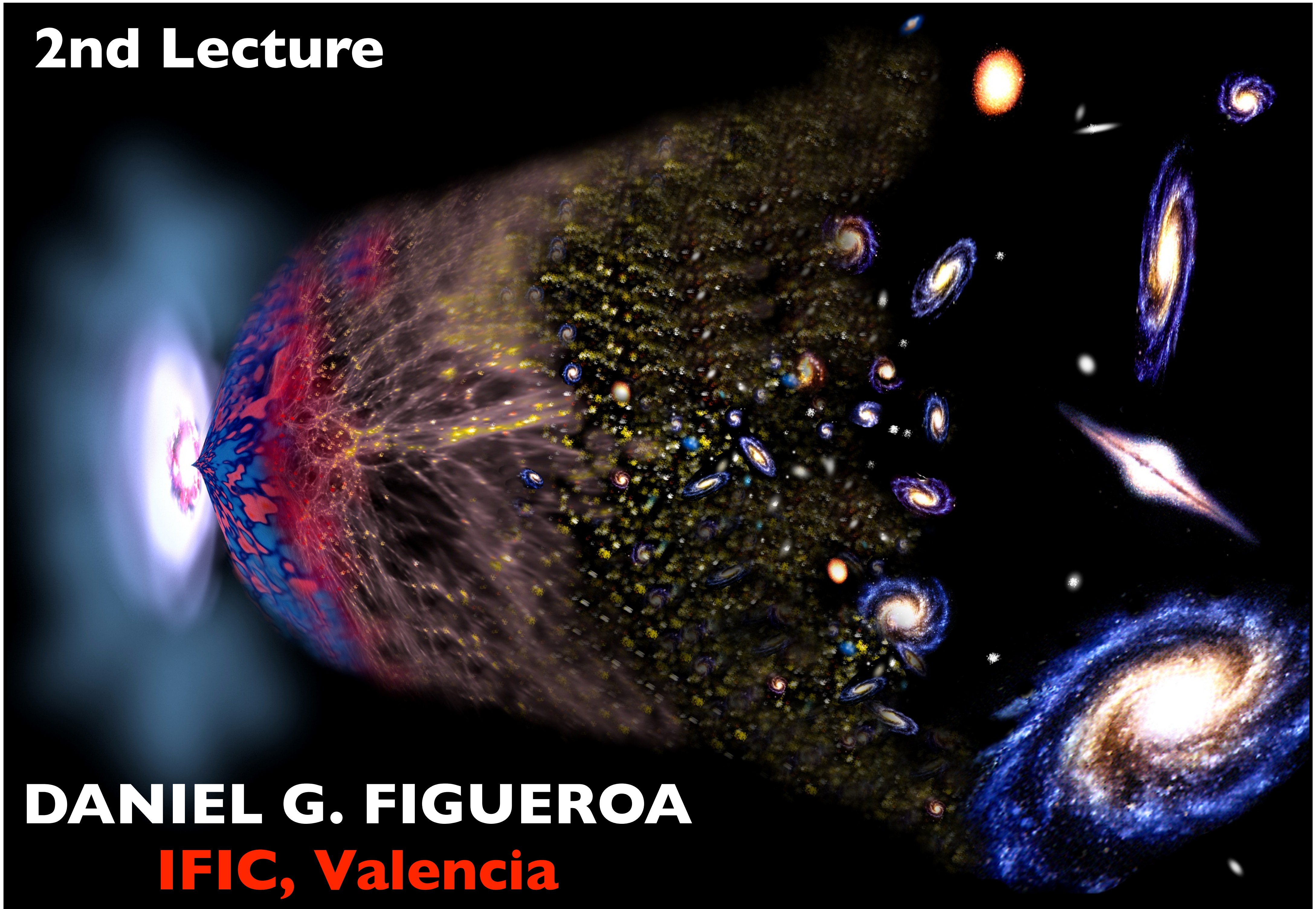
GRAVITATIONAL WAVE
— BACKGROUNDS —



ICTP's Summer School on Cosmology, 17-28 June 2024, Trieste, Italy

Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs) 1st Topic (Formal Th.)

2) GWs from Inflation

3) GWs from Preheating

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

6) Astrophysical Background(s)

7) Observational Constraints/Prospects

(Briefly)

Gravitational Wave Backgrounds

OUTLINE

1) Grav. Waves (GWs)

1st Topic
(Formal Th.)

Early Universe Sources

2) GWs from Inflation

3) GWs from Preheating

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

Core Topics
(Pheno Th.)

6) Astrophysical Background(s)

7) Observational Constraints/Prospects

(Briefly)

The Gravity of the Situation …

GW Propagation/Creation in Cosmology

FLRW: $ds^2 = a^2(-dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j)$, $\quad \mathrm{TT}: \begin{cases} h_{ii} = 0 \\ h_{ij,j} = 0 \end{cases}$

(conformal time)

GW Propagation/Creation in Cosmology

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(conformal time)

Creation/Propagation GWs in FLRW

Eom: $\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\mathrm{TT}}_{ij},}$

Source: Anisotropic Stress

$\boxed{\Pi_{ij} = T_{ij} - \langle T_{ij} \rangle_{\text{FLRW}}}$

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Source: Anisotropic Stress

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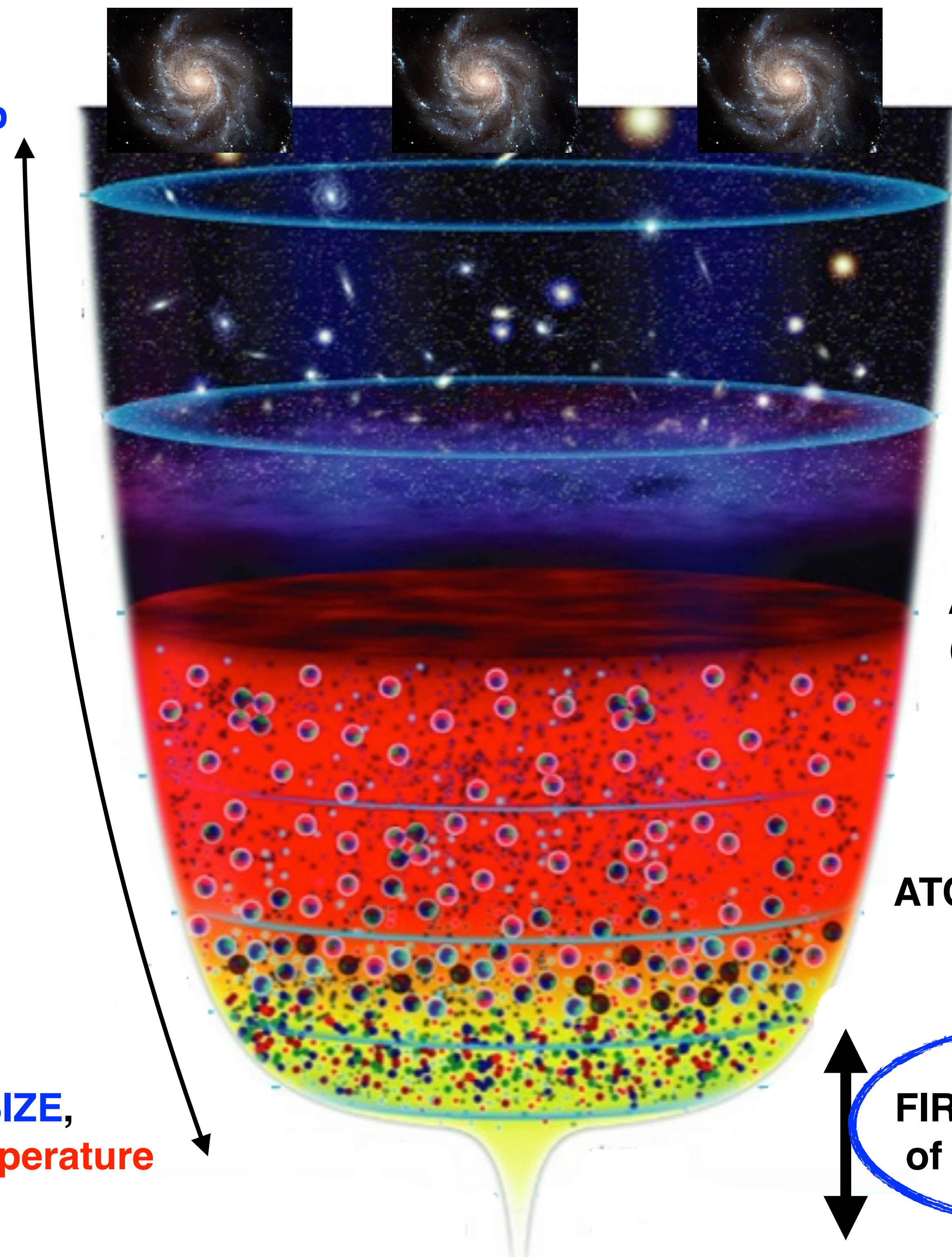
GW Source(s): (SCALARS , VECTOR , FERMIONS)

$$\Pi^{TT}_{ij} \quad \propto \quad \{\partial_i \chi^a \partial_j \chi^a\}^{TT}, \quad \{E_i E_j + B_i B_j\}^{TT}, \quad \{\bar{\psi}\gamma_i D_j \psi\}^{TT}$$

Cosmic History



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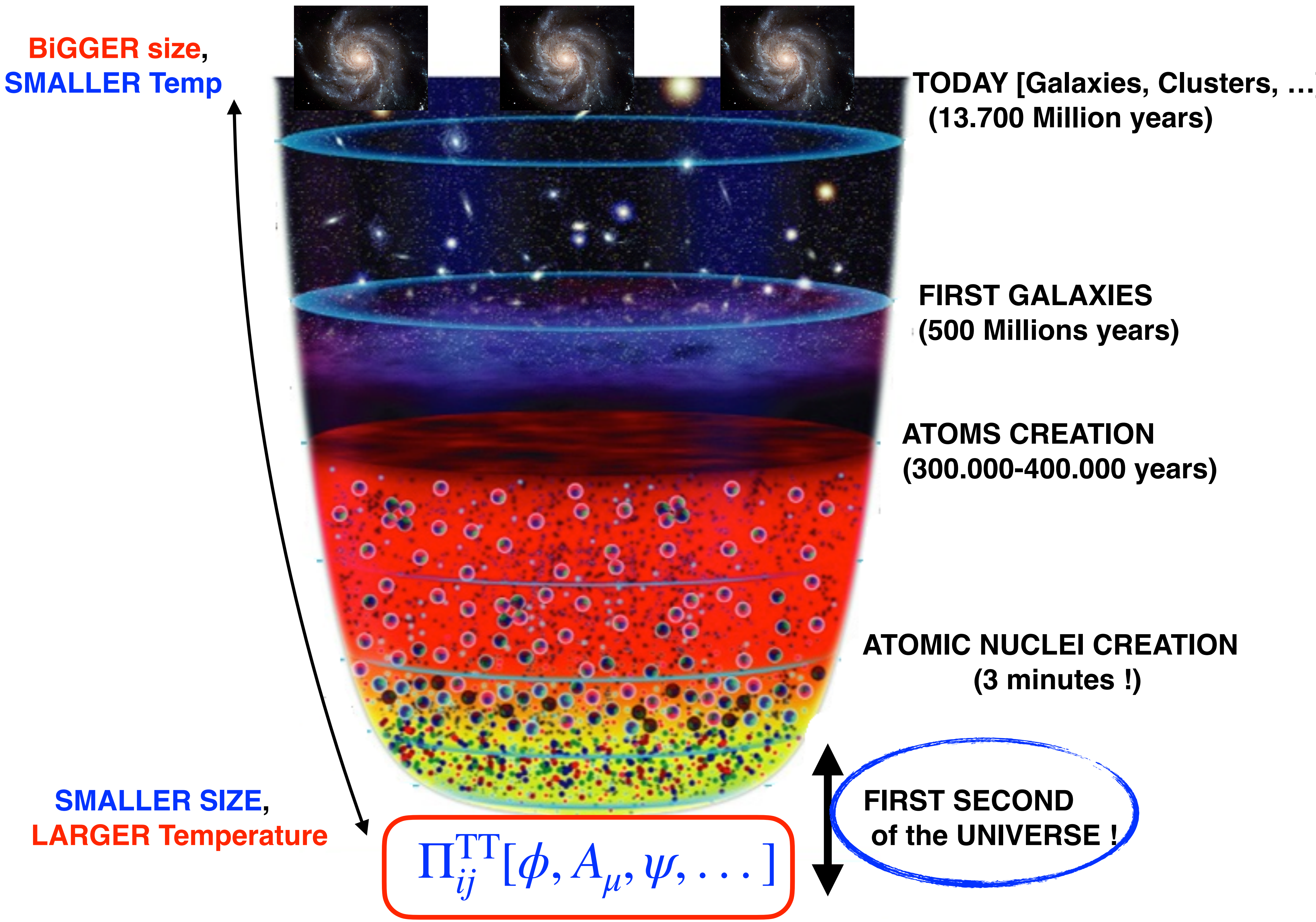

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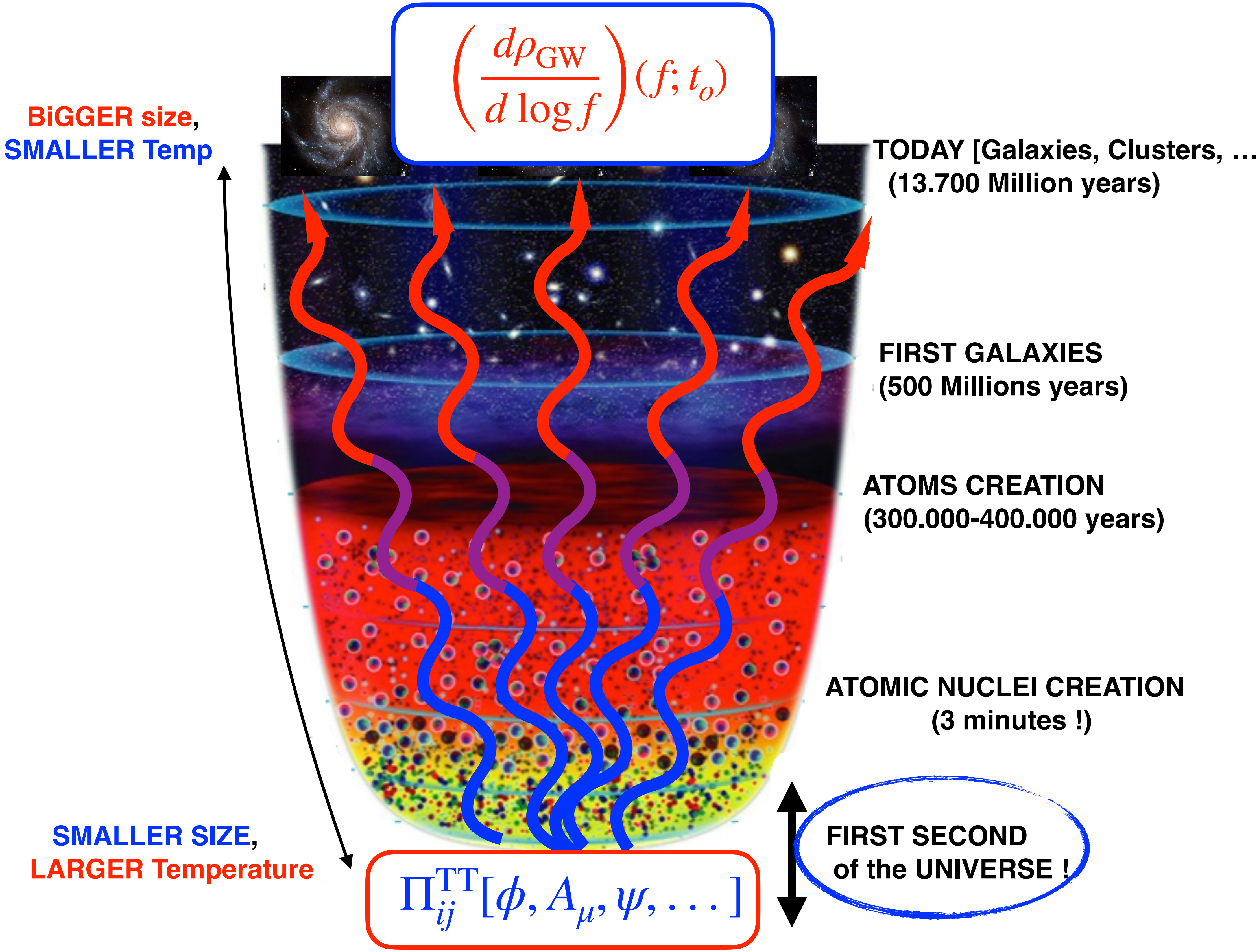

GW energy density spectrum

Recall, from previous lecture on

the energy-momentum of GW

GW energy density spectrum

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$$\rho_{\rm GW}(t) = \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V \qquad t: \textbf{conformal time}$$

$$\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)$$

GW energy density spectrum

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$$\rho_{\rm GW}(t) = \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V \qquad \boxed{(V^{1/3} \gg \lambda)}$$

$$\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)$$

GW energy density spectrum

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$$\begin{aligned}
\rho_{\rm GW}(t) &= \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V \\
&\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \\
&= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\, \dot{h}_{ij}(\mathbf{k},t)\dot{h}^*_{ij}(\mathbf{k}',t) \\
&\qquad \times \frac{1}{V} \int_V d\mathbf{x}\; e^{-i\mathbf{x}(\mathbf{k}-\mathbf{k}')}\,,
\end{aligned}$$

$\boxed{(V^{1/3} \gg \lambda)}$

GW energy density spectrum

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$$
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&\qquad \times \frac{1}{V}\int \; \boxed{\longrightarrow (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')}
\end{aligned}
$$

$$(kV^{1/3} \longrightarrow \infty)$$

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&= \frac{1}{32\pi G a^2(t) V}\int \frac{d\mathbf{k}}{(2\pi)^3}\,\dot h_{ij}(\mathbf{k},t)\dot h^*_{ij}(\mathbf{k},t)
\end{aligned}$$

$\boxed{(V^{1/3} \gg \lambda)}$

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$$\boxed{(V^{1/3} \gg \lambda)}$$

$$\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V}\int_V d\mathbf{x}\, \dot h_{ij}(\mathbf{x},t)\dot h_{ij}(\mathbf{x},t)$$

$$= \frac{1}{32\pi G a^2(t) V}\int \frac{d\mathbf{k}}{(2\pi)^3}\, \dot h_{ij}(\mathbf{k},t)\dot h^*_{ij}(\mathbf{k},t)$$

$$= \int d\log k \left(\frac{1}{(4\pi)^3 G a^2(t) V}\left\langle \dot h_{ij}(\mathbf{k},t)\dot h^*_{ij}(\mathbf{k},t)\right\rangle_{\Omega_k}\right)$$

$$\left[\left\langle |f(\mathbf{k})|^2\right\rangle_{\Omega_k} \equiv \frac{1}{4\pi}\int_{|\mathbf{k}'|=k} d\Omega_k\, |f(\mathbf{k}')|^2\right]$$

GW energy density spectrum

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$$\boxed{\rho_{\rm GW}(t)} = \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V$$

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Energy density Spectrum

$$\left[\left\langle |f(\mathbf{k})|^2 \right\rangle_{\Omega_k} \equiv \frac{1}{4\pi} \int_{|\mathbf{k}'| = k} d\Omega_k\, |f(\mathbf{k}')|^2 \right]$$

$$\equiv \left(\frac{d\rho_{\rm GW}}{d\log k} \right)(k,t)$$

Primer on Inflation

See Lectures
by J. Garriga

Inflation

(Brief review)

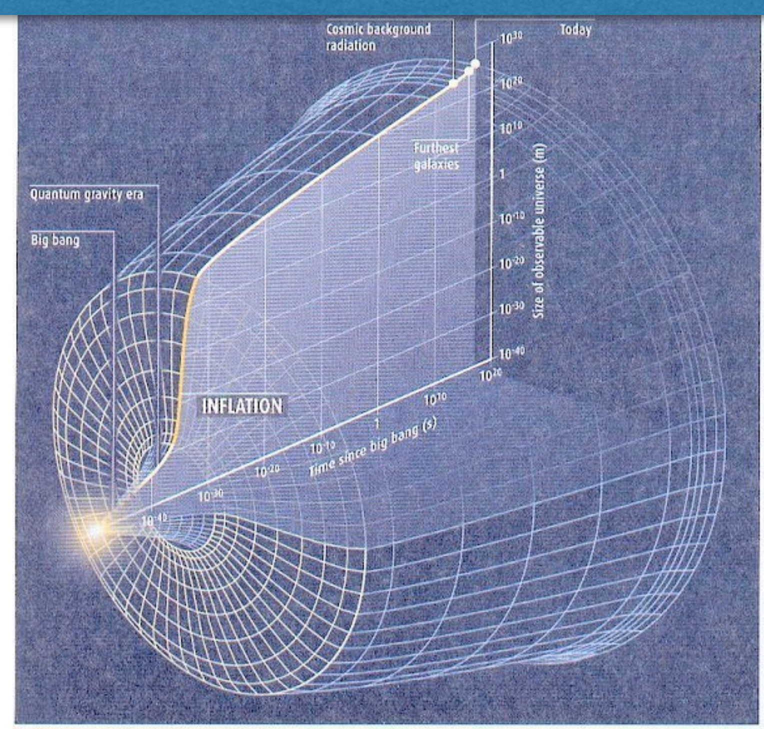
Inflation

INFLATIONARY COSMOLOGY

'cures' hBB $\Longrightarrow$ Cosmological Pple

Inflation

COSMIC INFLATION



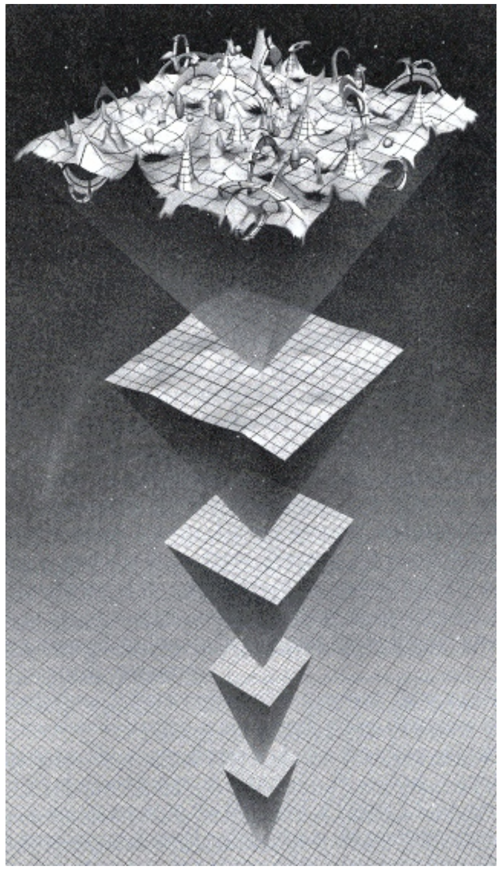
1 cm

10^{28} cm

INFLATIONARY COSMOLOGY

'cures' hBB $\Longrightarrow$ Cosmological Pple

Inflation $\rightarrow$ $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations $\begin{cases}\text{Scalar } \mathcal{R} \\ \text{Tensor (GWs) } h_{ij}\end{cases}$



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See
Lectures by
J. Garriga

INFLATIONARY COSMOLOGY

Inflation: Generator of Primordial Fluctuations

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~~**Minkowski**~~ $\rightarrow$ **Curved Space: (quasi)dS**

INFLATIONARY COSMOLOGY

Inflation: Generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

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$$B_i = \partial_i B - S_i$$

$$E_{ij} = 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}$$

INFLATIONARY COSMOLOGY

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Expanding U. $\longrightarrow$ **Vector Perturbations** $S_i, F_i \propto \frac{1}{a}$

INFLATIONARY COSMOLOGY

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$$\partial_i h_{ij} = h_{ii} = 0$$

(tensors = GWs)

INFLATIONARY COSMOLOGY

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$$\text{dof} \xrightarrow{\text{scalars}} \delta\phi + \{\Phi, B, \Psi, E\}$$

$$\text{dof} \xrightarrow{\text{tensors}} h_{ij}$$

INFLATIONARY COSMOLOGY

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dof

scalars: $\delta\phi + \{\Phi, B, \Psi, E\} \xrightarrow{\text{Diff.}} \delta\phi' + \{\Phi', B', \Psi', E'\}$ ~~Gauge Inv.~~

tensors: $h_{ij} \xrightarrow{\text{Diff.}} h_{ij}$ Gauge Inv. !

Diff.: $\boxed{x^\mu \;\to\; x^\mu + \xi^\mu}$

INFLATIONARY COSMOLOGY

Inflation: Generator of Primordial Fluctuations

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dof

scalars: $\delta\phi + \{\Phi, B, \Psi, E\}$

tensors: h_{ij}

$$\zeta \equiv -[\Psi + (H/\dot{\rho})\delta\rho_\phi] \xrightarrow{\text{Diff.}} \zeta$$

$$\mathcal{R} \equiv [\Psi + (H/\dot{\phi})\delta\phi] \xrightarrow{\text{Diff.}} \mathcal{R}$$

$$Q \equiv [\delta\phi + (\dot{\phi}/H)\Psi] \xrightarrow{\text{Diff.}} Q$$

Gauge Inv. !

INFLATIONARY COSMOLOGY

Inflation: Generator of Primordial Fluctuations

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dof $\xrightarrow{\text{scalars}}$ $\delta\phi + \{\Phi, B, \Psi, E\}$ $\longrightarrow$

dof $\xrightarrow{\text{tensors}}$ h_{ij}

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$$Q \equiv [\delta\phi + (\dot\phi/H)\Psi] \xrightarrow{\text{Diff.}} Q$$

Gauge Inv. !

Curvature Pert.

Tensor Pert. (GW)

Fixing Gauge: e.g. $E, \delta\phi = 0 \Rightarrow g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]$

Inflation & Primordial Perturbations

H & I

INFLATION

$$\boxed{g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]}$$

Inflation & Primordial Perturbations

INFLATION

$$g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

Scalar

$$\langle \mathcal{R}\mathcal{R}^* \rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta_{\mathcal{R}}^2(k)$$

Tensor

$$\langle h_{ij} h_{ij}^* \rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta_h^2(k)$$

Inflation & Primordial Perturbations

INFLATION ➡

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Tensor

$$\langle h_{ij} h^*_{ij} \rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta^2_{h}(k)$$

$$\left[\left\langle f(\mathbf{k}) f^*(\mathbf{k}') \right\rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta^2_f(k) \delta(\mathbf{k} - \mathbf{k}') \right]$$

Quantum fluctuations !

Inflation & Primordial Perturbations

INFLATION

$$g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2 \dot{\phi}^2} \left(\frac{k}{aH} \right)^{n_s - 1}$$

$$n_s - 1 \equiv 2(\eta - 2\epsilon)$$

$$\Delta^2_h(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p} \right)^2 \left(\frac{k}{aH} \right)^{n_t}$$

$$n_t \equiv -2\epsilon$$

Inflation & Primordial Perturbations

INFLATION →

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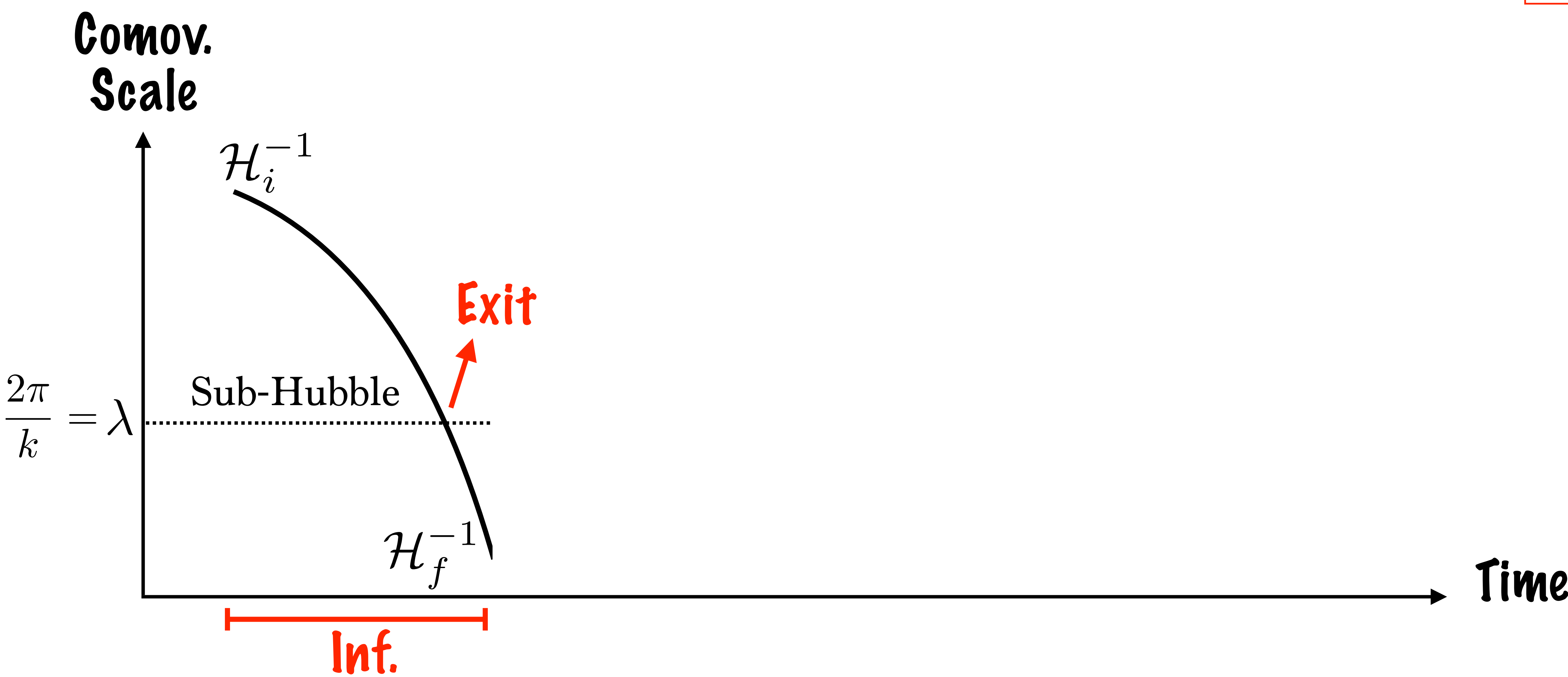

Inflation & Primordial Perturbations

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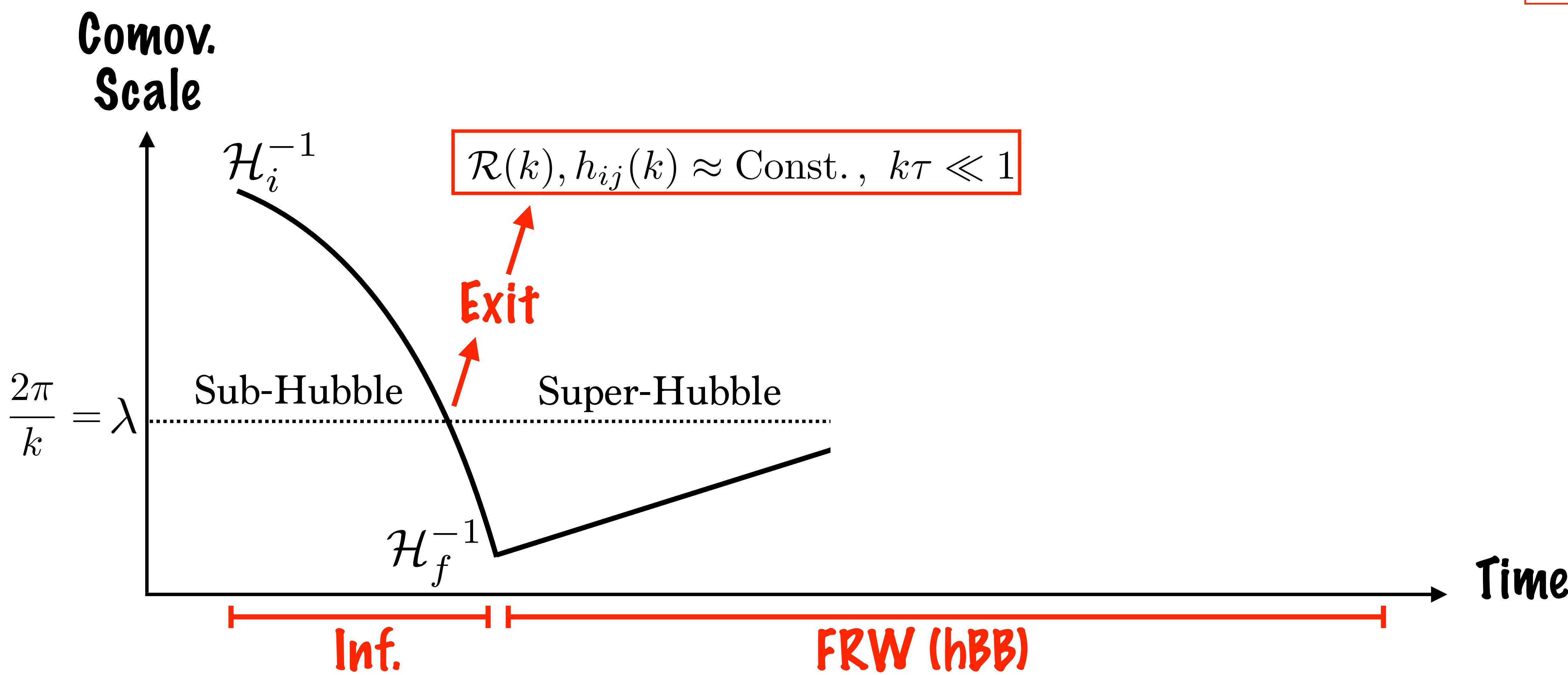

Inflation & Primordial Perturbations

INFLATION $\rightarrow$ $g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$

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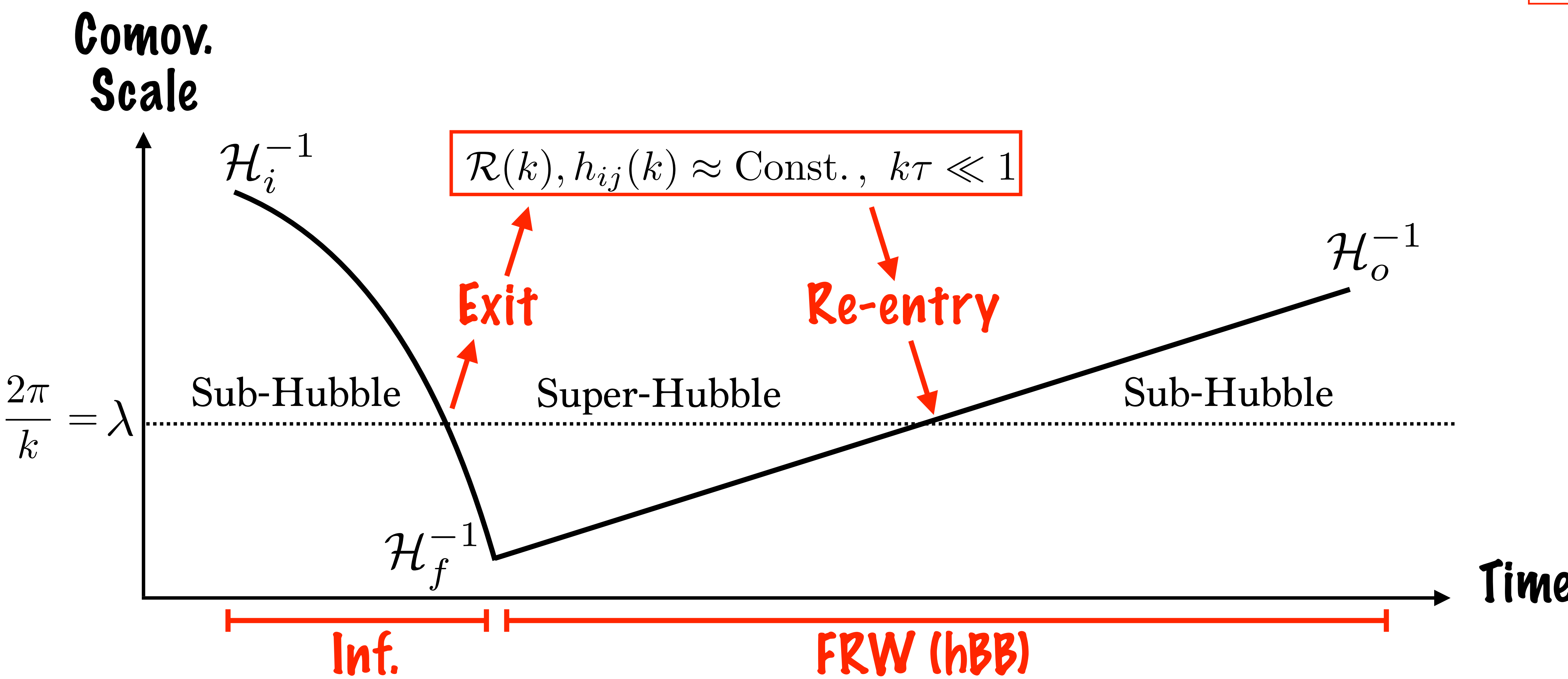

INFLATIONARY COSMOLOGY

'cures' hBB $\Rightarrow$ Cosmological Pple

Inflation $\rightarrow$ $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations $\begin{cases}\text{Scalar}\\ \text{Tensor} \Rightarrow \text{Stochastic GW Background}\end{cases}$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x}, \underset{\substack{\downarrow \\ \text{conformal time}}}{t}) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h^*_k(t) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r}\right)\, e^r_{ij}(\hat{\mathbf{k}})}$$

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quantum operators

Polarizations: +, x

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$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_V \longrightarrow$$ Volume/Time Average

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Tensors = GWs

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$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_{\rm QM} \longrightarrow \text{ensemble average}$$

Irreducible GW background from Inflation

Tensors = GWs

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$$= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\; e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')} \left\langle \dot{h}_{ij}\,(\mathbf{k},t)\, \dot{h}^*_{ij}\,(\mathbf{k}',t) \right\rangle$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(t)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(t) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r} \right)\, e^r_{ij}(\hat{\mathbf{k}})}$$

$$\rho_{\rm GW}(t) = \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_{\rm QM} \longrightarrow \text{ensemble average}$$

$$= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\, e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')} \left\langle \dot{h}_{ij}\,(\mathbf{k},t)\, \dot{h}^*_{ij}\,(\mathbf{k}',t) \right\rangle$$

$$\boxed{\left\langle \dot{h}_{ij}\,(\mathbf{k},t)\, \dot{h}^*_{ij}\,(\mathbf{k}',t) \right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot{h}}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

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$$\rho_{\rm GW}(t) = \frac{1}{(4\pi)^3 G a^2(t)} \int \frac{dk}{k}\; k^3\, \mathcal{P}_{\dot{h}}(k,t) \;\; = \int \frac{d\rho_{\rm GW}}{d\log k}\, d\log k$$

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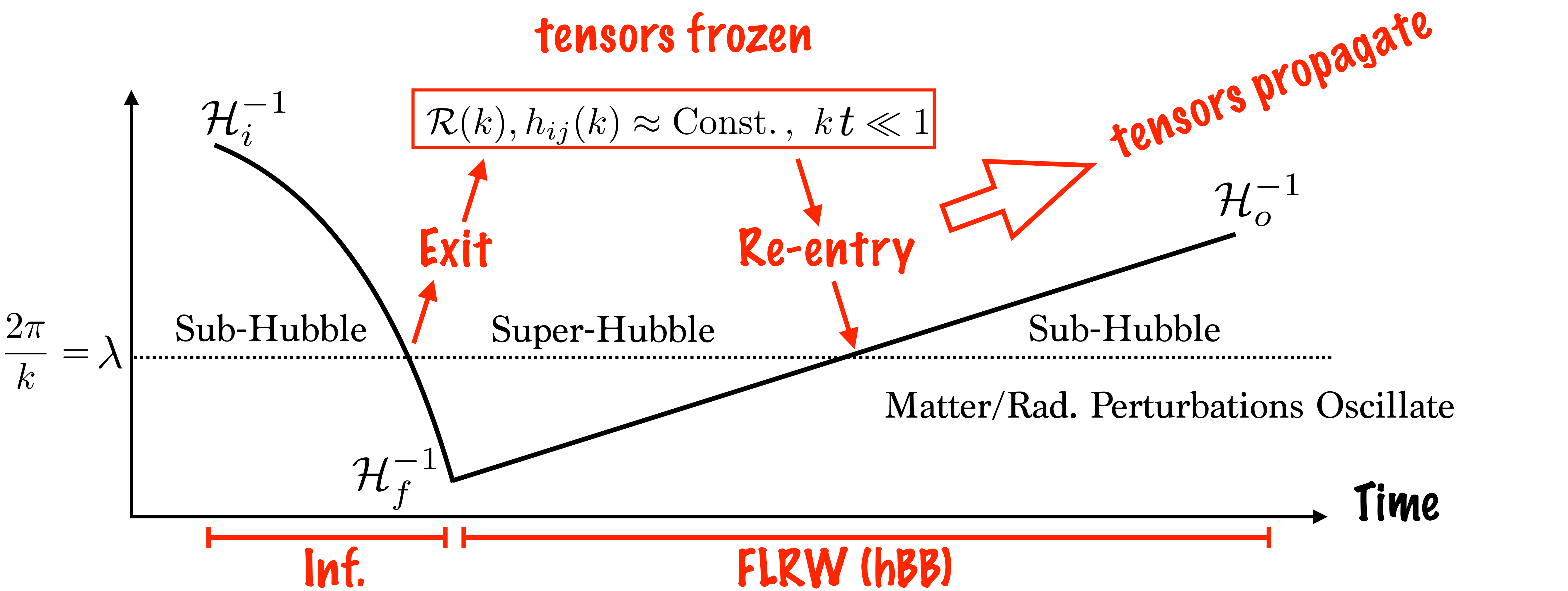

Irreducible GW background from Inflation

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Horizon Re-entry $\Longrightarrow$ tensors propagate

Rad Dom: $h_r(\mathbf{k},t) = \dfrac{A_r(\mathbf{k})}{a(t)}\, e^{ikt} + \dfrac{B_r(\mathbf{k})}{a(t)}\, e^{-ikt}$

Irreducible GW background from Inflation

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$$\boxed{\begin{array}{c} @\ \text{Horizon}: \begin{cases} h = h_* \\ \dot h_* = 0 \end{cases} \\ A = B = \frac{1}{2}a_* h_* \end{array}}$$

Irreducible GW background from Inflation

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$$\left\langle \dot{h}\dot{h}\right\rangle = k^2 \langle hh \rangle$$

After horizon re-entry

Irreducible GW background from Inflation

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After horizon re-entry

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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After horizon re-entry

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot{h}}(k,t)}$$

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After horizon re-entry

Redshift (re-entry)

Inflationary Tensor Spectrum !

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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Irreducible GW background from Inflation

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$\downarrow$

Scales as
Radiation !

(This happens for any GWB, once
freely propagating @ sub-H scales)

Irreducible GW background from Inflation

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$$(1+z_*)^{-2}_{\rm RD} = \Omega^{(o)}_{\rm Rad}\frac{a_o^2 H_o^2}{k^2} \Rightarrow \frac{d\rho_{\rm GW}}{d\log k} = \frac{\Omega^{(o)}_{\rm Rad}}{24}\left(\frac{a_o}{a}\right)^4 3m_p^2 H_o^2 \Delta^2_{h_*}$$

Irreducible GW background from Inflation

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RD: $(1+z_*)^{-2}_{\rm RD} = \Omega^{(o)}_{\rm Rad}\frac{a_o^2 H_o^2}{k^2} \Rightarrow \frac{d\rho_{\rm GW}}{d\log k} = \frac{\Omega^{(o)}_{\rm Rad}}{24}\left(\frac{a_o}{a}\right)^4 \overbrace{3m_p^2 H_o^2}^{\rho_c^{(o)}}\Delta^2_{h_*}$

$$\Omega^{(o)}_{\rm GW} \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o$$

Irreducible GW background from Inflation

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$$\boxed{\Omega^{(o)}_{\rm GW} \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o = \frac{\Omega^{(o)}_{\rm Rad}}{24}\Delta^2_{h_*}} \quad \textbf{(RD)}$$

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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(RD)

GW normalized energy density spectrum (today)

Inflationary tensor spectrum

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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(RD)

Transfer Funct

$$T(k) \equiv \frac{\Omega^{(o)}_{\rm GW}(k)}{\Delta^2_{h_*}(k)} \propto k^0\,(\mathrm{RD})$$

Irreducible GW background from Inflation

energy scale

$$\boxed{\Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

$$\boxed{n_t \equiv -2\epsilon}$$

Small red-tilt

(almost-) scale-invariant

Irreducible GW background from Inflation

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Transfer Funct.: $T(k) \propto k^0 \,(\mathrm{RD})$

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Small red-tilt

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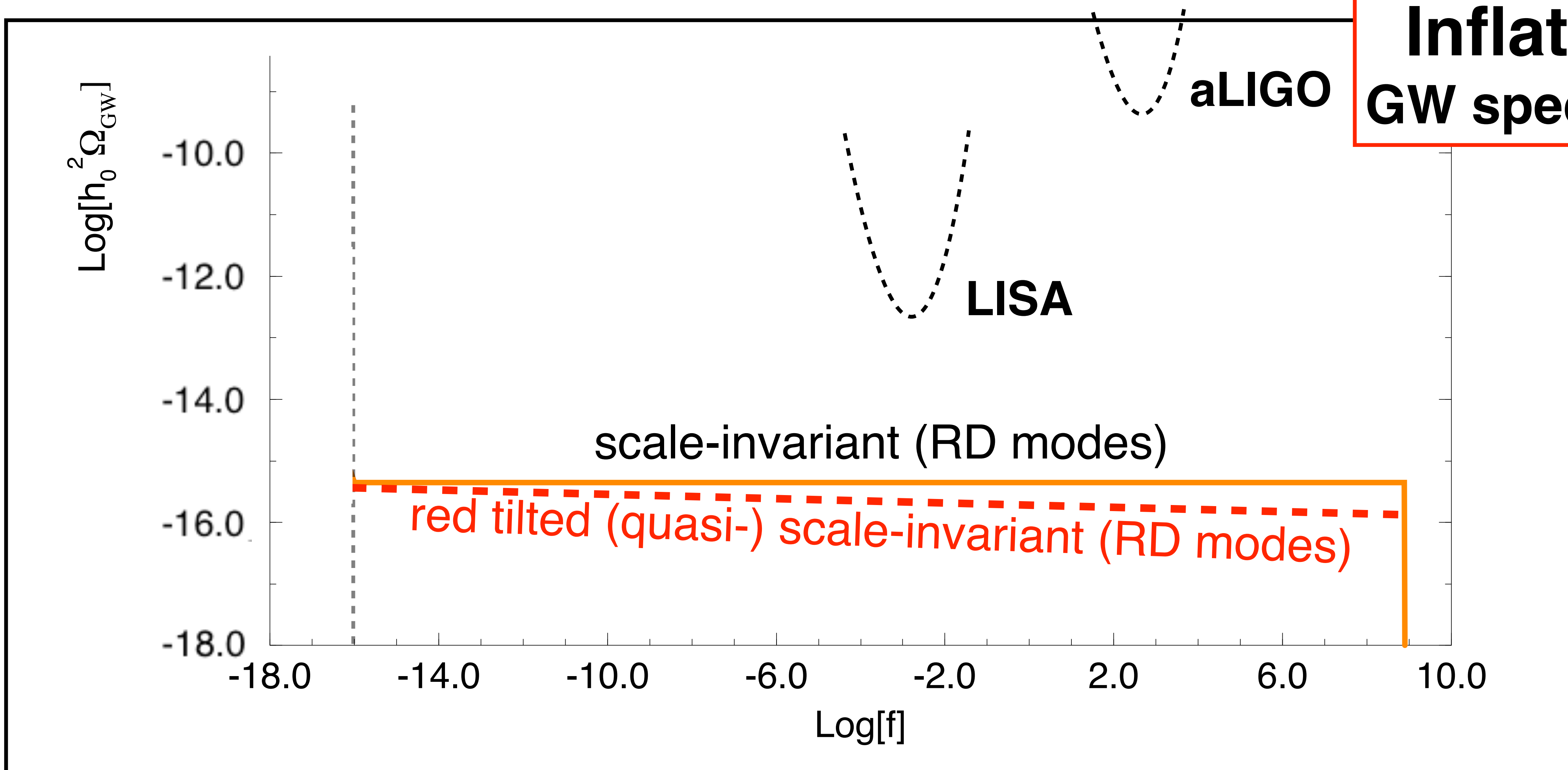
Inflation GW spectrum



Irreducible GW background from Inflation

$$\frac{\Omega_{\rm RD}}{24}\left(\frac{k_{\rm eq}}{k}\right)^2$$

$$\boxed{\Omega^{(o)}_{\rm GW}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o = \underbrace{\cancel{\frac{\Omega^{(o)}_{\rm Rad}}{24}}}\,\Delta^2_{h_*}(k)}$$

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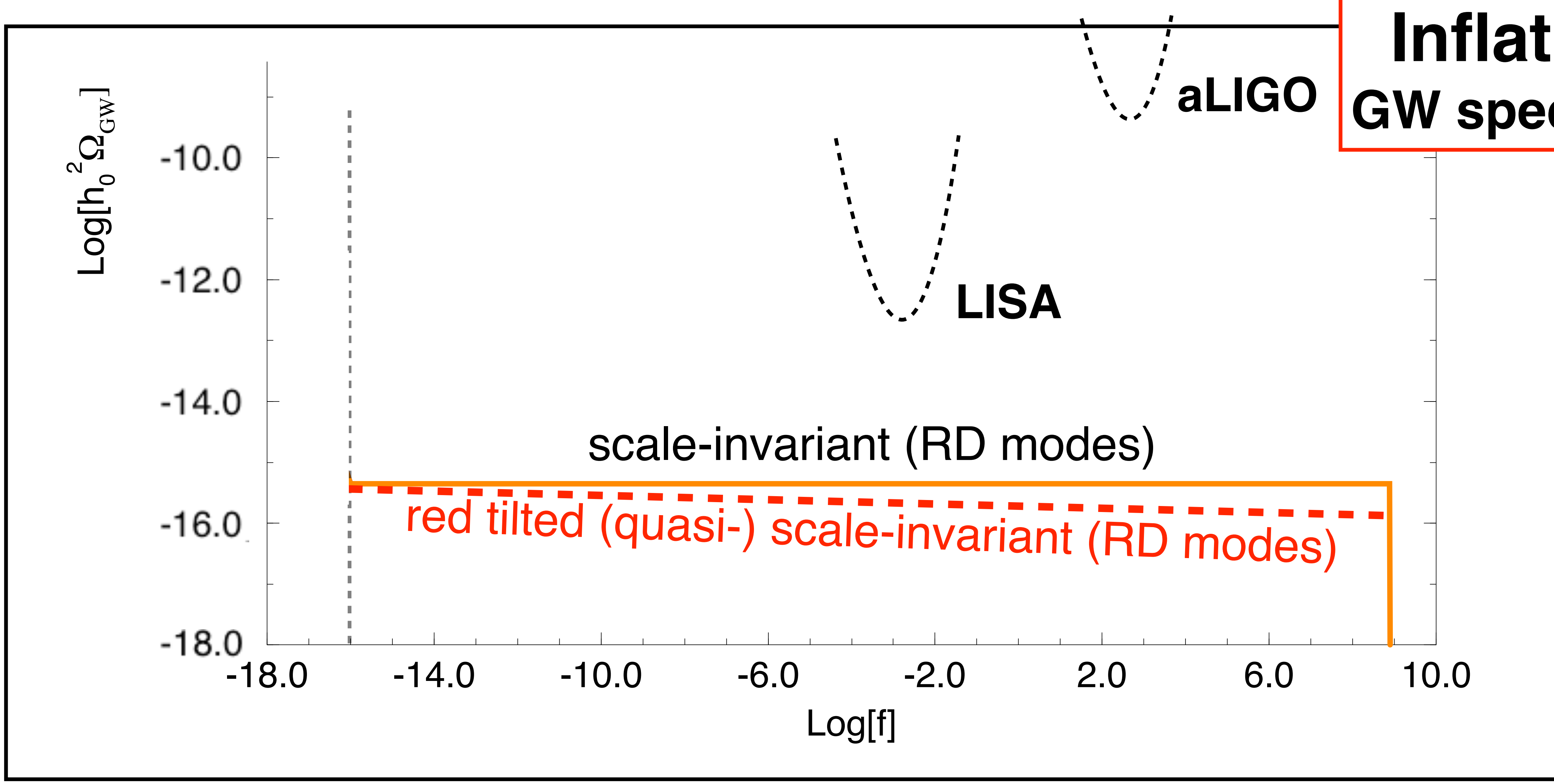
Small red-tilt

Transfer Funct.: $T(k) \propto k^{-2}(\mathrm{MD})$



Inflation GW spectrum

Irreducible GW background from Inflation

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Small red-tilt

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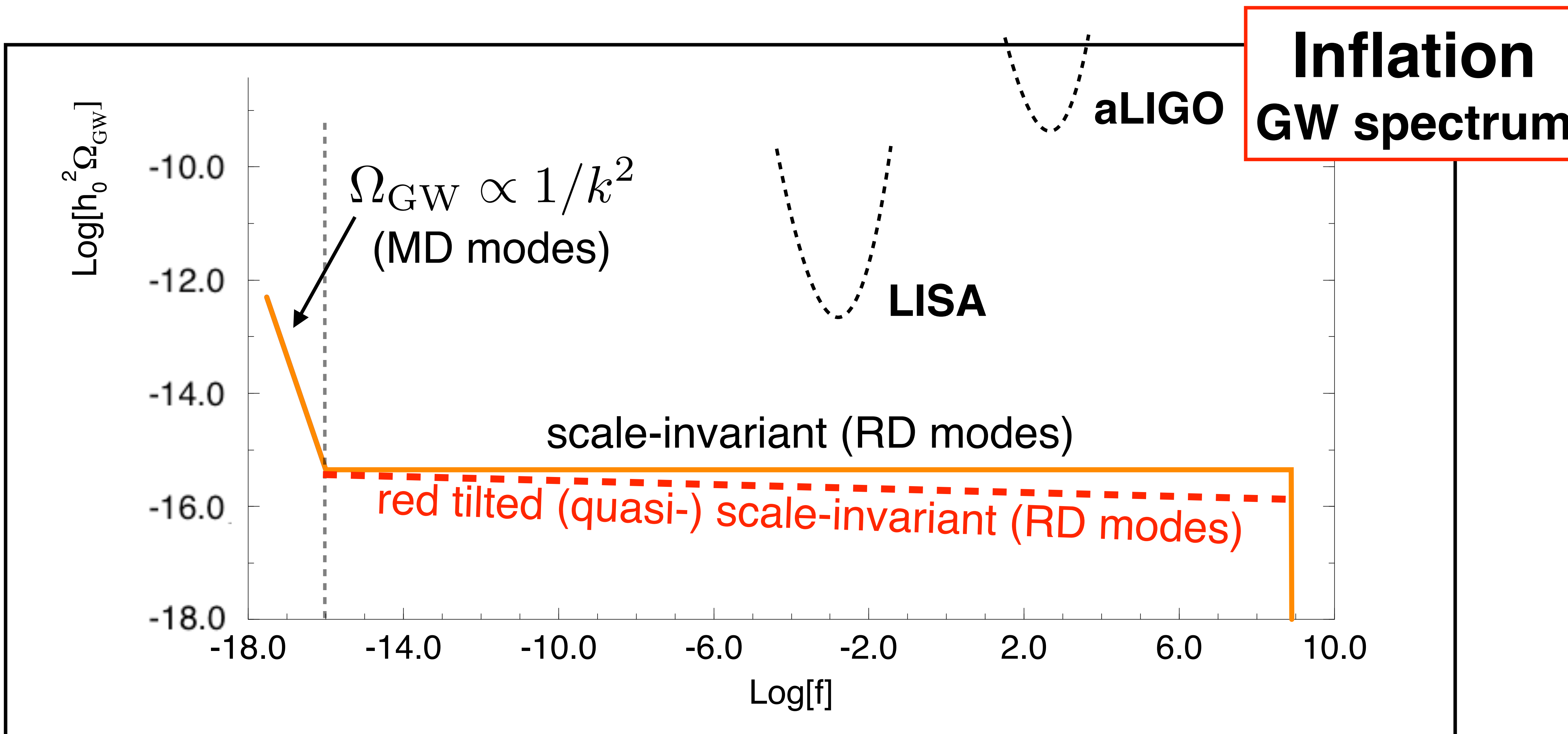


Irreducible GW background from Inflation

$$\Omega_{\rm GW}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}} \left(\frac{d\rho_{\rm GW}}{d\log k} \right)_o = \frac{\Omega_{\rm rad}^{(o)}}{24} T(k) \Delta^2_{h*}(k)$$

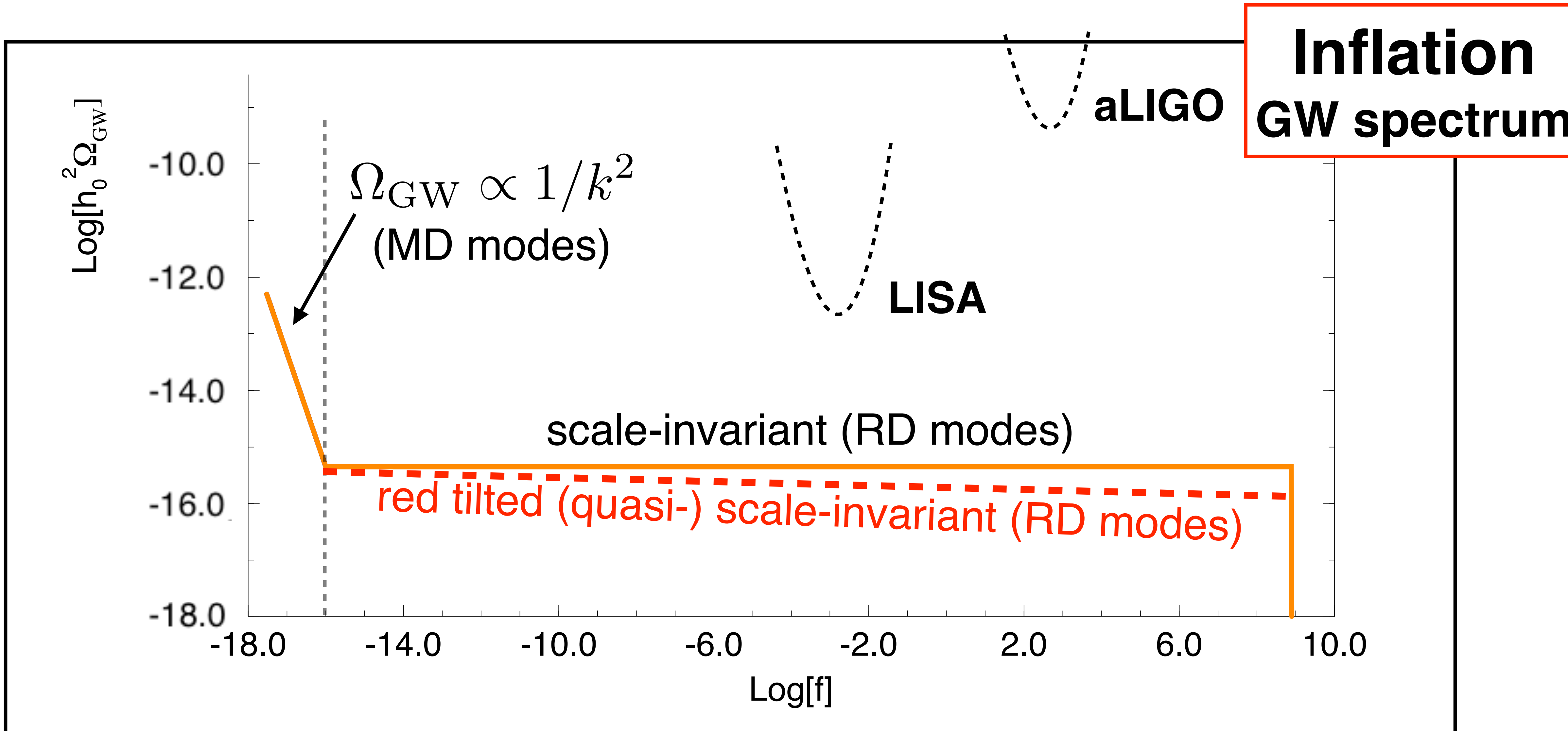
energy scale

$$\Delta_h^2(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p}\right)^2 \left(\frac{k}{aH}\right)^{n_t}$$

$$n_t \equiv -2\epsilon$$

Small red-tilt

Transfer Funct.: $T(k) = \begin{cases} 1 & ,\mathrm{RD} \\ (k_{\rm eq}/k)^2 & ,\mathrm{MD} \end{cases}$



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Small red-tilt

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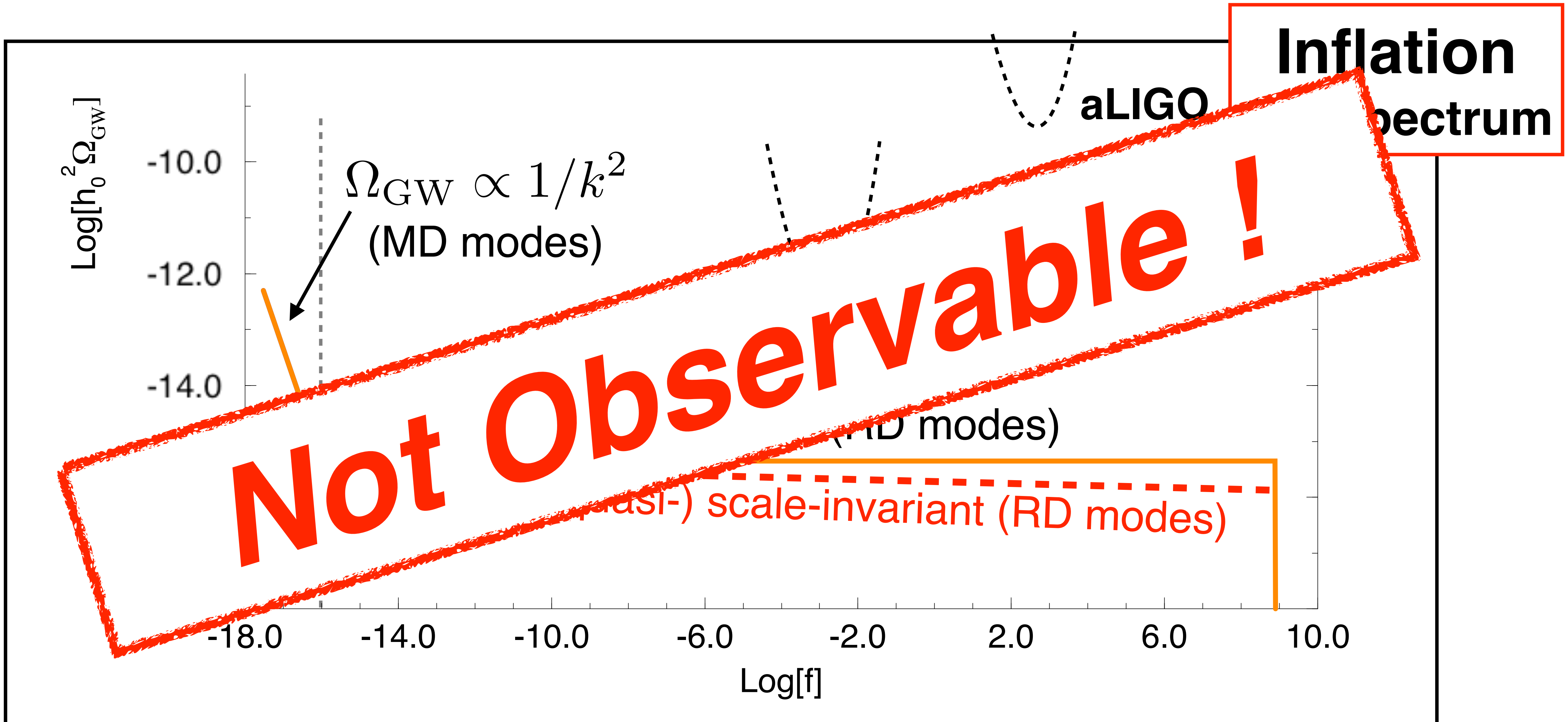

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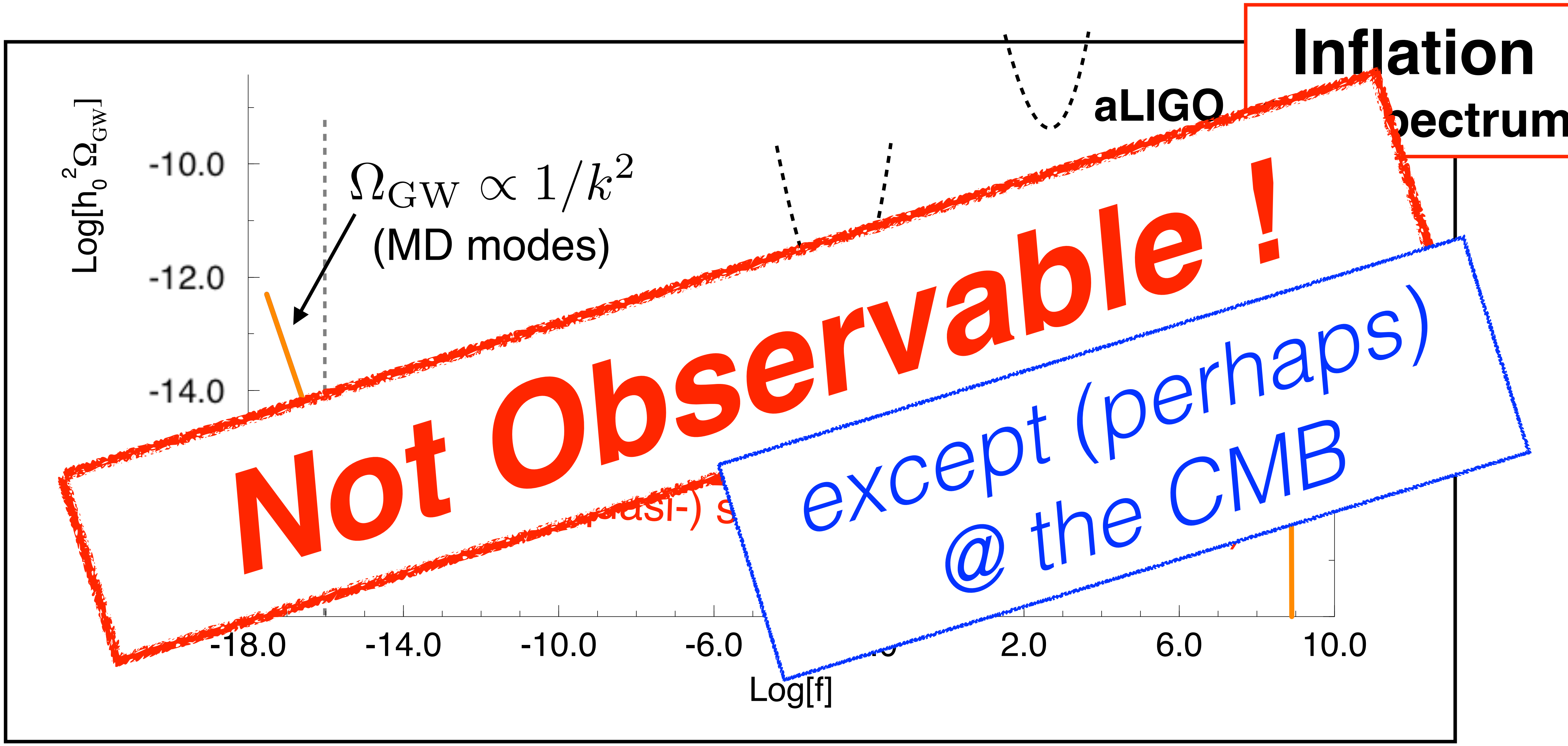


Irreducible GW background from Inflation

$$\Omega_{\rm GW}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o = \frac{\Omega_{\rm rad}^{(o)}}{24}T(k)\Delta^2_{h*}(k)$$

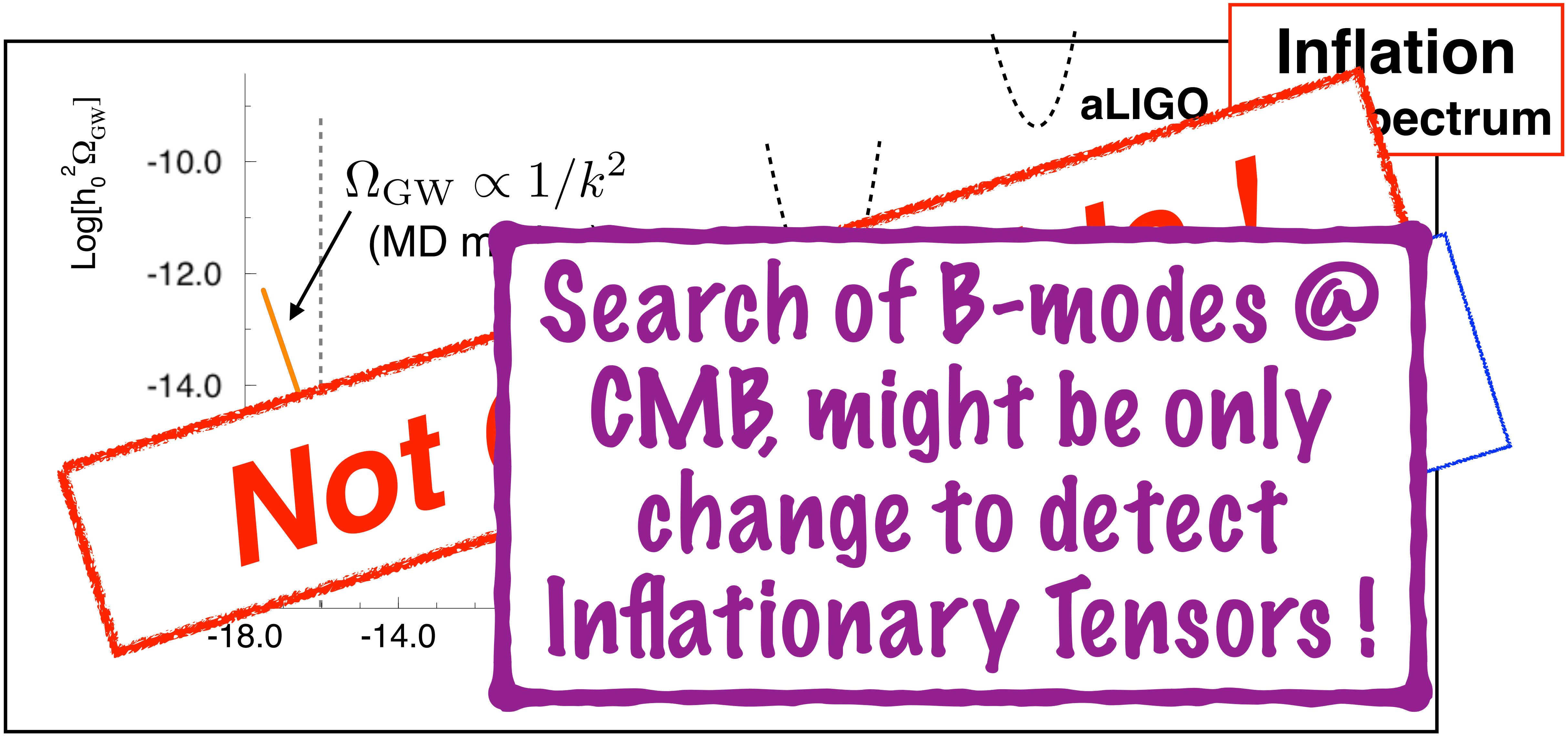
energy scale

$$\Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}$$

$$n_t \equiv -2\epsilon$$

Small red-tilt

Transfer Funct.: $T(k) = \begin{cases} 1 & ,\mathrm{RD} \\ (k_{\rm eq}/k)^2 & ,\mathrm{MD} \end{cases}$



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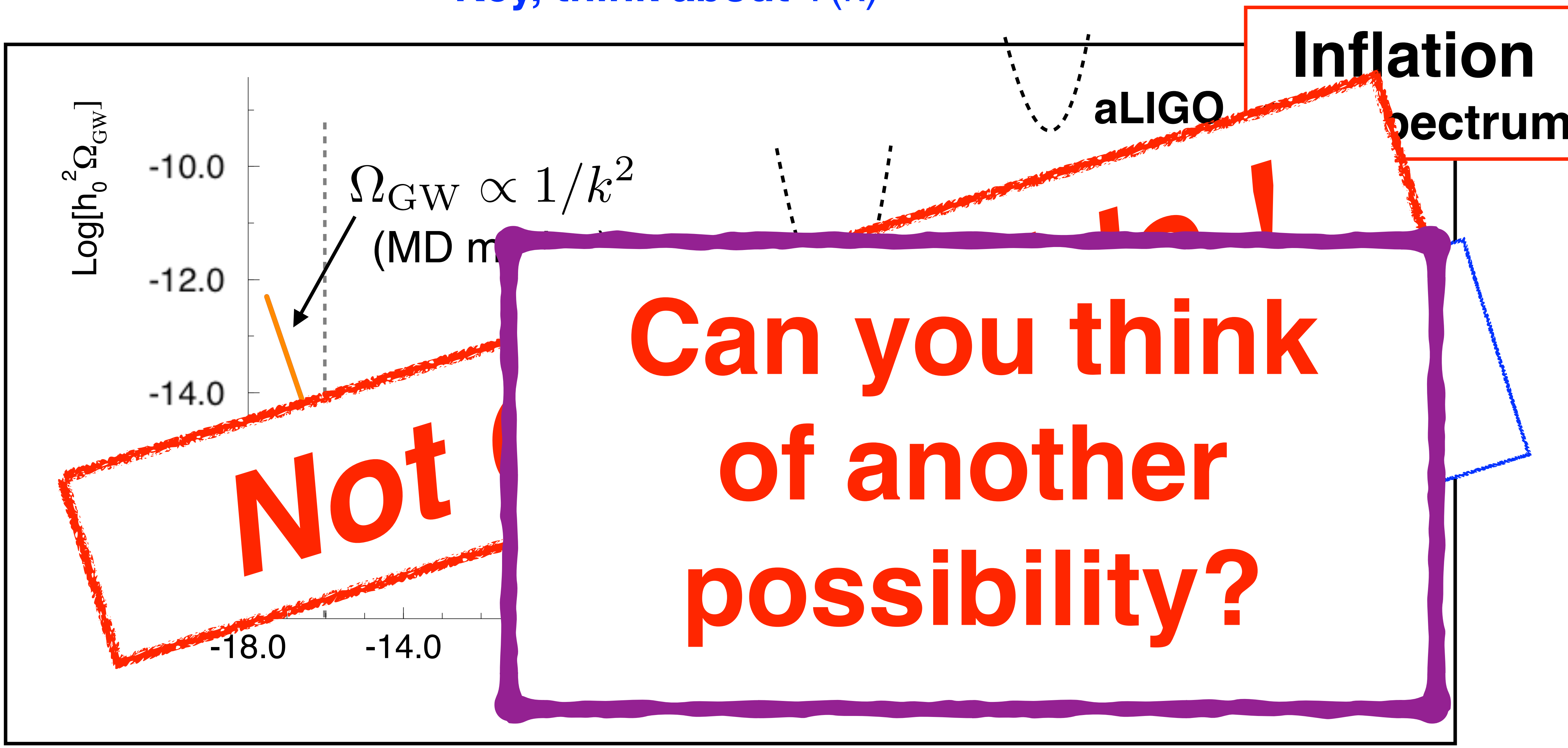
Key, think about T(k)



Can you think of another possibility?

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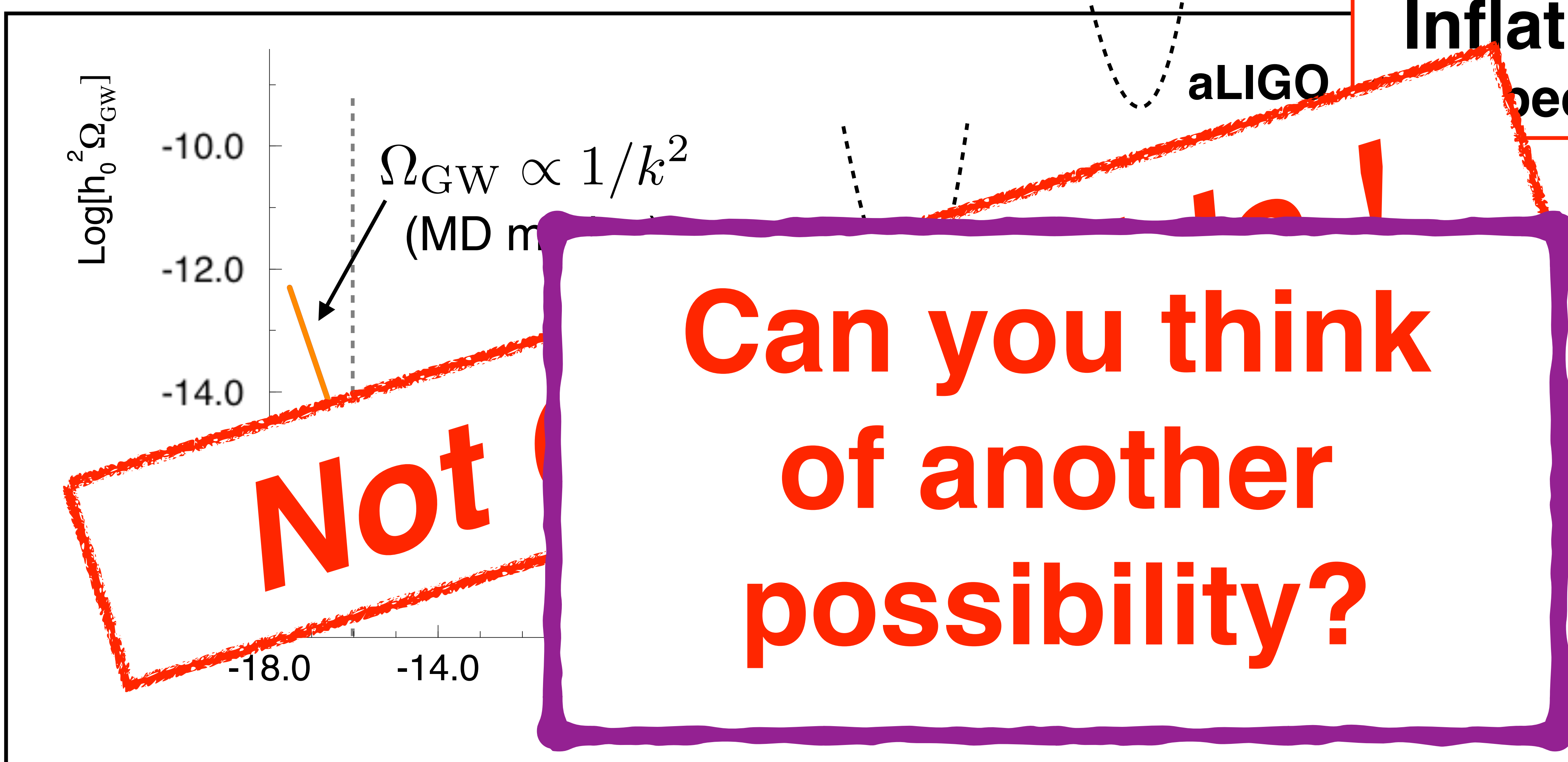

Inflation spectrum

Not

Can you think of another possibility?

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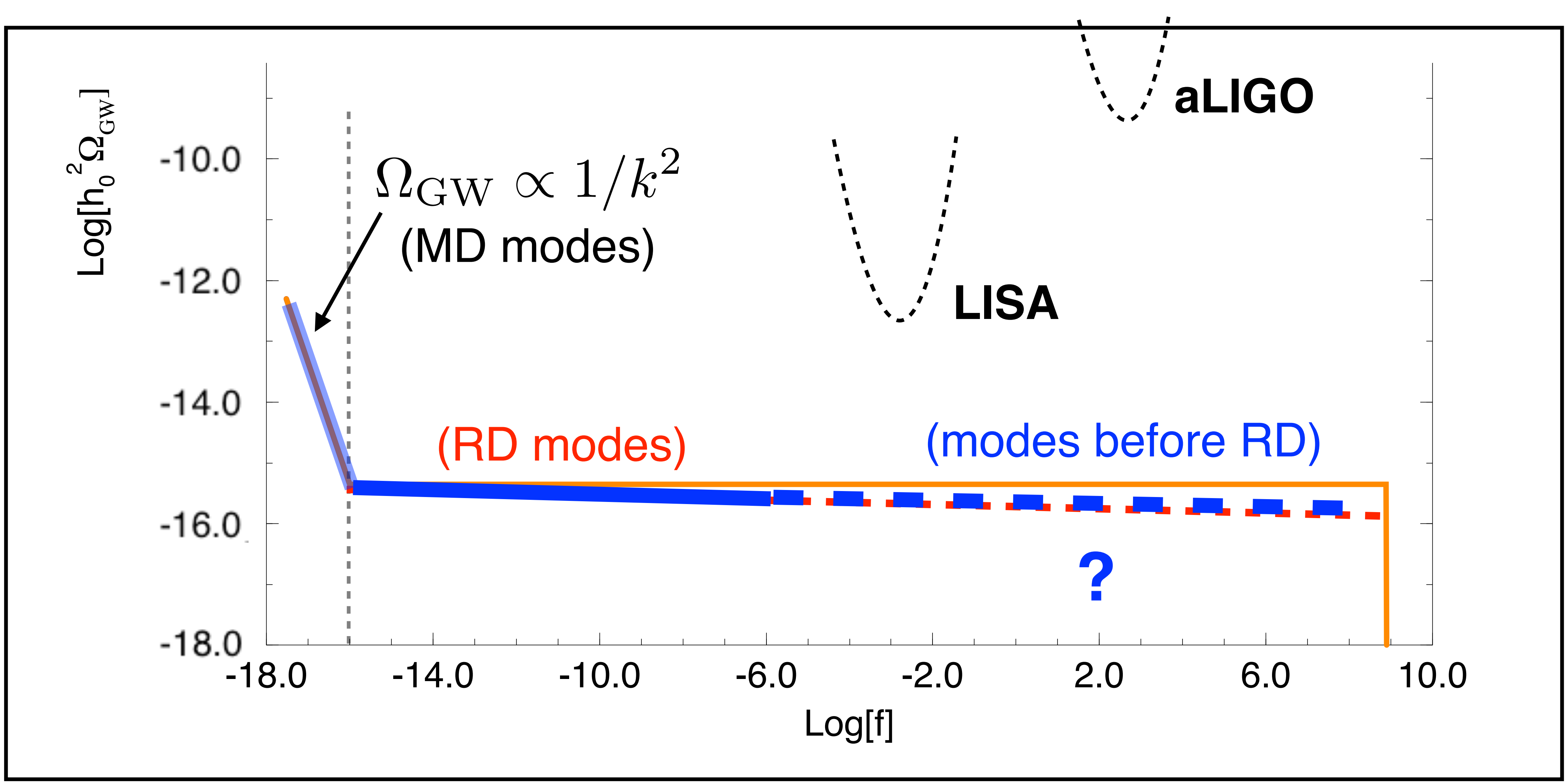

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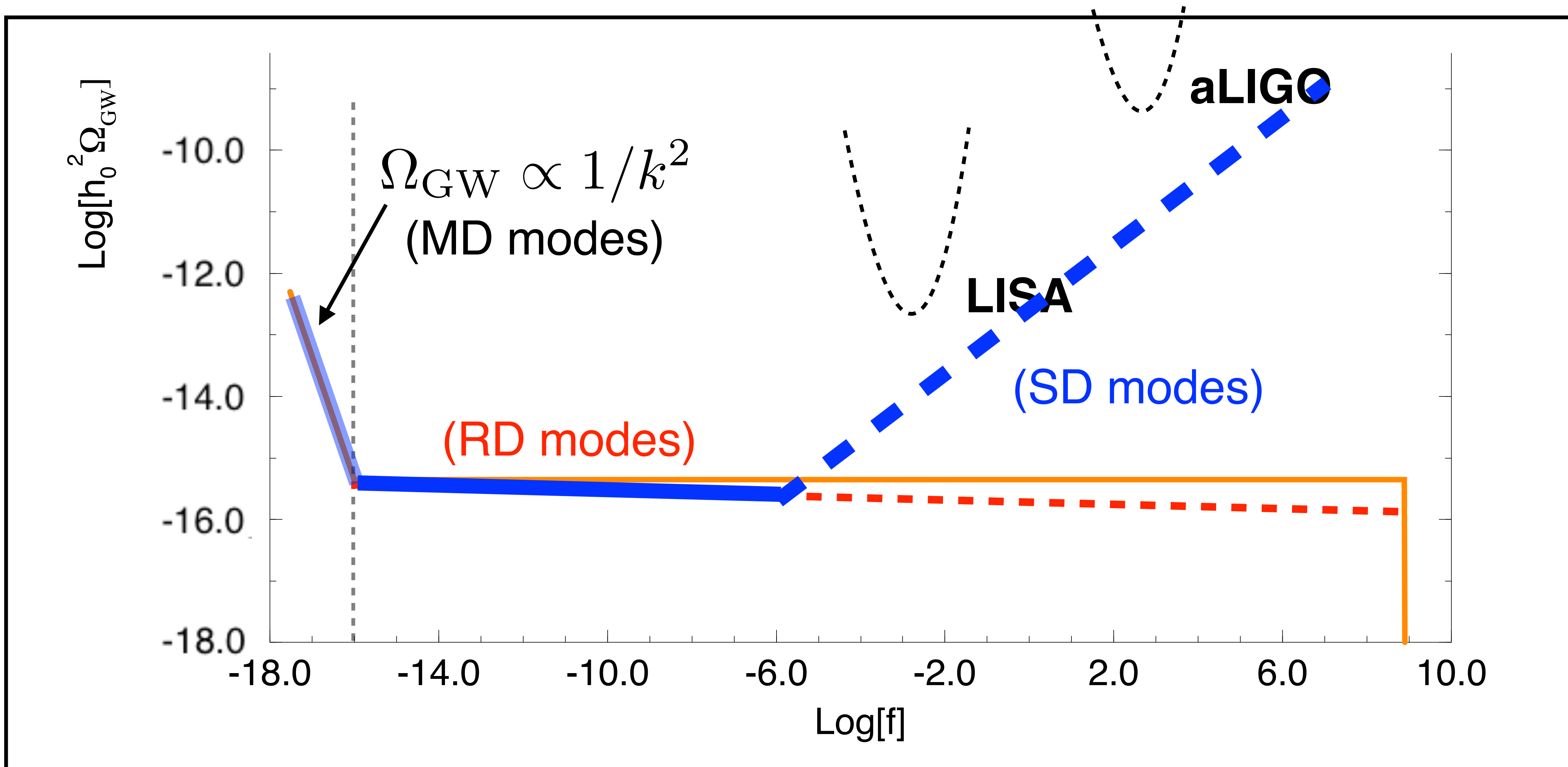

(

Realistic computation of Transfer function @ Stiff Domination —> —> Radiation Dom.

Inflationary GW background

$$\boxed{\Omega_{\rm GW}^{(0)}(f) = \underbrace{\Omega_{\rm GW}^{(0)}\Big|_{\rm plateau}}_{\textbf{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\rm RD}) \times \mathcal{A}_{\rm s}\left(\frac{f}{f_{\rm RD}}\right)^{n_t(w_s)}}_{\textbf{Transfer Funct. Stiff Period}}}\,, \quad \boxed{\Omega_{\rm GW}^{(0)}\Big|_{\rm plateau} \simeq 2\cdot 10^{-16}\left(\frac{H_*}{H_{\rm max}}\right)^2}$$

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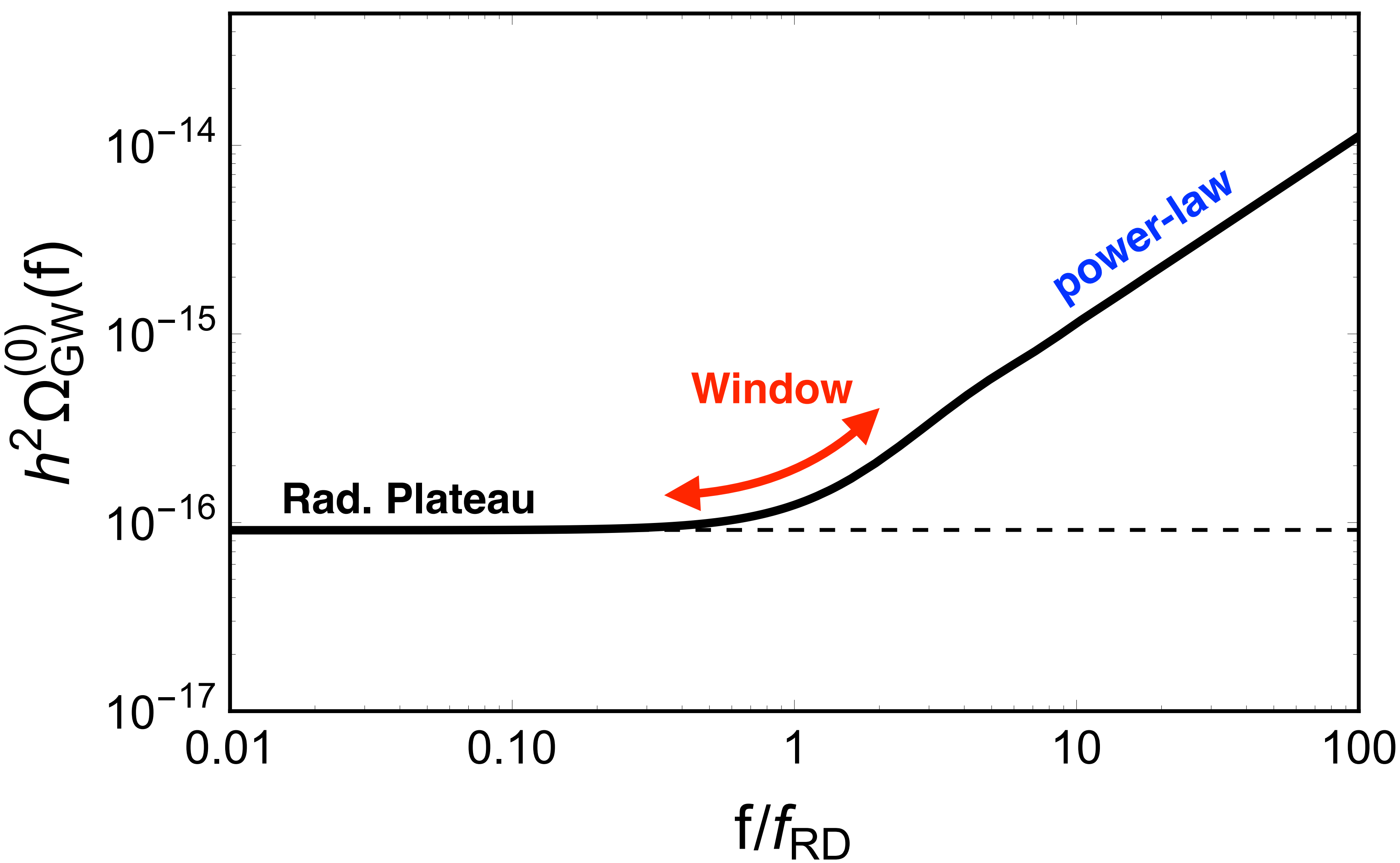
Window x *power-law*



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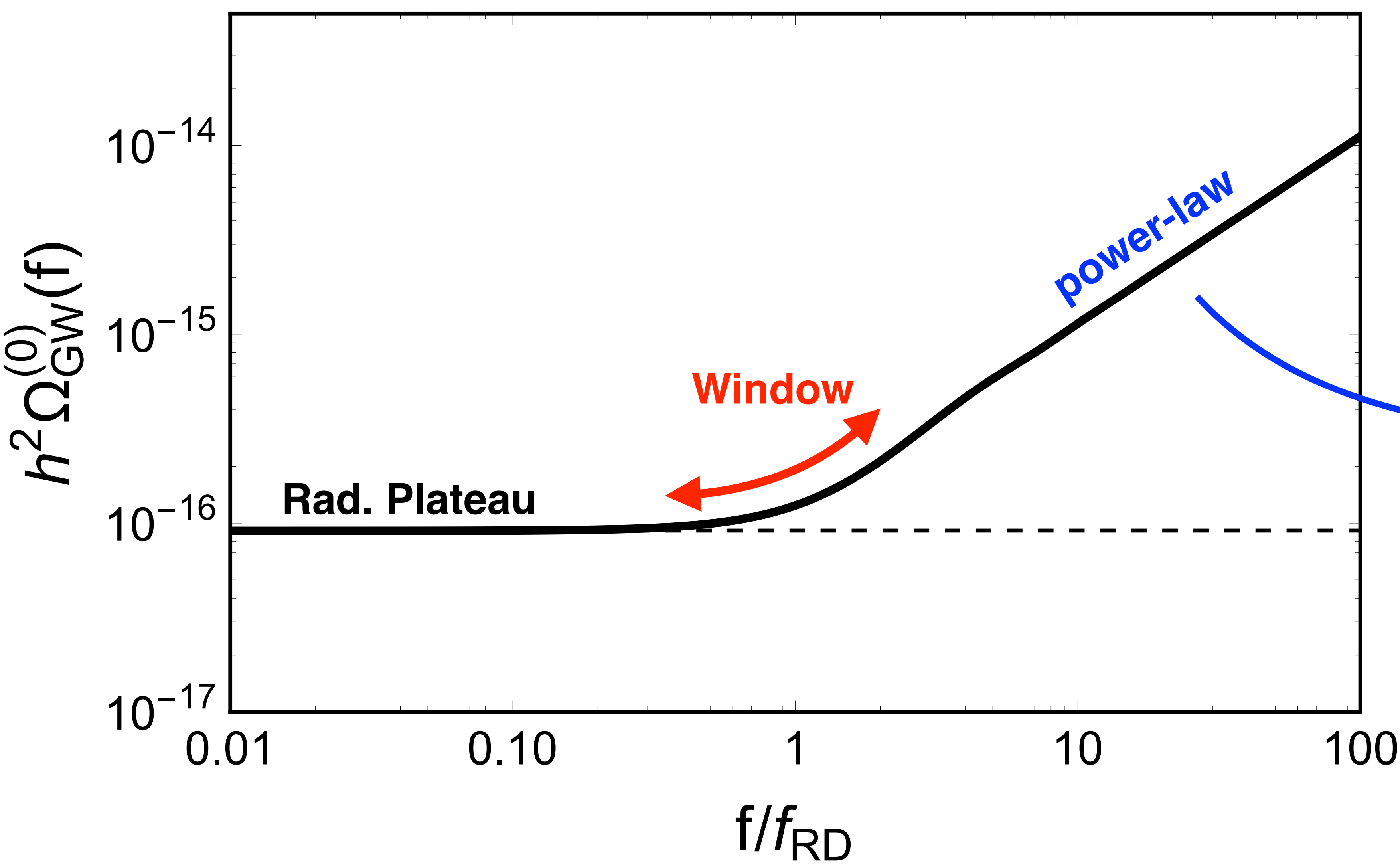

modes crossing during Stiff era

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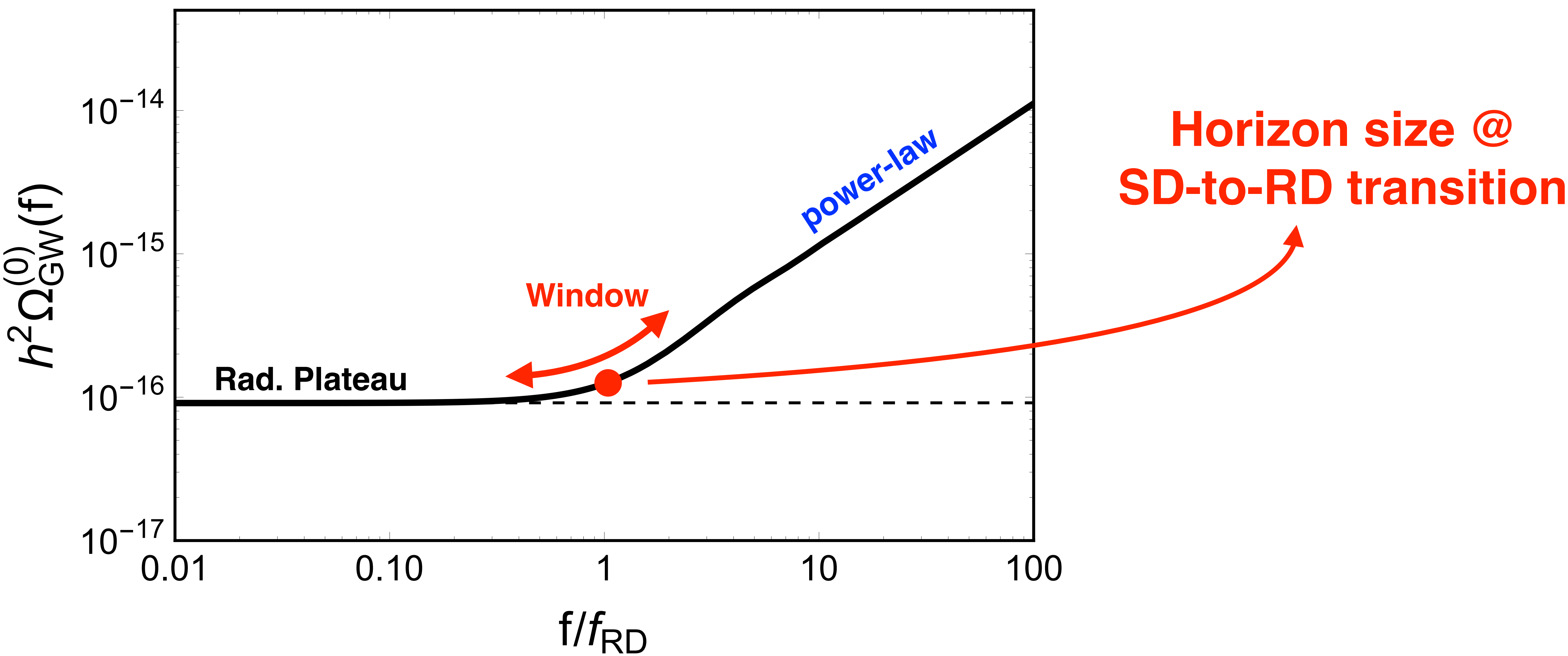

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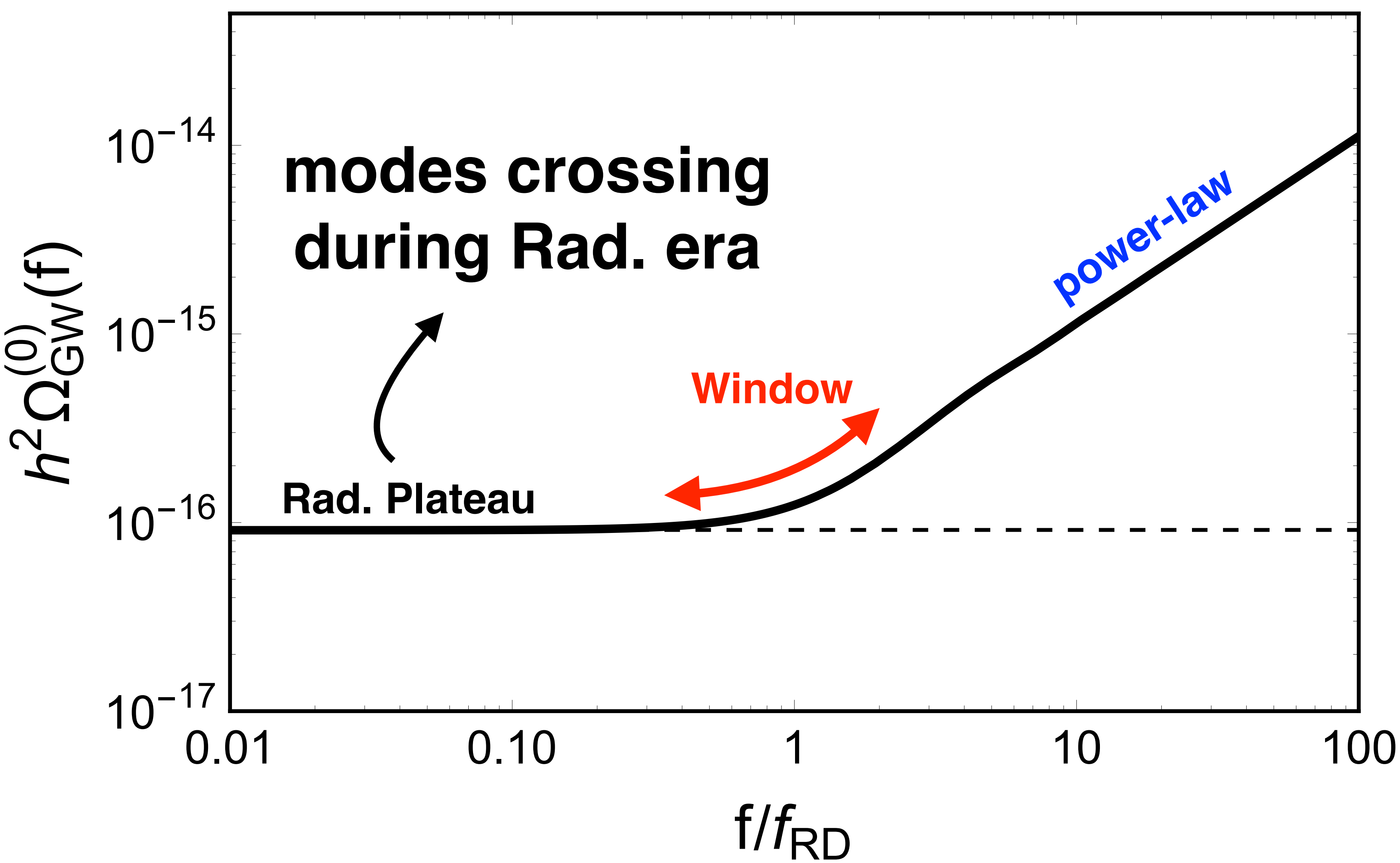

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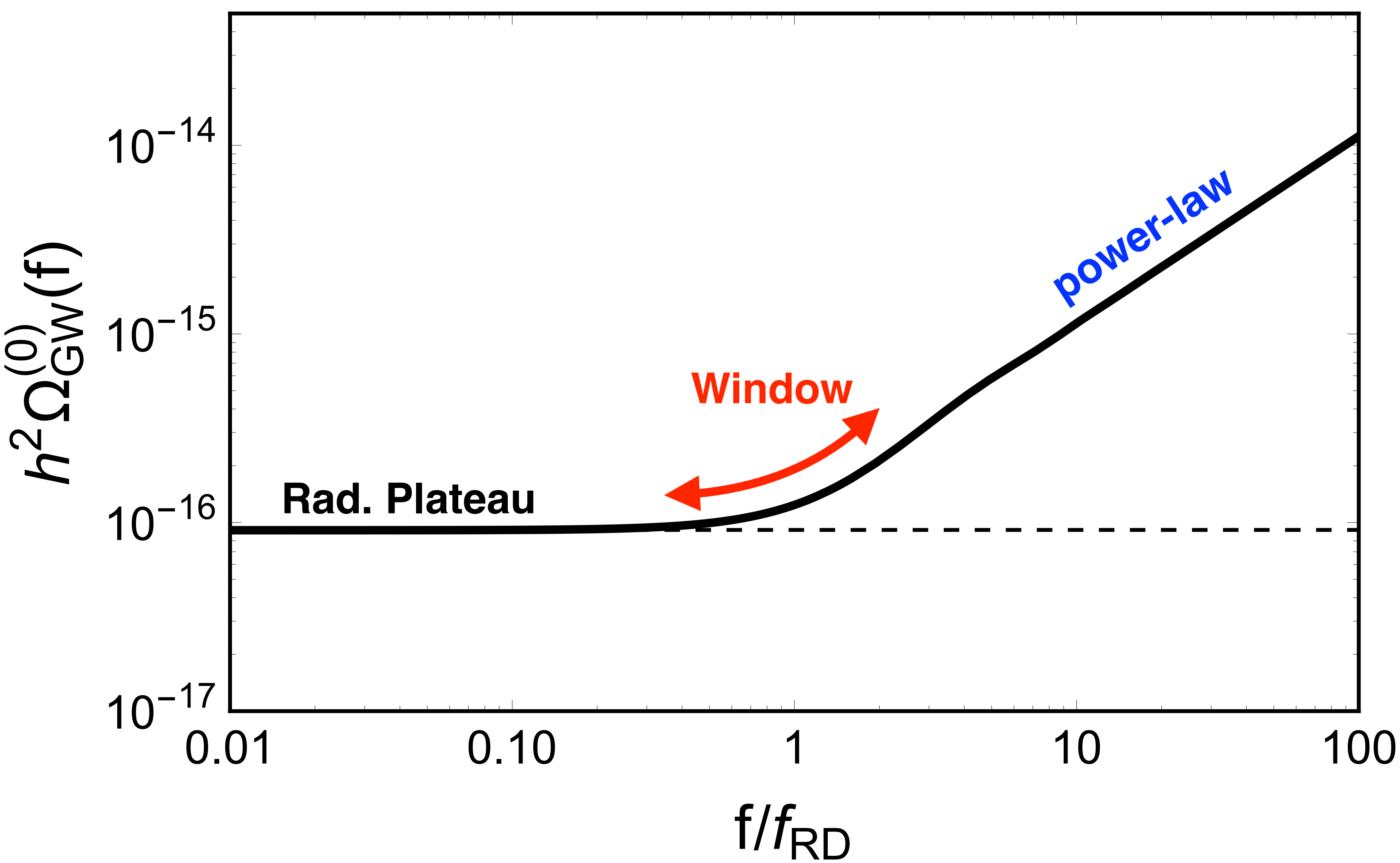

Is this the real GW spectrum ?

Inflationary GW background

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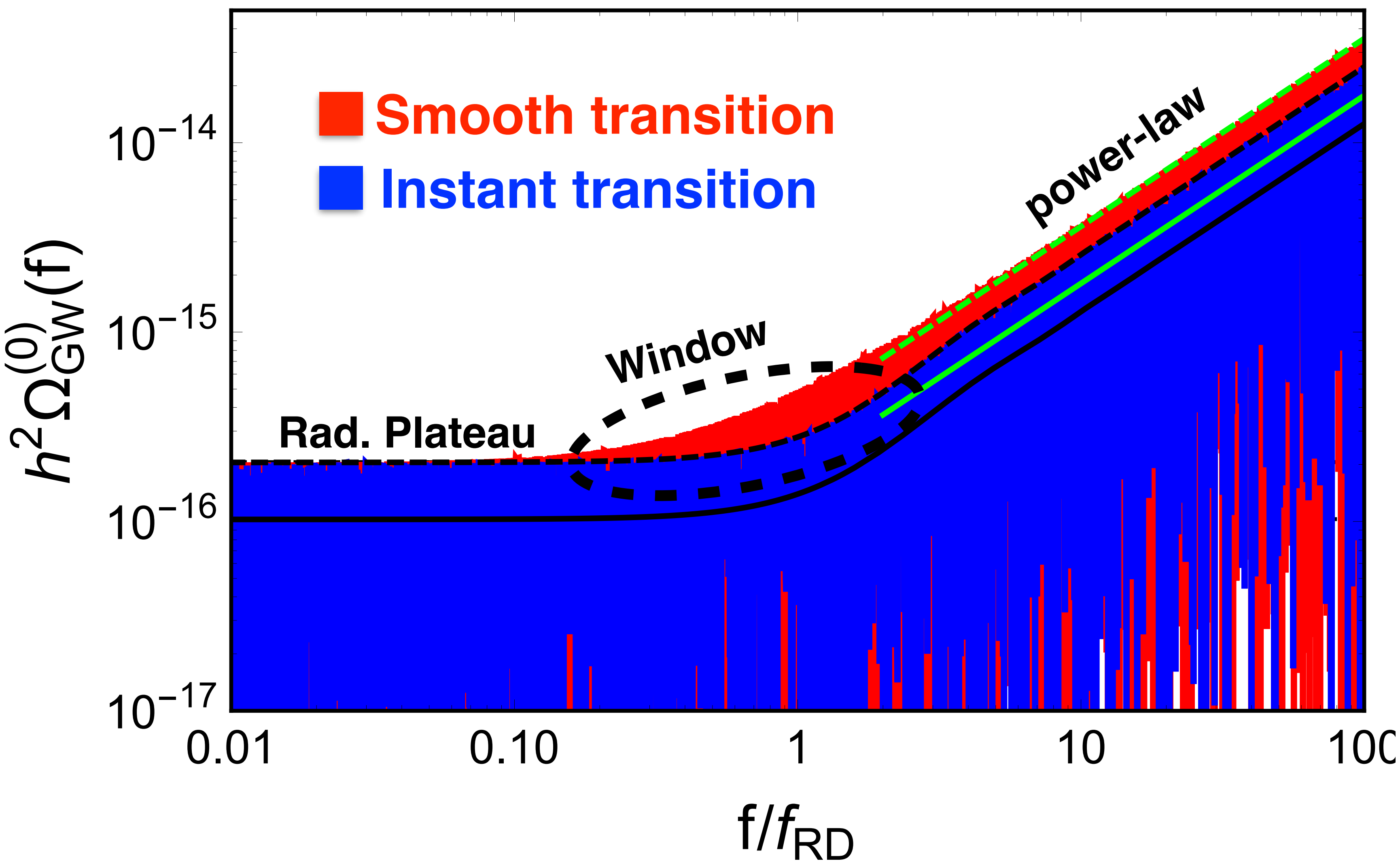

Real signal: highly oscillatory

Stochastic Signal: average measurement

$$\langle \dot{h}_{ij}(f)\dot{h}_{ij}(f)\rangle = \mathcal{P}_h(f)$$

Inflationary GW background

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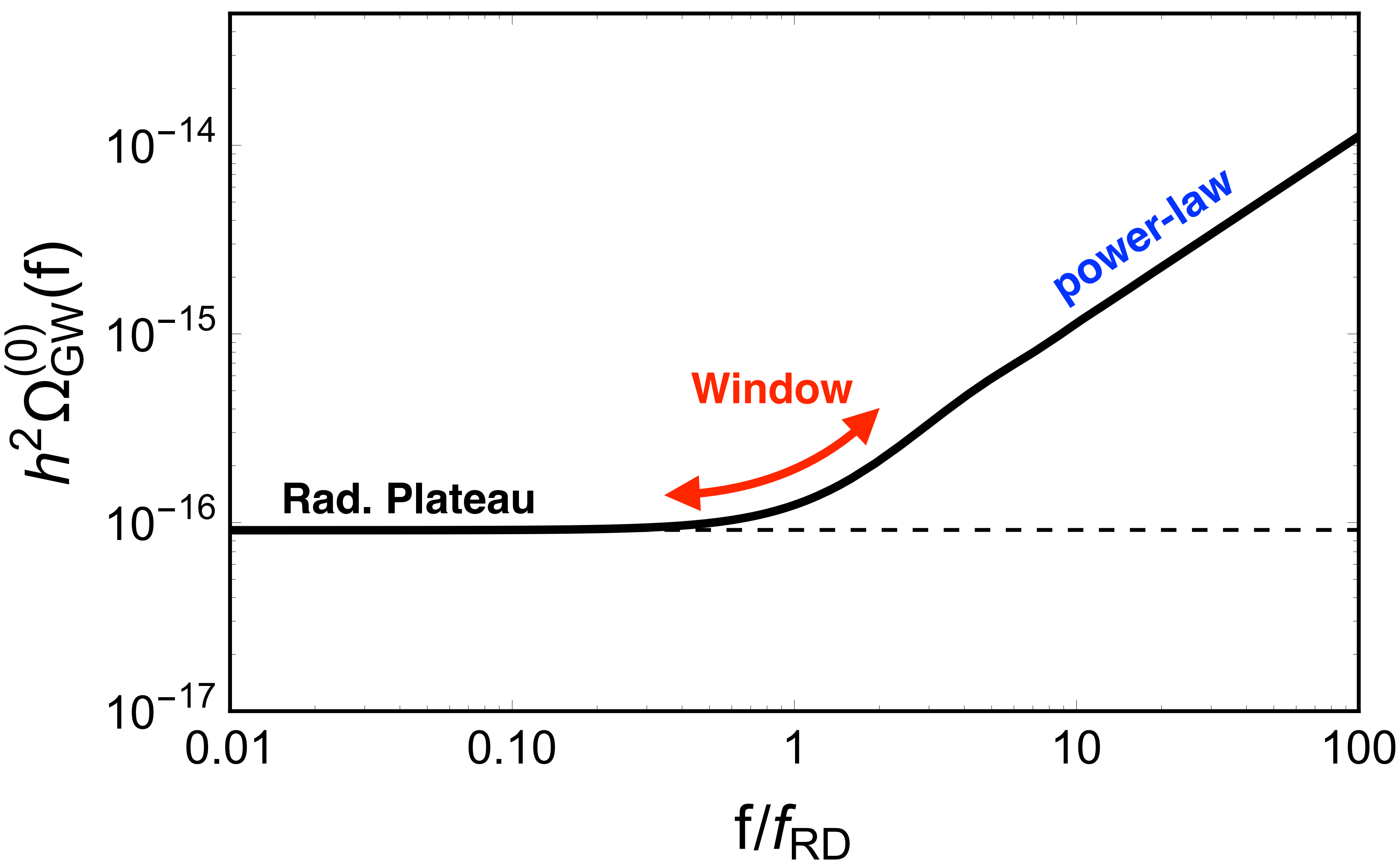

This is the oscillation averaged spectrum !

Inflationary GW background

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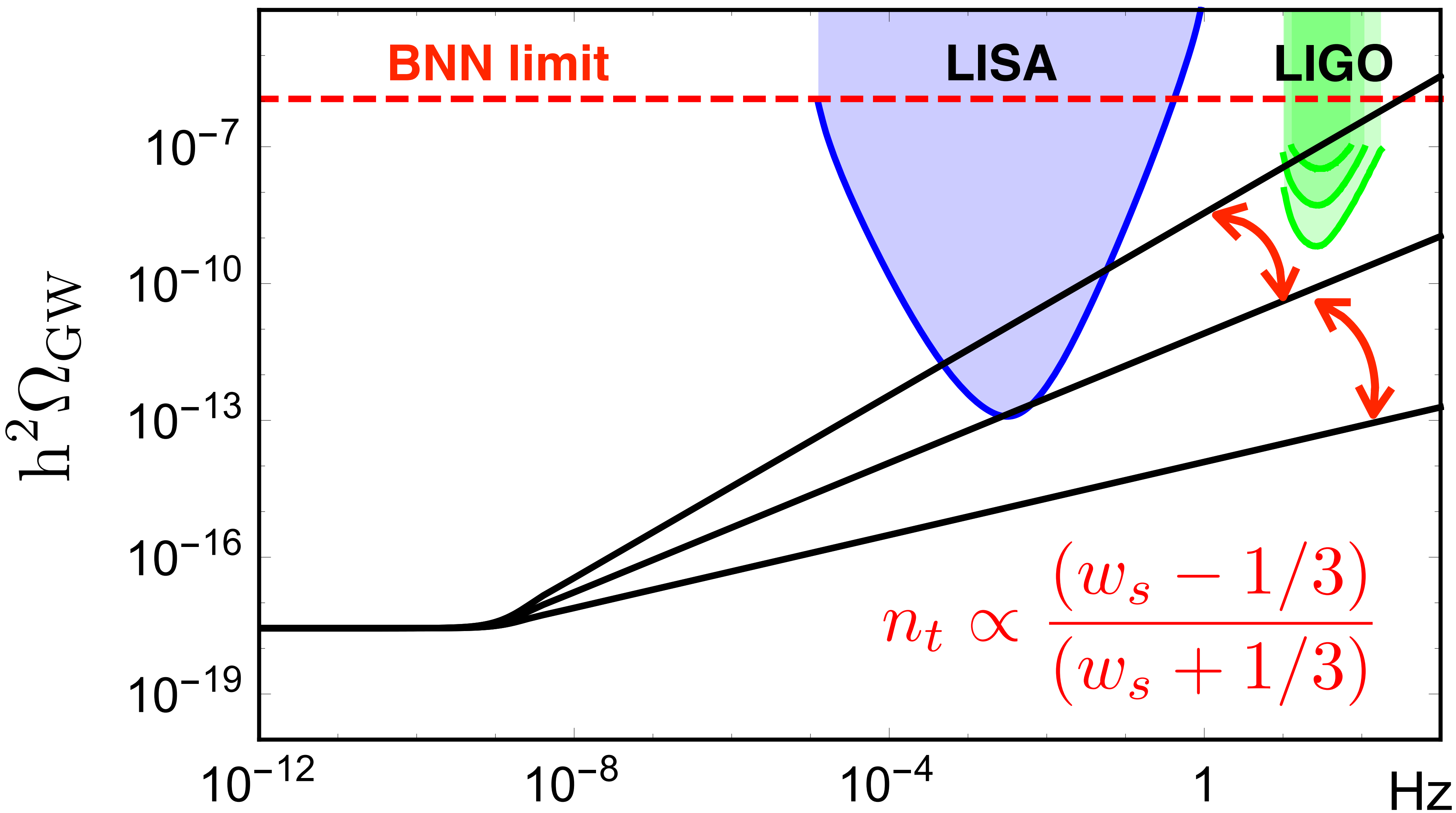

Slope/Tilt
(EoS Stiff Period)

Inflationary GW background

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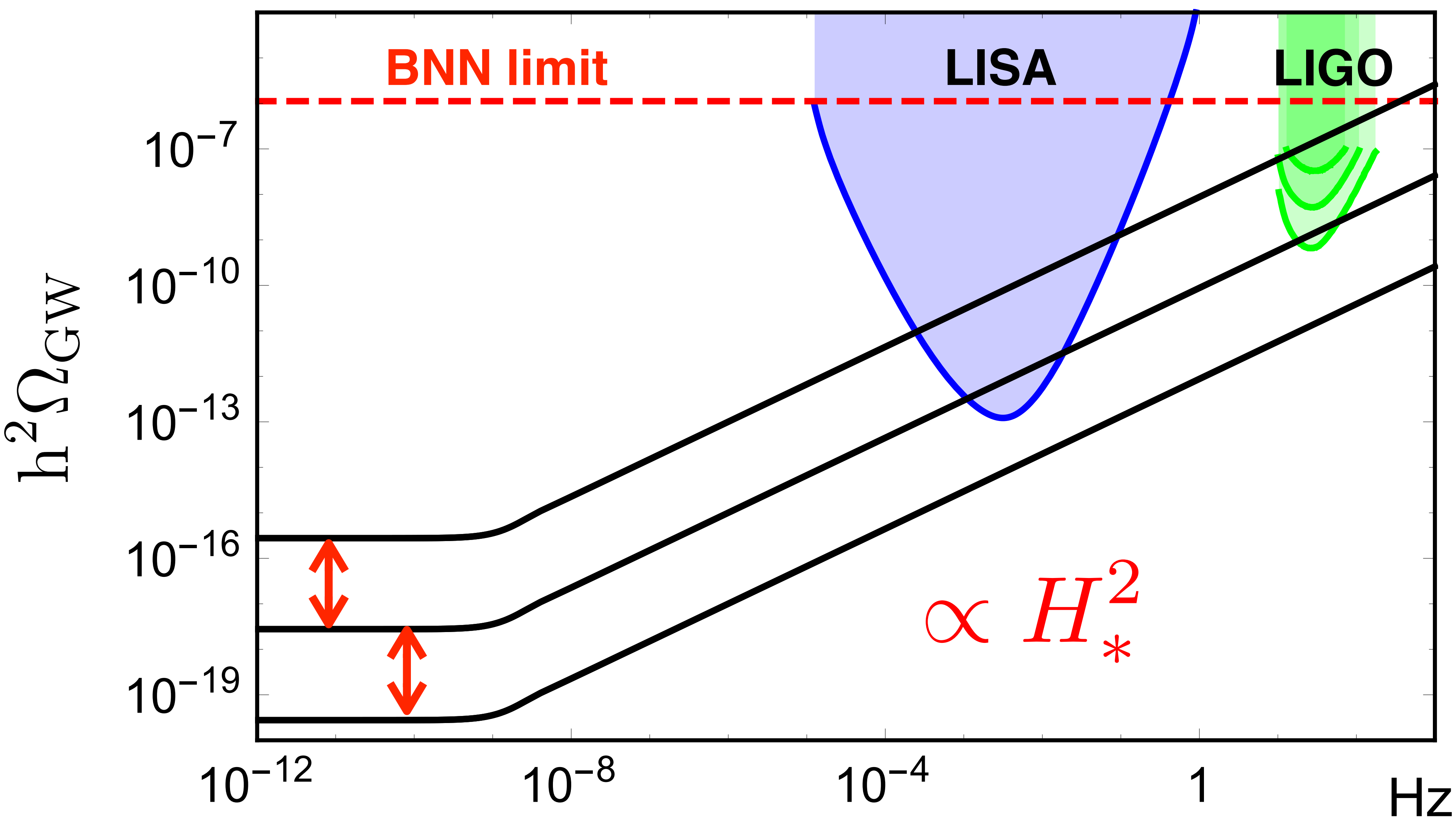

Overall Amplitude
(Energy Scale Inflation)

Inflationary GW background

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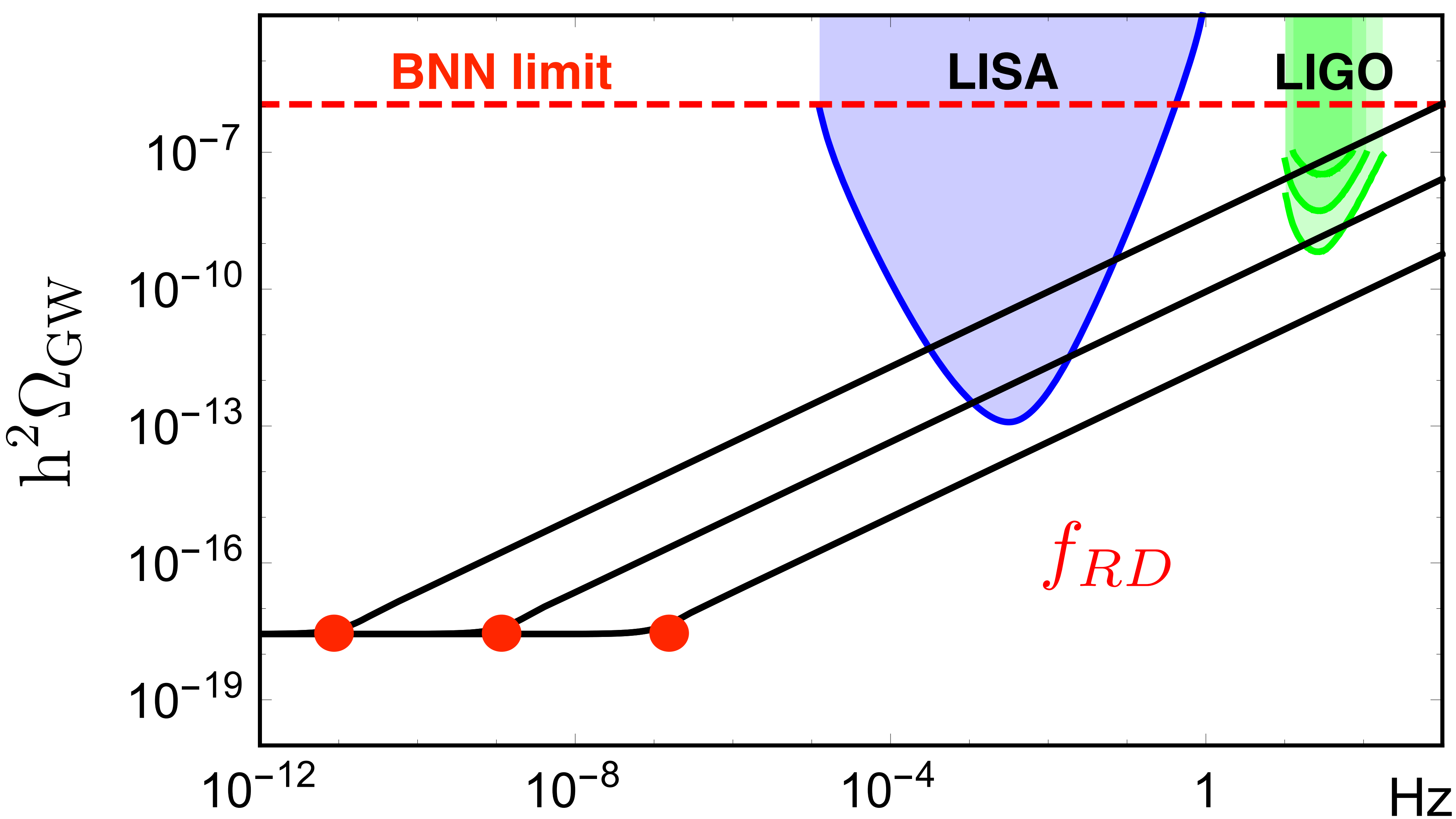


Freq. RD

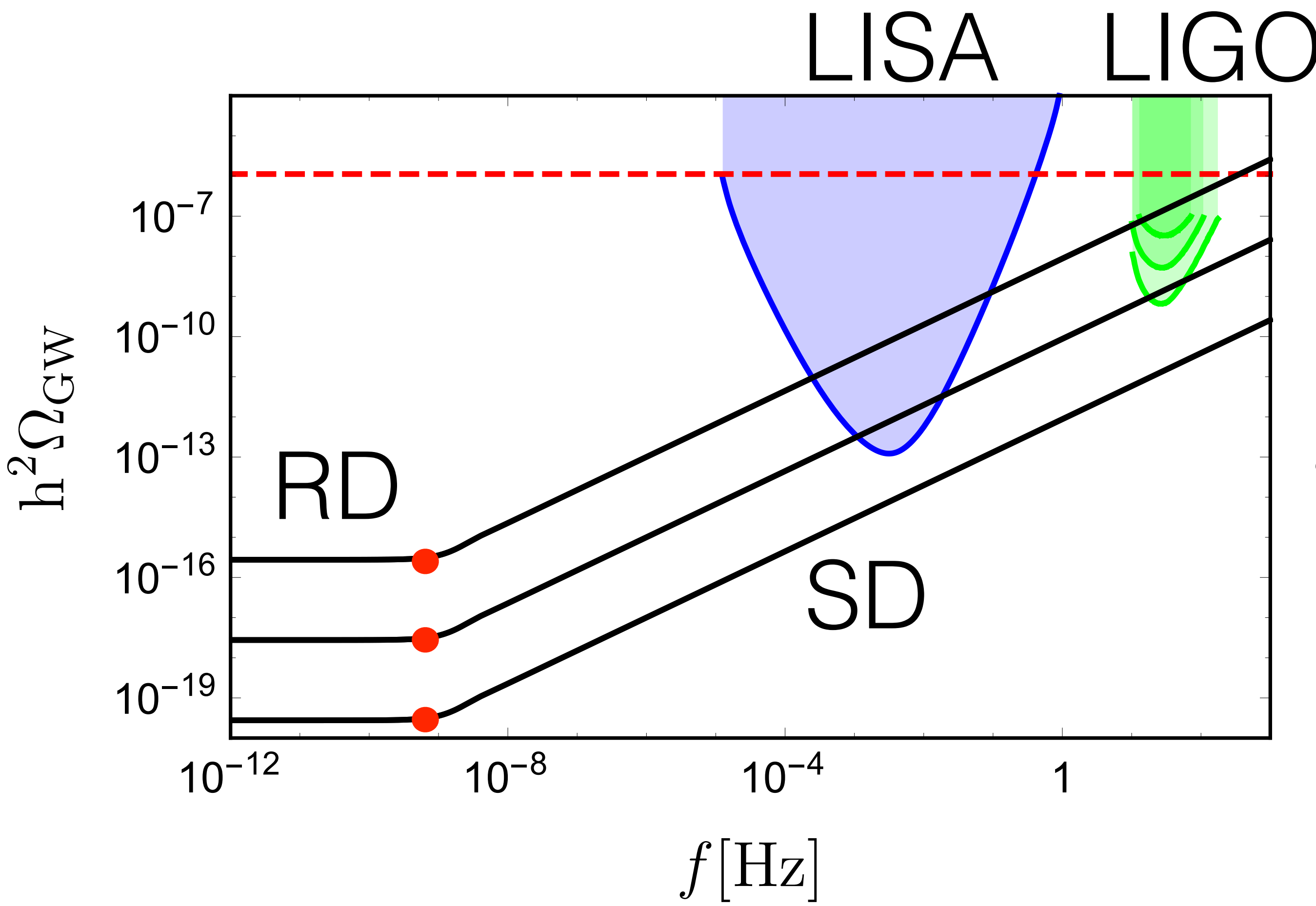
$$k_{\rm RD} = a_{\rm RD} H_{\rm RD}$$

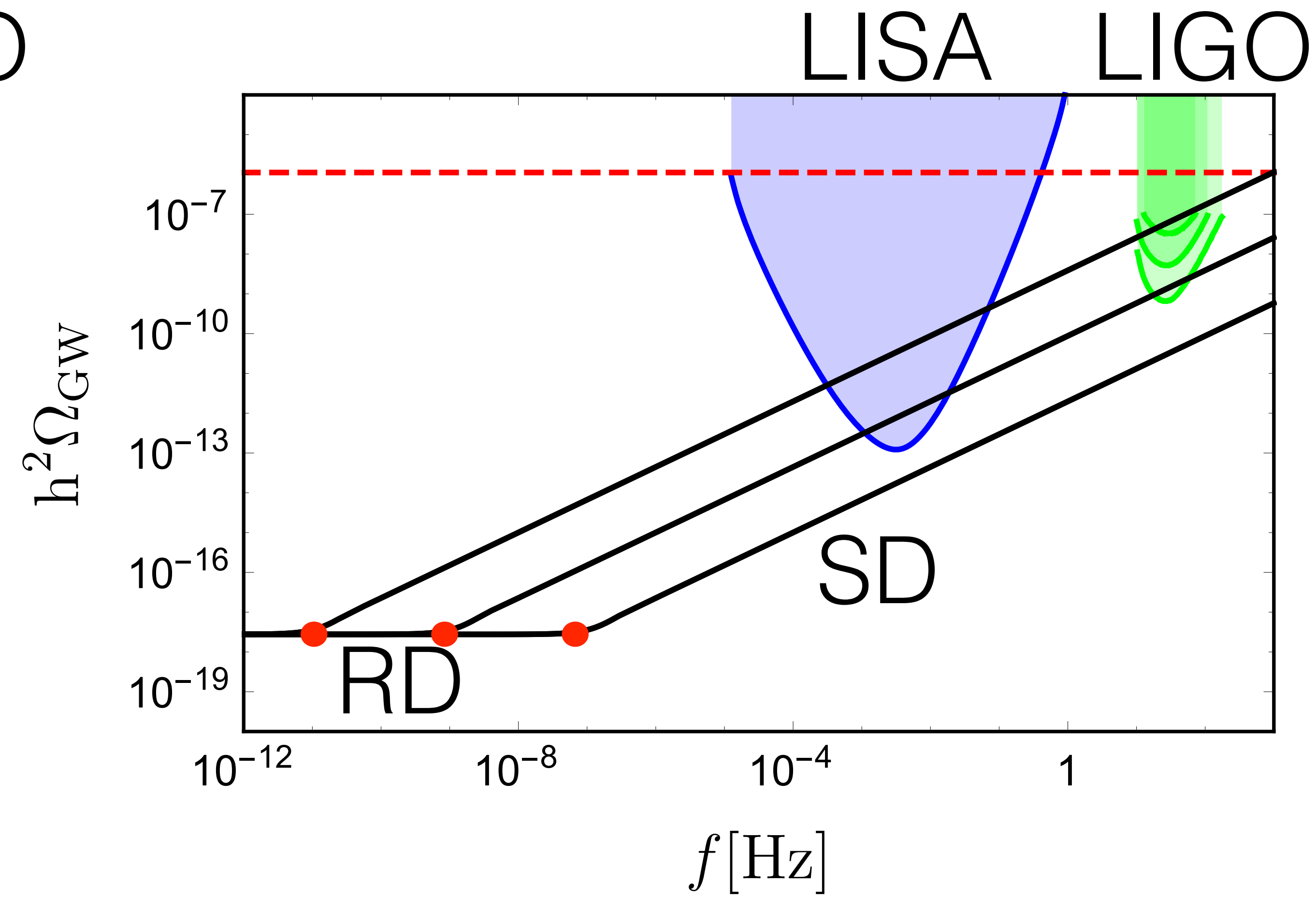
$$f_{\rm RD} \equiv k_{\rm RD}/(2\pi a_0)$$

SD-to-RD transition

)

STIFF EQ of STATE $(1/3 < \omega_s < 1)$





$$\Omega_{\rm GW}(f) \propto H_{\rm inf}^2 \left(\frac{f}{f_{\rm RD}}\right)^{\frac{2(w-1/3)}{(w+1/3)}}$$

Not Scale Invariant !

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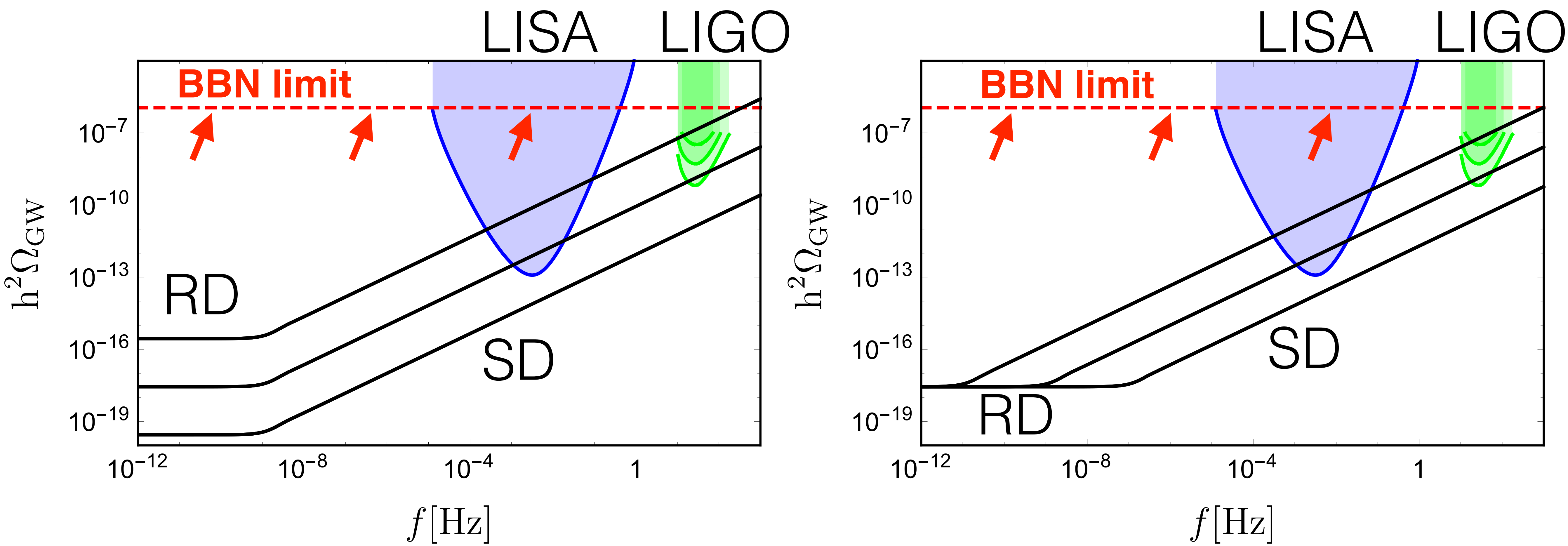

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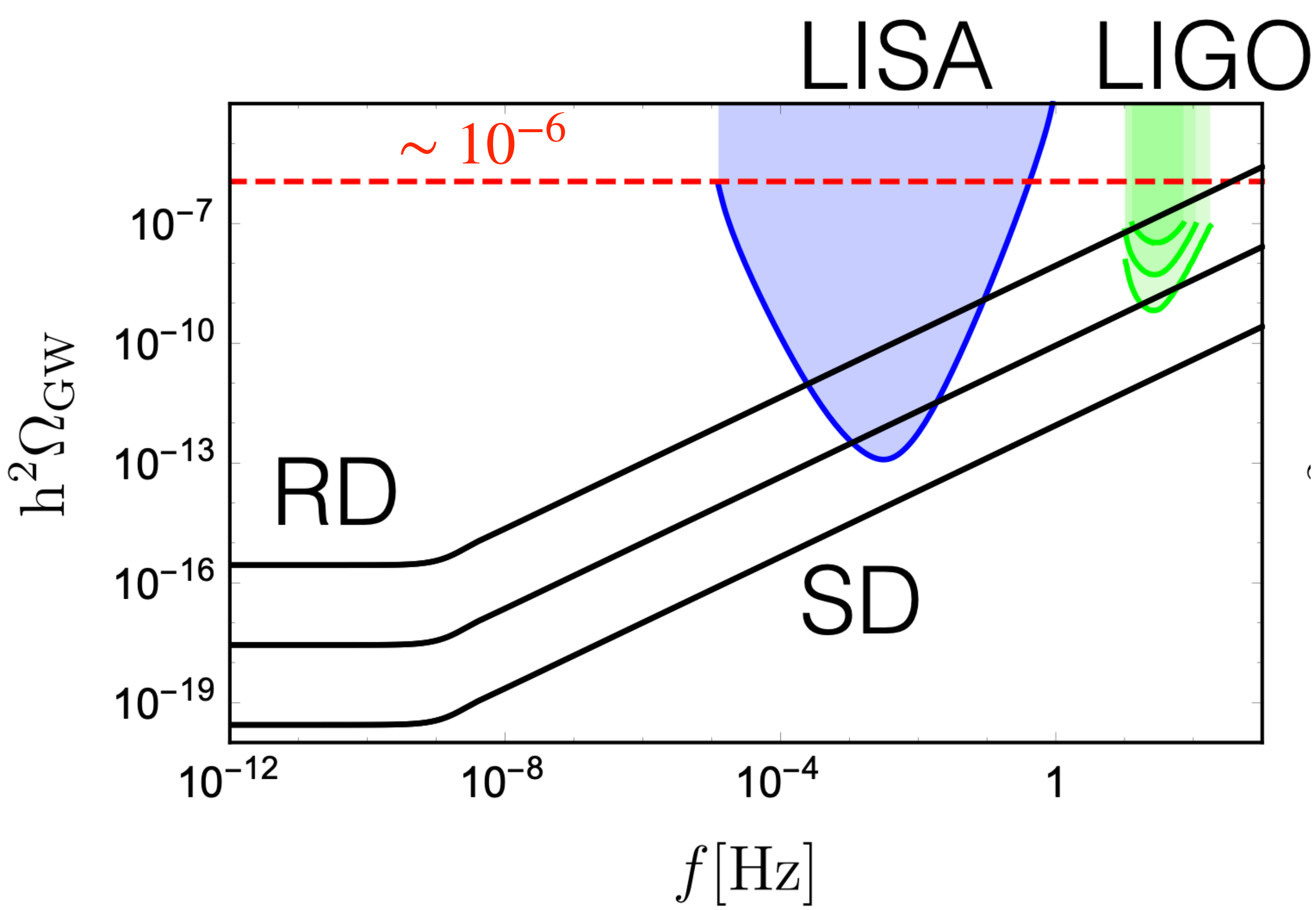

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DGF, Tanin '19, 1905.11960

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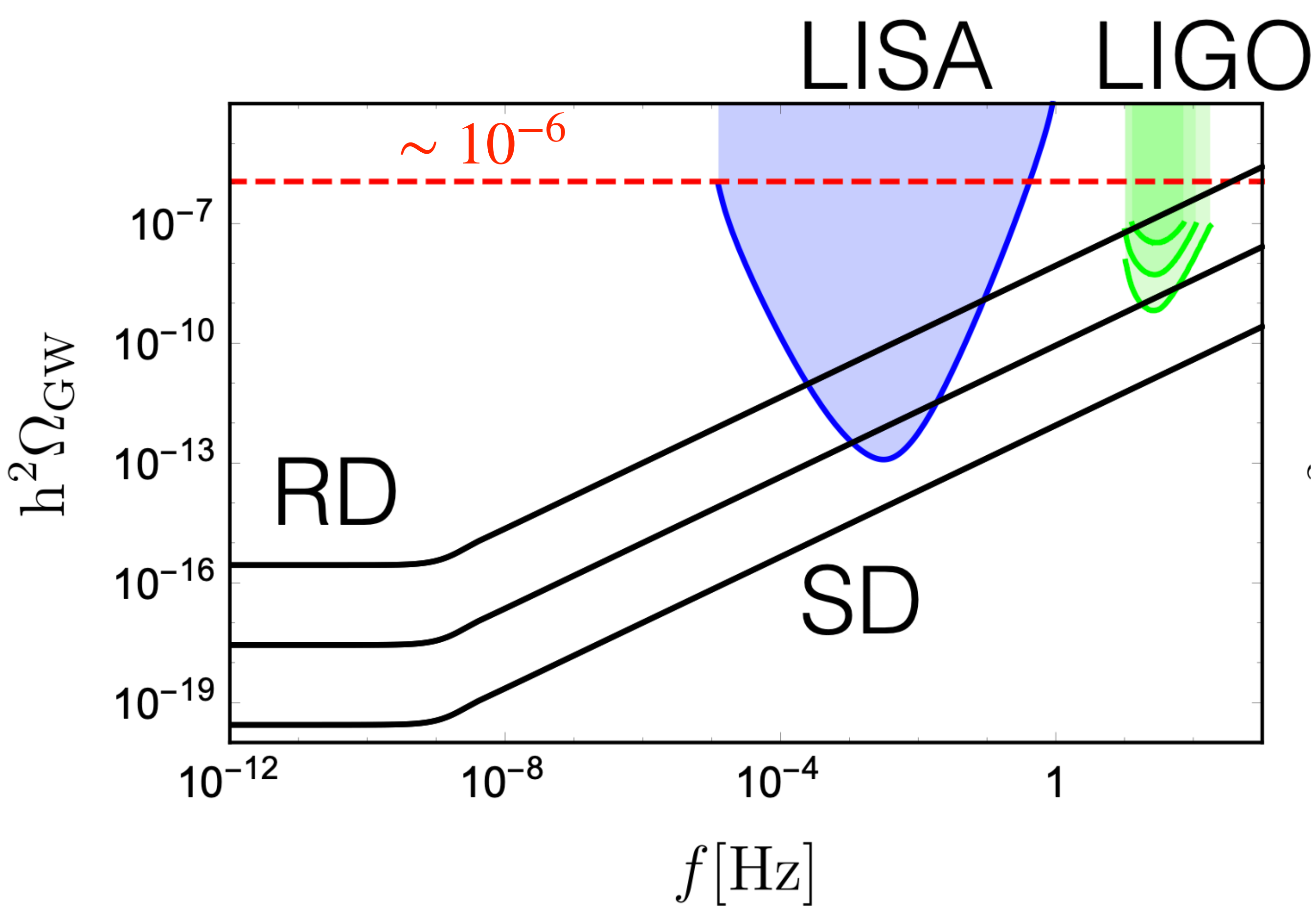

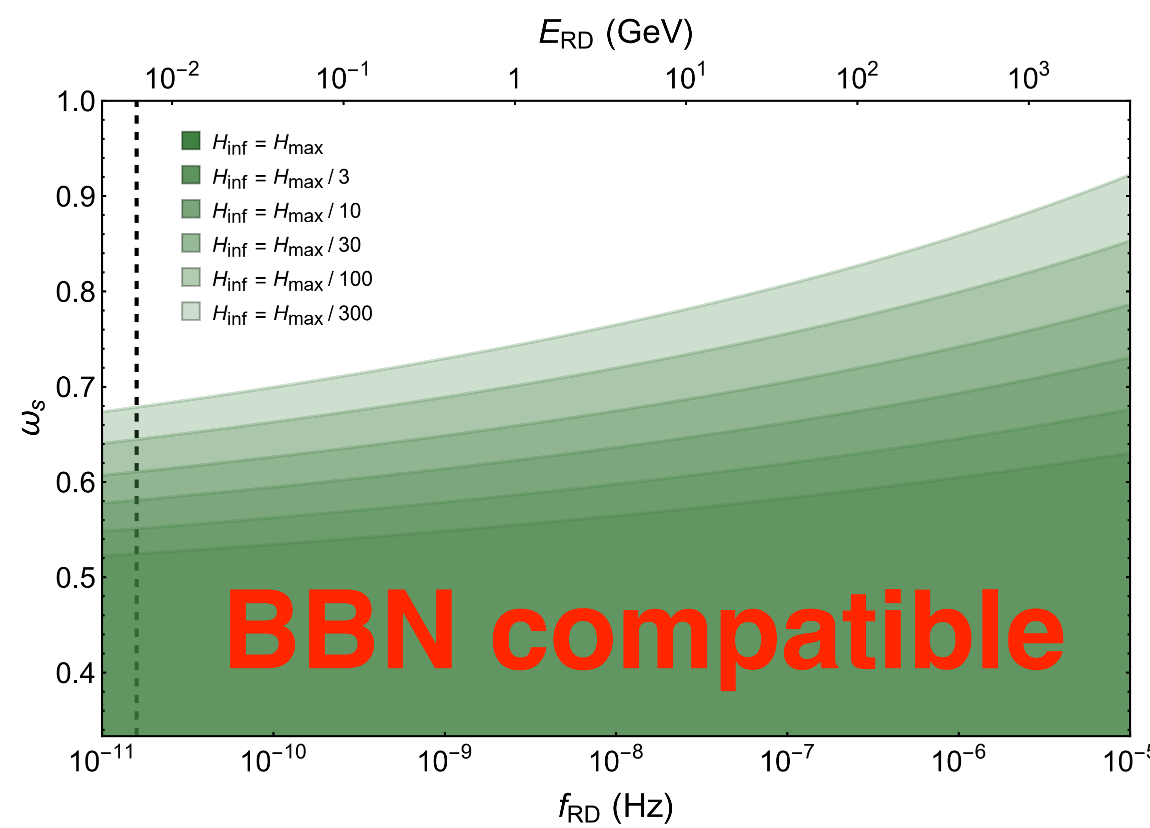

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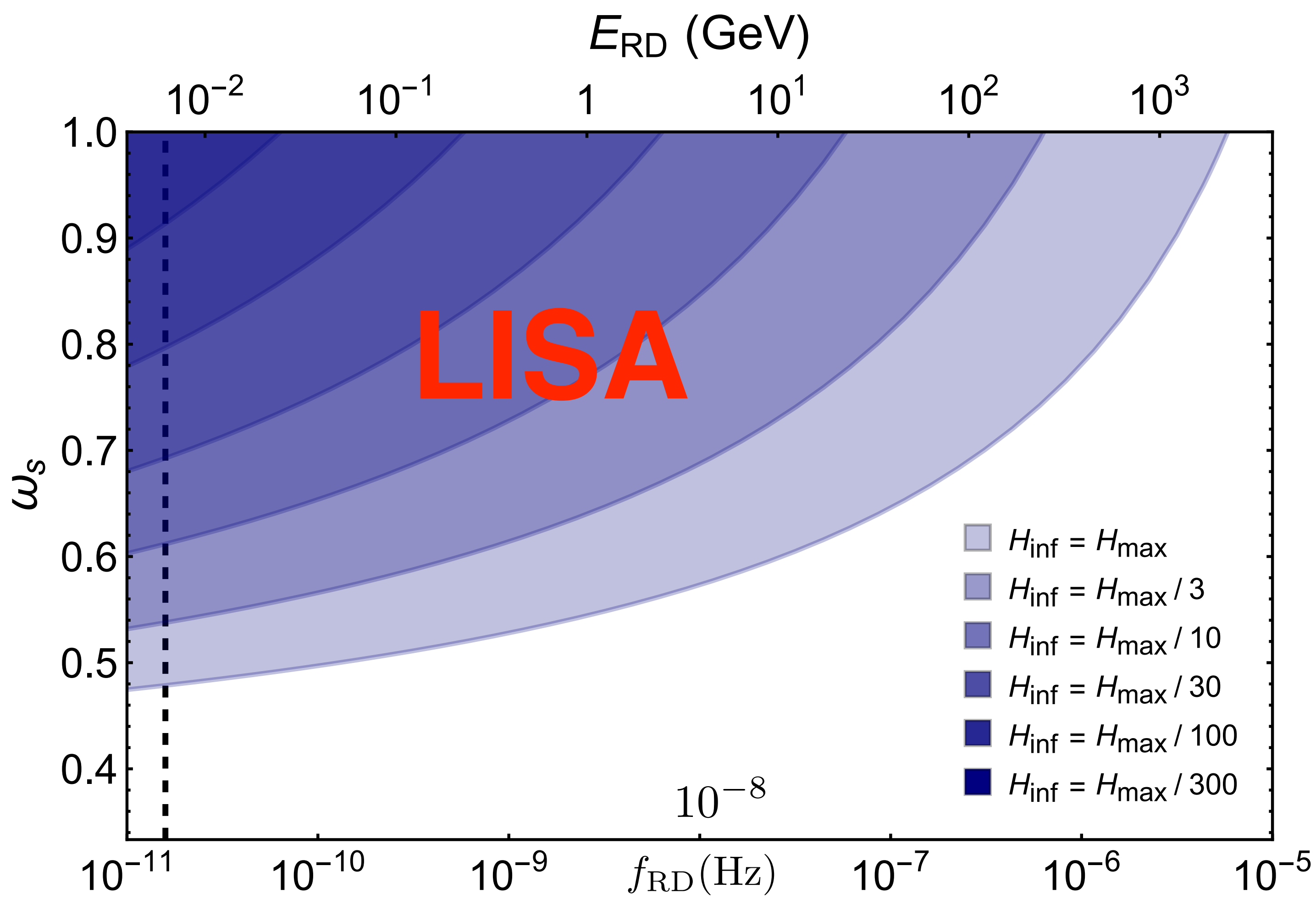

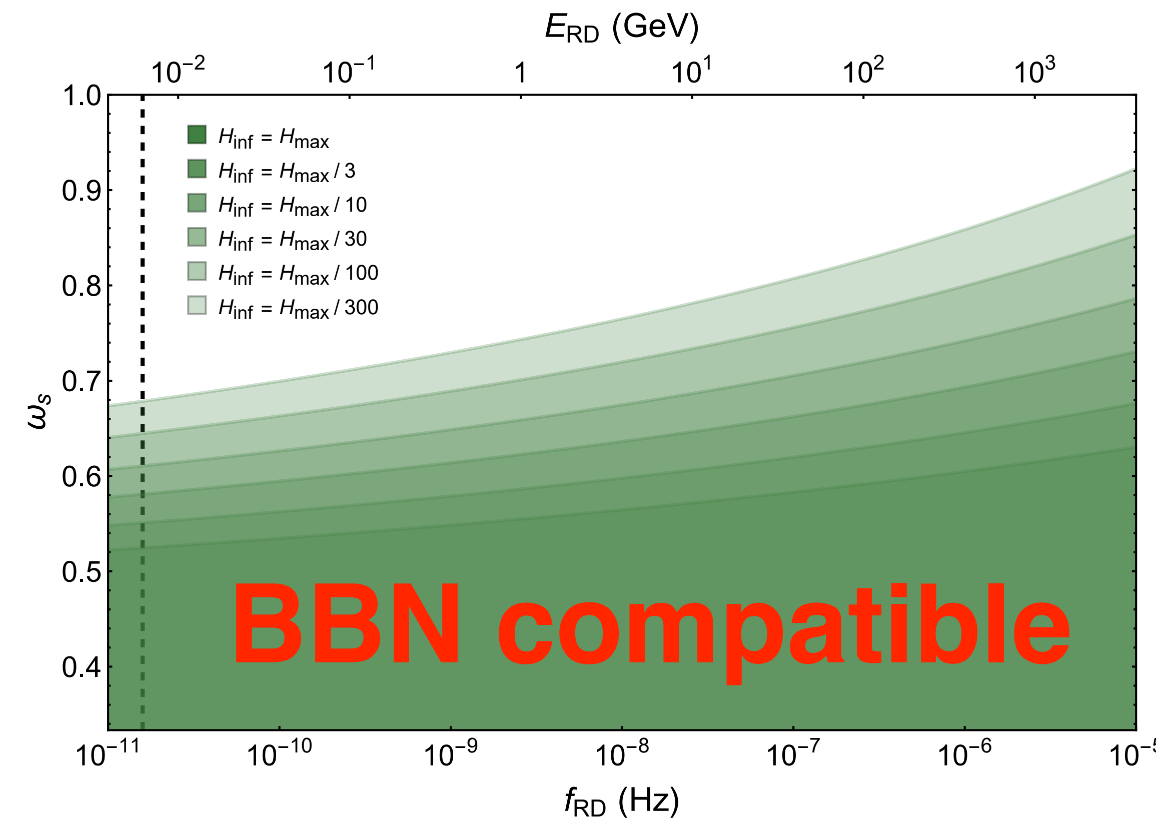

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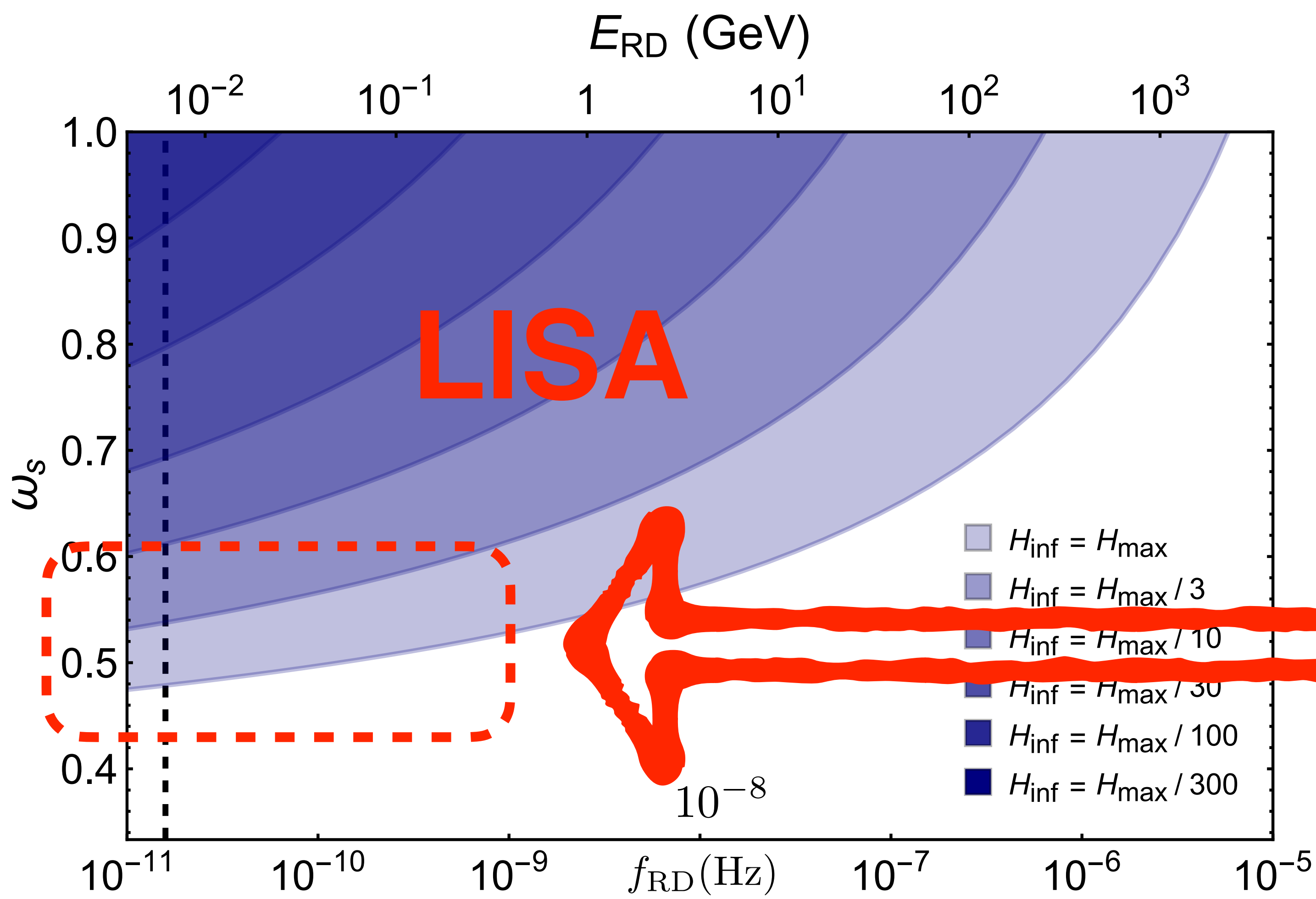

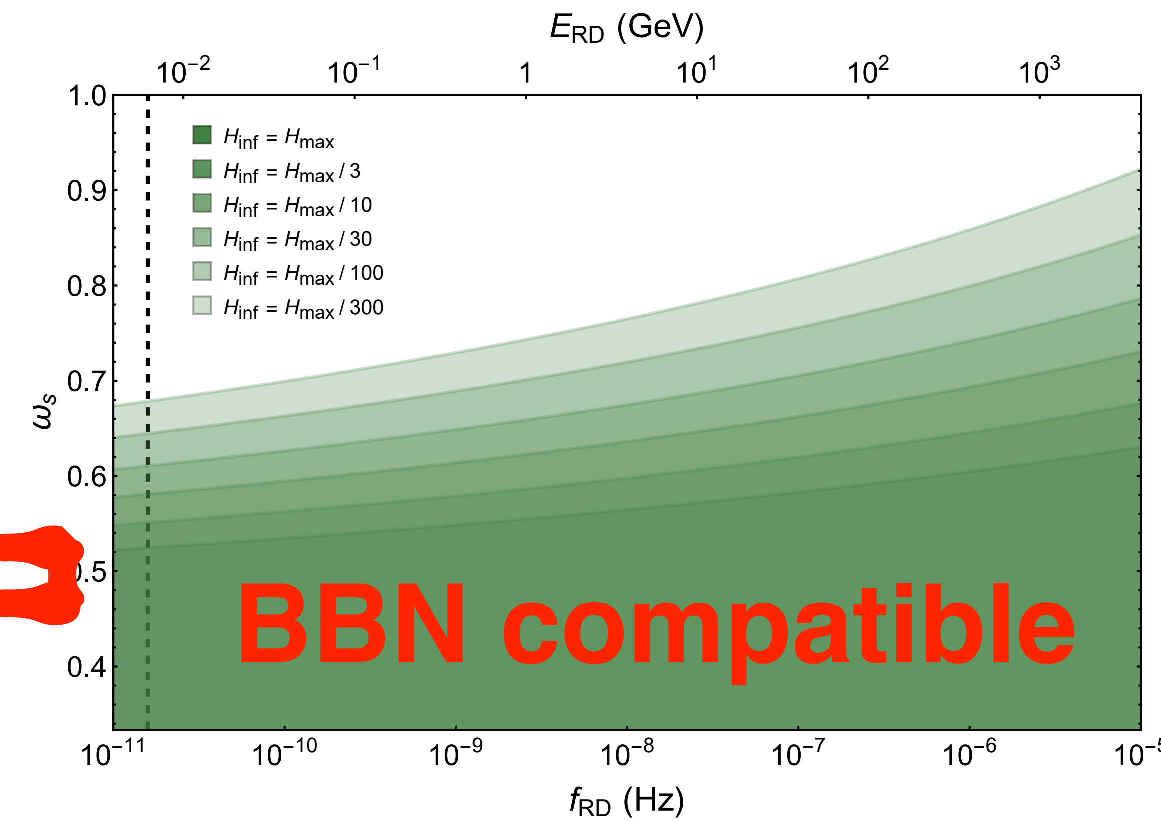

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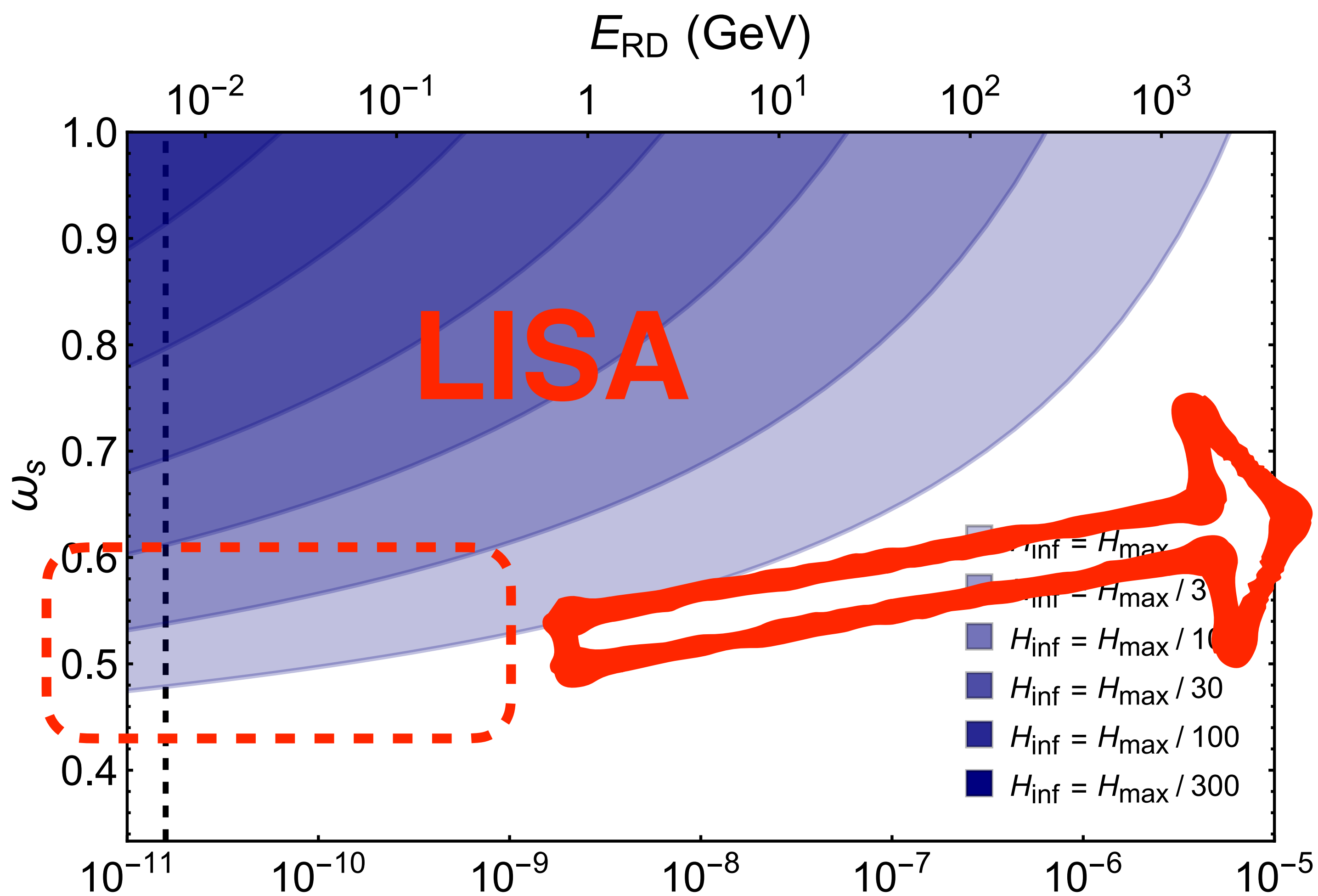
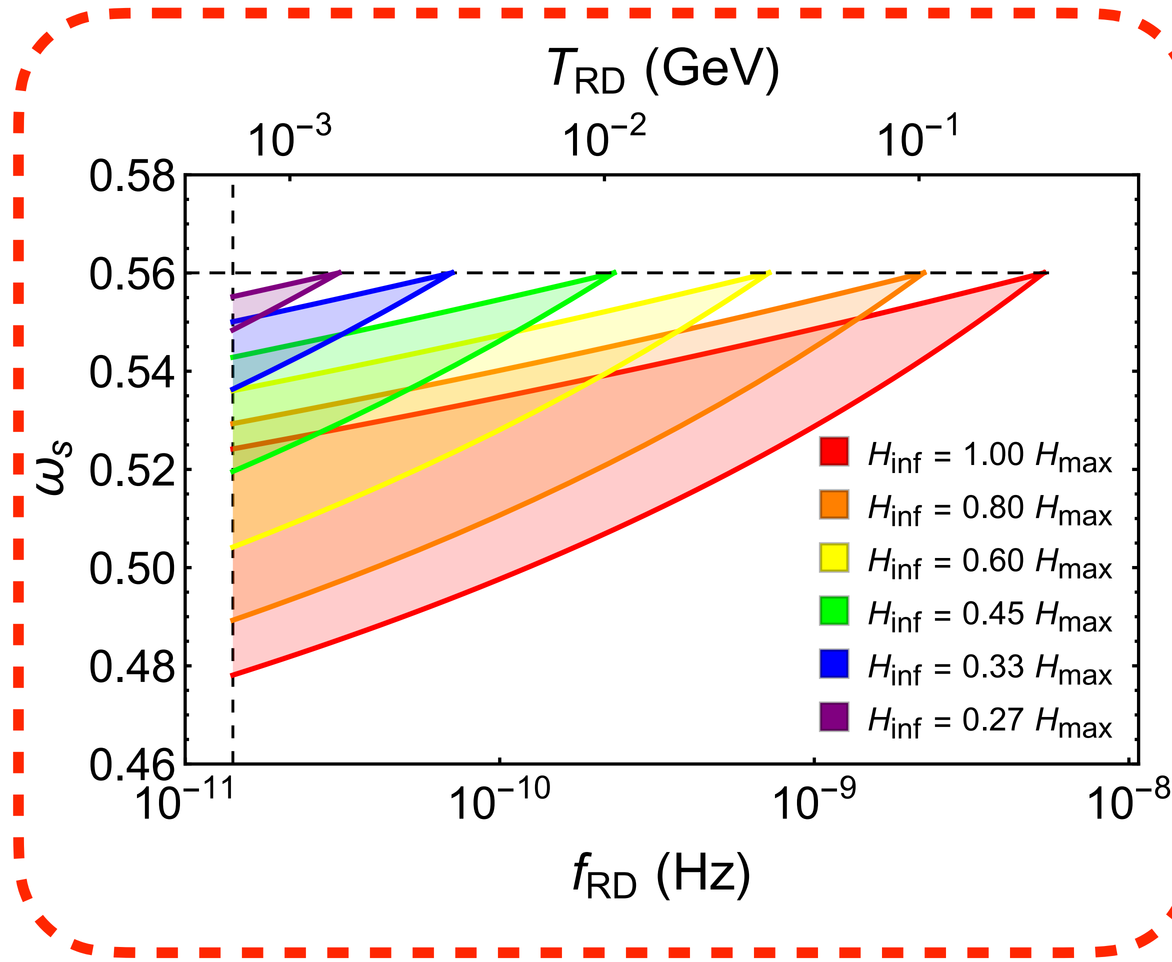


after BBN cut



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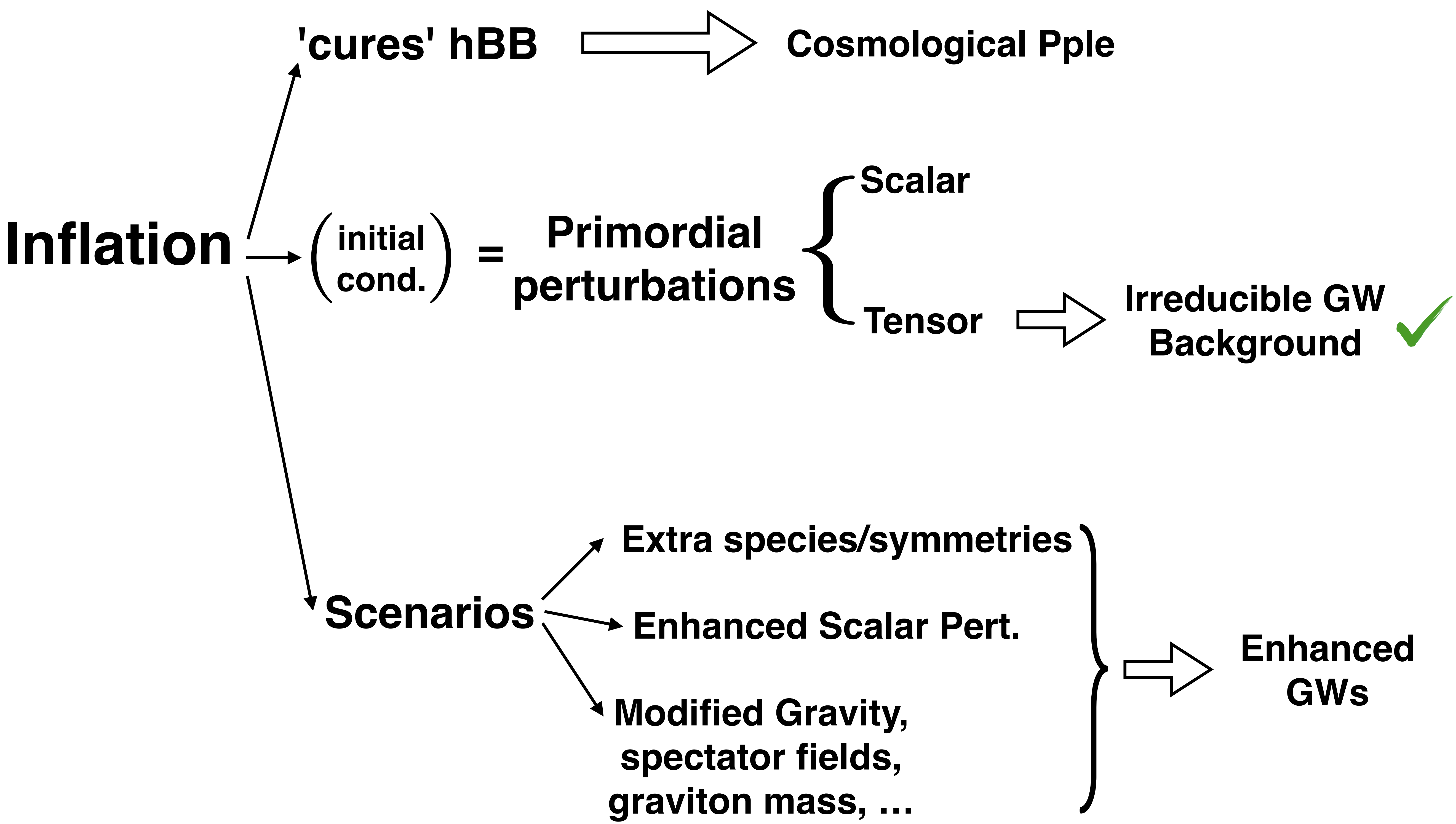
INFLATIONARY COSMOLOGY

Inflation $\rightarrow$ 'cures' hBB $\Rightarrow$ Cosmological Pple

Inflation $\rightarrow$ (initial cond.) = Primordial perturbations

- Scalar
- Tensor $\Rightarrow$ Irreducible GW Background ✓

INFLATIONARY COSMOLOGY



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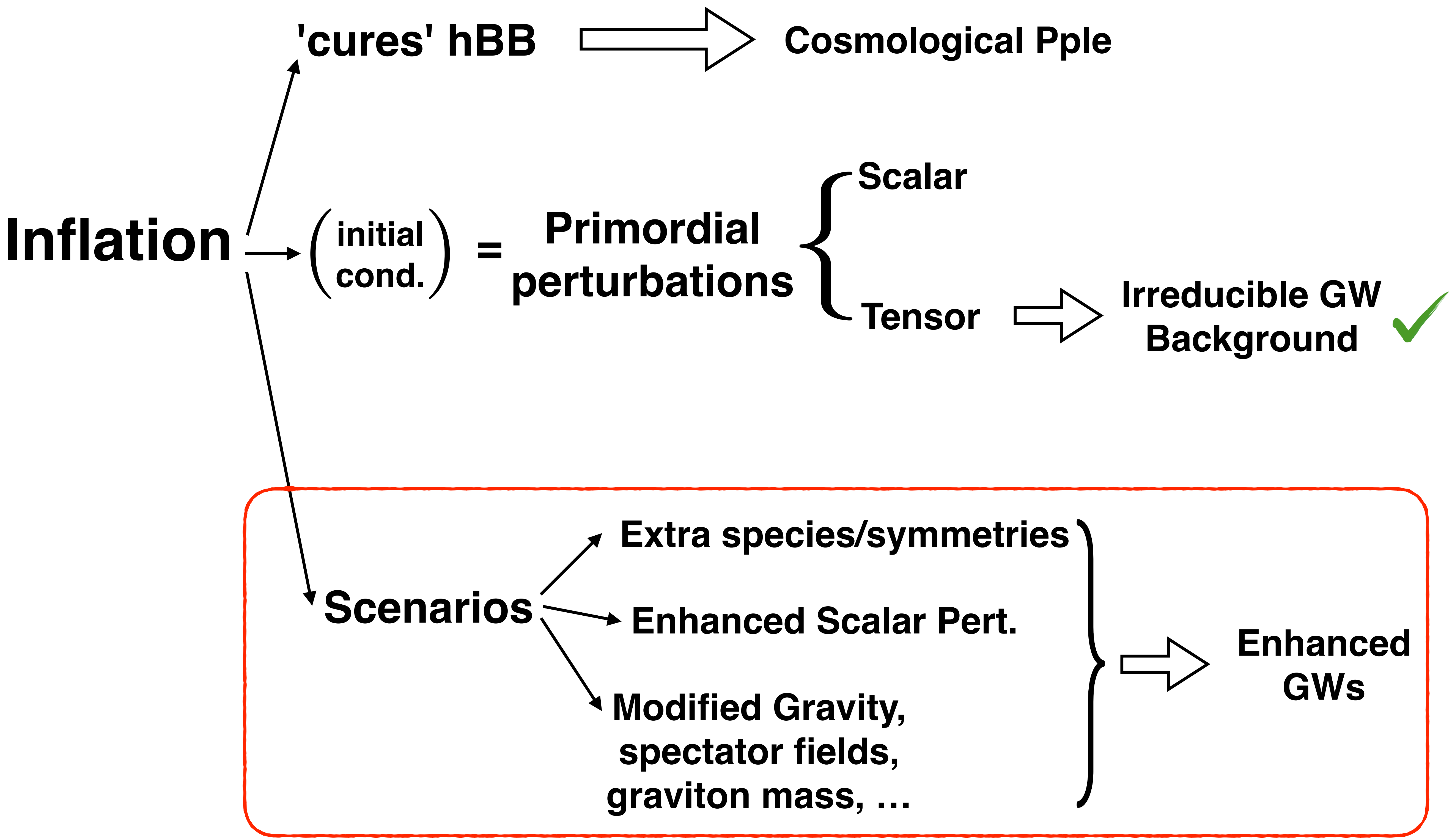

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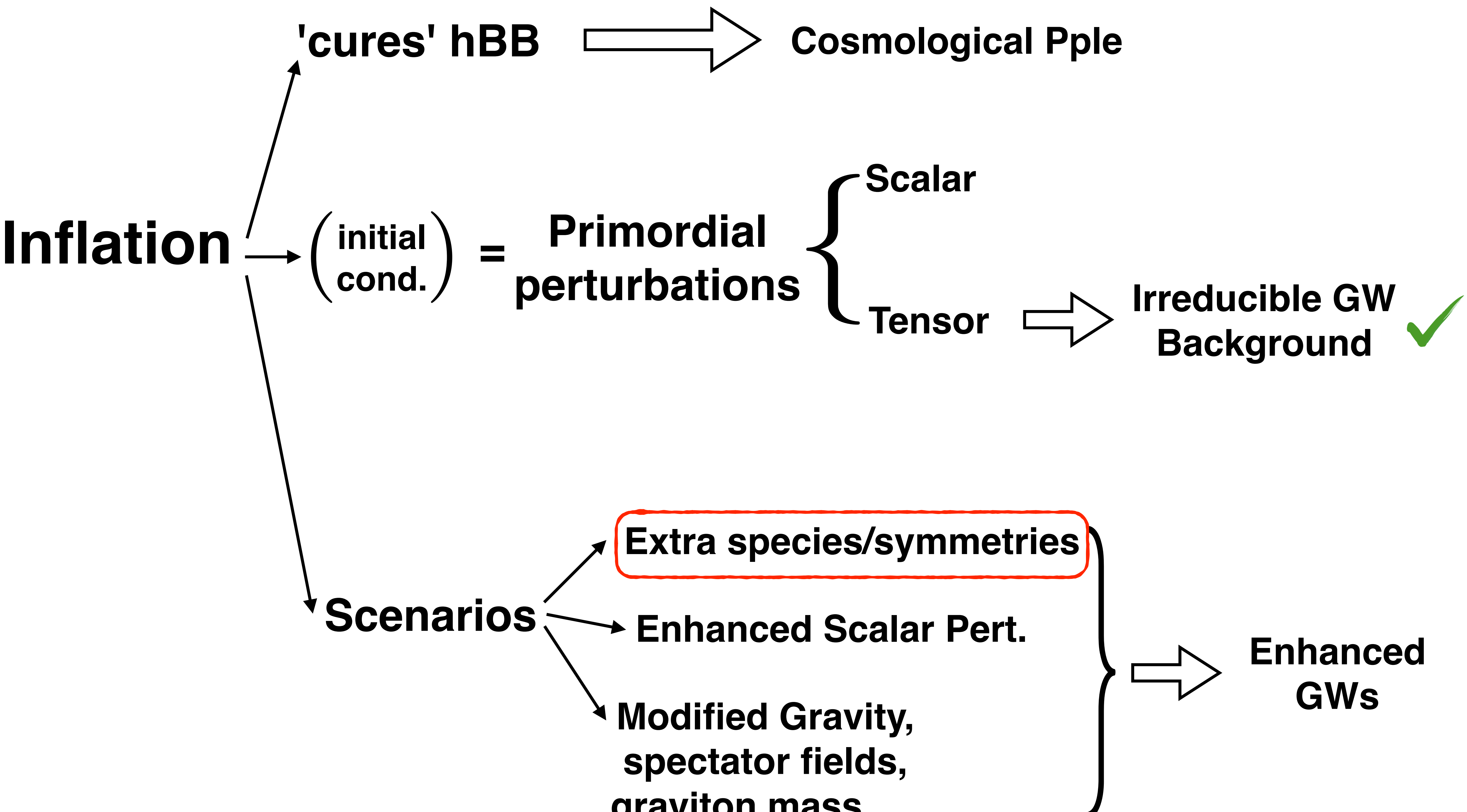

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + C$ on couplings to other fields

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derivative couplings to: fermions gauge fields

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Not the QCD axion;

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fermions   gauge fields

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fermions

gauge fields

(derivative couplings)

$= \partial_\mu K^\mu$

topological term

$[\phi \partial_\mu K^\mu = K^\mu \partial_\mu \phi]$

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breaks shift-symm

fermions

gauge fields

(derivative couplings)

$= \partial_\mu K^\mu$ topological term

$[\phi\partial_\mu K^\mu = K^\mu\partial_\mu\phi]$

With shift symmetry, $\Delta V \propto V_{\cancel{\text{shift}}}$ $\Rightarrow$ **Protected against radiative corrections !**

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

$$V(\varphi) + \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

inflaton ϕ = pseudo-scalar axion

INFLATIONARY MODELS

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inflaton ϕ = pseudo-scalar axion

Photon: 2 helicities

$$\vec{A}(\tau, \mathbf{x}) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} \left[\vec{\epsilon}_\lambda(\mathbf{k}) a_\lambda(\mathbf{k}) A_\lambda(\tau, \mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right]$$

$$\left[a_\lambda(\mathbf{k}), a^\dagger_{\lambda'}(\mathbf{k}') \right] = \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} - \mathbf{k}')$$

INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + const.$

$$V(\varphi) + \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

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$$\vec{A}'' - \nabla^2 \vec{A} - \frac{1}{f} \phi' \, \vec{\nabla} \times \vec{A} = 0 \quad \Rightarrow \quad \left[\frac{\partial^2}{\partial \tau^2} + k^2 \pm \frac{2k\xi}{\tau} \right] A_\pm(\tau, k) = 0,$$

$$\boxed{\xi \equiv \frac{\dot{\phi}}{2fH}}$$

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Chiral instability

$$\boxed{A_+(\tau, k) \cong \frac{1}{\sqrt{2k}} \left(\frac{k}{2\xi a H} \right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}}}$$

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$$A_+ \propto e^{\pi\xi}\,, \quad |A_-| \ll |A_+|$$

A_+ exponentially amplified,

A_- has no amplification

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Gauge field excitation creates chiral GWs !

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$$h''_{ij} + 2\mathcal{H} h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\rm TT}_{ij} \propto \; \{E_i E_j + B_i B_j\}^{TT}$$

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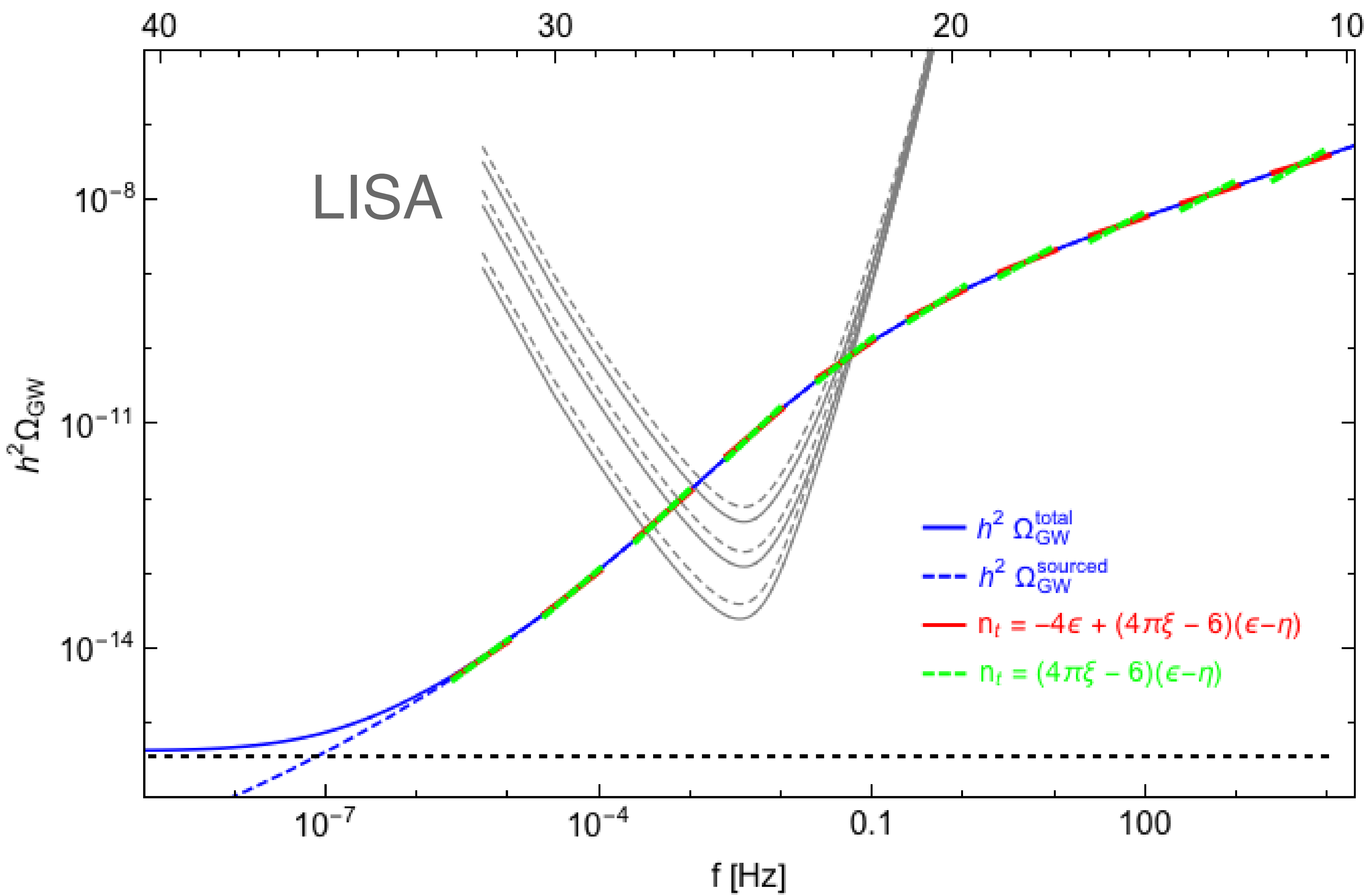
GW mostly* one-chirality

A_μ **Chiral**

(*why not exactly just one?)

INFLATIONARY MODELS
Axion-Inflation

GW energy spectrum today

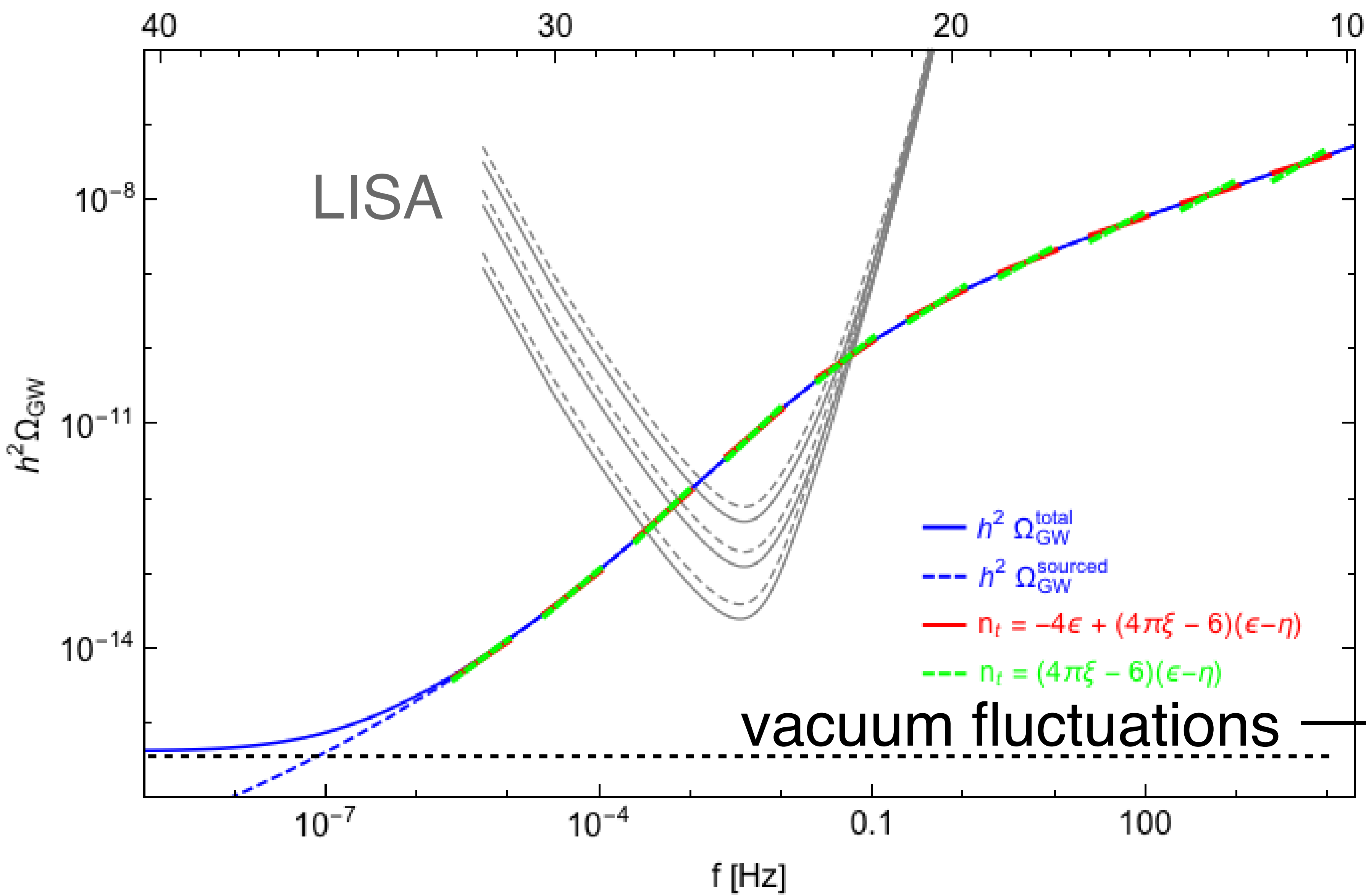


Gauge fields
source a
Blue-Tilted
+ Chiral
+ Non-G
GW background

INFLATIONARY MODELS

Axion-Inflation

GW energy spectrum today



$(\Omega_{\rm Rad}/24)\Delta_h^2$

Gauge fields source a Blue-Tilted + Chiral + Non-G GW background

INFLATIONARY MODELS

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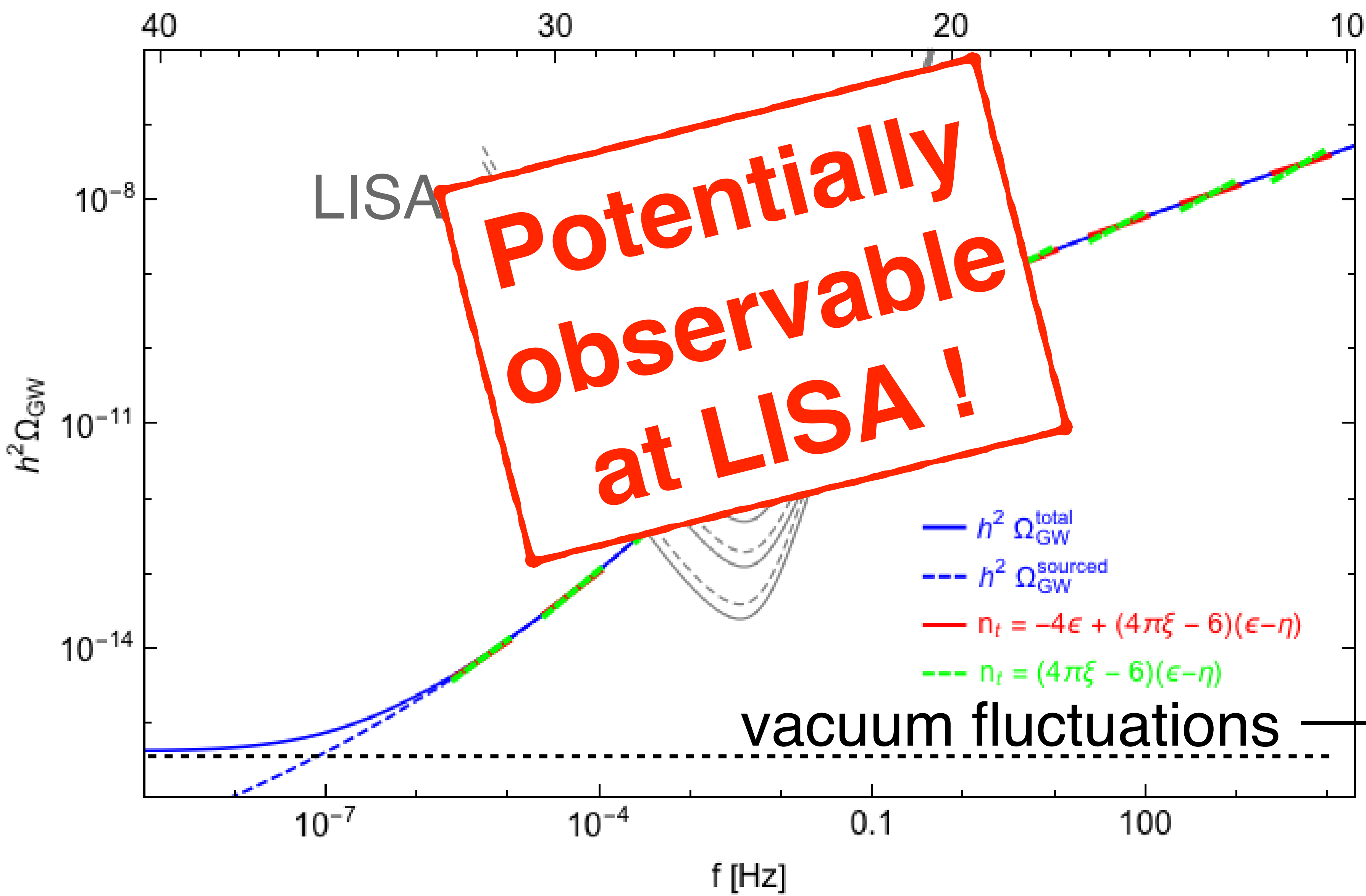

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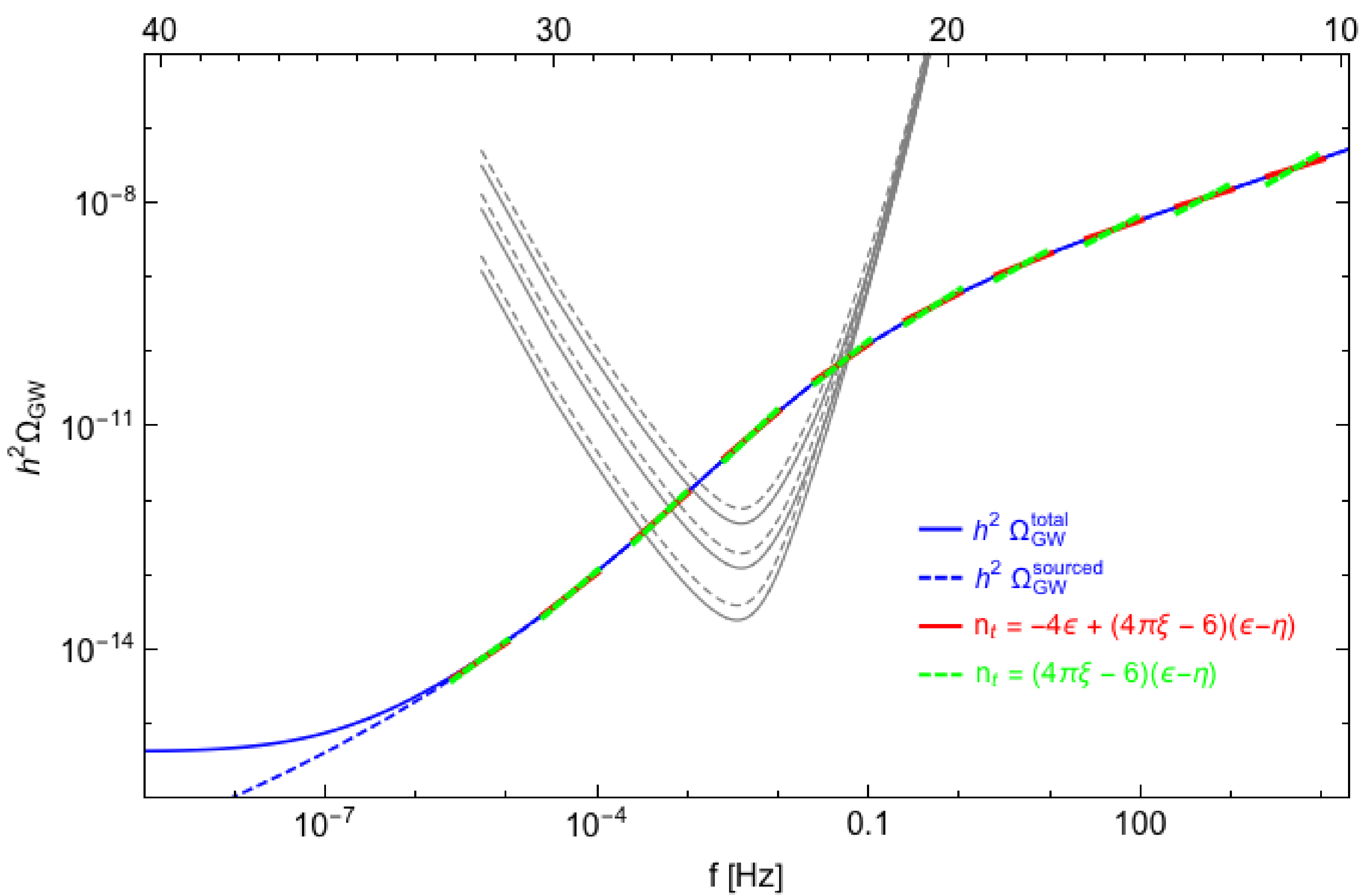
INFLATIONARY MODELS

Axion-Inflation

Bartolo et al ‘16

$$h^2\Omega_{\rm gw} = A_* \left(\frac{f}{f_*}\right)^{n_T}$$



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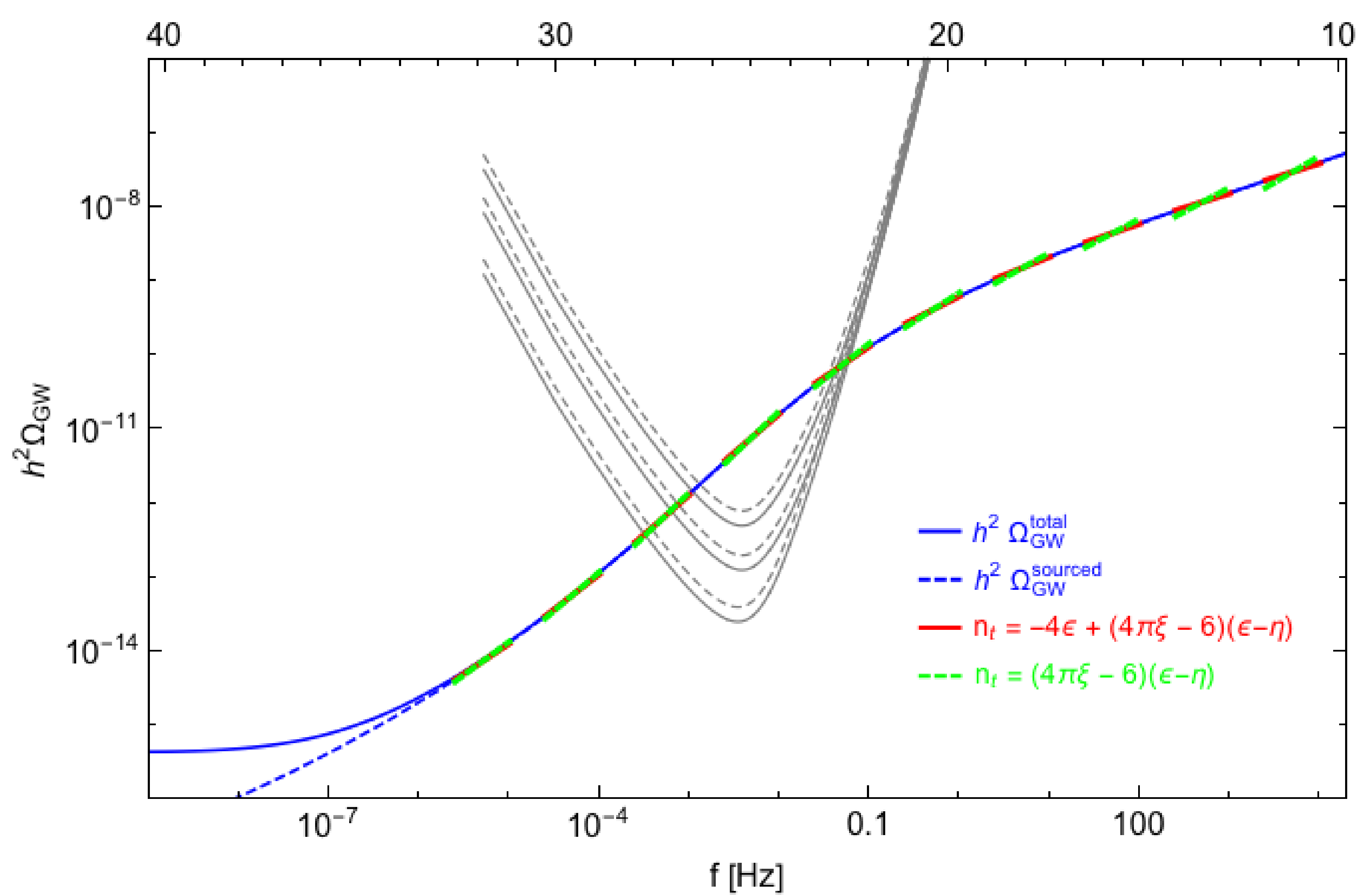
$$h^2\Omega_{\rm gw} = A_* \left(\frac{f}{f_*}\right)^{n_T} \longrightarrow \boxed{n_T \simeq (4\pi\xi - 6)(\epsilon_H - \eta)}$$

$$\boxed{\xi \equiv \frac{\alpha\dot\phi}{2fH}}$$

$$A_* = \boxed{1.5\cdot 10^{-13}\,\frac{H^4}{M_{Pl}^4}\,\frac{e^{4\pi\xi}}{\xi^6}}\,,$$

$\boxed{H,\ \xi,\ \epsilon_H - \eta}$ **(3 parameters!)**



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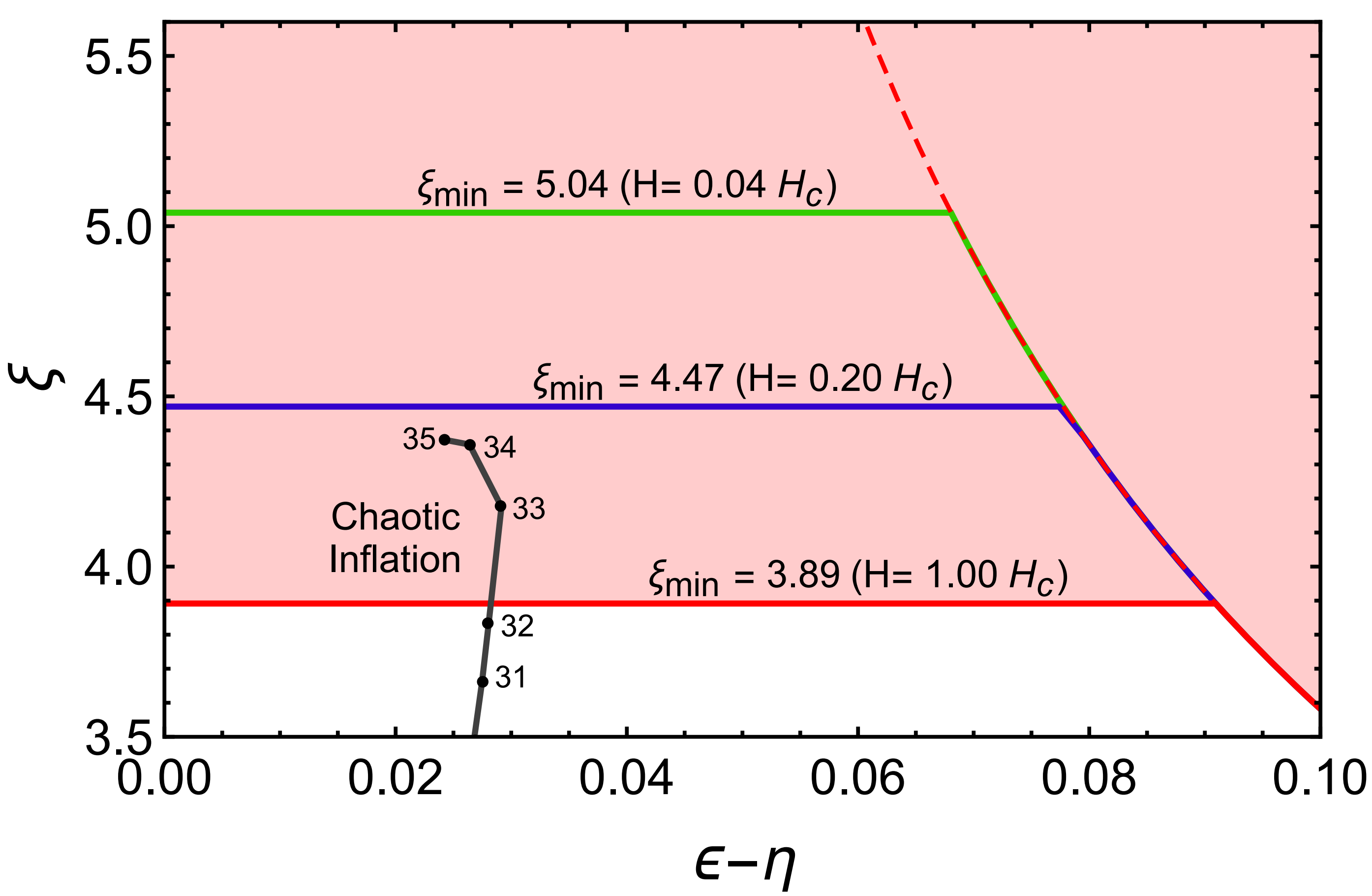
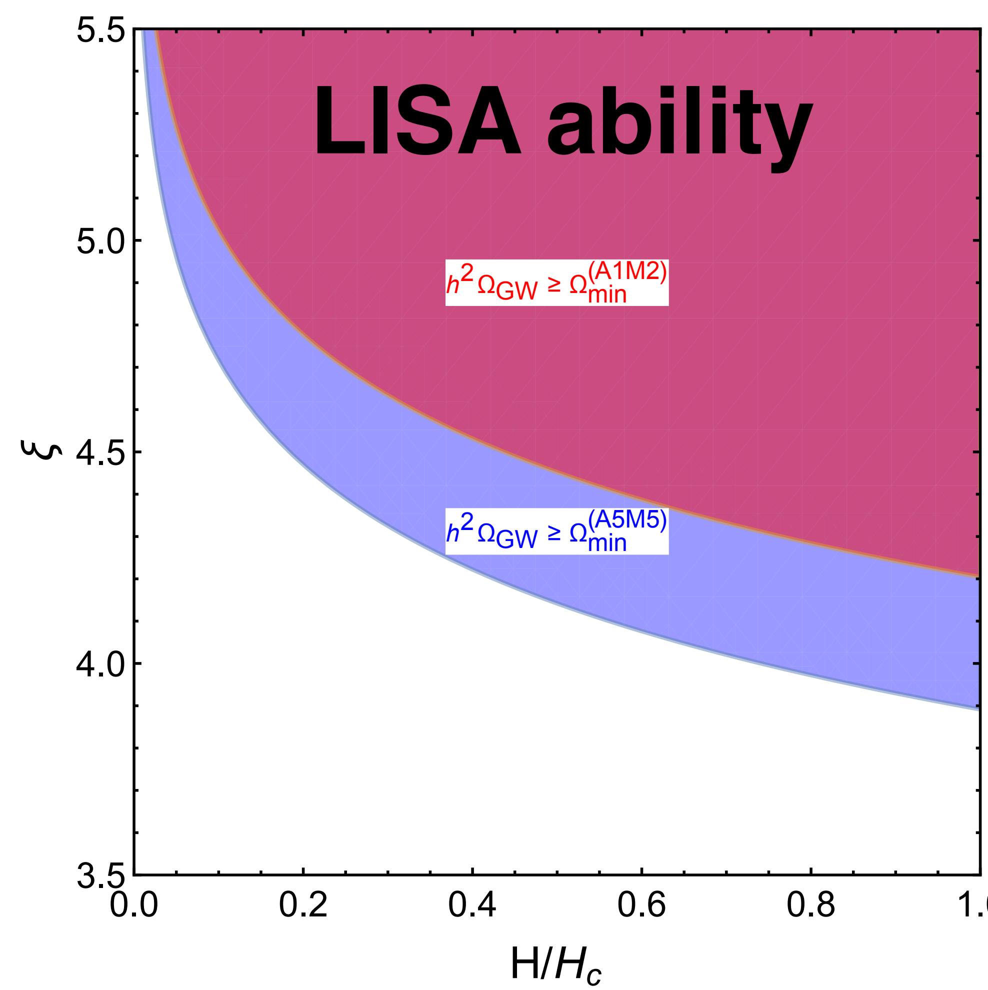
LISA ability





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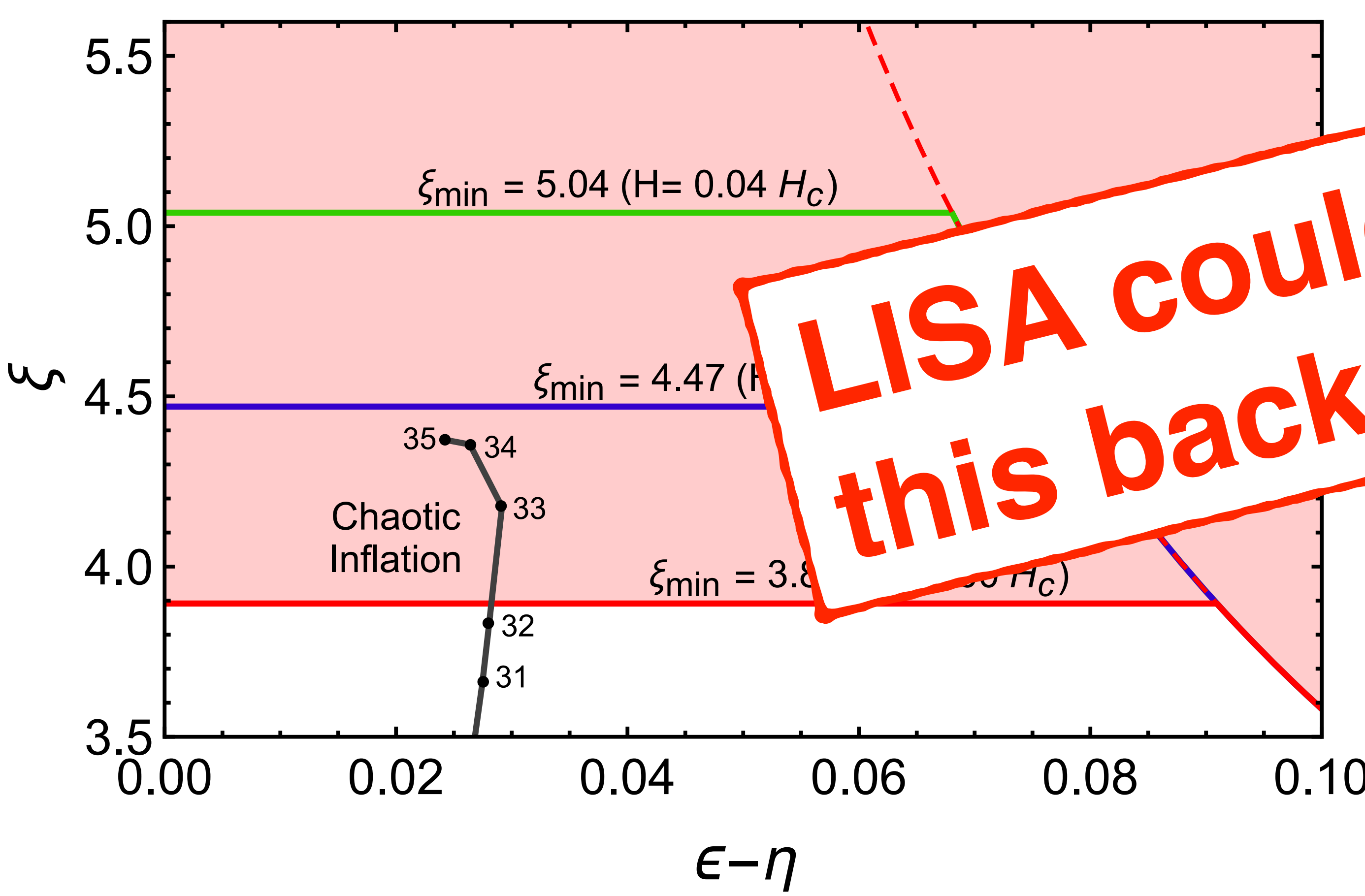

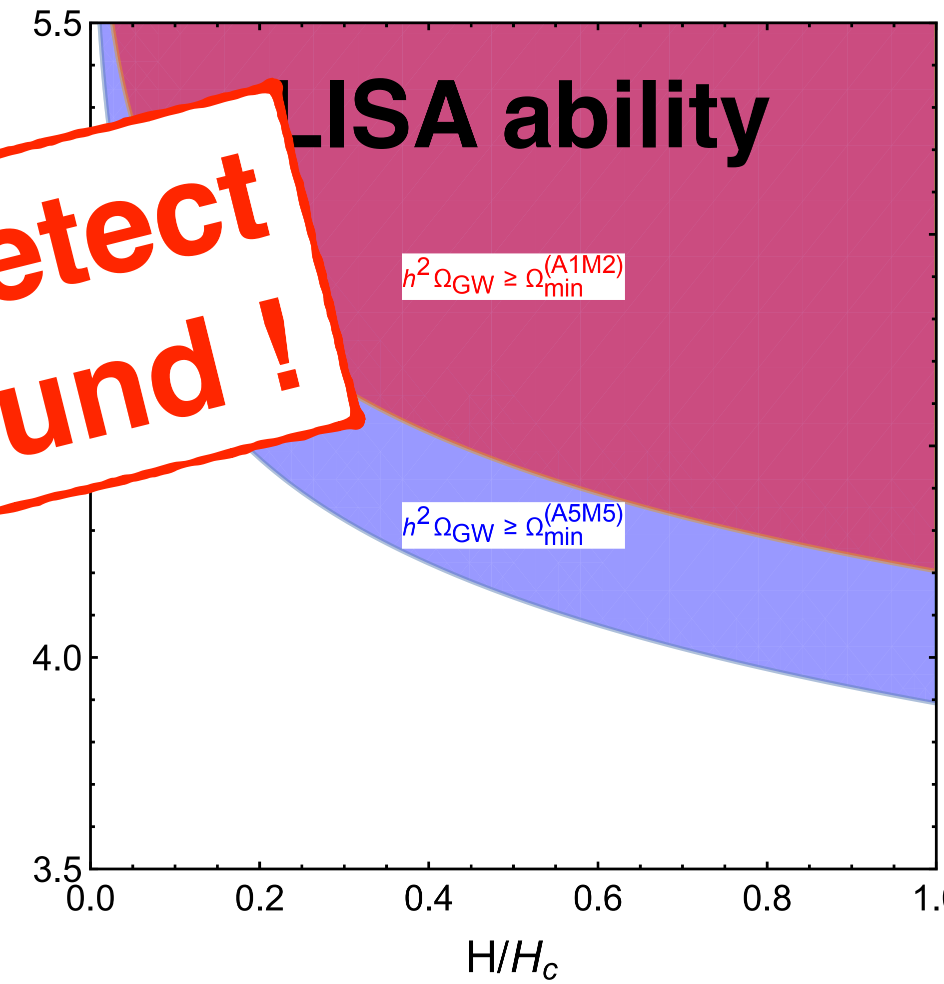

LISA could detect this background !

INFLATIONARY MODELS

Axion-Inflation: ***Shift*** **symmetry** $\longrightarrow$ **Natural (chiral) coupling to** A_μ

huge excitation of fields ! (photons)

INFLATIONARY MODELS

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What if there are arbitrary fields coupled to the inflaton ?

(i.e. no need of extra symmetry)

INFLATIONARY MODELS

Axion-Inflation: ***Shift*** **symmetry** → **Natural (chiral) coupling to A_μ**

huge excitation of fields ! (photons)

What if there are arbitrary fields coupled to the inflaton ? → **large excitation of these fields !?**

(i.e. no need of extra symmetry)

will they create GWs?

INFLATIONARY MODELS

fields coupled to the inflaton ? → **large excitation ?**

(i.e. no need of extra symmetry) **GW generation !?**

INFLATIONARY MODELS

fields coupled to the inflaton ? → large excitation ?

(i.e. no need of extra symmetry) GW generation !?

$$-\mathcal{L}_\chi = (\partial\chi)^2/2 + g^2(\phi - \phi_0)^2\chi^2/2$$ **Scalar Fld**

$$-\mathcal{L}_\psi = \bar{\psi}\gamma^\mu\partial_\mu\psi + g(\phi - \phi_0)\bar{\psi}\psi$$ **Fermion Fld**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - gA_\mu)\Phi)|^2 - V(\Phi^\dagger\Phi)$$ **Gauge Fld ($\Phi = \phi e^{i\theta}$)**

INFLATIONARY MODELS

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All 3 cases:

$$m = g(\phi(t) - \phi_0) \xRightarrow{\text{non-adiabatic}} \dot{m} \gg m^2 \quad \text{during} \quad \Delta t_{\rm na} \sim 1/\mu\,, \qquad \boxed{\mu^2 \equiv g\dot{\phi}_0}$$

$$\boxed{n_k = \mathrm{Exp}\{-\pi(k/\mu)^2\}}$$ **Non-adiabatic field excitation (particle creation)**

INFLATIONARY MODELS

fields coupled to the inflaton ? → large excitation ✓

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(spin-independent)

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Non-adiabatic field excitation (particle creation !)
(spin-independent)

In all three cases (scalars, fermions, and vectors)

GWs generated by anisotropic distribution of the created species

(Only $k \ll \mu$ long-wave modes excited)

INFLATIONARY MODELS

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GWs power spectrum: $\mathcal{P}_h^{(\mathrm{tot})}(k) \;=\; \mathcal{P}_h^{(\mathrm{vac})}(k) + \mathcal{P}_h^{(\mathrm{pp})}(k)$

from particle production

GW Source(s): (SCALARS , VECTOR , FERMIONS)

$$\Pi_{ij}^{TT} \;\propto\; \{\partial_i\chi^a\partial_j\chi^a\}^{TT},\;\; \{E_iE_j + B_iB_j\}^{TT},\;\; \{\bar\psi\gamma_i D_j\psi\}^{TT}$$

INFLATIONARY MODELS

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$\boxed{(W \lesssim 0.5)}$

N. Barnaby *et al.*, Phys. Rev. **D86**, 103508 (2012), [1206.6117].

J. L. Cook and L. Sorbo, Phys. Rev. **D85**, 023534 (2012), [1109.0022].

INFLATIONARY MODELS

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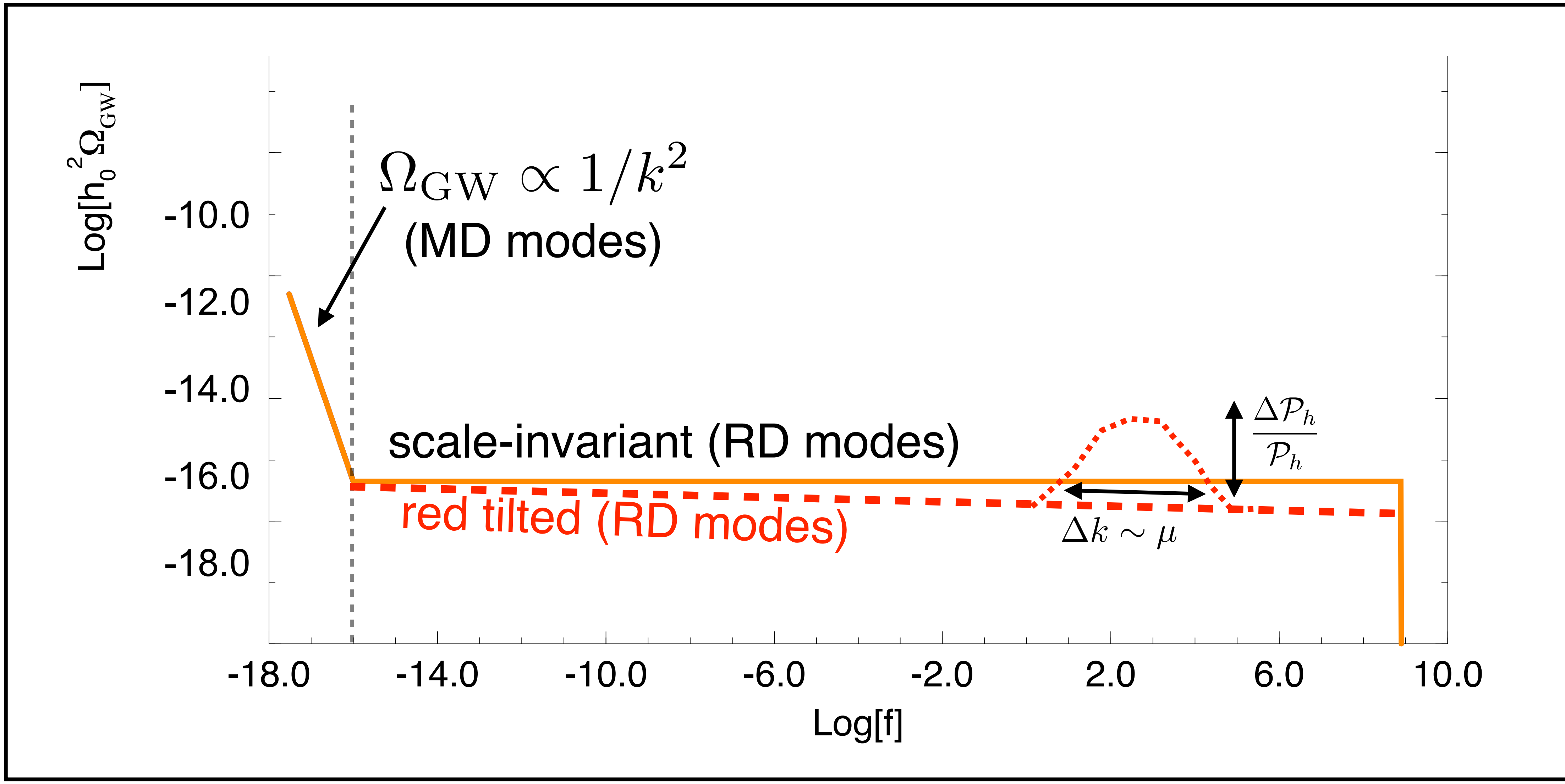
$$\boxed{\mu^2 \equiv g\dot{\phi}_0}$$

(Sorbo et al 2011, Peloso et al 2012-2013, Caprini & DGF 2018)



INFLATIONARY MODELS

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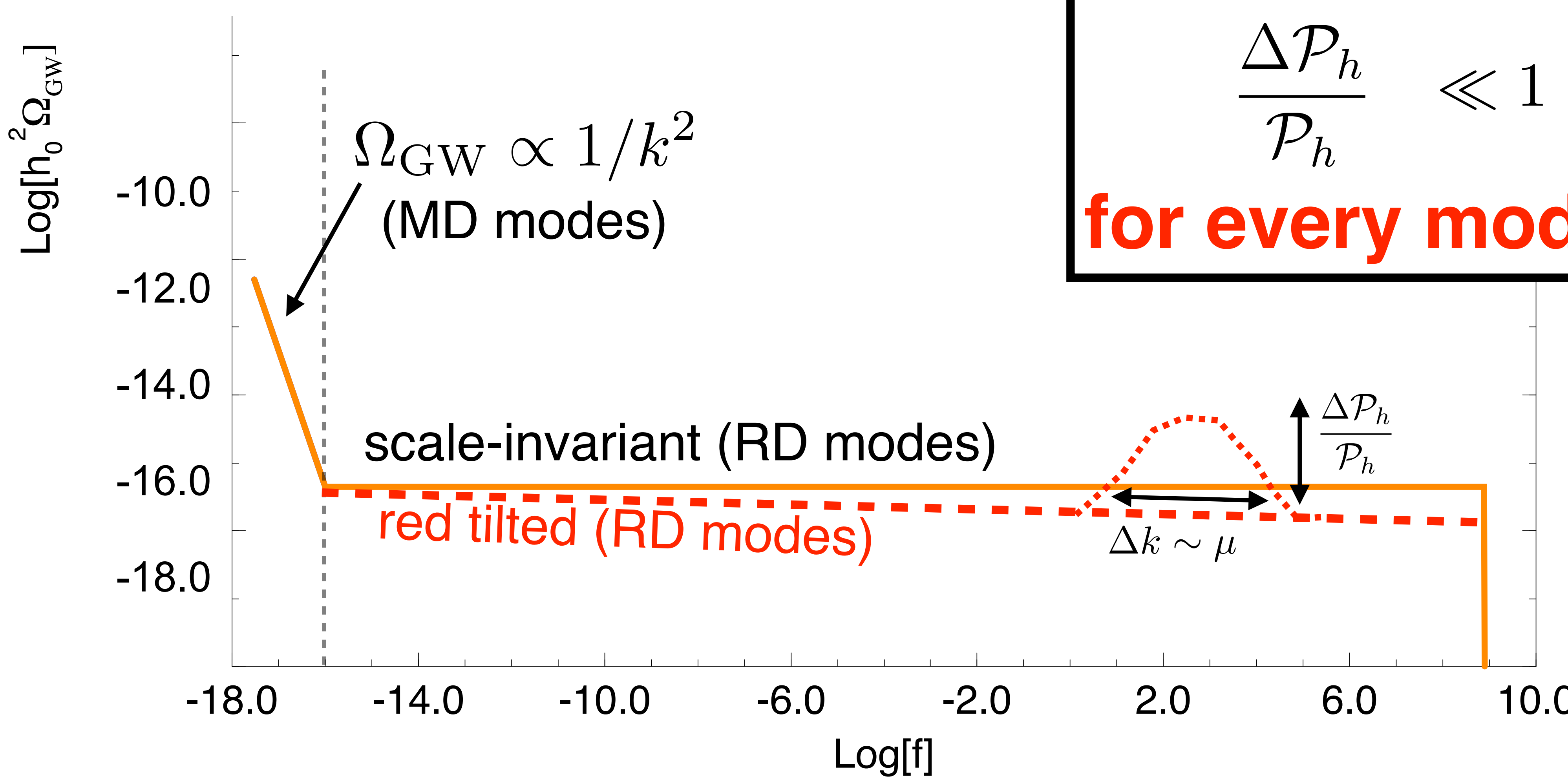
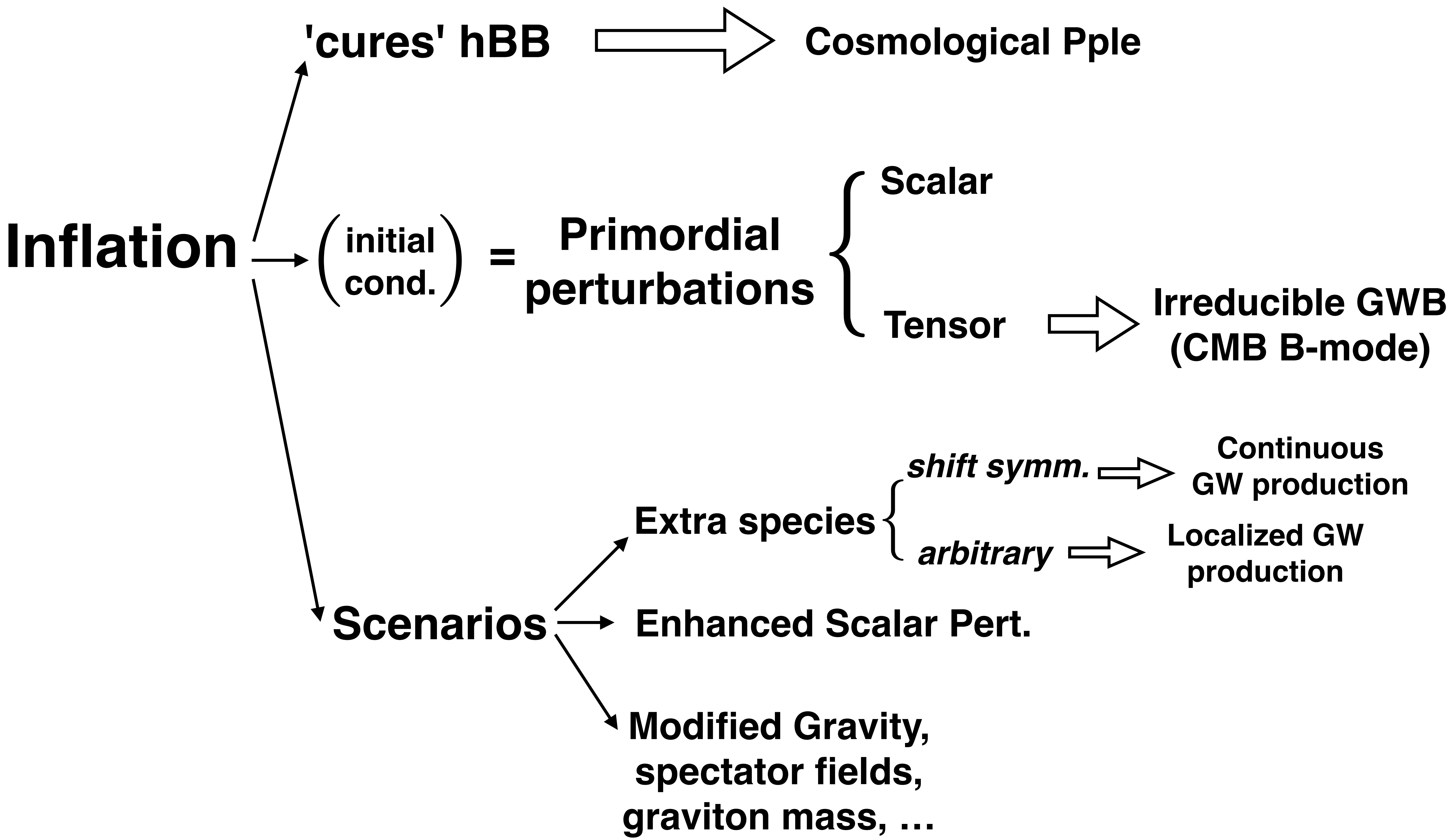


Unfortunately ...

$$\frac{\Delta \mathcal{P}_h}{\mathcal{P}_h} \ll 1$$

for every model !

INFLATIONARY COSMOLOGY



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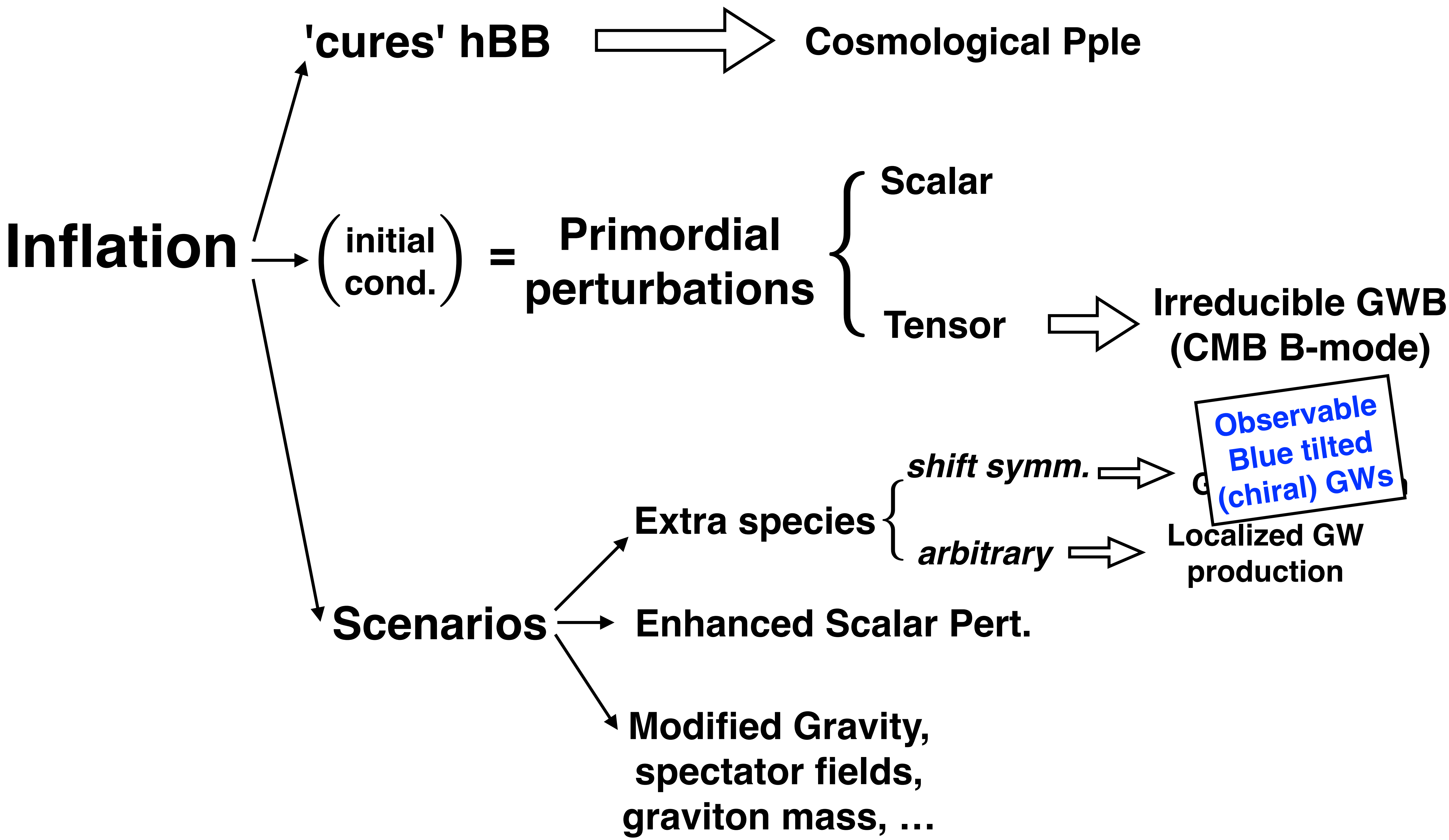

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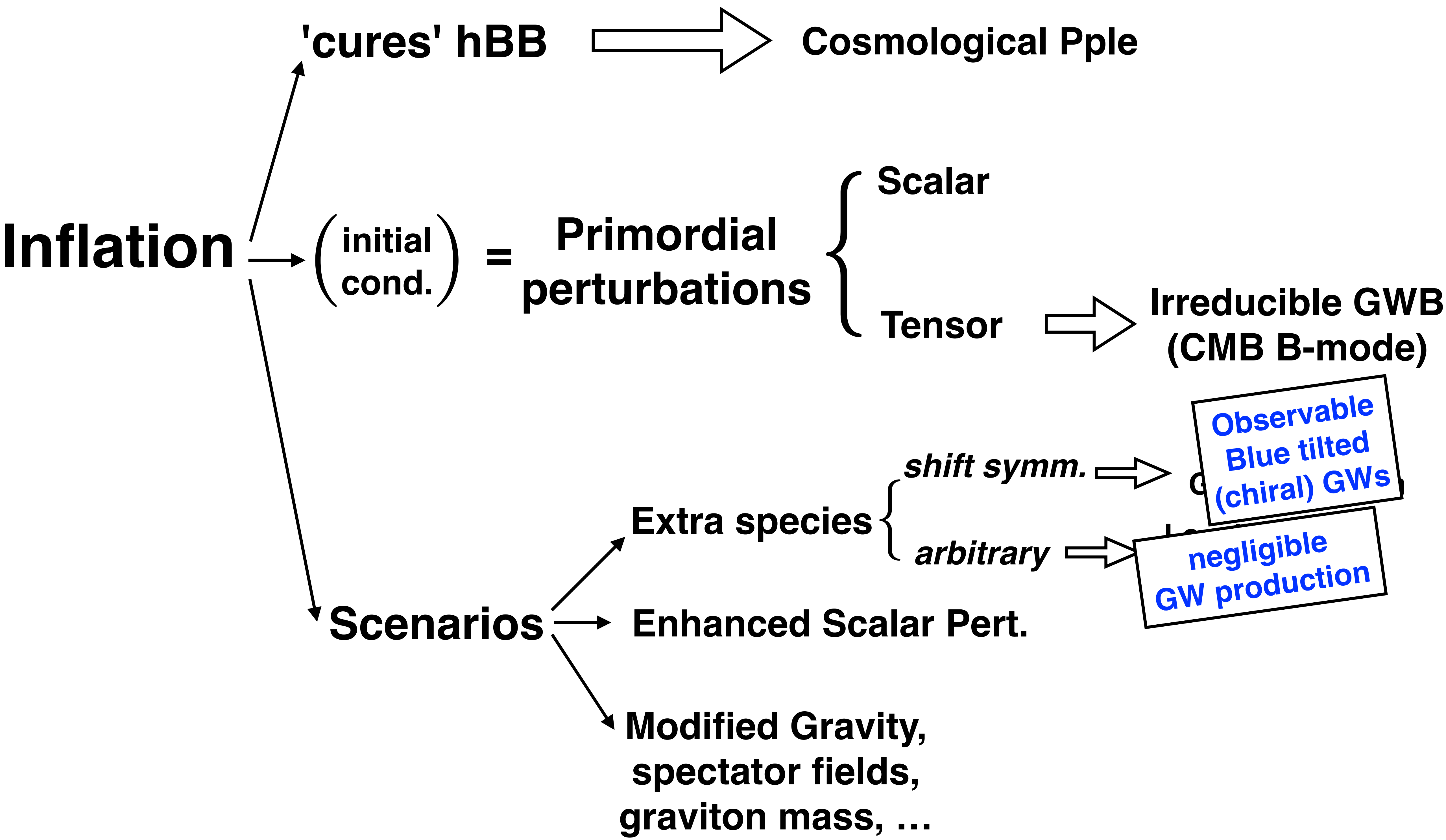

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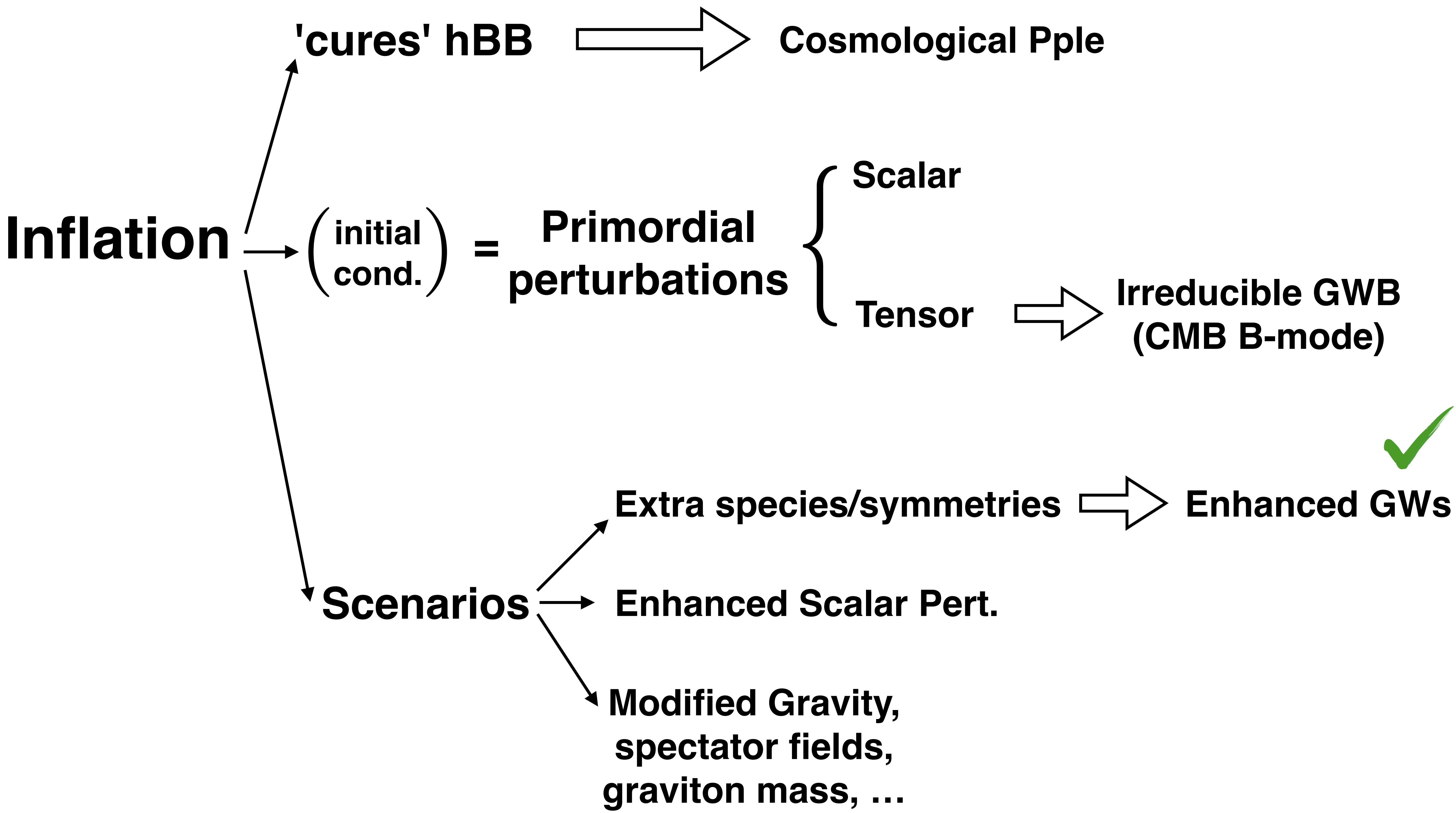

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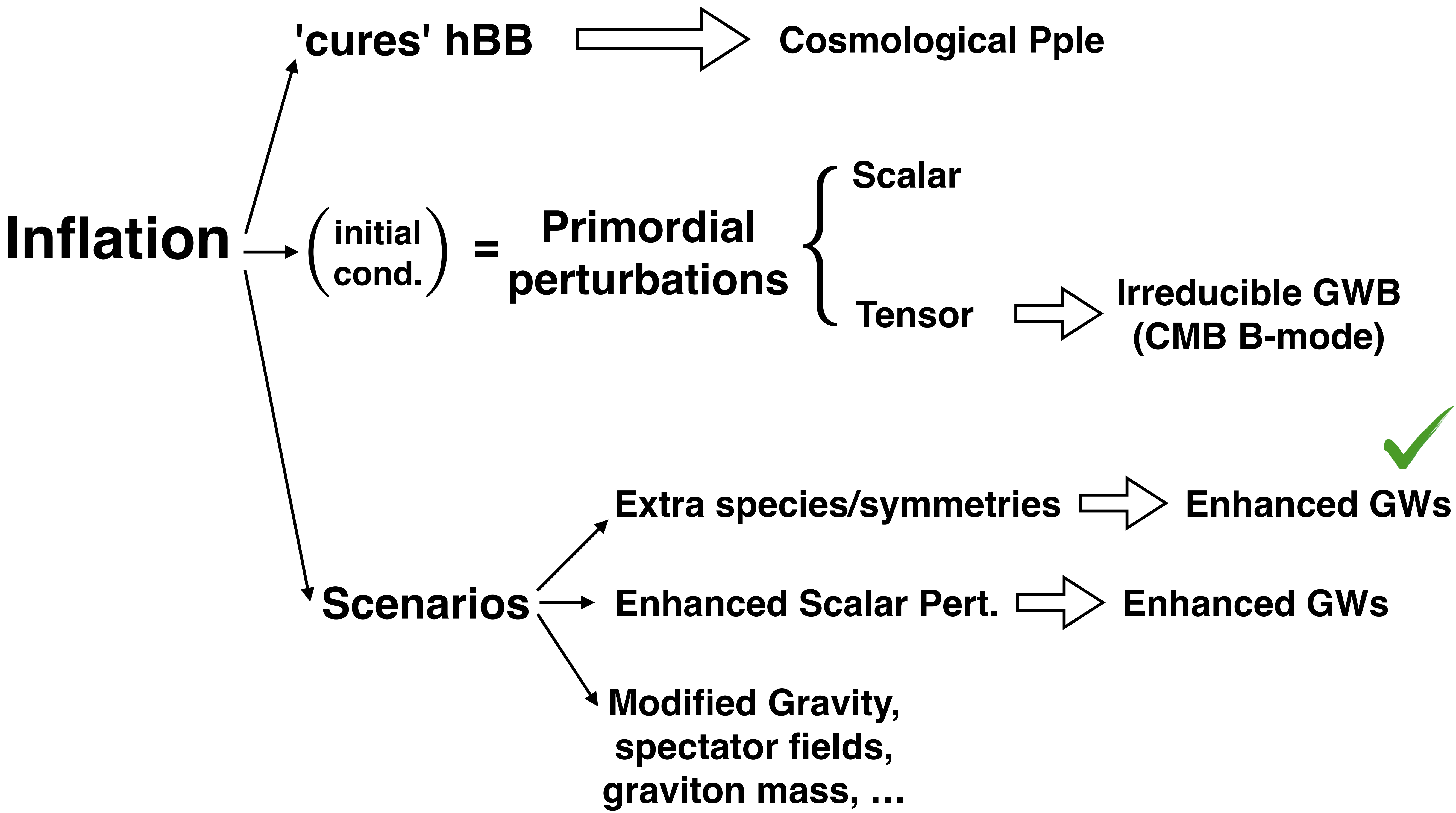

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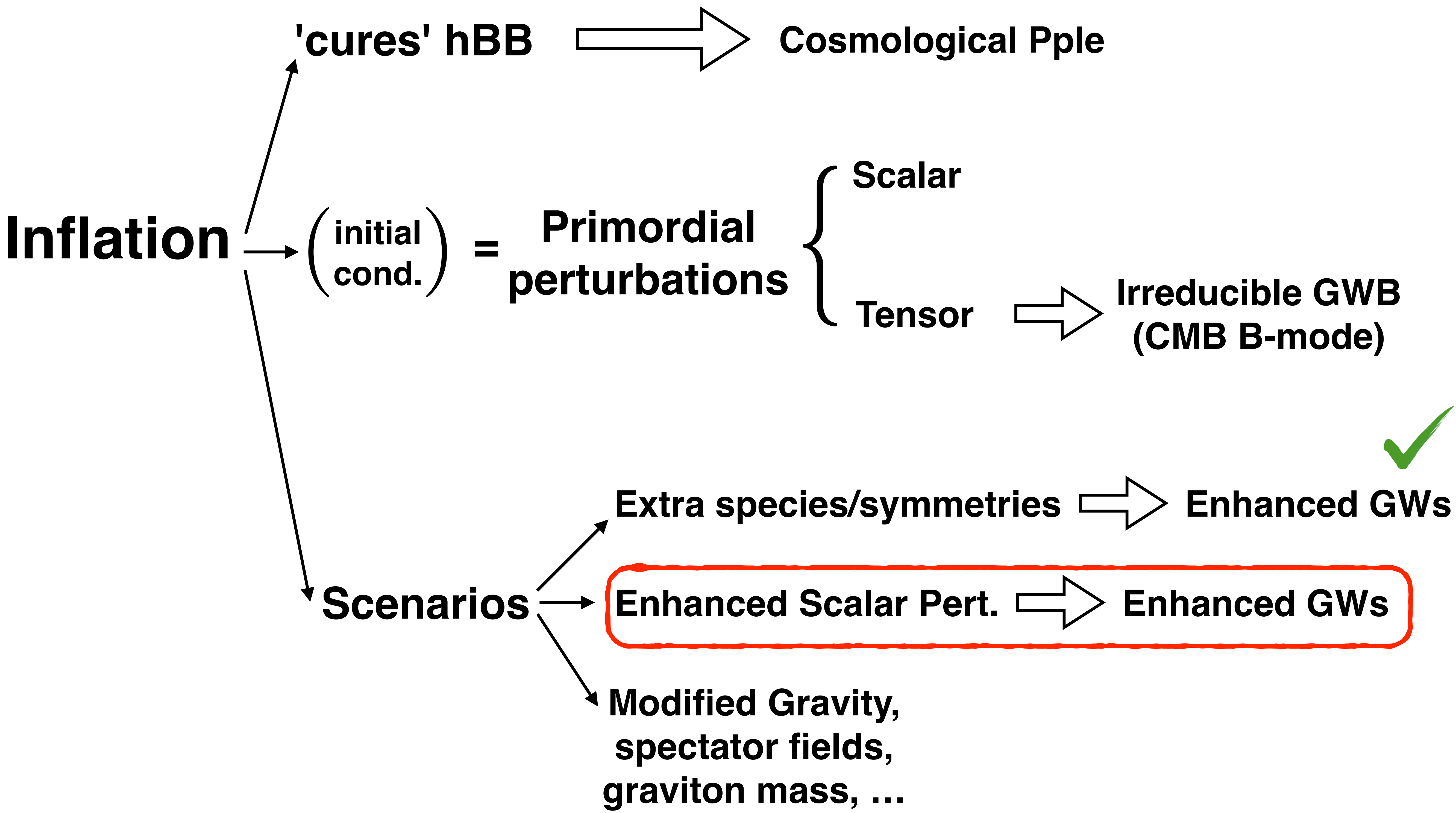


INFLATIONARY MODELS

INFLATION $\Rightarrow$ $g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}$$

$n_s - 1 \equiv 2(\eta - 2\epsilon)$

$$\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}$$

$n_t \equiv -2\epsilon$

(quasi-)scale invariance $\longleftrightarrow$ **Slow roll monotonic potentials**

INFLATIONARY MODELS

INFLATION $\rightarrow$ $$g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

$\Delta^2_{\mathcal{R}}(k)$ ... H^4 ... 1

$-2\epsilon)$

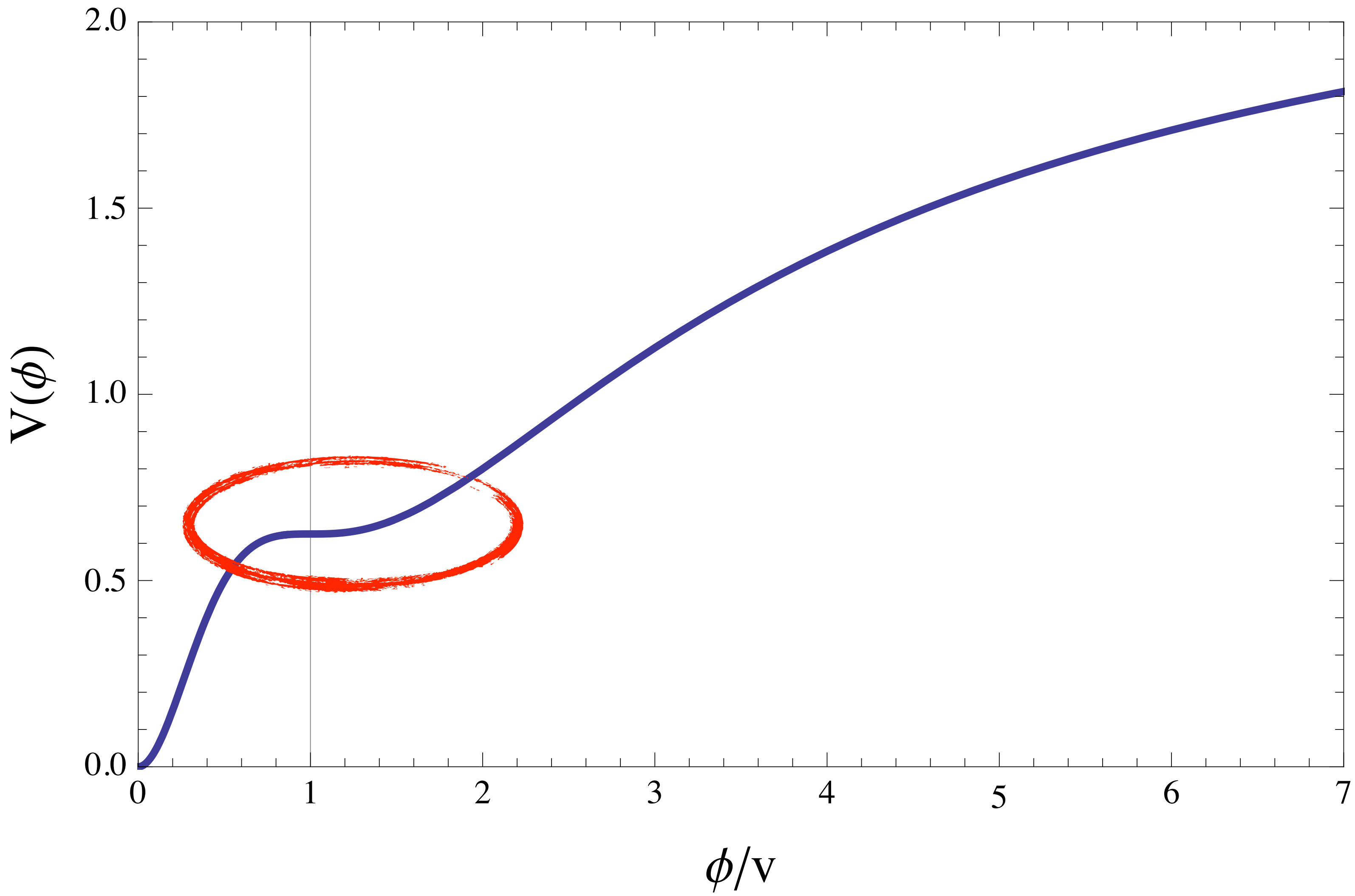
$\Delta^2_h(k$ ... (aH)

$n_t \equiv -2\epsilon$

Monotonic Single Field Slow Roll

(quasi-)scale invariance $\longleftrightarrow$ Slow roll monotonic potentials

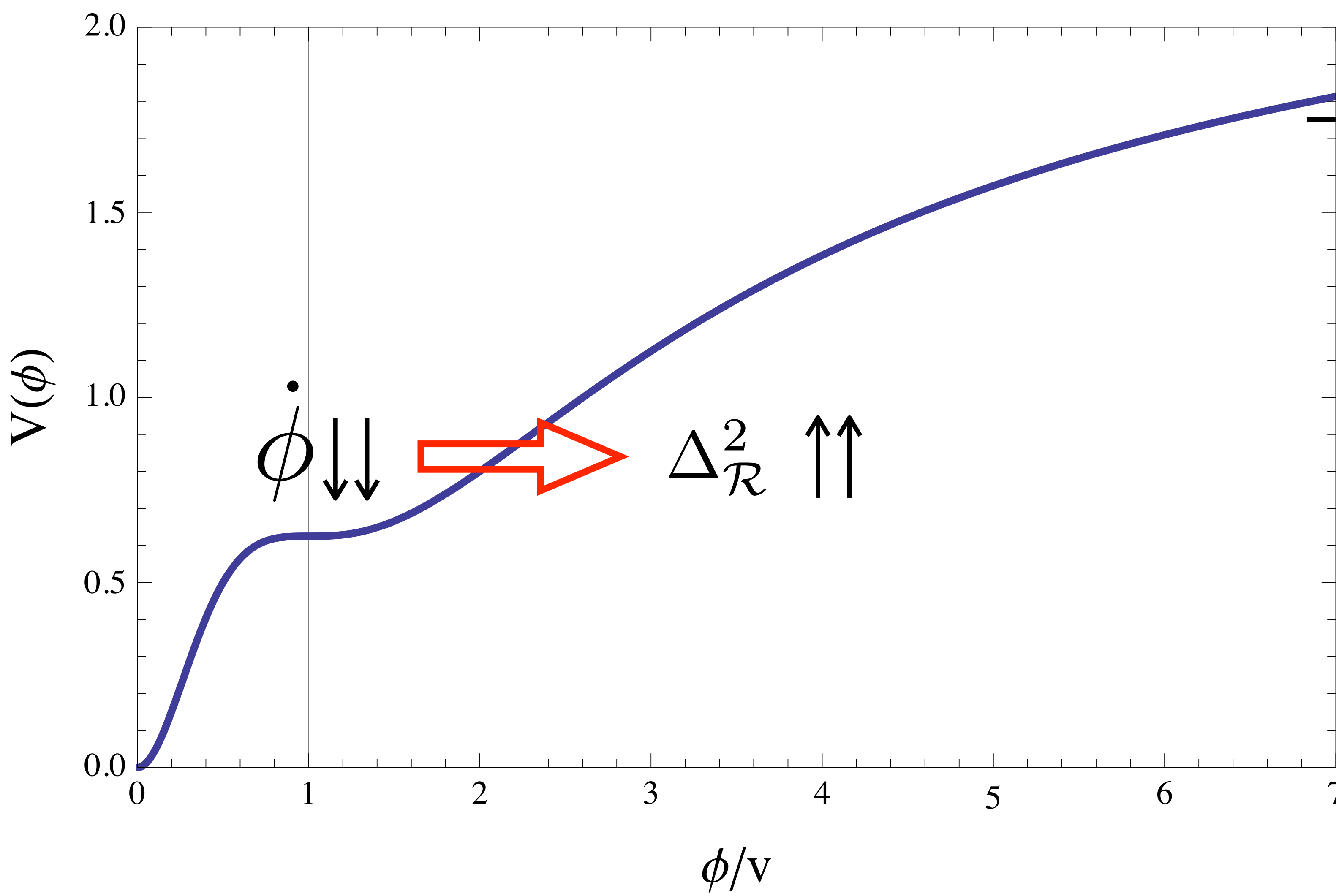
INFLATIONARY MODELS



What if the potential is not monotonic ?

INFLATIONARY MODELS

Ultra Slow-Roll Regime



$\Delta^2_{\mathcal{R}}\big|_{\rm CMB} \sim 3 \cdot 10^{-9}$

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INFLATIONARY MODELS

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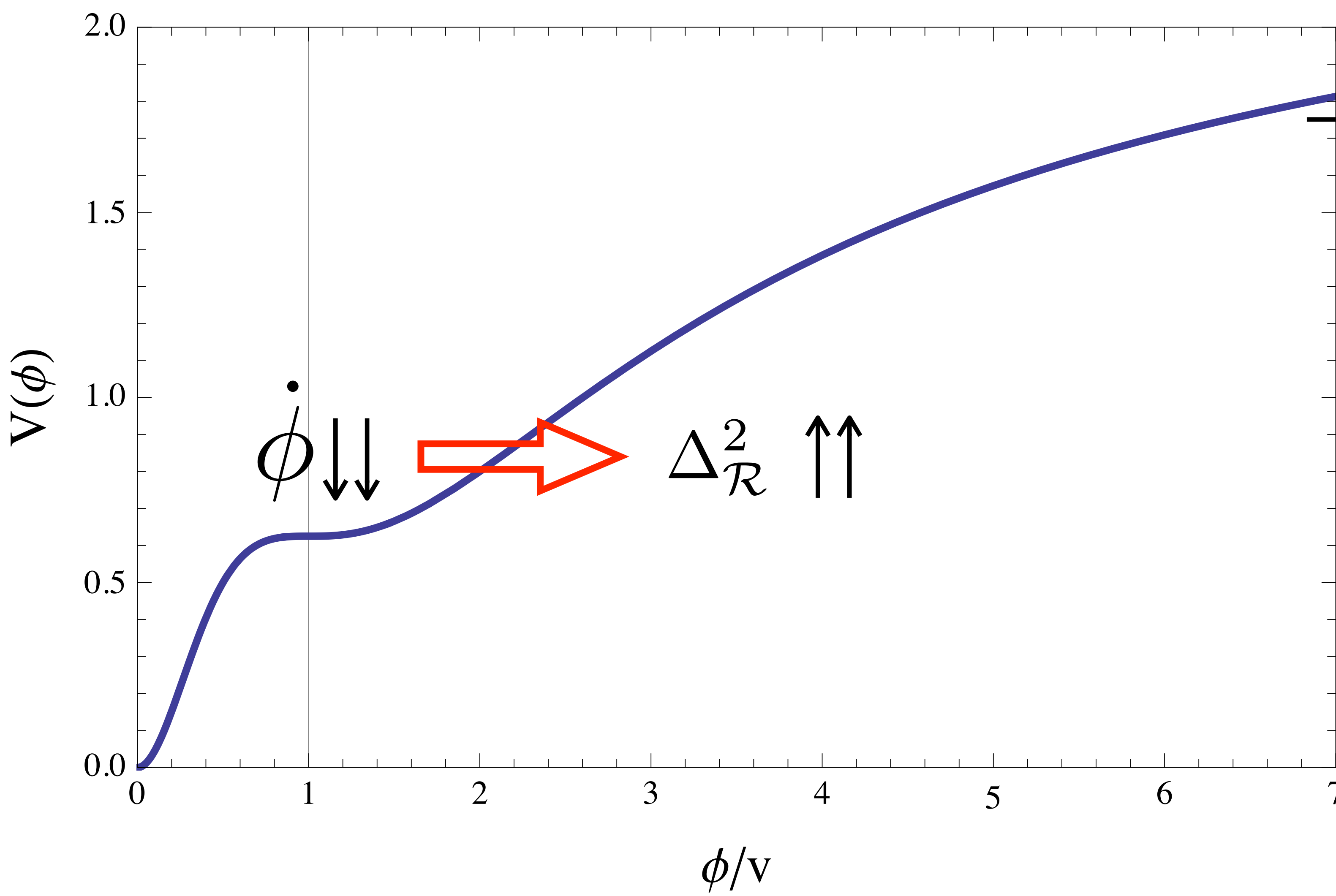

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What if the potential is not monotonic ?

$\Delta^2_{\mathcal{R}}$ **greatly enhanced!**
(at small scales)

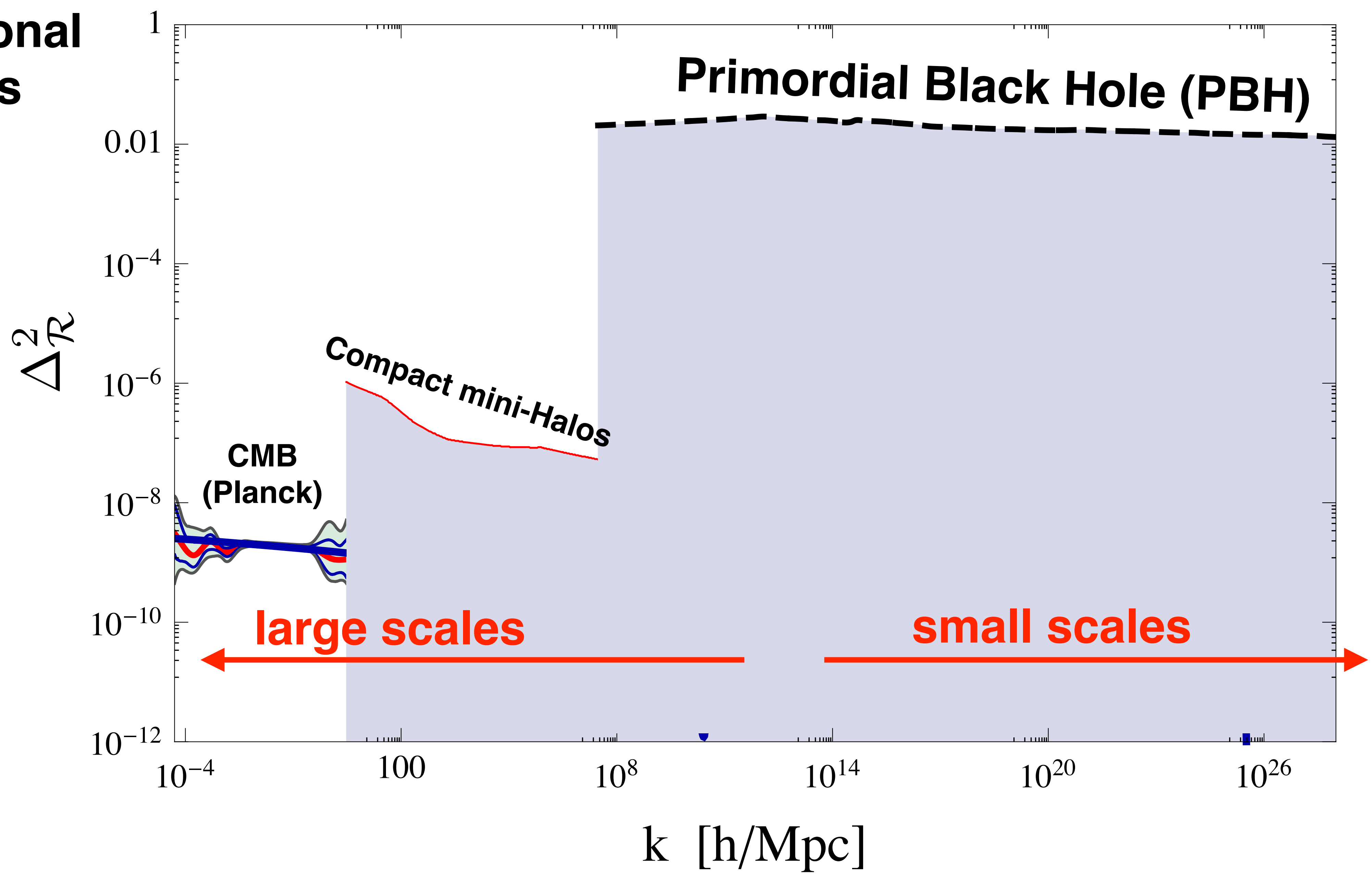
INFLATIONARY MODELS

INFLATION $\Rightarrow$ IF $\left\{\begin{array}{l}\text{non-monotonic}\\ \text{multi-field}\end{array}\right\}$ $\Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

INFLATIONARY MODELS

INFLATION ➡ IF { non-monotonic, multi-field } ⇨ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

Observational constraints



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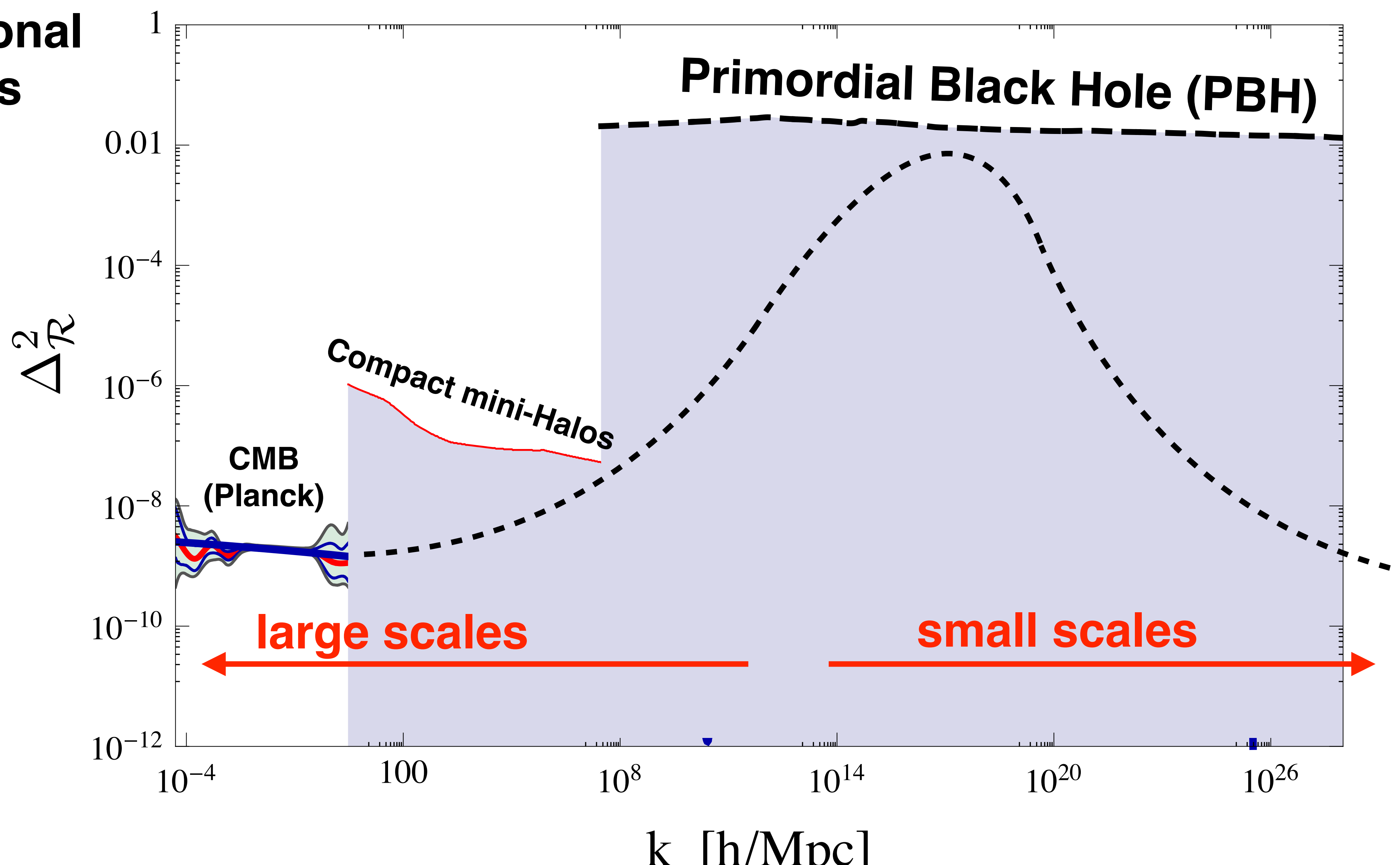

INFLATIONARY MODELS

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Let us suppose $\boxed{\Delta^2_{\mathcal{R}} \gg \Delta^2_{\mathcal{R}}\big|_{\rm CMB} \sim 3 \cdot 10^{-9}}$, @ small scales

$$ds^2 = a^2(\eta)[-(1+2\Phi)d\eta^2 + [(1-2\Psi)\delta_{ij} + 2F_{(i,j)} + h_{ij}]dx^i dx^j]$$

INFLATIONARY MODELS

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$$\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S^{TT}_{ij}} \sim \Phi * \Phi$$

(2nd Order Pert.)

$$S_{ij} = 2\Phi\partial_i\partial_j\Phi - 2\Psi\partial_i\partial_j\Phi + 4\Psi\partial_i\partial_j\Psi + \partial_i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi$$
$$-\frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Phi)\partial_j(\Psi' + \mathcal{H}\Phi)$$
$$-\frac{2c_s^2}{3w\mathcal{H}}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi)$$

D. Wands et al, 2006-2010
Baumann et al, 2007
Peloso et al, 2018

INFLATIONARY MODELS

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$$\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S^{TT}_{ij}} \sim \Phi * \Phi$$ **(2nd Order Pert.)**

$$\Omega^{(0)}_{\rm GW}(f) = \frac{\Omega^{(0)}_{\rm rad}\mathcal{G}(\eta_c)}{24}\left(\frac{2\pi f}{a(\eta_c)H(\eta_c)}\right)^2 \overline{\mathcal{P}^{\rm ind}_h(\eta_c, 2\pi f)}$$

INFLATIONARY MODELS

INFLATION $\Rightarrow$ IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ $\Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

Let us suppose $\boxed{\Delta^2_{\mathcal{R}} \gg \Delta^2_{\mathcal{R}}\big|_{\rm CMB} \sim 3 \cdot 10^{-9}}$, @ small scales

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$$\overline{\mathcal{P}^{\rm ind}_h(\eta, k)} = 2\int_0^\infty dt \int_{-1}^1 ds \left[\frac{t(2+t)(s^2-1)}{(1-s+t)(1+s+t)}\right]^2 \times \overline{I^2(u,v,k,\eta)}\, \Delta^2_{\mathcal{R}}(ku) \cdot \Delta^2_{\mathcal{R}}(kv),$$

INFLATIONARY MODELS

INFLATION → IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ $\Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

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$\Rightarrow$ $\Omega^{(0)}_{\rm GW} \propto (\triangle^2_{\mathcal{R}})^2$

INFLATIONARY MODELS

INFLATION $\rightarrow$ IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ $\Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

BBN $\Omega_{gw,0} < 1.5 \times 10^{-6}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 0.1$

LIGO $\Omega_{gw,0} < 6.9 \times 10^{-8}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 0.01$

PTA $\Omega_{gw,0} < 1 \times 10^{-9}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 5 \times 10^{-3}$

LISA $\Omega_{gw,0} < 10^{-13}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 1 \times 10^{-5}$

BBO $\Omega_{gw,0} < 10^{-17}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 3 \times 10^{-7}$

(Numbers not updated !)

INFLATIONARY MODELS

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IF $\Delta^2_{\mathcal{R}}$ very enhanced → Primordial Black Holes (PBH) may be produced!

INFLATIONARY MODELS

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PBH candidate for DM ? Yes !, for $\sim 10^{-15}$–$10^{-11} M_\odot$

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* If PBH are the DM, what is the GWB from 2nd O(Φ)? Bartolo et al, '18

Right in the middle of LISA !

INFLATIONARY MODELS

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Has LIGO detected PBH's ?

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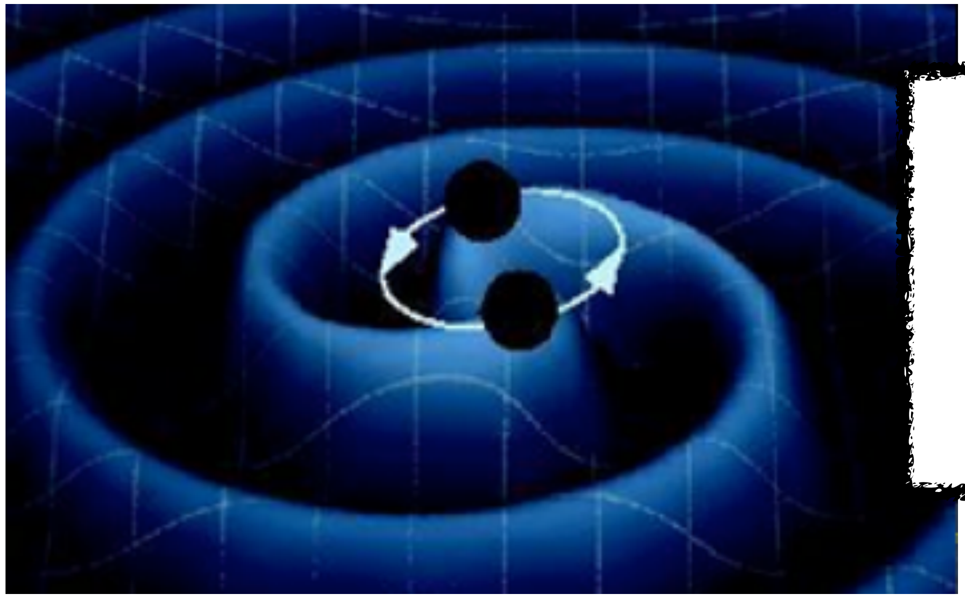

'We will know determining the mass/spin distribution'

(M. Fishbach (LIGO), Moriond'19)

e.g. **2102.03809**, **2105.03349**, **De Luca** *et al*

INFLATIONARY MODELS

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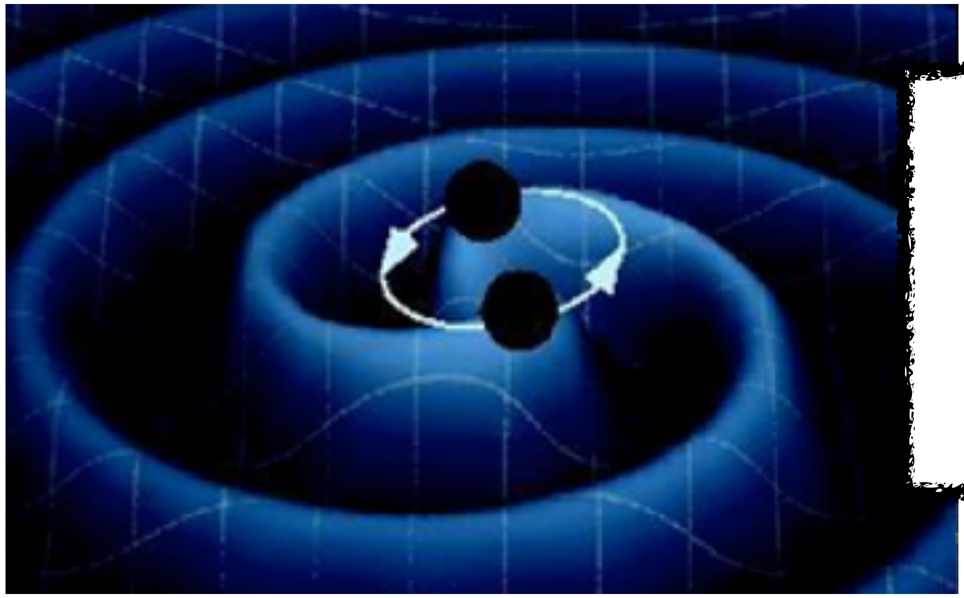
IF $\Delta^2_{\mathcal{R}}$ very enhanced → Primordial Black Holes (PBH) may be produced!

Has LIGO detected PBH's ? **Maybe P. Ajith will tell …**



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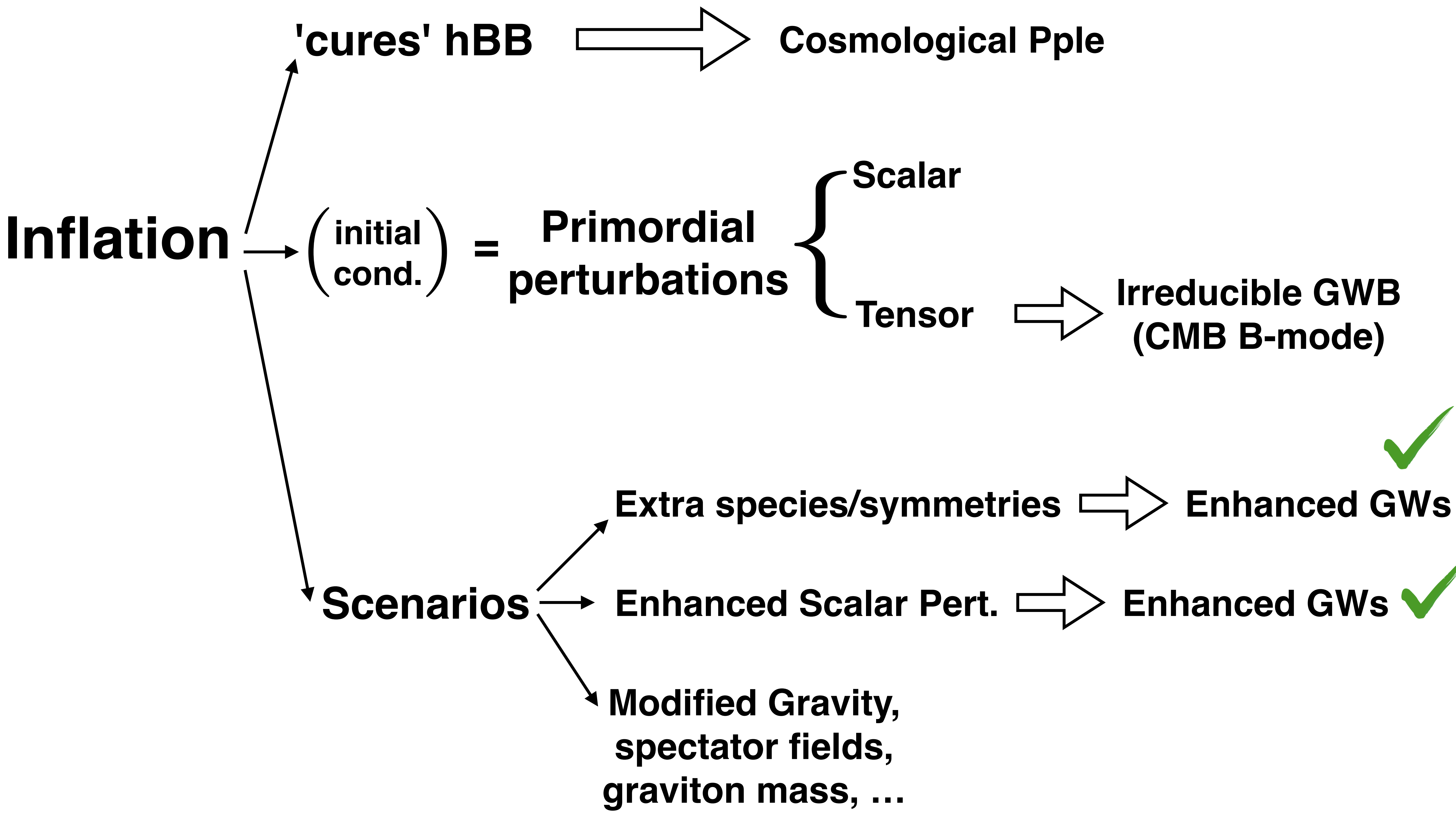
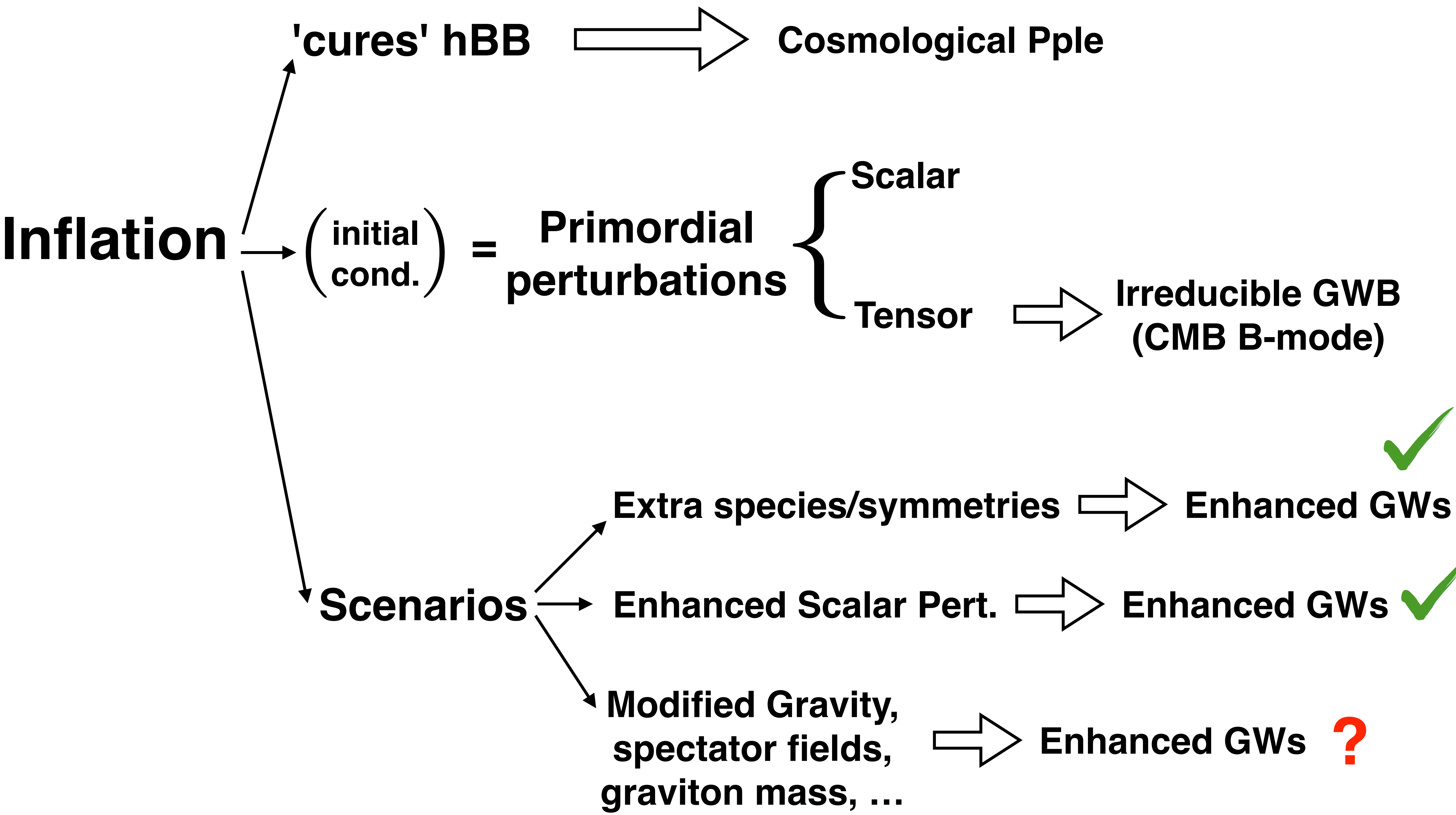
INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY



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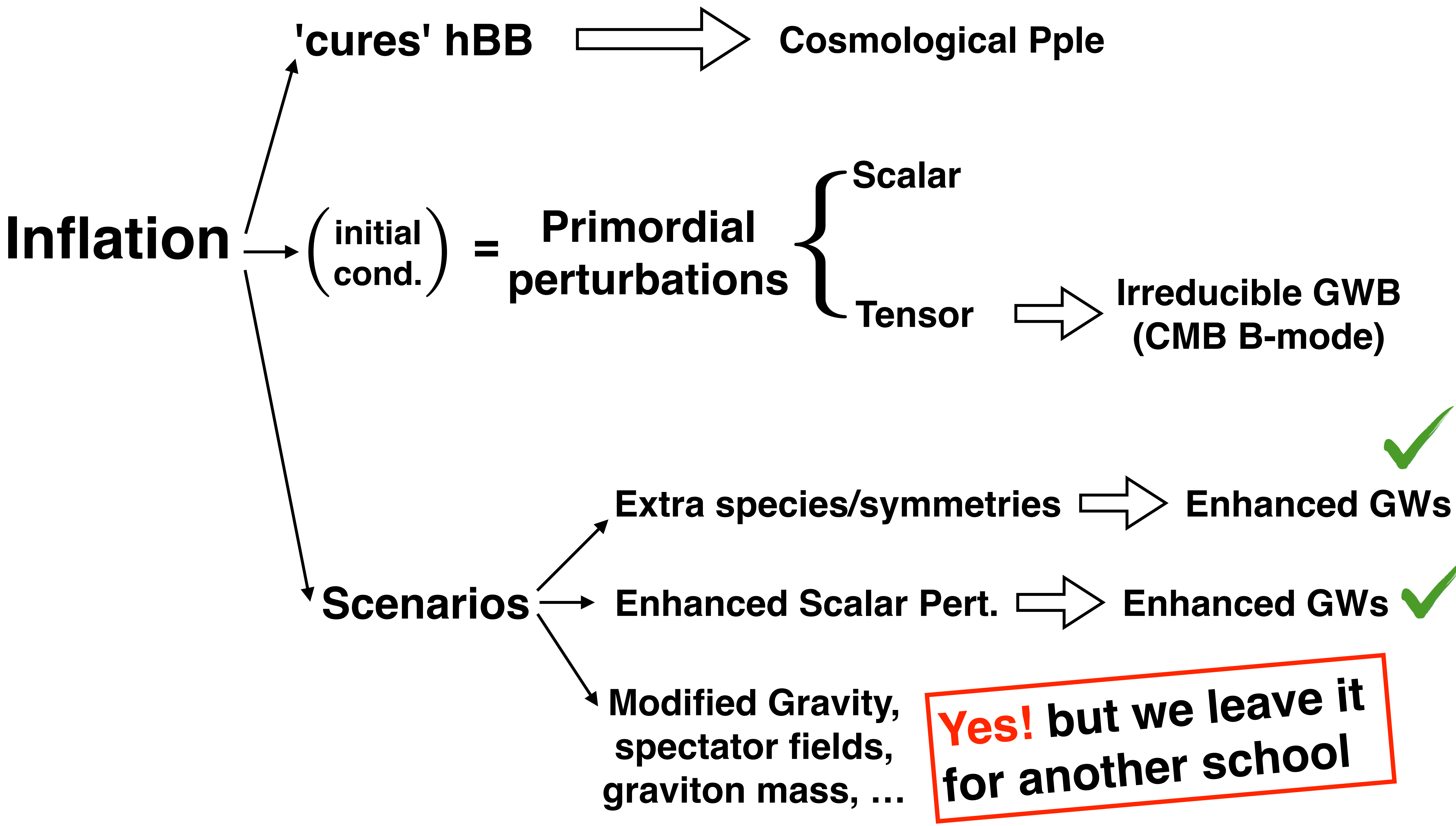
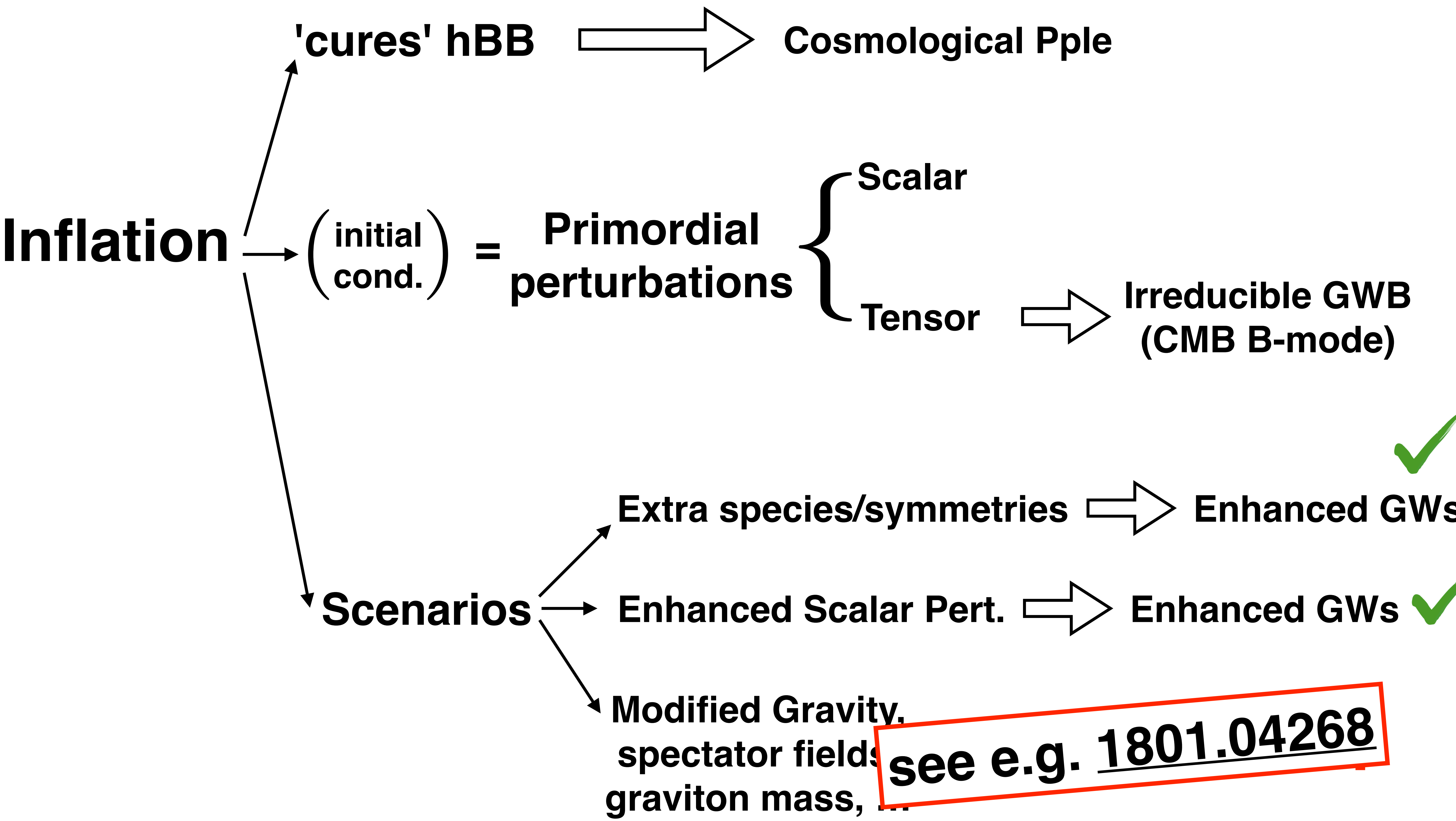


INFLATIONARY COSMOLOGY

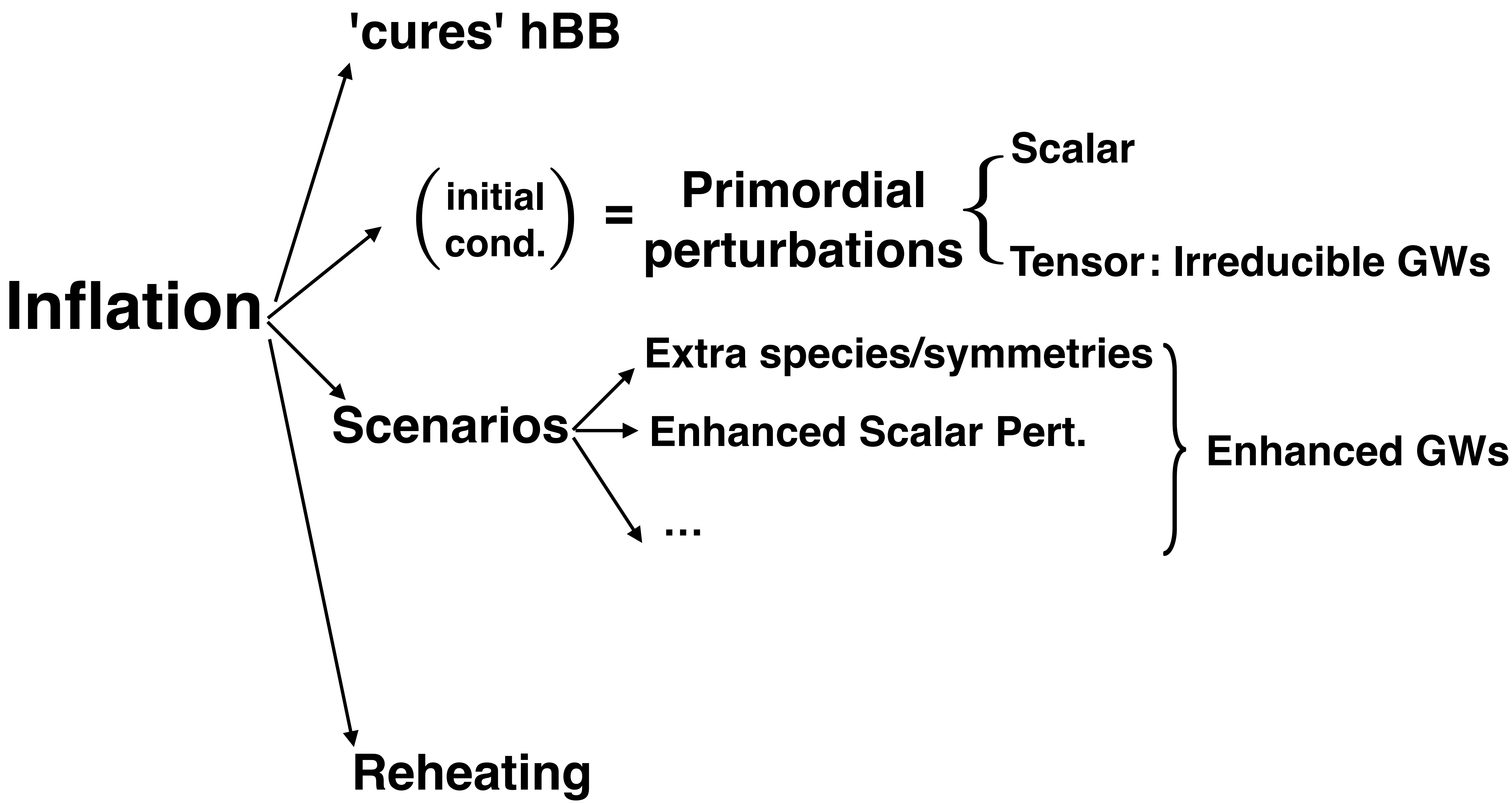


INFLATIONARY COSMOLOGY

Inflation
- 'cures' hBB
- $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations
  - Scalar
  - Tensor : Irreducible GWs
- Scenarios
  - Extra species/symmetries
  - Enhanced Scalar Pert.
  - ...

  Enhanced GWs

INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY