Structure Formation II

- Part 1: Halo and galaxy formation
- Part 2: Weak lensing
- Part 3: From pixels to parameters

Powerful surveys → excellent statistical power → systematic effects (astrophysics, observational) more significant. Correct cosmological interpretation of observations will require more and more astrophysics.

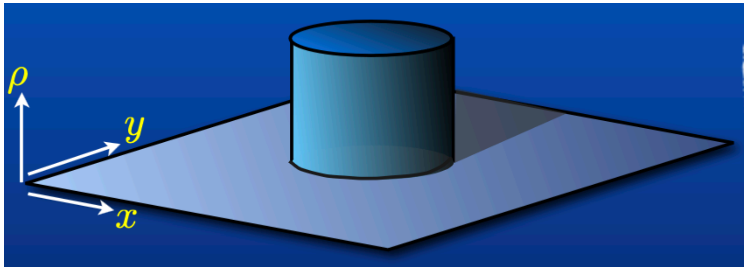
(Highly) Non-Linear Evolution

Last week, you learned to calculate the evolution of small density perturbations with (linear + higher-order) perturbation theory.

Today: oversimplified, but insightful, analytic models for the highly non-linear regime

- Halo formation through **gravitational collapse**.
- Assuming that all matter is distributed in halos, approximate the non-linear matter power spectrum with the **halo model**.
- Include "gastrophysics" to describe **galaxy formation**.

Spherical Collapse



- Start with small spherical overdensity.
- Overdensity grows through gravity.
  - Initially, linear growth: $\delta(t) = D(a)\delta_0$
  - When the linear overdensity passes some critical threshold δ_c, it undergoes gravitational collapse.
- Overdensity virializes, and forms a stable dark matter halo.

Spherical Collapse

To gain insight into the non-linear evolution of density perturbations, we now consider the highly idealized case of Top-Hat Spherical Collapse.

- Assume EdS cosmology.
- Assume homogeneous universe, except for a single, top-hat, spherical perturbation.

$$
\begin{aligned}
M(<r) &= \frac{4}{3}\pi r_{\rm i}^3 \bar{\rho}_{\rm i}\left[1+\delta_{\rm i}\right] \\
&= \frac{4}{3}\pi r^3(t)\bar{\rho}(t)\left[1+\delta(t)\right]
\end{aligned}
$$

Mass inside a shell r(t) is conserved until shell crossing.

$$
\boxed{\ddot{r} = -\frac{GM(r)}{r^2}}
$$

Equation of motion for shell radius(EdS)

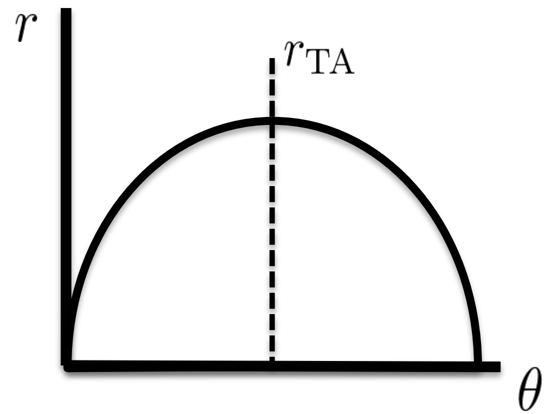
Λ would add to rhs, requiring numerical solution

Spherical Collapse

In EdS, equation of motion has parametric solution:

$$t(\theta) = B(\theta - \sin\theta) \quad ; \quad r(\theta) = A(1 - \cos\theta)$$

$$\frac{A^3}{B^2} = GM$$ (easily verified by evaluating $\ddot{r}(\theta)$)



This solution implies the following evolution:

- shell expands from $r=0$ at $\theta=0$ $(t=0)$,
- shell reaches a maximum radius r_{max} at at $\theta=\pi$ $(t=t_{\mathrm{max}})$,
- shell collapses back to $r=0$ at $\theta=2\pi$ $(t=t_{\mathrm{coll}}= 2t_{\mathrm{max}})$.

$$A = r_{\mathrm{max}}/2$$

$$B = t_{\mathrm{max}}/\pi$$

Spherical Collapse

Limiting behavior of the parametric solution:

Taylor expand $t(\theta)$, $r(\theta)$:

$$r(t) \approx \frac{A}{2}\left(\frac{6t}{B}\right)^{2/3}\left[1 - \frac{1}{20}\left(\frac{6t}{B}\right)^{2/3}\right]$$

Recover EdS background solution ($A, B \to \infty, \quad A^3/B^2 = GM$)

$$r_{\mathrm{EdS}}(t) = \frac{1}{2}\left(GM\right)^{2/3}\left(6t\right)^{2/3}$$

and linear evolution

$$r_{\mathrm{lin}}(t) = r_{\mathrm{EdS}}(t)\left[1 - \frac{1}{20}\left(\frac{6t}{B}\right)^{2/3}\right] \qquad \delta_{\mathrm{lin}}(t) = \frac{\rho_{\mathrm{lin}}(t)}{\rho_{\mathrm{EdS}}(t)} - 1 \approx \frac{3}{20}\left(\frac{6t}{B}\right)^{2/3}$$

recover linear growth $\propto$ D(t)

Spherical Collapse

Linear solution

$$\delta_{\rm lin}(t) = \frac{3}{20}\left(\frac{6\pi t}{t_{\rm max}}\right)^{2/3}$$

turn around

$t = t_{\rm max}$, $\theta = \pi$

$$\delta_{\rm lin}(t_{\rm max}) = \frac{3}{20}(6\pi)^{2/3} \approx 1.062$$

collapse

$t = 2t_{\rm max}$, $\theta = 2\pi$

$$\delta_{\rm lin}(t_{\rm coll}) = \frac{3}{20}(12\pi)^{2/3} \approx \boxed{1.686}$$

If initial density contrast evolved forward to time t in linear theory δ_c> **1.686**, then a region has collapsed by time t.
Only weak cosmology dependence.

Spherical Collapse

Linear solution

$$\delta_{\rm lin}(t) = \frac{3}{20}\left(\frac{6\pi t}{t_{\rm max}}\right)^{2/3}$$

Spherical collapse

$$\delta(t) = \frac{\rho(t)}{\rho_{\rm EdS}(t)} - 1 = \frac{9}{2}\frac{(\theta - \sin\theta)^2}{(1-\cos\theta)^3} - 1$$

turn around

t=$t_{\rm max}$, θ=π

$$\delta_{\rm lin}(t_{\rm max}) = \frac{3}{20}(6\pi)^{2/3} \approx 1.062$$

$$\delta(t_{\rm max}) = \frac{9\pi^2}{16} - 1 = 4.55$$

collapse

t=$2t_{\rm max}$, θ=2π

$$\delta_{\rm lin}(t_{\rm coll}) = \frac{3}{20}(12\pi)^{2/3} \approx \boxed{1.686}$$

$$\delta(t_{\rm coll}) = \infty$$

Spherical collapse model singular at collapse; proceed by considering *virialization* after turn around.

Virialization

What does it mean when I say that the collapsed region will be ***virialized***?

Virialization

What does it mean when I say that the collapsed region will be ***virialized***?

Kinetic and potential energy satisfy the virial theorem:

$$2T = -U$$

Assuming virialization, we can use conservation of energy to figure out the final radius of the overdensity.

$$E_{\rm vir} = U_{\rm vir} + T_{\rm vir} = \frac{1}{2}U_{\rm vir} = U_{\rm TA}$$

$$\frac{1}{2R_{\rm vir}} = \frac{1}{R_{\rm TA}} \longrightarrow R_{\rm vir} = \frac{1}{2}R_{\rm TA}$$

Assumes virialized region is uniform!

Virialization

Final virial radius: $R_{\rm vir} = \frac{1}{2} R_{\rm TA}$

$\rho_{\rm EdS}$ decreases by factor 4 during virialization ($t_{\rm max} \rightarrow 2t_{\rm max}$)

$$1 + \Delta_{\rm vir} = \frac{8\rho(t_{\rm max})}{\rho_{\rm EdS}(t_{\rm max})/4} = 18\pi^2 \approx 178$$

This is why halos are often defined using a ~200 overdensity criterion.

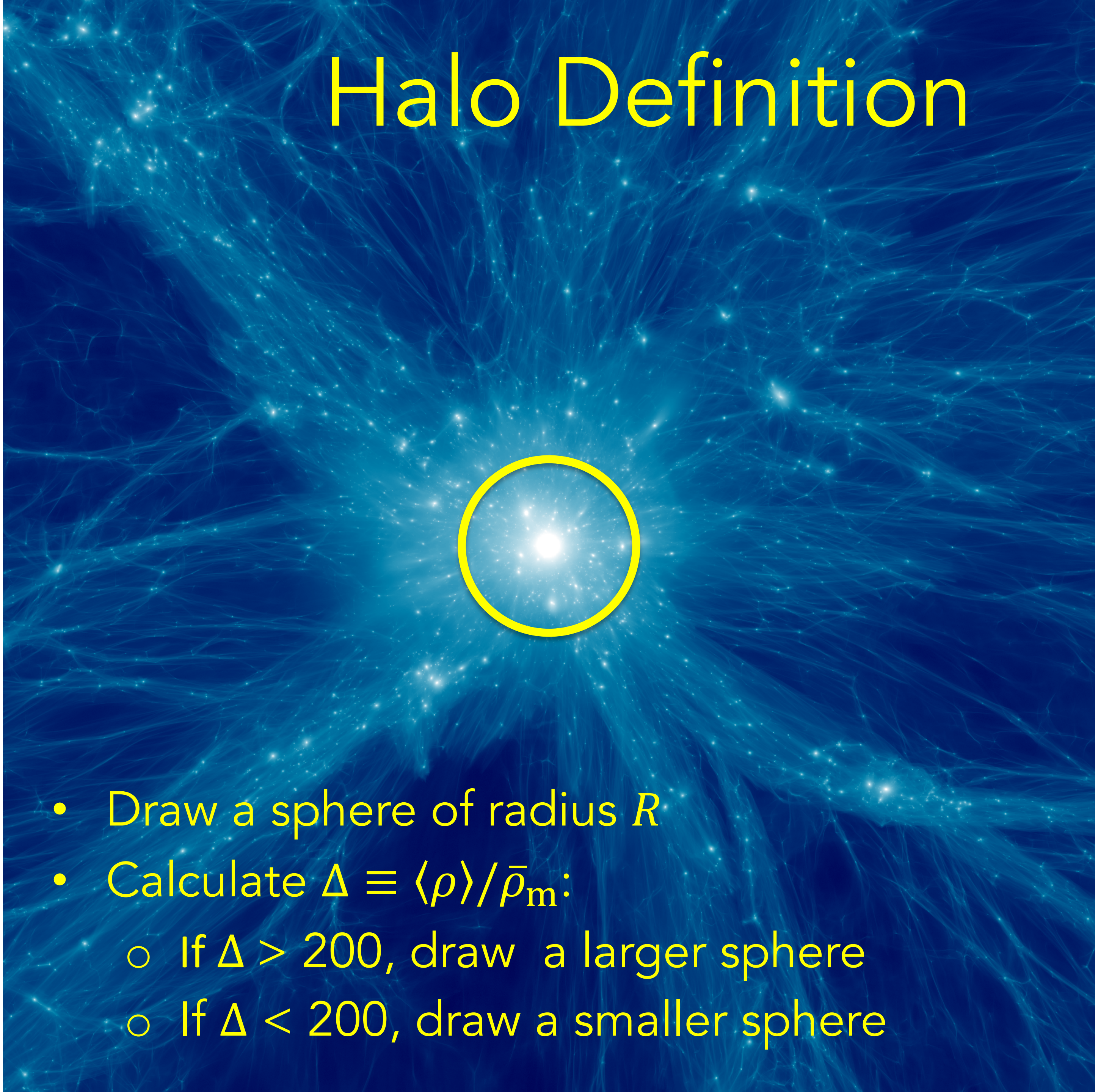
- Unlike the linear density contrast for collapse, δ_c=1.686, $\Delta_{\rm vir}$ depends notably on cosmology!

Spherical Collapse Model for Halo Formation

Spherical collapse predicts the end state as virialized halos given an initial density perturbation:

- At early times, overdensities grow as per linear theory.
- When the overdensity δ ~4.55, the overdensity undergoes gravitational collapse, resulting in a dark matter halo.
- Perturbations collapse when $\delta_{\text{lin}} > \delta_c = 1.686$, resulting in a virialized halo with a characteristic overdensity $\Delta \approx 200$.

Halo Definition



- Draw a sphere of radius R
- Calculate $\Delta \equiv \langle \rho \rangle / \bar{\rho}_{\mathrm{m}}$:
  - If $\Delta > 200$, draw a larger sphere
  - If $\Delta < 200$, draw a smaller sphere

Words of Warning

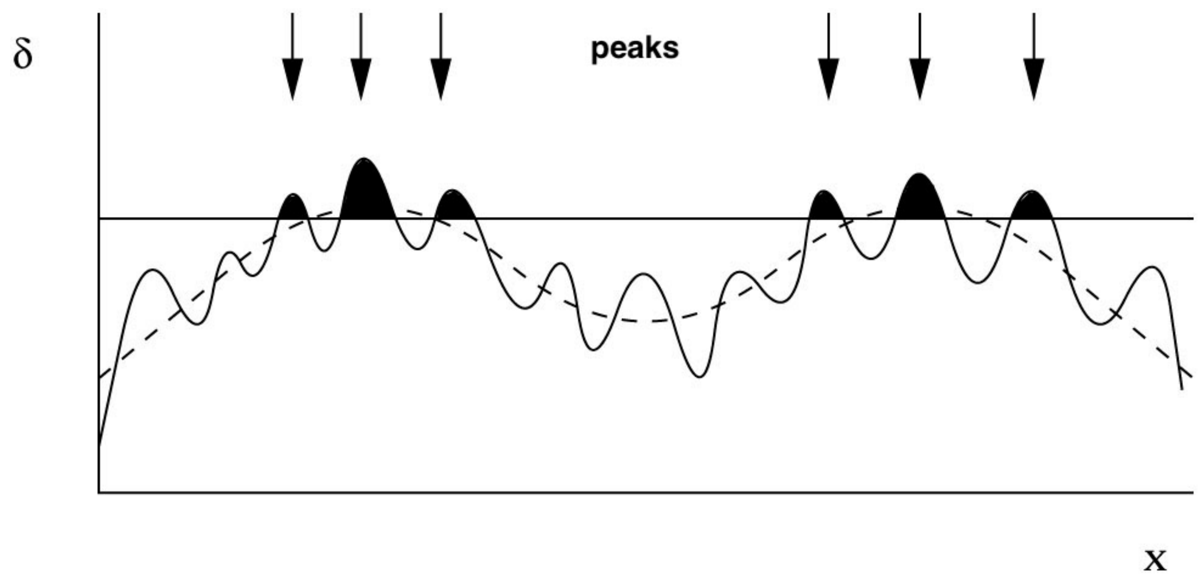
Mass definition is non-unique:

- Other overdensity criteria and/or reference densities can be used, e.g. M_{500c} vs M_{200m}.
- Spherical collapse calculation can be repeated for an LCDM cosmology:
    - results in a cosmology dependent overdensity.

The Halo Mass Function

Key statistic/(theorists' observable): density of massive halos as a function of halo mass.

Press & Schechter (1974): Compute volume in regions with smoothed linear density contrast $\delta_{\rm lin} > \delta_c$



Credit: John Peacock

The Halo Mass Function

Key statistic/(theorists' observable): density of massive halos as a function of halo mass.

Press & Schechter (1974) calculation of halo mass function:

- Lin. density contrast smoothed on scale $R(M) = (3M/4\pi\bar{\rho})^{1/3}$ is a Gaussian random field with mean zero, variance

$$\sigma(M,z) \equiv \mathrm{Var}(\delta, z) = D^2(z) \int \frac{\mathrm{d}^3 k}{(2\pi)^3} P(k, z=0) |W(kR(M))|^2$$

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- Fraction of space collapsed into halos above mass M at redshift z can be calculated as

$$F_{\mathrm{coll,PS}}(M,z) = 2 \frac{1}{\sqrt{2\pi}\sigma(M,z)} \int_{\delta_c}^{\infty} \mathrm{d}\delta \exp(-\delta^2/2\sigma^2(\mathrm{M},\mathrm{z}))$$

Fudge factor so that all matter is in collapsed halos as $M \rightarrow 0$
Rigorous derivation in Extended Press Schechter (Bond 1991)

The Halo Mass Function

- Fraction of space collapsed into halos above mass M at redshift z can be calculated as

$$F_{\rm coll,PS}(M,z) = 2\frac{1}{\sqrt{2\pi}\sigma(M,z)}\int_{\delta_{\rm c}}^{\infty} {\rm d}\delta\,\exp(-\delta^2/2\sigma^2({\rm M},{\rm z}))$$

$$= 2\frac{1}{2}{\rm erfc}\left(\nu/\sqrt{2}\right),\ \ \nu = \delta_{\rm c}/\sigma(M,z)$$

- Differentiate in M to find fraction in range dM and multiply by $\bar{\rho}/M$ the number density of halos if all of the mass were composed of such halos → differential number density of halos

$$\frac{dn(M,z)}{d\ln M} = \frac{\bar{\rho}}{M}\left|\frac{dF_{\rm coll,PS}}{d\ln M}\right|$$

$$= \frac{\bar{\rho}}{M}f_{\rm PS}(\nu)\left|\frac{d\ln\sigma(M,z)}{d\ln M}\right|,\quad f_{\rm PS}(\nu) = \sqrt{\frac{2}{\pi}}\nu e^{-\nu^2/2}$$

The Halo Mass Function

The Press-Schechter model builds intuition for the halo mass function and its cosmology + redshift dependence:

$$\frac{dn(M,z)}{d\ln M} = \frac{\bar{\rho}}{M}\sqrt{\frac{2}{\pi}}\nu e^{-\nu^2/2}\left|\frac{d\ln\sigma(M,z)}{d\ln M}\right|$$

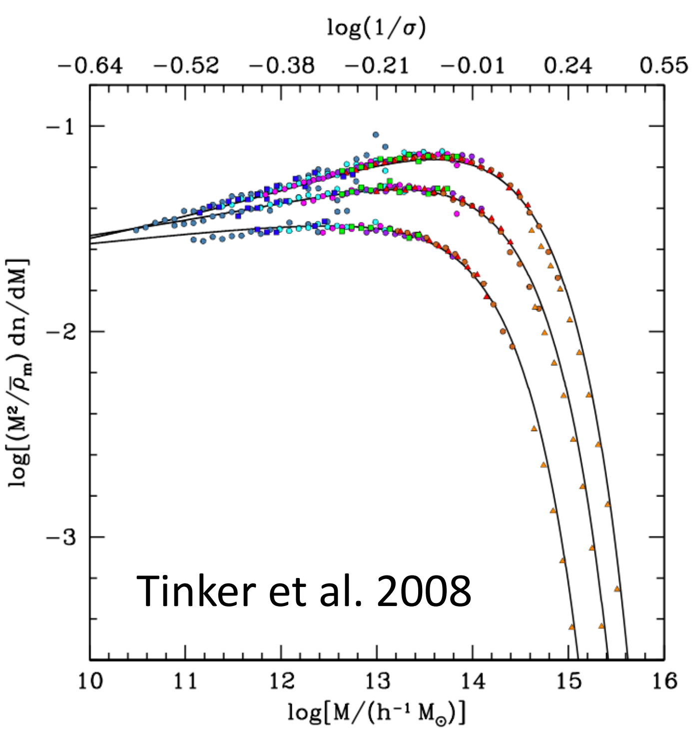
- **High mass**: exponential cut off above M^* where $\sigma(M^*) = \delta_c$

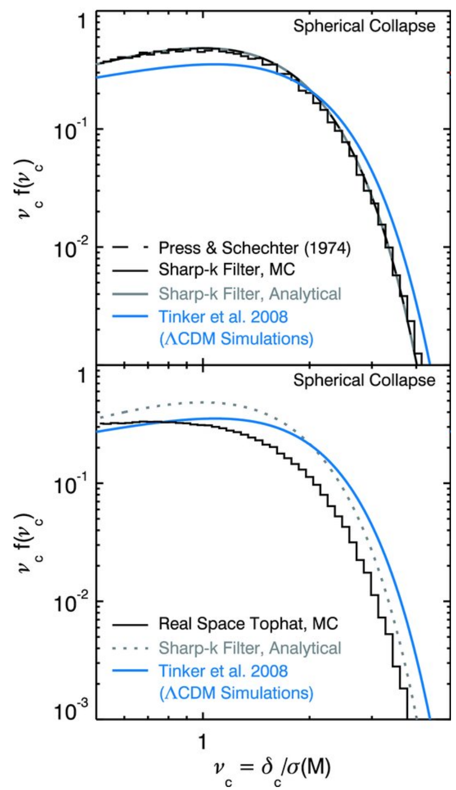
  $M^*(z=0) \sim 10^{13}h^{-1}M_\odot$

The Halo Mass Function

The Press-Schechter model builds intuition for the halo mass function and its cosmology dependence.

For higher precision, measure f(v) in simulations:





Robertson et al. 2009

Hierarchical Merging

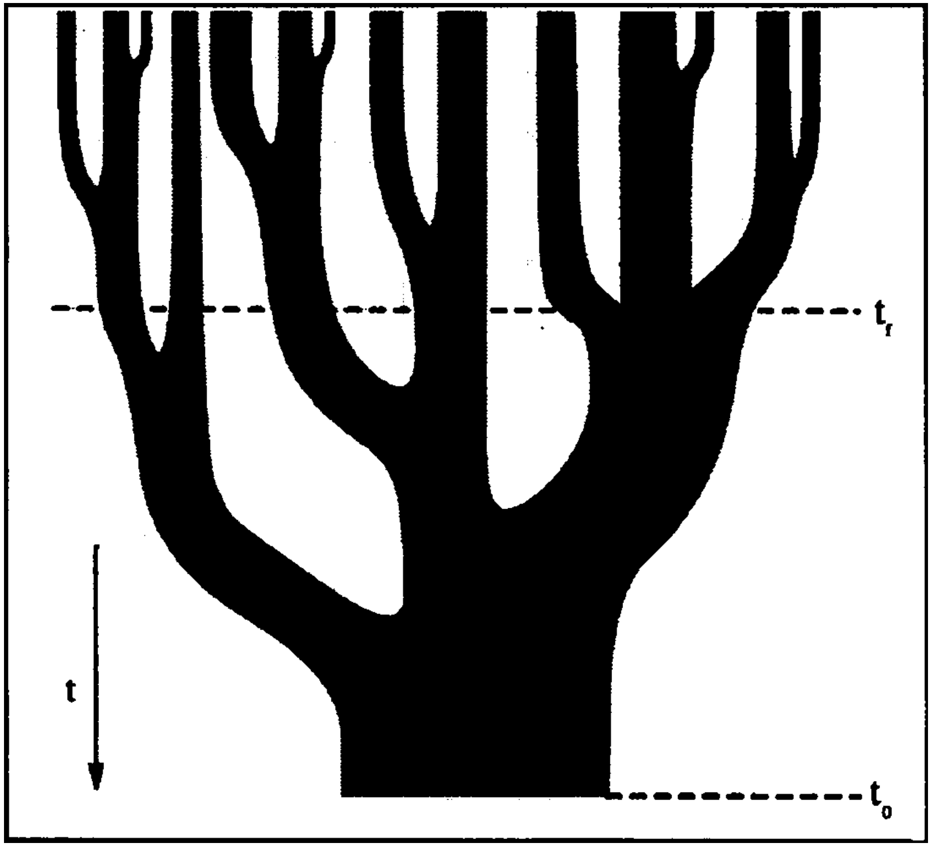


Illustration of a merger tree, depicting the growth of dark matter halos as the results of a series of mergers. A horizontal slice through the tree gives the mass distribution of progenitor halos at a given time.
(Lacey & Cole 1993)

Cluster Cosmology

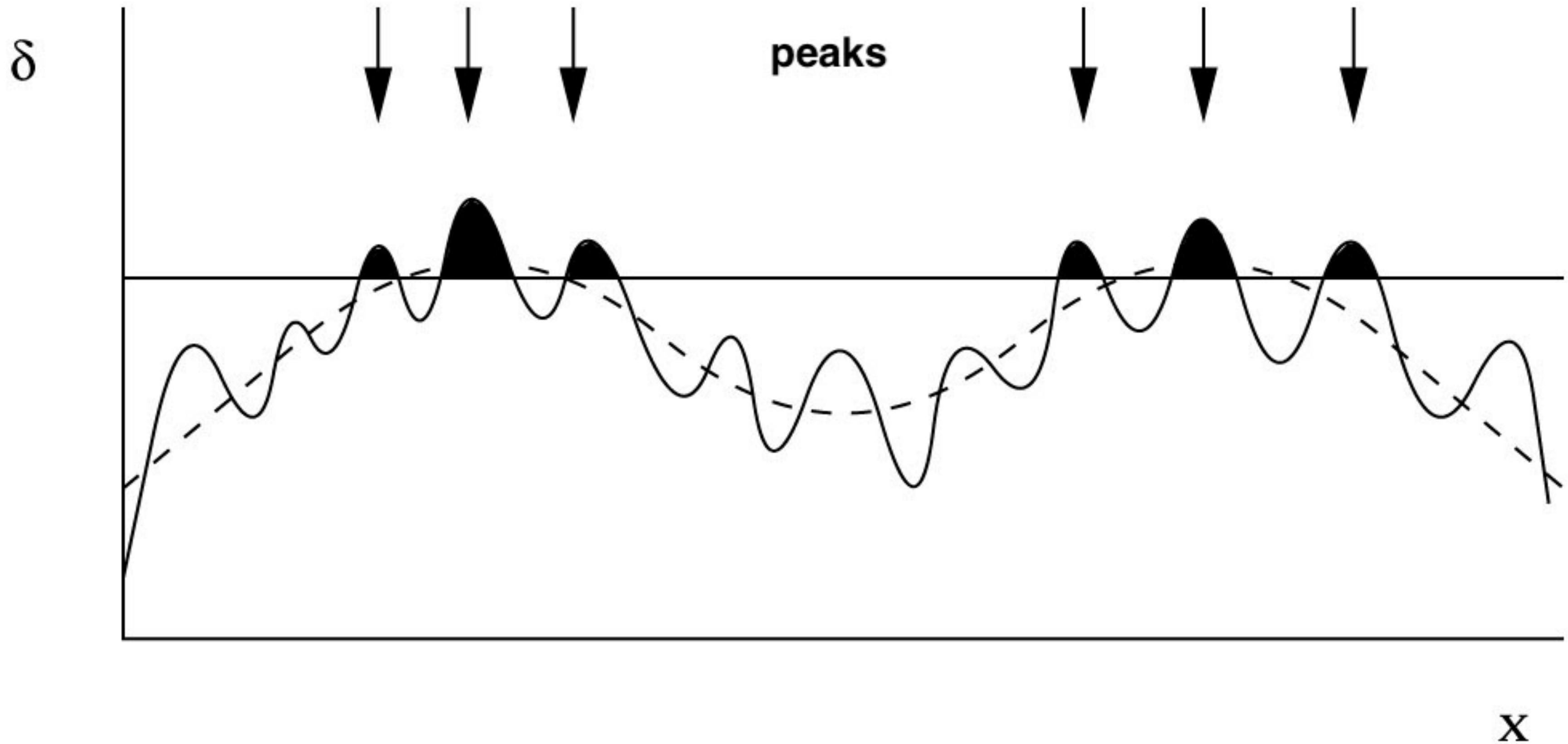
- Halo abundances can be predicted from the overdensities in the initial linear density field.
  - Halo abundances are a cosmological probe in their own right!
  - The information in cluster abundances is complementary to that in two point functions.

- All cluster cosmology experiments are limited by their ability to measure cluster masses.

- Selection effects must by adequately modeled, which is especially important for optically selected clusters.

The Peak-Background Split

To compute spatial distribution of halos, split δ_{m} into large-scale and small-scale fluctuations.



Credit: John Peacock

Halo Clustering

Modify spherical collapse to incorporate for large-scale matter fluctuations, compute *linear* halo bias

Key idea:

Split δ_{m} into large and small scale fluctuations.

$$\delta_{\mathrm{m}} = \delta_{\mathrm{m,short}} + \delta_{\mathrm{m,long}}$$

Halo bias is relative to large scale overdensities.

$$\delta_{\mathrm{h}} = b(M)\delta_{\mathrm{m,long}}$$

Effective barrier is lower in high density regions.

$$\delta_{\mathrm{c,eff}} = \delta_{\mathrm{c}} - \delta_{\mathrm{m,long}}$$

Halo Bias

Number of halos in an overdensity $\delta_{\mathrm{m,long}}$ is:

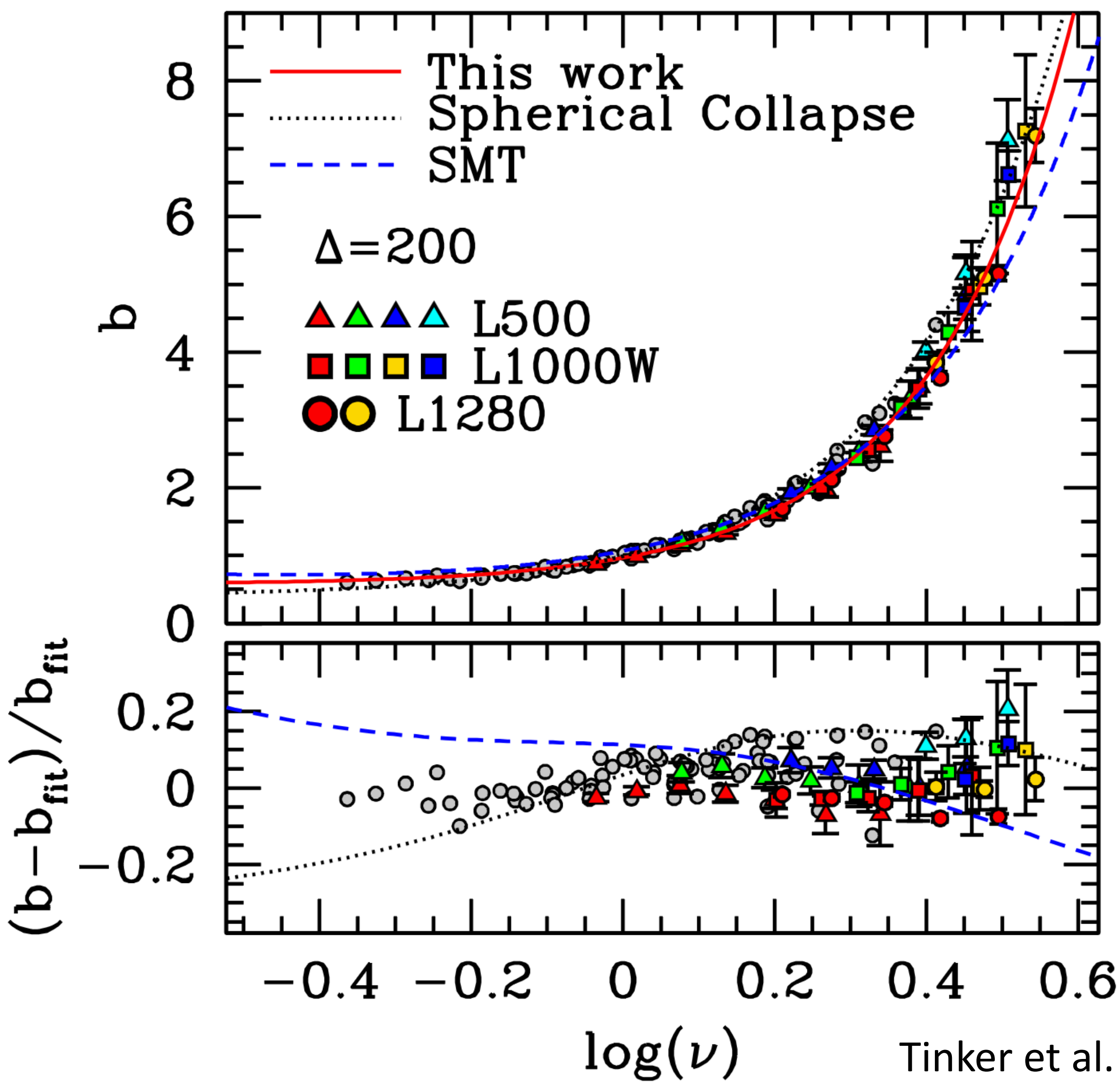
$$\left.\frac{dn(M,z)}{d\ln M}\right|_{\delta_{\mathrm{m,long}}} = \frac{\bar{\rho}}{M} f_{\mathrm{PS}}\left(\frac{\delta_{\mathrm{c}} - \delta_{\mathrm{m,long}}}{\sigma(M,z)}\right)\left|\frac{d\ln\sigma(M,z)}{d\ln M}\right|$$

$$= \left.\frac{dn(M,z)}{d\ln M}\right|_{0}\left[1 - \frac{d\ln f_{\mathrm{PS}}}{d\nu}\frac{\delta_{\mathrm{m,long}}}{\sigma(M,z)}\right]_{\nu=\delta_{\mathrm{c}}/\sigma(M,z)}$$

$$\delta_{\mathrm{h,long}} = \frac{dn(M,z)/d\ln M|_{\delta_{\mathrm{m,long}}}}{dn(M,z)/d\ln M|_{\delta_0}} \equiv b_1(M,z)\delta_{\mathrm{m,long}}$$

Press-Schechter: $b(M) = \dfrac{\nu^2 - 1}{\sigma_M \nu}$

Halo Bias



Tinker et al. 2010

The Halo Model

The concept of halos is a useful way of modeling the density of the Universe.

Assume that all mass is in dark matter halos.

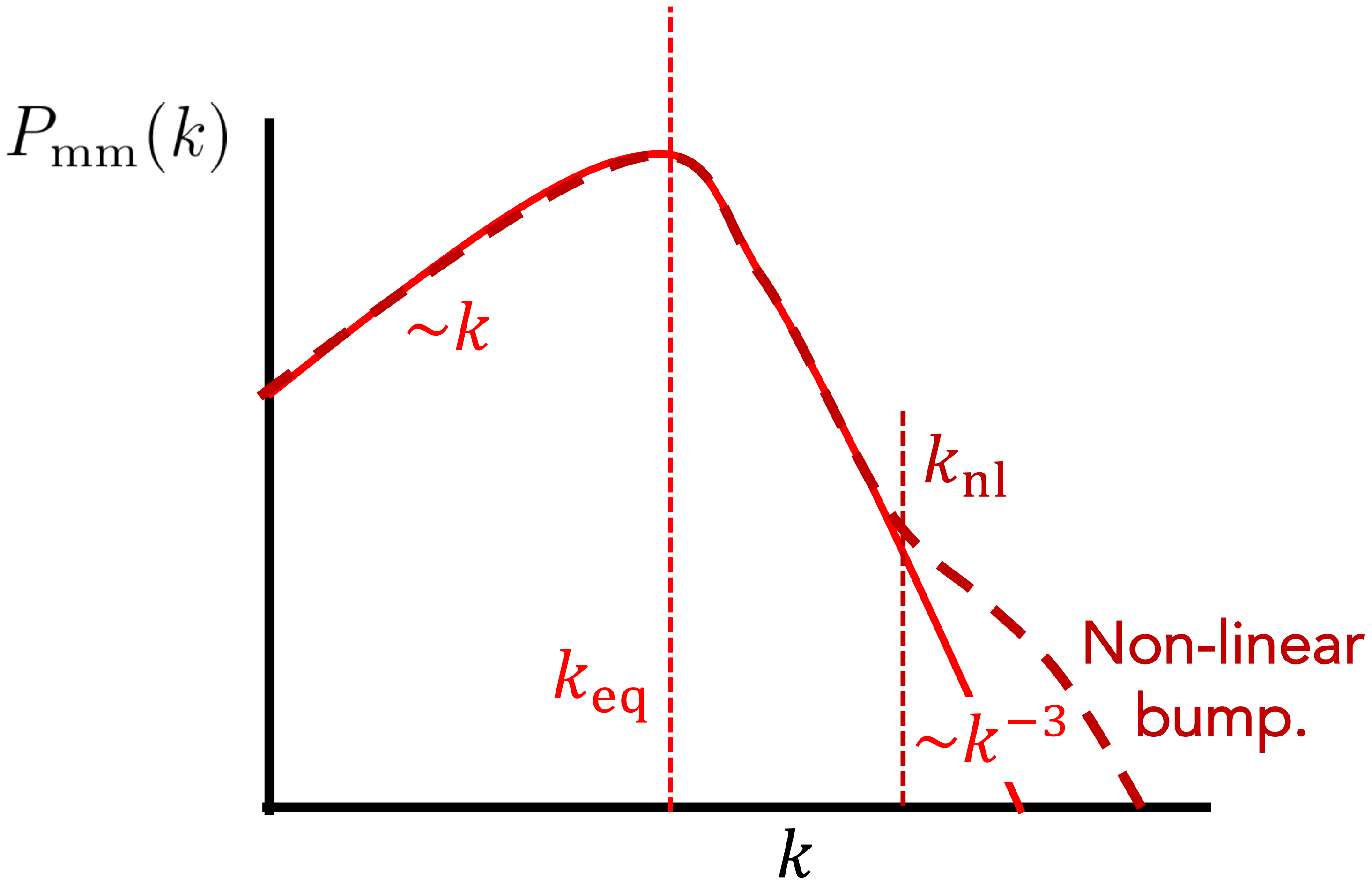
If we know:

1. How halos cluster.
2. The mass distribution around each halo.

Then we can figure out how *ALL* matter is clustered!

We can write an expression for $P_{\rm NL}(k)$!

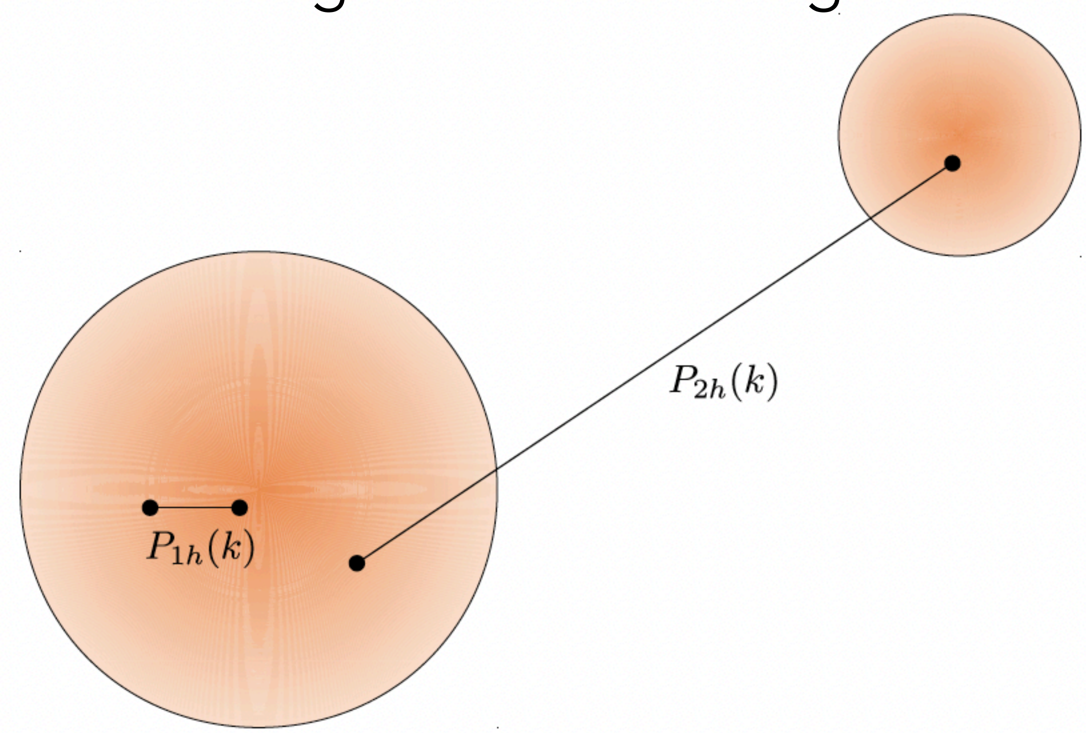
We can use the idea of halos to model the non-linear bump!



The Halo Model

Computation of matter clustering splits into

- 1-halo term: clustering of matter within one halo
- 2-halo term: large-scale clustering of halos



Dodelson/Schmidt: Modern Cosmoloy

The Halo Model: 1h Term

Write mass profile as $\rho(r, M) = MU(r, M)$

$U(r|M)$ is normalized to unity over halo volume (e.g., R_{200}).

On small scales, the two particles will always be in the same halo → single mass integral

$$P^{\rm 1h}_{\rm mm}(k) = \frac{1}{\bar{\rho}^2} \int_0^\infty M^2 \hat{U}^2_{\rm m}(M, k) n(M)\, \mathrm{d}M$$

Note that mass function must obey $\int_0^\infty M n(M)\, \mathrm{d}M = \bar{\rho}$

(all mass is contained in halos)

The Halo Model: 2h Term

For two halo term, halos may have different masses

$$P_{\rm hh}(M_1, M_2, k) = b(M_1)b(M_2)P^{\rm lin}_{\rm mm}(k)[1 + \cancel{\beta^{\rm nl}(M_1, M_2, k)}]$$

non-linear corrections,
→ Asgari+(2023) review

→ double mass integral

$$P^{\rm 2h}_{\rm mm}(k) = P^{\rm lin}_{\rm mm}(k) \times \left[\frac{1}{\bar{\rho}}\int_0^\infty M\hat{U}_{\rm m}(M,k)b(M)n(M)\,{\rm d}M\right]^2$$

Note that bias function must obey $\displaystyle\int_0^\infty Mb(M)n(M)\,{\rm d}M = \bar{\rho}$

(halos are on average unbiased)

Halo Profile

Empirically, the Navarro-Frenk-White profile describes density profile of simulated halos across remarkable mass and redshift range.

Free parameter: halo concentration **c(M)** requires fitting function

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Empirically, the Navarro-Frenk-White profile describes density profile of simulated halos across remarkable mass and redshift range.

Free parameter: halo concentration **c(M)**

requires fitting function

Reference	Definition	Notes
Navarro et al. (1997)	200c	Depends on a cosmology-dependent halo-collapse redshift that is calculated semi-analytically.
Bullock et al. (2001)	virial	Two relations presented in paper: a simple model where c is a power-law in M (although scaled by a cosmology-dependent non-linear mass) and a more complicated model where c is related to a cosmology-dependent halo formation redshift, which is calculated semi-analytically.
Eke et al. (2001)	virial	Depends on a cosmology-dependent halo-collapse redshift that is calculated semi-analytically.
Neto et al. (2007)	200c	Only considered the Millennium Springel et al. (2005) cosmology at $z = 0$.
Macciò et al. (2008)	virial	Modified version of the Bullock et al. (2001) algorithm.
Duffy et al. (2008)	200, 200c, virial	Simple $c(M)$ power-law relations are presented that are fitted to simulations of *WMAP* 5 cosmology. Explicit z dependence. Separate relations for 'relaxed' and 'full' samples of haloes.
Prada et al. (2012)	200c	'Cosmology dependent' relation presented as a function of $\sigma(M, a)$. Upturn in halo concentration for high-mass haloes.
Kwan et al. (2013)	200c	Emulated relation for a variety of wCDM cosmologies.
Ludlow et al. (2014)	200c	Relates halo concentration to mass-accretion history.
Klypin et al. (2014)	200c	Parametrised in terms of ν.
Diemer & Kravtsov (2015)	200c	Present a semi-analytical, cosmology-dependent model parametrised in terms of ν and n_{eff} – the effective slope of the power spectrum on collapse scales. Demonstrates that concentration–mass relation is 'most universal' when masses are defined via 200c.
Correa et al. (2015)	200c	Relates halo concentration to mass-accretion history. Only applies to relaxed haloes.
Okoli & Afshordi (2016)	200c	Focusses on relaxed low-mass haloes using analytical arguments. Cosmology dependence incorporated via ν dependence.
Ludlow et al. (2016)	200c	Applies for WDM as well as for CDM cosmologies. Depends on a collapse redshift that is calculated semi-analytically.
Child et al. (2018)	200c	Power-law relation but scaled via the cosmology-dependent non-linear mass. Also consider Einasto profiles. Individual and stacked halo profiles considered separately.
Diemer & Joyce (2019)	200c	Improved version of Diemer & Kravtsov (2015) with additional dependence on the logarithmic linear growth rate to capture non-standard expansion histories.
Ishiyama et al. (2021)	200c, virial	Uses the same functional form as Diemer & Joyce (2019) but fitted to a larger simulation resulting in up to 5% errors for a wide range of masses and redshifts.

Non-exhaustive list of c(M) fitting functions. Note different definitions!

Asgari+2023

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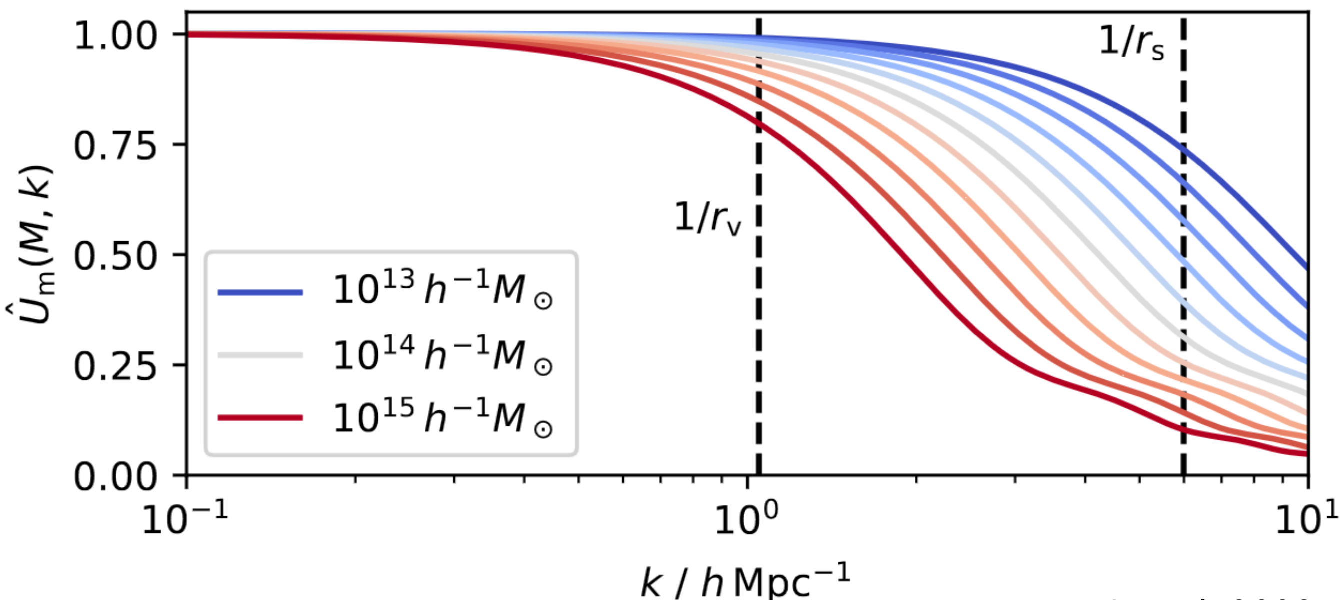
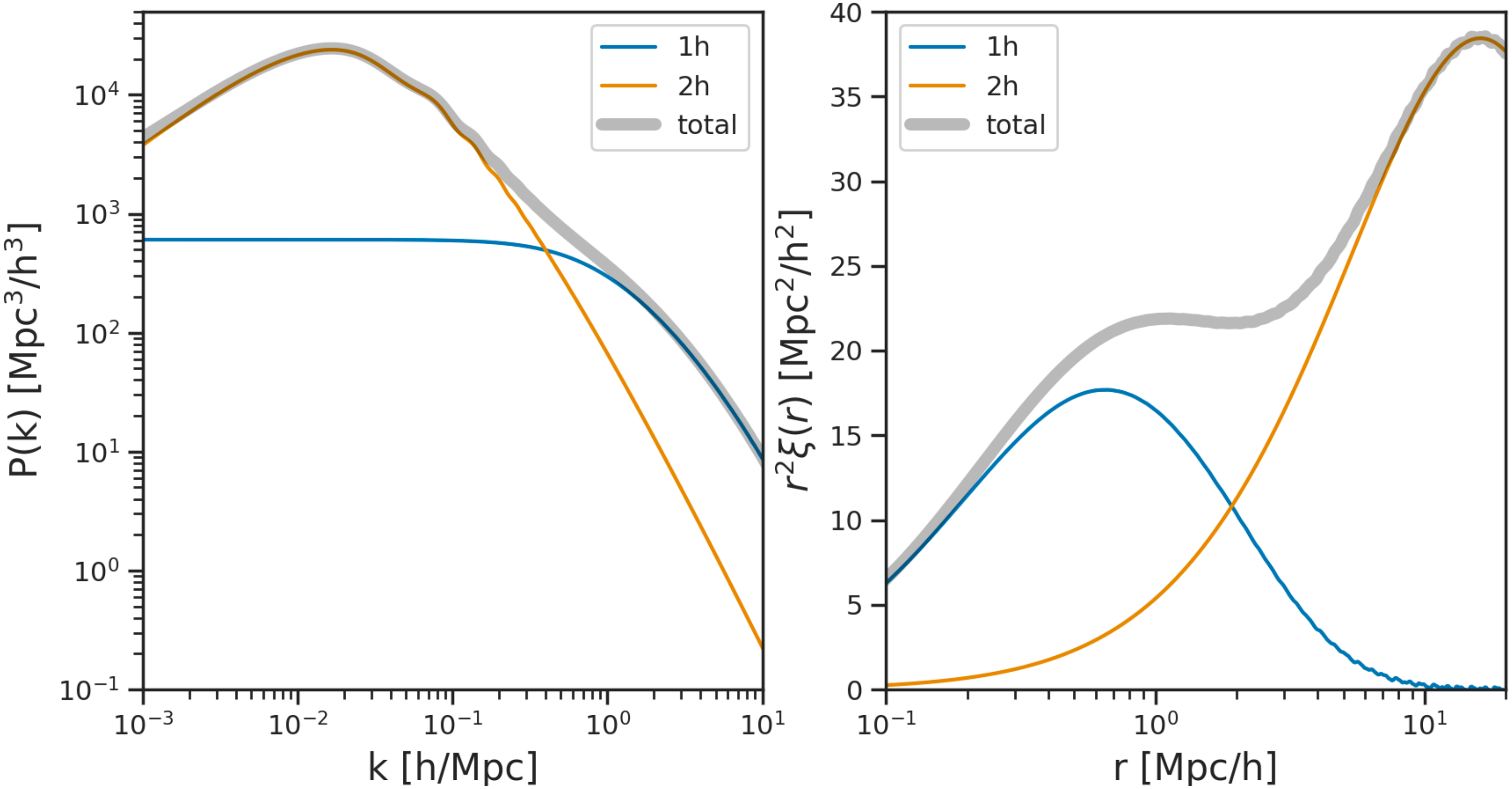
Empirically, the Navarro-Frenk-White profile describes density profile of simulated halos across remarkable mass and redshift range.

Free parameter: halo concentration **c(M)** requires fitting function



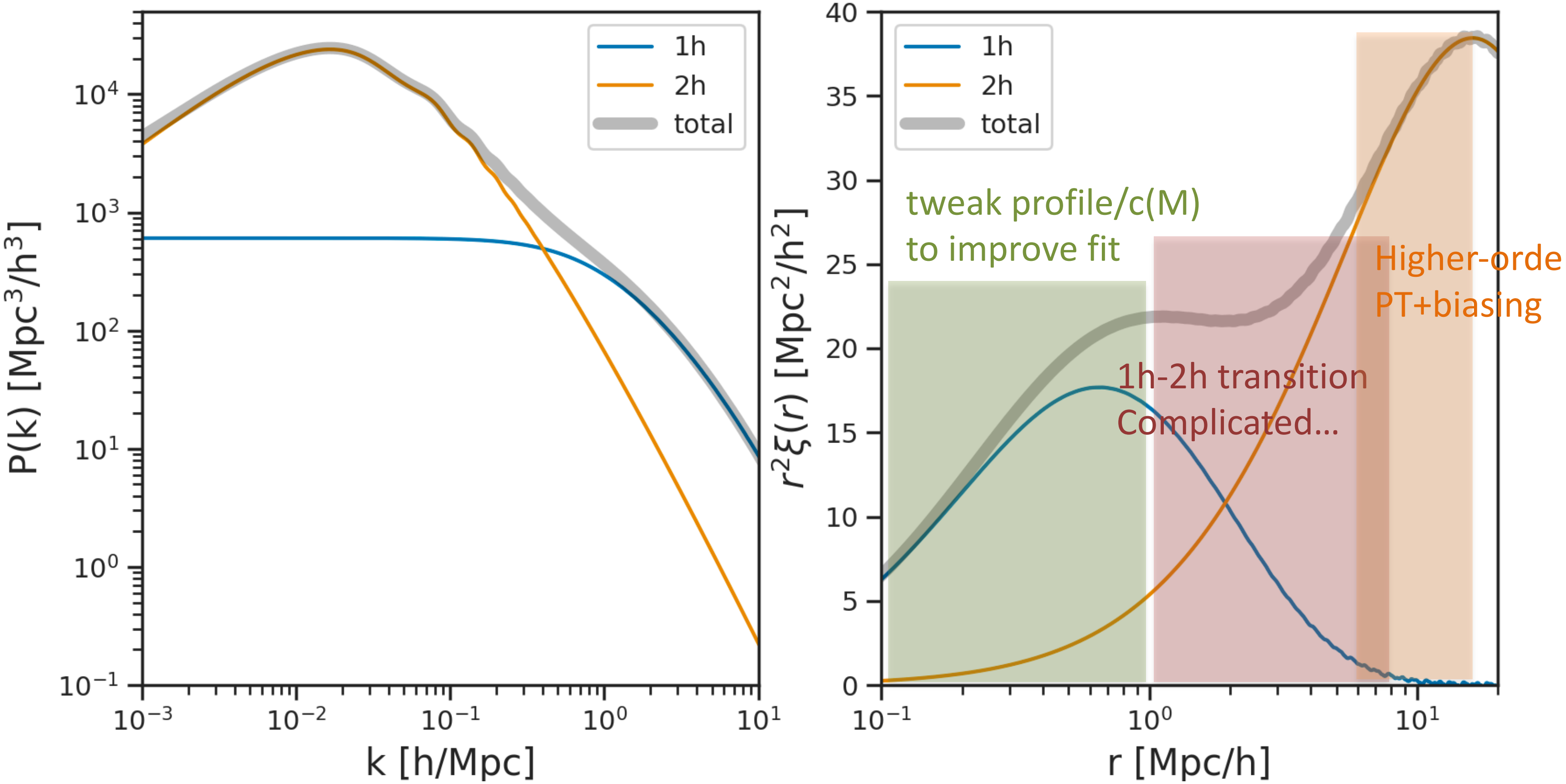
Asgari+2023

The Halo Model



Typically agrees with simulations at 10s of % level.

The Halo Model



Typically agrees with simulations at 10s of % level.

Galaxy Formation: a Classic Recipe

1. Structure formation is driven by gravitational instability.
2. Dark-matter proto haloes get a spin due to tidal torques.

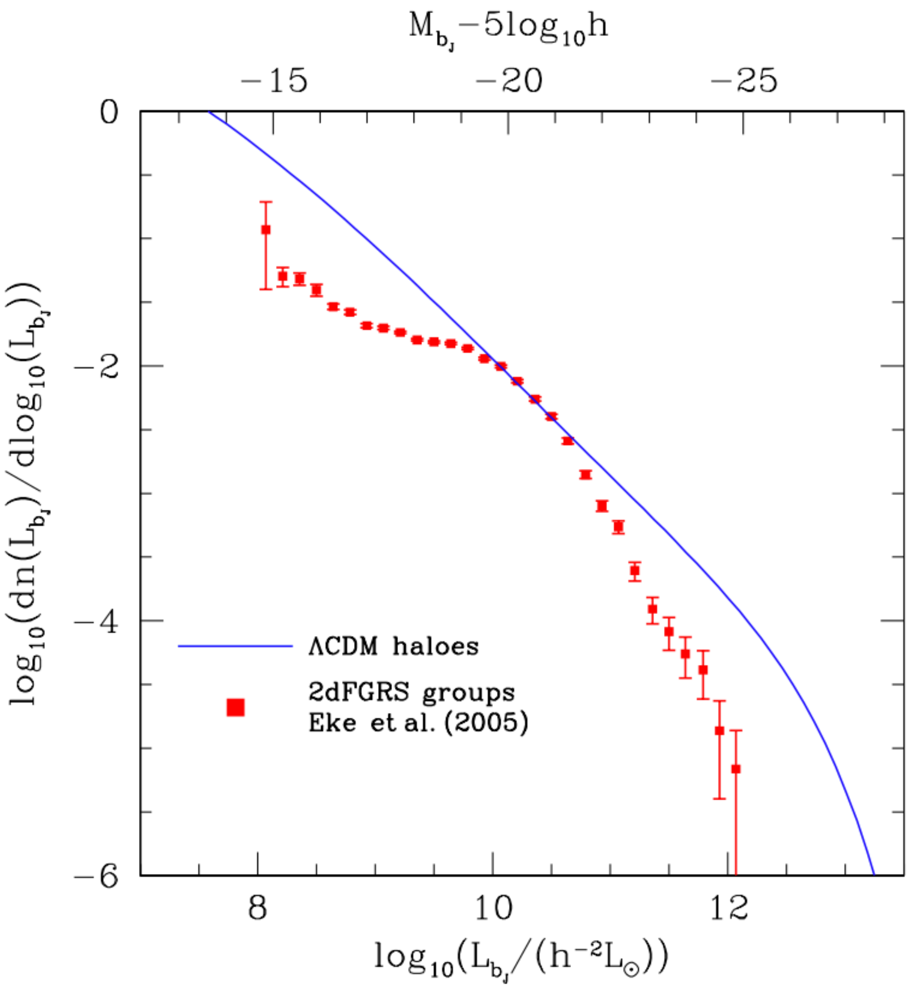
   Hoyle 1949
3. Galaxies form inside dark-matter haloes, via a two-stage collapse:
   a. Dissipationless collapse of the dark-matter haloes,
   b. dissipative collapse of gas: baryons collapse into the halo potential well and get shock heated.

   Rees & Ostriker 1977, White & Rees 1978
4. Gas cools mainly by radiative transitions: typical mass of a galaxy is set by cooling arguments .

   Hoyle 1953, Silk 1977, Binney 1977, Rees & Ostriker 1977
5. The formation of disk galaxies can be understood by the cooling of gas to the DM-halo centres via conservation of the angular momentum of the DM halo.

   Fall & Efstathiou 1980
6. (Elliptical galaxies form via the merger of disk galaxies)

From Halos to Galaxies

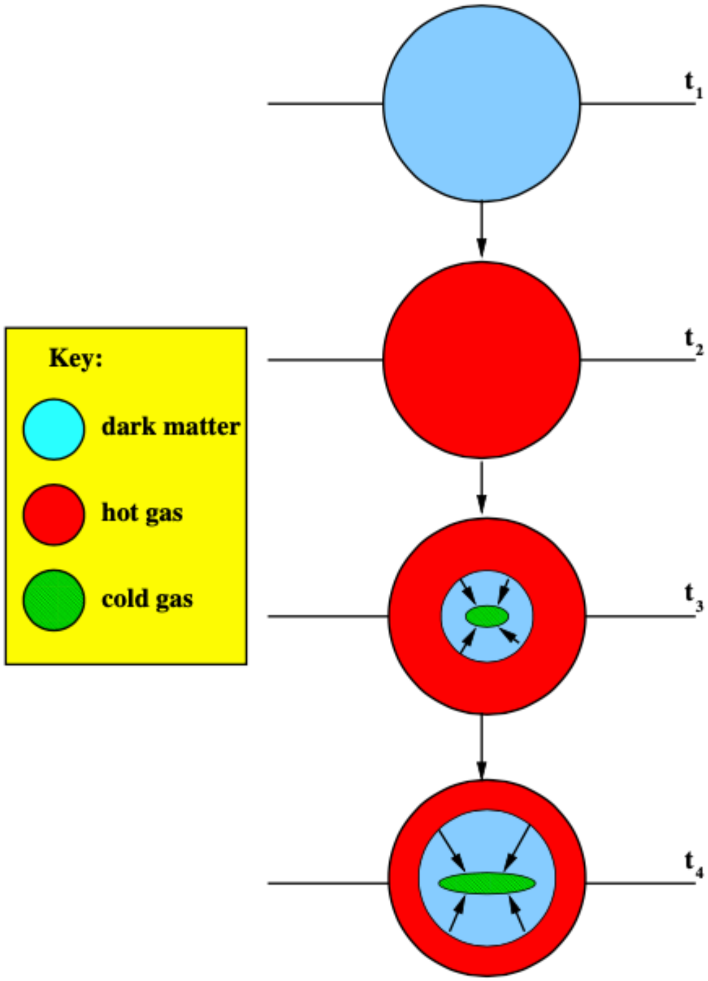


Simple prediction for the LF of galactic compared with the group luminosity function measured from the 2dFGRS (Eke+06).

Obvious discrepancy between the shape of the observed luminosity function and the one expected if all halos had the same M/L ratio.

Halos of different mass must have different efficiency with which baryons are converted into stars.

From Halos to Galaxies



t1: Baryons fall into the potential well of a dark matter halo

t2: Baryons are shock-heated to the halo temperature.

t3: The hot halo gas cools, and cool gas falls towards the halo center where it forms stars.

t4: The radius within which gas had sufficient time to cool increases with time.

(From Baugh 2007)

From Halos to Galaxies

t2: Baryons are shock-heated to the halo temperature.

Consider a gas cloud of mass M_{gas} falling into a halo of mass M_h with v_{in}.

If the gas falls in from large distances (where $\Phi(r)$ ~0), and has negligible initial velocity, then **$v_{in}^2 \sim v_{esc}^2(r_{sh}) \sim 2|\Phi(r_{sh})|$.**

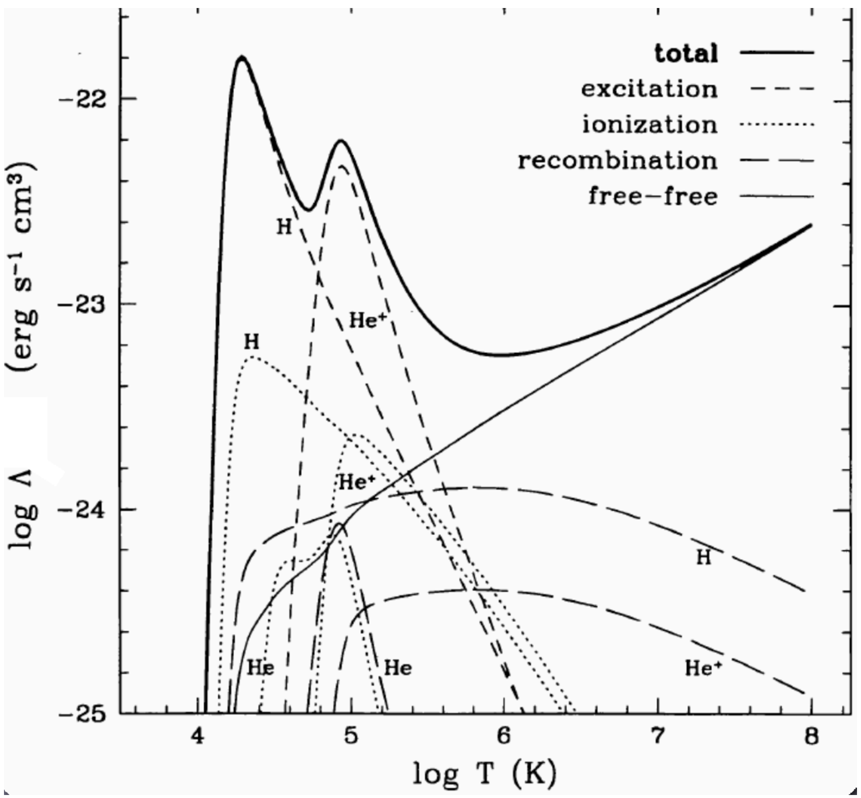
Assuming hydrostatic equilibrium (+ideal gas, spherical halo) and virial theorem, and ignoring external pressure, magnetic fields, etc., one can define the **virial temperature**

$$T_{\rm vir} = \frac{\mu m_{\rm p}}{2k_{\rm B}} V_{\rm c}^2 \simeq 3.6 \times 10^5\,{\rm K} \left(\frac{V_{\rm c}}{100\,{\rm km\,s^{-1}}} \right)^2$$

$$T_{\rm vir} \simeq 4 \times 10^4 \left(\frac{\mu}{1.2} \right) \left(\frac{M_{\rm h}}{10^8 h^{-1} {\rm M}_\odot} \right)^{\frac{2}{3}} \left(\frac{1+z}{10} \right) {\rm K}\,,$$

From Halos to Galaxies

In order to compute the cooling function $\Lambda(T)$ for a certain gas, one first needs to determine the densities of the various ionic species. In the case of a pure H/He mixture (the simplest, relevant case), these are
n_e , n_{H0} , n_{H+} , n_{He0} , n_{He+} , n_{He++}.



In ionization equilibrium, the ionic abundances are determined by simple algebraic equations.

If there is no photo-ionization, then, under CIE, the relative abundances of ionic species depend only on temperature.

From Katz+ 1996.

From Halos to Galaxies

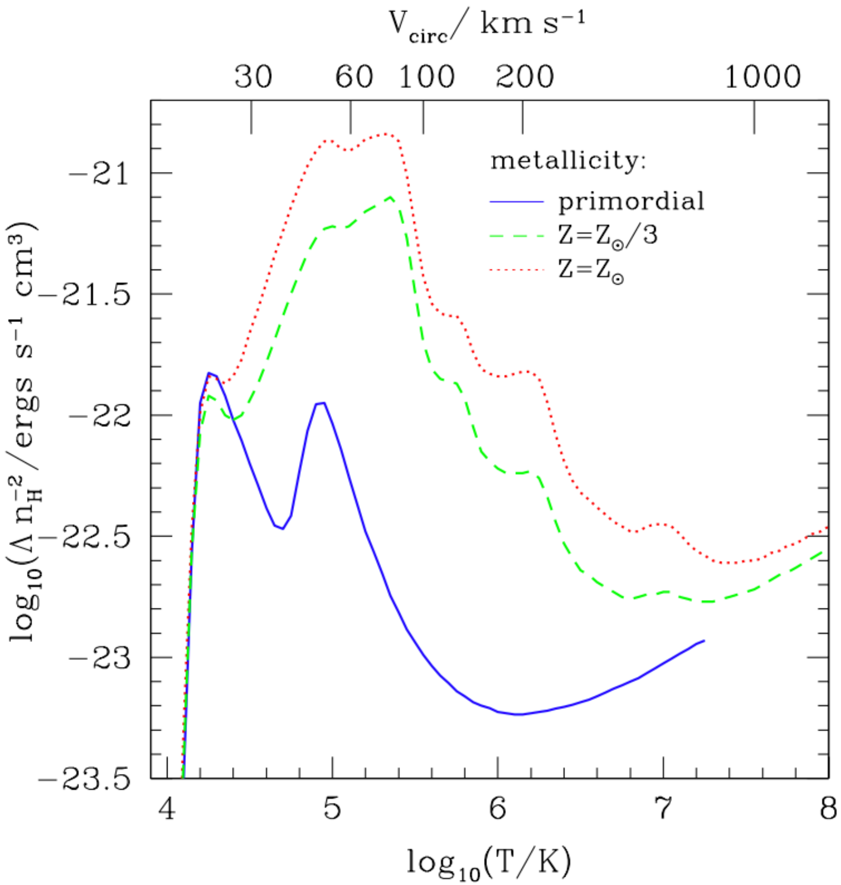
t3: The hot halo gas cools, and cool gas falls towards the halo center where it forms stars.

$$t_{\rm cool}(r) = \left(\frac{3}{2}\frac{\rho_{\rm gas}(r)k_{\rm B}T_{\rm vir}}{\mu m_{\rm H}}\right) / \left(\rho^2_{\rm gas}(r)\Lambda(T_{\rm vir}, Z_{\rm gas})\right)$$

- **Molecular cooling:** Collisions excite vibrational and rotational energy levels in H2. The subsequent decay of the excited states removes energy from the gas. Most important in halos with virial temperature below $T \sim 10^4$ K.
- **Atomic cooling:** Partly ionized atoms cool through the excitation and radiative decay of higher energy levels. Contributes while gas is partly, but not fully ionized (10^4 K < T < 10^6 K for primordial gas). Metal enrichment enhances the cooling efficiency at higher temperatures where gas of primordial composition is almost completely ionized.
- **Bremsstrahlung:** As electrons are accelerated in an ionized plasma they loose energy due to the emission of Bremsstrahlung. Dominant cooling process in cluster halos with $T > 10^7$ K.

From Halos to Galaxies

t3: The hot halo gas cools, and cool gas falls towards the halo center where it forms stars.



Cooling rate as a function of halo temperature and gas metallicity. Bremsstrahlung becomes the dominant process at $T > 10^7$ K. Atomic line cooling is most efficient at lower temperatures and causes the peaks at 15000 K (H) and 10^5 K (He+).

From Sutherland & Dopita 1993.

From Halos to Galaxies

t4: The radius within which gas had sufficient time to cool increases with time.

$$t_{\rm cool}(r) = \left(\frac{3}{2}\frac{\rho_{\rm gas}(r) k_{\rm B} T_{\rm vir}}{\mu m_{\rm H}}\right) / \left(\rho^2_{\rm gas}(r) \Lambda(T_{\rm vir}, Z_{\rm gas})\right)$$

We can implicitly define the as the radius at which the cooling time is equal to the dynamical time (or free-fall time) of the halo

$$t_{\rm cool}(r_{\rm cool}) = t_{\rm dyn}$$

If $t_{cool} > t_H$: Cooling is not important. Gas is in hydrostatic equilibrium, unless it was recently disturbed.

If $t_{dyn} < t_{cool} < t_H$: System is in quasi-hydrostatic equilibrium. It evolves on cooling time scale. Gas contracts slowly as it cools, but system has sufficient time to continue to re-establish hydrostatic equilibrium.

If $t_{cool} < t_{dyn}$: Gas cannot respond fast enough to loss of pressure. Since cooling time decreases with increasing density, cooling proceeds faster and faster. Gas falls to center of dynamic system on free-fall time.

From Halos to Galaxies

t4: The radius within which gas had sufficient time to cool increases with time.

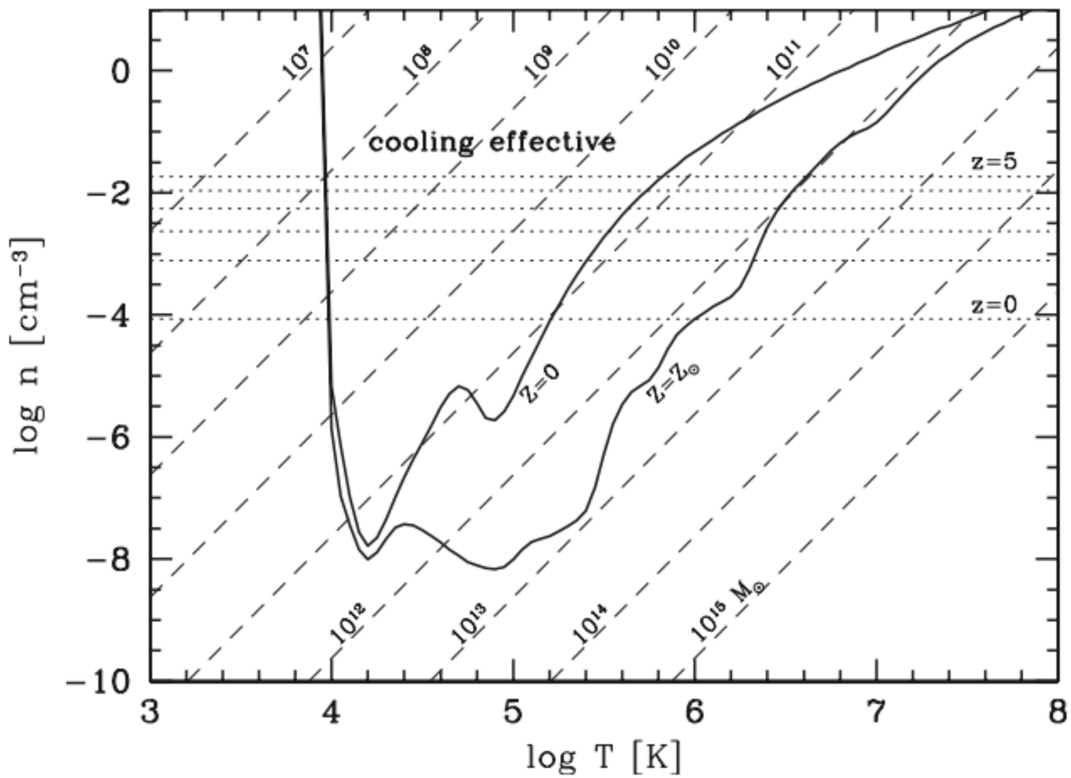


Fig. 8.6. Cooling diagram showing the locus of $t_{\rm cool} = t_{\rm ff}$ in the n–T plane. The upper and lower curves correspond to gas with zero and solar metallicity, respectively. The tilted dashed lines are lines of constant gas mass (in $M_\odot$), while the horizontal dotted lines show the gas densities expected for virialized halos ($\delta = 200$) at different redshifts. All calculations assume $f_{\rm gas} = 0.15$ $\Omega_{m,0} = 0.3$, and $h = 0.7$. Cooling is effective for clouds with n and T above the locus.

From Halos to Galaxies

t4: The radius within which gas had sufficient time to cool increases with time.

$$t_{\rm cool}(r) = \left(\frac{3}{2}\frac{\rho_{\rm gas}(r) k_{\rm B} T_{\rm vir}}{\mu m_{\rm H}}\right) / \left(\rho^2_{\rm gas}(r) \Lambda(T_{\rm vir}, Z_{\rm gas})\right)$$

We can implicitly define the as the radius at which the cooling time is equal to the dynamical time (or free-fall time) of the halo

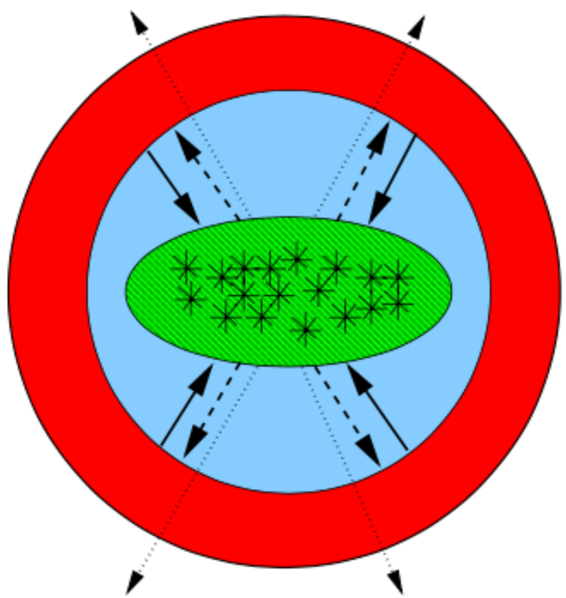
$$t_{\rm cool}(r_{\rm cool}) = t_{\rm dyn}$$

At late times and in massive systems, the cooling radius typically is smaller than the virial radius. The gas outside r_{cool} forms a quasi-static hot halo, while gas from the central regions ($r < r_{cool}$) cools and is accreted onto the center. The accretion rate can be estimated from a continuity equation

$$\dot{m}_{\rm cool} = 4\pi \rho_{\rm gas}(r_{\rm cool}) r^2_{\rm cool} \dot{r}_{\rm cool} = \frac{m_{\rm hot} \dot{r}_{\rm cool}}{R_{\rm vir}} \sim \frac{m_{\rm hot} r_{\rm cool} v_{\rm vir}}{R^2_{\rm vir}} \propto f_{\rm b} r_{\rm cool} v^2_{\rm vir} H(z) G^{-1}$$

At $z = 0$ the transition between halos that are rapidly cooling and those forming a static hot halo occurs at a halo virial mass of about $\sim 2 - 3 \times 10^{11} M_\odot$.

Star Formation & Feedback



Feedback from supernova-driven winds. Hot halo gas cools and builds up a reservoir of cold gas in the galactic disk (solid arrows). The cold gas is turned into stars. Supernova explosions reheat a fraction of the cold gas and returns it to the hot phase (dashed arrows) or eject material from the halo (dotted arrows).

(Illustration from Baugh 2007)

$$\Delta m_{\rm reheat} = \frac{4}{3}\epsilon \frac{\eta_{\rm SN}\, E_{\rm SN}}{v_{\rm vir}^2} \Delta m_*$$

Galaxy Transformations

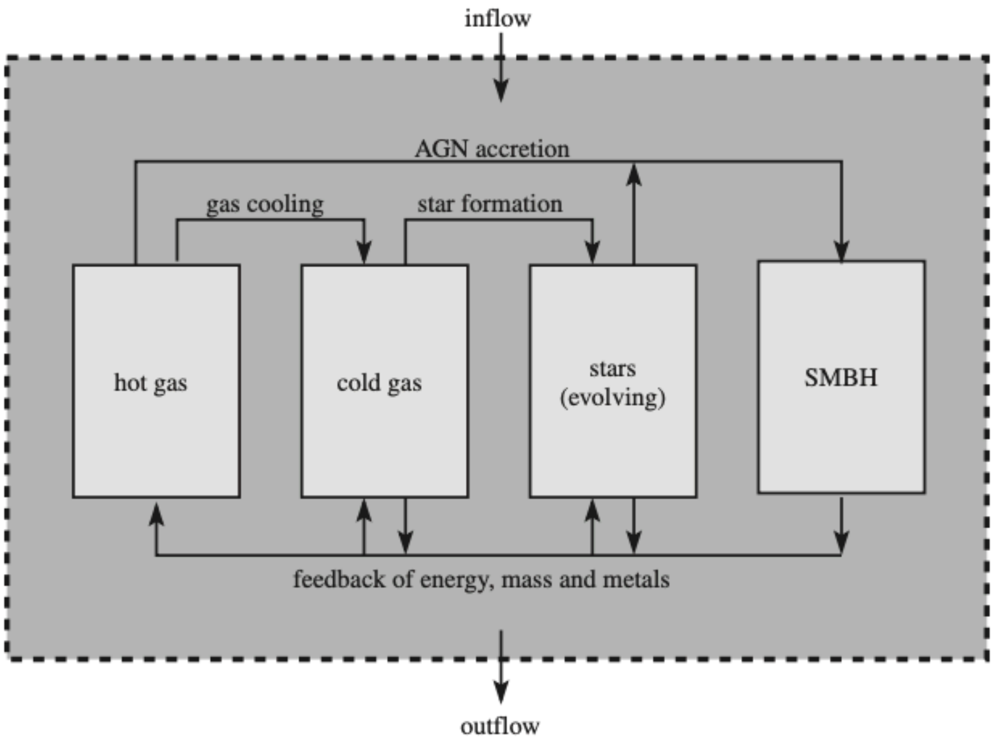


Fig. 1.2. A flow chart of the evolution of an individual galaxy. The galaxy is represented by the dashed box which contains hot gas, cold gas, stars and a supermassive black hole (SMBH). Gas cooling converts hot gas into cold gas, star formation converts cold gas into stars, and dying stars inject energy, metals and gas into the gas components. In addition, the SMBH can accrete gas (both hot and cold) as well as stars, producing AGN activity which can release vast amounts of energy which affect primarily the gaseous components of the galaxy. Note that in general the box will not be closed: gas can be added to the system through accretion from the intergalactic medium and can escape the galaxy through outflows driven by feedback from the stars and/or the SMBH. Finally, a galaxy may merge or interact with another galaxy, causing a significant boost or suppression of all these processes.

Simulating Galaxy Evolution

- Gravity & hydrodynamics
- Atomic processes (radiative cooling of the gas):
  - Heavy elements (metals = beyond He)
  - Molecules (H_2, CO, …)
- Star formation
  - Stellar evolution
  - Metal production and enrichment
  - Feedback: from supernovae, stellar winds, …
- (SuperMassive) Black Holes:
  - Formation
  - Growth: merging and gas accretion
  - Feedback: radiative, thermal, momentum
- Radiation (RHD):
  From stars, BHs, diffuse gas, reionization
- Magnetic Fields (MHD)
- Relativistic particle populations (cosmic rays)
- Dust (i.e. very large molecules)
- Plasma Physics (thermal conduction, …)

Essentially, all astrophysical phenomena are unresolved

(occur below the physical resolution of the sims)

They require some level of "subgrid" modelling:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \nabla \cdot \boldsymbol{F} = \cancel{0}$$
$$= \mathrm{S}$$

Messy astrophysics adds complex, poorly understood source terms

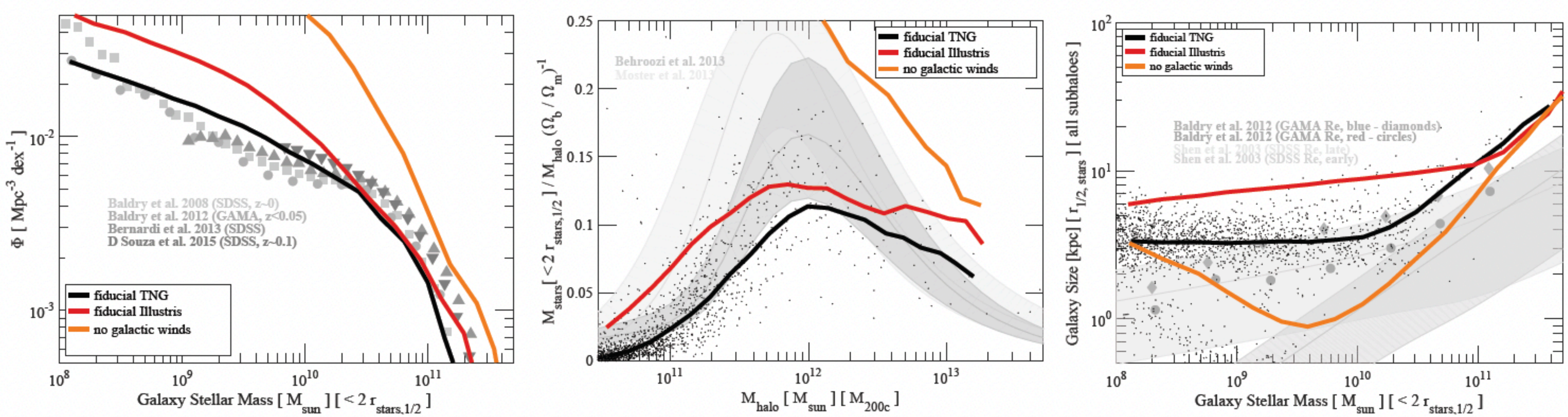
"Laws" suggested by observations and tailored theoretical models are invoked and implemented

Via look-up "tables"

into surrounding gas

Simulating Star Formation

Different Simulation – Different Recipe



Pillepich+2018

Without ‘a’ form of feedback acting at all masses, i.e., a mechanism regulating conversion of gas into star, halos would host way too massive and compact galaxies.

AGN Feedback

Supermassive black holes, residing in all galactic spheroids, play an important role in the formation of massive galaxies. Black holes deplete a galaxy's gas supply by accretion of gas, and injecting heat into the ISM. At times where a galaxy is not undergoing a major merger, the black hole quiescently accretes gas which is assumed to come from the hot halo.

A simple phenomenological model for the accretion rate is

$$\dot{m}_{\mathrm{BH}} = \frac{\kappa_{\mathrm{AGN}}}{10G} m_{\mathrm{BH}}\ f_{\mathrm{hot}}\ v_{\mathrm{vir}}^3\ ,$$

This accretion rate can be motivated by noting that $f_{hot}v_{vir}^3/H(z)/(10G)$ is the total mass of hot gas, so that in case of unit accretion efficiency the black hole will accrete all hot gas within a Hubble time. We then assume that a fixed fraction **η** of the accreted energy is released radiatively

$$L_{\mathrm{BH}} = \eta\ \dot{m}_{\mathrm{BH}}\ c^2$$

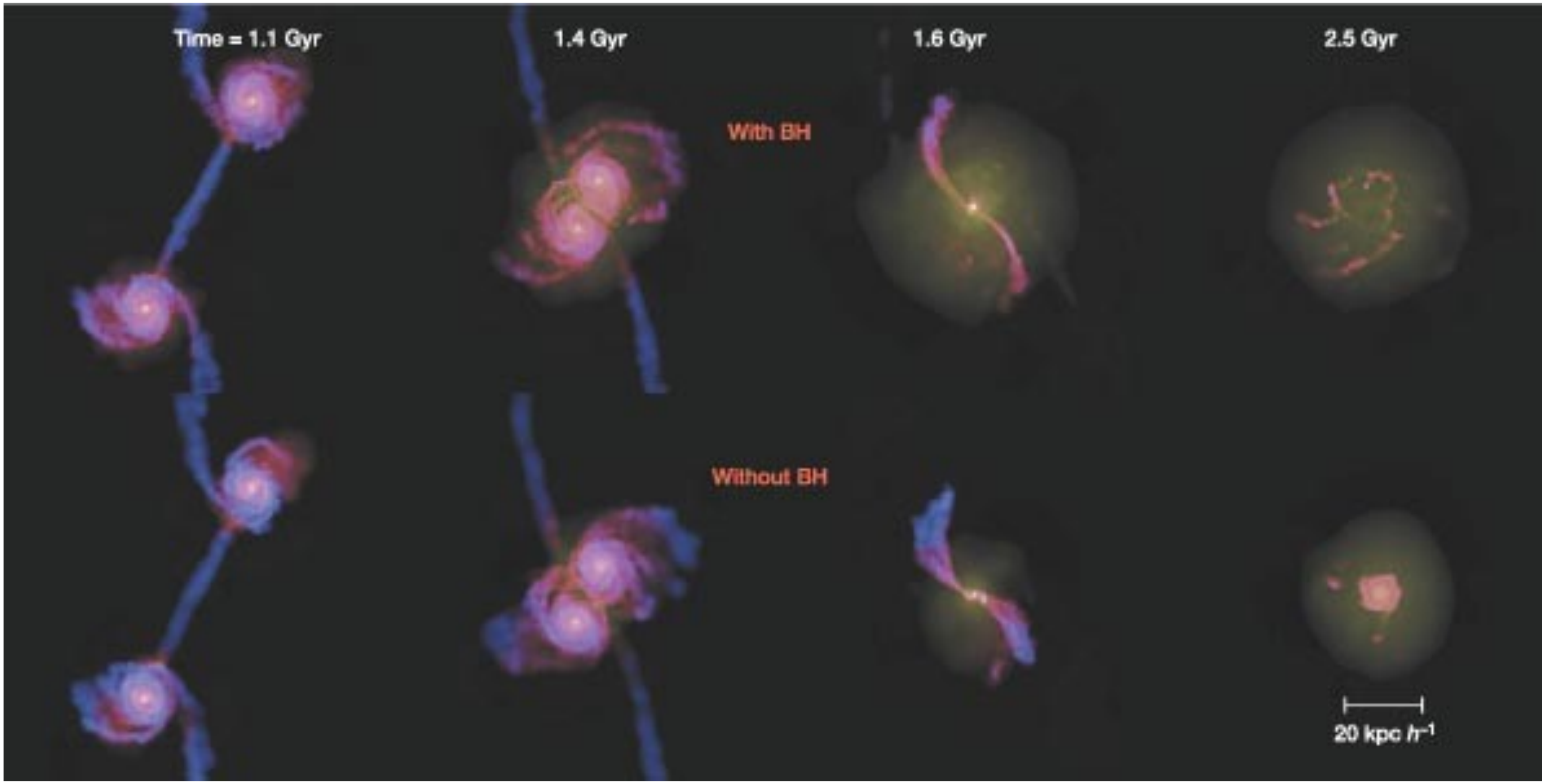
and injected into the ISM where it compensates in part for cooling, giving rise to a modified infall rate

$$\dot{m}'_{\mathrm{cool}} = \dot{m}_{\mathrm{cool}} - \frac{L_{\mathrm{BH}}}{\frac{1}{2}\ v_{\mathrm{vir}}^2} = \dot{m}_{\mathrm{cool}} - \frac{\eta\ \kappa_{\mathrm{AGN}}}{5G}\ m_{\mathrm{BH}}\ c^2\ f_{\mathrm{hot}}\ v_{\mathrm{vir}}\ .$$

AGN Feedback in Simulations

Di Matteo, Springel & Hernquist 2005:

Simulations of galaxy mergers with and without AGN feedback



AGN Feedback in Simulations

Di Matteo, Springel & Hernquist 2005:

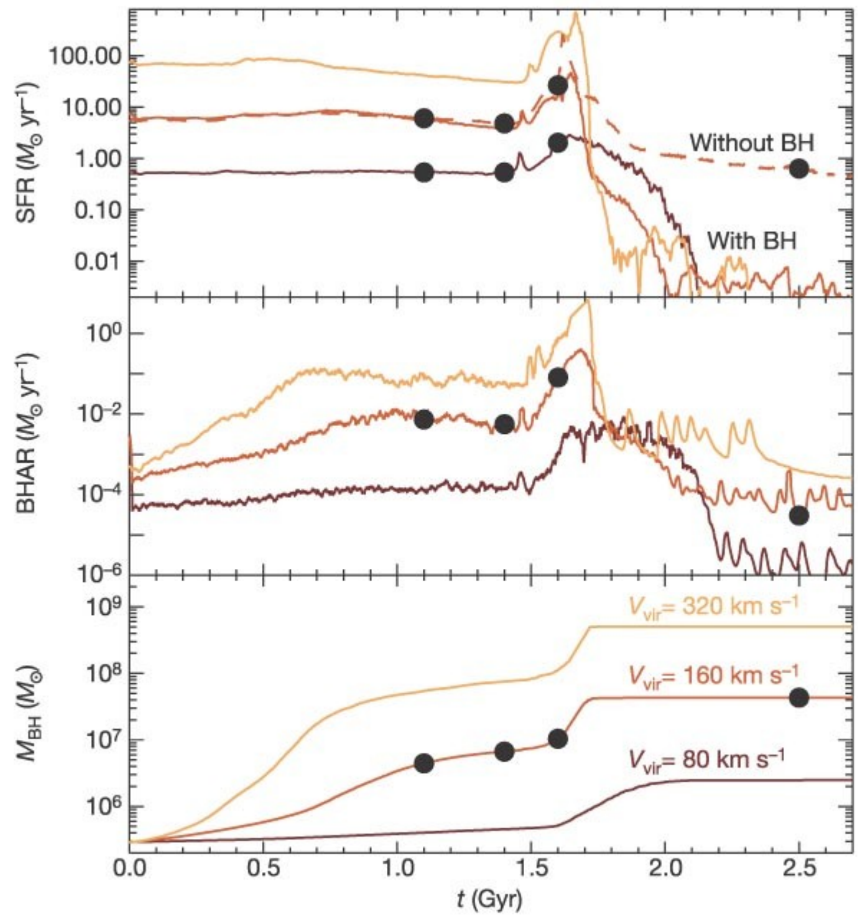
Simulations of galaxy mergers with and without AGN feedback

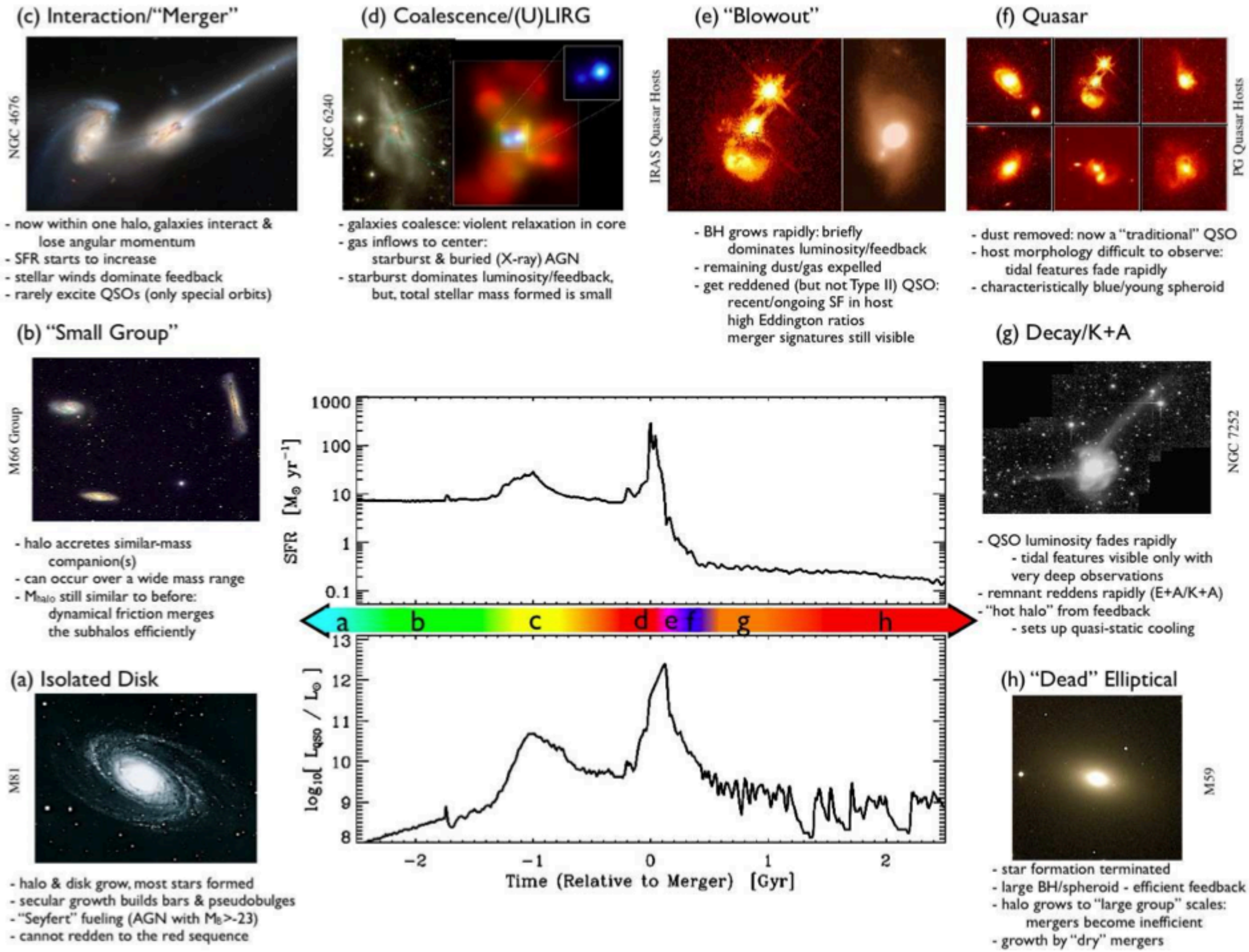
Merger leads to inflow of gas, burst of star formation.

Merger also leads to strong inflows that feed gas to the supermassive black hole and thereby power the quasar.

The energy released by the quasar expels enough gas to quench both star formation and further black hole growth.

Simulated merger remnants with AGN feedback follow observed early-type galaxy scaling relations!





Hopkins et al. 2008

Simulating AGN Feedback

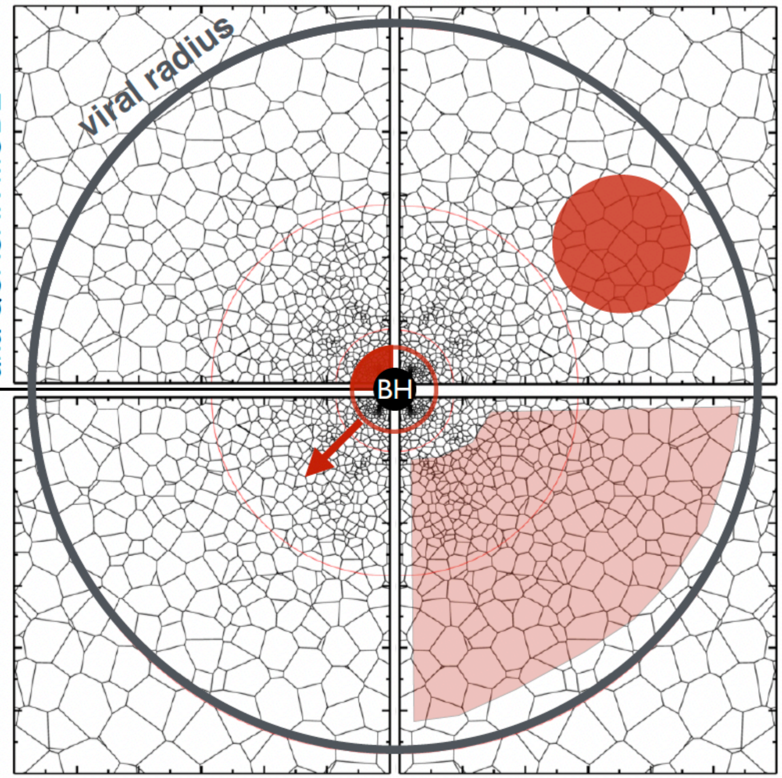
Different Simulation – Different Recipe

Thermal Dump (near the BH)

Continuous?
- yes e.g. Illustris, HorizonAGN
- no e.g. Eagle

Only at high accretion rates?
- yes e.g. Illustris
- no e.g. Eagle (all the time)

aka QUASAR MODE



aka RADIO MODE

Thermal Dump (bubbles)

Very sporadic, energetic bubbles: Illustris
More frequent, "smaller bubbles": Auriga

Only at low accretion rates?
- yes e.g. Illustris
- no e.g. Auriga (all the time)

Isotropic?
- no e.g. TNG, each time in different dirs
- no: bipolar e.g. HorizonAGN

Continuous?
- ~ e.g. TNG, each time in different dirs

Only at low accretion rates?
- yes e.g. TNG, HorizonAGN

Kinetic Kick or BH-driven winds

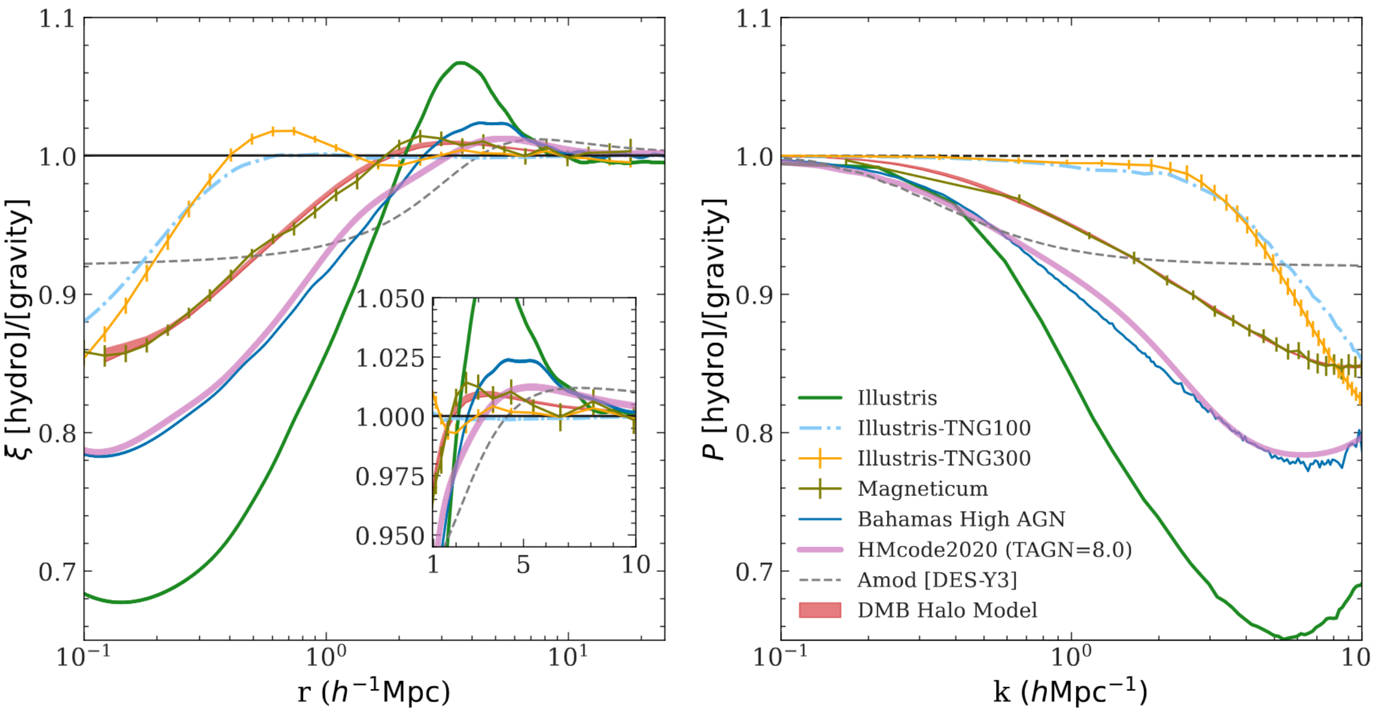
See also Choi et al. 2012, 2014, 2015; Dubois et al. 2010, 2012; Weinberger et al 2017

Affecting only non-self shielded gas
e.g. Mufasa, NIHAO variations

"By Hand" heating of the gaseous halo

All simulations also require prescription for forming initial SMBHs…

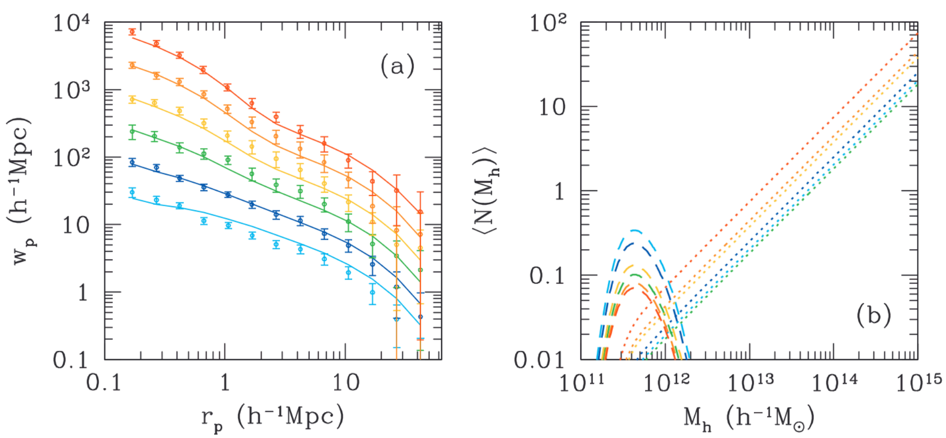
Impact of Feedback on the Matter Distribution



Halo Occupation Distribution

Despite the rich astrophysics of feedback, the empirical halo occupation distribution (HOD) has been successful to model 2pt clustering of many different galaxy samples (so far…):

- Split galaxies into central galaxies (c) and satellites (s)
- Introduce empirical forms for halo occupation $N_{c/s}(M)$ and radial galaxy distribution.
- Compute galaxy power spectrum $P_{\rm gg}(k) = P_{\rm cc}(k) + 2P_{\rm cs}(k) + P_{\rm ss}(k)$



Zehavi et al. 2010

Halo + HOD model fits to luminosity bin galaxy samples in SDSS (w_p measurements offset for clarity).
To match precision of recent data, many extensions needed already…

Summary

- We developed a cartoon model of halo formation:
  - At early times, overdensities grow as per linear theory.
  - When the overdensity $\delta = \delta_c$, the overdensity undergoes gravitational collapse, resulting in a dark matter halo.
  - Assuming the final halo is virialized, we used conservation of energy to find the final radius of the perturbation.
    - Halos have a characteristic overdensity $\Delta \approx 200$.
- We can predict the abundance of halos!
  - To find the number of halos of mass M, split the Universe into spheres with mass M.
  - Calculate the fraction of spheres with $\delta_{\rm lin} > \delta_{\rm c} = 1.686$

Summary

We can use the halo model to describe the clustering of matter (and galaxies, other tracers) in the Universe.

- Halo bias can be described with ~5% precision using the peak-background split.
  - Start with the abundance function: $n(M|\delta_{\mathrm{c}})$
  - The effective threshold for collapse is $\delta_{\mathrm{eff}} = \delta_{\mathrm{c}} - \delta_{\mathrm{long}}$
  - $\dfrac{\delta n}{n} = -\dfrac{d\ln n}{d\delta_{\mathrm{c}}}\delta_{\mathrm{long}}$

Summary

Halo model:

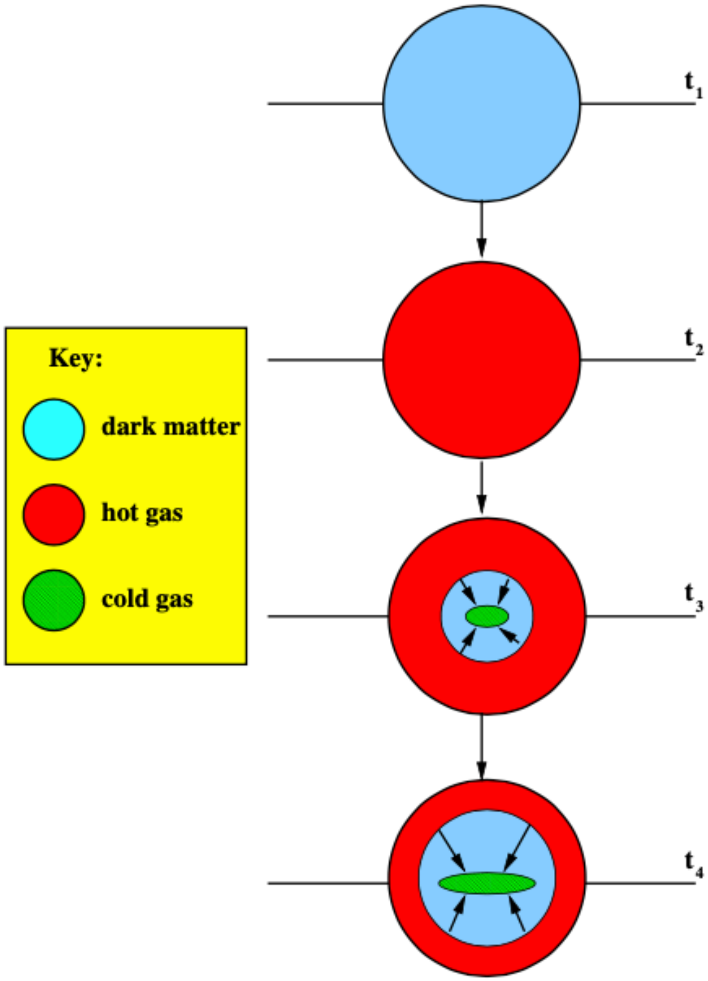
- Splits clustering into a 1-halo and a 2-halo term.
  - 1-halo term is described by the halo profile
  - 2-halo term is described using a linear bias model

$$P_{1-\mathrm{halo}}(k) = \frac{1}{\bar{\rho}_{\mathrm{m}}} \int dM \; n(M) M^2 u^2(k)$$

$$P_{2-\mathrm{halo}}(k) = \langle b \rangle^2 P_{\mathrm{lin}}(k) \qquad \langle b \rangle = \frac{1}{\bar{\rho}_{\mathrm{m}}} \int dM \; n(M) M b(M)$$

$$P(k) = P_{1-\mathrm{halo}}(k) + P_{2-\mathrm{halo}}(k)$$

Summary



t1: Baryons fall into the potential well of a dark matter halo

t2: Baryons are shock-heated to the halo temperature.

t3: The hot halo gas cools, and cool gas falls towards the halo center where it forms stars.

t4: The radius within which gas had sufficient time to cool increases with time.

(From Baugh 2007)