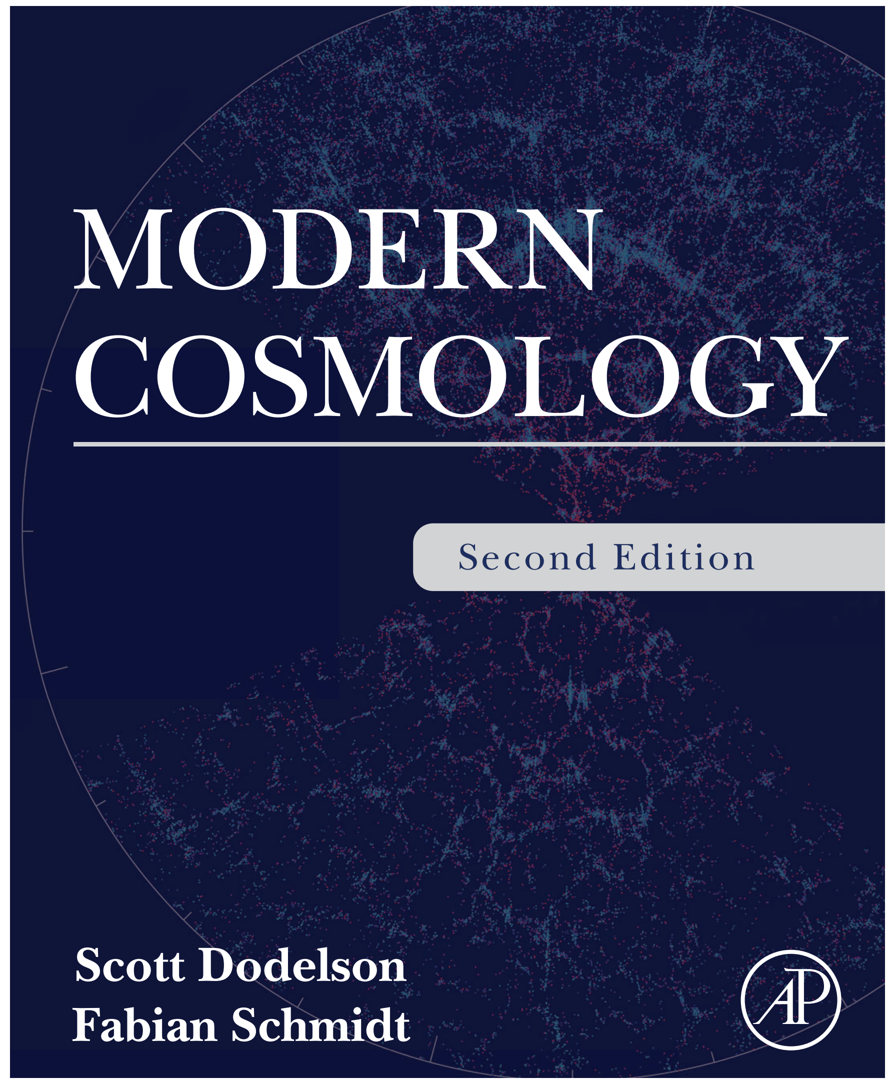
Exercises on Structure Formation I

Note: new exercises might be added during the course of the week…

Including material from:



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12.0: take the moments of the Boltzmann equation to derive the fluid equations. Use:

$$\frac{1}{m}\left\langle p^i p^j \right\rangle_{f_{\mathrm{m}}} = \rho_{\mathrm{m}} u^i_{\mathrm{m}} u^j_{\mathrm{m}} + \sigma^{ij}_{\mathrm{m}}. \qquad \text{Eq (12.17)}$$

12.1 Show that the stress tensor σ^{ij}_{m} [Eq. (12.17)] vanishes for a “cold” distribution function of the form Eq. (12.9).

12.2 Use Eq. (12.23) to derive an equation for the vorticity $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}_{\mathrm{m}}$ of the matter velocity. Show that no vorticity is generated if it is absent in the initial conditions. How does an initial vorticity evolve in time at linear order?

12.3 Fill in the missing steps of the transformation of the Euler–Poisson system into Fourier space, Eq. (12.31).

Equation numbers refer to *Modern Cosmology, second edition.*
However, the relevant ones are given either here, or in the lecture slides.

12.6 Derive the leading contribution to the matter bispectrum, Eq. (12.51). How does this look in the diagram form of Fig. 12.3?

12.7 In Sect. 12.2, we developed perturbation theory based on the density field. An alternative, *Lagrangian* approach is based on the equations of motion for N-body "particles," Eq. (12.57). In this exercise, you will derive the lowest-order result, known as *Zel'dovich approximation*. The solution to Eq. (12.57) is a particle trajectory $\boldsymbol{x}(\eta)$. We write this as

$$\boldsymbol{x}(\eta) = \boldsymbol{q} + \boldsymbol{s}(\boldsymbol{q}, \eta), \tag{12.106}$$

where $\boldsymbol{q}$ is the initial position at $\eta = 0$, when all perturbations were negligible. Hence $\boldsymbol{s}(\boldsymbol{q}, 0) = 0$. Rewrite Eq. (12.57) as an equation for $\boldsymbol{s}$. Now expand to linear order in $\boldsymbol{s}$. Solve the equation by using the solution of the Poisson equation for Ψ at linear order. Your result should relate $\boldsymbol{s}^{(1)}(\boldsymbol{k}, \eta)$ to $\delta^{(1)}(\boldsymbol{k}, \eta)$. This result can be used to obtain the initial small displacements of particles to start an N-body simulation. We also need their initial momenta p_c^i. Derive these in terms of the displacement as well.

$$p_c^i = a p^i$$

$$\frac{dx^i}{dt} = \frac{p_c^i}{ma^2}, \tag{12.57}$$

$$\frac{dp_c^i}{dt} = -m \frac{\partial \Psi}{\partial x^i}.$$

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12.11 Scratched.

12.12 Derive the second-order perturbation theory kernel for the galaxy density $\delta_g^{(2)}$.

(a) Define the scaled tidal field through

$$K_{ij}(\boldsymbol{x},\eta) = \frac{1}{4\pi G a^2(\eta)}\left[\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right]\Psi(\boldsymbol{x},\eta). \tag{12.115}$$

Use this definition to relate K_{ij} is to the matter density in real and Fourier space.

(b) Begin with the real-space expression of the second-order galaxy density,

$$\delta_g^{(2)}(\boldsymbol{x},\eta) = b_1\delta^{(2)} + \frac{1}{2}b_2(\delta^{(1)})^2 + b_{K^2}K_{ij}^{(1)}K^{(1)ij}, \tag{12.116}$$

where on the right-hand side all fields are evaluated at $(\boldsymbol{x},\eta)$, and the bias parameters b_1, b_2, b_{K^2} are defined at η. Why does the tidal field only appear at second order and in this particular combination? Now pull out the time dependence contained in the growth factors, and Fourier transform Eq. (12.116) to arrive at Eq. (12.87).

$$F_{g,2}(\boldsymbol{k}_1,\boldsymbol{k}_2;\eta) = b_1(\eta)F_2(\boldsymbol{k}_1,\boldsymbol{k}_2) + \frac{1}{2}b_2(\eta) + b_{K^2}(\eta)\left[\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2} - \frac{1}{3}\right]. \tag{12.87}$$

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