Structure Formation
Lecture 3

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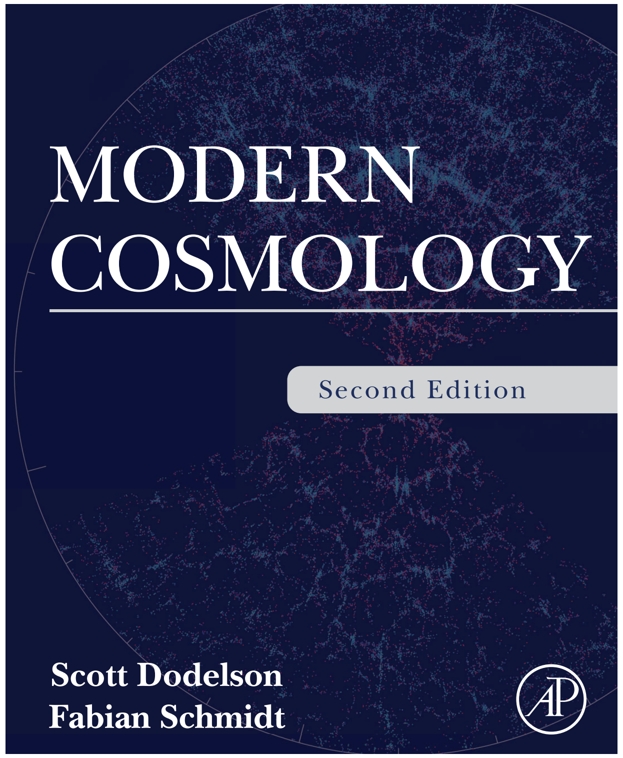
All figures taken from *Modern Cosmology, Second Edition*, unless otherwise noted



ICTP School on Cosmology, June 2024

Notation

$$ds^2 = -(1+2\Psi(\boldsymbol{x},t))dt^2 + a^2(t)(1+2\Phi(\boldsymbol{x},t))d\boldsymbol{x}^2$$

- Comoving coordinates: $d\boldsymbol{r} = a(t)d\boldsymbol{x}$
- Conformal time: $d\eta = \frac{dt}{a(t)} = \frac{da}{a^2H(a)} = \frac{d\ln a}{aH(a)}.$
    - Comoving distance: $d\chi = -d\eta = \frac{dz}{H(z)}$
- Particle velocity/momentum: $\boldsymbol{v} = \frac{\boldsymbol{p}}{m} = a\frac{d\boldsymbol{x}}{dt} = \boldsymbol{x}'$
- Fluid velocity; divergence: $\boldsymbol{u}; \quad \theta = \partial_i u^i$
- Gravitational potential: Ψ

Recap

- We then derived the *linear approximation*, when all of δ,θ,Ψ are small:

$$\delta^{(1)}(\boldsymbol{x},\eta) = D(\eta)\delta_0(\boldsymbol{x})$$

$$D'' + aHD' = \frac{3}{2}\Omega_\mathrm{m}(\eta)(aH)^2 D(\eta)$$

$$\Omega_\mathrm{m}(\eta) = \frac{\rho_\mathrm{m}(\eta)}{\rho_\mathrm{cr}(\eta)}$$

Time-dependent density parameter;
=0.3 today, =1 in the past

The density at all points in (real or Fourier) space evolves independently!

Going beyond linear theory

- Let's go back to full fluid equations
- They contain nonlinear terms, specifically quadratic terms, moved here to the r.h.s.:

$$\delta_{\rm m}{}' + \theta_{\rm m} = -\delta_{\rm m}\theta_{\rm m} - u_{\rm m}^j \frac{\partial}{\partial x^j}\delta_{\rm m},$$

$$\theta_{\rm m}{}' + aH\theta_{\rm m} + \nabla^2\Psi = -u_{\rm m}^j \frac{\partial}{\partial x^j}\theta_{\rm m} - (\partial_i u_{\rm m}^j)(\partial_j u_{\rm m}^i).$$

$$\nabla^2\Psi = \frac{3}{2}\Omega_{\rm m}(\eta)(aH)^2\delta_{\rm m}.$$ is just linear!

Going beyond linear theory

- That structure suggests iterative approach: plug in linear solution to nonlinear source terms, and solve for second order:

$$\delta^{(2)\prime} + \theta^{(2)} = -\delta^{(1)}\theta^{(1)} - (u^{(1)})^j \frac{\partial}{\partial x^j}\delta^{(1)},$$

$$\theta^{(2)\prime} + aH\theta^{(2)} + \frac{3}{2}\Omega_{\rm m}(\eta)(aH)^2\delta^{(2)} = -(u^{(1)})^j \frac{\partial}{\partial x^j}\theta^{(1)} - [\partial_i(u^{(1)})^j][\partial_j(u^{(1)})^i],$$

where we have used the Poisson equation for $\nabla^2\Psi^{(2)}$

Perturbation theory

- Idea: expand all fields according to:

$$\delta_{\rm m}(\boldsymbol{x},\eta) = \delta^{(1)}(\boldsymbol{x},\eta) + \delta^{(2)}(\boldsymbol{x},\eta) + \cdots + \delta^{(n)}(\boldsymbol{x},\eta)$$
$$\theta_{\rm m}(\boldsymbol{x},\eta) = \theta^{(1)}(\boldsymbol{x},\eta) + \theta^{(2)}(\boldsymbol{x},\eta) + \cdots + \theta^{(n)}(\boldsymbol{x},\eta)$$

- Each order collects all terms that have the same number of *linear fields* $\delta^{(1)}$, $\theta^{(1)}$
- This approach is expected to work as long as each successive term in the series is smaller than the previous one
- Of course, in practice we always stop at some *n*

Second order

- So let's proceed with solving at second order:

$$\delta^{(2)\prime} + \theta^{(2)} = -\delta^{(1)}\theta^{(1)} - (u^{(1)})^j \frac{\partial}{\partial x^j}\delta^{(1)},$$

$$\theta^{(2)\prime} + aH\theta^{(2)} + \frac{3}{2}\Omega_{\mathrm{m}}(\eta)(aH)^2\delta^{(2)} = -(u^{(1)})^j \frac{\partial}{\partial x^j}\theta^{(1)} - [\partial_i(u^{(1)})^j][\partial_j(u^{(1)})^i],$$

- R.h.s. involves derivatives and velocity u: more easily solved in *Fourier space*

- The linear velocity is given by

$$(u^{(1)})^i(\boldsymbol{k},\eta) = \frac{ik^i}{k^2}aHf\delta^{(1)}(\boldsymbol{k},\eta) \qquad f \equiv d\ln D/d\ln a$$

Second order

- Fourier transform, and pull out time dependence of source term (important that we can do that!)

$$\delta^{(2)\prime}(\boldsymbol{k},\eta) + \theta^{(2)}(\boldsymbol{k},\eta) = aHfD^2(\eta)S_\delta(\boldsymbol{k})$$

$$\theta^{(2)\prime}(\boldsymbol{k},\eta) + \frac{3}{2}\Omega_m(\eta)(aH)^2\delta^{(2)}(\boldsymbol{k},\eta) = (aHf)^2D^2(\eta)S_\theta(\boldsymbol{k})$$

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$$S_\delta(\boldsymbol{k})=\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\mathrm{D}}^{(3)}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2)\times\left[1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1^2}\right]\delta_0(\boldsymbol{k}_1)\delta_0(\boldsymbol{k}_2),$$

$$S_\theta(\boldsymbol{k})=-\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\mathrm{D}}^{(3)}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2)\times\left[\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1^2}+\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2}\right]\delta_0(\boldsymbol{k}_1)\delta_0(\boldsymbol{k}_2)$$

where we used

$$\delta^{(1)}(\boldsymbol{k},\eta)=D(\eta)\delta_0(\boldsymbol{k})$$

Second order

- So we can separate the time- and **k**-dependent parts even at second order!

$$\delta^{(2)\prime}(\boldsymbol{k},\eta)+\theta^{(2)}(\boldsymbol{k},\eta)=aHfD^2(\eta)S_\delta(\boldsymbol{k})$$

$$\theta^{(2)\prime}(\boldsymbol{k},\eta)+\frac{3}{2}\Omega_m(\eta)(aH)^2\delta^{(2)}(\boldsymbol{k},\eta)=(aHf)^2D^2(\eta)S_\theta(\boldsymbol{k})$$

$$S_\delta(\boldsymbol{k})=\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\rm D}^{(3)}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2)\times\left[1+\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1^2}\right]\delta_0(\boldsymbol{k}_1)\delta_0(\boldsymbol{k}_2),$$

where we used

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Second order

- Solving coupled set of sourced first-order ODE using standard techniques* yields:

$$\delta^{(2)}(\boldsymbol{k},\eta) = D_+^2(\eta)\int \frac{d^3k_1}{(2\pi)^3}\int \frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\rm D}^{(3)}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2) \times F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\delta_0(\boldsymbol{k}_1)\delta_0(\boldsymbol{k}_2),$$

$$F_2(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{5}{7} + \frac{2}{7}\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2} + \frac{1}{2}\boldsymbol{k}_1\cdot\boldsymbol{k}_2\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)$$

time-independent perturbation theory kernel

- Velocity divergence θ obeys similar equation

* Assume matter domination when integrating equations; accurate to better than 1%.

Second order

- Solving coupled set of sourced first-order ODE using standard techniques* yields:

grows twice as fast as linear density

$$\delta^{(2)}(\boldsymbol{k},\eta) = D_+^2(\eta) \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} (2\pi)^3 \delta_{\rm D}^{(3)}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2) \times F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\delta_0(\boldsymbol{k}_1)\delta_0(\boldsymbol{k}_2),$$

$$F_2(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{5}{7} + \frac{2}{7}\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2} + \frac{1}{2}\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)$$

time-independent perturbation theory kernel

Note error in Mod. Cosmology, Ch. 12 !

- Velocity divergence θ obeys similar equation

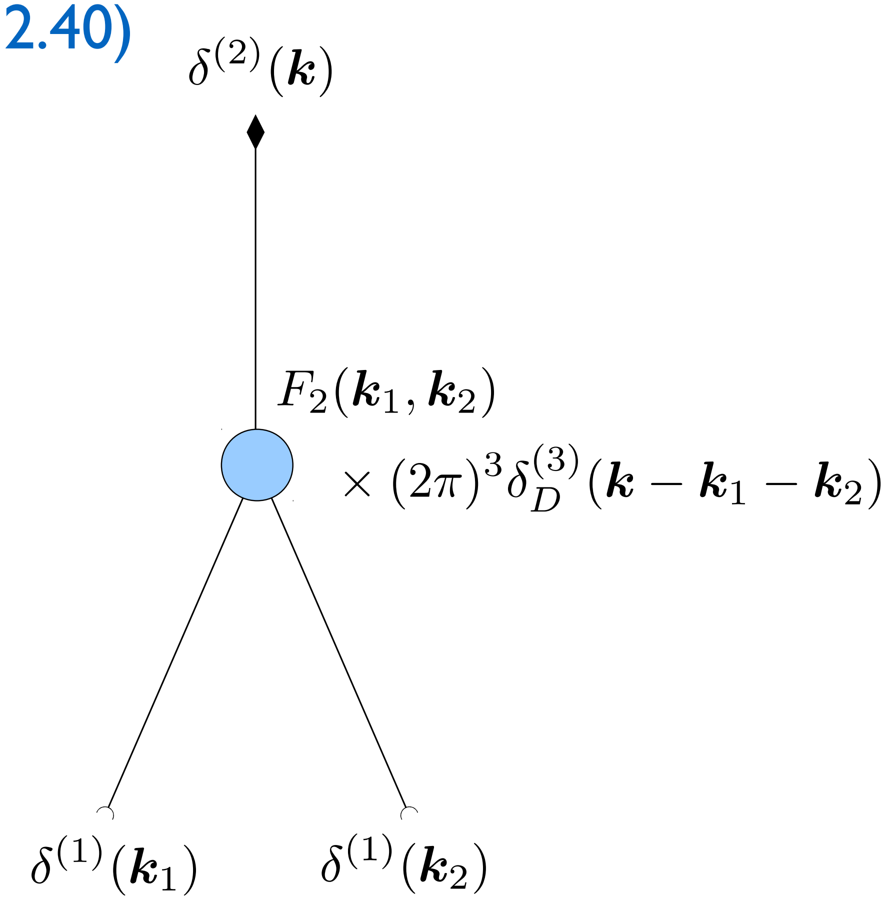
* Assume matter domination when integrating equations; accurate to better than 1%.

Diagrammatic representation

- F_2 corresponds to interaction vertex (with 3-momentum conservation) coupling two incoming δ_0

$$\delta^{(2)}(\boldsymbol{k},\eta) = D_+^2(\eta)\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}(2\pi)^3\delta_{\mathrm{D}}^{(3)}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2) \times F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\delta_0(\boldsymbol{k}_1)\delta_0(\boldsymbol{k}_2),$$

Eq. (12.40)

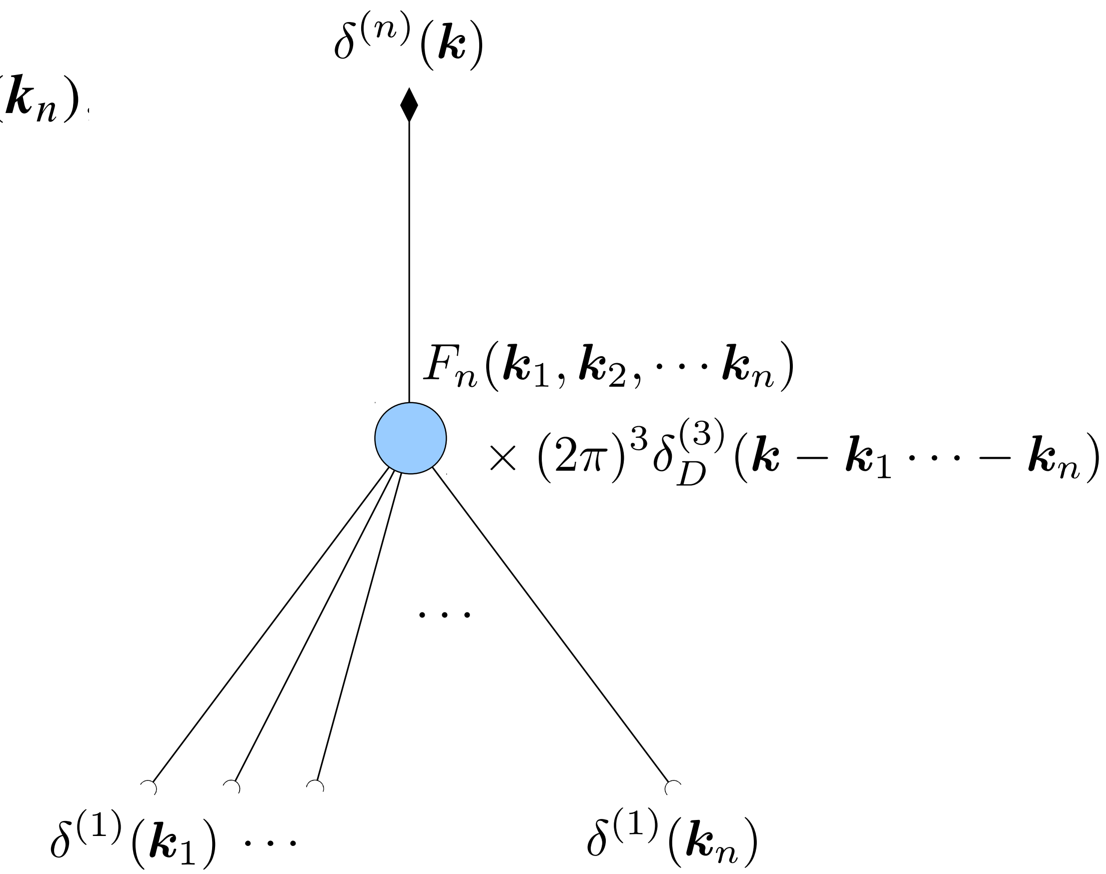


Diagrammatic representation

- Similarly, we can go to higher orders:

$$\delta^{(n)}(\boldsymbol{k},\eta) = D_+^n(\eta)\left[\prod_{i=1}^{n}\int \frac{d^3k_i}{(2\pi)^3}\right](2\pi)^3\delta_{\rm D}^{(3)}\left(\boldsymbol{k}-\sum_{i=1}^{n}\boldsymbol{k}_i\right) \times F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n).$$

PT kernels F_n obey recursion relation.



Matter power spectrum

- Since we don't know the initial conditions at the field level, let's compute statistics
- Power spectrum:

$$\left\langle\delta_{\mathrm{m}}(\boldsymbol{k},\eta)\delta_{\mathrm{m}}(\boldsymbol{k}',\eta)\right\rangle = D_+^2(\eta)\left\langle\delta_0(\boldsymbol{k})\delta_0(\boldsymbol{k}')\right\rangle + \left\langle\delta^{(2)}(\boldsymbol{k},\eta)\delta^{(2)}(\boldsymbol{k}',\eta)\right\rangle + 2\left\langle\delta^{(1)}(\boldsymbol{k},\eta)\delta^{(3)}(\boldsymbol{k}',\eta)\right\rangle + \cdots .$$

Eq. (12.42)

- Why these terms and not others?
  - Count terms that have equal numbers of δ_0
  - Terms with odd number of δ_0 vanish

Matter power spectrum

- Terms with odd number of δ_0 vanish because δ_0 is *Gaussian*

(this can be generalized to include small amount of *primordial non-Gaussianity*)

- For terms with even number, we use *Wicks' theorem:*

$$\langle \delta_0(\boldsymbol{k}_1)\delta_0(\boldsymbol{k}_2)\delta_0(\boldsymbol{k}_3)\rangle = 0$$

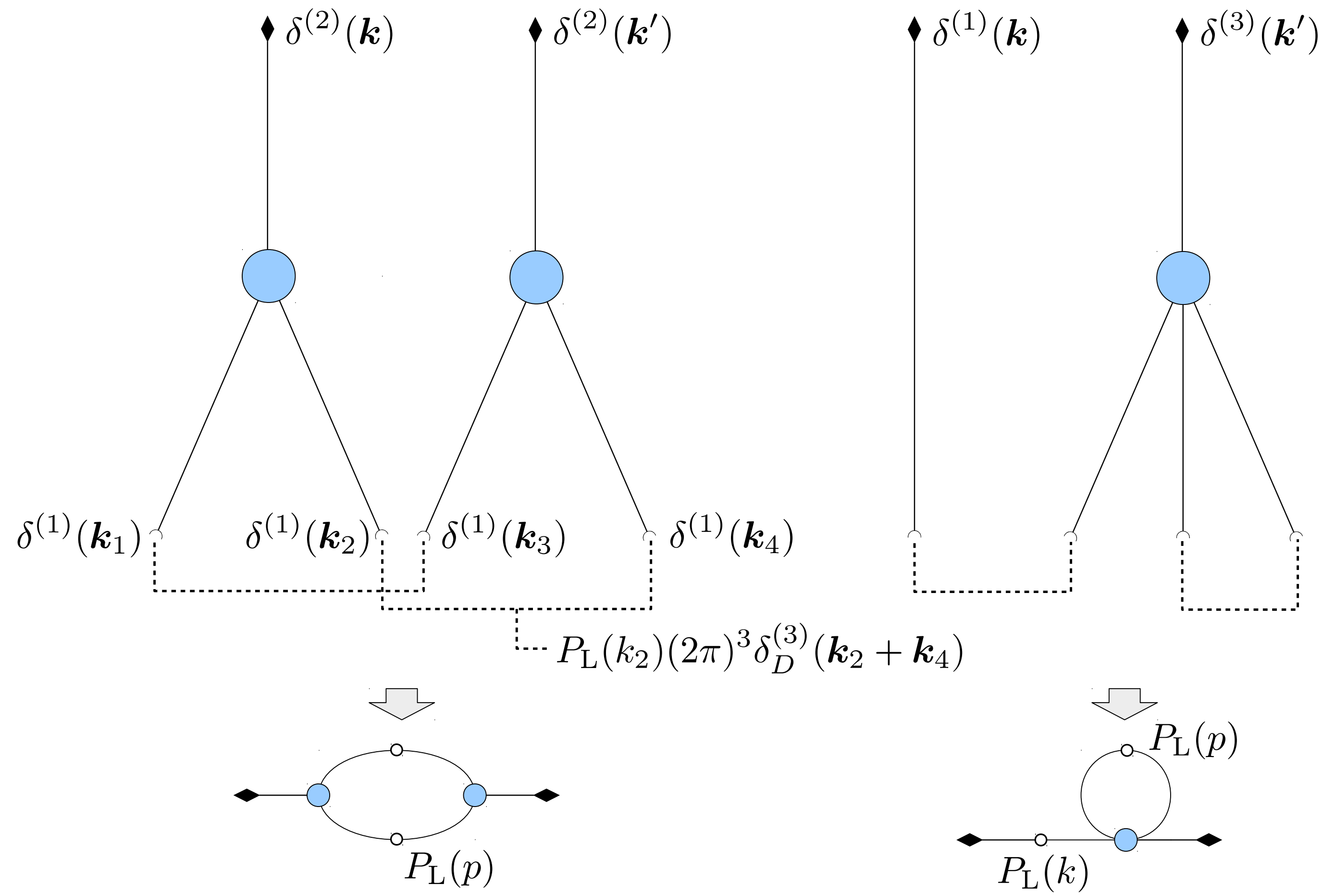
$$\begin{aligned}\langle \delta_0(\boldsymbol{k}_1)\delta_0(\boldsymbol{k}_2)\delta_0(\boldsymbol{k}_3)\delta_0(\boldsymbol{k}_4)\rangle &= (2\pi)^6\delta_{\rm D}^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2)\delta_{\rm D}^{(3)}(\boldsymbol{k}_3+\boldsymbol{k}_4)P(\boldsymbol{k}_1)P(\boldsymbol{k}_3)\\ &+(2\pi)^6\delta_{\rm D}^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_3)\delta_{\rm D}^{(3)}(\boldsymbol{k}_2+\boldsymbol{k}_4)P(\boldsymbol{k}_1)P(\boldsymbol{k}_2)\\ &+(2\pi)^6\delta_{\rm D}^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_4)\delta_{\rm D}^{(3)}(\boldsymbol{k}_2+\boldsymbol{k}_3)P(\boldsymbol{k}_1)P(\boldsymbol{k}_2).\end{aligned}$$

leads directly to

$$\begin{aligned}\left\langle \delta_{\rm m}(\boldsymbol{k},\eta)\delta_{\rm m}(\boldsymbol{k}',\eta)\right\rangle &= D_+^2(\eta)\left\langle \delta_0(\boldsymbol{k})\delta_0(\boldsymbol{k}')\right\rangle\\ &+\left\langle \delta^{(2)}(\boldsymbol{k},\eta)\delta^{(2)}(\boldsymbol{k}',\eta)\right\rangle + 2\left\langle \delta^{(1)}(\boldsymbol{k},\eta)\delta^{(3)}(\boldsymbol{k}',\eta)\right\rangle + \cdots .\end{aligned}$$

Matter power spectrum

- Nicely represented using diagrams:



$$\left\langle\delta_{\rm m}(\boldsymbol{k},\eta)\delta_{\rm m}(\boldsymbol{k}',\eta)\right\rangle = D_+^2(\eta)\left\langle\delta_0(\boldsymbol{k})\delta_0(\boldsymbol{k}')\right\rangle + \left\langle\delta^{(2)}(\boldsymbol{k},\eta)\delta^{(2)}(\boldsymbol{k}',\eta)\right\rangle + 2\left\langle\delta^{(1)}(\boldsymbol{k},\eta)\delta^{(3)}(\boldsymbol{k}',\eta)\right\rangle + \cdots .$$

Eq. (12.42)

Matter power spectrum

- Use Feynman rules, or just plug in kernels to obtain:

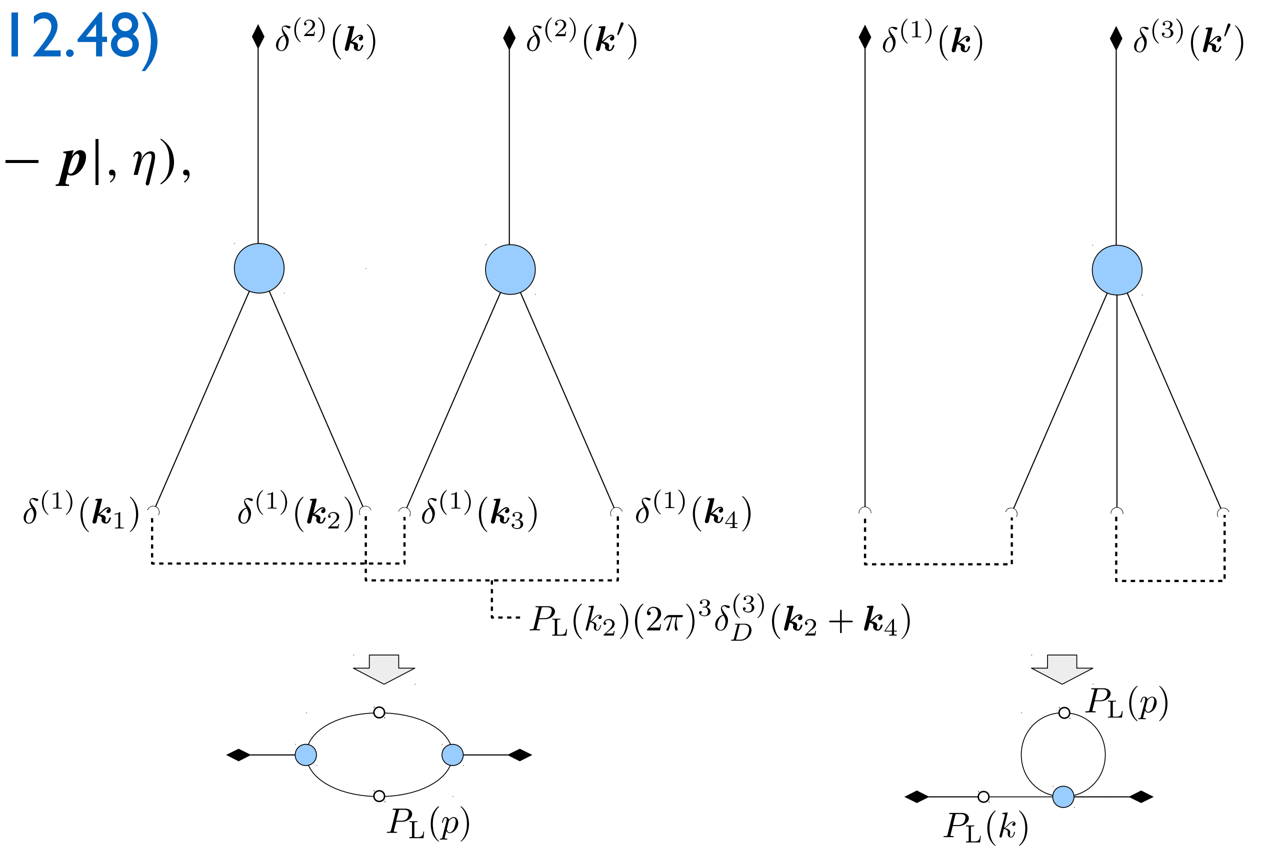
$$P(k,\eta) = P_{\rm L}(k,\eta) + P^{\rm NLO}(k,\eta) + \cdots\,,$$

$$P^{\rm NLO}(k,\eta) = P^{(22)}(k,\eta) + 2P^{(13)}(k,\eta),$$

Eq. (12.48)

$$P^{(22)}(k,\eta) = 2\int \frac{d^3p}{(2\pi)^3}\left[F_2(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p})\right]^2 P_{\rm L}(p,\eta)P_{\rm L}(|\boldsymbol{k}-\boldsymbol{p}|,\eta),$$

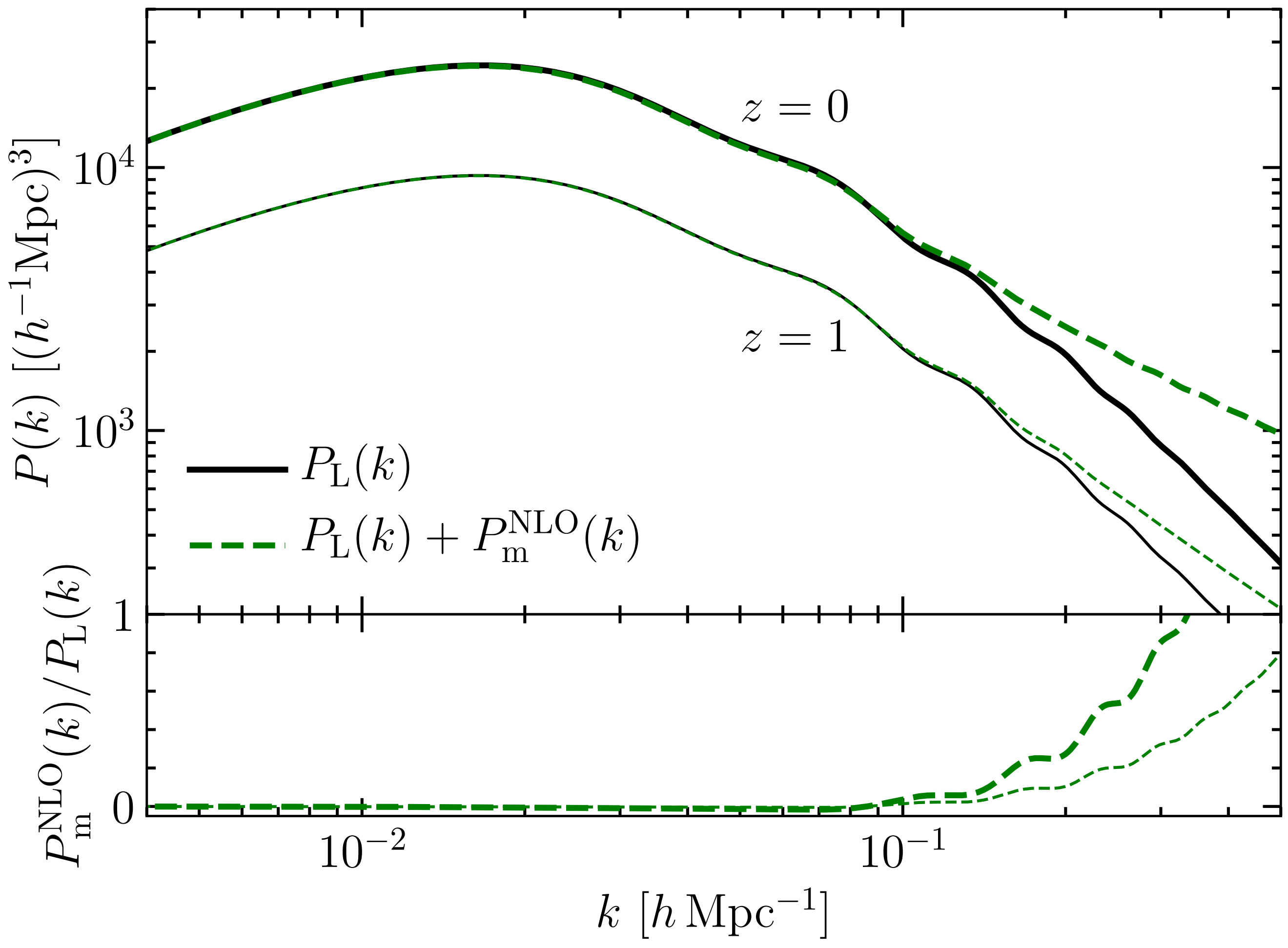
$$P^{(13)}(k,\eta) = 3P_{\rm L}(k,\eta)\int \frac{d^3p}{(2\pi)^3} F_3(\boldsymbol{p},-\boldsymbol{p},\boldsymbol{k})P_{\rm L}(p,\eta).$$



Matter power spectrum

- Then let computer do the work…

$$P(k,\eta) = P_{\rm L}(k,\eta) + P^{\rm NLO}(k,\eta) + \cdots\,,$$
$$P^{\rm NLO}(k,\eta) = P^{(22)}(k,\eta) + 2P^{(13)}(k,\eta),$$



Bispectrum (exercise)

- The bispectrum, or three-point function of δ_0 vanishes, but not that of the evolved field δ_m, thanks to nonlinear evolution:

$$\langle \delta_\mathrm{m}(\boldsymbol{k}_1, \eta)\delta_\mathrm{m}(\boldsymbol{k}_2, \eta)\delta_\mathrm{m}(\boldsymbol{k}_3, \eta)\rangle = (2\pi)^3 \delta_\mathrm{D}^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \times \left[2F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) P_\mathrm{L}(k_1, \eta) P_\mathrm{L}(k_2, \eta) + 2\ \text{perm.}\right]$$

Eq. (12.51)

At leading order; there are also "next-to-leading" (NLO) contributions - try writing down the diagram for the leading three-point function as well as the NLO one!

Beyond the fluid approximation

- So far, did well-defined perturbation theory, but of the ***wrong equation:*** collisionless matter is not a fluid
- Rather, the correct equation is the collisionless Boltzmann equation
- What is the error we are making?
- Recall that we neglected the velocity dispersion, or stress tensor σ_m, which adds force term to the Euler equation, $\rho_{\rm m}^{-1}\partial_j\sigma_{\rm m}^{ij}$

$$\frac{1}{m}\left\langle p^i\,p^j\right\rangle_{f_{\rm m}} = \rho_{\rm m}u_{\rm m}^i u_{\rm m}^j + \sigma_{\rm m}^{ij}.$$

- What is the effect of the stress tensor? Can we incorporate it?

Beyond the fluid approximation

- Idea: treat stress tensor as effective quantity, and parametrize it, at the background and perturbation level:

$$\sigma_{\rm m}^{ij}(\boldsymbol{x},\eta) = \bar{\sigma}_{\rm m}(\eta)\delta^{ij}\left[1 + c_\sigma(\eta)\delta_{\rm m}(\boldsymbol{x},\eta) + \ldots\right]$$

- We can't predict the coefficients from within the fluid picture - leave them free for now

$$\bar{\sigma}_{\rm m}(\eta),\ c_\sigma(\eta)$$

Beyond the fluid approximation

$$\sigma_{\mathrm{m}}^{ij}(\boldsymbol{x},\eta) = \bar{\sigma}_{\mathrm{m}}(\eta)\delta^{ij}\left[1 + c_{\sigma}(\eta)\delta_{\mathrm{m}}(\boldsymbol{x},\eta) + \ldots\right]$$

- Insert into Euler equation:

$$u_{\mathrm{m}}^{i}{}' + aHu_{\mathrm{m}}^{i} + \partial^{i}\Psi + \frac{c_{\sigma}\bar{\sigma}_{\mathrm{m}}}{\bar{\rho}_{\mathrm{m}}}\partial^{i}\delta_{\mathrm{m}} = (\text{unchanged 2nd-order terms})$$

Notice that constant, background stress has no dynamical effect.

Beyond the fluid approximation

$$\sigma^{ij}_{\rm m}(\boldsymbol{x},\eta) = \bar{\sigma}_{\rm m}(\eta)\delta^{ij}\left[1 + c_\sigma(\eta)\delta_{\rm m}(\boldsymbol{x},\eta) + \ldots\right]$$

- Insert into Euler equation, take divergence again:

$$\theta'_{\rm m} + aH\theta_{\rm m} + \nabla^2\Psi + \frac{c_\sigma\bar{\sigma}_{\rm m}}{\bar{\rho}_{\rm m}}\nabla^2\delta_{\rm m} = (\text{unchanged 2nd-order terms})$$

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Beyond the fluid approximation

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Notice that constant, background stress has no dynamical effect.

- Additional contribution is suppressed on large scales: two additional derivatives, ~k^2 in Fourier space
- Hence, can take into account stress tensor at leading order by adding one term to equations, *at the price of an unknown, free coefficient* $\bar{\sigma}_{\mathrm{m}}c_\sigma$

Cold collisionless matter = *effective fluid*

- At leading order, this is just another linear term in the equations (but with more derivatives)
- By redefining coefficient, correction to final density field can be written as (with free coefficient C_s^2)

$$\delta^{(1)}(\boldsymbol{k},\eta) \to \left[1 - C_s^2(\eta)k^2\right] D_+(\eta)\delta_0(\boldsymbol{k}) \quad ; \quad P_{\rm NLO}(k) \to P_{\rm NLO}(k) - 2C_s^2(\eta)k^2 P_{\rm L}(k)$$

Similar size as $P_{\rm NLO}(k)$

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Similar size as $P_{\rm NLO}(k)$

- In fact, theoretical consistency forces us to introduce C_s^2 (~ eff. sound horizon) as *counterterm:*

$$P^{(13)}(k,\eta) = 3P_{\rm L}(k,\eta) \int \frac{d^3p}{(2\pi)^3} F_3(\boldsymbol{p},-\boldsymbol{p},\boldsymbol{k}) P_{\rm L}(p,\eta)$$

$$\propto k^2/p^2 \text{ for } p \gg k$$

Yes, P_{22} also leads to a counterterm, but that one is much smaller.

Effective Field Theory of Structure Formation

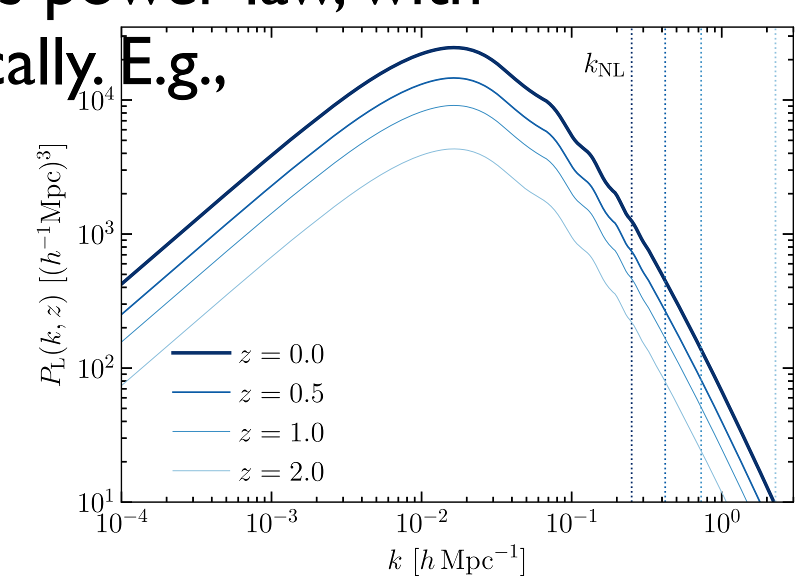
- Idea: allow for all counterterms in effective fluid equations consistent with symmetries: general covariance; mass and momentum conservation
- Order different contribution according to their scaling with k
- Only one relevant scale: k_{NL}, where (roughly) matter density field becomes fully nonlinear:

$$k_{\rm NL}^{-2} = \int \frac{d^3p}{(2\pi)^3} p^{-2} P_{\rm L}(p)$$

- For this ordering, we typically approximate *$P_L(k) \sim k^n$* as power law, with *$n \sim -1.5$*, allowing us to compute loop integrals analytically. E.g.,

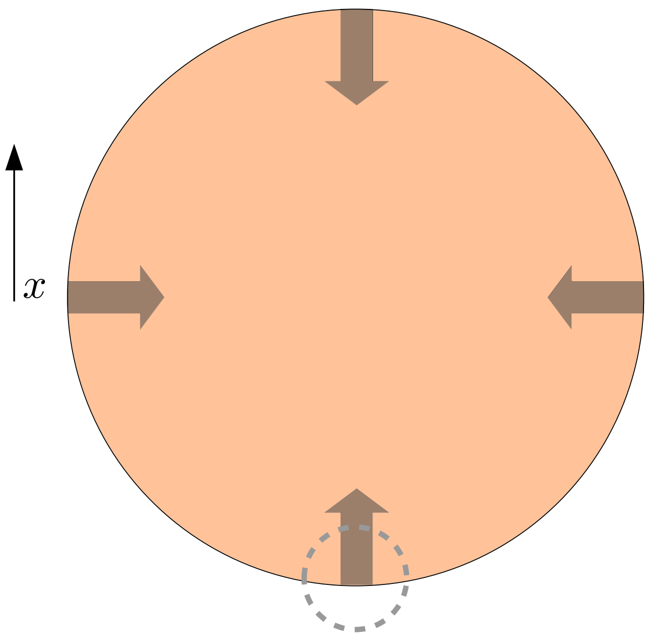
$$P_{\rm NLO}(k) \sim \left(\frac{k}{k_{\rm nl}}\right)^{3+n} P_{\rm L}(k)$$

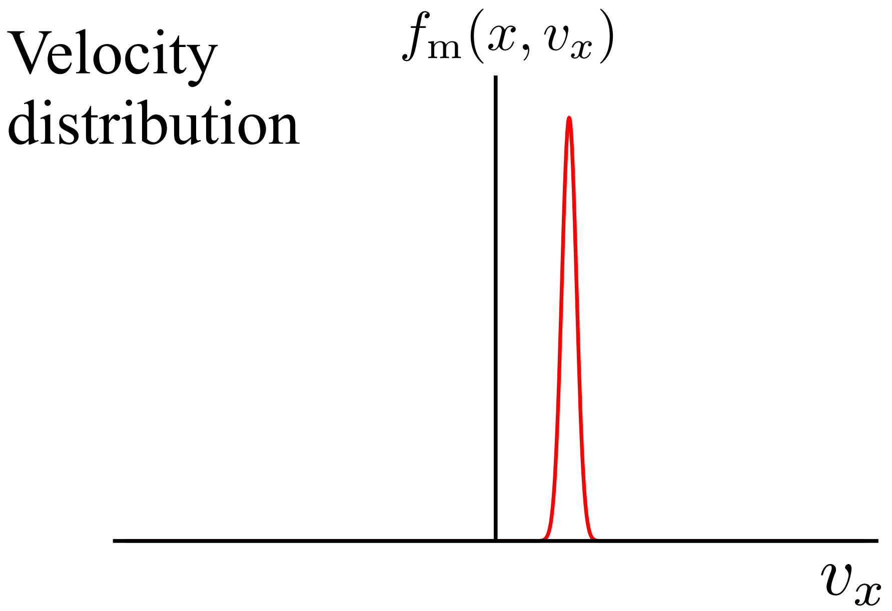
$$P_{C_s^2}(k) \sim \left(\frac{k}{k_{\rm nl}}\right)^{2} P_{\rm L}(k)$$

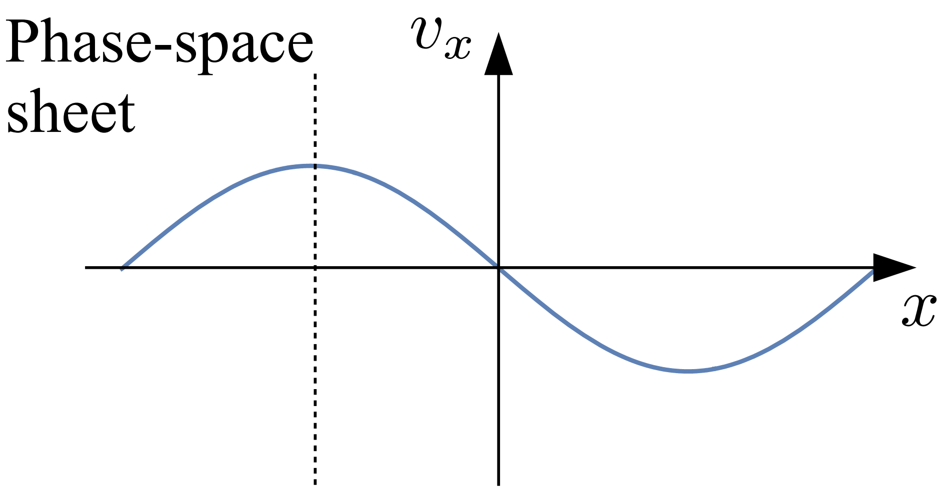


Phasespace view of structure formation

- Initial stages of collapse of overdense region

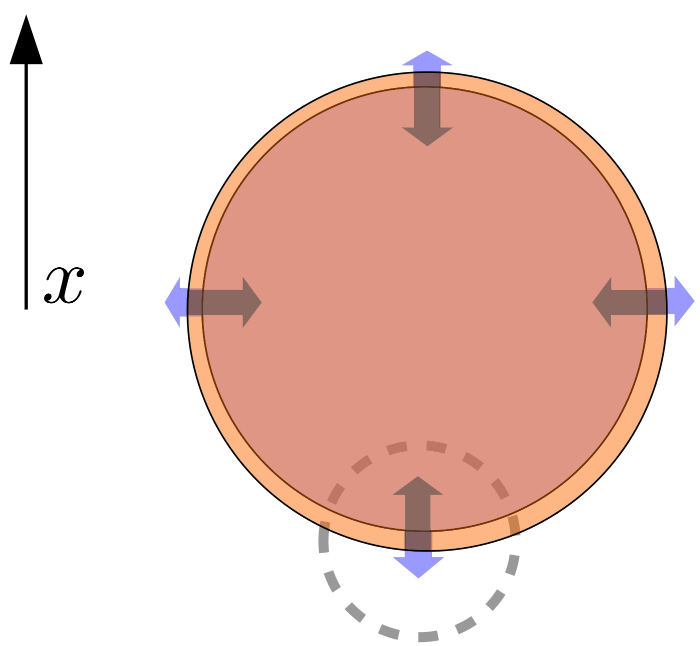


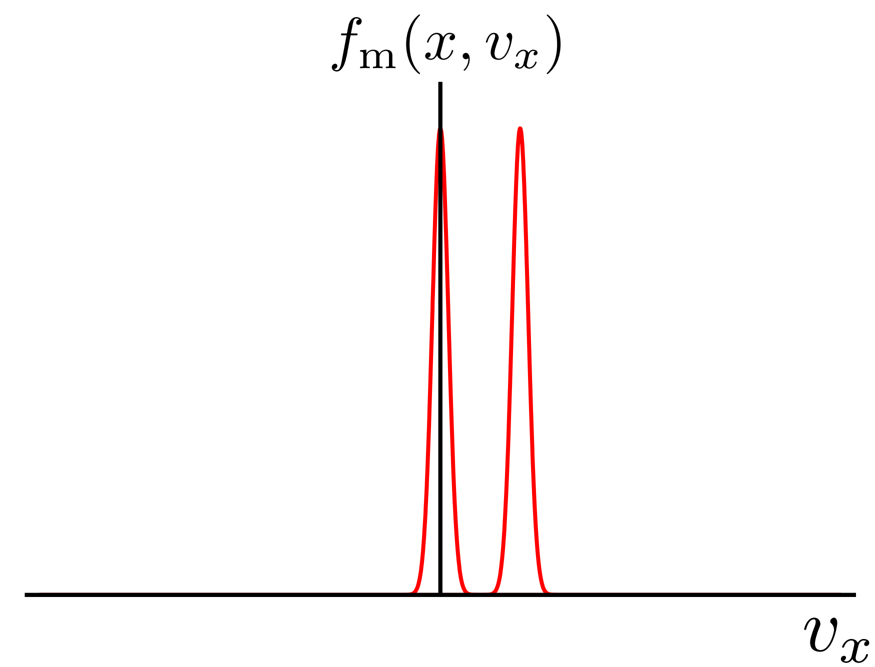


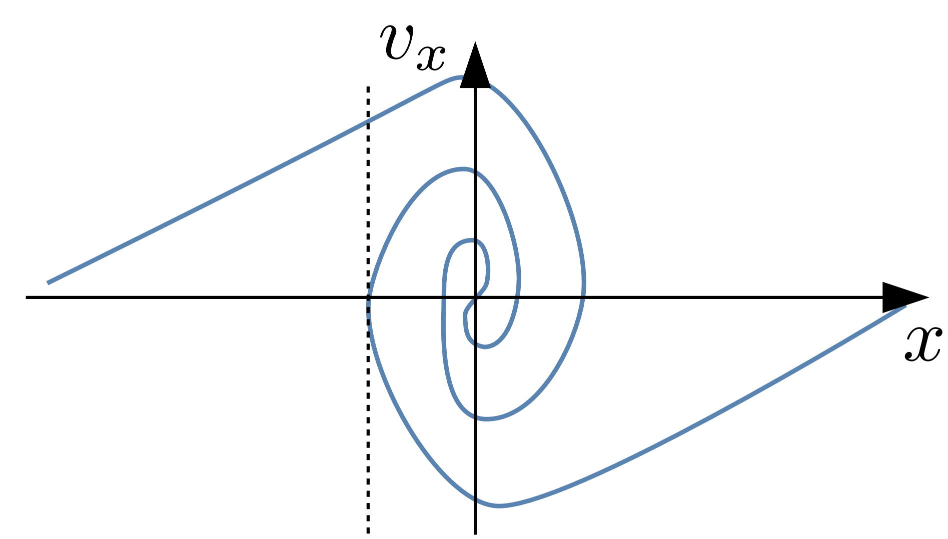


Phasespace view of structure formation

- Later stages of collapse of overdense region







Structure formation beyond perturbation theory

- In order to take this phasespace evolution into account properly, need to go beyond fluid picture and perturbation theory.
- Back to collisionless Boltzmann equation!