What is radiation

Radiation is an electromagnetic wave characterized by frequency f.

Convention is to define other quantities as in a vacuum: speed c, wavelength $\lambda = c/f$, wavenumber $\nu = 1/\lambda$

The wave travels in space and carries energy (Poynting vector), so

radiation is a flow or flux of energy, depending on position, direction and characterized by a spectral coordinate (here ν).

Fundamental unit is spectral intensity (radiance): energy flux per unit area per unit time per unit direction per spectral interval

$$I(\mathbf{x}, \Omega, \nu) = \frac{d^6 E_{\text{rad}}(\mathbf{x})}{d^2 A dt d^2 \Omega d\nu}$$

A and Ω are perpendicular; $I \propto E^2$ with E as electric field strength

Intensity has units of e.g. W m^{-2} sr^{-1} cm^{-1}

Coordinates

Make dependence on location implicit, use subscript to denote spectral dependence, consider normalized transport (per unit area)

Typically we we will use Cartesian coordinates for $\mathbf{x}$ with z *positive down* so that radiation in the positive z mean energy absorbed by the planet

[Draw a picture]

Spherical coordinates to define direction: Polar angle θ, azimuthal angle ϕ, $d^2\Omega = \sin\theta d\theta d\phi$. We'll often use $\mu = \cos\theta$

For our purpose we'll ignore variations in x, y so there's a single spatial dimension (the vertical)

Integral quantities

To understand how radiation affects flows we account for all directions and all wavenumbers

Define spectral flux as the amount of energy crossing some value of z - this is weighted by the projection on the vertical (i.e. weighted by μ) e.g. for downward flux with positive μ

$$F_\nu^- = \int_0^{2\pi} \int_0^1 \mu I_\nu d\mu d\phi$$

Units of W m^{-2} cm^{-1}

If we want to know the total energy transport we have to add up all the colors/frequencies/wavenumbers to get the *broadband* flux

$$F^- = \int_0^\infty F_\nu^- d\nu$$

Units of W m^{-2}

Source of radiation

Radiation is produced by Planck or "blackbody" emission:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T} - 1}$$

This is derived by considering radiation in equilibrium with its environment, relying on the equipartition theorum but allowing for discretized values of energy depending on wavenumber. h and k_B are Planck and Boltzmann constants

B_ν is *isotropic*

[Draw $B_\nu(T)$, $B_\nu(T + \Delta T)$]

Things to notice:

- $B_\nu(T + \Delta T) > B_\nu(T) \; \forall \; \nu$
- wavenumber of max B_ν increases with T, i.e. hotter objects emit at higher wavenumbers/shorter wavelengths

Spectral radiation on earth

Two sources of radiation on earth and other planets: planet and star. On earth these temperatures are so different ($T_\oplus \approx 288$K, $T_\odot \approx 5777$K) that the spectra are essentially disjoint

- terrestrial or longwave or infrared radiation
- solar or shortwave or visible++ radiation

Blackbody flux

Intensity depends on temperature; so does spectrally-integrated flux:

$$F = \int_0^\infty \pi B_\nu d\nu$$

π because radiation is isotropic. Integral is

$$F = \sigma T^4$$

with

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} = 5.67 \times 10^{-8}$$

W/m^2-K^4 is the Stefan-Boltzmann constant

The fate of radiation I

So what happens to radiation? Consider the fate of a pencil of radiation passing through a medium. [Draw: pencil of radiation along a path ds between S_1 and S_2.]

Use geometry and the assumption of homogeneity to write $ds = dz/\cos\theta = dz/\mu$ in our Cartesian coordinate system. This is the *plane parallel* assumption.

What is the change in intensity dI_ν across this path? Energy/intensity might be *removed* or *extinguished* (via absorption or scattering) or *added* (via emission and/or scattering). The intensity should change in proportion to the path length traversed.

Make ν dependence implicit. Introduce an *extinction coefficient* $\beta_e = \beta_a + \beta_s$, in units of inverse length, to relate physical distance to (frequency-dependent) optical distance.

Define *optical depth* as our coordinate:

$$\tau = \int_0^z \beta_e dz$$

Or better, use a *mass extinction coefficent* in length^2 per mass

$$k_m = \beta_e/\rho$$

where ρ is mass density.

Then we have

$$\tau = \int_0^z \rho k_m dz$$

Optical depth combines frequency-dependent opacity with physical distance.

The amount removed is proportional to the amount you start with (equation is linear), so

$$\mu \frac{dI}{d\tau} = -I + S$$

Where S represents the source of radiation.

This describes the spectral intensity as a function of (spectrally-dependent) optical depth.

Limit: Absorption

If there are no sources of radiation along the path, radiation decays exponentially with optical depth, i.e.

$$\frac{dI}{d\tau} = -I/\mu$$

has solution

$$I(\tau) = I(0)e^{-\tau/\mu} = I(0)\mathcal{T}$$

where $\mathcal{T} = e^{-\tau/\mu}$ is the transmissivity. It doesn't take much (in units of tau) to be optically thick i.e. to have essentially the same behavior as $\tau = \infty$

This is the Beer-Lambert-Bouguer law

Limit: Absorption and blackbody emission

For spectral regions in which the source of radiation is dominated by blackbody emission (e.g. the longwave) we can solve Schwarzchild's equation with an integrating factor $e^{\tau/\mu}$ and collecting terms in I_ν

$$e^{\tau/\mu}\frac{dI_\nu}{d\tau_\nu} + e^{\tau/\mu}\frac{I_\nu}{\mu} = e^{-\tau/\mu}\frac{B_\nu}{\mu}$$

or

$$\frac{d}{d\tau_\nu}e^{\tau/\mu}I_\nu = e^{\tau/\mu}\frac{B_\nu}{\mu}$$

Integrate from $\tau' = 0$ from (the top of the atmosphere) to some vertical location τ

$$e^{\tau/\mu}I_\nu(\tau) - I_\nu(0) = \int_{\tau'=0}^{\tau} e^{\tau'/\mu}\frac{B_\nu(\tau')}{\mu}d\tau'$$

Add $I_\nu(0)$ both sides and mutiply by $e^{-\tau/\mu}$

$$I_\nu(\tau) = I_\nu(0)e^{-\tau/\mu} + \frac{1}{\mu}\int_{\tau'=0}^{\tau} e^{-(\tau-\tau')/\mu}B_\nu(\tau')d\tau'$$

Phyiscal content: radiation emerging along a given direction is sum of

1. the boundary value, attenuated by $e^{-\tau/\mu} = \mathcal{T}(\tau)$, and
2. the source at each height, attenuated by the atmosphere between the source location and the "observation" location $\mathcal{T}(\tau - \tau')$.

We might have anticpated this from Beer-Lambert-Bouguer: the boundary term is the same, but now each increment of atmosphere also adds to the radiation by an amount that depends on local temperature, which is then attenuated by the medium between τ and the point of emission.

For reasons I won't go into we can use this equation for flux, too, by using a "representative" value of $\bar{\mu} \approx 3/5$

We can define a *brightness temperature* $T_b = B^{-1}(I)$ meaning the temperature at which a blackbody would produce a given (normally observed) intensity (or flux). T_b can be monochromatic, or narrow band, or spectrally integrated (in which case $T_b = \sqrt[4]{\frac{F}{\sigma}}$ if we measured flux).

Radiative heating

Radiation is a form of energy; gradients in the net flow are balanced by heating. Let's define the optical or non-thermodynamic heating rate as

$$\mathcal{H} = \frac{dF^{net}}{d\tau}$$

in W/m^2

When can scale these later to thermodynamic heating rates with denisty and heat capacity.

Grey radiative equilibrium

"Convection acts to stabilize the atmosphere made unstable by radiative cooling" - let's understand why an atmosphere in radiative equilibrium is dynamically unstable

For simplicity imagine no spectral variation in k_m or τ_{nu} - a grey atmosphere; this lets us solve the equation once, using Stefan-Boltzmann as the source. Changing variables in the Schwatzchild equation to a "flux optical depth" and integrating overy upwelling (+) and downwelling (-) hemispheres we can write separate equations for the up and down flux:

$$\frac{dF^-}{d\tau} = -F^- + \sigma T^4$$

$$\frac{dF^+}{d\tau} = F^+ - \sigma T^4$$

where the minus sign on the right hand sign in the second equation comes from moving to smaller τ. Here T depends on the local value of τ

The heating rate, or the vertical gradient of net flux, is 0 in radiative equilibrium

$$\frac{d(F^- - F^+)}{d\tau} = -(F^- + F^+) + 2\sigma T^4 = 0$$

from which we can infer the temperaure in terms of total flux $F^- + F^+$

Adding the two equations together

$$\frac{d(F^- + F^+)}{d\tau} = -(F^- - F^+)$$

The right hand side of this equation (the net flux) is constant, since radiative equilibrium requires it to have zero gradient. So

$$F^- + F^+ = -(F^- - F^+)\tau + C$$

The net flux at $\tau = 0$ is the (as yet unspecified) outgoing longwave radiation -OLRI since it's constant that's the value everywhere. The boundary condition on total flux at $\tau = 0$ is $F^- + F^+ = OLR$, so

$$F^- + F^+ = OLR(1 + \tau) = 2\sigma T^4$$

We can invert this for the temperature profile as a function of τ

$$T(\tau) = \sqrt[4]{\frac{OLR}{2\sigma}} \sqrt[4]{1 + \tau}$$

.

To map this back to a physical problem we might imagine a constant mass absorption coefficient in a hydrostatic atmosphere so that

$$\tau = \tau_0 e^{-z/H}$$

where H is some scale height.

$$T(z) = \sqrt[4]{\frac{OLR}{2\sigma}} \sqrt[4]{1 + \tau_0 e^{-z/H}}$$

.

The temperature of an atmosphere in pure radiative equilibirum decreases very quickly with height; the rate of change is largest at the surface. This thermodynamically unstable profile is what convection works to decrease.

[Draw a picture]

Connecting the dots

Starting with the fundamental decription of radiation, varying in position, direction, and frequency, we used

- the ODE describing the fate of radiation
- re-interpreted as an equation for angularly-integraed flux
- and the spectrally-integrated Planck function, i.e. the Stefan-Boltmann relation to determine the temperature profile of an atmosphere in radiative equailibirum... which is unstable to convection.

We'll make more use of the solutions to Schwartchild's equation in the second lecture.