$\boxed{L4}$ Galaxies as tracers of LSS

What's different from matter? 1) Galaxies are not conserved $\rightsquigarrow$ source term to continuity equation 2) Galaxies experience non-gravitational forces: (effective) pressure term to Euler equation.

Formally, can derive this by starting from Boltzmann eqn. for galaxy distr. fct. $f_g(\vec{x}, \vec{p}, t)$, which now has collision term $\mathcal{C}$:

$$\frac{Df_g}{Dt} = \mathcal{C}[f_g, f_m] .$$

But $\mathcal{C}$ is unknown nonlinear functional ... so let's try to take moments again.

$$\delta_g' + \theta_g + \delta_g \theta_g + u_g^i \partial_i \delta_g = C(\vec{x}, \eta)$$

$$\boxed{{}' = \frac{d}{d\eta}}$$

$$u_g'^i + aH u_g^i + u_g^j \partial_j u_g^i + \frac{1}{a} \partial^i \Psi = F^i(\vec{x}, \eta)$$

$C(\vec{x}, \eta)$: 0'th moment of collision term:
effective galaxy formation rate per unit volume.

$F^i(\vec{x}, \eta)$: 1st moment of collision term:
effective (non.-grav.) force density.

Goal: do perturbation theory for galaxies, as for matter, being agnostic to galaxy formation physics encoded by
$C(\vec{x}, \eta)$; $\vec{F}(\vec{x}, \eta)$.

→ expand them in perturbations

Fields available for perturbative expansion:

$\boxed{\delta,\ u^i,\ \Psi,\ \text{and space- and time derivatives thereof.}}$

However, I can remove Ψ, u^i, $\partial^i\Psi$ at any given point with a coordinate transf.

$\Rightarrow$ not locally observable, and cannot appear in local galaxy formation rate & force term (see Sec. 2.10.2 of arXiv: 1611.09787)

$\Leftrightarrow$ equivalence principle: galaxies and matter fall at the same rate under large-scale gravitational acceleration $\partial^i\Psi$.

$\Rightarrow$ hence, we have $\boxed{\delta,\ \partial_i u^j,\ \partial_i\partial_j\Psi}$, and derivatives thereof.

* (higher) spatial derivatives : $\partial^i \delta$, $\nabla^2 \delta$, ...

Fourier space $\Longrightarrow$ $ik^i \delta$, $-k^2 \delta$, ... $\Rightarrow$ suppressed on large scales.

* time derivatives : <u>not</u> suppressed; no reason why they should be ... but we won't need to expand.

Begin w/ <u>force term $F^i(\vec{x}, \eta)$, i.e. galaxy Euler eqn.</u>:

$$F^i(\vec{x}, \eta)\Big|_{\text{linear order}} = \underbrace{f_{\nabla\delta}(\eta)\, \partial^i \delta^{(1)}(\vec{x}, \eta)}_{\substack{\text{eff. force} \\ \text{induced by large-} \\ \text{scale density pert.}}} + \underbrace{\varepsilon^i(\vec{x}, \eta)}_{\substack{\text{random force} \\ \text{component due} \\ \text{to small-scale} \\ \text{modes;} \\ \text{uncorrelated w/} \\ \delta,\ \partial_i u^i, \ldots}}$$

$\Rightarrow$ exactly equivalent to eff. pressure for matter, $f_{\nabla\delta} \Leftrightarrow c_s^2$, but not the same value.

$\Rightarrow$ $u_g^i - u^i$ sourced by $\left(f_{vs}(\eta) - c_s^2(\eta)\right) \partial^i \delta(\vec{x}, \eta)$

$\Rightarrow$ suppressed by k^2 $\sim \partial^i \theta \sim \nabla^2 u^i$

$$\boxed{\text{Galaxy \& matter velocities are the same on large scales (up to } k^2 \text{ corrections) } (\Leftrightarrow \text{"higher derivative"})}$$

In following, set $u_g^i = u^i$ (consider h. deriv. later)

Go back to galaxy & matter cont. eqn.s:

$$\frac{D}{D\eta} \delta_g + (1 + \delta_g)\, \theta = C(\vec{x}, \eta)$$

$$\frac{D}{D\eta} \delta + (1 + \delta)\, \theta = 0$$

Now expand source term:

$$C(\vec{x},\eta) = c_\delta(\eta)\,\delta^{(1)}(\vec{x},\eta) + \tilde{\varepsilon}(\vec{x},\eta)$$
$$+ \text{(second order)} + \text{(h. deriv.)}$$

Note: no need to include say $\Theta^{(1)}$ or $\nabla^2\Psi^{(1)}$: can be expressed in terms of $\delta^{(1)}$ (and $\delta^{(1)\prime}$; later).

linearize continuity equation:

$$\frac{\partial}{\partial\eta}\delta_g^{(1)} + \underbrace{\Theta^{(1)}}_{= -\frac{\partial}{\partial\eta}\delta^{(1)}} = c_\delta(\eta)\,\delta^{(1)}(\vec{x},\eta)$$

can verify that solution (w/ $\delta_g = 0$ at $\eta = 0$)

is $\delta_g(\vec{x},\eta) = b(\eta)\,\delta^{(1)}(\vec{x},\eta)$

where $b(\eta) = 1 + \frac{1}{D(\eta)}\int_0^\eta d\eta'\, D(\eta')\, c_\delta(\eta')$,

using that $\delta^{(1)}(\vec{x},\eta) = D(\eta)\,\delta_0(\vec{x})$

$\Rightarrow$ absorb <u>entire formation history</u> <u>in single free bias coefficient b.</u>

$\Rightarrow$ At fixed time η, $\delta_g^{(1)}(\vec{x},\eta) \propto \delta^{(1)}(\vec{x},\eta)$

Finally, include random, "stochastic" source. Leads to constant contribution to gal. power spectrum (locality).

$\rightsquigarrow$ $P_{gg}^{(1)}(k) = b^2 P_{\delta\delta}^{(1)}(k) + P_\varepsilon$

↳ matter power spectrum

$\rightsquigarrow$ Two free parameters b, P_ε to be determined from observed galaxies.

But can still use shape of P_{gg} to probe shape of $P_{\delta\delta}$.