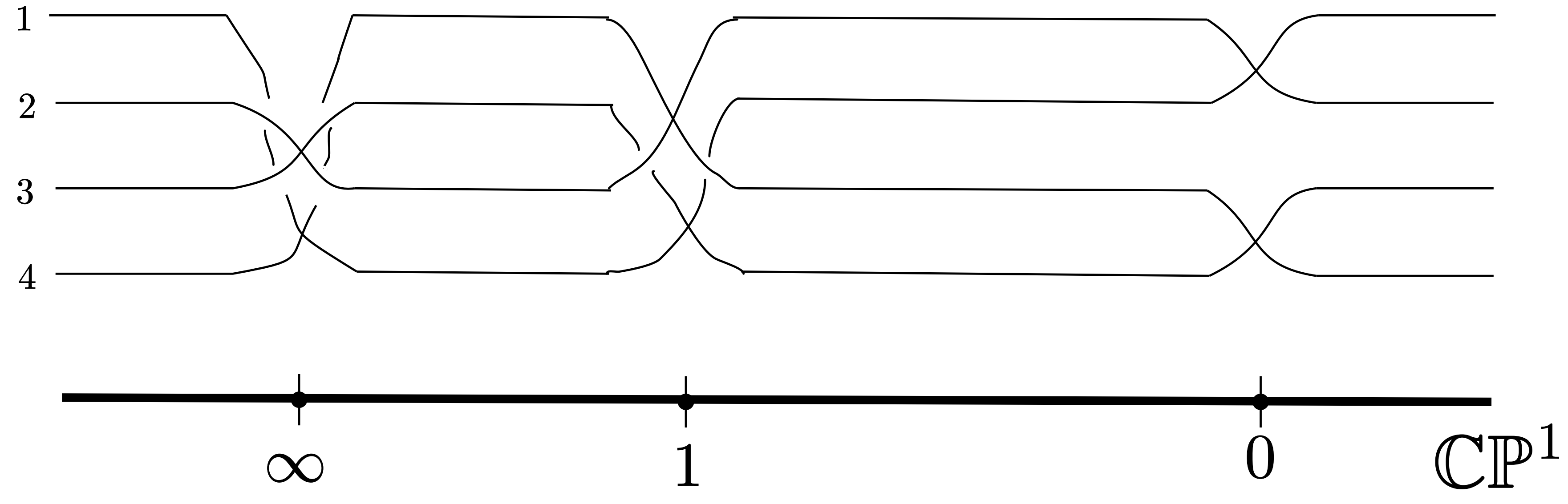
- The topological operators are constructed from branes. Can we find a map from the category of branes (such as the derived category of coherent sheaves in B-model) in topological string theory to categorical symmetries?

Deriving the Simplest Gauge/String Duality

Strings from Feynman Diagrams







With M. Gaberdiel
& R. Gopakumar

Today's Focus:
Moments in the Gaussian Matrix Model as Stringy Correlators

Edward Mazenc (ETH Zürich) - June 13th, 2024 - String Math 2024

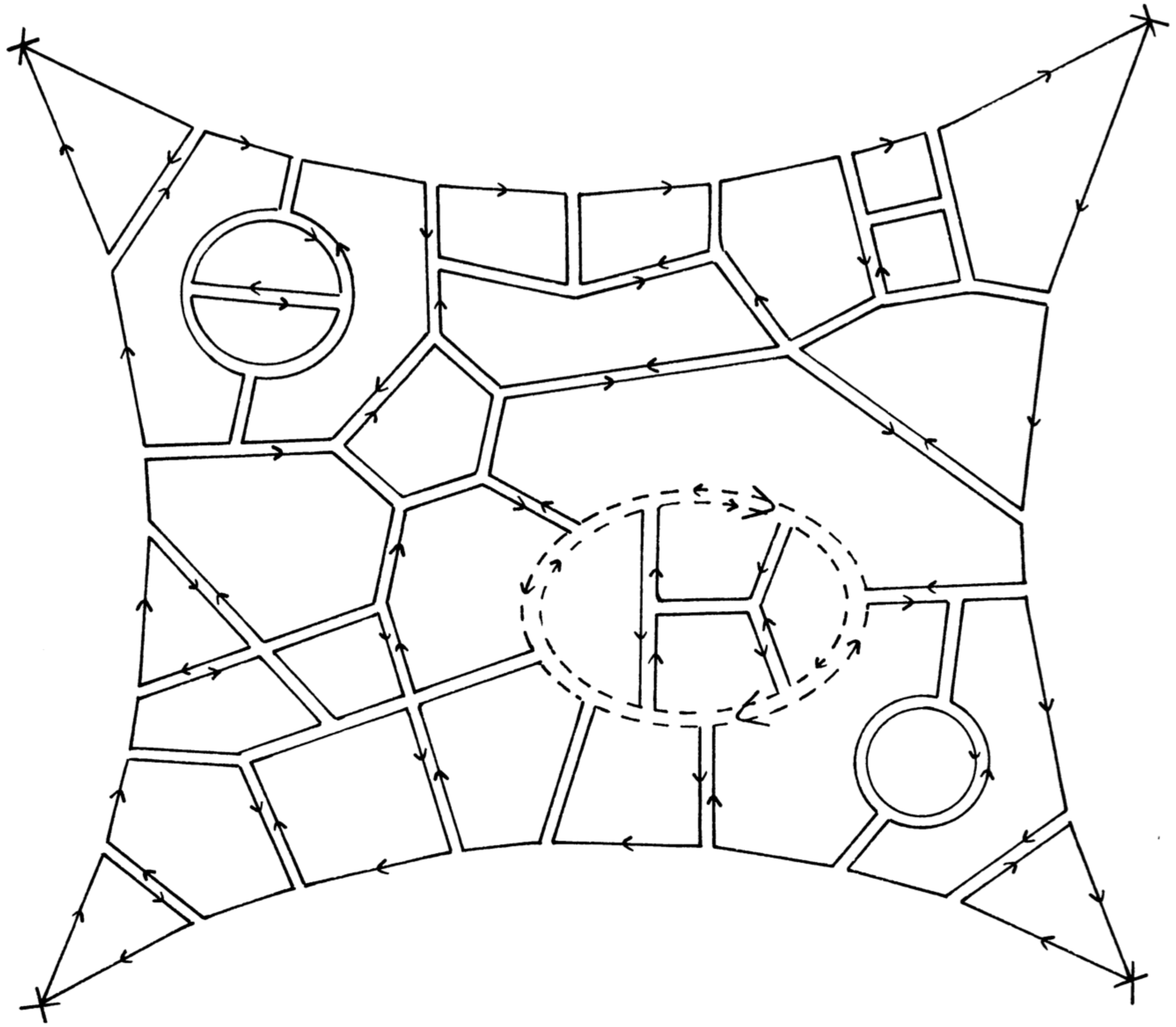
A PLANAR DIAGRAM THEORY FOR STRONG INTERACTIONS

Ref.TH.1786-CERN

13 December 1973

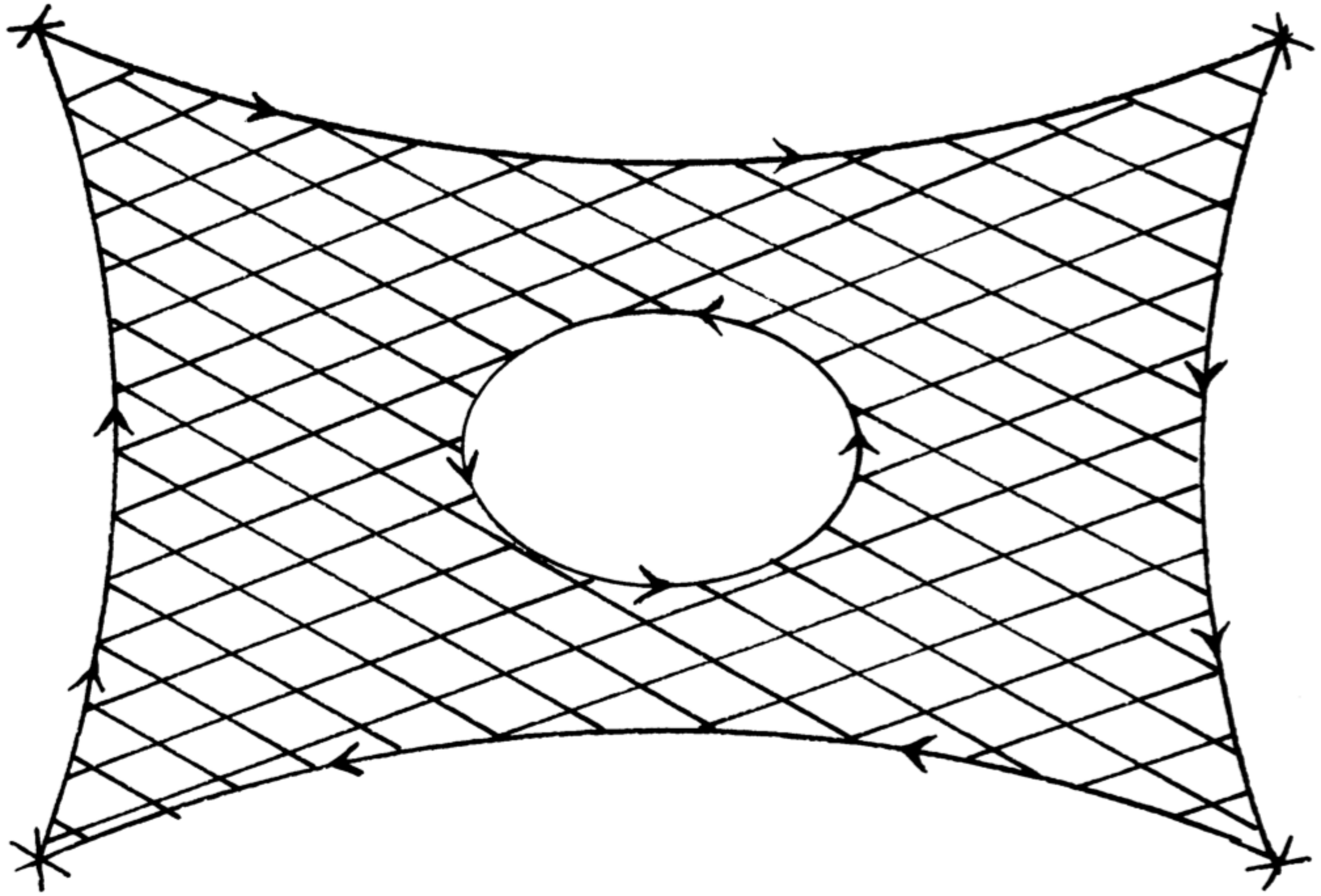
G. 't Hooft

CERN - Geneva



- Figure 3 -

$\stackrel{?}{=}$



(a)

TODAY'S SIMPLIFICATION:

„*SET PROPAGATORS TO 1*" → MATRIX INTEGRAL

The Simplest Gauge Theory

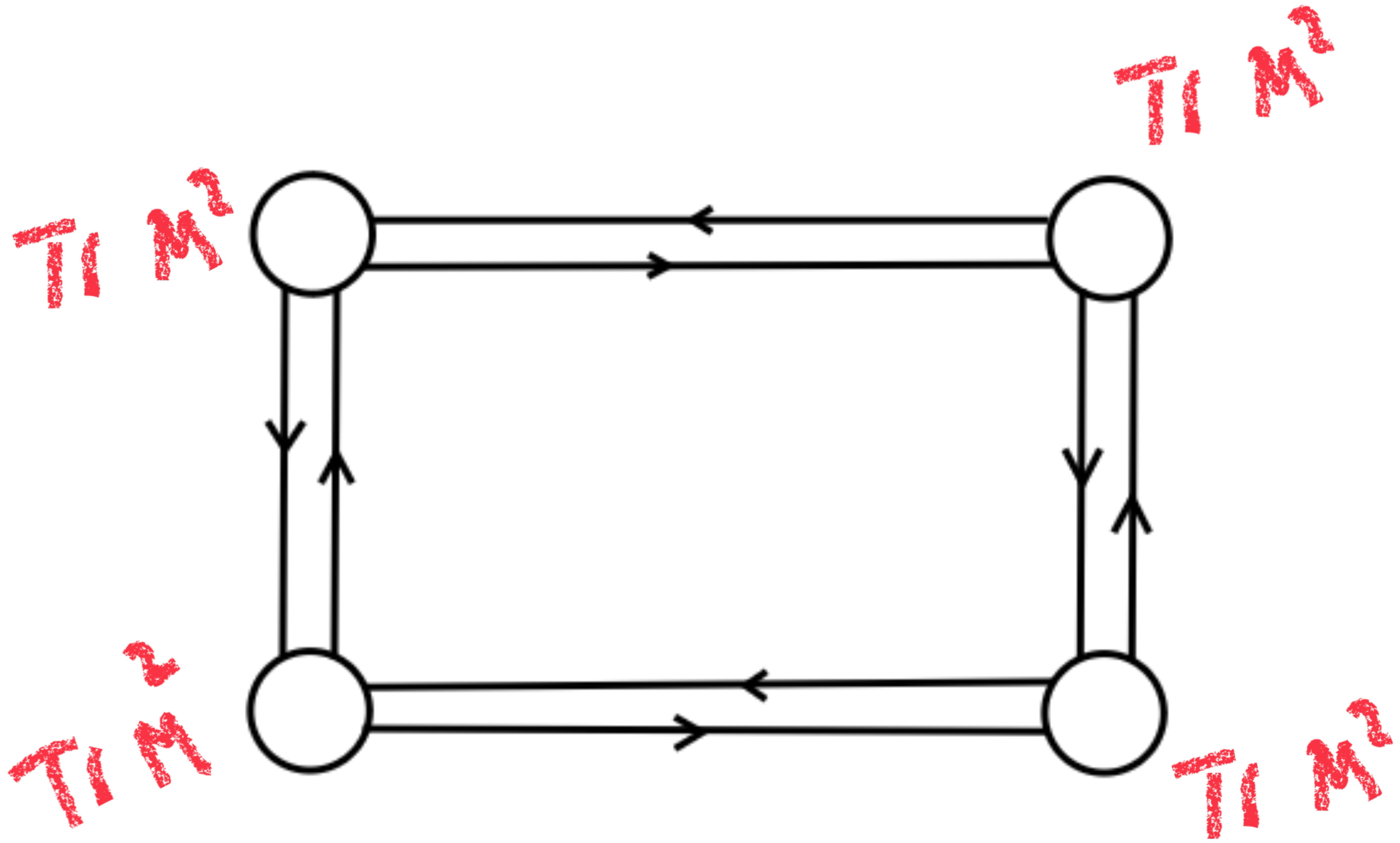
& The simplest Feynman diagram

$$Z_N = \frac{1}{Vol\ U(N)} \int dM_{N\times N}\ e^{-\frac{N}{2t_2}TrM^2}$$

Gauge-invariant observables built from $Tr(M^n)$

Example (free theory):

$$\langle Tr(MM)Tr(MM)Tr(MM)\ Tr(MM)\rangle_{con.}$$



The Main Result

An explicit all-genus mapping of GUE correlators

$$\sum_h \left(\frac{1}{N}\right)^{2h-2} \left\langle \prod_{i=1}^{n} \frac{1}{Nk_i} Tr(M^{k_i}) \right\rangle_h^{conn.} = \sum_h g_{st}^{2h-2} \left\langle \prod_{i=1}^{n} V_{k_i} \right\rangle_h^{conn.}$$

Where $g_{string} = \frac{1}{N}$

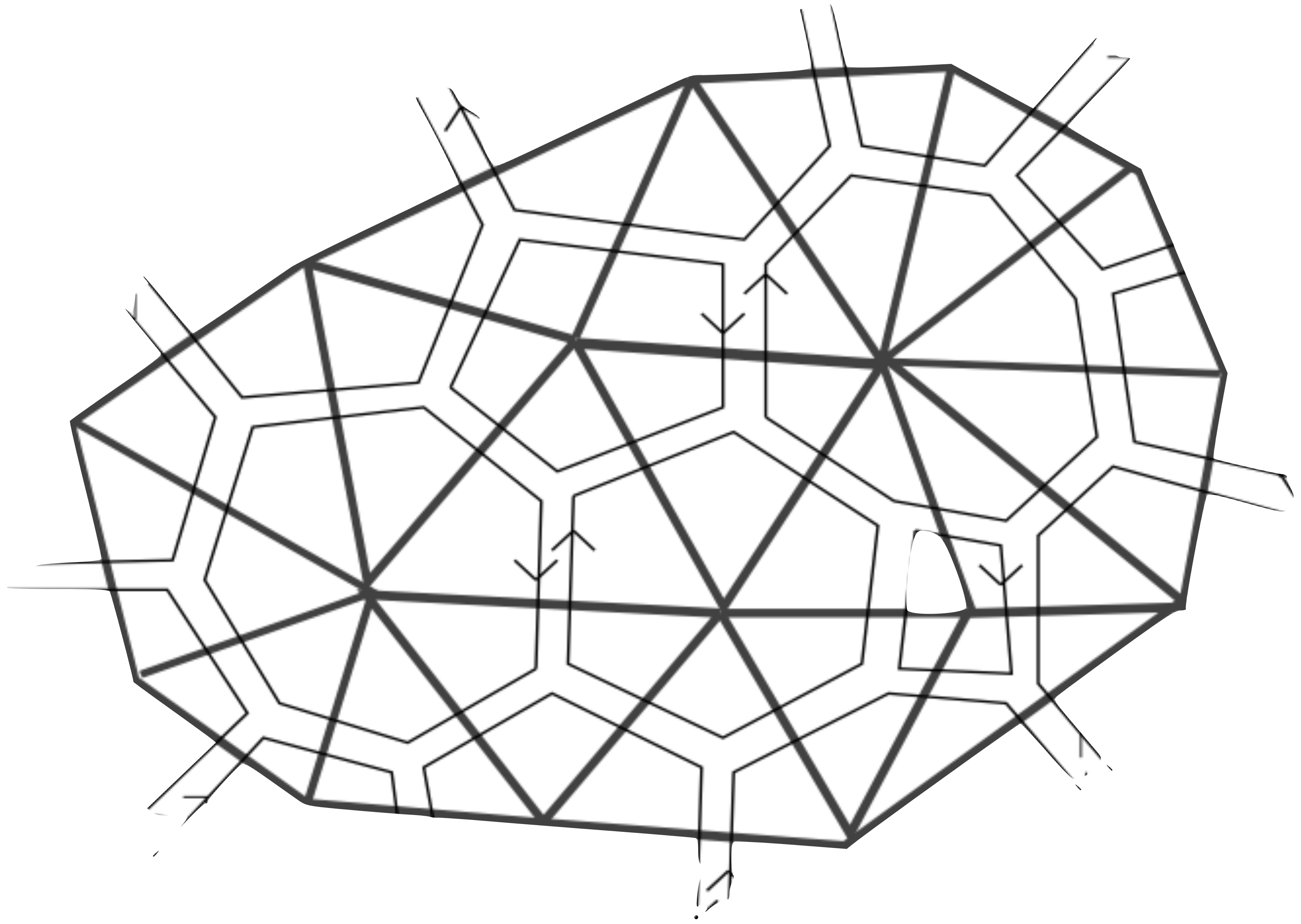
Matrix Models and Strings

What is new here?

- **Previously focused on double-scaling limit**: Feynman diagrams as "latticization" of the worldsheet

  [Cf. Gross-Migdal; Douglas-Shenker; Brezin-Kazakov]

Not 't Hooft limit as in AdS/CFT!



- **Dijkgraaf-Vafa**: matrix integrals from localized holomorphic Chern-Simons Theory (i.e. open string field theory on branes in non-compact CYs)

Focus here on worldsheet theory
Find operator dictionary/correlators

[Cf. Aganagic-Dijkgraaf-Klemm-Marino-Vafa]

What Replaces the Minimal String?

3 equivalent closed string descriptions for the GUE

A-Model :

Kazama-Suzuki $\dfrac{SL(2,\mathbb{R})_1}{U(1)}$

(With mom. +2 deformation)

* Derived via Feynman Diagrams

$$\frac{1}{Nk}Tr(M^k) \leftrightarrow V^k_{j=\frac{1}{2}}(X=\infty, z)$$

[Cf. Witten, Mukhi-Vafa, Ashok-Murthy-Troost, Nakamura-Niarchos Eberhardt-Gaberdiel-Gopakumar]

↔

B-Model :

Topological Landau-Ginzburg with superpotential

$$W_{LG}(Y) = \frac{1}{Y} + t_2 Y = X_{sc}(Y)$$

* Derived via Top. Recursion

$$\lim_{k\to\infty} \frac{1}{Nk} : Tr(M^k) :\leftrightarrow e^{-\frac{k^2}{2}\psi}$$

[Cf. Vafa, Ghoshal-Vafa, Ghoashal-Imbimbo-Mukhi, Hanany-Oz-Plesser, Aganagic-Dijkgraaf-Klemm-Marino-Vafa, Eynard, DOSS]

↔

$c = 1$ string at self-dual radius:

Liouville + compact boson (with mom. +2 Tachyon background)

*Derived via Duality with Imbimbo-Mukhi MM

$$\frac{1}{Nk}Tr(M^k) \leftrightarrow T_{-k}$$

[Cf. Aganagic-Dijkgraaf-Klemm-Marino-Vafa, Moore-Plesser-Rangoolam, Imbimbo-Mukhi]

The Derivation

***Why* do these closed string theories appear?**

The A-Model

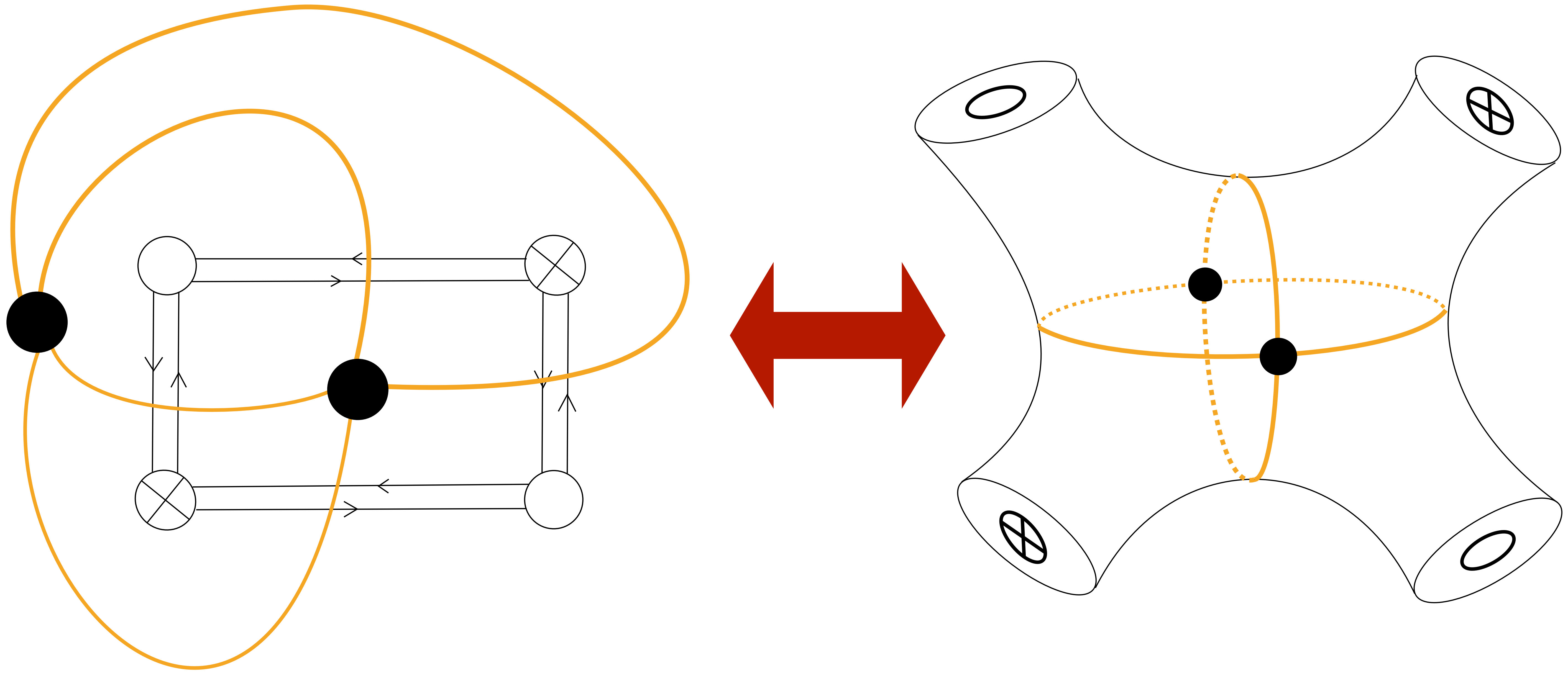
How do we see the holomorphic maps from the WS to the TS from the matrix?

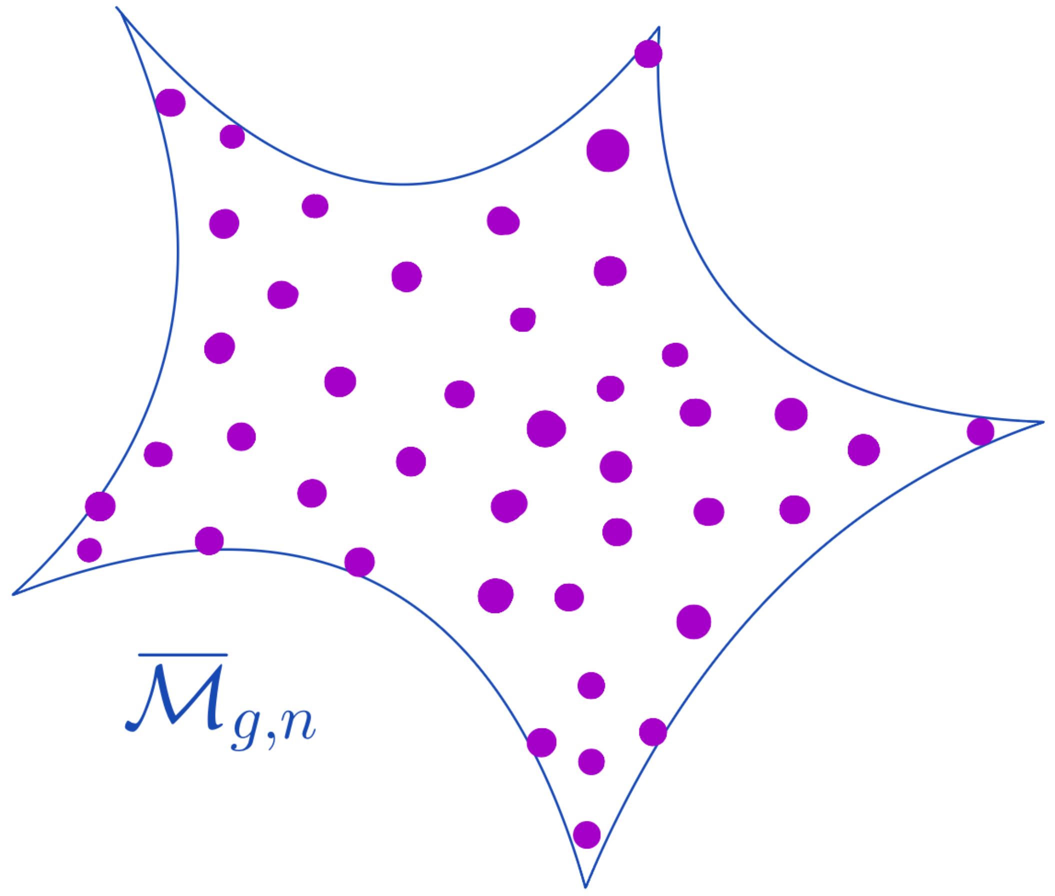
I. Reconstructing the Worldsheet

[Cf. Strebel, Kontsevich, Gopakumar]

Use the Strebel Parametrization of $\mathscr{M}_{g,n} \times \mathbb{R}^n_+$

→An exact equivalence between genus g metrized ribbon graph with n faces & $p \in \mathscr{M}_{g,n} \times \mathbb{R}^n_+$





"Each gauge theory FD as a string worldsheet"

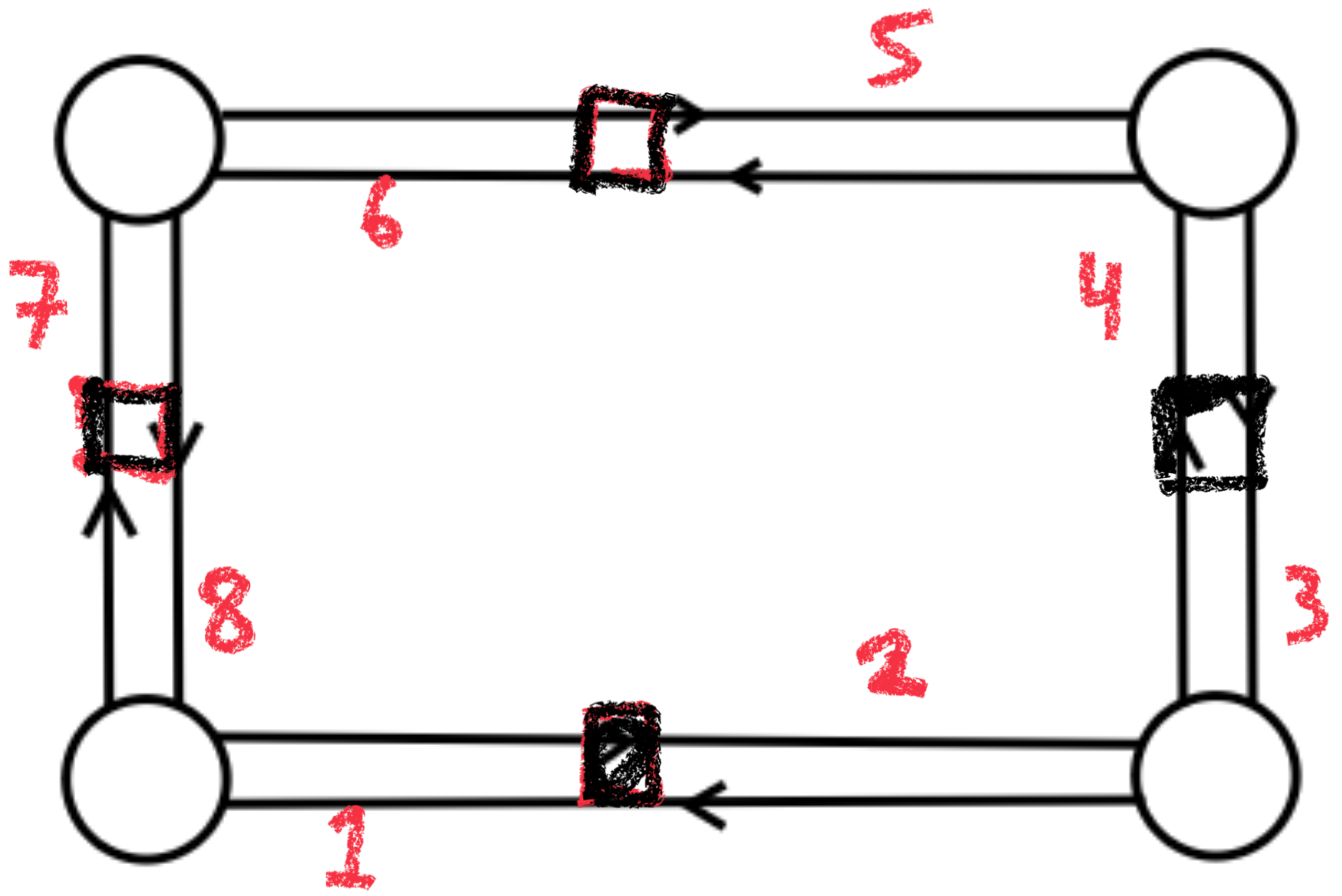
1. →"Strebel Graph" is dual to Feynman diagram
2. → Lengths assigned to edges = number of (homotopic) Wick contractions $\in \mathbb{Z}$
3. →Localization to discrete "arithmetic" surfaces (only worldsheets admitting Belyi maps)
4. → GUE correlators count lattice points on moduli space

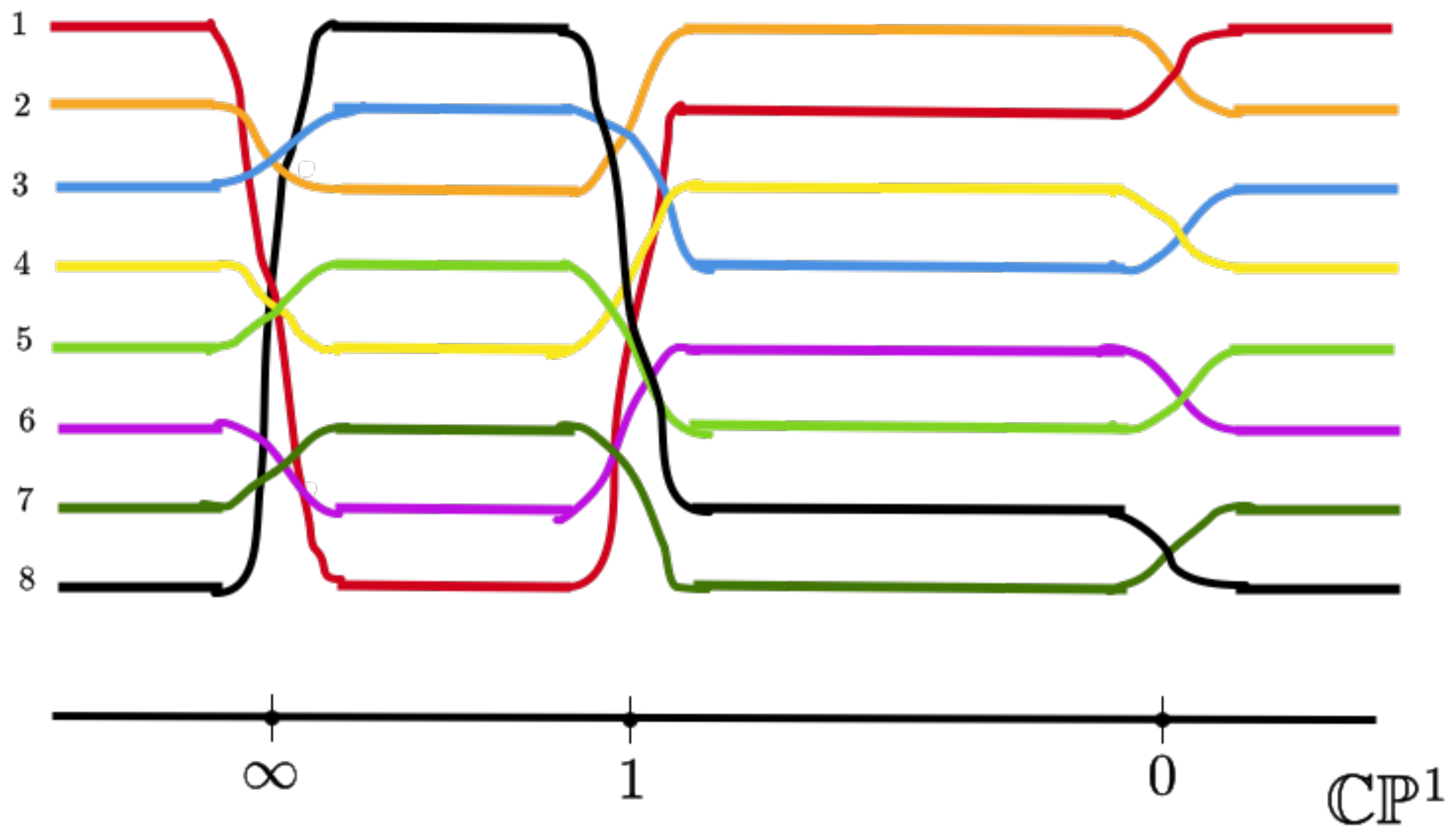
[Cf. Norbury, Chekhov et al.]

II. Reconstructing the Embedding Map

Our Simplest Diagram as a Belyi Map

Cf. e.g. Belyi, Grothendieck, Zagier, Lando-Zvonkin, Pakman-Rastelli, Gopakumar, Rangoolam





Vertices :

$\sigma_\infty = (18)(23)(45)(67)$

Edges:

$\sigma_0 = (12)(34)(56)(78)$

Faces:

$\sigma_1 = (1753)(2468)$

Such localization to holomorphic covering maps of $\mathbb{CP}^1$ indeed already seen in $\frac{SL(2,\mathbb{R})_1}{U(1)}$ string!

[Cf. Eberhardt, Dei, Gaberdiel, Gopakumar, Knighton, Maity]

- See Poster for 2-Matrix Model Story -

Thank You!

See Longer Talk at KITP
“What is String Theory?” Program

Role of double-scaling?

Imbimbo-Mukhi vs. Kontsevich

V-type
$N \times N$ Hermitian MM

“$N \to \infty$” Double-scaling

$$\epsilon \to 0$$

[Hashimoto-Huang-Klemm-Shih]

$$V_p(z) = \sum_{k=1}^{p} \frac{1}{k}(z+1)^k$$

$$g = \frac{1}{N} = \epsilon^{p+1}$$

$$A = -1 + \epsilon Z$$

$$X = \epsilon^p \tilde{X}$$

V-type
Double-scaled Hermitian MM
(“On ZZs”)

[$p = 2$: Cf. Maldacena-Moore-Seiberg-Shih]

OCO Triality

F-type
$Q \times Q$
Imbimbo-Mukhi MM

OCO Triality

F-type
$Q \times Q$
Kontsevich MM
(“On FZZTs”)

$$Z_{IM} \propto \int dA_{Q\times Q} e^{-\frac{1}{g}Tr\left(V_p(A) - AX\right) - (N+Q)Tr\log(A)}$$

“N” Double-scaling
Q fixed!

$$Z_{Kontsevich} \propto \int dZ_{Q\times Q} e^{Tr\left(\frac{Z^{p+1}}{p+1} + Z\tilde{X}\right)}$$

[Gaiotto-Rastelli]

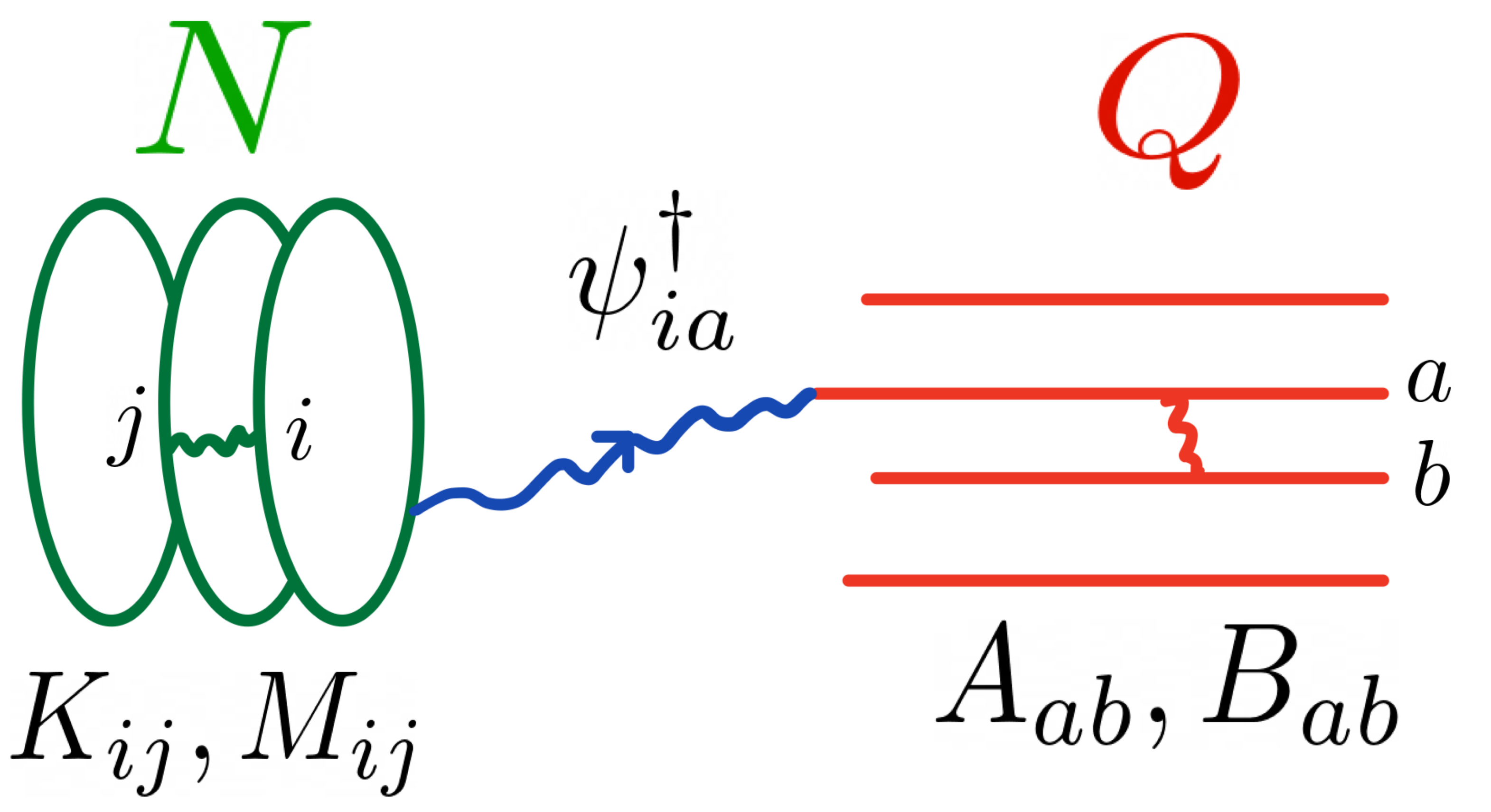
p=2: OSFT c=28 Liouville +c=-2 on Q FZZT

The Even Bigger Picture

As a topological subsector of "standard" AdS/CFT

A New Equality of 2 Matrix Integrals

Open-Closed-Open Triality as Verification



$$Z(X,Y) = \frac{1}{Z_N}\int dKdM_{N\times N} e^{+\frac{1}{g}Tr\left(V(K) - K(M-Y)\right)} \prod_{a=1}^{Q} \det(x_a - M)$$
$$= \frac{(-1)^{NQ}}{Z_Q}\int dAdB_{Q\times Q} e^{-\frac{1}{g}Tr\left(V(A) + A(B-X)\right)} \prod_{i=1}^{N} \det(y_i - B)\,.$$

Key Steps

$$Z(X,Y) = \frac{1}{Z_N}\int dKd\psi d\psi^\dagger e^{\frac{1}{g}Tr_N\left(V(K)+KY\right)+\psi^\dagger_{ia}X_{ab}\psi_{ib}} \int dM e^{-\frac{1}{g}M_{ij}\left(K_{ji} - g\psi^\dagger_{ia}\psi_{ja}\right)}$$
$$= \int dKd\psi d\psi^\dagger e^{\frac{1}{g}Tr_N\left(V(K)+KY\right)+\psi^\dagger_{ia}X_{ab}\psi_{ib}} \delta\left(K_{ji} - g\psi^\dagger_{ia}\psi_{ja}\right)$$
$$= \int d\psi d\psi^\dagger e^{\frac{1}{g}Tr_N V[(-g\psi\psi^\dagger)] + \psi^\dagger_{ia}\left(X_{ab}\delta_{ij} - \delta_{ab}Y_{ij}\right)\psi_{jb}}$$

[Cf. Maldacena-Moore-Seiberg-Shih, Aganagic-Dijkgraaf-Klemm-Marino-Vafa, Goel - H. Verlinde, Altland-Sonner]

“Color-Flavor Transformation”

$$Tr_N\left[(\psi\psi^\dagger)^k\right] = (-1)^{2k-1} Tr_Q\left[(\psi^\dagger\psi)^k\right]$$

→ Reverse Steps

$$A_{ba} = -g\psi^\dagger_{ia}\psi_{ib}$$

The Imbimbo-Mukhi Matrix Model

Traces as tachyon modes in c=1 at self-dual radius

[Cf. "Kontsevich-Penner-Model", Chekhov et al., Bonora-Xiong, Moore-Plesser-Rangoolam]

$$\frac{1}{Z_N}\int dKdM_{N\times N}e^{-\frac{1}{g}\mathrm{Tr}\left(V_p(K)-K(M-X)\right)}\prod_{a=1}^{Q}\det(x_a-M)$$
$$=$$
$$\frac{1}{Z_Q}\int dAdB_{Q\times Q}e^{+\frac{1}{g}\mathrm{Tr}\left(V_p(A)+A(B-X)\right)}\prod_{i=1}^{N}\det(y_i-B)$$

$$\frac{1}{Z_N}\int dKdM_{N\times N}e^{+NTr\left(-\sum_n t_nK^n-KM+\sum_{k=1}^{\infty}\bar{t}_nM^n\right)}$$
$$=$$
$$\det(X)^{-N}\int dA_{Q\times Q}e^{NTr\left(\sum_n t_nA^n+AX\right)-(N+Q)Tr\log(A)}\times(\text{Penner Model})$$

$$Z_{IM}(t_n,\bar{t}_n)=\det(X)^{-i\mu}\int dA_{Q\times Q}e^{+i\mu\sum_{n=1}t_nTr(A^n)+i\mu Tr(AX)-(i\mu+Q)Tr\log(A)}$$

$N=i\mu$ **Genus-expansion = large N expansion**

$$\bar{t}_n=\frac{1}{n}Tr_Q\left(X^{-n}\right)$$

$$\frac{1}{Nn}TrM^n\leftrightarrow\frac{\partial}{\partial\bar{t}_n}\leftrightarrow T_{-n}$$
$$\frac{1}{Nn}TrK^n\leftrightarrow\frac{\partial}{\partial t_n}\leftrightarrow T_{+n}$$

Generating Function of "Tachyon" correlators in "c=1 2d-string theory"

The « BMN-Limit »

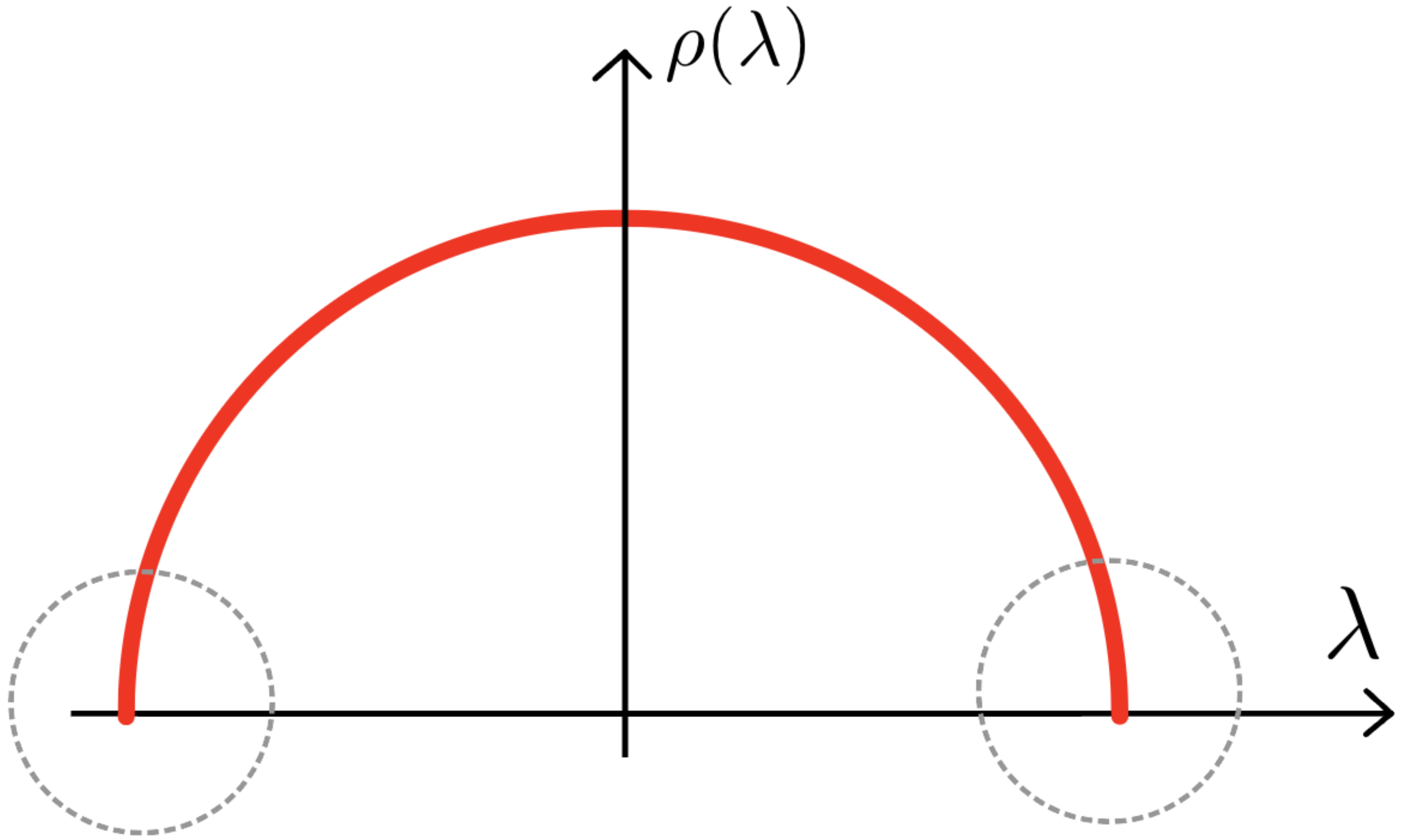
A New Perspective on Double-Scaling

$$\omega_{Kontsevich} = \sum_{i=1}^{n} k_i^2 \psi_i$$

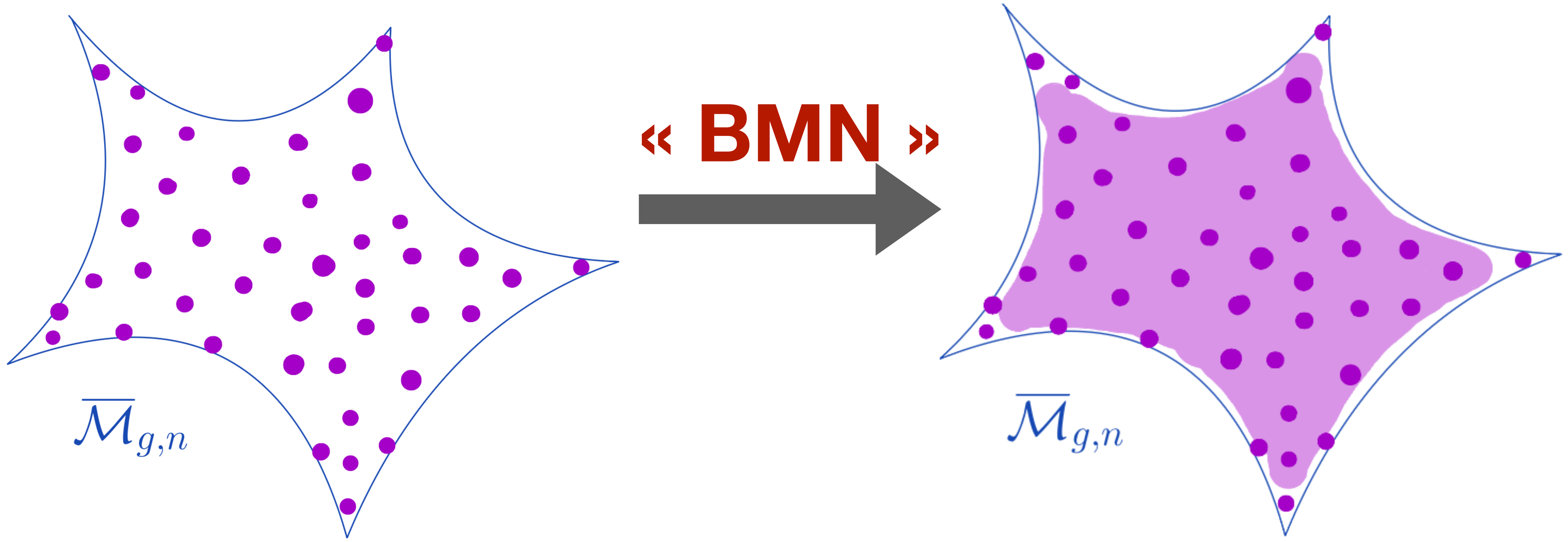
[Cf. Ehrhart, Norbury]

$$\lim_{k_i \to \infty} \langle \prod_{i=1}^{n} \frac{1}{Nk_i} : TrM^{2k_i} : \rangle_c = \lim_{k_i \to \infty} N_{g,n}(2k_1, \ldots, 2k_n) \to Vol_{Kontsevich}(2k_1, \ldots, 2k_n)$$

Pure 2d top gravity in BMN limit



Wigner Semicircle ↔ full AdS
Edge Region (Airy) ↔ pp-Wave geometry



Cover all of moduli space!

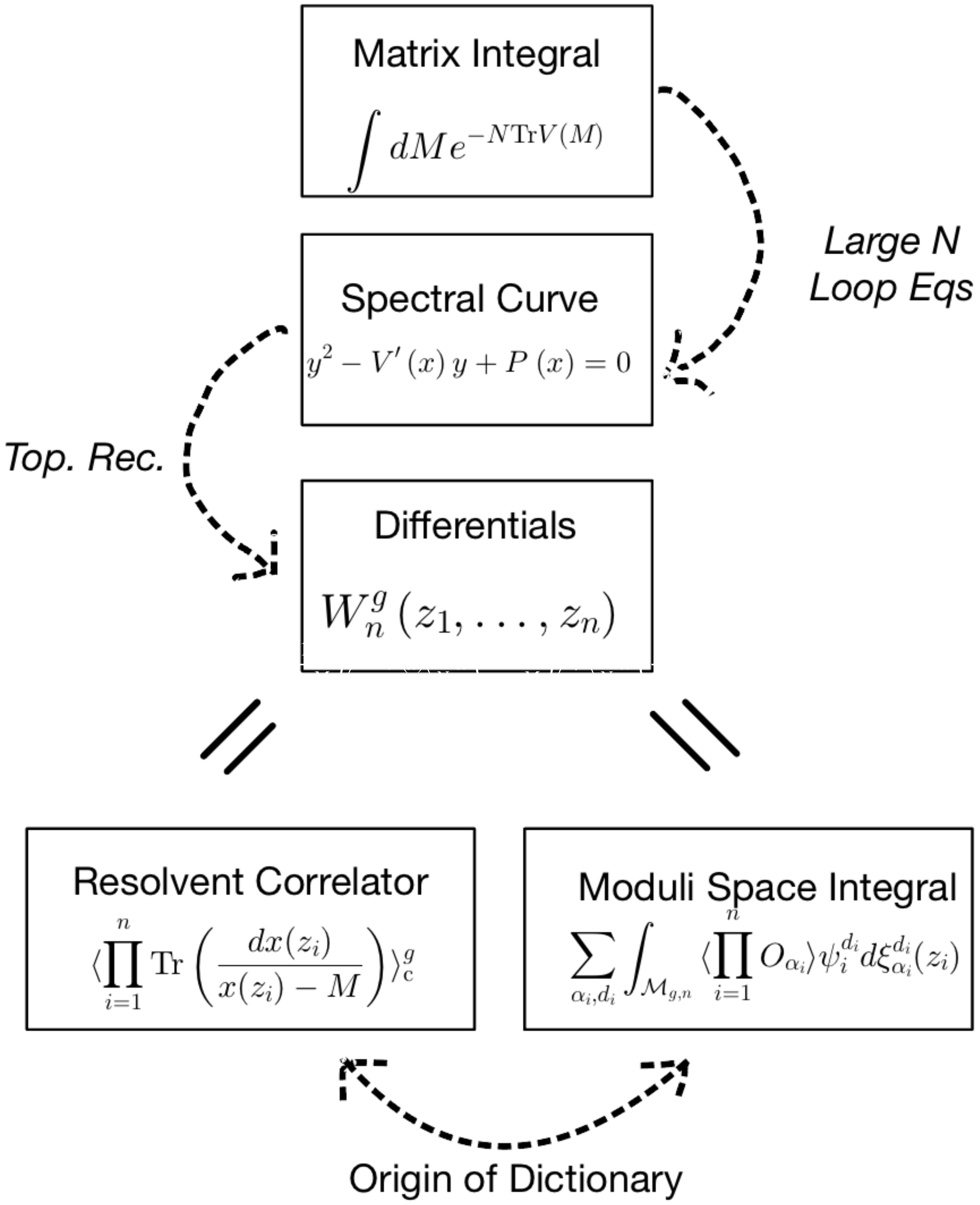
The B-Model

How do we see the constant maps from the WS to the critical points of the super potential?

Finding a B-Model in Disguise

The Many Faces of Topological Recursion

[Cf. Eynard-Orantin; Eynard; DOSS
KS : Dijkgraaf-Vafa; Post, v.d. Heijden, E Verlinde
LG : Dunin-Barkowski, Norbury, Orantin, Popolitov, Shadrin]



Gaussian Model Spectral Curve

$$x = \frac{1}{y} + ty$$

Landau-Ginsburg Superpotential

$$W(Z) = \frac{1}{Z} + tZ$$

(Cf. Dijkgraaf-Vafa)

Branchpoints of Spectral Curve

$$dx = 0$$

Critical Points of Superpotential

$$dW = 0$$

Topological Recursion: Residues at branchpoints of spec curve
B-model string: localization to constant maps into critical points of W

Integrate out matter first: moduli space integral & intersection numbers

$\updownarrow$

Integrate out « gravity » first (cf. Losev): Top. Recursion as matter residue calculus with new contact terms

[$WS\ Sol$: Cf. E. & H. Verlinde
Dijkgraaf-Verlinde-Verlinde]

CohFT Correlators

Traces as Matter Primaries + Gravitational Descendants

$$\int dM_{N\times N}\, e^{-NTrV(M)} \prod_{i=1}^{N} TrM^{k_i}$$

Potential determines matter theory

large N expansion = genus expansion

$$\sum_g N^{2-2g-n} \sum_{\alpha_i,d_i} c^{k_i}_{\alpha_i,d_i} \int_{\overline{\mathscr{M}}_{g,n}} \langle O_{\alpha_1} \ldots O_{\alpha_n} \rangle \prod_{i=1}^{n} \psi_i^{d_i}$$

Extra psi-class Insertions

$$TrM^{2k} \leftrightarrow \mathbf{O}_+ \sum_{d=0}^{k-1} \frac{(2k)!}{(k-d)!(k-1-d)!}\psi^{2d} + \mathbf{O}_- \sum_{d=0}^{k-1} \frac{(2k)!}{(k-1-d)!(k-1-d)!}\psi^{2d+1}$$

Main tool: TR as CohFT
(Eynard 2011 + DOSS 2014 + Giachetto Thesis+…)

Matter Primaries =
Edges of Eigenvalue Distribution

Sanity Checks:
g=0 3pt & 4pt, and g=1 1pt correlators from explicit moduli-space integrals

Very Explicit Universal Operator Dictionary!

Two B-Model Perspectives on 4-pt Fn.

"Pure Matter" vs. Intersection Theory Computations of $N_{0,4}(2k_1,\ldots,2k_4)$

Matrix Model Answer: $\langle \prod_{i=1}^{n=4} \frac{1}{N2k_i} : TrM^{2k_i} :\rangle_c^{g=0} = N_{g=0,n=4}(2k_1,..,2k_4) = k_1^2 + k_2^2 + k_3^2 + k_4^2 - 1$

"Pure Matter" LG w/ contact terms (cf. Losev)

$$W(Z) = \frac{1}{Z} + Z \qquad \frac{1}{Nk} : TrM^k : \leftrightarrow \mathcal{O}_k \equiv \frac{1}{Z^{k+1}}$$

Start with 3-pt Fn.
(Cf. Vafa)

$$\begin{aligned}
N_{g=0,n=3}(k_1,k_2,k_3) &= \langle \mathcal{O}_{k_1}\mathcal{O}_{k_2}\mathcal{O}_{k_3}\rangle \\
&= \oint \frac{1}{W'(z)}\frac{1}{z^{k1+1}}\frac{1}{z^{k2+1}}\frac{1}{z^{k3+1}} \\
&= \mathrm{Res}_{z\to 1}\frac{z^2}{(z^2-1)}\frac{1}{z^{k1+1}}\frac{1}{z^{k2+1}}\frac{1}{z^{k3+1}} + \mathrm{Res}_{z\to -1}\frac{z^2}{(z^2-1)}\frac{1}{z^{k1+1}}\frac{1}{z^{k2+1}}\frac{1}{z^{k3+1}} \\
&= \left(\frac{1}{2}\right) + (-1)^{k1+k2+k3}\left(\frac{1}{2}\right)
\end{aligned}$$

Matter Theory as Iterated Residue Calculus

B-Model "after integrating out gravity"

"Pure Matter" LG w/ contact terms (cf. Losev)

$$C_W(\mathcal{O}_{k_i}, \mathcal{O}_{k_j}) = \frac{d}{dz}\left(\frac{\mathcal{O}_{k_i}\mathcal{O}_{k_j}}{W'(z)}\right)_{-} = \sum_{l=1}^{k_i+k_j} 2l\, \mathcal{O}_{2l}$$

$$\langle \mathcal{O}_{2k_1}\mathcal{O}_{2k_2}\mathcal{O}_{2k_3}\mathcal{O}_{2k_4}\rangle = \frac{d}{dt}\langle \mathcal{O}_{2k_1}\mathcal{O}_{2k_2}\mathcal{O}_{2k_3}\rangle_{W+t\mathcal{O}_{2k_4}}\Big|_{t=0} + \sum_{i=1}^{3}\langle C_W(\mathcal{O}_{2_k4}, \mathcal{O}_{2k_i})\prod_{j\neq i}^{3}\mathcal{O}_{2k_j}\rangle$$

$$= -(2k_4+1)\big(k_1+k_2+k_3+k_4+1\big) + k_1(1+k_1) + k_2(1+k_2) + k_3(1+k_3) + 2k_4(k_1+k_2+k_3) + 3k_4(1+k_4)$$

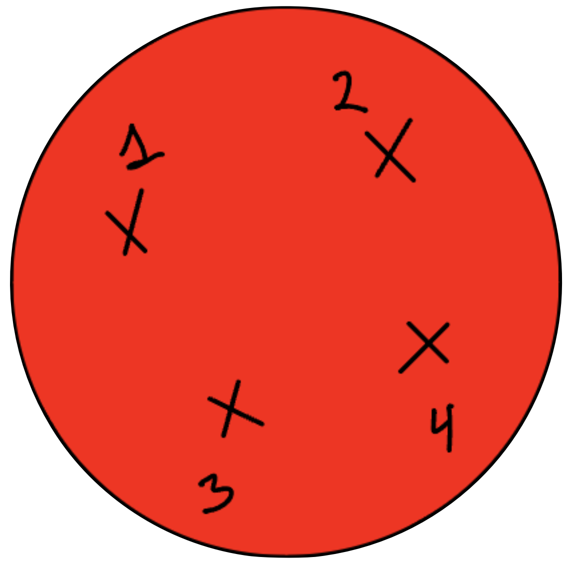
Contributions from deformed 3-pt fn

Contributions from contact terms

$$= k_1^2 + k_2^2 + k_3^2 + k_4^2 - 1 = N_{g=0,n=4}(2k_1, \ldots, 2k_4)$$

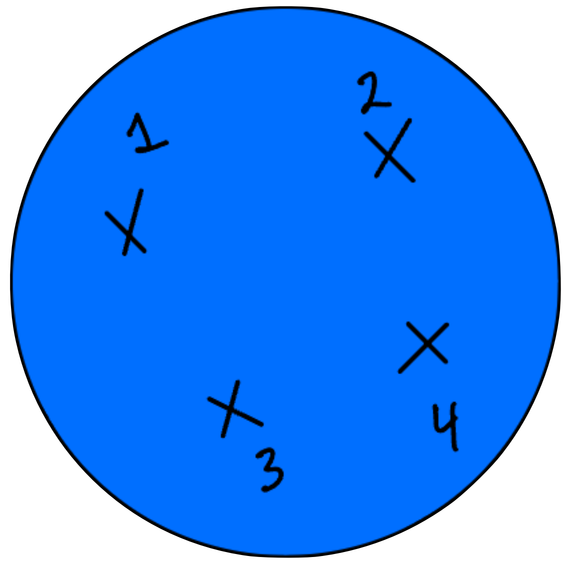
4-pt Function from Moduli Space Integral

B-Model "after integrating out matter"



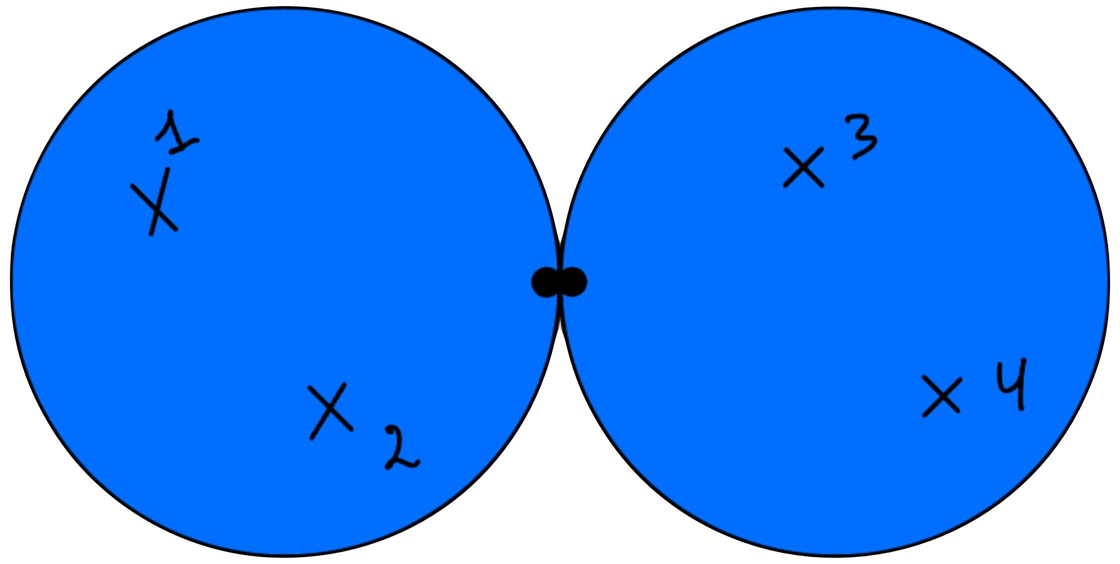
$$\frac{1}{2}(k_1^2 - \frac{1}{16})\langle \psi_1 \rangle_{\mathscr{M}_{0,4}}$$

+Perm(1,2,3,4)

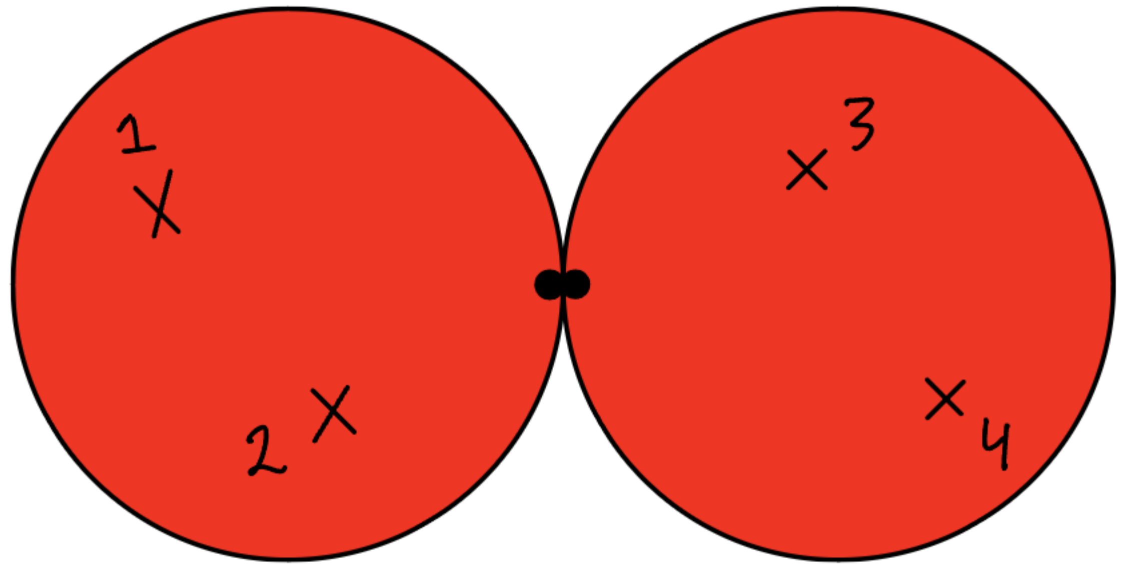


$$\frac{1}{2}(k_1^2 - \frac{1}{16})\langle \psi_1 \rangle_{\mathscr{M}_{0,4}}$$

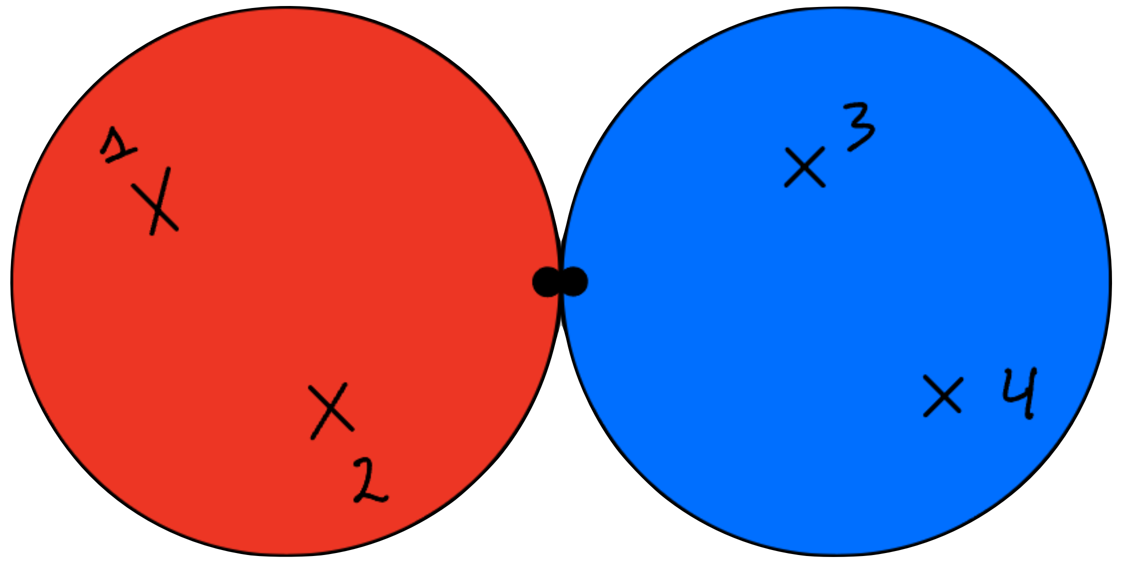
+Perm(1,2,3,4)



$$-\frac{3}{64}\langle \psi_1^0 \psi_2^0 \psi^0 \rangle_{\mathscr{M}_{0,3}} \langle \psi_3^0 \psi_4^0 \psi'^0 \rangle_{\mathscr{M}_{0,3}}$$



$$-\frac{3}{64}\langle \psi_1^0 \psi_2^0 \psi^0 \rangle_{\mathscr{M}_{0,3}} \langle \psi_3^0 \psi_4^0 \psi'^0 \rangle_{\mathscr{M}_{0,3}}$$



$$-\frac{3}{32}\langle \psi_1^0 \psi_2^0 \psi^0 \rangle_{\mathscr{M}_{0,3}} \langle \psi_3^0 \psi_4^0 \psi'^0 \rangle_{\mathscr{M}_{0,3}}$$

From operator insertions

Contributions from boundary of moduli space (cf. contact terms)

$$-\frac{3}{64}\langle \kappa_1 \rangle_{\mathscr{M}_{0,4}} \qquad -\frac{3}{64}\langle \kappa_1 \rangle_{\mathscr{M}_{0,4}}$$

From "background" dual to matrix potential

Coefficients fixed both by local behavior of spectral curve & Bergmann kernel near branchpoints) **and** our new operator dictionary

$$= k_1^2 + k_2^2 + k_3^2 + k_4^2 - 1 = N_{g=0,n=4}(2k_1, \ldots, 2k_4)$$

Fermionic Chern–Simons theory on $S^2 \times S^1$ at large-N in the 'temporal' gauge

String-Math Conference 2024, ICTP, Trieste, Italy

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Objective

We look at a fully non-perturbative, finite-temperature solution to a non-supersymmetric, non-abelian gauge theory with vector fermions in curved space – fermionic Chern–Simons theory on $S^2 \times S^1$.

Background

- Light-cone gauge calculations demonstrate Bose–Fermi duality [AGAY12] at the level of the thermal free energy;

- Reproduced the free energy calculations for the fermionic theory on $\mathbb{R}^2 \times S^1$ using the *'temporal' gauge* [MNTV23];

- Set up the calculation amenable to general genus-g Riemann manifolds.

Motivation

To establish Bose–Fermi duality on genus-g Riemann manifolds (possibly at finite N)!

Path integral of our theory

The finite temperature partition function of $U(N)$ Chern–Simons theory coupled to fundamental fermionic matter is defined by the Euclidean path integral,

$$Z = \int \mathcal{D}\left[A_\mu\right] \mathcal{D}\left[\psi\right] \mathcal{D}\left[\bar{\psi}\right] e^{-S_E}, \quad (1)$$

where the Euclidean action S_E is given by,

$$S_E = \frac{i\kappa}{4\pi} \int_{\Sigma\times S^1} \mathrm{Tr}\left\{A \wedge \mathrm{d}A - \frac{2i}{3} A \wedge A \wedge A\right\} + \int_{\Sigma\times S^1} \bar{\psi}(\not{D} + M)\psi. \quad (2)$$

We study the path integral of this theory on $\Sigma \times S^1$, where Σ is an arbitrary Riemann surface, and set up the problem at finite N and temperature $T = \beta^{-1}$, where β is the circumference of the S^1.

Gauge conditions adopted

- The *'temporal' gauge* is defined by $\partial_3 A_3(x) = 0$, which imposes on the holonomy field $U(x)$,

$$\partial_3 U(x) = \partial_3 e^{i\beta A_3(x)} = 0; \tag{3}$$

- The remaining two-dimensional $U(N)$ is reduced to a two-dimensional $U(1)^N$ by simultaneously diagonalizing the holonomy field $U(\vec{x})$ for every point $\vec{x}$ on Σ [BT93, JMS+13];
- Complete gauge is fixed by imposing the Coulomb gauge on the time-independent (zero mode) diagonal elements of A_1, A_2, or equivalently,

$$(A_{\dot{\mu}})_{\sigma}^{\ \sigma} = \sum_{j=1}^{N(g)=2g} \alpha_j^{\sigma} a_{\dot{\mu},j} + \epsilon_{\dot{\mu}\dot{\nu}} \partial_{\dot{\nu}} \chi^{\sigma}, \qquad \dot{\mu}, \dot{\nu} = 1, 2, \tag{4}$$

where $\{a_j\}$ is a basis for the nontrivial flat one-forms on the genus g Riemann surface Σ (the number of such one-forms is $N(g)$).

Exact path integral on a general manifold Σ at finite N and volume

In summary, the exact (in N and volume) partition function down to an unfixed two-dimensional $U(1)^N$ is given by,

$$Z = \left(\prod_{\alpha=1}^{N}\sum_{n_\alpha=1}^{\infty}\right)(-1)^{(N-1)\sum_\alpha n_\alpha}\int\prod_{\alpha=1}^{N}\left(\mathcal{D}\left[A_3^\alpha(\vec{x})\right]\mathcal{D}\left[(A_{\dot{\mu}})_\alpha{}^\alpha(\vec{x})\right]\prod_{m\in\mathbb{Z}+\frac{1}{2}}\mathcal{D}\left[\bar{\psi}_m^\alpha(\vec{x})\right]\mathcal{D}\left[\psi_{\alpha,m}(\vec{x})\right]\right)e^{-S_{U(1)^N}}, \quad (5)$$

where $\dot{\mu} = 1, 2$, and the complicated yet completely local (in two dimensions) action $S_{U(1)^N}$ is:

$$S_{U(1)^N} = \frac{i\kappa\beta}{2\pi}\sum_\alpha\int_\Sigma A_3^\alpha (F_{12})_\alpha{}^\alpha - \frac{1}{8\pi}\int_\Sigma R\ln V(\vec{x}) + \frac{1}{\beta}\int_\Sigma\sum_{\substack{m\in\mathbb{Z}+\frac{1}{2}\\ \alpha}}\bar{\psi}_{-m}^\alpha(\vec{x})\left\{\tilde{\not{D}} + M + i\gamma^3\left(\frac{2\pi m}{\beta} - A_3^\alpha(\vec{x})\right)\right\}\psi_{\alpha,m}(\vec{x})$$

$$-\frac{2\pi}{\kappa\beta^3}\int_\Sigma\sum_{\substack{l\in\mathbb{Z}\\ m,n\in\mathbb{Z}+\frac{1}{2}\\ \alpha,\sigma\\ \alpha\neq\sigma \text{ at } l=0}}\frac{1}{\frac{2\pi l}{\beta} - A_3^\alpha(\vec{x}) + A_3^\sigma(\vec{x})}\bar{\psi}_{-m}^\alpha(\vec{x})\gamma^1\psi_{\sigma,m-l}(\vec{x})\bar{\psi}_{-n}^\sigma(\vec{x})\gamma^2\psi_{\alpha,n+l}(\vec{x}). \quad (6)$$

Effectively, (5) describes two-dimensional $U(1)^N$ gauge fields interacting with N neutral scalar fields A_3^α and an infinite number of fermionic fields $\psi_{\alpha,n}(\vec{x})$ via the action (6).

Specialization to the case $\Sigma = S^2$

Specializing to a sphere (genus $g = 0$), fixing the residual two-dimensional $U(1)^N$ gauge freedom on a two-sphere, and integrating out the (remaining) Abelian A_1, A_2 fields from (5),

$$Z = \left(\prod_{\alpha=1}^{N}\sum_{n_\alpha=1}^{\infty}\right)(-1)^{(N-1)\sum_\alpha n_\alpha}\int\prod_{\alpha=1}^{N}\left(\mathcal{D}\left[A_3^\alpha(\vec{x})\right]\prod_{m\in\mathbb{Z}+\frac{1}{2}}\mathcal{D}\left[\bar{\psi}_m^\alpha(\vec{x})\right]\mathcal{D}\left[\psi_{\alpha,m}(\vec{x})\right]\right)e^{-S_{\text{exact}}}, \quad (7)$$

reducing the action $S_{U(1)^N}$ in (5) to the exact (in N and volume) action S_{exact}, given by,

$$\begin{aligned} S_{\text{exact}} =& i\kappa\sum_\alpha n_\alpha\lambda_\alpha - \frac{1}{8\pi}\int_{S^2} R\ \ln V(\vec{x}) + \frac{1}{\beta}\int_{S^2}\sum_{\substack{m\in\mathbb{Z}+\frac{1}{2}\\ \alpha}}\bar{\psi}^\alpha_{-m}(\vec{x})\left\{\tilde{\not{D}} + M + i\gamma^3\left(\frac{2\pi m}{\beta} - A_3^\alpha(\vec{x})\right)\right\}\psi_{\alpha,m}(\vec{x}) \\ & -\frac{2\pi}{\kappa\beta^3}\int_{S^2}\sum_{\substack{l\in\mathbb{Z}\\ m,n\in\mathbb{Z}+\frac{1}{2}\\ \alpha,\sigma\\ \alpha\neq\sigma \text{ at } l=0}}\frac{1}{\frac{2\pi l}{\beta} - A_3^\alpha(\vec{x}) + A_3^\sigma(\vec{x})}\bar{\psi}^\alpha_{-m}(\vec{x})\gamma^1\psi_{\sigma,m-l}(\vec{x})\bar{\psi}^\sigma_{-n}(\vec{x})\gamma^2\psi_{\alpha,n+l}(\vec{x}). \end{aligned} \quad (8)$$

$V(\vec{x})$ is solely given in terms of $A_3^\alpha(\vec{x})$.

Simplifications at large N

In the 't Hooft large-N limit, the local Vandermonde factor $V(\vec{x})$ becomes global, and the A_3 fields become space-independent (expressed in terms of eigenvalues $\{\lambda_\alpha\}$), leading to,

$$Z = \left(\prod_{\alpha=1}^{N} \sum_{n_\alpha=1}^{\infty}\right) (-1)^{(N-1)\sum_\alpha n_\alpha} \int \left(\prod_{\alpha=1}^{N} \mathrm{d}\lambda_\alpha\right) \; V \; \exp\left(-i\kappa \sum_\alpha n_\alpha \lambda_\alpha - S_{\text{eff}}\left(\{\lambda_\alpha\},\{n_\alpha\}\right)\right), \tag{9}$$

where,

$$e^{-S_{\text{eff}}(\{\lambda_\alpha\},\{n_\alpha\})} = \prod_{\alpha=1}^{n} \int \prod_{m\in\mathbb{Z}+\frac{1}{2}} \mathcal{D}\left[\bar{\psi}^{\alpha}_{m}(\vec{x})\right] \mathcal{D}\left[\psi_{\alpha,m}(\vec{x})\right] \, e^{-S_{\text{f}}}, \tag{10}$$

$S_{\text{eff}}\left(\{\lambda_\alpha\},\{n_\alpha\}\right)$ is the renormalized effective action after integrating out the fermionic fields,

$$S_{\text{f}} = \frac{1}{\beta}\int_{S^2} \sum_{\substack{m\in\mathbb{Z}+\frac{1}{2} \\ \alpha}} \bar{\psi}^{\alpha}_{-m}(\vec{x}) \left\{\tilde{\not{D}} + M + i\gamma^3 \left(\frac{2\pi m}{\beta} - \frac{\lambda_\alpha}{\beta}\right)\right\} \psi_{\alpha,m}(\vec{x}) - \frac{2\pi}{\kappa\beta^3}\int_{S^2} \sum_{\substack{l\in\mathbb{Z} \\ m,n\in\mathbb{Z}+\frac{1}{2} \\ \alpha,\sigma \\ \alpha\neq\sigma \text{ at } l=0}} \frac{1}{\frac{2\pi l}{\beta} - \frac{\lambda_\alpha}{\beta} + \frac{\lambda_\sigma}{\beta}} \bar{\psi}^{\alpha}_{-m}(\vec{x})\gamma^1\psi_{\sigma,m-l}(\vec{x})\bar{\psi}^{\sigma}_{-n}(\vec{x})\gamma^2\psi_{\alpha,n+l}(\vec{x}). \tag{11}$$

The spatial covariant derivative $\tilde{D}$ in (11) is taken in the background of the gauge fields corresponding to the constant fluxes in the $U(1)^N$.

Finite temperature gap equation

Following the Schwinger–Dyson procedure, the finite temperature gap equation is obtained as,

$$\Sigma_T(P_{m\alpha,3}) = -\frac{1}{2\kappa\beta R^2}\sum_{n,\sigma}\frac{1}{P_{m\alpha,3}-Q_{n\sigma,3}}\,H\left(\sum_{j=|q_\sigma|-\frac{1}{2}}^{\infty}\frac{2j+1}{i\not{Q}_{n\sigma j}+M+\Sigma_T(Q_{n\sigma,3})}\right), \qquad (12)$$

where $H(C) \equiv \gamma^1 C\gamma^2 - \gamma^2 C\gamma^1$, $P_{m\alpha} = \left(\vec{p}, \frac{2\pi m - \lambda_\alpha}{\beta}\right)$, $q_\alpha = \frac{n_\alpha}{2}$, and,

$$\not{Q}_{m\alpha j} = \begin{pmatrix} Q_{m\alpha,3} & \frac{1}{R}\sqrt{j(j+1)-\left(q_\alpha+\frac{1}{2}\right)\left(q_\alpha-\frac{1}{2}\right)} \\ \frac{1}{R}\sqrt{j(j+1)-\left(q_\alpha+\frac{1}{2}\right)\left(q_\alpha-\frac{1}{2}\right)} & -Q_{m\alpha,3} \end{pmatrix}. \qquad (13)$$

In close analogy, for the flat space case, we obtain,

$$\Sigma_T(P_{n\sigma,3}) = -\frac{2\pi}{\kappa\beta}\sum_{\substack{m\in\mathbb{Z}+\frac{1}{2} \\ \alpha \\ \alpha\neq\sigma \text{ at } m=n}}\int\frac{\mathrm{d}^2q}{(2\pi)^2}\frac{1}{P_{n\sigma,3}-Q_{m\alpha,3}}\,H\left(\frac{1}{i\not{Q}_{m\alpha}+M+\Sigma_T(Q_{m\alpha,3})}\right). \qquad (14)$$

Solution of the finite temperature gap equation

(14) is solved component-wise,

$$\Sigma_{T,I}(P_{m\alpha,3}) = P_{m\alpha,3}\,\sin\left(\frac{4\pi}{\kappa}\Omega_T(M_T, P_{m\alpha,3})\right) + M_T\,\cos\left(\frac{4\pi}{\kappa}\Omega_T(M_T, P_{m\alpha,3})\right), \tag{15}$$

$$\Sigma_{T,3}(P_{m\alpha,3}) = P_{m\alpha,3}\left(\cos\left(\frac{4\pi}{\kappa}\Omega_T(M_T, P_{m\alpha,3})\right) - 1\right) - M_T\,\sin\left(\frac{4\pi}{\kappa}\Omega_T(M_T, P_{m\alpha,3})\right). \tag{16}$$

where,

$$\Omega_T(M_T, P_{m\alpha,3}) = \frac{1}{4\pi\beta R^2}\sum_{\substack{n\in\mathbb{Z}+\frac{1}{2}\\ \sigma\\ \sigma\neq\alpha \text{ at } n=m}}\sum_{j=|q_\sigma|-\frac{1}{2}}^{\infty}\frac{2j+1}{(Q_{n\sigma,3})^2 + \frac{j(j+1)-\left(q_\sigma-\frac{1}{2}\right)\left(q_\sigma+\frac{1}{2}\right)}{R^2} + M_T^2}\frac{1}{P_{m\alpha,3} - Q_{n\sigma,3}}, \tag{17}$$

$$\Phi_T(M_T) = \frac{1}{4\pi\beta R^2}\sum_{\substack{\sigma\\ n\in\mathbb{Z}+\frac{1}{2}}}\sum_{j=|q_\sigma|-\frac{1}{2}}^{\infty}\frac{2j+1}{(Q_{n\sigma,3})^2 + \frac{j(j+1)-\left(q_\sigma-\frac{1}{2}\right)\left(q_\sigma+\frac{1}{2}\right)}{R^2} + M_T^2}, \tag{18}$$

and the thermal mass M_T obeys the mass gap equation,

$$M_T = M - \frac{4\pi}{\kappa}\Phi_T(M_T). \tag{19}$$

Evaluation of thermal free energy

For the flat space case, subtracting the zero point energy to remove the UV-divergences, we get the holonomy-dependent thermal free energy functional $S_{\rm eff}$ as,

$$S_T - S_0 = -\frac{4\pi\beta V_2}{\kappa}\left(\Phi_T(M_T)\right)^2\left(\frac{4\pi}{3\kappa}\Phi_T(M_T) + M_T\right) + \frac{4\pi\beta V_2}{\kappa}\left(\Phi_0(M_0)\right)^2\left(\frac{4\pi}{3\kappa}\Phi_0(M_0) + M_0\right)$$
$$-V_2\sum_{\substack{\alpha \\ n\in\mathbb{Z}+\frac{1}{2}}}\int\frac{\mathrm{d}^2 q}{(2\pi)^2}\ln\left((Q_{n\alpha})^2 + M_T^2\right) + V_2\beta N\int\frac{\mathrm{d}^3 q}{(2\pi)^3}\ln\left(q^2 + M_T^2\right) - \frac{V_2 N}{6\pi\beta^2}\left((\beta M_0)^3 - (\beta M_T)^3\right). \tag{20}$$

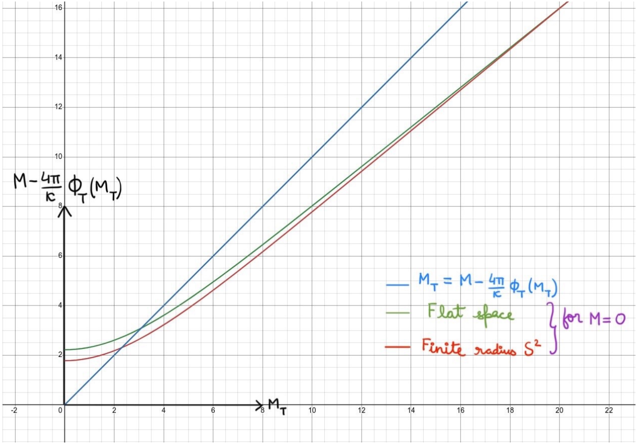
$\Phi_T(M_T)$ is a simple explicit function for the flat space case,

$$\Phi_T(M_T) = -\frac{1}{4\pi\beta}\sum_{\sigma}\ln\left|2(\cosh(\beta M_T) + \cos(\lambda_\sigma))\right|. \tag{21}$$

Using the finite temperature and finite volume analogues of the **'symmetrization' identities** of [MZJ15], it is possible to evaluate all the integrals/summations in (20) in the large-N limit.

$\Phi_T(M_T)$ for the finite volume case

$$\Phi_T(M_T) = \frac{1}{4\pi\beta R^2} \sum_{\substack{\sigma \\ n\in\mathbb{Z}+\frac{1}{2}}} \sum_{j=|q_\sigma|-\frac{1}{2}}^{\infty} \frac{2j+1}{(Q_{n\sigma,3})^2 + \frac{j(j+1)-\left(q_\sigma-\frac{1}{2}\right)\left(q_\sigma+\frac{1}{2}\right)}{R^2} + M_T^2}. \tag{22}$$



Discussion and future directions

- The thermal effective action is not independent of fluxes at finite volume - holonomy eigenvalues are not quantized - Bose–Fermi duality has to work in a mathematically different way (compared to flat space);
- At finite volume, the fermion determinant receives flux contributions from different sectors, and the free energy will be presented in terms of a sum over these fluxes (in addition to the summation over Kaluza–Klein momenta and the spin-weighted spherical harmonic labels), which is seemingly similar to the calculations of the superconformal index [MMPS22];
- First-principles computation of S-matrices [MMP$^+$22] may be possible with our choice of gauge that could shed new light on their unusual crossing symmetry properties;
- Computation of the free energy for bosons has some subtleties related to the $\phi^2 A^2$ coupling.

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