The Cosmic Microwave Background

Lecture 1: Homogeneous Universe, Thermal History, and Anisotropy Basics

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CMB: Why?

- Key evidence of the hot Big Bang origin of our universe

- Snapshot of the "initial" conditions for the later formation of all cosmic structure

- The most powerful single probe of cosmology:
  - Extremely well-described by *linear* perturbation theory
  - Can determine all cosmological parameters (age, composition, evolution)
  - Clean, direct probe of the initial conditions set by inflation (or other early-universe theories)
  - Robust testing ground for new physics, e.g., at ultra-high energies during inflation or via feeble (possibly "dark") interactions later in the universe

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CMB: Outline

- Lecture 1: Homogeneous Universe, Thermal History, and Anisotropy Basics
- Lecture 2: Anisotropy Basics, Line-of-Sight Projection, and Acoustic Physics
- Lecture 3: Line-of-Sight Projection, Acoustic Physics, and CMB Power Spectrum
- Lecture 4: CMB Power Spectrum and Parameter Sensitivity + Polarization Intro
- Lecture 5: CMB Polarization, Spectral Distortions, and Secondary Anisotropies

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CMB: References

- S. Dodelson & F. Schmidt, "Modern Cosmology", 2nd Ed.
- D. Baumann, "Cosmology"
- D. Huterer, "A Course in Cosmology: From Theory to Practice"
- S. Weinberg, "Cosmology"
- Online notes:
  - W. Hu & M. White: https://arxiv.org/abs/astro-ph/9706147
  - A. Challinor & H. Peiris: https://arxiv.org/abs/0903.5158
  - Many others!

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Basic Facts

- We look back in time (speed of light is finite ~ 0.3 pc/yr)
- The universe is expanding — the distances between galaxies are growing with time
- The universe is cooling — recall $E = hc/\lambda$

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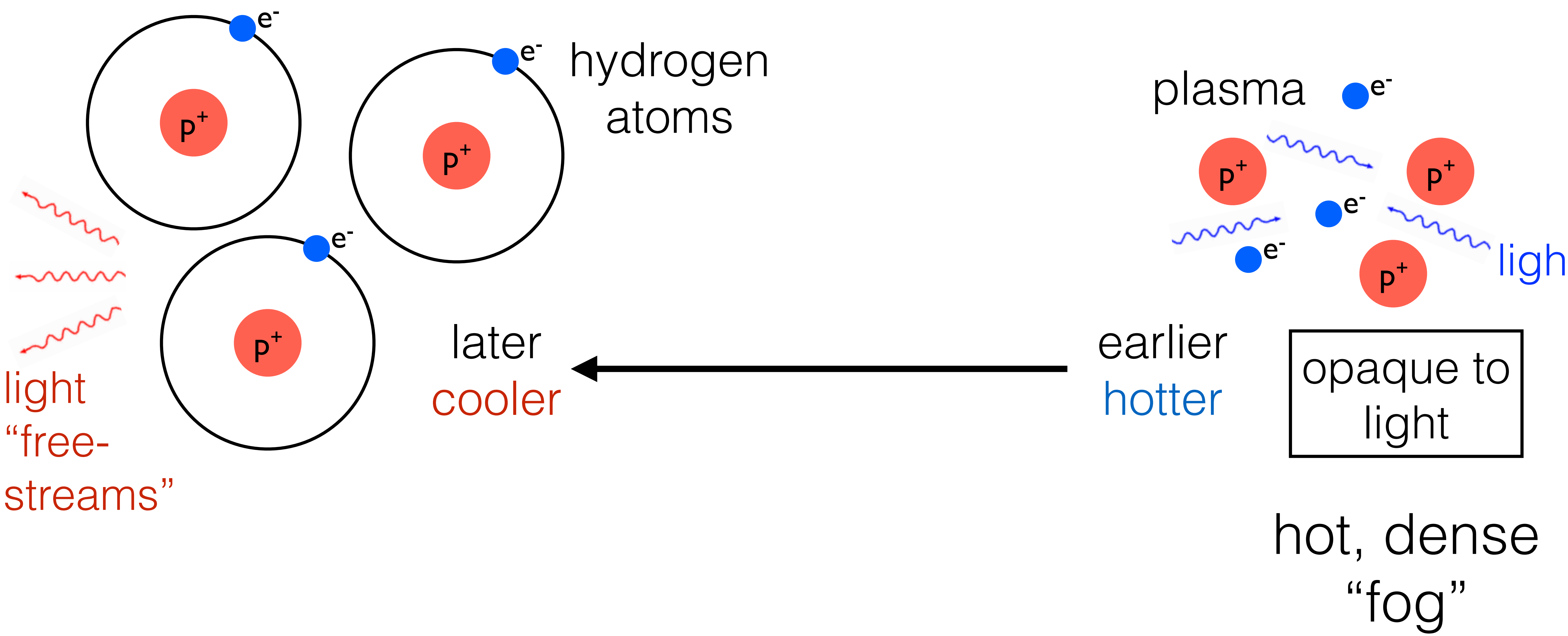
The Cosmic Microwave Background

- The universe is expanding — thus the average density is decreasing and its contents are cooling

What does this tell us about conditions at earlier times?



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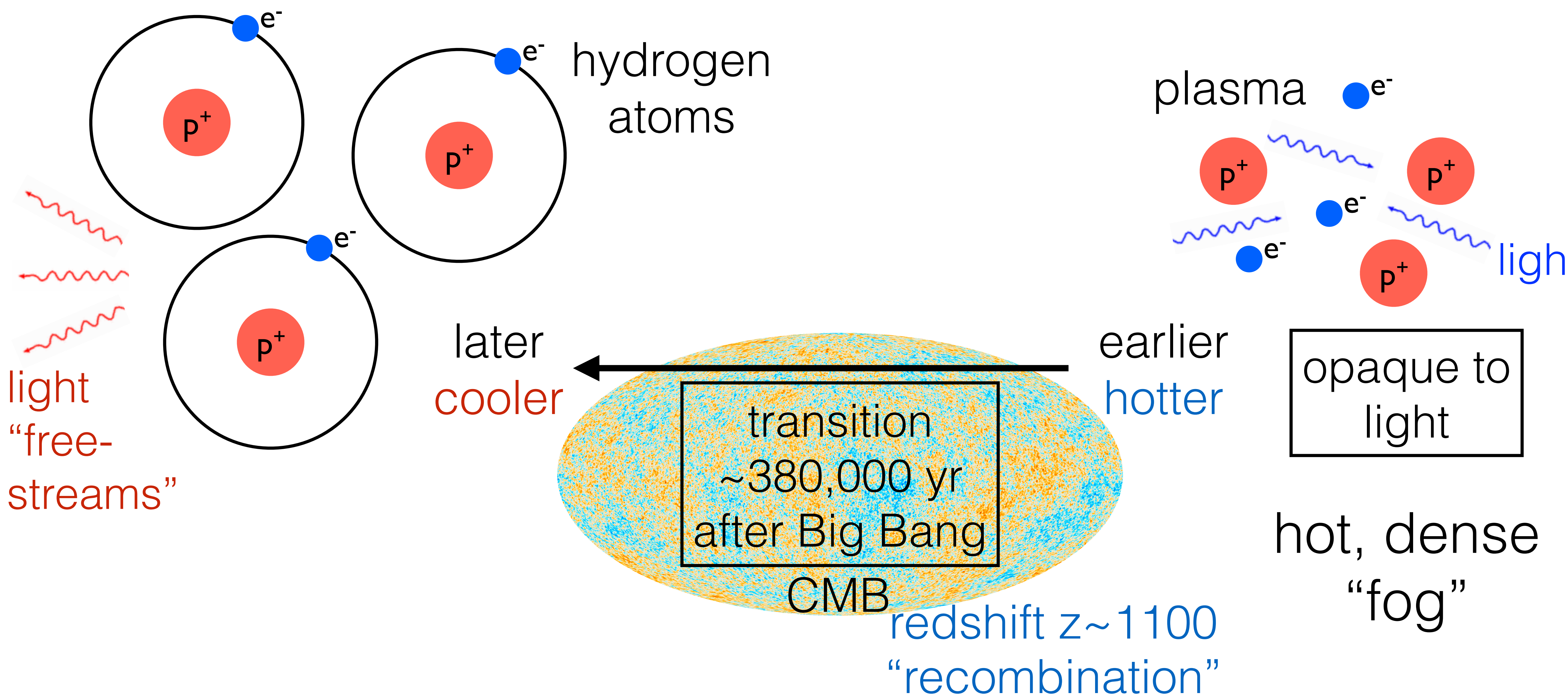

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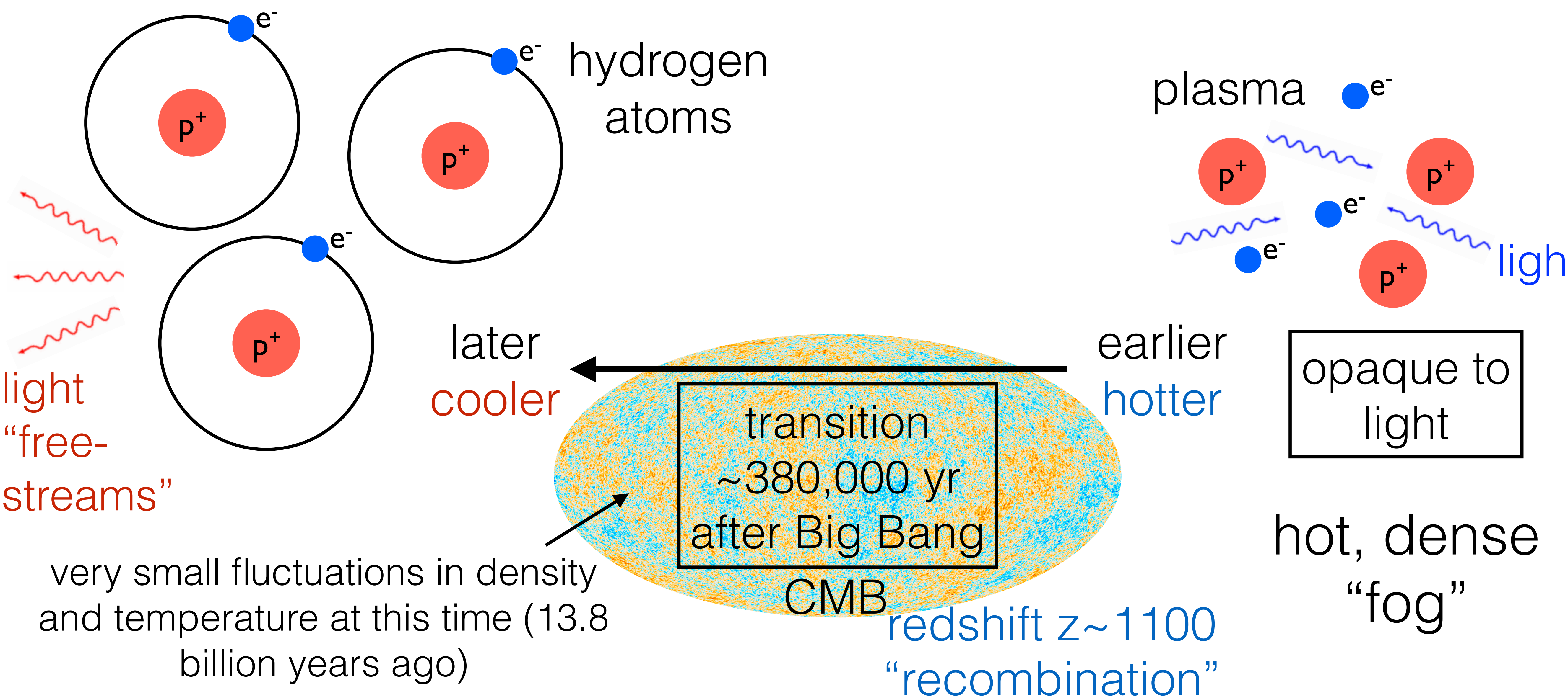

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"actual" CMB
T = 2.7255 +/- 0.0006 K

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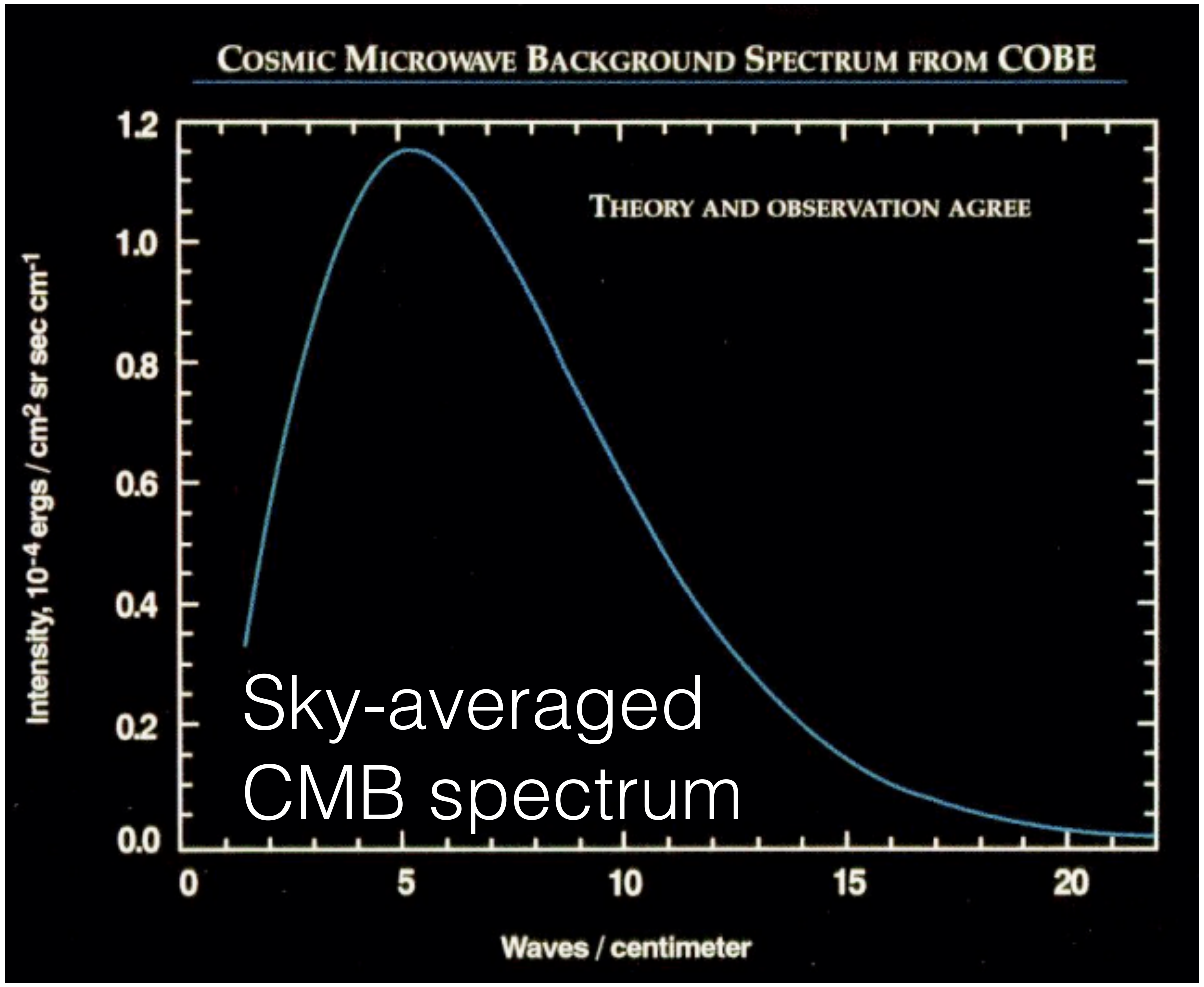

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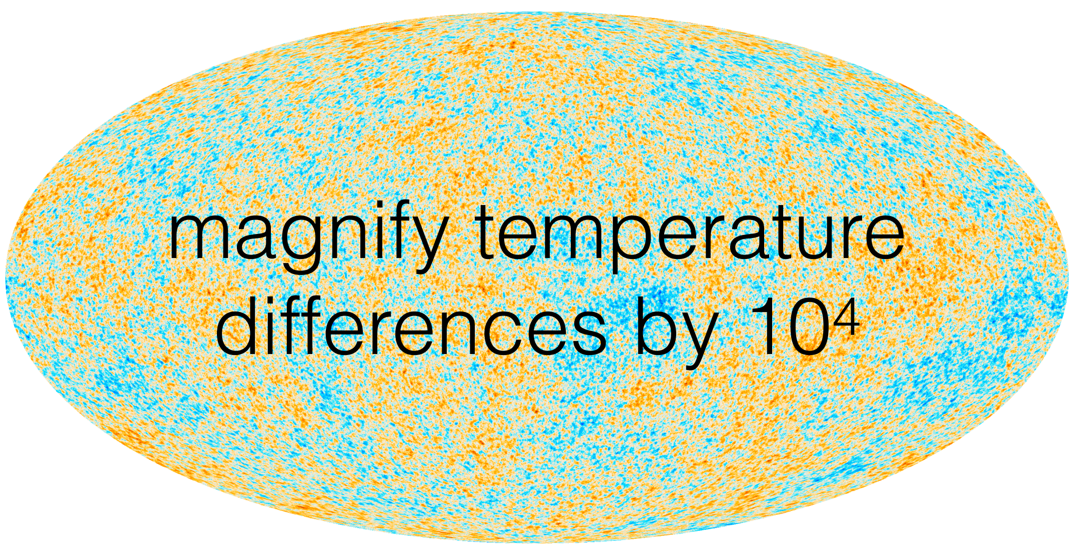

very small fluctuations in density and temperature

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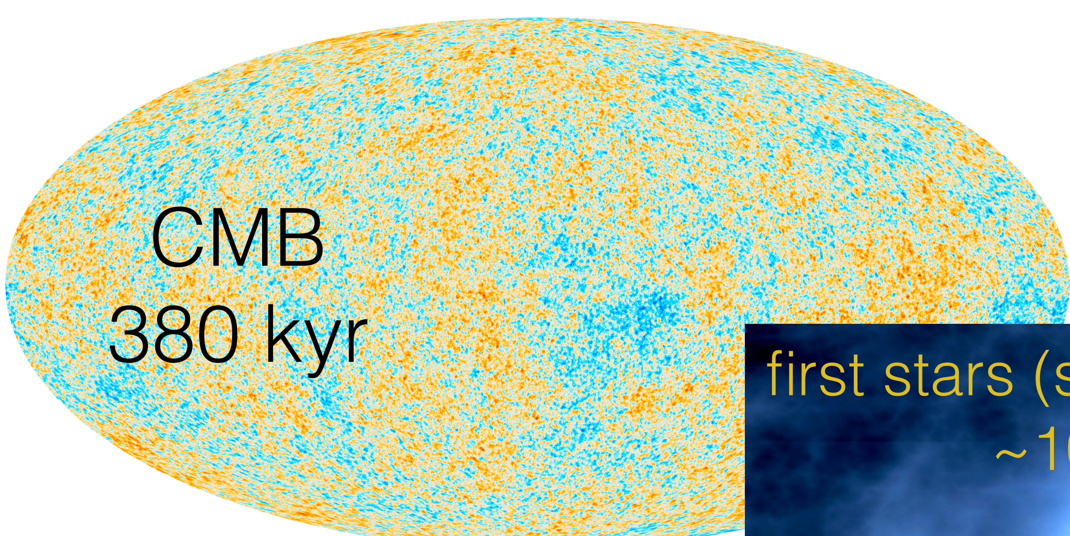

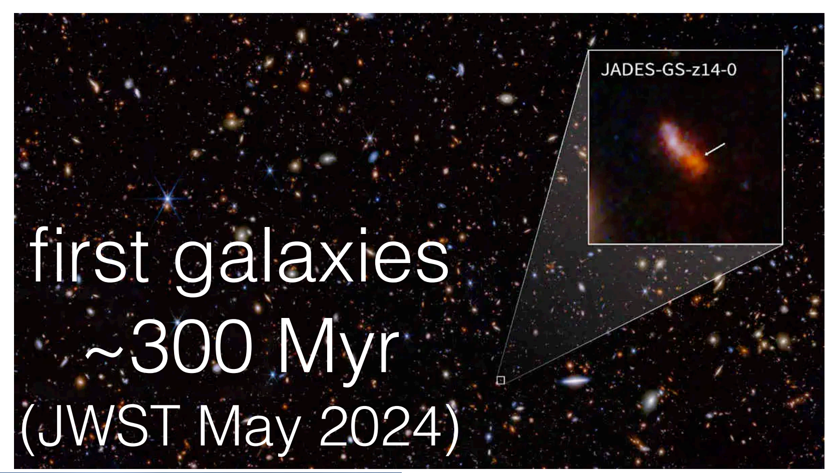

earlier
smoother

later
clumpier

Cosmic Timeline

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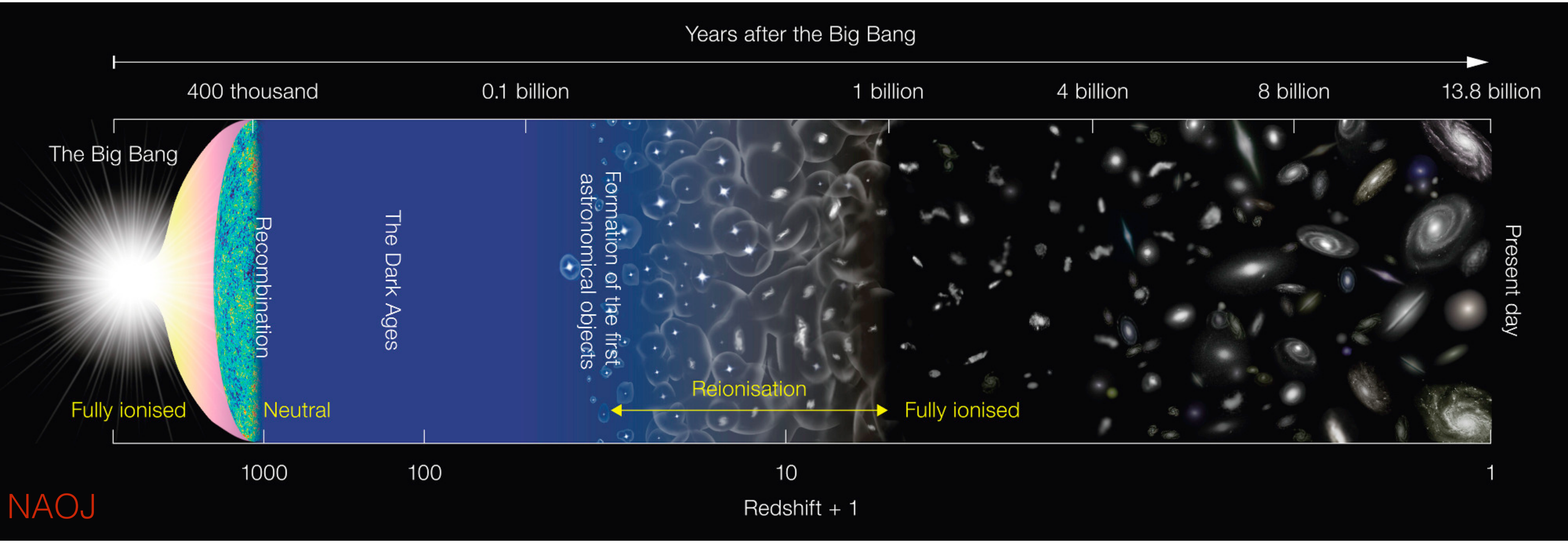
- Galaxies and other structures formed much later out of the initial, small fluctuations in density that we see in the CMB



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The Homogeneous Universe

- Convention: $c=1$
- For simplicity, we will consider only a spatially flat universe (unless stated otherwise)
- (Flat) Robertson-Walker metric: $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$
- Conformal time: $d\eta \equiv \frac{dt}{a} \longrightarrow ds^2 = a^2(t)(-d\eta^2 + d\vec{x}^2)$

  scale factor $a = 1/(1+z)$
- Friedmann equations:

$$H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda\right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

The Homogeneous Universe

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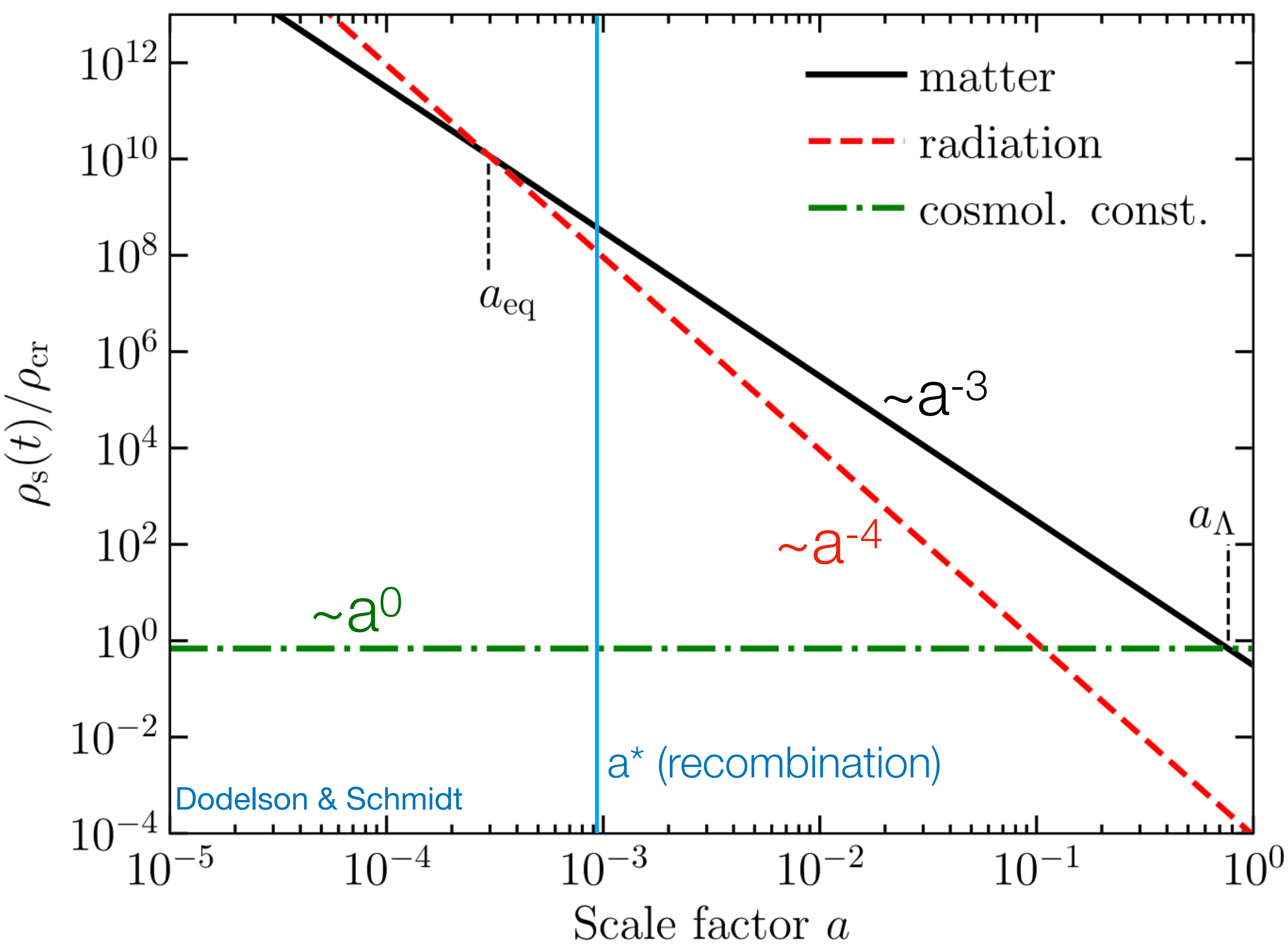
- Continuity equation: $$\frac{d\rho}{dt} + 3\left(\frac{\dot{a}}{a}\right)(\rho + P) = 0$$

$$\rho_{\rm cr} \equiv \frac{3H_0^2}{8\pi G}$$

critical density



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The Homogeneous Universe

$$H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda\right)$$

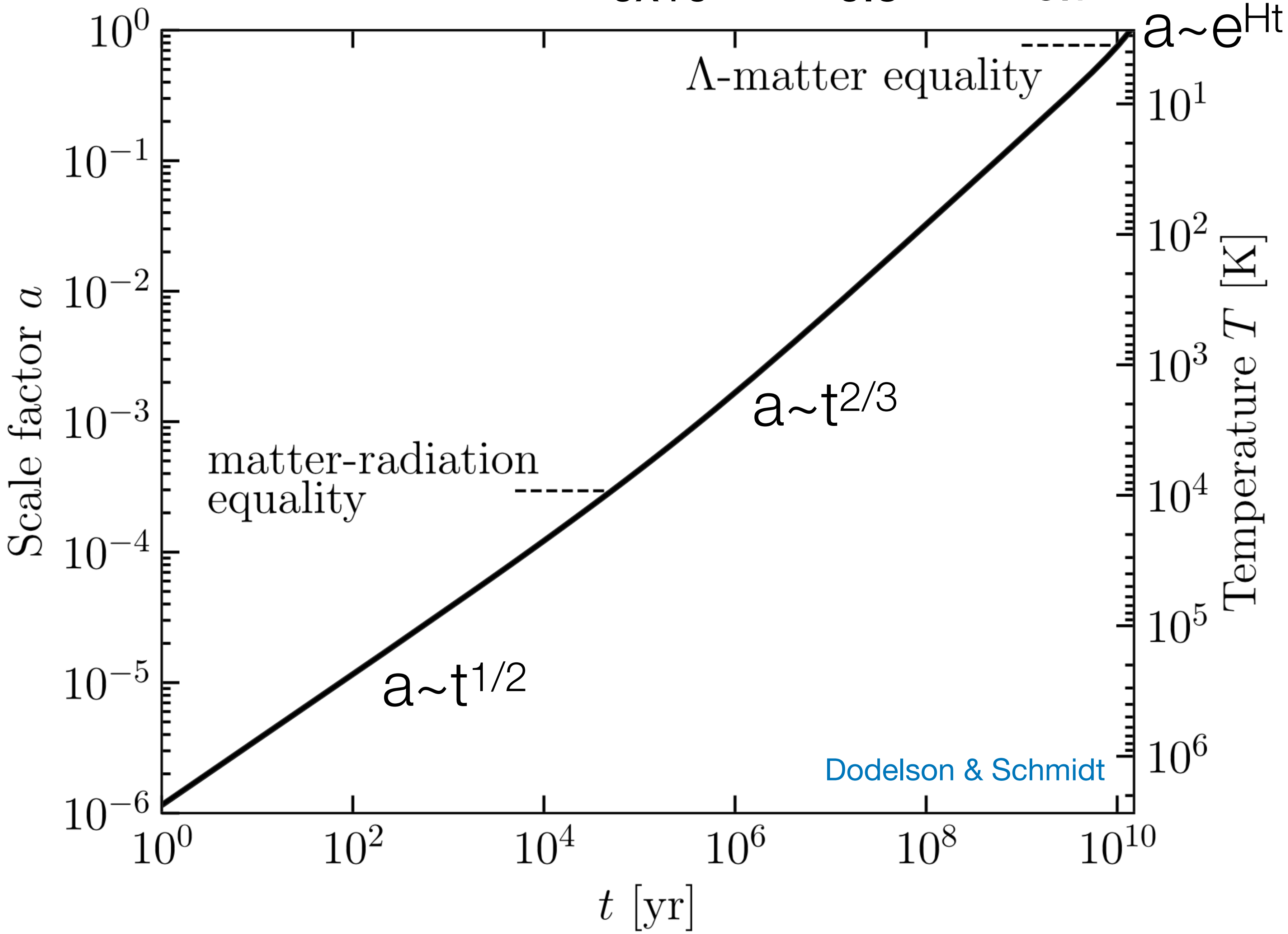
$$\Omega_i \equiv \frac{\rho_{i,0}}{\rho_{\mathrm{cr},0}}$$

Hubble rate

radiation density today ~6x10^{-5}

matter density today ~0.3

c.c. density today ~0.7



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Thermal History

- The 0th component of the unperturbed geodesic equation determines scaling of the average temperature of the thermal bath with scale factor a: $T \sim 1/a \sim (1+z)$

- Since the energy of every photon scales in the same exact way with a, the form of the photon distribution function is preserved, and we maintain a Planck blackbody spectrum with temperature $T \sim 1/a$

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Thermal History

Event	Time	Redshift	Temperature
Electroweak phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Neutrino decoupling	1 s	6×10^{9}	1 MeV
e^+e^- annihilation	6 s	2×10^{9}	500 keV
Big Bang nucleosynthesis	3 min	4×10^{8}	100 keV
Epoch of matter-rad. equality	50 kyr	3400	0.8 eV
Recombination	260–380 kyr	1300–1100	0.30–0.25 eV
Photon (CMB) decoupling	380 kyr	1090	0.25 eV
Reionization	~200 Myr – 1 Gyr	~20–6	4.9–1.6 meV
Λ-matter equality epoch	8–9 Gyr	0.3–0.4	0.33 meV
Today	13.8 Gyr	0	0.235 meV

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Recombination Era

At 10 keV $\gtrsim T \gtrsim 1$ eV: plasma consisting of γ, e^-, H nuclei, ^{4}He nuclei, and the decoupled ν and DM.

γ and e^- tightly coupled by Compton scattering.

e^- and H (i.e., p^+) tightly coupled by Coulomb scattering.

Very little neutral H around — plasma $T \gg 13.6$ eV.

Recombination Era

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As T decreased, eventually $e^- + p^+ \rightarrow H + \gamma$, i.e., e^- and p^+ combine to form neutral H ("recombination").

$\Rightarrow n_e$ decreased sharply

$\Rightarrow \gamma$ decoupled from the baryonic matter (photon decoupling) ← no longer in thermal equil.

$\Rightarrow$ mean free path of photons became larger than the horizon

$\Rightarrow$ photons "free-stream": universe became transparent

$\Rightarrow$ these photons comprise the cosmic microwave background today.

Three-stage process:

1) Recombination (sharp decrease in n_e)
2) Photon-matter decoupling
3) Freeze-out of residual free electron fraction

Thermal History

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- Evolution of the free electron fraction: Saha's equation

$$\frac{1 - X_e}{X_e^2} = \frac{2\zeta(3)}{\pi^2}\eta\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{I_H/T}$$

- Here:

the equilibrium ionization $X_e = n_e/n_b$

the baryon-to-photon ratio $\eta = n_b/n_\gamma$ ~ 6×10^{-10}

$n_b \approx n_H + n_p$ the baryon number density (ignoring helium)

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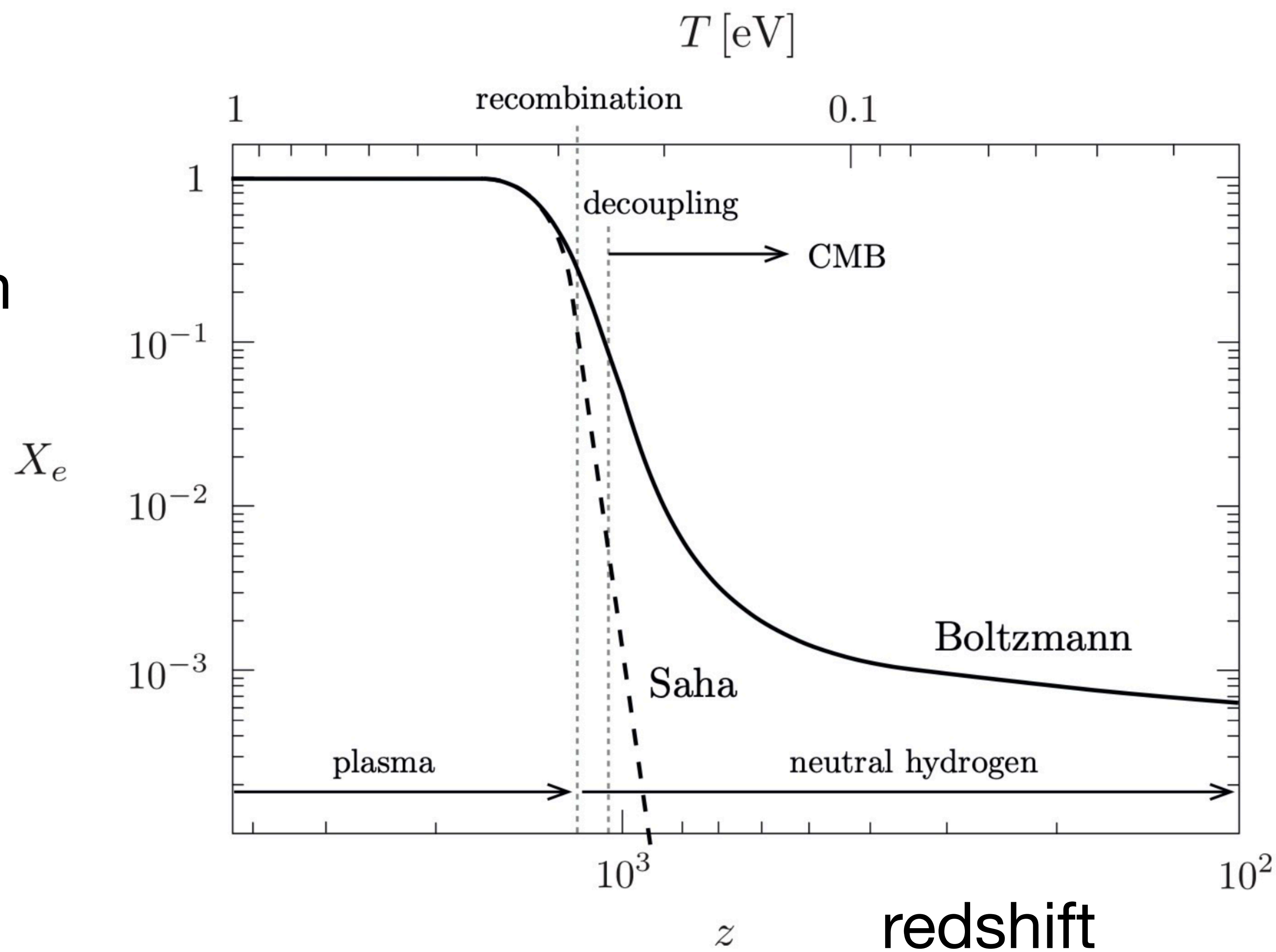
Thermal History

- Evolution of the free electron fraction

free electron fraction



redshift

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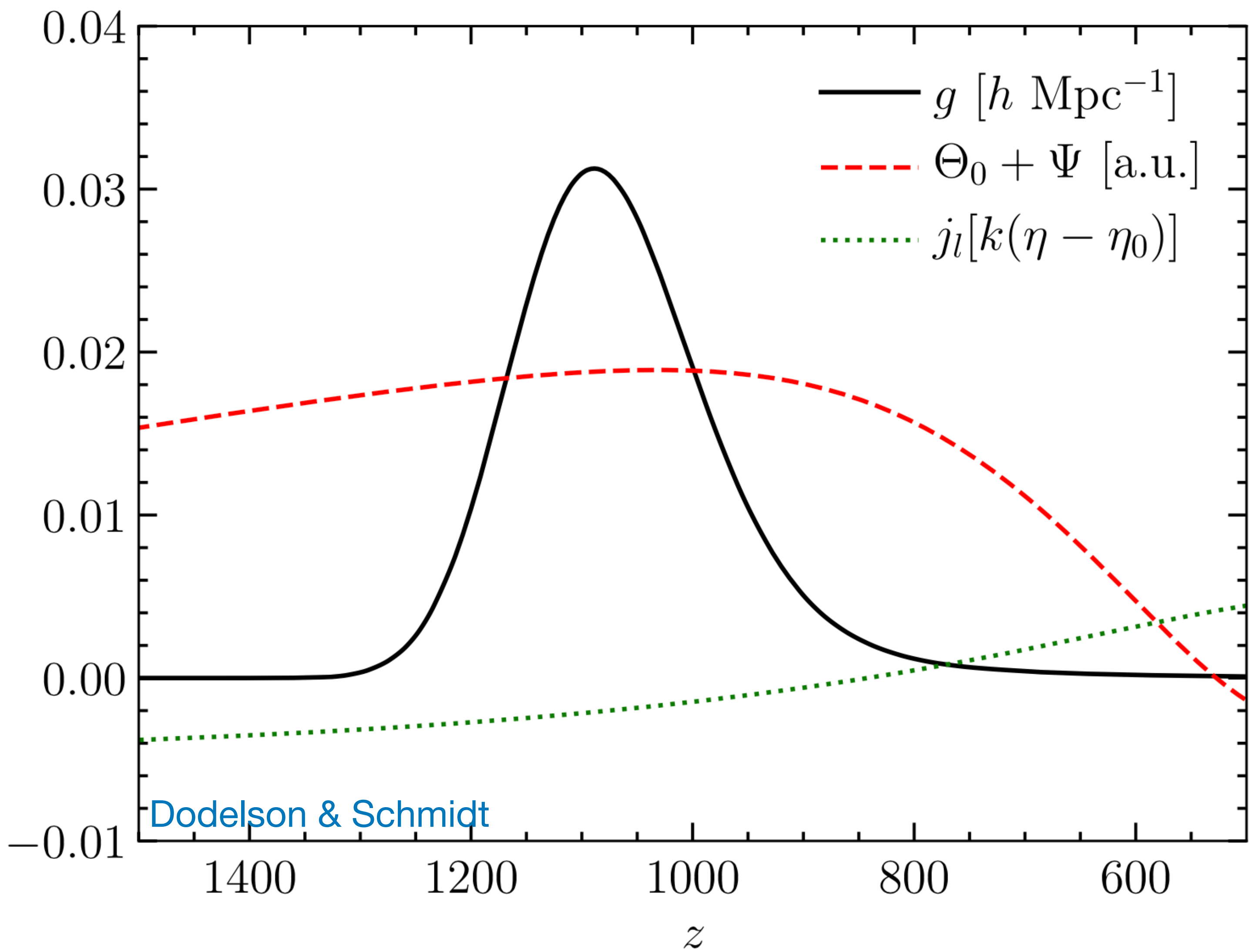
Thermal History

- As recombination nears completion, the photons decouple from the electrons, when the Thomson scattering rate $\Gamma \sim n_e \sigma_T$ drops below Hubble rate H

visibility function (black)



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Thermal History

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- Saha results: $z_{\mathrm{rec}} \sim 1380$, $z_{\mathrm{dec}} \sim 1180$

- Precise results: $z_{\mathrm{rec}} \sim 1270$, $z_{\mathrm{dec}} \sim 1090$

- We infer these redshifts (rather than only temperatures/energies) via the known CMB temperature at z=0: T_{CMB} = 2.726 +/- 0.001 K

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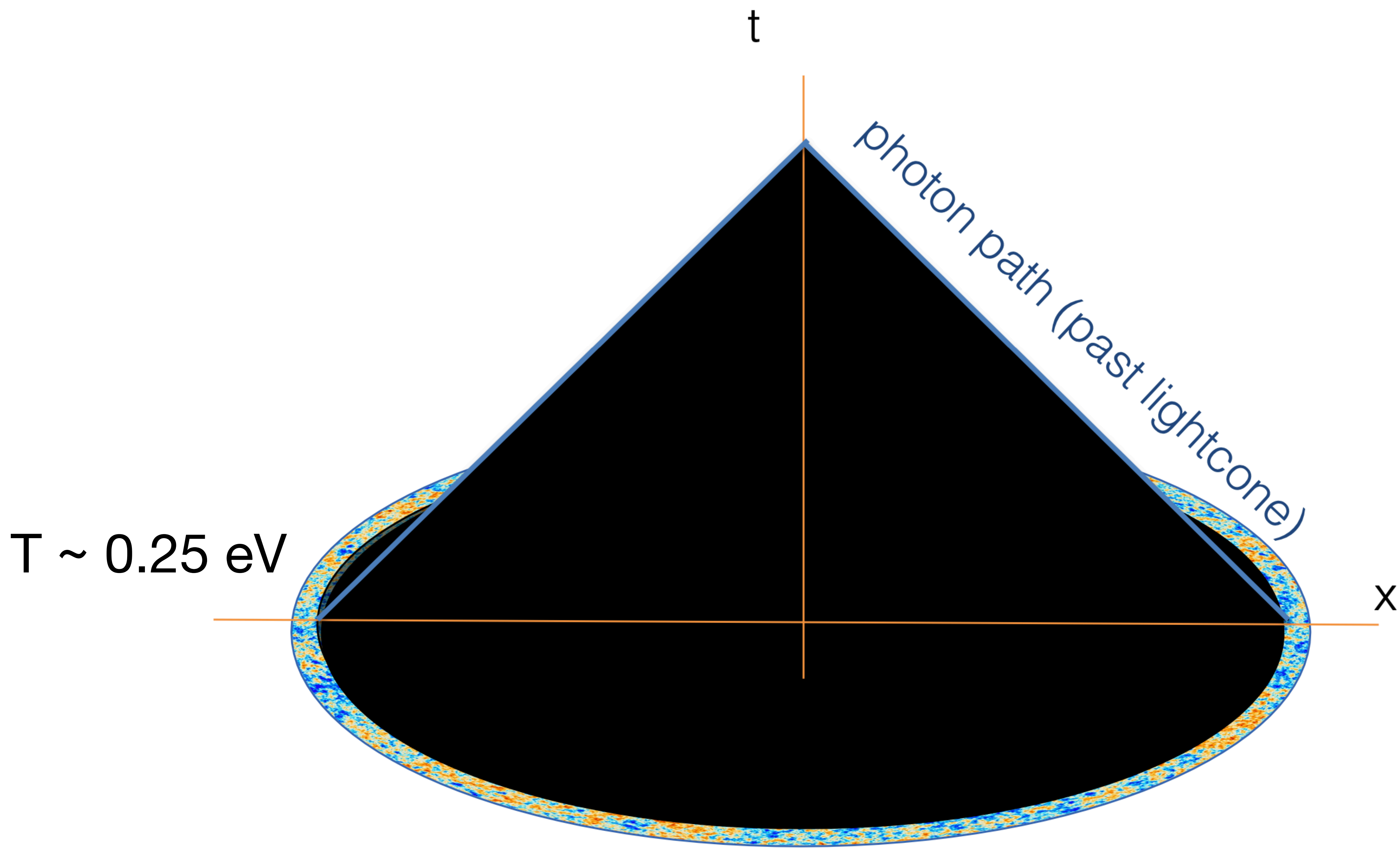
Surface of Last Scattering



Last-scattering is a 2D spherical surface in both time and space