Anisotropy Basics: Randon Fields on the Sphere Basis for radan fields on the sphere: spherical => $f(\hat{n}) = \sum_{k=0}^{\infty} \sum_{n=-k}^{k} f_{\ell n} Y_{\ell n}(\hat{n}) = \sum_{k,n} f_{\ell n} Y_{\ell n}(\hat{n})$
Recall $Y_{\ell n}$ from $a.m. :$ eigenstates (in position space) of $\nabla^2 \chi_{\text{em}} = -l(l+1) \gamma_{\text{em}}$ 26220 $|m| \leq \ell$ $\partial_{\phi}Y_{\ell m} = im\gamma_{\ell m}$ Orthonometits: $\int d^{2}x \chi_{m}(\hat{a}) \chi_{m}^{*}(\hat{a}) = \delta_{\ell \ell} \delta_{mn'}$ Phase convertion: $\gamma_{en}^{*} = (-1)^{n} \gamma_{en}$ => If field is real (f(2) ER) => $f_{kn}^* = (-1)^n f_{l-n}$ Statistical isotropy: implication for 2-pt. comelator of $f_{\ell n}$? => Turns out that <fenter > = $\delta_{ll'}\delta_{m'}C_{l}$ $=7 C_2 \equiv$ angular power spectrum" of $f(x)$ Implization for 2-pt. correlator in position space? $\langle f(\hat{n})f(\hat{n}')\rangle=\sum_{lm}\sum_{l'm}\langle f_{lm}f_{lm}^{*}\rangle\chi_{lm}(\hat{n})\chi_{lm}^{*}(\hat{n}')$

 $C(e) = \langle f(\hat{\lambda})f(\hat{\lambda}') \rangle$
 $C(e) = \langle f(\hat{\lambda})f(\hat{\lambda}') \rangle$ $=\sum_{1}^{\infty}\frac{2\ell+1}{4\pi}C_{\ell}P_{\ell}(\cos\theta)$ Daddition theorem for sphersed hemond =>2-1+ correlation in real space => total variouse in field = $C(e=0) = 5\frac{u+1}{4\pi}C$ Using orthogonality of Pe(use), can show that $C_{\ell} = 2\pi \int d\mu s \rho (s) P_{\ell}(\mu s)$ N. B. if we consider $R > 24$ = $\frac{22+1}{4\pi} \approx \frac{2+1}{2\pi}$
and $\Sigma \rightarrow \int \frac{dI}{L} \cdot l = \int l \, dl$ \Rightarrow $\frac{2l+1}{l}C_{e}$ \rightarrow $\int dlnL C_{e} \frac{l(l+1)}{2\pi}$ => Def. $D_{\ell} = \frac{l(l+1)}{2T}$ (e => contribution per decode a ℓ to total various of the field.

Power spectrum intuition (slides) Why the power spectin ? CMB is extremely wellapproximated as a Gaussian randon field (GRF)
 $\hat{f}(\vec{x}) \equiv$ randon field with zero mean <f(2)>=0
Probability of some field configuration is a
functional of f(2) : $P[f(\vec{x})]$
RF=7 $Pr[f(\vec{x})]$ is a Gaussian functional of $f(\vec{x$ $f(\vec{x}) =$ randon field with zero mean <f(2)>=0 Probability of some field configuration is a functional of $f(z)$: $P[f(z)]$ $GRF \Rightarrow Pr[F(z)]$ is a Goussion functional of $f(\vec{x})$ Consider discretizing field f(2) in N pixels (voxels) Consider discreti 2.15 field $f(z)$ in N pixels (voxels)

=> represent as N-dim. vector $\hat{f} = [f(\hat{z}_1), f(\hat{z}_2), ..., f(\hat{z}_n)]^\top$

=> PDF for \hat{f} is a multi-variate Gaussian which is fully specified by the 2-pt. correlation function: f ¹ f_0 specified by 1
 $\langle f_t^2 f_0^2 \rangle = 3 (1 \dot{z}_i \vec{z}_j$ l $)$ = $\vec{\Sigma}_j$ $f_i = f(\vec{x}_i)$ $\langle f_{\hat{i}} f_{\hat{j}} \rangle = \frac{f}{\lambda} (\vec{x}_i - \vec{x}_j)$
 $f_{\hat{i}} = f(\vec{x}_i)$ $-\frac{1}{2} f_{\hat{i}} \frac{f_{\hat{j}} - f_{\hat{j}}}{f_{\hat{j}}}$ $M-dm.$ Vech

is a multi-

1. by the
 $\frac{1}{2}$ (1 \vec{x}_i - \vec{x}_j l)
 $-\frac{1}{2}$ f_i $\frac{1}{2}$ fi $\frac{1}{2}$
 $\sqrt{det(3_{ij})}$ $-\frac{1}{2}f_{\hat{i}}\xi_{\hat{j}}^{-1}f_{\hat{j}}$ \Rightarrow $Pr[F]\propto\sqrt{\sqrt{\det(3_{ij})}}$
Since $f(\vec{k})$ is thear in $f(\vec{k})$ Hear in $F(\vec{x})$, $Pr[F(\vec{k})]$ is also a
Gonssion: $Pr[F(\vec{k})] \propto e^{-f^2(k)/2P_f(k)}/\sqrt{\det(P_f(k))}$ $m\nu H - varizte forssion: Pr [f(k)] \propto e^{-f^2(k)/2P_f(k)}/\sqrt{\det(P_f(k))}$ Since different Fourier modes are uncorrelated, they are statistically independent for GRFs .

Relevance to cosmology/CMB: - Inflation predicts initial perturbations are very close to Goussian (as required by actual data) - Linear evolution preserves Gaussianity - Linear evolution provertes non-Gaussianity (NG) - Non-linear eventua geventes non-vooringzy voor)
- Searching for primodial NG is a very active research area Goals for next ~2 lectures: develop inderstanding - searching to primarial NG is a vag agive reason of
als for next ~2 lectures: develop understanding
of the physics underlying the CMB angular of the physics underlying the CMB argular
power spectrum -Inflation pressures minimally of the fourth of the second several of the physics underlying
- Searching for primoted NG is
Goals for next ~2 lectures:
of the physics underlying
Power spectrum
The Inhomogeneous Universe
C The Inhomogeneous Universe Conformed Newtonian gauge metric: $\Phi = \Phi(r, \vec{x}) = N$ eut. potenti 10^2 Newtown 3^{2} metric. $2^{2} = 2^{2}$
ds² = al(2)(-(1+2 Φ)dg² + (1-2 Φ)dz²) Consider photon propagation along geodesic as above: define λ via $p^*=\frac{dx^{\mu}}{d\lambda}$ (hoton Important difference ^w.r.t . homog . case : the Tenegy E measured by an observer in their local inertial frame nor differ from p^o! In general, p^r components are def. in coord. Frame while physically we care about pr' reasured in local inertial frame of observer $(w!)$: $P^{\mu} = (E_{\mu} p^{i})$ at $P^{\mu} = (p^{\mu}, p^{i})$.). Franc ultile physically we concerned in local intitial frame.
reasonal in local intitial frame.
): $P^{\mu} = (E, \rho^i)$ and $\rho^{\mu} = (\rho^i, \rho^i)$. These are related via:

 $ds^2 = ds^2$ (invariat interval) \Rightarrow $\gamma_{\mu;\nu}$, $p^{\mu}e^{\nu'} = g_{\mu\nu}p^{\mu}p^{\nu}$. Take obs. to be Take obs. to be
at rest and
on'estation of Minksuski
Minksuski
Surte Good , systems => - $E^{2} + \delta_{ij} p^{i'} p^{j'} = g_{oo} (p^{0})^{2} + g_{ij} p^{i} p^{j}$ to align $=$ $E = \sqrt{-3.0}$ $p^{\circ} = \sqrt{a^2 (1+25)}$ p° \approx $a(1+\Phi)p^{\circ}$ $V=3.0 \text{ } \rho^2 = \sqrt{\frac{\lambda^2 (1+25)}{\rho^2}} = \frac{\alpha (1+5)}{\alpha (1+5)}$
 $\Rightarrow \rho^0 = \frac{E}{\alpha (1+5)}$ $\Rightarrow \rho^0 = \frac{E}{\alpha} (1-5)$ $\left(\Phi \right)$ and $p = \frac{1}{2} (1 + \frac{1}{2}) \hat{p}$ is not nector in propagation direction Note: we could also intritively "gress" the p° result ote: we could also intritively "gress" the p° result
by noting that I looks like a local perturbation of the scale factor: $\tilde{a}(n,\vec{x}) = a(n)(1 + \Phi(n,\vec{x}))$ Now use these results in the geodesiz eg. to determine evolution of E : Note (like before) that $\frac{dp^{\circ}}{d\lambda} = \frac{dp^{\circ}}{d\lambda} \frac{dp}{d\lambda} = \frac{dp^{\circ}}{d\lambda} \frac{dx}{d\lambda} = \frac{dp^{\circ}}{d\lambda} \frac{dx^{\circ}}{d\lambda} = \frac{dp^{\circ}}{d\lambda} \frac{dx^{\circ}}{d\lambda}$ Leodesic eg .: + $\Gamma^{\circ}_{\mu\nu}$ ρ^{μ} ρ^{ν} = 0 $e q$.: $\frac{1}{11} + \frac{1}{11} - \frac{1}{11} + \frac{1}{11} = \frac{1}{11} + \frac{1}{1$ $\Rightarrow \frac{d\rho^{\circ}}{d\eta} + \frac{1}{\rho^{\circ}} \int_{A\nu}^{\rho} \rho^{A} \rho^{\prime} = 0$
 $\Rightarrow \frac{d\rho^{\circ}}{d\eta} + \Gamma^{\circ}_{\circ} \rho^{\circ} + 2\Gamma^{\circ}_{\circ} \rho^{\circ} + \frac{1}{\rho^{\circ}} \Gamma^{\circ}_{\circ} \rho^{\circ} \rho^{\circ} = 0$

Exercise: compute $L^{\circ}_{\mu\nu}$ for this metric, to obtain the $\frac{\partial}{\partial x}$ rise: compute $L^{\circ}_{\mu\nu}$ for this metric,
following: Here $W \equiv \frac{d\alpha/d\alpha}{\alpha} = \frac{\alpha}{\alpha}$ Exercise: compute \vec{r} in for this metric, to obtain the
following: Here $\vec{r} = \frac{dn}{n}$
= $\frac{dp}{dr} + (u + \partial_{\eta} \vec{q})p^{\circ} + (2\partial_{\xi} \vec{q})p^{\dot{\iota}} + (-3u + \partial_{\eta} \vec{q})d_{\dot{\iota}j}$ \vec{r}
is \vec{r} or $= \frac{F}{n}(1-\vec{q})$ and $p^{\dot{\$ $\Rightarrow \frac{d\rho^o}{d\eta} + (\gamma + \partial_{\eta} \Phi) \rho^o + (2 \partial_{i} \Phi) \rho^i + (-3\gamma + \partial_{\eta} \Phi) \delta_{ij} + \frac{\rho^i \rho^j}{\rho^o}$
Using $\rho^o = \frac{E}{\rho} (1 - \Phi)$ and $\rho^o = \frac{E}{\rho} (1 + \Phi) \rho^o$ here and keeping terms to first order (Ex: do this), we obtain $\frac{dE/d\eta}{E} = -H$ + (23.9)
and p^2
first ed
fit dg de
- H + dg de
delighted) $+\sqrt{2\pi r}$

and $\rho^2 = \frac{E}{\pi} (1 + \frac{\pi}{4}) \hat{\rho}^2$

first ender $(\varepsilon_x : d_0 \text{ the})$
 $-\frac{H + \partial_2 \Phi}{\partial \rho} - \hat{\rho}^2 \partial_2 \Phi$ to (potated) expansion redshifting \Rightarrow $\frac{dE/d\eta}{E} = -H + \frac{\partial_2 E}{\partial_2 E}$
 $\frac{dh(aE)}{dq} = \frac{\partial_2 E}{\partial_2 E} - \hat{p}^2 \partial_2 E$ (To obtain this, = $\partial_1 \Phi - \hat{p}^2 \partial_2 \Phi$
also used $\frac{dF}{d\eta} = \partial_1 \Phi + (\partial_2 \Phi) \frac{d\mu}{d\eta}$
= $\partial_4 \Phi + (\partial_3 \Phi) \hat{p}^2$ $a_1\overline{b}$ + (e: s)
 $a_2\overline{b}$ + (e: s) $= \partial_{\hat{\imath}} \, \overline{\mathcal{L}} + (\mathfrak{z}, \overline{\mathfrak{z}}) \, \hat{\mathfrak{e}}^2 \, \Big)$ The grav. redshifting term can be rewritten as: \Rightarrow => $\frac{dh(aE)}{dq} = \frac{\partial q}{\partial x}$

=> $\frac{dh(aE)}{dq} = \frac{\partial q}{\partial x}$

=> $\frac{dh(aE)}{dq} = \frac{\partial q}{\partial x}$
 $\frac{\partial^2 q}{\partial x^2} = \frac{d q}{dq} - \frac{\partial q}{\partial x}$
 $\frac{dh(aE)}{da} = 2 \partial q \Phi$ $\frac{dh(aE)}{dq} = \partial_1 E - \hat{p}$
 $dH^{\perp\perp}$ the drop used
 $g_{\overline{c}W}$. redshiftly term
 $\partial_{\hat{c}} E = \frac{dF}{dq} - \partial_1 E$
 $dH = 2 \partial_1 E - \frac{dF}{dq}$
 $dH = 2 \partial_1 E - \frac{dF}{dq}$
 $dH = 2 \partial_1 E - \frac{dF}{dq}$ Generalization of $G^2 \frac{1}{a}$ to perturbed minere Integrate: (from n_e to n_o) $=$ $\frac{1}{2}$ $\ln(a_0E_0) - \ln(a_0E_0) = \underline{\Phi}_e - \underline{\Phi}_o + 2\frac{\Phi_0}{2}$ Recall $a_0 = 1$ 7e

Note that $\Phi_o=$ local gravitational potential, which can only affect the nonopole $(l=0 \mod d)$ and is here de that Φ_o = local gravitational patential, which is
only affect the nonopole (l=0 mode) and is her
unobservable and con be set to zero WLOG. Note: perturbation grantities here can be evaluated at the unperturbed last-scattering time (η_*) , since unobservable and con be set to zero blog.
Note: pertubation grantities here con be evaluated
at the unperturbed last-scattering time (η_*), since
corrections would be seemd order. \Rightarrow $ln(a_{o}E_{o}) = ln(a_{*}E_{*}) + \Phi_{*} + 2\int_{a_{*}}^{a_{o}}d\eta(a_{0}\Phi)$ For photons, note that $E\propto T$ and that the distribution function (Base-Eistely) is only a function of $\frac{\varepsilon}{T}$, thus: $aE \propto aT \propto a(T+\Delta T) \propto aT(1+\frac{\Delta T}{T})$ \Rightarrow $ln(aE) = ln(aT(1 + \frac{AT}{T})) + cwt.$ = $ln(n\bar{T}) + ln(1 + \frac{\Delta T}{\bar{T}}) + ln(\frac{1}{2} + \frac{\Delta T}{\bar{T}})$ + cost.
 $ln(n\bar{T}) + ln(T - \frac{\Delta T}{\bar{T}})$ Note that $\Phi_s = \text{load}$ graphofology prioritional prioritional control only established and consider the section of the control of th $= ln (aT) + \frac{15}{5} + cwt.$ Alternate derivation that clarifies connection to local perturbation in scale factor : $S_{tr}art from $\frac{a_0E_o}{a_0E_a}$$ = - $\frac{1}{e^{\frac{1}{2}}}\int_{0}^{\pi} \frac{1}{e^{2}} \, d\theta \left(\frac{\partial}{\partial \theta} \Phi \right)$

Taylor expand RMS (first order in perturbations) : Taylor expend RHS (fix
=> $\frac{E_{\rm e}}{R_{\rm e}E_{\rm e}}$ = 1- $\Phi_{\rm e}$ + $\Phi_{\rm e}$ + $\frac{E_{o}}{a_{e}E_{e}}$ = 1 - Φ_{o} + Φ_{e} + 2 $\int_{r_{e}}^{r_{o}} dr$ ($\partial_{r} \Phi$) Note that $\Phi_o=$ local gravitational potential, which can only affect the nonopole (R=0 mode) and is here de that Φ_o = local gravitational patential, which is
only affect the Monopole (R=0 mode) and is her
unobservable and con be set to zero WLOG. => E. = $a_{e}E_{e} (1+\Phi_{e}+2\int_{a_{e}}^{a_{e}}d\eta(\theta,\Phi))$
Note: pertubution grantities on RHS can be evaluated
ot the importanted last-scattering time (η_{*}) , since
corrections would be seend order. Not true for a. Note : perturbation quantities on RHS can be evaluated at the unperturbed last-scattering time (γ_{*}) , since corrections would be
=> $E_o = e_{e}E_{*}$ (1+ E_{*} + $2\int_{\gamma_{*}}^{\gamma_{o}}d\gamma$ ($\partial_{\gamma}\overline{\Phi}$) For photom, note that EXT, so we have \Rightarrow T. $=$ acT. (1+ Φ_{+} + 2 $\int_{\gamma_{+}}^{\gamma_{0}} d\gamma \, (\partial_{\gamma} \Phi)$) $=5F_{0}+DT_{0} \approx (a_{*}+Da)F_{*}(1+E_{*}+$ $(3, 4)$
2 $\int_{\gamma_{*}}^{\gamma_{0}} d\gamma \, (\partial_{\gamma} 4)$ $T_s = (a_* + \Delta a) T_* (1 + \cancel{b}_* + 2)$
 $\approx a_* T_* (1 + \cancel{b}_* + 2) T_* d_2 (a_2 \cancel{b})$
 $+ T_* \Delta a_*$
 $+ T_* \Delta a_*$

Readl that \overline{T} of $\frac{1}{4}$ so \overline{T} = \overline{A}

 $\Rightarrow \Delta T_e \approx a_{\star} \overline{T}_{\star} \left(\overline{\Phi}_{\star} + 2 \int_{q_{\star}}^{q_{\star}} d\eta \left(\partial_{\gamma} \overline{p} \right) \right) + \overline{T}_{\star} \Delta q_e$ $\Rightarrow \frac{\Delta T_{o}}{T_{a}} = \mathcal{I}_{a} + 2 \int_{\gamma_{a}}^{\gamma_{o}} d\gamma \, (\partial_{\gamma} \vec{\Phi}) + \frac{\Delta a_{c}}{a_{*}}$ What is Δa_2 ? Must be determined by the local
feng. being ~0.3 eV (- \overline{T}_f). $\Rightarrow (\overline{f}_r + \overline{f}_r)(r_{r} + \Delta r_{r}) = \overline{f}_r(r_{r}) = \sigma T_r^{\gamma}$ $\frac{d}{dt}$ => $\bar{J}_r(t_{1+}) + \bar{S}_{fr}(t_{1+}) + \bar{J}_r' I_{n_{+}} b_2 = \bar{J}_r(t_{1+})$ => $S_{3r}(q_{4}) = -\overline{g}_{r}'|_{q_{4}} Dq \Rightarrow \Delta q = -\frac{S_{r}(q_{4})}{\overline{J}_{r}'|_{q_{4}}}$ => $\Delta q = \frac{-\delta f_c}{\frac{\Delta f_c}{\Delta a}} = \frac{-\delta f_c}{\delta_{r_o}(-4a^{-5})a'} = \frac{\delta f_c}{\delta_{r} \cdot 4(\frac{a'}{a})} = \frac{\delta_r}{4H}$ where $S_r \equiv \frac{S_f r}{S_r}$ as usual. Thus: $a_{*}b_{k} = a_{*} + a_{*}b_{k}$ => $\frac{a_{2}}{a_{*}} = (1 + \frac{a_{*}'}{a_{*}}(\frac{\delta^{*}}{4\pi}))$ $\frac{a_{*}+b a_{e}}{a_{*}}=1+\frac{\delta_{r}^{*}}{4}$ $\Rightarrow \frac{a_e}{a_*} = 1 + \frac{f'_r}{4} \Rightarrow$ $\Rightarrow \frac{\Delta a_e}{a_*} = \frac{\delta f^*}{4}$

 $\Rightarrow \frac{\Delta I_{0}}{T_{0}} = \bar{L}_{+} + 2 \int_{\gamma_{+}}^{\gamma_{0}} d\gamma \, (\partial_{\gamma} \bar{L}) + \frac{\delta_{i}^{+}}{4} \sqrt{\frac{2}{\gamma_{+}^{+}}}$ \Rightarrow shows immediately that $\frac{\Delta T_{\cdot}}{T_{\cdot}} = \frac{\delta r^*}{T}$ (by comparison to our result above) In fact, in full generality there are two contributions to ΔT_f $dy \frac{dy}{dt}$
that
result
garealis
 $\frac{\Delta T_{f}}{T_{f}}$:
 $\frac{\Delta T_{f}}{T_{f}}$: 1) Rediation desity perturbations: <> local section factor J_r d T'' = J_{r} d $4\overline{T}^{3}\overline{\delta T}$ d $4\overline{T}^{4}\overline{T}^{2}$ d $4\overline{T}^{5}$ \Rightarrow $\frac{5T}{T} \propto \frac{5R}{4g}$ $=$ $\frac{1}{x}$ $\frac{1}{y}$ $\frac{1}{r}$ $\frac{1}{r$ B ut physically I think more webel to think in \Rightarrow $\frac{5T}{T}$ a $\frac{5r}{15}$ \Rightarrow $\frac{5T}{T}$ a $\frac{5r}{1}$
 \neq physically I think now weted to think
terms of local perturbation to scale factor, since the local photon tespective at decoply \int_{C} \int_{T} $\frac{1}{16}$ ~ 0.25 eV always : $\frac{\Delta a_{*}}{a_{*}} = \frac{\Delta r}{4} = \frac{5T}{7}$ => paints of higher so (positive do) reach 0.25 eV $|iter|$, heree CRB last-scatter later (thus with larger a) , thus redshifting less to today, and thus yielding positive observed temp. fluctuation (and vice versa for regative dr). => surface of last scatterly is "wrinkled" (in terms of a) [in Newtonian garge! this is men prim J a statement

2) Doppler contribution: perturbations in T sourced by bulk velocities of e^- at last scattering $=$) e^- noving toward us: positive $\frac{dT}{T}$ e noving trust s. \int_{0}^{∞} away from us: regative $\frac{5T}{T}$ Let $\hat{n} \equiv \dot{m}$ t vector pointly from us to point on last-scatterly surface $=$) e^{-} novits
 e^{-} "
 $=$ "
 e^{-} "

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 $=$ "
 e^{-} "

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 $=$ "
 $=$ $\sqrt{2}$
 $=$ $\sqrt{2}$
 $=$ $\sqrt{2}$ is moving in $-n$ direction) 5 \Rightarrow $\frac{\delta \Gamma}{\delta} = \vec{v_e} \cdot (-\hat{n})$ $(\hat{v_i})$ Note that $\vec{v_e} \approx \vec{v}_b$ since plasma is tightly \Rightarrow $\frac{\delta T}{\delta} = -\hat{n} \cdot \vec{v}$ (This can also be derived from first principles via Boltzman approach .) Note that $\vec{v}_e \approx \vec{v}_h$ since plusing is the
 $\Rightarrow \vec{v}_h = -\hat{n} \cdot \vec{v}_h$
 $(\vec{v}_h \cdot \vec{v}_h \cdot \vec{v}_h \cdot \vec{v}_h)$
 $(\vec{v}_h \cdot \vec{v}_h \cdot \vec{v}_h$ $\frac{\Delta T_{o}}{T_{o}} = \bar{\mathbf{L}}_{r} + 2 \int_{\gamma_{*}}^{\gamma_{o}} d\gamma \, (\partial_{\gamma} \bar{\mathbf{L}}) + \frac{\delta_{r}^{*}}{4} - (\hat{n} \cdot \vec{v}_{b})_{*}$ Interpretation : $1)$ I_* : gravitational redshift of C^{18} r as they climb out of potentials at last scattering $clmb$ out at potentials at last
 $CE_{\star} < 0 \Rightarrow m$ matter overleasing $\Rightarrow -\delta T/T$ > 0 \Rightarrow matrice overally \Rightarrow + $\delta T/T$)

 $2)$ $\frac{\sqrt{5}}{4}$: intrinsic temp. parts. (es scale factor perts.) 2) $\frac{5x}{4}$: intrinsic temp. puts. (as scale factor puts.)
=> the combination $S_t \equiv \Phi_t + \frac{5x}{4}$ is called the Sachs-Wolfe term [this combination is gargethe
arge-
<u>Mand</u> independent] $3)$ $2\int_{9}^{9} dr \, (3\pi \bar{B})$: "integrated Sochs-Wolfe" term \Rightarrow if $\partial_1 \Phi \neq 0$, e.g., dre to dark energy or radiation domination, then this ferm is non r distin derivation, the This part is $-DE$ generates this at $z \leq 1 \implies$ late ISW Potentials decaying due to $DE \Rightarrow \Phi$ $\overline{\Phi}$ ' 70 = + $\delta T/\overline{T}$ in collapsed regions 4) - $(n \cdot \vec{v}_b)$: Doppler term (I' < 0 in voids =? 5F/F) e^- velocity toward us => + ST/\tilde{T} $\frac{1}{2}$ velocity toward us => +07/T Notes : - The S-W and Doppler terms are 3D fields evaluated at last scattering (at time η_* and $(n.\vec{v}_b);$ Doppler term
 e^- relation tournal we
 \therefore The S-W and Dopposition (2-21) is

position (2-21) is

distance to lest where $\eta_o - \eta_* =$ χ_* = 14 Gpc is the distance to last scattering (in flat minere)) Which contributions dominate? Let's focus on large scales $(z \approx 100)$: · s rocks on the sence (x ----).
ISW generally small since universe has been matter-dominated for most of CMB-releast hitag

· Doppler small since \vec{v}_b varishes on superhoritor => S-W will dominate What term wins in $S-W$? s-w un dernaux
at term uns de 5-W?
Syerhorizon limit: -25 = $\delta_n = \frac{3}{4}\delta_r \Rightarrow \delta_r = \frac{3}{3}\Phi$ \Rightarrow $(\frac{5}{4} + \overline{1})_{+} = (-\frac{2}{3} \overline{1} + \overline{1})_{+} = \frac{\overline{1} + \overline{1} + \$ => grar, redshift term $=\frac{1}{5}R \quad \left(\frac{\gamma e c d l}{2-\frac{3}{5}R}\right)$ $y = (\frac{26}{9} + \frac{1}{2})_+ = (-\frac{2}{3} + \frac{1}{2})_+ = \frac{24}{3}$
 $y = \frac{2}{3}R$ $(\frac{2}{3} - \frac{3}{3})_+$
 $y = \frac{2}{3}R$ $(\frac{2}{3} - \frac{3}{3})_+$
 $y = \frac{24}{3}$
 $y =$ $=$ > overdensity at lest scattering (\int_{r}^{*} >0, \int_{r}^{*} >0, $y:abb - \frac{\delta T}{T}$ (cold spot) in CMB, $\Phi_{*})$ and vize versa for underdensity => these are the hot and cold spots you see by eye is eye in full-sky CMB maps (Plack, WAAP)
=> inage of grav, field at last seattenig! => inage of graw. field at last seatteng!
Initial Conditions Inflationary theories predict statistics of the by eye in tull-sky LMB Angr (
=) inage of grow, field at last
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conoving curvature perturbation R.
conoving curvature perturbation R. consuitg curvature perturbation R.
R is constant on superhorizon scales, thus allowing us to correct inflationary physics (when Forier modes go outside the horizon) to late-time observables (after modes re-enter).