Workshop on Dynamical Systems | (SMR 4026)

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P01

The Lyapunov spectrum as the Newton-Raphson method for interval Makov maps.

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In this poster, we will define MRL maps (Markov-Renyi-Luroth), a family of transformations of the unit interval with countable ramifications that can have parabolic fixed points. We will present the thermodynamic formalism of MRL maps. The aim of the poster is to study the Lyapunov spectrum of MRL transformations, a map that encodes a fractal decomposition of the repeller through the Hausdorff dimension of the sets that compose it. The sets of the decomposition are the level sets of the map that assigns to each point the value of its Lyapunov exponent. The study of the Lyapunov spectrum dates back to Weiss's work in '99 [8, 6]. Contributions in the compact context are [7, 2, 4], and in the non-compact context see, for example, [7, 5, 3]. We will present a study of the Lyapunov spectrum of MRL maps that generalizes several results from the area. We will see that the spectrum of an MRL map can be expressed in terms of the Legendre transform of the topological pressure with respect to the potential $-t \log |T'|$ as a function of t, where T denotes the transformation. Additionally, it is shown that the Lyapunov spectrum coincides with a function that generalizes the Newton map given by the Newton-Raphson method applied to the topological pressure with respect to the potential $-t \log |T'|$ (see the paper relater [1]).

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P02

Weak physical measures and some statistical properties of non-statistical maps

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I will present recent work joint with I. Melbourne and A. Talebi in which we consider a class of dynamical systems f which preserve an *infinite* absolutely continuous ergodic measure μ, but that are *non-statistical* (or *historic*) and so do *not* admit any physical measures (for example, our results apply to the intermittent interval maps described in [1] and [2]). Using techniques from operator renewal theory we prove that even though there are no physical measures there *does* exist an invariant probability measure ν such that the push forward $f_*^n\lambda$ of any absolutely continuous probability measure λ converges to ν. Moreover, our techniques yield a precise asymptotic description of the distribution of the sequences of empirical measures $e_n(x) = \frac{1}{n}\sum_{k=0}^{n-1}\delta_{f^k(x)}$ showing that despite the fact e_n does not converge almost surely, it does converge in *distribution* to a non-degenerate limit.

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[2] D. Coates and S. Luzzatto. Persistent non-statistical dynamics in one-dimensional maps, *To appear in Commun. Math. Phys.* 2024 2024.

P03

Abstract template for poster

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Here we address the problem of constructing partially hyperbolic geodesic flows that are not of Anosov type. This problem was firstly approached by Fernando Carneiro and Enrique Pujals. Their construction is based on deforming the Riemannian metric along a closed geodesic in order to break the hyperbolic behavior. Initially based their work, we propose a new construction that has several advantages in order to analyze the remaining hyperbolic behavior and we hope to obtain information about ergodicity and transitivity of those examples. Our construction is based on finding an appropriate function h on a compact negative curved Riemannian manifold (M, g) such that the new metric $g = e^h g$ has a partially hyperbolic geodesic flow. For conformal metrics we have the following relations

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P04

A New Characterization of Sturmian Subshift

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Let w be a bispecial word in the language of a Sturmian subshift $\mathcal{S}$. In the Rauzy graph $\Gamma_{|w|}$ of order $|w|$, the bispecial word w is connectned to two disjoint loops, associated with factors u and v. Any word in the language of $\mathcal{S}$ generates a path over $\Gamma_{|w|}$. However the order of the factor loops u and v remains uncertain. In this article, we aim to establish a connection between the order of factor loops u and v and the standard Sturmian coding.

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P05

Rate of memory loss in non-stationary dynamical systems with some hyperbolicity

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We study the notion of statistical memory loss in non-stationary dynamical systems described by time-dependent compositions $T_n \circ \cdots \circ T_1$, where each map $T_i : X \to X$ is a self-map of a bounded metric space equipped with a reference probability measure m. Memory is said to be lost (in the strong sense) if for any two sufficiently regular initial densities ρ_0 and ρ_0',

$$\lim_{n\to\infty} \int |\rho_n - \rho_n'| \, dm \to 0,$$

where ρ_n and ρ_n' denote the time-evolutions of ρ_0 and ρ_0', respectively. This notion was introduced by Ott, Stenlund, and Young [5] in a context of uniformly expanding maps, and subsequently explored in the literature, for example in [1, 2, 4].

In [3], we developed a coupling approach to analyze the rate of memory loss in non-uniformly expanding non-stationary dynamical systems. We considered an abstract framework where the time-dependent trajectory makes frequent returns to a reference set $Y \subset X$, with first return dynamics that have "good" distortion properties. We derived polynomial (stretched exponential) rates of memory loss depending on the tails of return times, which were assumed to decay at a uniform polynomial (stretched exponential) rate with respect to the chosen sequence of maps $T_1, T_2, \ldots$.

In work in progress (joint with A. Korepanov), we further expand our approach to handle non-uniformly decaying tails of return times. This enables us to obtain sharper estimates on the rate of memory loss for sequences in which "good maps" occur sufficiently frequently. Our framework includes piecewise expanding interval maps exhibiting neutral fixed points and/or singularities, and we apply our results to derive (nearly) sharp rates of memory loss for random ergodic compositions of such maps. These estimates are integral tools for deriving more advanced limit theorems, such as concentration inequalities and quantitative central limit theorems.

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[2] Gupta, C., Ott, W., & Török, A. Math. Res. Lett. 20.1: 141-161 (2013).
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P06

On the uniqueness of the maximizing measure for n-dimensional compact visibility manifolds without conjugate points

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The geodesic flow of a compact Riemannian manifold of negative curvature is a classical example of Anosov flow of geometric origin. Many dynamical and ergodic properties are well-known for this class of Riemannian manifolds. In particular, the existence and uniqueness of the maximizing measure was proved by Margulis and Bowen in the early 1980s [1, 2]. There was interest in extending this ergodic property to Riemannian metrics more general than negative curvature metrics. In 1998, Knieper [3] extended this property to compact rank-1 manifolds of non-positive curvature using the so-called Patterson Sullivan measure. In 2018, Gefert and Ruggiero [4] proved the same conclusion for compact higher genus surfaces without focal points using an expansive factor flow of the geodesic flow. The first author extends this result to the case of compact higher genus surfaces without conjugate points using the same strategy [5]. In the present work, we generalize Gelfert-Ruggiero's approach to n-dimensional compact manifolds without conjugate points assuming the so-called gap-entropy and a special global geometry property of the manifold called visibility condition. This is a joint work with Rafael Ruggiero [6].

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