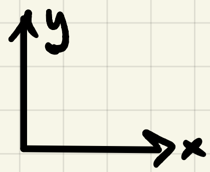
$$= \bar{T}\left(1 + \frac{\Delta T}{\bar{T}} e^{-\tau}\right) \equiv T_{obs}$$

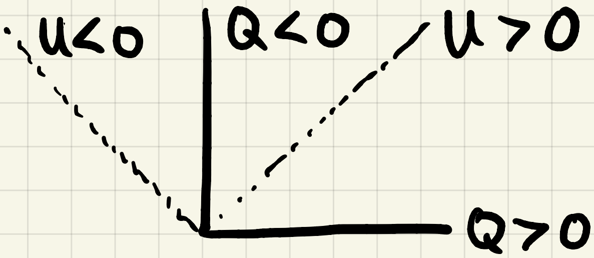
$\Rightarrow$ We observe anisotropy $\frac{\Delta T}{\bar{T}} e^{-\tau}$

$\Rightarrow$ C_ℓ^{TT} is suppressed by $e^{-2\tau}$ for all modes within the horizon during reionization ($\ell \gtrsim 50$)

CMB Polarization: E and B



Recall Stokes Q and U:



Compact representation: $Q \pm iU$

Rotate coord. system by ϕ:

$$Q \pm iU \longrightarrow \tilde{Q} \pm i\tilde{U} = e^{\mp 2i\phi}(Q \pm iU)$$

$\Rightarrow$ Q, U are coord.-dep. $\Rightarrow$ not good for all-sky CMB pol. analysis

Fix: define coord.-indep. quantities E, B

Most easily done in the flat-sky approx.:

Expand $Q + iU$ in Fourier basis:

$$(Q+iU)(\vec{x}) = \int \frac{d^2\ell}{(2\pi)^2} a_{\vec{\ell}}\, e^{i\vec{\ell}\cdot\vec{x}}$$

Here $\vec{\ell} \equiv$ 2D wavevector in plane of sky

Def. $\vec{\ell} = (\ell\cos\phi_{\vec{\ell}}, \ell\sin\phi_{\vec{\ell}})$ s.t. $\ell = |\vec{\ell}|$ and $\phi_{\vec{\ell}}$ = angle of $\vec{\ell}$ w.r.t. x-axis of coord. system

Rotation of coords. by angle ϕ still changes $a_{\vec{\ell}}$ here (we haven't yet done anything clever).

Trick: define new quantity $_2a_{\vec{\ell}} = -e^{-2i\phi_{\vec{\ell}}} a_{\vec{\ell}}$

$$\Rightarrow a_{\vec{\ell}} = -e^{2i\phi_{\vec{\ell}}} \, {}_2a_{\vec{\ell}}$$

$$\Rightarrow (Q+iU)(\vec{x}) = -\int \frac{d^2\ell}{(2\pi)^2} \, {}_2a_{\vec{\ell}} \, e^{2i\phi_{\vec{\ell}}} e^{i\vec{\ell}\cdot\vec{x}}$$

Now note that if we rotate coord. system by ϕ:

$$\phi_{\vec{\ell}} \to \phi_{\vec{\ell}} - \phi$$

So under such rotation, our eq. above transforms as:

$$(Q+iU)e^{-2i\phi} = -\int \frac{d^2\ell}{(2\pi)^2} \, {}_2a_{\vec{\ell}} \, e^{2i(\phi_{\vec{\ell}}-\phi)} e^{i\vec{\ell}\cdot\vec{x}}$$

$$= -e^{-2i\phi} \int \frac{d^2\ell}{(2\pi)^2} \, {}_2a_{\vec{\ell}} \, e^{2i\phi_{\vec{\ell}}} e^{i\vec{\ell}\cdot\vec{x}} = e^{-2i\phi}(Q+iU)$$

$\Rightarrow$ thus the $_2a_{\vec{\ell}}$ Fourier-space coefficients are unchanged under coord. rotations!

(analog on the curved sky involves using spin-lowering and spin-raising operators in a similar way after expanding $Q \pm iU$ in spin-2 spherical harmonics)

Thus, we have

$$(Q \pm iU)(\vec{x}) = -\int \frac{d^2\ell}{(2\pi)^2} \, {}_{\pm 2}a_{\vec{\ell}} \, e^{\pm 2i\phi_{\vec{\ell}}} e^{i\vec{\ell}\cdot\vec{x}}$$

Def.: $\;{}_{\pm 2}a_{\vec\ell} \equiv -\left(E_{\vec\ell} \pm i B_{\vec\ell}\right)$

$$\Rightarrow \quad (Q \pm iU)(\vec x) = \int \frac{d^2\ell}{(2\pi)^2} \left(E_{\vec\ell} \pm i B_{\vec\ell}\right) e^{\pm 2i\phi_{\vec\ell}}\, e^{i\vec\ell\cdot\vec x}$$

i.e.,

$$Q(\vec x) = \int \frac{d^2\ell}{(2\pi)^2} \left(E_{\vec\ell}\cos(2\phi_{\vec\ell}) - B_{\vec\ell}\sin(2\phi_{\vec\ell})\right) e^{i\vec\ell\cdot\vec x}$$

$$U(\vec x) = \int \frac{d^2\ell}{(2\pi)^2} \left(E_{\vec\ell}\sin(2\phi_{\vec\ell}) + B_{\vec\ell}\cos(2\phi_{\vec\ell})\right) e^{i\vec\ell\cdot\vec x}$$

Exercise: invert these expressions to obtain $E_{\vec\ell}$ and $B_{\vec\ell}$ in terms of Fourier transform of $Q(\vec x)$ and $U(\vec x)$.

$\Rightarrow$ Conclusion: E and B are coord.-indep. (scalar and pseudo-scalar, respectively) quantities that fully describe the pol. field

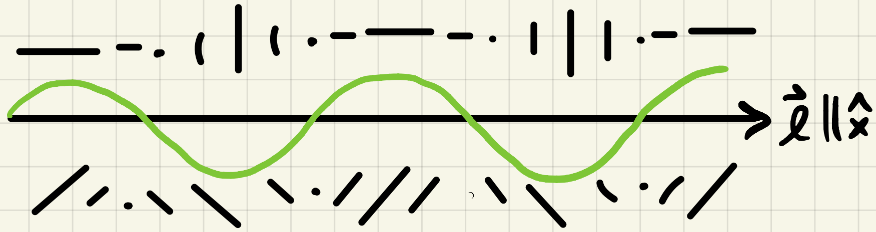
What did we really do? We rotated the frame in which Q, U were originally defined to have its x-axis now aligned with the wavevector $\vec\ell$, which yields E, B.

Simple example: a single Fourier mode aligned with x-axis

$$\Rightarrow \quad \phi_{\vec\ell} = 0 \quad \Rightarrow \quad \begin{aligned} Q(\vec x) &= E_{\vec\ell}\, e^{i\vec\ell\cdot\vec x} \\ U(\vec x) &= B_{\vec\ell}\, e^{i\vec\ell\cdot\vec x} \end{aligned}$$

Consider pure E-mode:

Pure B-mode:



Photons are coming out of the page

Polarization Generation:

Scalar perturbations produce only E-modes

(slides)

Note: since γ diffusion from hot and cold regions is responsible for quadrupolar anisotropy that generates polarization, we should expect that the source term involves the diffusion scale (l_p above) and the baryon-photon velocity:

Consider scattering at $\vec{x}_0$: scattered γ originally came from $\vec{x} = \vec{x}_0 + l_p \hat{n}$ Recall $l_p = \frac{1}{a n_e \sigma_T}$

Velocity: $\vec{v}_b(\vec{x}) \simeq \vec{v}_b(x_0) + l_p \hat{n} \cdot \vec{\nabla} \vec{v}_b(\vec{x}_0)$

Temp. seen by scatterer is Doppler-shifted:

$$\frac{\Delta T(\vec{x}_0, \hat{n})}{T} = \hat{n} \cdot \left(\vec{v}_b(\vec{x}) - \vec{v}_b(\vec{x}_0) \right) \simeq l_p \hat{n}_i \hat{n}_j \partial_i v_j^b \quad \text{to first order}$$

$$\overset{\text{Fourier}}{\Longrightarrow} \frac{\Delta T(\vec{k}, \hat{n})}{T} \simeq l_p \hat{n}_i \hat{n}_j (i k_i)(i \hat{k}_j v^b) = -k l_p v_b \underbrace{(\hat{k} \cdot \hat{n})^2}_{\text{quadrupolar}}$$

$\Rightarrow$ large for $k \gtrsim l_p^{-1}$

and proportional to v_b $\Rightarrow$ out of phase w.r.t. density perts.

$\Rightarrow$ transfer function $\sim k l_p \sin(k r_s^*)$

CMB Spectral Distortions

Blackbody $\Rightarrow$ described by T alone : $\rho = \rho_{pl}(T) = \sigma T^4$ and $N = N_{pl}(T) \propto T^3$

Recall the blackbody photon occupation number:

$$n_{pl}(\nu, T) = \frac{1}{e^{h\nu/kT} - 1} \equiv \frac{1}{e^x - 1} = \frac{c^2}{2h\nu^3} B(\nu, T)$$

Simple question: how do we change the temperature from T to T'?

Suppose we add some energy: $\epsilon = \frac{\Delta\rho}{\rho_{pl}(T)} = \frac{\rho' - \rho_{pl}(T)}{\rho_{pl}(T)}$ $\Leftrightarrow 1 + \epsilon = \frac{\rho'}{\rho_{pl}(T)}$

If still blackbody, then $\rho' \propto T'^4$

so $1 + \epsilon = \left(\frac{T'}{T}\right)^4$

$$\Rightarrow T' = T(1+\epsilon)^{1/4} \Leftrightarrow \underbrace{\frac{T' - T}{T}}_{\equiv \frac{\Delta T}{T}} = (1+\epsilon)^{1/4} - 1$$

$$\Rightarrow \frac{\Delta T}{T} \simeq \frac{1}{4}\epsilon \Rightarrow \frac{\Delta T}{T} \simeq \frac{1}{4} \frac{\Delta\rho}{\rho_{pl}(T)}$$

But clearly the new spectrum would not be a blackbody if we did not also change the photon number density appropriately:

Similar calculation yields: $$\frac{\Delta N}{N_{pl}} = \left(\frac{T'}{T}\right)^3 - 1$$
$$= (1+\epsilon)^{3/4} - 1 \simeq 3\frac{\Delta T}{T}$$

Thus: $\frac{\Delta N}{N_{Pl}} = \frac{3}{4}\frac{\Delta \rho}{\rho_{Pl}}$ $\Rightarrow$ requirement to avoid creating non-blackbody spectrum

<u>But</u>: we haven't said anything (yet) about how the new photons should be distributed in frequency! Clearly they can't all be in a spectral line...

We need to consider the photon occupation number directly: to preserve blackbody spectrum, require

$$\Delta n(\nu) = n_{Pl}(\nu, T') - n_{Pl}(\nu, T) \qquad \text{Suppose } \left|\frac{T'-T}{T}\right| << 1$$

$$= \frac{1}{e^{x'}-1} - \frac{1}{e^{x}-1}$$

$$\simeq (T'-T)\frac{\partial}{\partial T} n_{Pl}(\nu, T) + \mathcal{O}\left(\frac{\Delta T}{T}\right)^2$$

$$= -x\frac{\Delta T}{T}\frac{\partial}{\partial x} n_{Pl}(x)$$

$$= \frac{x e^x}{(e^x-1)^2}\frac{\Delta T}{T} \equiv G(x)\frac{\Delta T}{T}$$

$\Rightarrow$ energy spectrum of a small temperature shift is thus

$$T\frac{\partial}{\partial T}B(\nu, T) \propto \frac{x^4 e^x}{(e^x-1)^2} \qquad \left(x \equiv \frac{h\nu}{kT}\right)$$

$\Rightarrow$ this is the SED of the CMB temperature anisotropies!

(cf. plot in slides)

How can spectral distortions be produced?

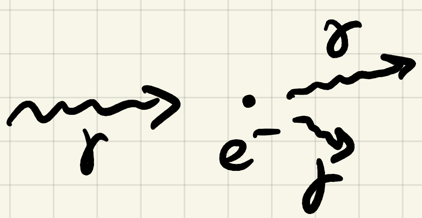
Need to study what happens if energy or entropy is injected into primordial plasma. Can processes efficiently restore blackbody spectrum (at new T')?

Important: need processes that both produce new photons and can redistribute them across energy efficiently.

Very early universe ($z > 10^8$): very efficient thermalization via many processes — no distortion generated.

Three important processes to consider at $z \lesssim 10^8$:

- Double-Compton emission:

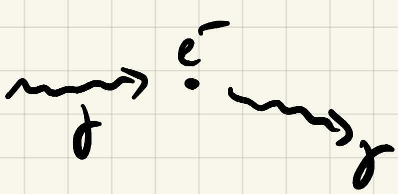


$\Rightarrow$ produces new (soft) photons

- Bremsstrahlung:

$\Rightarrow$ also produces new γ

- Compton scattering:



$\Rightarrow$ redistributes γ across freq. (in particular from low freq. to high freq.)

Each of these processes has a rate Γ^I associated with it, which depends on T, n_e, etc. As usual, when $\Gamma^I \lesssim H$, the process becomes inefficient and falls out of equilibrium.

<u>Simple example: y-type distortion</u>

Consider mixing of blackbodies at two different temps.: what is the spectrum afterward (if none of the three processes above are operating efficiently)?

Carry out our calculation above to $\mathcal{O}\left(\frac{\Delta T}{T}\right)^2$:

Consider ensemble of blackbodies at temperatures $\underset{\uparrow \text{ mean temp.}}{T_0} + \Delta T_i$ where $\langle \Delta T_i \rangle = 0$ but $\langle \Delta T_i^2 \rangle \equiv \sigma_T^2 \neq 0$.

If they mix, what is the photon occupation number?

$$\langle n_{pl}(T_0+\Delta T_i)\rangle \equiv \left\langle \frac{1}{e^{h\nu/k(T_0+\Delta T_i)}-1} \right\rangle$$

$$\simeq \left\langle n_{pl}(T_0) + \ln\left(1+\frac{\Delta T_i}{T_0}\right)\frac{\partial n_{pl}}{\partial \ln T}\bigg|_{T_0} + \frac{1}{2}\ln^2\left(1+\frac{\Delta T_i}{T_0}\right)\frac{\partial^2 n_{pl}}{\partial \ln T^2}\bigg|_{T_0} + \cdots \right\rangle$$

$$= \underbrace{n_{pl}(T_0) + \Bigg(\underbrace{\frac{\langle \Delta T_i\rangle}{T_0}}_{=0} + \frac{\langle \Delta T_i^2\rangle}{T_0^2}\Bigg) T_0 \frac{\partial n_{pl}}{\partial T}\bigg|_{T_0}}_{\simeq\, n_{pl}\left(T_0 + \frac{\sigma_T^2}{T_0}\right)} + \underbrace{\frac{1}{2}\frac{\langle \Delta T_i^2\rangle}{T_0^2} T_0 \frac{\partial}{\partial T}\left(T\frac{\partial n_{pl}}{\partial T}\right)\bigg|_{T_0}}$$

$\uparrow$ blackbody at higher temp.

$$= \frac{1}{2}\frac{\sigma_T^2}{T_0}\left(\frac{\partial n_{pl}}{\partial T} + T\frac{\partial^2 n_{pl}}{\partial T^2}\right)\bigg|_{T_0}$$

$$= \frac{1}{2}\frac{\sigma_T^2}{T_0^2}\underbrace{\left(T\frac{\partial n_{pl}}{\partial T} + T^2\frac{\partial^2 n_{pl}}{\partial T^2}\right)\bigg|_{T_0}}_{\equiv Y(x)}$$

not blackbody

Here $Y(x) = \frac{xe^x}{(e^x-1)^2}\left(x\,\frac{e^x+1}{e^x-1} - 4\right)$ $\qquad x \equiv \frac{h\nu}{kT_0}$

$$= G(x)\left(x\coth\left(\frac{x}{2}\right) - 4\right)$$

$\Rightarrow$ this is the famous spectrum of the Sunyaev-Zel'dovich effect (the y-type distortion)

It is the generic distortion resulting from the addition of Planck spectra without Comptonization or photon-number-changing processes.