Stochastic Lattice Models for Clouds and Parameterization of Organized Convection

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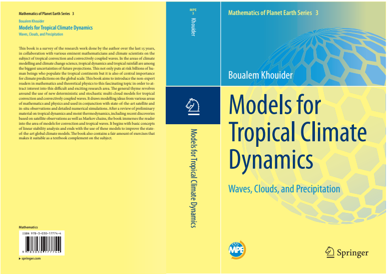


- Lecture 1: Stochastic modelling, Markov processes, Theory and Simulation (9 am - 10:30 am); Chapter 4 and Section 10.2.
- Lecture 2: The stochastic multicloud model (11 am - 12:30 pm) ; Chapter 10: Sections 10.3 to 10.6
- Hands-on activities (2:30 pm - 3:45 pm):
- Lecture 3: Waves and convective organization in the SMCM (4:00 pm - 5:15 pm) ; Chapter 11.

Lecture 1

Monte Carlo Simulation, Markov Chains, and the Birth-Death Process

- Discrete-time Markov chains
- Limiting distribution and detailed balance
- Random Walk
- Poisson Process
- Continuous time Markov Chains
- Kolmogorov backward and forward equations
- Birth and death process

A stochastic model for convective inhibition

- The microscopic stochastic model for CIN: the Ising model
- The coarse grained stochastic model for CIN
- The transition probability matrix and Gillespie's Algorithm
- Numerical results

Coupling the CIN model to a Toy GCM

- Mean-field regime and effect of CIN on CC-Waves
- Effect of stochastic fluctuations on climate dynamics

Monte Carlo Simulation

- Markov Chain Monte Carlo: sample from complex or unknown distribution (Steward 1994; Robert and Cassella 2007)
- Construct a Markov process whose equilibrium distribution is the target distribution
- First need to generate random numbers. The typical command "rand" generates pseudo-random numbers; periodic sequences of floating-point numbers with very large periods!
- Monte Carlo Integration

Monte Carlo Integration

- **Law of large numbers:**
  $X_1, X_2, \cdots, X_n, n \geq 2$ I.I.D's with common mean μ and common standard deviation, σ.

$$\text{Sample mean:} \qquad \mu \approx \bar{X}_n = \frac{1}{n}\sum_{j=1}^{n} X_j. \qquad (1)$$

$E(\bar{X}_n) = \mu$ (unbiased estimation).

$$Var[\bar{X}_n] = \frac{1}{n^2}\sum_{j=1}^{n} Var[X_j] = \frac{\sigma^2}{n} \text{Slow Convergence: } O(1/\sqrt{n})$$

- For integrals on [0,1]:

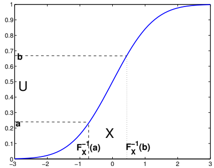
$$I = \int_0^1 g(x)dx = E[g(U)], U \sim \mathcal{U}([0,1]).$$

- For general $\int_a^b h(x)dx$? $[a, b]$ can be infinite!

Sampling arbitrary distributions

- Inverse tranform method: for a pdf f on $[a, b]$. CDF is a uniform distribution.

$$U = F_X(X) = \int_{-\infty}^{X} f_X(x)dx \sim \mathcal{U}([0,1])$$



Conversely $X = F^{-1}(U) \sim f$:

$$P(\{X \leq x\}) = P(\{F^{-1}(U) \leq x\}) = P(\{U \leq F(x)\}) = F(x).$$

- Acceptance-rejection: When $F(x)$ is not easy to invert:
  - Take a sample Y from $g(x)$ such that $Kg(x) \geq f(x)$;
  - Accept sample if $U = G^{-1}(Y) \leq \frac{f(Y)}{Kg(Y)}$, reject otherwise.

K sets the acceptance rate: $K \sim 1/r$ (See Exercise 1.)

Discrete-time Markov chains

- A sequence X_t of random variables is called a stochastic processes. Can be continuous or discrete.
- Discrete Markov chain (Markov/ memoryless property)

$$P\{X_{n+1} = x_j / X_n = x_i, X_{n-1} = x_k, \cdots, X_0 = x_l\} = P\{X_{n+1} = x_j | X_n = x_i\}$$

- Homogeneous or stationary Markov chain:

$$P\{X_{n+1} = x_j | X_n = x_i\} \equiv P\{X_1 = x_j | X_0 = x_i\}$$

- Transition Probability Matrix

$$P = [P_{ij}]_{i,j \geq 0} = P\{X_{n+1} = x_j | X_n = x_i\}$$

- P is a stochastic matrix: rows sum to 1 & entries≥ 0.
- Chapmann-Kolmogorov

$$P_{ij}^{(n+m)} = \sum_{k=0}^{\infty} P_{ik}^{(n)} P_{kj}^{(m)}, i, j = 0, 1, 2, \cdots.$$

(n-step transition matrix) $P^{(n)} = P^n = P \times P \times \cdots \times P$.

Equilibrium Distribution, Time reversibility, detailed balance and limiting distribution

- A Markov chain is in equilibrium if
  $\text{Prob}\{X_n = x_j\} = \text{Prob}\{X_m = x_j\}, \; n \neq m.$
- Limiting distribution
  $\pi_j = \lim_{n \longrightarrow \infty} \text{Prob}\{X_n = x_j | X_0 = x_i\} \equiv (P^n)_{ij}$
- When π_j exists, it satisfies $\pi_j = \sum_{i=0}^{\infty} \pi_i P_{ij}$ or $\pi = \pi P$.
- Consider reversed time process $\cdots, X_{n+1}, X_n, X_{n-1}, \cdots$.
  - It is a Markov chain with transition matrix

$$Q_{ij} \equiv P\{X_m = x_j / X_{m+1} = x_i\} = \frac{\pi_j}{\pi_i} P_{ji}.$$

  - The Markov chain is reversible is said to be $Q_{ij} = P_{ij}$

$$\Longleftrightarrow \; \pi_i P_{ij} = \pi_j P_{ji} : \quad \text{detailed balance}$$

    i.e, in the long run the rates for transitions from i to j and back are equal.

Random Walk

- Random steps to left or right ... $M+1$ positions: $0, 1, 2, \cdots, M$

  $P_{i,i+1} = \alpha_i,\ P_{i,i-1} = 1 - \alpha_i, i = 1, \cdots, M-1; P_{0,1} = 1, P_{M,M-1} = 1.$

- Limiting distribution Satisfies Detailed balance:

  $\pi_0 = (1-\alpha_1)\pi_1,\ \pi_i\alpha_i = \pi_{i+1}(1-\alpha_{i+1}),\ 1 \leq i \leq M-2,\ \pi_{M-1}\alpha_{M-1} = \pi_M.$

- Equilibrium/limiting distribution:

$$\pi_0 = \left[1 + \sum_{j=1}^{M} \prod_{l=1}^{j} \frac{\alpha_{l-1}}{1-\alpha_l}\right]^{-1} \text{ and } \pi_j = \pi_0 \prod_{l=1}^{j} \frac{\alpha_{l-1}}{1-\alpha_l}.$$

- When $\alpha_j = 0.5, j = 1, 2, \cdots, M-1$, we get $\pi_0 = \pi_M = 1/2M$ and $\pi_j = 1/M$, for $1 \leq j \leq M-1$.

Poisson process

- A counting process N_t that counts the number of events of a certain type that occur by some time $t > 0$.
- Sequence of Poisson random variables with mean λt.
- Time increments are (independent) Poisson random variables:

$$P\{N_{t+s} - N_t = k\} = P\{N_s - N_0 = k\} = P\{N_s = k\} = e^{-\lambda s} \frac{(\lambda s)^k}{k!}.$$

- For a small time increment:

$$P\{N_{t+h} - N_t = 1\} = \lambda h + o(h) \text{ and } P\{N_{t+h} - N_t \geq 2\} = o(h)$$

- Inter-arrival times ($T_1, T_2, \cdots, T_n$ time increments between successive events) are i.i.d exponential with parameter λ (mean $1/\lambda$: average waiting time between events).
- Event waiting times $S_n = \sum_{j=1}^{n} T_j$ are $\Gamma(n, \lambda)$
- Memoryless: Exponential is only distribution such that

$$P\{T > t + s\} = P\{X > t\} P\{X > s\}.$$

Continuous time Markov Chains

- Stochastic process $X_t,\ t \in [0, +\infty)$ s.t. (Markov property)

  $P\{X_{t+s} = x_j / X_s = x_i, X_u = x_u, 0 \le u \le s\} = P\{X_{t+s} = x_j / X_s = x_i\}.$

- Homogeneous/stationary if for all $s > 0$,

  $$P\{X_{t+s} = x_j / X_s = x_i\} = P\{X_t = x_j / X_0 = x_i\} \equiv P_{i,j}(t)$$

  transition probability matrix $P(t)$—(P^n in the discrete case.)
- Waiting time is exponential R.V. $\iff$ Markov property:

  $$T_i = \inf\{s > 0 \text{ such that } X_{t+s} \neq x_i \text{ given that } X_t = x_i\}$$

- Times T_{ij} of transitions $x_i \longrightarrow x_j$ exponential R.V.'s.
- We have $T_i = \min_{j \neq i} T_{ij}$, by construction
- If we denote by q_{ij} the rates of the T_{ij}'s and v_i the rate of T_i, then $v_i = \sum_{j \neq i} q_{ij}$.
- For small time increment

  $P_{ii}(h) = P\{T_i > h\} = 1 - v_i h + o(h);\ P_{ij}(h) = P\{T_{ij} < h\} = q_{ij} h + o(h).$

- Infinitesimal generator matrix $R = [R_{ij}]$:

  $$R_{ii} = -v_i \text{ and } R_{ij} = q_{ij},\ \text{when } i \neq j,$$

Kolmogorov backward and forward equations

- Chapman-Kolmogorov equations

$$P_{ij}(t+h) - P_{ij}(t) = \sum_{k=0}^{\infty} P_{ik}(h)P_{kj}(t) - P_{ij}(t) = \sum_{k \neq i} P_{ik}(h)P_{kj}(t) - [1 - P_{ii}(h)]P_{ij}(t).$$

- Divide both sides by h, $h \longrightarrow 0$ yields Backward equations

$$\frac{d}{dt}P_{ij}(t) = \sum_{k \neq i} q_{ik}P_{kj}(t) - v_i P_{ij}(t).$$

- Equivalently C-K: $P_{ij}(t+h) = \sum_{k=0}^{\infty} P_{ik}(t)P_{kj}(h)$, which yields

$$\frac{d}{dt}P_{ij}(t) = \sum_{k \neq j} q_{kj}P_{ik}(t) - v_j P_{ij}(t) : \quad \text{Forward Eqns.}$$

- In matrix form

$$P' = RP \quad \text{(backward eqns.)}; \& P' = PR \text{(forward eqns.)}$$

- With initial conditions $P_{ij}(0) = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$ we have the solution

  $P(t) = \exp(tR)$ (Matrix exponential: Hard to evaluate when R is large.).

Limiting distribution and detailed balance

- Limiting distribution: $\lim_{t\longrightarrow\infty} P_{ij}(t) = P_j$.
- Steady state of forward equations:

$$\sum_{k\neq j} q_{kj}P_k = v_jP_j,\ 0\leq P_j\leq 1,\ \ \sum_{j=0}^{\infty} P_j = 1.\ \text{Equilibrium distribution}$$

- The Markov chain is said to be time reversible if

$$P_iq_{ij} = P_jq_{ji},\ \ i,j = 0,1,2,\cdots \quad \text{(Detailed balance)}$$

- **Queueing theory example:** When in service, a certain machine breaks down at a time rate $\mu > 0$. When broken the repair shop waiting time has rate $\lambda > 0$. Find the fraction of time the machine is in service.
- Answer: Set State 1: machine in service & State 0: Machine at the repair shop. Detailed balance:
  $\mu p_1 = \lambda p_0 \implies p_1 = \frac{\lambda}{\lambda+\mu}$.
  (Probability of the quicker to go first!)

Birth and death process

- Customers arrive in a shop according to a Poisson process with rate $\lambda > 0$. Shop has m tellers and each serve customers with a service time rate $\mu > 0$. Customers leave the shop as soon as they are served.
- The number of customers in the shop at time t, X_t, is a Markov chain. Transition rates:

$$q_{n,n+1} = \lambda, \text{ for } n \geq 0, q_{n,n-1} = n\mu \text{ for } 1 \leq n \leq m,$$
$$q_{n,n-1} = m\mu \text{ for } n \geq m+1.$$

$v_0 = \lambda, v_n = \lambda + n\mu,$ for $1 \leq n \leq m$, and $v_n = \lambda + m\mu$ if $n \geq m+1$

- Infinitesimal matrix is tridiagonal. State space is infinite. It becomes bounded if we set $q_{N,N+1} = 0$.
- We can assume in general $q_{n,n+1} = \lambda_n$ (depends on n).

Birth and death process–continued.

- Forward equations at steady state:

$$\lambda_0 P_0 = \mu_1 P_1,$$
$$(\lambda_n + \mu_n) P_n = \mu_{n+1} P_{n+1} + \lambda_{n-1} P_{n-1}, n \geq 1.$$

- Yields detailed balance for birth and death process:

$$\lambda_n P_n = \mu_{n+1} P_{n+1}, \;\; n \geq 0$$

- This yields the equilibrium solution

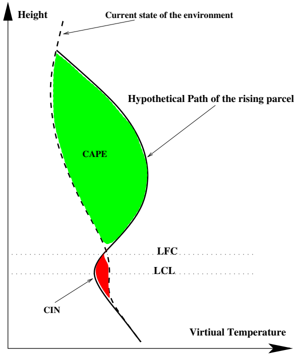
$$P_n = \frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_0}{\mu_n\mu_{n-1}\cdots\mu_1} P_0, \quad P_0 = \left[1 + \sum_{j=0}^{\infty} \frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_0}{\mu_n\mu_{n-1}\cdots\mu_1}\right]^{-1}.$$

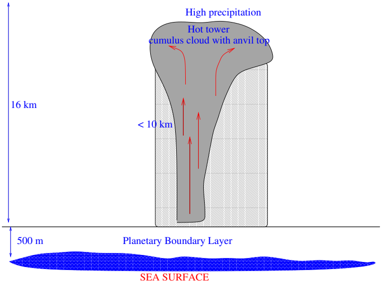
- Necessary condition for limiting distribution to exist

$$\sum_{j=0}^{\infty} \frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_0}{\mu_n\mu_{n-1}\cdots\mu_1} < \infty$$

- For the queueing theory example $\iff \lambda/m\mu < 1$!

A Stochastic Model for Convective Inhibition (CIN)



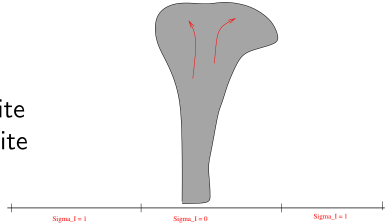


CIN is an energy barrier for convection.

The microscopic stochastic model for CIN: the Ising model

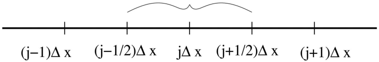
- Define an order parameter on (microscopic) lattice

$$\sigma_I(x) = \begin{cases} 1, & \text{at a CIN site} \\ 0, & \text{at a PAC site} \end{cases}$$



- Area coverage for GCM grid cell of size Δx:

$$\bar{\sigma}_I(j\Delta x) = \frac{1}{\Delta x} \int_{(j-1/2)\Delta x}^{(j+1/2)\Delta x} \sigma_I(x)dx.$$



- Intuitive rules
  - A) If a CIN site is surrounded by mostly CIN sites, then it has higher probability to remain a CIN site.
  - B) If a PAC site is surrounded by mostly CIN sites, then it has higher probability to switch to a CIN site.
  - C) The large-scale flow, $\vec{u}_j$, supplies an external potential $h(\vec{u}_j)$ that can modify the microscopic dynamics according to whether external conditions favour CIN or PAC.

Hamiltonian energy function and Gibbs measure

- Microscopic energy for CIN:

$$H_h(\sigma_I) = -\frac{1}{2}\sum_{x}\sum_{y\neq x} J(|x-y|)\sigma_I(x)\sigma_I(y) - h\sum_{x}\sigma_I(x).$$

- Nearest neighbour interactions: $J(r) = \begin{cases} J_0 & \text{if } r < 1 \\ 0 & \text{otherwise} \end{cases}$
- For $J_0 > 0$, $\sigma_I(x) = 1$ is the configuration with lowest energy and $\sigma_I(x) = 0$ has the highest energy (so are checkerboards). The opposite happens when $J_0 < 0$.
- External potential $h \equiv h(\vec{u}_j)$ modifies *this ground-state.*
- Hamiltonian dynamics: Grand canonical Gibs equilibrium measure

$$G(\sigma_I) = \frac{1}{Z}e^{-\beta H_h(\sigma_I)},$$

β is the inverse temperature Z is partition function (hard to compute!).

Spin-flip rules and Arrhenius dynamics

- A configuration flips randomly, one site at a time

$$\sigma_I^x(y) = \begin{cases} 1 - \sigma_I(x) & \text{if } y = x \\ \sigma_I(y) & \text{if } y \neq x \end{cases} = \begin{cases} q_{01} \\ q_{10} \end{cases}$$

- Markov-jump process with Arrhenius rates

$$c(\sigma_I, x) = \begin{cases} \frac{1}{\tau} e^{-\beta V(x)}, & \sigma_I(x) = 1 \\ \frac{1}{\tau}, & \sigma_I(x) = 0 \end{cases}$$

$$V(x) \equiv \Delta H = H_{\{x=0\}} - H_{\{x=1\}} = \tfrac{1}{2} \textstyle\sum_{z \neq x} J(|x - z|)\sigma_I(z) + h$$

- Detailed balance satisfied!

$$q_{01} G(H_h(\{x = 0\}) = \frac{1}{\tau} e^{-\beta(H_h(\{x=1\}) - H_h(\{x=0\})} G(H_h(\{x = 0\})$$

$$= q_{10} G(H_h(\{x = 1\})$$

Guarantees that G is the limiting distribution of the Markov process.
This is how MCMC works!

The coarse grained stochastic model for CIN

- Fine and Coarse lattices: $\Lambda \equiv \frac{1}{mq}\mathbb{Z} \cap [0,1]$ and $\Lambda_c \equiv \frac{1}{m}\mathbb{Z} \cap [0,1]$.

$$D_k \equiv \frac{1}{q}\{1,2,...,q\}, \forall k = 1,...,m.$$

- Coarse-grained process: $\eta_t(k) = \sum_{y \in D_k} \sigma_{I,t}(y) \ \in \ \{0,1,...,q\}^{\Lambda_c}$.
- Coarse grained Hamiltonian:

$$\bar{H}(\eta) = -\frac{1}{2}\sum_{l \in \Lambda_c}\sum_{k \in \Lambda_c,\, k \neq l} \bar{J}(k,l)\eta(k)\eta(l) - \frac{1}{2}\bar{J}(0,0)\sum_{l \in \Lambda_c}\eta(l)(\eta(l)-1) - h\sum_{l \in \Lambda_c}\eta(l).$$

- Birth-death process:

$$\begin{aligned}
\text{Prob}\{\eta_{t+\Delta t}(k) = n+1 | \eta_t(k) = n\} &= C_a(k,n)\Delta t + o(\Delta t) \\
\text{Prob}\{\eta_{t+\Delta t}(k) = n-1 | \eta_t(k) = n\} &= C_d(k,n)\Delta t + o(\Delta t) \qquad (2) \\
\text{Prob}\{\eta_{t+\Delta t}(k) = n | \eta_t(k) = n\} &= 1 - (C_a(k,n) + C_d(k,n))\Delta t + o(\Delta t)
\end{aligned}$$

$$C_a(k,n) = \frac{1}{\tau_I}[q - \eta(k)]; \qquad C_d(k,n) = \frac{1}{\tau_I}\eta(k)e^{-\beta \bar{V}(k)}$$

$$\bar{V}(k) = \sum_{l \in \Lambda_c,\, l \neq k} \bar{J}(k,l)\eta(k) + \bar{J}(0,0)\big(\eta(k) - 1\big) + h; \quad \bar{J}(0,0) = \frac{J_0}{q-1}$$

The transition probability matrix and Gillespie's Algorithm

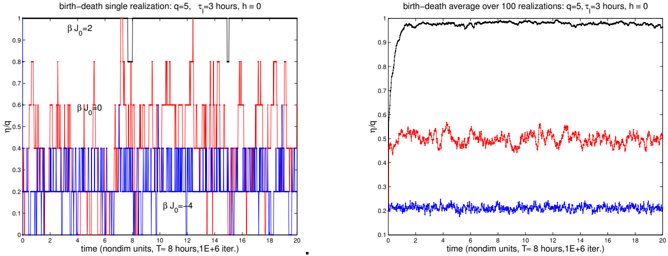
- The transition probabilities:

$$P'_{i,j}(t) = C_d(j+1,k)P_{i,j+1}(t) + C_a(j-1,k)P_{j,j-1}(t) \\ -\big(C_a(j,k) + C_d(j,k)\big)P_{i,j}(t),\ j = 0, \cdots, q \qquad (3)$$

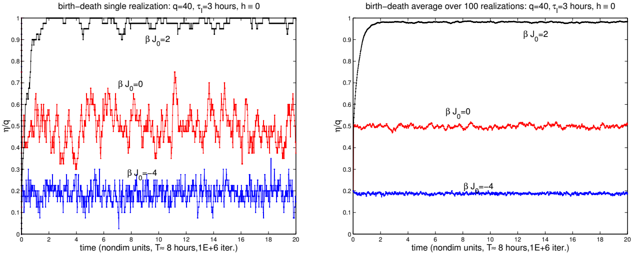
Solution: $[p_t(j,j')] = e^{tA}$ with A tri-diagonal matrix etc.

- This can be expansive to compute.
- Gillespie's exact algorithm:
  1) Given the state η_t of the process at time $t, 0 \leq t \leq \Delta T$.
  2) Draw a uniform random number r_1 from $[0,1]$ and set $s = -\frac{1}{\lambda+\mu}\ln(r_1)$.
  3) If $s + t > \Delta T$, then set $t = \Delta T$ and terminate the algorithm. Otherwise (the transition is accepted) we draw a second uniform random number r_2 in $[0,1]$.
  4) If $r_2 < \lambda/(\lambda+\mu)$, set $\eta_{t+s} = \eta_t + 1$. otherwise set $\eta_{t+s} = \eta_t - 1$.
  5) Set $t = t + s$. If $t < \Delta T$ goto 1.

Numerical tests for the coarse graining process



Evolution in time of the random process η_t/q. single realization v.s. ensemble average (100): $\tau_I = 3$ hours, $q = 5$



Increasing q — $q = 40$

Convergence to Mean field dynamics.

Coupling the CIN model to a Toy GCM

- 2d-Shallow water equations (Majda and Shefter, 2001)

$$\frac{Du_1}{Dt} - \bar{\bar{\alpha}}\frac{\partial\theta}{\partial t} = -C_D u_1 \qquad q_1 = MM_c, \quad M_c = \sigma_c (CAPE)^{1/2}$$

$$\frac{D\theta_1}{Dt} - \bar{\alpha}\frac{\partial u_1}{\partial x} = q_1 + \frac{1}{1+s}Q_R^0 - \frac{1}{1+s}\frac{\theta_1}{\tau_D} \qquad m_+ = (1-\mu)m_c + \mu m_s$$

$$h\frac{\partial\theta_{eb}}{\partial t} = D(\theta_{eb} - \theta_{em}) + C_\theta(\theta_{eb}^* - \theta_{eb}), \qquad D = m_e - m_-\, ; m_- = \frac{1-\Lambda}{\Lambda}m_+,$$

$$\frac{\partial q_2}{\partial t} = \frac{1}{\tau_s}(sq_1 - q_2) \qquad m_s = M^{-1}q_2, \, m_e = (1-\sigma_c)w_e^- \equiv -(m_c + H_m u_x)^+$$

  - $+$ A dynamically slaved second baroclinic mode to includes effect of stratiform evaporative cooling
  - μ is the stratiform instability parameter!
  - Wind enhanced surface evaporation and friction (WISHE): $C_x = \frac{C_x^0}{h}\sqrt{u_0^2 + u_1^2}$
- σ_c convection area coverage– important stability parameter; CCW's Instability when $\sigma_c \gtrsim 0.0014$.
- Two-way coupling with the mesoscopic CIN model:

$$h = -[\tilde{\alpha}\theta_{eb} + (1-\tilde{\alpha})m_s], \quad \sigma_c = \sigma_c^+ - \sigma_I(\sigma_c^+ - \sigma_c^-)$$

Mean-field regime

Majda and Khouider (PNAS, 2002)

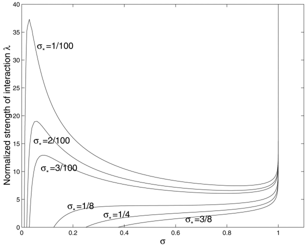
- The mean-field equation for (mesocopic CIN coverage)

$$\frac{\partial \sigma_I}{\partial t} = \frac{1}{\tau_I}\left\{1 - \sigma_I\left[1 + e^{(-\beta h - \beta J * \sigma)}\right]\right\}$$

$$J * \sigma_I \approx J_0\left[\frac{1}{4}\sigma_I(x_j - \Delta x) + \frac{1}{2}\sigma_I(x_j) + \frac{1}{4}\sigma_I(x_j + \Delta x)\right]$$

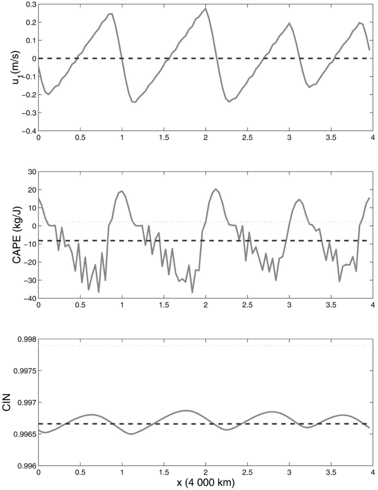
- Multiple equilibria: $h = \bar{h}$ Radiative convective equilibrium state.

$$F(\bar{\sigma}_I) \equiv e^{-\beta \bar{h}}(1 - \bar{\sigma}_I) - \bar{\sigma}_I e^{-\lambda J_0 \bar{\sigma}_I} = 0; \quad \sigma_* = \frac{1}{1 + e^{-\beta \bar{h}}}.$$



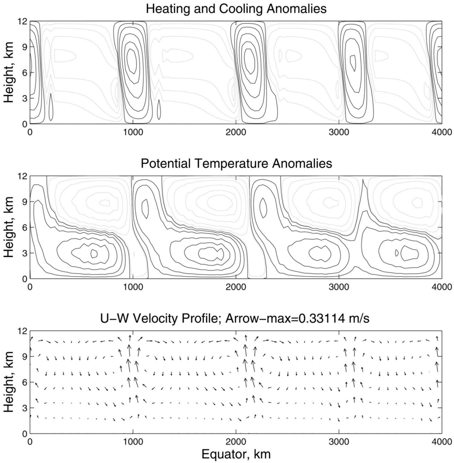
Interaction of CIN with CC-waves

- Start with an RCE states in which CCW's are unstable, $\bar{\sigma}_c = 0.002$, $\tau_I = 72\text{h}$; **Use the mean field equation for CIN.**
- 100 day simulation: Wave train of nonlinear waves (13 m/s).
  - Higher Mean CAF: $\bar{\sigma}_c \approx 0.0035$.
  - CIN fluctuation further destabilize the system.
  - CIN and CAPE are out of phase as in a predator-prey system.
  - CAPE is in phase with rising air (convergence).



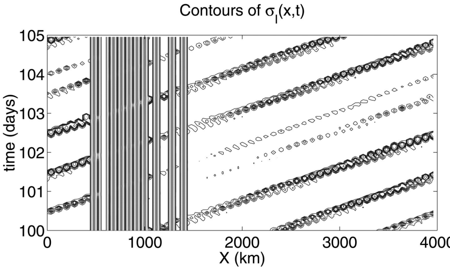
Wave structure

- Front to rear downward tilted heating, temperature, and wind anomalies
- Warm lower troposphere in suppressed phase of wave characterized by descending motion
- Warm upper troposphere coinci with active convection (heating and rising motion.



Multiple equilibria regime

- Initialize one half of domain with high CIN equilibrium and the other with a low CIN equilibrium: $\sigma_c^- = 0.001,\ \sigma_c^+ = 0.01$.
- Simulation results depend on τ_I
  - For large τ_I (72h): similar results as above; CCWaves wipe out CIN.
  - For small τ_I (3h): steady state pattern with spikes and plateaus of CIN and CCWaves that **do not carry CIN**.
  - For intermediate τ_I (12h): Mixture of plateaus and spikes of CIN and CCWaves that carry CIN as before

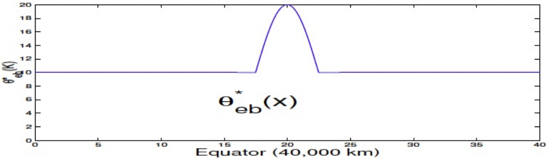


Stochastic effects on large scale/climate dynamics

Khouider, Majda and Katsoulakis (PNAS, 2003)

- Couple the Toy GCM with the coarse-grained stochastic model for CIN
- No stratiform contribution $\mu = 0$ (No instability when WISHE is off)
- Mimic Western Pacific/Indian Ocean warm pool

$$\frac{\theta^*_{eb}(x)}{\theta^{*,0}_{eb}} = \begin{cases} 1 + A_0 \cos\left(\frac{\pi(x-x_0)}{L_0}\right), & |x - x_0| < \frac{L_0}{2} \\ 1, & |x - x_0| \geq \frac{L_0}{2} \end{cases}$$



Interaction Potential Affects Wave fluctuations

- Moderate Walker Forcing: $A_0 = 0.5$

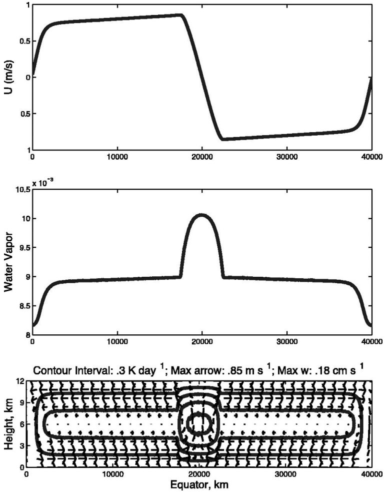
Interaction potential, βU_0	Time, τ_I, days	Climate, m·s^{-1}		Fluctuation, m·s^{-1}		Mean area fraction, $\bar{\sigma}_c$	Standard deviation, $\langle(\sigma_c - \bar{\sigma}_c)^2\rangle^{1/2}$
		$\bar{u}_-$	$\bar{u}_+$	u'_-	u'_+		
1	5	-0.856	0.855	-0.207	0.214	4.55×10^{-4}	3.00×10^{-4}
1	20	-0.855	0.856	-0.214	0.208	4.55×10^{-4}	2.96×10^{-4}
0.01	5	-1.047	1.046	-0.508	0.486	9.96×10^{-4}	3.18×10^{-4}
0.01	20	-1.048	1.040	-0.804	0.676	9.96×10^{-4}	3.15×10^{-4}
-0.01	5	-1.047	1.049	-0.603	0.572	1.00×10^{-3}	3.15×10^{-4}
-0.01	10	-0.923	0.920	-4.497	4.429	1.00×10^{-3}	3.14×10^{-4}
-0.1	5	-0.816	0.867	-4.820	4.727	1.04×10^{-3}	3.11×10^{-4}
-0.1	10	-0.824	0.877	-4.861	4.737	1.04×10^{-3}	3.12×10^{-4}

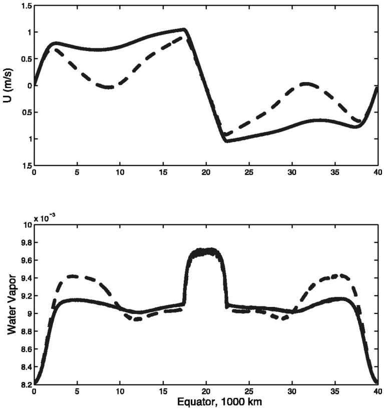
- Strong Walker Forcing: $A_0 = 1$

Interaction potential, βU_0	Time, τ_I, days	Climate, m·s^{-1}		Fluctuation, m·s^{-1}		Mean area fraction, $\bar{\sigma}_c$	Standard deviation, $\langle(\sigma_c - \bar{\sigma}_c)^2\rangle^{1/2}$
		$\bar{u}_-$	$\bar{u}_+$	u'_-	u'_+		
1	5	-1.417	1.417	-0.536	0.436	4.56×10^{-4}	3.00×10^{-4}
1	20	-1.415	1.417	-0.330	0.546	4.56×10^{-4}	3.00×10^{-4}
0.01	5	-1.692	1.691	-1.196	1.603	9.96×10^{-4}	3.17×10^{-4}
0.01	20	-1.692	1.691	-1.180	1.266	9.96×10^{-4}	3.17×10^{-4}
-0.01	5	-1.693	1.693	-1.421	1.470	1.00×10^{-3}	3.15×10^{-4}
-0.01	10	-1.693	1.693	-1.277	1.243	1.00×10^{-3}	3.16×10^{-4}
-0.1	5	-1.700	1.699	-0.990	1.092	1.04×10^{-3}	3.10×10^{-4}
-0.1	10	-1.700	1.700	-1.447	1.269	1.04×10^{-3}	3.07×10^{-4}

- SST forcing doesn't influence stochastic CIN (controlled by local interactions)
- Stronger climate has smaller stochastic fluctuations...more deterministic!

Effect of CIN on Walker circulation



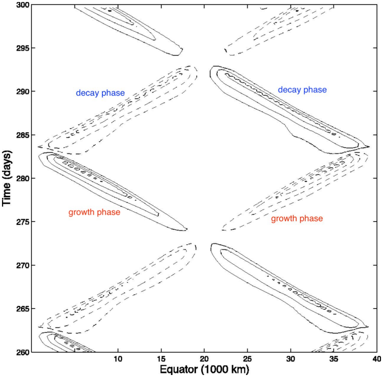


$A_0 = 0.5 \quad \tau_I = 20$ days, $\beta J_0 = 1$

$A_0 = 0.5, \quad \tau_I = 5$ days (solid), $\tau_I = 10$ days (dash),

$\beta J_0 = -0.01$

Circumnavigating WISHE waves



$A_0 = 0.5, \quad \tau_I = 10$ days (dash), $\quad \beta J_0 = -0.01$

(Low level) Easterlies destabilize eastward moving waves and Westerlies destabilize westward moving waves.

This is not physical due to Lack of Persistent Easterlies over Indian Ocean!