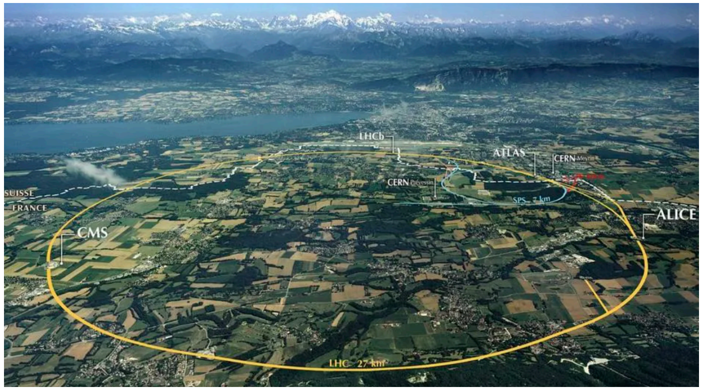
Categorical Symmetries and Scattering Amplitudes

Shota Komatsu

CERN

Based on **2403.04835** + WIP with Christian Copetti (Oxford) & Lucía Córdova (CERN)

CERN



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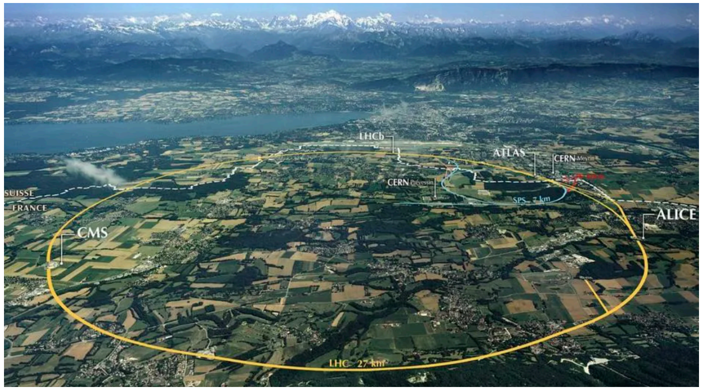
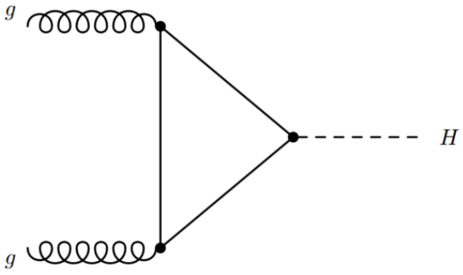
CERN



- I work at CERN.
- Accelerate particles to high energy, collide them and measure the outcome.
- Compare against theoretical predictions.
- Theoretical prediction: Scattering Amplitudes (S-matrix)

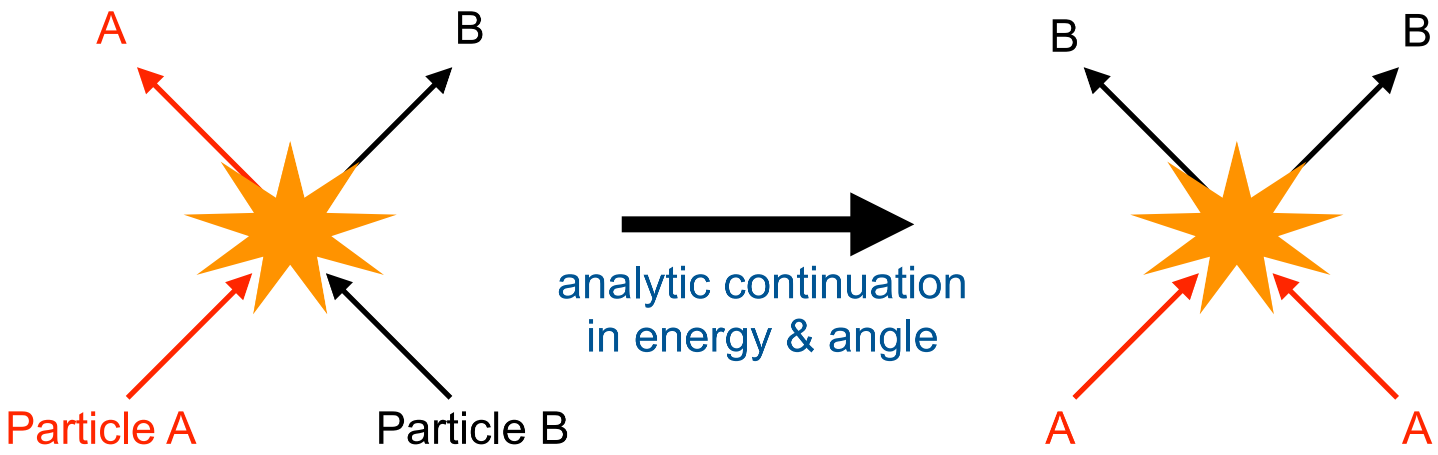


Feynman diagram:
compute probability of a given outcome

Crossing symmetry

- Scattering amplitudes exhibit an interesting property.

- **Crossing symmetry**



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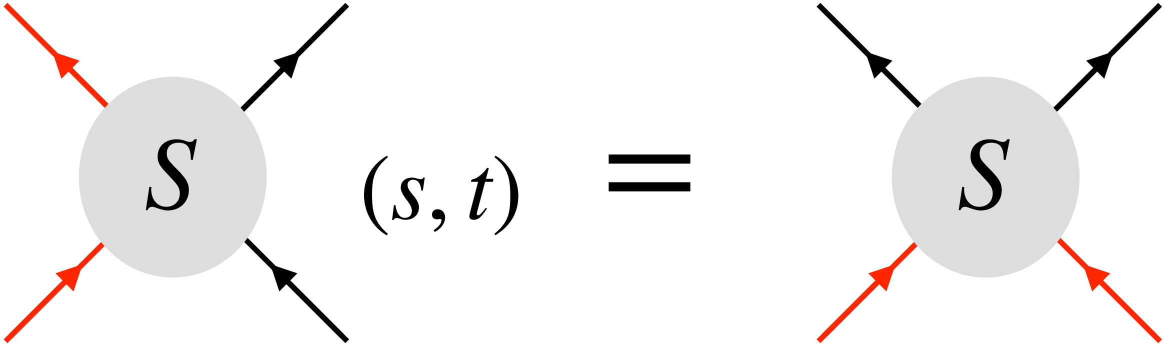

$$S(s,t) = S(t,s)$$

$s \sim$ energy $\quad t \sim$ angle

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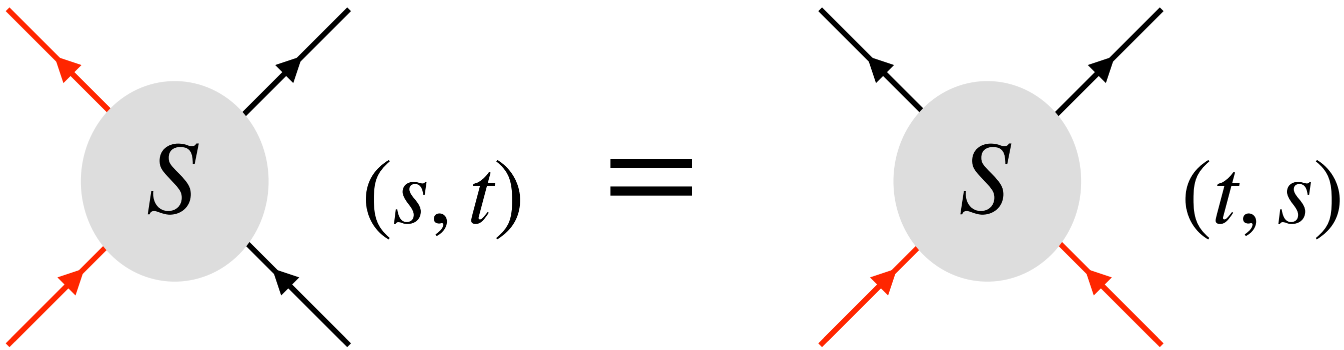


$s \sim$ energy $\quad t \sim$ angle

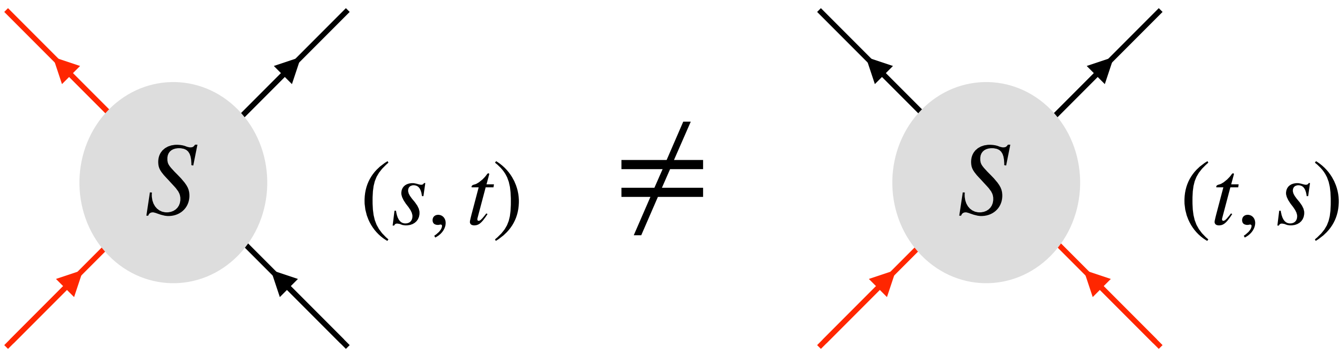
- Useful for the computation.

- First found through the computation (~50's)

- Rigorous proof in some cases. [Bros, Epstein, Glaser], ….., [Mizera]

Punchline of the talk

In the presence of certain* categorical symmetries, crossing symmetry of S-matrix is modified.



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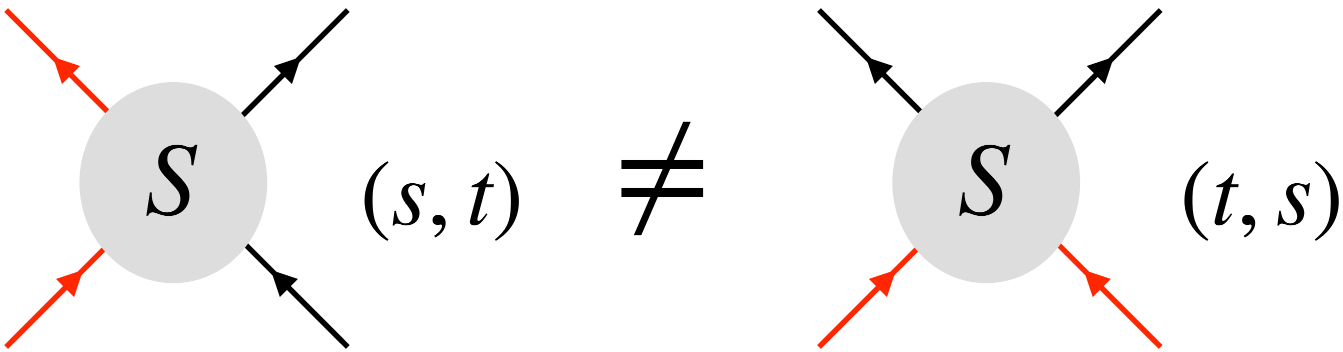

- Theories in 1+1 dimensions. (Comments on higher d at the end)
- Use integrable theories to check the claim but applies to non-integrable as well.

Plan

1. Categorical symmetries in 1+1 dim

2. Integrable flow from tricritical Ising and S-matrix

3. Derivation of modified crossing rules

4. Conclusion

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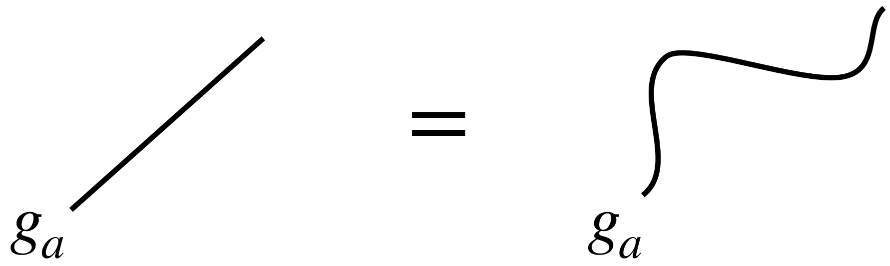
Categorical Symmetries in 1+1 dim

- Symmetry generators in QFT: (d-1)-dim topological operators

[Gaiotto, Kapustin, Seiberg, Willet]



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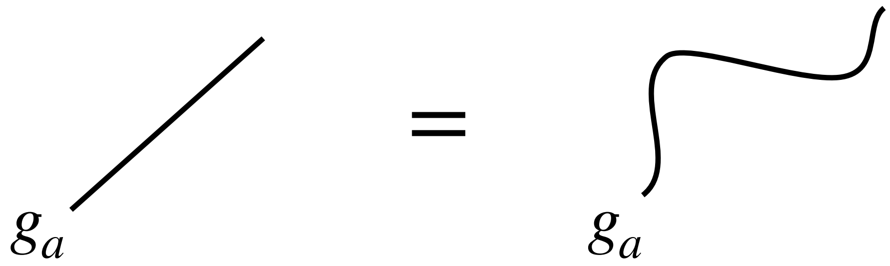

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- Categorical symmetries (in 1+1 dim):

Fusion category $$\mathscr{L}_a \mathscr{L}_b = \sum_{\mathscr{L}_c} N_{ab}{}^c \mathscr{L}_c \qquad (N_{ab}{}^c \in \mathbb{Z}_{\geq 0})$$

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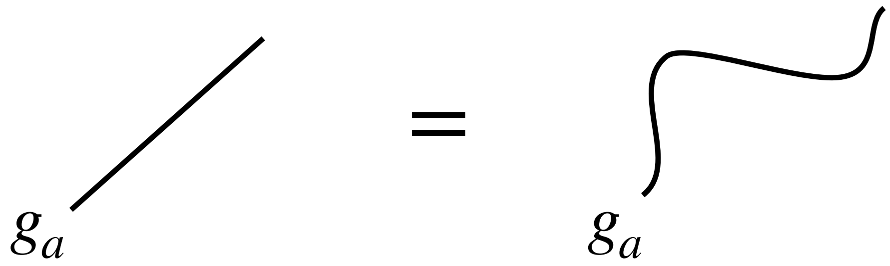

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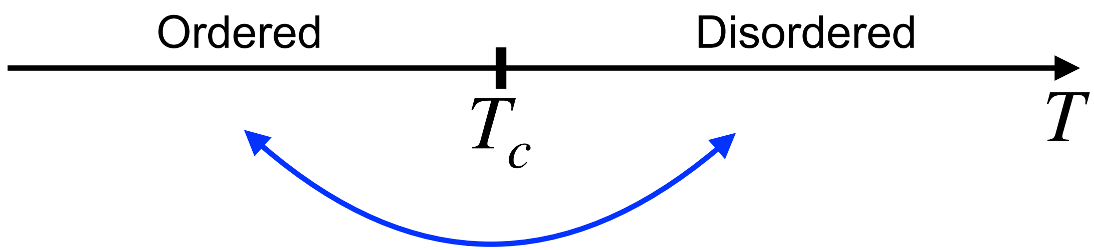
- Simplest 1+1-d system with non-invertible symmetry:

Ising model at critical temperature (Ising CFT)

Non-Invertible Kramers-Wannier

- 2d Ising model: $$Z = \sum_{\{\sigma\}} \exp\left[\beta \sum_{<ij>} \sigma_i \sigma_j\right]$$



Kramers-Wannier duality $$e^{-2\beta_{\text{low}}} \leftrightarrow \tanh \beta_{\text{high}}$$

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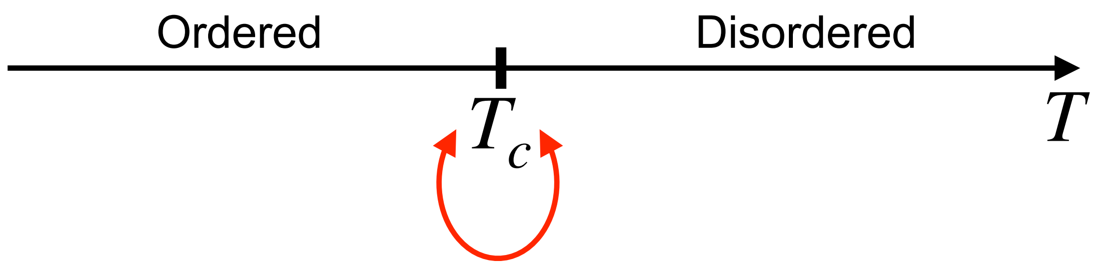

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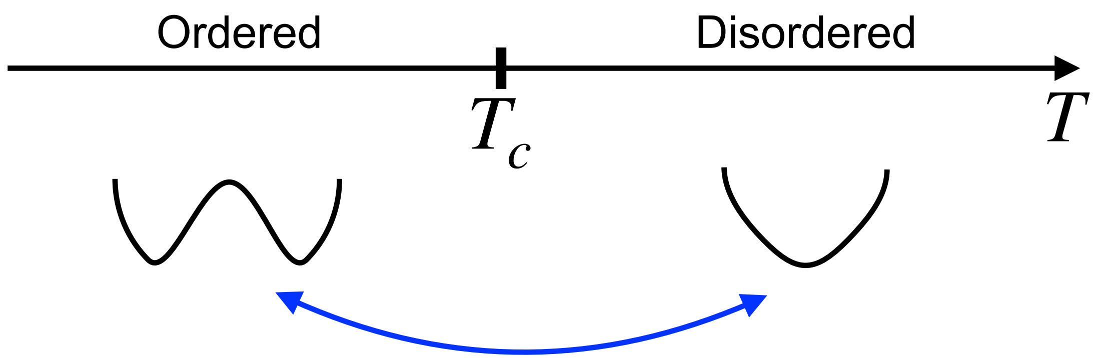

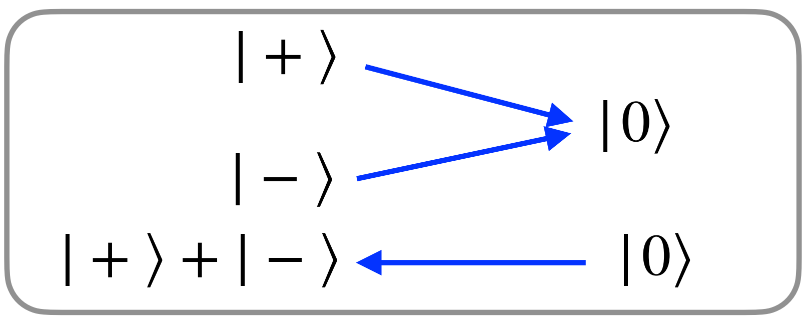

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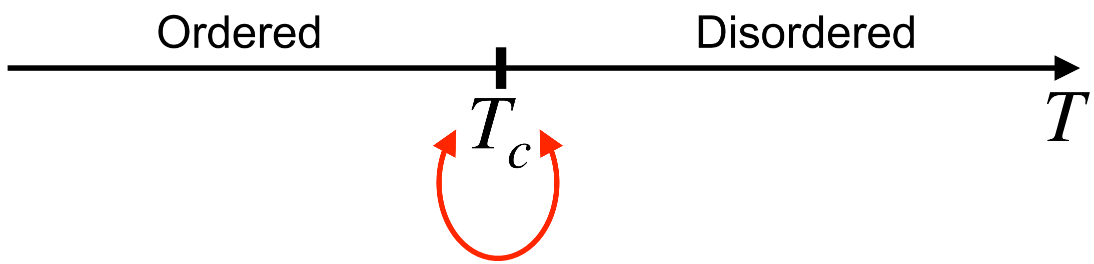
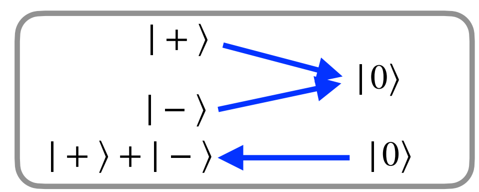


Kramers-Wannier **symmetry** $\mathcal{N}$

- Symmetry of 2d Ising CFT

$$\eta^2 = 1\,, \qquad \mathcal{N}^2 = 1 + \eta\,, \qquad \mathcal{N}\eta = \eta\mathcal{N} = \mathcal{N}$$



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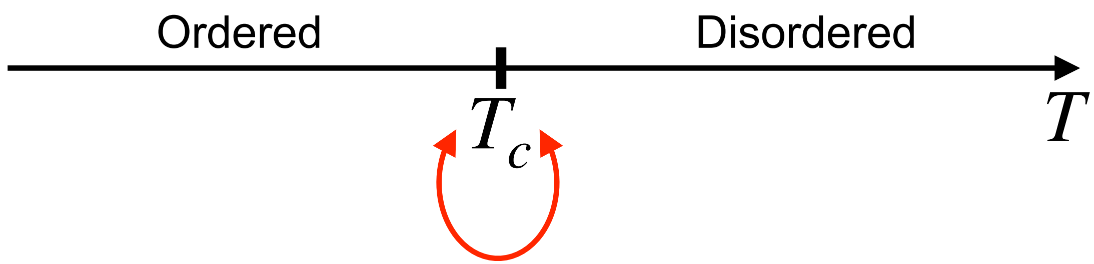

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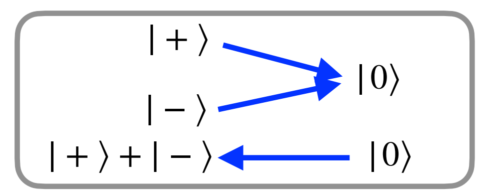

- In Ising CFT, scattering amplitudes are trivial: $S(s,t) = -1$
- KW symmetry broken by (relevant) deformation
- Nontrivial theory preserving KW symmetry…?

Flow from tricritical Ising

- Ising CFT is the simplest unitary minimal model ($\mathscr{M}_{3,4}$).

- Next simplest is tricritical Ising CFT ($\mathscr{M}_{4,5}$).

- Categorical symmetric deformation of tricritical: [Chang, Lin, Shao, Wang, Yin]

Tricritical Ising

$\downarrow$ $+\phi_{1,3}$

Ising

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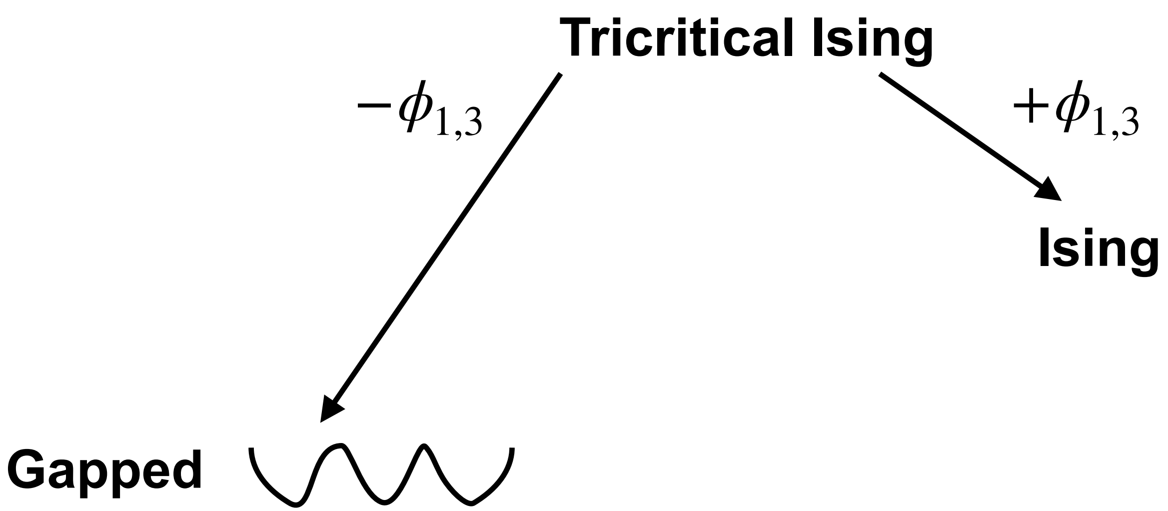

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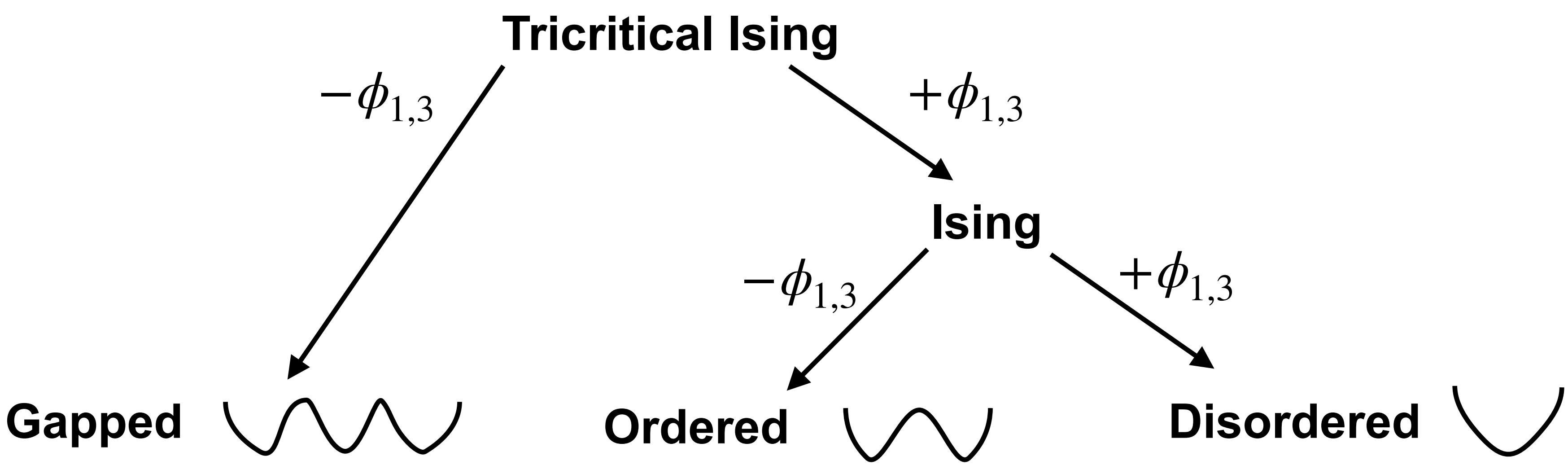
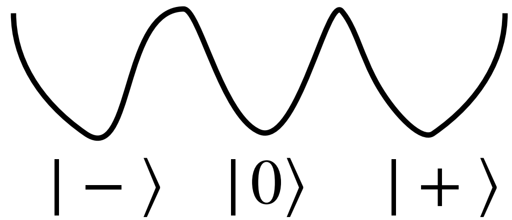


Action on vacua



- $\mathbb{Z}_2$-defect exchanges $|+\rangle$ and $|-\rangle$

$$\eta : |+\rangle \leftrightarrow |-\rangle$$

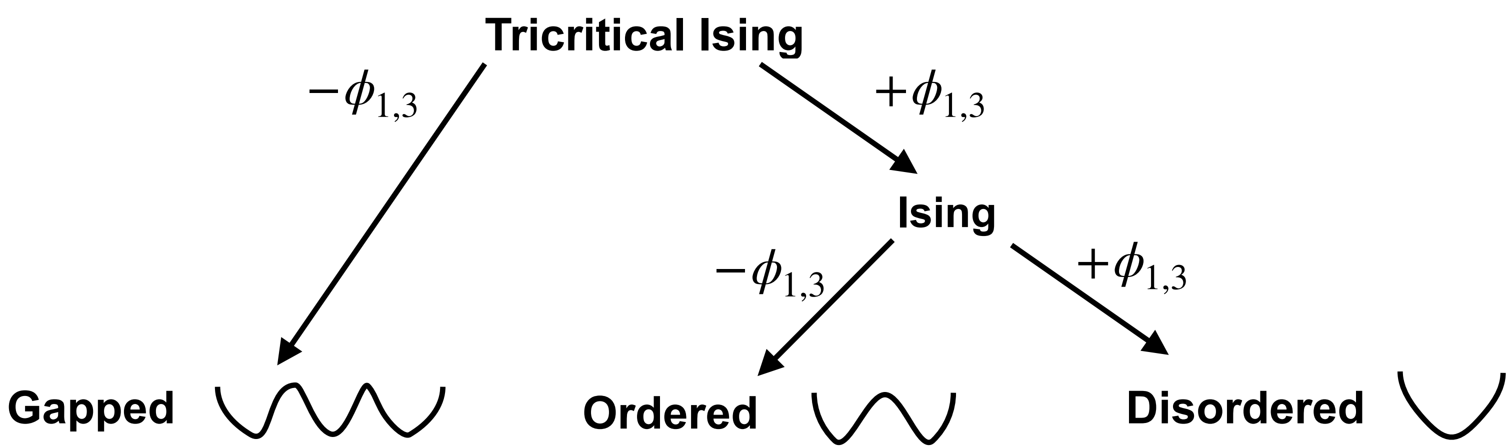
- $\mathcal{N}$-defect:

$$\mathcal{N}|0\rangle = |+\rangle + |-\rangle$$

$$\mathcal{N}|+\rangle = \mathcal{N}|-\rangle = |0\rangle$$

- “Superposition” of disordered and ordered vacua in Ising

Remarks



- 3 is the minimal number of vacua allowed by categorical sym.

$$\mathbb{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad \mathcal{N} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} , \quad \eta = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\eta^2 = 1 , \qquad \mathcal{N}^2 = 1 + \eta , \qquad \mathcal{N}\eta = \eta\mathcal{N} = \mathcal{N}$$

Entries need to be non-negative integers

- Similar pattern persists for higher minimal models

$\mathscr{M}_{n,n+1} - \phi_{1,3} \to$ gapped with (n-1) vacua

$\mathscr{M}_{n,n+1} + \phi_{1,3} \to \mathscr{M}_{n-1,n}$

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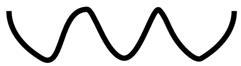
Integrability along the flow

Tricritical Ising $\xrightarrow{-\phi_{1,3}}$ 

- At UV CFT fixed point, there exist ∞ many higher spin charges.

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  [Zamolodchikov 1989]

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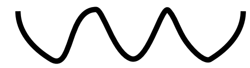
Tricritical Ising $\xrightarrow{-\phi_{1,3}}$ 

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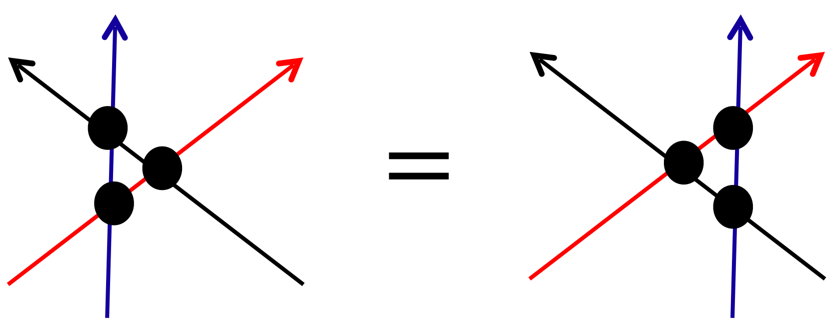
  [Zamolodchikov 1989]
- Multi-particle S-matrix factorizes to 2→2 S-matrices.
  [Shankar, Witten] [Parke]
- 2→2 S-matrices satisfy the Yang-Baxter equation.



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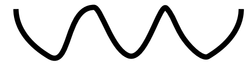
Tricritical Ising $\xrightarrow{-\phi_{1,3}}$ 

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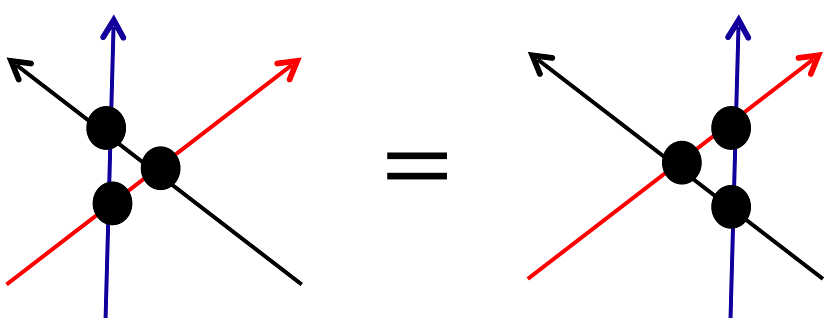
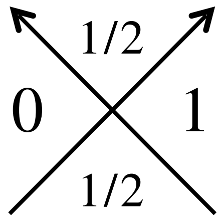


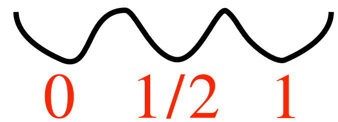
- Imposing unitarity, crossing & YB, the S-matrix can be almost uniquely “**bootstrapped**”.
  [Zamolodchikov 1991]

S-matrix by Zamolodchikov

- Particles in IR = kinks interpolating between adjacent vacua.



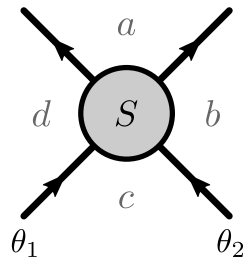
etc .



*Warning: Change of notation!

- It also depends on total energy: $s = (p_1 + p_2)^2 = 4m^2 \cosh^2(\theta/2)$

$S_{dc}^{ab}(\theta) =$



$$\theta = \theta_1 - \theta_2$$

$$s + t = 4m^2 \qquad t = -4m^2 \sinh^2(\theta/2)$$

S-matrix by Zamolodchikov

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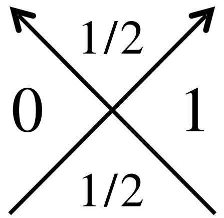

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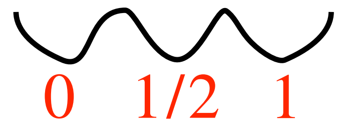

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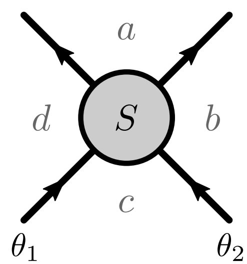
$S^{ab}_{dc}(\theta) =$ 

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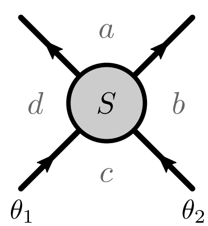
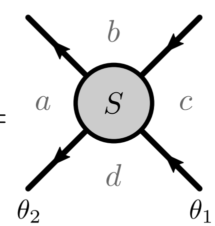
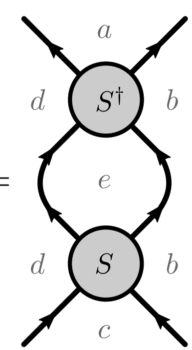
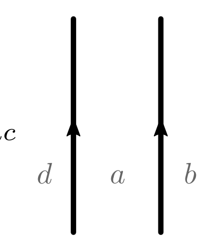
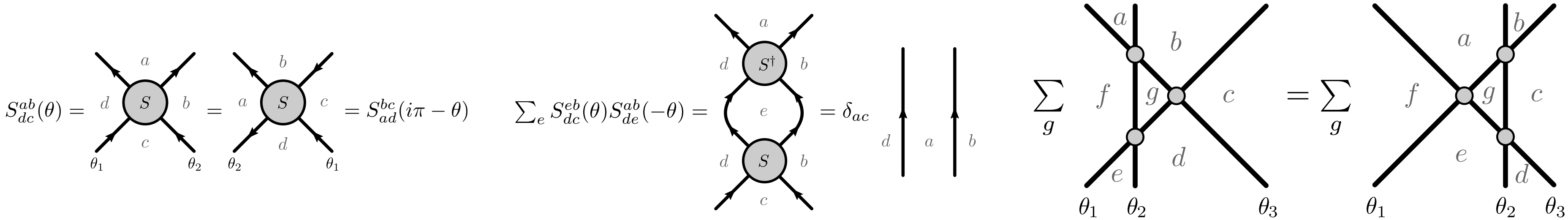
- Crossing symmetry & unitarity

$S^{ab}_{dc}(\theta) =$  $=$  $= S^{bc}_{ad}(i\pi - \theta)$

$\sum_e S^{eb}_{dc}(\theta) S^{ab}_{de}(-\theta) =$  $= \delta_{ac}$ 

S-matrix by Zamolodchikov

- Imposing crossing & unitarity & YB,



we can fix S-matrix uniquely (up to overall "CDD" factor) as

$$\widehat{S}^{ab}_{dc}(\theta) = Z(\theta) \left(\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}\right)^{\frac{i\theta}{2\pi}} \left[\sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}} \sinh\left(\frac{\theta}{n}\right) \delta_{bd} + \sinh\left(\frac{i\pi - \theta}{n}\right) \delta_{ac}\right]$$

$$(n = 3, \quad \mathrm{d}_0 = \mathrm{d}_1 = 1, \quad \mathrm{d}_{1/2} = \sqrt{2})$$

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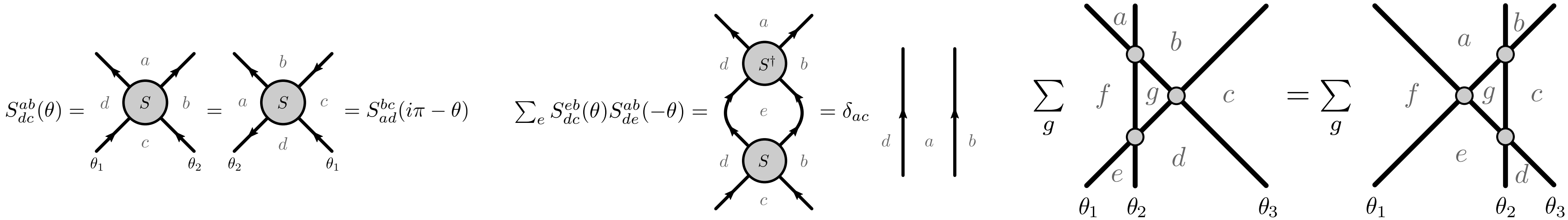

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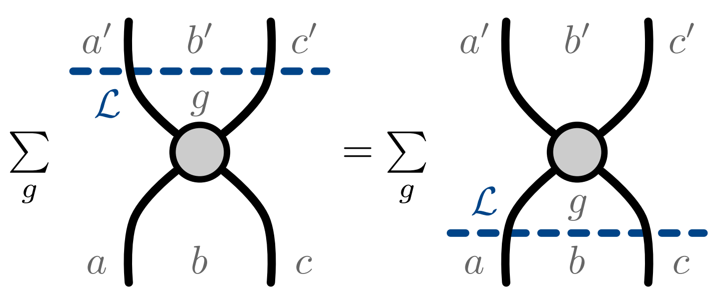
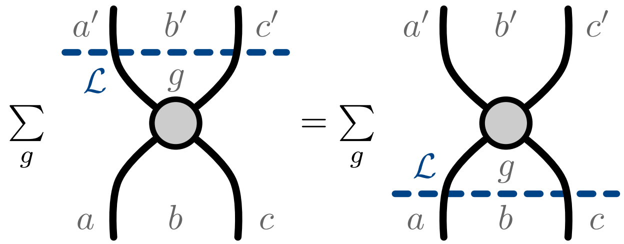
- We expect that it also preserves categorical symmetries:



?

Paradox



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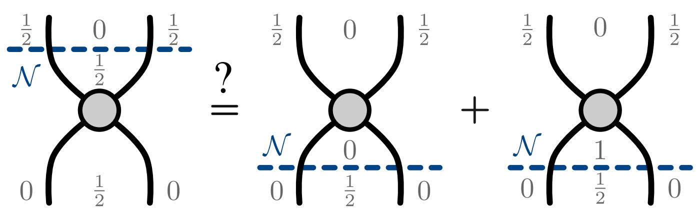
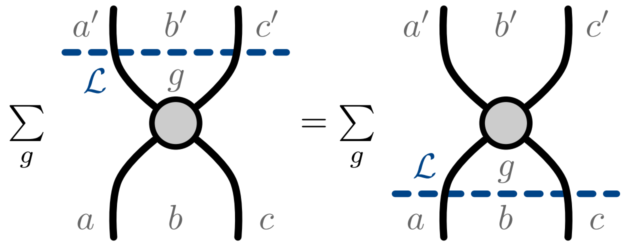
- We found that $\widehat{S}$ commutes with η ($\mathbb{Z}_2$), but not with $\mathcal{N}$.



$$\mathcal{N}: \quad S^{\frac{1}{2}\,0}_{0\,\frac{1}{2}}(\theta) \overset{?}{=} S^{0\,\frac{1}{2}}_{\frac{1}{2}\,0}(\theta) + S^{0\,\frac{1}{2}}_{\frac{1}{2}\,1}(\theta)$$

Paradox



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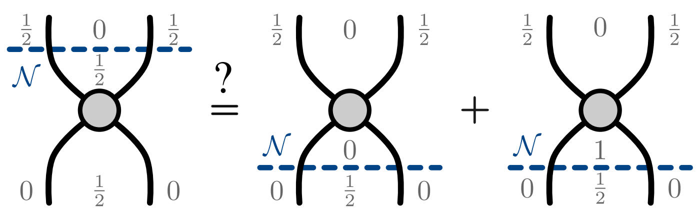
$$\widehat{S}^{ab}_{dc}(\theta) = Z(\theta)\left(\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}\right)^{\frac{i\theta}{2\pi}}\left[\sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}}\,\sinh\left(\frac{\theta}{n}\right)\delta_{bd} + \sinh\left(\frac{i\pi-\theta}{n}\right)\delta_{ac}\right]$$

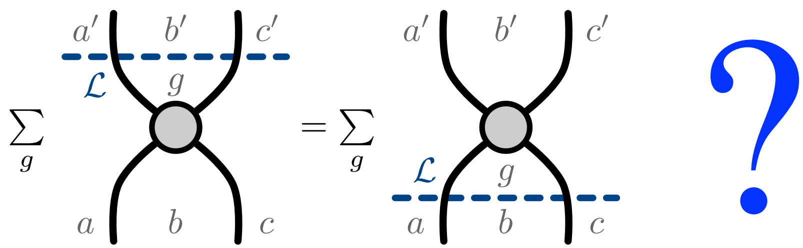
- We found that $\widehat{S}$ commutes with η ($\mathbb{Z}_2$), but not with $\mathcal{N}$.



$$\mathcal{N}: \quad S^{\frac{1}{2}0}_{0\frac{1}{2}}(\theta) \overset{?}{=} S^{0\frac{1}{2}}_{\frac{1}{2}0}(\theta) + S^{0\frac{1}{2}}_{\frac{1}{2}1}(\theta)$$

- The following 4 properties are mutually incompatible.
  - Unitarity
  - Crossing
  - Integrability (YB)
  - Categorical symmetry

Paradox



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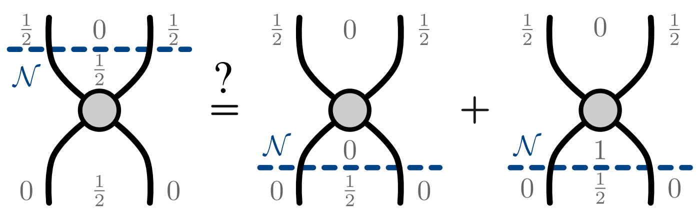

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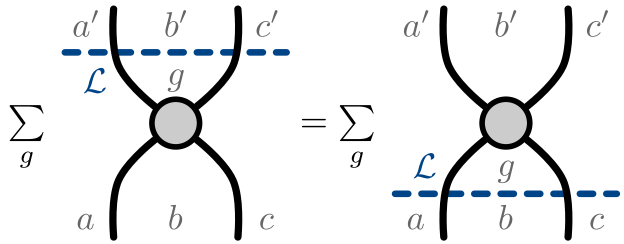

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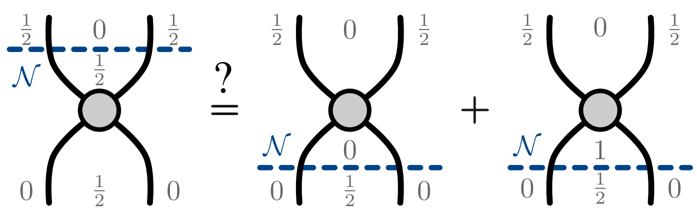

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  - Integrability (YB) ✓ deformation of CFT
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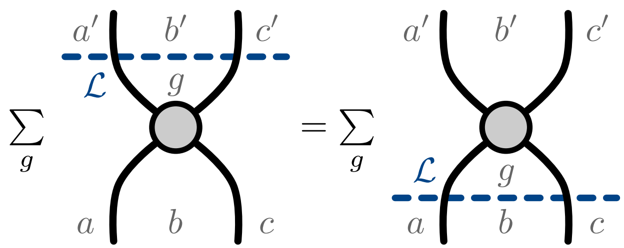
Paradox



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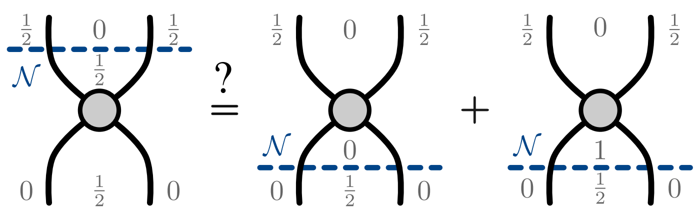

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- The following 4 properties are mutually incompatible.
  - Unitarity ✓ basic principle
  - Crossing
  - Integrability (YB) ✓ deformation of CFT
  - Categorical symmetry ✓ deformation of CFT
- The only viable option is to give up crossing.

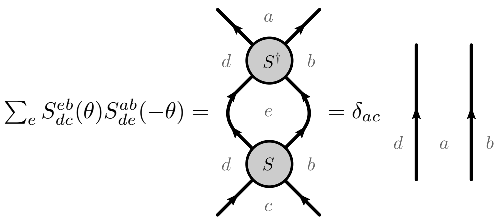
New proposal

$$S^{ab}_{dc}(\theta) = Z(\theta)\left[\sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}}\ \sinh\left(\frac{\theta}{n}\right)\ \delta_{bd} + \sinh\left(\frac{i\pi-\theta}{n}\right)\ \delta_{ac}\right]$$

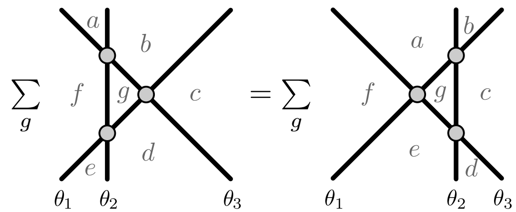
$$\widehat{S}^{ab}_{dc}(\theta) = Z(\theta)\left(\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}\right)^{\frac{i\theta}{2\pi}}\left[\sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}}\ \sinh\left(\frac{\theta}{n}\right)\ \delta_{bd} + \sinh\left(\frac{i\pi-\theta}{n}\right)\ \delta_{ac}\right]$$

New proposal

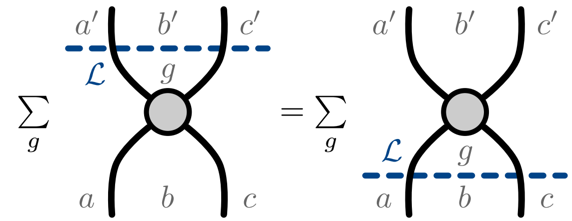
$$S_{dc}^{ab}(\theta) = Z(\theta)\left[\sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}}\ \sinh\left(\frac{\theta}{n}\right)\ \delta_{bd} + \sinh\left(\frac{i\pi - \theta}{n}\right)\ \delta_{ac}\right]$$



Unitarity



Yang-Baxter



Categorical

New proposal

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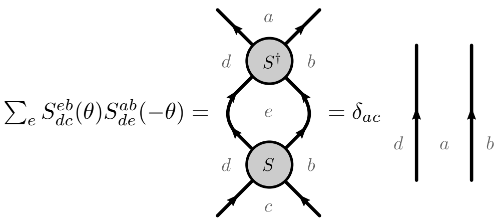
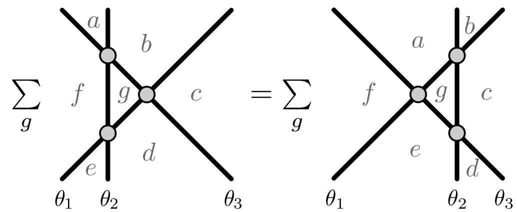


Unitarity



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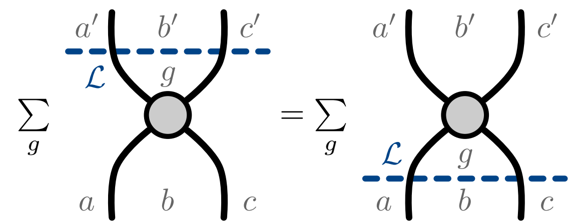


Categorical

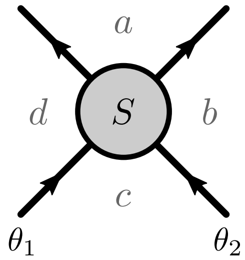
- Crossing symmetry is modified:

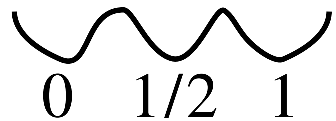
$$S_{dc}^{ab}(\theta) = \sqrt{\frac{\mathrm{d}_a\mathrm{d}_c}{\mathrm{d}_b\mathrm{d}_d}}\,S_{ad}^{bc}(i\pi-\theta)$$

- Physical origin? (The topic of the rest of the talk)

1. Non-invertible symmetries in 1+1 dim
2. Integrable flow from tricritical Ising and S-matrix
3. Derivation of modified crossing rules
4. Conclusion

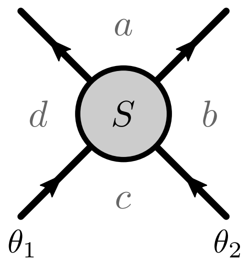
Key physical input

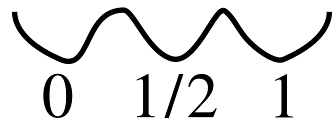
$S_{dc}^{ab}(\theta) =$ 



- In the IR, the action of a kink on vacua = the action of symmetry line $\mathcal{N}\,(=: v)$

Key physical input

$S^{ab}_{dc}(\theta) =$ 



- In the IR, the action of a kink on vacua = the action of symmetry line $\mathcal{N} (=: v)$

- The vacua are in 1-to-1 correspondence with symmetry lines.

$$|0\rangle \leftrightarrow \mathbf{1} \qquad |1/2\rangle \leftrightarrow \mathcal{N} \qquad |1\rangle \leftrightarrow \eta$$

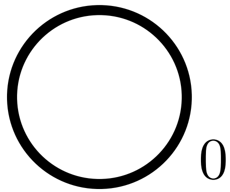
(Regular representation of fusion category)

- All the vacua can be obtained from $|0\rangle$ by acting symmetry lines.

$$\mathcal{N}|0\rangle = |1/2\rangle \qquad \eta|0\rangle = |1\rangle$$

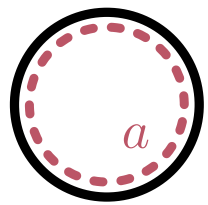
Warm up: action of symmetry line

- Consider a path integral on a large disk with BC “ 0 ”

$$\langle\langle 0|0\rangle\rangle =$$  $$= 1$$

(State in open Hilbert space of TQFT)

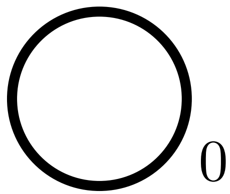
- Other vacua can be obtained by acting symmetry lines.

$$|a\rangle\rangle =$$  $,\quad \langle\langle a|a\rangle\rangle =$  $$= \mathrm{d}_a$$

$$\langle \mathcal{L}_a \rangle =$$  $$= \mathrm{d}_a$$ quantum dim

Warm up: action of symmetry line

- Consider a path integral on a large disk with BC “ 0 ”

$\langle\langle 0|0\rangle\rangle =$  $= 1$ (State in open Hilbert space of TQFT)

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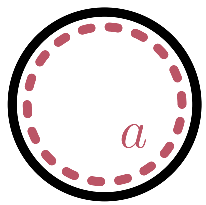
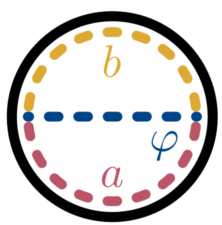
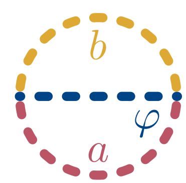
$|a\rangle\rangle =$  , $\langle\langle a|a\rangle\rangle =$  $= \mathrm{d}_a$

$\langle \mathcal{L}_a \rangle =$  $= \mathrm{d}_a$ quantum dim

- Matrix element of symmetry line between normalized states

$$\frac{\langle\langle b|\mathcal{L}_\varphi|a\rangle\rangle}{\sqrt{\langle\langle a|a\rangle\rangle\langle\langle b|b\rangle\rangle}} = \frac{1}{\sqrt{\mathrm{d}_a \mathrm{d}_b}}$$ 

 $= \sqrt{\mathrm{d}_a \mathrm{d}_b \mathrm{d}_\varphi}\, N_{\varphi\, a}{}^{b}$

Warm up: action of symmetry line

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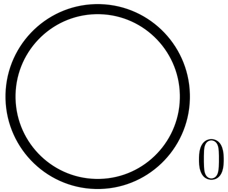
$\langle\!\langle 0|0\rangle\!\rangle =$  $= 1$ (State in open Hilbert space of TQFT)

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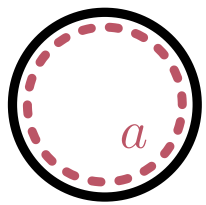
$|a\rangle\!\rangle =$  , $\langle\!\langle a|a\rangle\!\rangle =$  $= \mathrm{d}_a$

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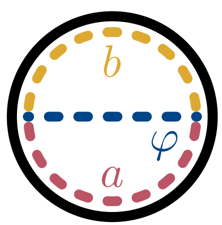
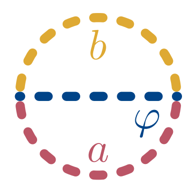
$$\frac{\langle\!\langle b|\mathcal{L}_\varphi|a\rangle\!\rangle}{\sqrt{\langle\!\langle a|a\rangle\!\rangle\langle\!\langle b|b\rangle\!\rangle}} = \frac{1}{\sqrt{\mathrm{d}_a\mathrm{d}_b}}$$ 

 $= \sqrt{\mathrm{d}_a\mathrm{d}_b\mathrm{d}_\varphi}\, N_{\varphi\, a}{}^{b}$

- To correctly realize the symmetry algebra, we need "renormalization"

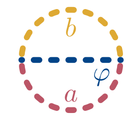
$$\hat{\mathcal{L}}_\varphi = \frac{1}{\sqrt{\mathrm{d}_\varphi}}$$ 

Warm up: action of symmetry line

- Now consider a state with a kink interpolating a and b vacua

$|a;b\rangle\rangle =$ 

$$\langle\langle a;b\,|\,a;b\rangle\rangle = \sqrt{\mathrm{d}_a \mathrm{d}_b \mathrm{d}_v}$$



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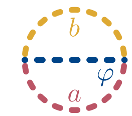
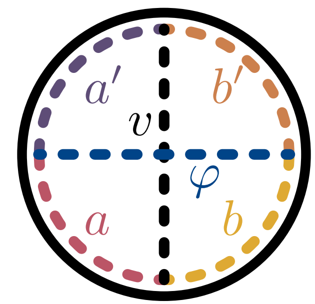


$= \sqrt{\mathrm{d}_a\mathrm{d}_b\mathrm{d}_\varphi}\, N_{\varphi\, a}{}^{b}$

- Matrix element between single-kink states:

$$\frac{\langle\langle a';b'\,|\hat{\mathcal{L}}_\varphi|\,a;b\rangle\rangle}{\sqrt{\langle\langle a;b|a;b\rangle\rangle\,\langle\langle a';b'|a';b'\rangle\rangle}} = \left(\mathrm{d}_a\mathrm{d}_{a'}\mathrm{d}_b\mathrm{d}_{b'}\mathrm{d}_v^2\mathrm{d}_\phi^2\right)^{-\frac{1}{4}}$$



$$= \left(\mathrm{d}_a\mathrm{d}_{a'}\mathrm{d}_b\mathrm{d}_{b'}\right)^{1/4}\begin{bmatrix}\varphi & a' & a\\ v & b & b'\end{bmatrix}$$

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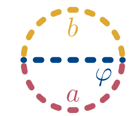

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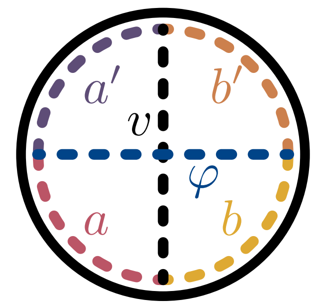


$$= \left(\mathrm{d}_a \mathrm{d}_{a'} \mathrm{d}_b \mathrm{d}_{b'}\right)^{1/4} \begin{bmatrix} \varphi & a' & a \\ v & b & b' \end{bmatrix}$$

F-symbol

- Our S-matrix commutes with this action of non-invertible symmetries.

Modified crossing for S-matrix

- The IR dynamics is described by a nontrivial TQFT.
- Normalizations of in- and out-states are corrected by TQFT.
- Corrections depend on the channels we consider.

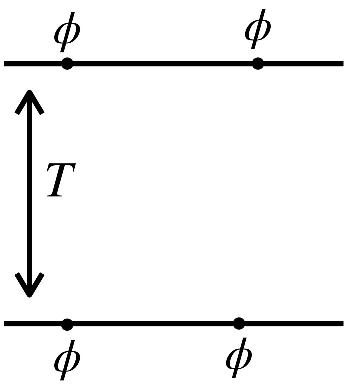
Modified crossing for S-matrix

- Kink creation operators are generally non-local.

Unclear how to get S-matrix from LSZ reduction.

- Use alternative (discussed in Itzykson-Zuber Ch.5)

$$S(\{p_i\}) = \lim_{T\to\infty} \int \prod_j dv_j e^{ip_j x(v_j)} \prod_k (n_k \cdot \overleftrightarrow{\partial}_{x_k}) G(\{x(v_k)\})$$



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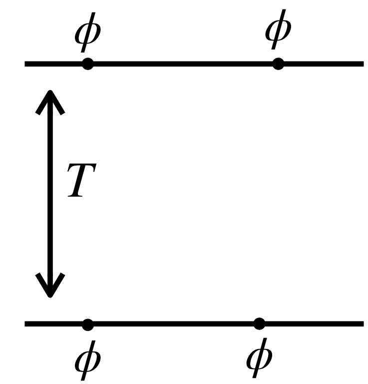
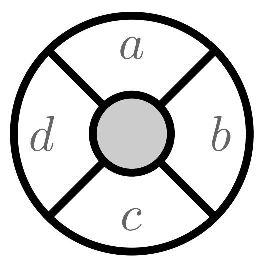
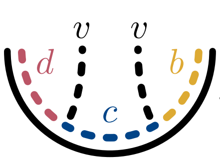


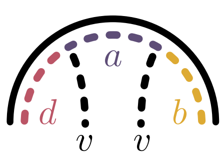
- In our case,

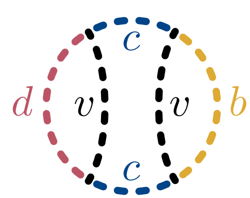
$$S^{ab}_{dc}(\theta) \propto$$ 

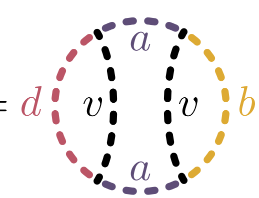
$\Big|_{\text{analyt. cont.}}$

- But we need to take into account normalization.

$|\text{in}\rangle\rangle =$ 

, $|\text{out}\rangle\rangle =$ 

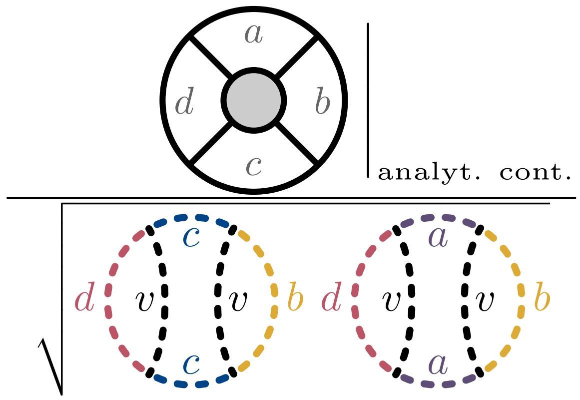
$\langle\langle \text{in}|\text{in}\rangle\rangle =$ 

$\langle\langle \text{out}|\text{out}\rangle\rangle =$ 

Modified crossing for S-matrix

- Thus the correct expression should be

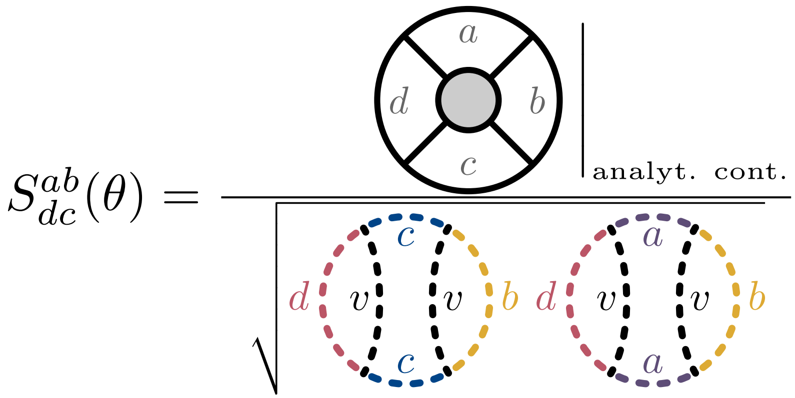
$$S_{dc}^{ab}(\theta) =$$



- Numerator : disk correlation function, crossing symmetric.
- Denominator : depend on the channel we consider.

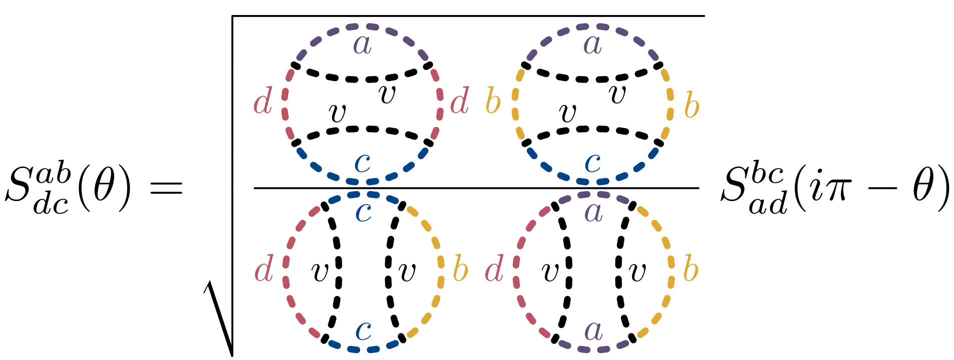
Modified crossing for S-matrix

- Thus the correct expression should be



- Numerator : disk correlation function, crossing symmetric.
- Denominator : depend on the channel we consider.

- Modified crossing:



$$S^{ab}_{dc}(\theta) = \sqrt{\frac{\mathrm{d}_a \mathrm{d}_c}{\mathrm{d}_b \mathrm{d}_d}}\, S^{bc}_{ad}(i\pi - \theta)$$

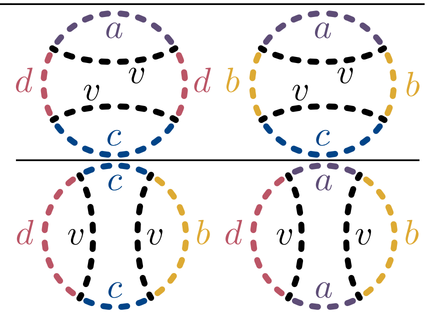
1. Non-invertible symmetries in 1+1 dim

2. Integrable flow from tricritical Ising and S-matrix

3. Derivation of modified crossing rules

4. Conclusion

Conclusion

$S^{ab}_{dc}(\theta) = \sqrt{}$  $S^{bc}_{ad}(i\pi - \theta)$

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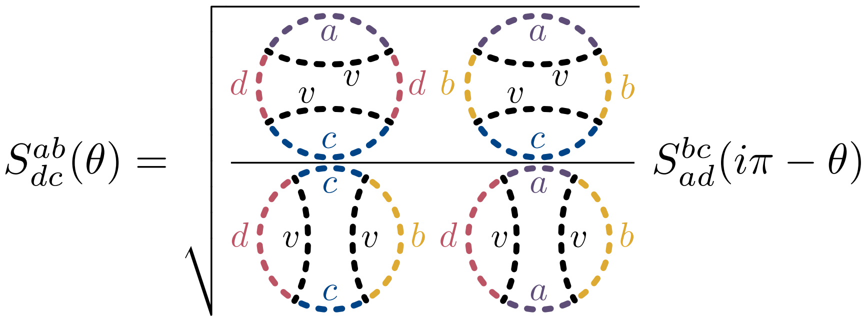
- In RG flows to gapped phase in 1+1 dim, non-invertible symmetries and anomalies lead to modified crossing symmetry.

- Physically, it comes from corrections to norms of in- and out-states due to TQFT dynamics.

Conclusion



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- Many other examples: $\phi_{1,3}$-deformed $\mathscr{M}_{n,n+1}$, $\phi_{2,1}$-deformed tricritical

  [Zamolodchikov] vs [Klassen-Melzer] vs [Smirnov] [Colomo, Koubek, Mussardo]

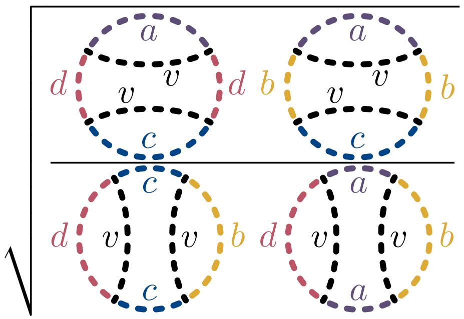
- Similar modified crossing observed in Chern-Simons matter in 2+1 d.

  [Mehta, Patel, Prakash, Minwalla, Sharma],…

In our examples, 1. braiding is not important. 2. TQFT d.o.f is more hidden.

Future

$$S^{ab}_{dc}(\theta) =$$



$$S^{bc}_{ad}(i\pi - \theta)$$

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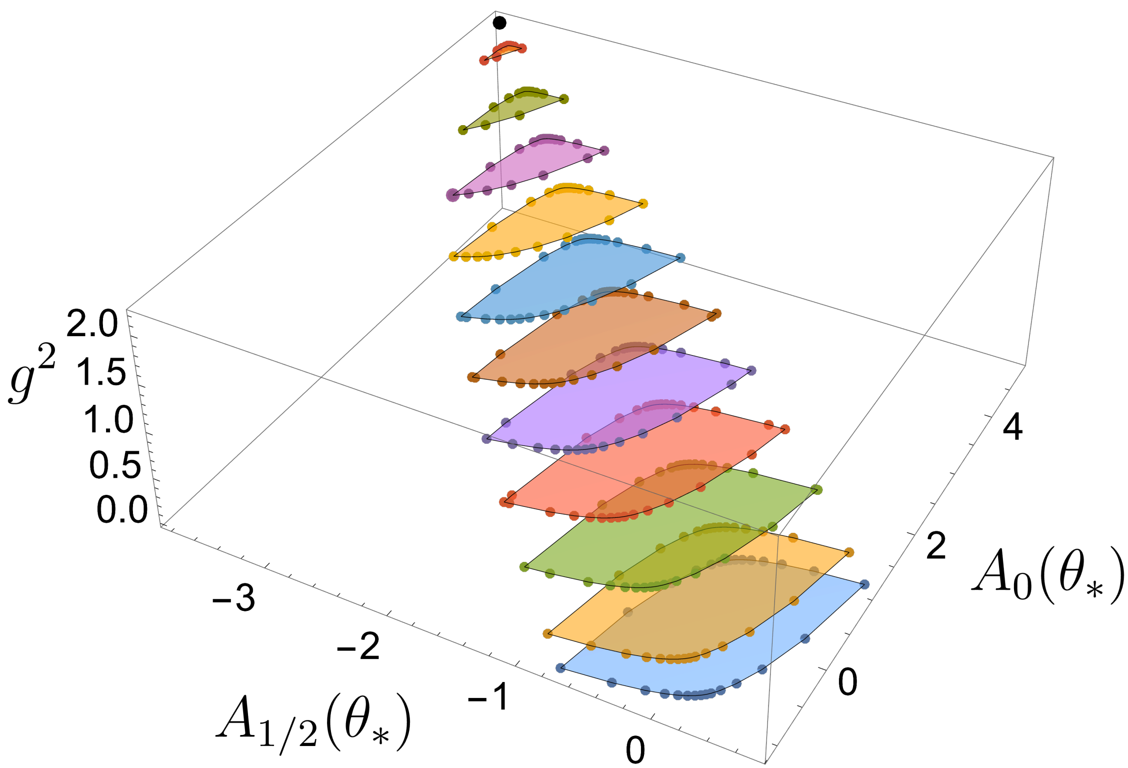
- S-matrix bootstrap with categorical symmetry (Haagerup fusion category)

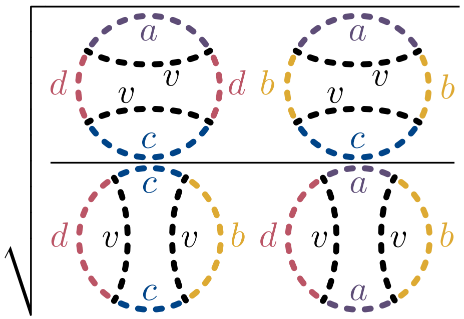
[To appear with Copetti, Cordova] [WIP]

Fibonacci



- Found integrable theories at the cusps of the allowed region.

Future

$S^{ab}_{dc}(\theta) =$ 

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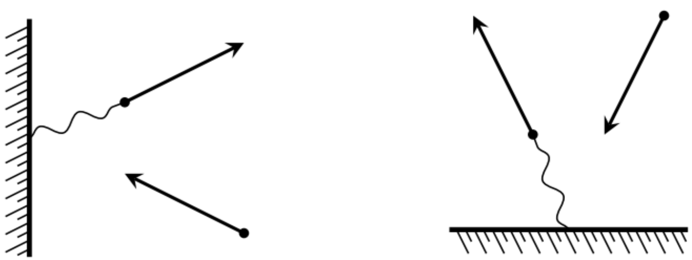
- Modified crossing in monopole scattering?



[Csaki, Hong, Shirman, Telem, Terning, Waterbury]

[van Beest, Boyle Smith, Delmastro, Komargodski, Tong]

[van Beest, Boyle Smith, Delmastro, Mouland, Tong]

- Toy model for IR effects (Faddeev-Kulish) in gravity and gauge theory?

Back ups

Why are Symmetries Important?

- Symmetries: Fundamental concepts in physics.

- Constrain the Renormalization Group flows.
  - once you know your theory, it's relatively easy to deduce consequence on observables ($\simeq$ experiment)

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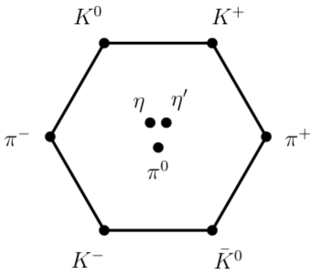

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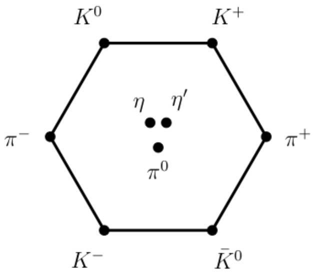

- A variety of modern refinements / generalizations.

  higher-form symmetries, categorical (non-invertible) symmetries, 2-groups,…

  [Gaiotto, Kapustin, Seiberg, Willet], [Frolich, Fuchs, Runkel, Schwiegert], [Tachikawa, Bhardwaj], [Chang, Lin, Shao, Wang, Yin], [Benini, Cordova, Hsin], [Cordova, Dumitrescu, Intriligator]….. and many others

However, the implication of modern symmetries on scattering amplitudes has not been sufficiently explored.