Organisation in parcel-based simulations of cumulus clouds

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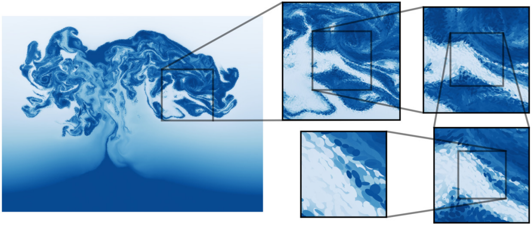

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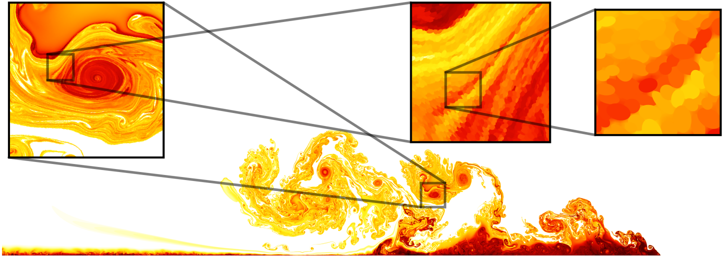
Parcel-In-Cell (PIC) models

- These are (almost fully) based on parcels. Very different approach!
- Important: these parcels must interact (mix on small scale).
- Clean distinction between resolved dynamics and small-scale mixing.

Domantas Dilys, using EPIC's predecessor MPIC

- We use parcels carrying any number of (conserved) attributes and vorticity (evolved). Monotonic by design and globally conservative.
- Parcels have a volume V_i and a shape matrix $\mathbf{B}_i$ in order to determine gridded fields needed to construct velocity on the grid.
- Incompressible, Boussinesq, evaporation and condensation but no precipitation yet.
- There are 2D (below) and 3D versions.



Mixing via splitting and merging

Splitting and merging conserve

- total centroid
- total area/volume
- total second moments (approximately)

Split criterion:

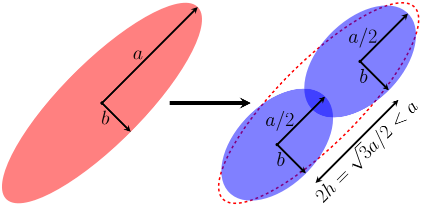
$$\lambda = \frac{a}{b} > \lambda_{\max} \quad [= 4]$$

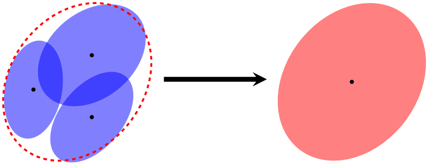
$a_n > a_{\max}$

Merge criterion:

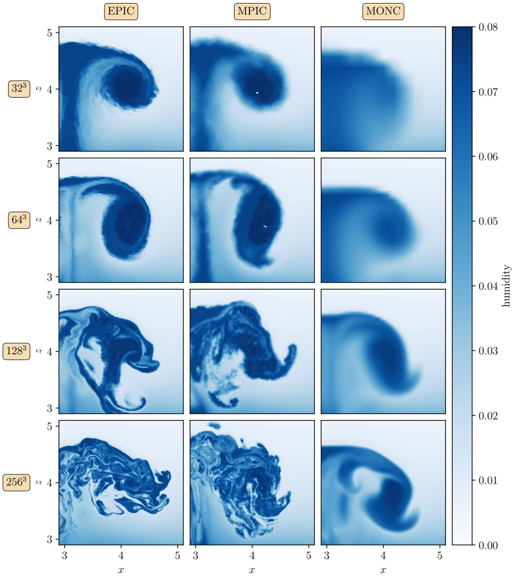
$$V_n < V_{\min}$$

Nearest neighbour merging

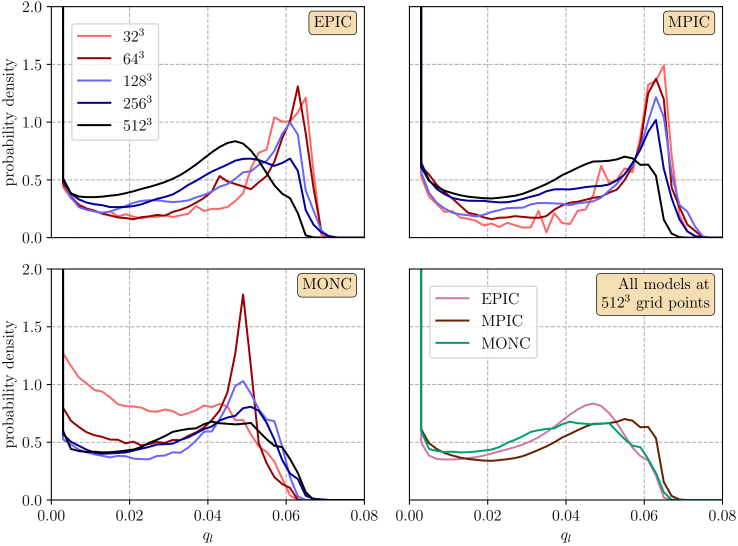




Resolution dependence



Histograms of liquid water

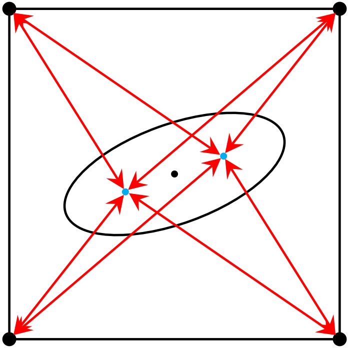


Enhanced mixing via the grid

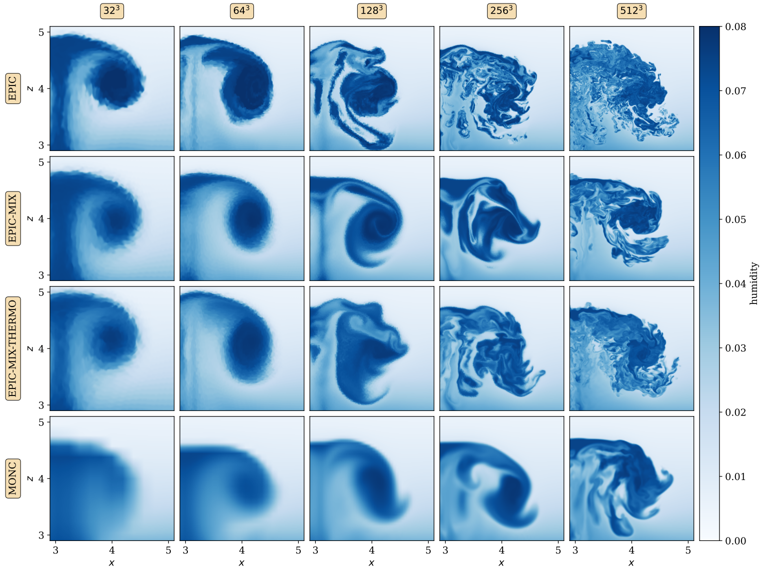
- Basic idea: nudge parcel value to grid value, which represents a mean of neighbours.

$$\phi' = \phi_p - \phi_g = \phi'(0)e^{-\beta|\mathbf{S}|t}$$

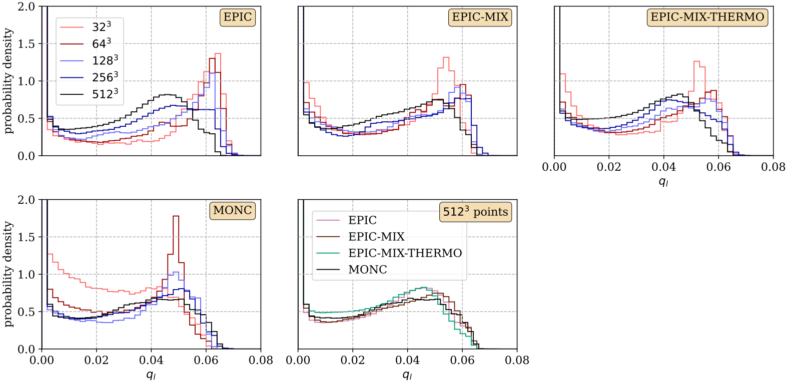
- β plays similar role to C_s in Smagorinsky scheme.
- Apply to grid point contributions to ensure conservation!



Application to moist thermal

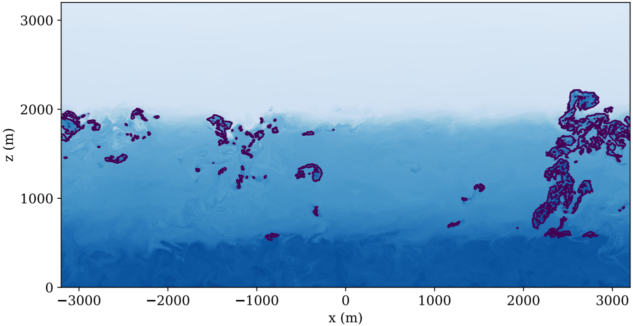


Moist thermal statistics

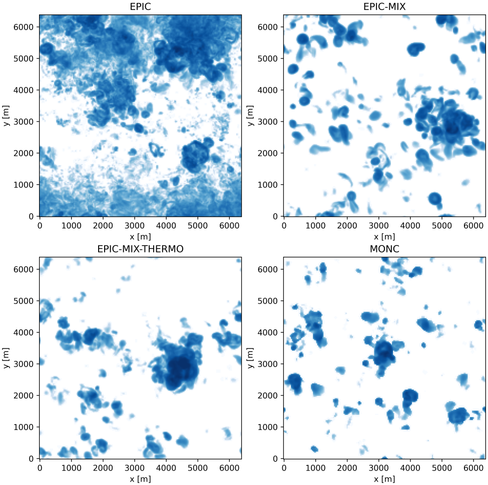


Prototype BOMEX simulations

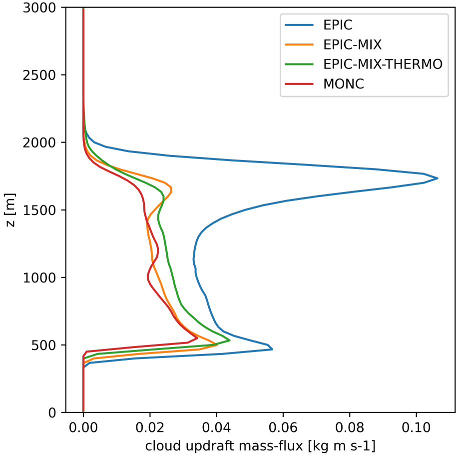
- Realistic thermodynamics: potential temperature, liquid water and water vapour.
- Includes large-scale forcings and surface fluxes.
- Mean velocity profile nudged at this point. 33m isotropic grid.



Liquid water path

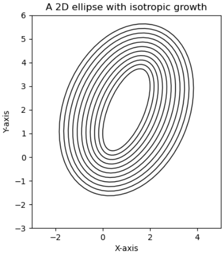


Time-mean mass flux in BOMEX



Summary

- Clean separation between resolved dynamics and parameterised mixing.
- Turbulent mixing on small scales a challenge, even in Lagrangian models.
- Alternative approaches to be explored.



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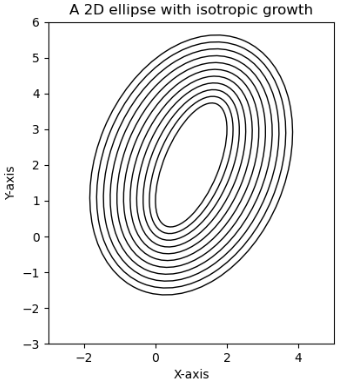
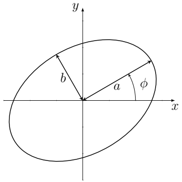
Alternative mixing formulations?

- Parcels grow and dilute over time.
- Parcels move stochastically (Brownian/Langevin/SDE).
- Parcels stretch stochastically (Brownian/Langevin/SDE for deformation tensor).
- Use parcel properties more efficiently?



Parcel-based model based on ellipses



Semi-major/-minor axes a and b

$$\mathbf{x}^\mathsf{T}\mathbf{B}^{-1}\mathbf{x} = 1$$

where $\mathbf{B} = \begin{pmatrix} B_{11} & B_{12} \\ B_{12} & B_{22} \end{pmatrix}$ is symmetric.

Ellipse area preserved $\Rightarrow$ Possible to store 2 (3D: 5) values per parcel (but may want one more for symmetry)!

Motion of a fluid ellipse in a linear background flow[1]:

Given a linear background straining flow

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{S}(\mathbf{x}, t)\mathbf{x}(t)$$

where

$$\mathbf{S}(\mathbf{x}, t) = \nabla\mathbf{u}(\mathbf{x}, t) = \begin{pmatrix} u_x(\mathbf{x}, t) & u_y(\mathbf{x}, t) \\ v_x(\mathbf{x}, t) & v_y(\mathbf{x}, t) \end{pmatrix},$$

then reformulating the time derivative of the ellipse equation results in

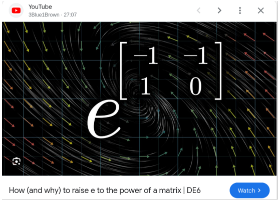
$$\frac{d\mathbf{B}}{dt} = \mathbf{B}\mathbf{S}^\mathsf{T} + \mathbf{S}\mathbf{B}.$$

Note: This is also valid in 3D!

[1] McKiver, W. J. & Dritschel, D. G. (2003). The motion of a fluid ellipsoid in a general linear background flow. Journal of Fluid Mechanics, 474, 147-173.

Deformation with matrix exponentials?

Computational cost is a challenge. Can we increase time step?



Tendency calculations, previously use RK4 directly on

$$\frac{d\mathbf{B}}{dt} = \mathbf{B}\mathbf{S}^{\mathsf{T}} + \mathbf{S}\mathbf{B}.$$

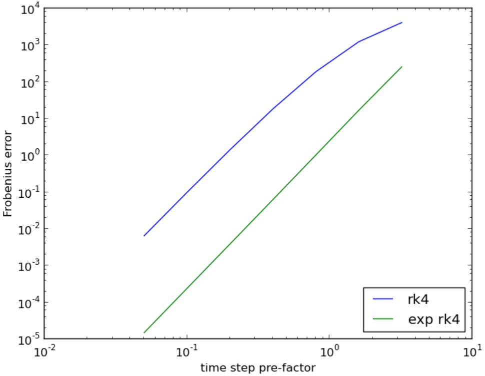
Solution for constant **S**, which preserves volume (because **S** is traceless)

$$\mathbf{B}(t + \Delta t) = e^{\mathbf{S}\Delta t}\mathbf{B}(t)e^{\mathbf{S}^{\mathsf{T}}\Delta t}.$$

- Bit expensive, but can limit matrix multiplications needed to 3 for 8th order Taylor expansion (Bader et al. 2019).
- Possibly combine with scaling and squaring (Ward, 1977).
- Combines with low-storage RK schemes (Bazavov, 2020).

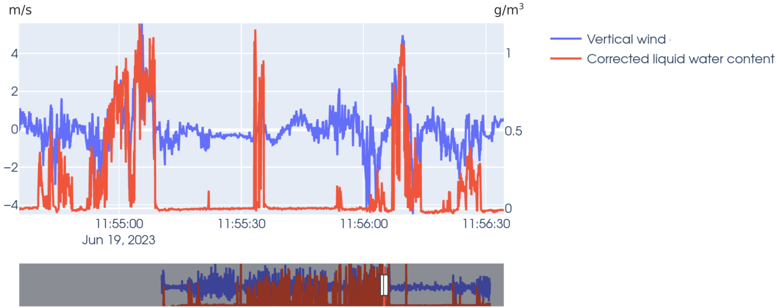
Test

- Pick random symmetric **B** arrays.
- Pick random traceless $\mathbf{S}_{\mathbf{start}}$ and $\mathbf{S}_{\mathbf{end}}$ arrays.
- Calculate time-step pre-factor (using worst case).
- Calculate error as function of time-step.



Plans: microphysics

- Plan: prognostic supersaturation and two-moment cloud droplets.
- Non-precipitating vs. precipitating microphysics.
- Approach also suitable for approach using droplet bin sizes or superdroplets.
- Comparison against WOEST flight data.



Superdroplets

UNIVERSITY OF LEEDS

QUARTERLY JOURNAL OF THE ROYAL METEOROLOGICAL SOCIETY
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Published online 19 June 2009 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/qj.441

RMetS Royal Meteorological Society

The super-droplet method for the numerical simulation of clouds and precipitation: A particle-based and probabilistic microphysics model coupled with a non-hydrostatic model

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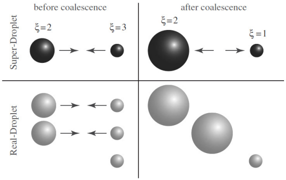

Figure 1. Schematic view of the coalescence of super-droplets. Two super-droplets with multiplicity 2 and 3 undergo coalescence (upper left panel). This represents the coalescence of two droplet pairs (lower panels). As a result, the super-droplet with multiplicity 2 becomes larger and the multiplicity of the other super-droplet decreases $3 \rightarrow 1$ (upper right panel).

Meteorol Atmos Phys
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ORIGINAL PAPER

Investigation of droplet dynamics in a convective cloud using a Lagrangian cloud model

Junghwa Lee · Yign Noh · Siegfried Raasch · Theres Riechelmann · Lian-Ping Wang

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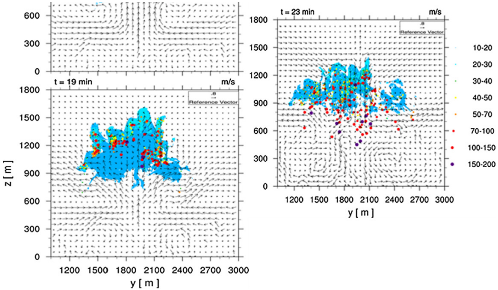
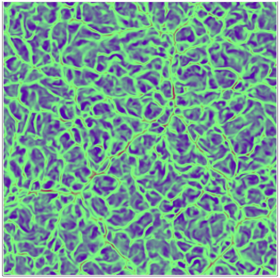


Fig. 7 Super-droplet distribution at the vertical cross section with the band thickness 100 m (500 m $< x <$ 600 m), Vertical velocity field is overlapped for the AW case. Super-droplets are shown in different colors depending on its size (t = 15, 17, 19, 21, 23 min)

Plans: surface exchange

- Current simulations use free-slip boundary conditions, constant fluxes, and nudging of mean wind.
- Convective Boundary Layer study ongoing (Sam Wallace, St Andrews).

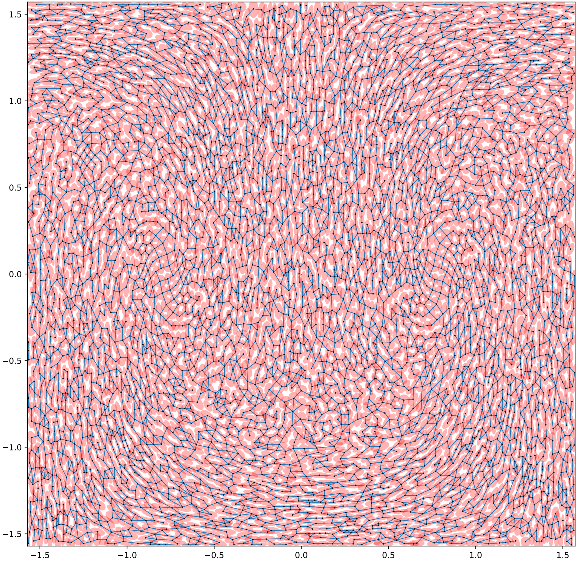


- Wind speed would determine interactive surface fluxes.
- It is not clear a priori how to implement atmospheric boundary conditions.
- Formulate Monin-Obukhov in terms of vorticity? Alternatives?

Future

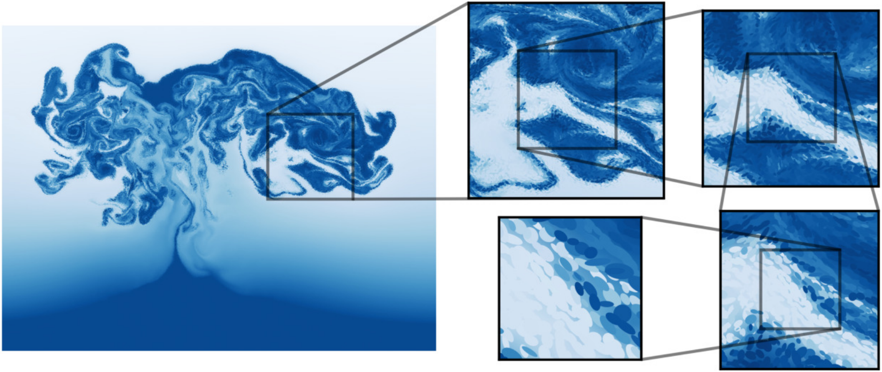
- Currently Cartesian grid, incompressible.
- However, in principle these aren't restrictions.
    1. Calculate flow field from parcel properties.
    2. Move and deform parcels.
    3. Split and merge parcels.
- Needs thinking about corrections. Local deviations from incompressible typically small in atmosphere.
- Hybrid gridded-Lagrangian options?

Exploit parcel (near) overlap?

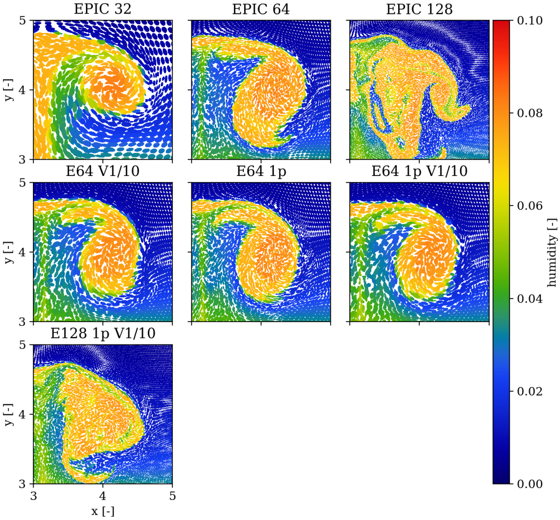


Uses algorithm of Gilitschenski and Hanebeck, IEEE, 2012

Parcel-level behaviour



Computational cost considerations



Energy

Number of grid cells	Energy loss (‰)	
	EPIC	PS3D
$48^2 \times 12$	0.310	0.514
$64^2 \times 16$	0.124	0.217
$96^2 \times 24$	0.040	0.065
$128^2 \times 32$	0.017	0.029
$256^2 \times 64$	0.002	0.005

Table 3: Relative total energy loss (in per mille) for the internal wave test case between the initial time $t = 0.0$ and the final time $t = 4\pi/\sqrt{2}$ versus the grid resolution. Note that the simulation values are interpolated to match the final time.