Conference on Arithmetic Geometry and Applications | (SMR 3950)

01 Jul 2024 - 05 Jul 2024
ICTP, Trieste, Italy

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T01

Bad L-Factors of Hypergeometric Motives

Asem Abdelraouf

Scuola Internazionale Superiore di Studi Avanzati

In this talk, I will sketch the relation between the monodromy of the hypergeometric differential equation and the bad L-factors of the associated motive. In the case of curves, I will show how one can combinatorially describe these local L-factors using toric models

T02

Locally recoverable codes with availability from fiber products of curves

María Chara

Researcher of CONICET at Universidad Nacional del Litoral, Argentina

In this talk, we will present a construction of locally recoverable codes with availability, that is, codes in which each coordinate can be recovered by using more than one recovery set. We will provide an introductory overview of constructing these types of codes using fiber products of curves and determining the minimum distance for various families, supported by a general theorem that establishes the minimum distance of such codes under certain conditions. Consequently, we will show that fiber product codes can achieve arbitrarily large rates and arbitrarily small relative defects.

This talk is based on a joint work with Sam Kottler (University of Wisconsin), Beth Malmskog (Colorado College), Bianca Thompson (Westminster College), and Mckenzie West (University of Wisconsin).

[1] **"Minimum Distance and Parameter Ranges of Locally Recoverable Codes with Availability from Fiber Products of Curves"** M.Chara, S. Kottler, B. Malmskog, B. Thompson and M. West. Des. Codes Cryptogr. 91: 2077—2105, (2023).

T03

A HYPERELLIPTIC STATISTICS ON MODULI SPACES

SAMPA DEY

Abstract

In this talk, we will discuss the distribution of the $\mathbb{F}_q$-rational points on the various moduli spaces while taking a large family of hyperelliptic curves as the probability space with the uniform probability measure. This is based on joint work with Dr. Arijit Dey and Dr. Anirban Mukhopadhyay.

Reference: Statistics of Moduli Space of Vector Bundles, Dey A., Dey S., and A. Mukhopadhyay, Bull.Sci.Math, Vol 151 (2019), 13-33.
https://doi.org/10.1016/j.bulsci.2018.12.003

T04

Title: Fermat-type equations of signature (p, p, q) via Hypergeometric Motives

<u>Franco Golfieri</u>[1], **Ariel Martin Pacetti** [1,2]

[12] *Universidade de Aveiro, Portugal*

Abstract: Let p and q be two different primes numbers and let consider the generalised Fermat equation $x^p + y^p = z^q$ of signature (p, p, q). The general approach to prove the non-existence of solutions to this equation consists of the modularity method. The first step of this method is attach to a putative solution a geometric object satisfying that its Galois representation modulo a well chosen prime p has very small ramification. Darmon attaches to the equation of signature (p, p, q) some hyperelliptic curves of GL_2-type that satisfy this property. In this talk, we will present an approach to deal with this equation replacing these geometric objects by Hypergeometric Motives and we will see how these motives behave in all steps of the modularity method.

[1] Franco Golfieri and Ariel Pacetti. Hypergeometric motives and the generalized Fermat equation. In preparation, 2024.

T05

Conference on Arithmetic Geometry and Applications"

Vadym Kurylenko[1]

[1]*SISSA, Trieste*

Let A be a point configuration. Its Gale transform is another point configuration defined by the affine-linear relations between the points of A. I will explain how we can use a Gale transform of A to express Ehrhart-theoretic invariants of the convex hull of A. Moreover, we will also discuss operations preserving local h*-vector.

T06

Perfect powers in elliptic divisibility sequences

Maryam Nowroozi[1], **Samir Siksek**

[1]*(Presenting author underlined) University of Warwick*

Let E be an elliptic curve over the rationals given by an integral Weierstrass model and let P be a rational point of infinite order. The multiple nP has the form $(\frac{A_n}{B_n^2}, \frac{C_n}{B_n^3})$ where A_n, B_n, C_n are integers with A_nC_n and B_n coprime and B_n positive. The sequence (B_n) is called the elliptic divisibility sequence generated by P. In this talk we answer the question posed in 2007 by Everest, Reynolds and Stevens: does the sequence (B_n) contain only finitely many perfect powers?

[1] Maryam Nowroozi and Samir Siksek, Perfect powers in elliptic divisibility sequences. Preprint, 2023. `arXiv:2312.08997`

T07

Mahler measure of Laurent polynomials over arbitrary tori

Subham Roy[1]

[1] *Université de Montréal*

The (logarithmic) Mahler measure of a non-zero rational function P in n variables is defined as the arithmetic mean of $\log|P|$ restricted to the standard n-torus ($\mathbb{T}^n = \{(x_1, \ldots, x_n) \in (\mathbb{C}^\times)^n : |x_i| = 1, \forall\, 1 \leq i \leq n\}$). The Mahler measure has been related to special values of L-functions, and this has been explained in terms of regulators. In 2018, Lalín and Mittal [1] considered a generalization of the Mahler measure (where the mean is taken over n-tori with arbitrary radii) and computed this for certain polynomials mentioned in Boyd's work. This generalization is also related to the Ronkin function of Amoebas associated to the corresponding polynomials.

This talk will explore the generalized Mahler measure for Laurent polynomials in two or more variables that do not vanish on the integration torus. We will present results [2] relating the standard and generalized Mahler measures for such polynomials. Time permitting, we will discuss applications to families of polynomials whose Newton polygon has a single interior lattice point.

[1] M. N. Lalín, T. Mittal, *The Mahler measure for arbitrary tori*, Res. Number Theory. **4**, no. 2, Art. 16, 23 (2018).

[2] S. Roy, *Generalized Mahler measures of Laurent polynomials*, Ramanujan J. **64**, no. 3, 581–627 (2024).

T08

Abstract

The Mahler measure of multivariate polynomials, introduced by Mahler in 1962, has connections to special values of L-functions and the Riemann zeta function. The main goal of the talk is to study these relations using cohomological tools.

The idea of using Goncharov's polylogarithmic complexes to study Mahler measures was considered by Lalín for the polynomial (x+1)(y+1)+z. We generalize her result for more general exact polynomials. More precisely, under some conditions and Beilinson's conjecture, we express the Mahler measure of three-variable exact polynomials in terms of L-values of elliptic curves and values of the Bloch-Wigner dilogarithm function. Using the work of De Jeu on the polylogarithmic complexes, we construct explicitly an element in the motivic cohomology of the Maillot variety such that its Beilinson regulator is related to the Mahler measure.

More generally, we express the Mahler measure of an exact polynomial in arbitrarily many variables in terms of the Deligne-Beilinson cohomology of the Maillot variety. We apply this method in the case of the four-variable exact polynomial (x+1)(y+1)(z+1)+t. Under Beilinson's conjecture, we prove that the Mahler measure of this polynomial is a rational linear combination of the L-function of a cusp form of weight 3 and level 7, and the Riemann zeta function.

T09

The modular method for Diophantine equations

Lucas Villagra Torcomian

Universidad Nacional de Córdoba

In this short talk we will present the modular method, followed by Wiles to prove Fermat's Last Theorem. Then we will see how can be used to tackle different Diohantine equations. More specifically, we will se its application to the problem of perfect powers in arithmetic progression [1].

[1] L. Villagra Torcomian. On the sum of fifth powers in arithmetic progression (2024). Available at arXiv:2404.03457.