Schematic to inderstand effect on CMB temp. power spectrum: consider function  $f(x) \propto cos(x)$  and its square  $f^2(x) \propto cos^2(x)$ , as<br>well as shifted function  $(f(x)-\Delta)$  and its square  $(f(x)-\Delta)^2$ well as shifted function  $(f(k)-\Delta)$  and its square  $(f(k)-\Delta)^2$  $f - \Delta$ ↑  $\frac{\text{shift}}{8}$ chenatic to indextand et<br>consider finction  $f(x) \propto \cos(x)$ <br>well as shifted forching<br>f<br>for the scaler (at decay)<br>int consider evolution<br>bsence of baryons ( 2 and its square f '(x) x cas' (x), x<br>
and its square f '(x) x cas' (x), x<br>
(x) -  $\Delta$ ) and its square (f(x) -  $\Delta$ )<sup>2</sup><br>
f -  $\Delta$  and its square (f(x) -  $\Delta$ )<sup>2</sup><br>
f -  $\Delta$ <br>
(x) -  $\Delta$ <br>
(x) -  $\Delta$ <br>
(x) =  $\Delta$  and  $\Delta$  and  $b - \Delta$  $S_{\text{mod}}$  scales (at decoy(ing):  $k > k_{eg}$ First consider evolution of rad. pets, in the First consider evolution of rad. pots. in the<br>absence of bayons (in the rad. -don. era).  $\rho_{oisson}: \quad \nabla^2 \Phi = \frac{3}{2} H^2 \Delta_r \qquad (\Delta_r \equiv \omega m \omega n g_{\omega} d\omega r F)$  $\Rightarrow$   $\Delta_r(\eta, \vec{k}) = -\frac{2}{3}(k\eta)^2 \Phi(\eta, \vec{k})$  using  $\mathcal{H} = \frac{1}{\eta}$  dry  $U_0 = \sum_{r=1}^{n} (r_1, k) = -\frac{1}{3} (k_1) 0 (r_1, k)$  with  $W =$ <br>Use our sol. for  $D$  during rad. don. to obtain.  $s$ e ov $\Delta$ r(n,  $|s-1|$  for  $\Phi$  during rad don. to obtain:<br> $\vec{k}) = -4R_c^k \frac{sin(kn\sqrt{s}) - (kn\sqrt{s})cos(kn\sqrt{s})}{(kn\sqrt{s})}$  $(kn/\sqrt{s})$  cos  $(kn/\sqrt{s})$  valid  $\sqrt{(k_1/\sqrt{3})}$  scales

 $Super-horiton$  limit:  $km < 1$  $s$ -horizon limit:  $k_{1}$  >>1  $-\frac{v}{2}R_{i}^{k}(k_{1})^{2}$  super-horizon  $\Rightarrow$   $\Delta_r(n,\vec{k}) \approx$  $\begin{cases} \frac{1}{\sqrt{2}} & \text{if } k \leq 1 \end{cases}$  sub-haiton  $=$  $A_r$ d  $\eta^2$ da<sup>2</sup> on super-horizo scales (De oscillates with constant amplitude on sub-hoiten corresponds ↓ Or = -EMs on super-horizon scaly ↳Newtonian grange Now consider the matter-dom. era:<br>Pertoted cant. eq.:  $S_r' = -\frac{p}{3}\vec{v}\cdot\vec{v}_r + 4\vec{B}$ Perturbed cont.  $N_{\bullet\bullet\bullet}$  consider the matter -<br>Perturbed cont. eq. :  $\delta_{\bullet} = -\frac{1}{3}$ <br>Perturbed Enter eq. :  $\overrightarrow{v}_{\bullet} = -\frac{1}{3}$ Perturbed Enler ez. :  $\vec{v}_r = -\frac{1}{7}\vec{P}\hat{s}_r - \vec{P}\vec{P}$  $=$   $S_1$ " -  $\frac{1}{3}$  $S^2$  $S_r$  =  $\frac{1}{3}$  $S^2$  $\cancel{E}$  +  $4\cancel{I}$ "  $MD: grows$  mode sol. for  $F =$  is  $F =$  const. on all scales MD: growing made sol. for  $9$  is  $9 =$  const. on all seal<br>=>  $5'' - \frac{1}{3} 9^2 f_r = \frac{4}{3} 7^2 f =$  const. =>  $5'' + \frac{1}{3} k (5 + 49) = 0$  $\Rightarrow$   $S_{r}$ " -  $\frac{1}{3}$   $S^{2}$   $S_{r}$  =<br> $\Rightarrow$  EOM of a S.H.  $P$   $E$ OM of a  $S.H.o.$  with a constant driving force Thus, sub-horizon perts. In rad. density ascillate with a => EOM of a S.H.O. with a constant driving force<br>Thus, sub-hoizon puts. In rad. density ascillate with a<br>invitant anplitude around an offset equilibrium point, constant amplitude around an affect equilibrium point,<br>given by -4 $\underline{\Phi}_o(\vec{k})$ , where  $\underline{\Phi}_o(\vec{k}) \equiv$  amplitude of potential in MD. given by  $-\Psi \Phi_o(\vec{k})$ , where  $\Phi_o(\vec{k}) \equiv$  anglitude of potential in 10.<br>Note that  $\Phi_o(\vec{k})$  is  $\vec{k}$ -dep, due to ILs and the non-trivial scale dependence of growth during RD era ("transfer function").  $\Rightarrow$  sol.:  $S_r(n, \vec{k}) = C(\vec{k}) cos(\frac{kn}{\sqrt{3}}) + D(\vec{k}) sin(\frac{kn}{\sqrt{3}}) - 4\vec{e}_o(\vec{k})$ 

On deep sub-horizon scales, Dr <sup>=</sup> dr  $(A_r = \frac{1}{8} (S_f + v_r \overline{S_r}'))$ and modes all entered during rad. dow., and by<br>decoupling  $\Phi$  had fully decayed away, so sol. is  $S_r(r, \vec{k}) \simeq 4R_{\tilde{c}}(\vec{k}) \cos(\frac{k_1}{\sqrt{3}})$ Sol. on intermediate scales is obtained by addy the carst. potential affect above (or we also fund warlier).  $S-W$ :  $S \equiv \frac{dr}{y} + \Phi$  $F=0$  on very small scales at decoupling  $S \simeq \frac{5\pi}{9}$  ("radiation dividig" booting applitude  $\frac{1}{6}$ perts . don't so  $S \approx \frac{3}{7}$  (religion driving pooring  $r$  in have to fight gahst grow· potential  $\Rightarrow$   $S_r(r,\vec{k}) \approx R_c(\vec{k}) \cos(kr_s(r))$  a the bouce back a thos bounce back  $\frac{50}{50}$ <br> $\frac{5 \text{ of the } i}{500}$ Synthesis :  $S(\vec{k}, \gamma^*)$ ) =  $\begin{cases} \frac{1}{5}R_i(\vec{k})((1+3R)\omega_s(kr_s^*)-3R) \\ R_i(\vec{k})\omega_s(kr_s^*) \end{cases}$ ,  $k$ <sup> $\ll k$ </sup>eq , krke  $\equiv T_s(k,t)R_c(\vec{k})$  $S(\vec{k}, \gamma^*) = \begin{cases} 5k_0(k)(1+3k)\omega_1(k\gamma_1)-3k) & k \le k_0, \\ R_c(\vec{k})\omega_2(k\gamma^*) & k > k_0, \end{cases}$ <br>=  $T_s(k_1\gamma^*) R_c(\vec{k})$ <br>=  $\begin{cases} \frac{1}{5}((1+3k)\omega_3(k\gamma^*)-3k) & k \le k_0, \\ \omega_3(k\gamma^*) & k > k_0, \end{cases}$  $\begin{cases}\n\frac{1}{5}((1+3R)\cos(kr_5^+) - \cos(kr_5))\n\end{cases}$ 1 kee kee  $cos(kr_s^+)$  , k  $n = k_e$ 

Danping : Diffusion

Danping : Diffusion<br>Important feature not captured in our sinple analysis: "diffusion damping" ("Silk damping")<br>=> arises because & diffuse out of high-density regions into low-density regions, which was<br>which reduce<br>scales (4) reduces of density contrast)  $\Rightarrow$  suppresses  $\delta$  on very small scales (high k)  $\Rightarrow$ treatment requires going beyond tight-coupling approx. Sarises because of diffuse and of high-density relations (which related by (dustry)<br>into low-density regions, which relates by (dustry)<br>and present requires going beyond tright-coupling approx.<br>Appox.: I mean free path  $R$  $\Rightarrow$  number of scatterize = dm/lp = N  $\Rightarrow$  number at scorrengs = mean free path  $l_p = \frac{1}{\alpha n_e r_p} = |r|^{r-1}$ <br>
sutterigs = dr /  $l_p = N$ <br>
sutterigs = dr /  $l_p = \sqrt{\frac{M}{l_p}} l_p \approx \sqrt{d_1 |\tau'|^{-1}}$ <br>
= Lo<br>
scale :  $h^{-2}$  and  $r = \sqrt{n_e}$ ,  $1 \sqrt{1 - \frac{1}{n_e}}$ => damping scole :  $k_0^{-2} \approx L_0^2 = \int_0^{n_x} dq \, |z'|^{-1}$  $Result: k_0^{+1} = 7$  Me = 0 =  $k_0^{+}$  $R_0^*$  sente ·  $R_0^* \approx L_0 \approx \int_0^{\pi} dq |q'|$ <br>  $\therefore$   $R_0^{*-1} = 7$  Mpc  $\Rightarrow$   $R_0 \approx \frac{k_0^{*-1}x_*}{\sqrt{2}}$ \*- \*  $= 7$  Mpc  $\Rightarrow \ell_{p} \approx \frac{k_{0}^{2} + \gamma_{p}}{\sqrt{2}} \approx$  $\approx$  1400 VE => really a contation of thermal conduction and photon viscosity => really<br>Planging :<br>Decompl Danping: Landau Decoupling is not actually instantaneous For high-k modes, the dwartion of recombination is comparable to the fluctuation wavelength Goussion approx, to width of visibility function  $s(q)$ :<br>  $\sigma_3 \simeq 16$  Mpc Have to average S-W (and other) source terms over  $th3$  finite with  $\Rightarrow$  leads to darping of high-le modes

 $\Rightarrow$   $k_t^{-1} \approx \frac{c_5^+ \sigma_3}{\sqrt{c}} = \frac{\sigma_3}{\sqrt{6(1+\rho_*)}} \approx 5$  Mpc Combine Silk + Londan:  $\Rightarrow$   $k_{\text{slow}}^{* - 1} = \sqrt{k_{\text{0}}^{* - 2} + k_{\text{C}}^{* - 2}} = 9$  Mpc -1  $\Rightarrow$   $k_{c}^{2} \approx$ <br>  $\Rightarrow$   $k_{cm}^{*}$ <br>  $\Rightarrow$   $k_{cm}^{*}$  $v_{\text{day}} = \frac{k_{\text{day}}^2 \chi_{\mu}}{\sqrt{2}} = 1100$ Include in transfer function:  $\int_{S}$   $(k,\eta)$ \*  $\Rightarrow k_{t}^{-1} \approx \frac{455}{12} = \frac{5}{\sqrt{6(118.5)}} \approx 5$  Mpc<br>
Carline Silk + London:<br>  $\Rightarrow k_{t-1}^{*1} = \sqrt{k_{t-1}^{*1} + k_{t-2}^{*2}} \approx 9$  Mpc<br>  $\Rightarrow k_{t-1}^{*1} = \sqrt{k_{t-1}^{*1} + k_{t-2}^{*2}} \approx 9$  Mpc<br>  $\Rightarrow \int_{\frac{1}{2}\sqrt{k_{t-1}}} \frac{k_{t-1}^{*1}x_{t}}{\sqrt{k_{t-1}}} \approx 1100$ i VI = 100<br>
in fransfer function:<br>  $= e^{\frac{hc^2}{kT}} \sum_{cs(kr_s^+)} \frac{1}{s} ((1+3R)cos(kr_s^+)-3R)$ , k  $x = k_{eg}$ <br>  $\sum_{cs(kr_s^+)}$ , k  $x = k_{eg}$  $\Rightarrow$  (appointed)  $S-W$  transfer function  $I_s(R_1r) = e$ <br>  $\Rightarrow$  (appointed)<br>
Revonization :<br>
Consider  $\gamma$ Consider 8 coming from some direction with temperature  $T = \bar{T}(1+\frac{\Delta T}{T}).$  $T = T(1 + \frac{1}{T}).$ <br>They scatter off free  $e^-$  during reionization: only  $e^+e^-$ <br>get through to us. set through to us.<br>A fraction (1-e<sup>-c)</sup> get scattered into this line of sight from all other directions (photon runber sight from all other directions (photo runker have temperature F on average.  $\Rightarrow$  results  $\top$  we see =  $\overline{T}(1+\frac{\Delta T}{T})e^{-T} + \overline{T}(1-e^{-T})$ 

=  $\overline{T}(1 + \frac{\Delta T}{T}e^{-T}) = T_{obs}$ => Me observe anisotrogy  $\frac{\Delta T}{T}e^{-T}$ <br>=>  $C_z^{TT}$  is suggessed by  $e^{-2\tau}$  for all modes with the horizon during reinvisorfien (e 250)