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On the automorphism group of gap shift

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Abstract

Let (X_S, ρ) be a gap shift, and $G = Aut(X_S)$ denote the group of homeomorphisms of X_S commuting with the shift map. We investigate the algebraic properties of the countable group G and the dynamics of its action on X_S and associated spaces.

Keywords: automorphism group, shift of finite type, gap shift, low complexity, entropy

References

[1] M. Boyle and Scott Schmieding. Finite group extensions of shifts of finite type: K-theory, Parry and Livˇsic. Ergodic Theory Dynam. Systems, 37(4):1026–1059, 2017.

[2] M. Boyle and J. B. Wagoner. Positive algebraic K-theory and shifts of finite type. In Modern dynamical systems and applications, pages 45–66. Cambridge Univ. Press, Cambridge, 2004.

[3] Mike Boyle and Sompong Chuysurichay. The mapping class group of a shift of finite type. J. Mod. Dyn., 13:115– 145, 2018.

[4] Mike Boyle, Douglas Lind, and Daniel Rudolph. The automorphism group of a shift of finite type. Trans. Amer. Math. Soc., 306(1):71–114, 1988.

[5] Mike Boyle and Scott Schmieding. Strong shift equivalence and the generalized spectral conjecture for nonnegative matrices. Linear Algebra Appl., 498:231–243, 2016.

[6] Mike Boyle and Scott Schmieding. Strong shift equivalence and algebraic K-theory. J. Reine Angew. Math., 752:63–104, 2019.

[7] Mike Boyle and Scott Schmieding. Symbolic dynamics and the stable algebra of matrices. arXiv e-prints, page arXiv:2006.01051, June 2020.

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P02

Intrinsic ergodicity for non-invertible DA maps

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We will provide a result so that the isotopic maps to Anosov endomorphisms, known as Derived from Anosov (DA), have a unique measure of maximal entropy. In addition, we will give an example that satisfies the hypotheses of such a theorem.

[1] Álvarez, C.F., Cantarino, M. Equilibrium States for Partially Hyperbolic Maps with One-Dimensional Center. J Stat Phys 190, 194 (2023).
[2] Aoki, N., and Hiraide, K. Topological theory of dynamical systems, vol. 52 of North-Holland Mathematical Library. North-Holland Publishing Co., Amsterdam, 1994. Recent advances.
[3] Costa, J. S. C., and Micena, F. Some generic properties of partially hyperbolic endomorphisms. Nonlinearity 35, 10 (2022), 5297–5310.
[4] Mañé, R. Contributions to the stability conjecture. Topology 17, 4 (1978), 383–396.
[5] Przytycki, F. Anosov endomorphisms. Studia Math. 58, 3 (1976), 249–285.
[6] Qian, M., Xie, J.-S., and Zhu, S. Smooth ergodic theory for endomorphisms, vol. 1978 of Lecture Notes in Mathematics. Springer-Verlag, Berlin, 2009.

P03

Abstract for a Poster

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In this work, we address the decay of correlations for dynamical systems that admit an induced weak Gibbs Markov map (not necessarily full branch). Additionally, we yield results concerning the Central Limit Theorem and Large Deviations.

This is a joint work with Helder Vilarinho.

P04

Complex dynamics and chaos control of a Bertrand duopoly game with homogeneous players

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The complex dynamic behavior of the Bertrand duopoly game involves the intricate interplay of actions and reactions among competing firms within the market. Our study addresses this complexity by examining how firms, operating with limited information, determine pricing decisions influenced by various factors. Initially, we formulate a model to illustrate how homogeneous players, operating with limited information, determine pricing decisions influenced by various factors. Subsequently, we analyze equilibrium points and discover that as firms adjust their strategies more rapidly, the stable state of the system, known as Nash equilibrium, becomes unstable, resulting in unpredictable (chaotic) behavior. We then explore the chaotic dynamics that ensue from this instability, leading to unpredictable market outcomes. Our numerical analyses confirm these findings. Finally, we apply state feedback control to stabilize the system, particularly at the Nash equilibrium point. This stabilization helps guide the market towards stability amidst dynamic fluctuations.

[1] Elsadany AA, Agiza HN, Elsadany EM. Complex dynamics and chaos control of heterogeneous quadropoly game. Appl Math Comput, 219 (2013), 11110–8.
[2] Y. Peng, Q. Lu, Y. Xiao, X, Wu, Complex dynamics analysis for a remanufacturing duopoly model with nonlinear cost, Phys. A, 514 (2019), 658–670.
[3] S. S. Askar, On complex dynamics of cournot-bertrand game with asymmetric market information, Appl. Math. Comput., 393 (2021), 125823.

P05

On orbit complexity of dynamical systems: intermediate value property and level set related to a Furstenberg problem

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For symbolic dynamics with some mild conditions, we solved the lowering topological entropy problem for subsystems and determine the Hausdorff dimension of the level set with given complexity, where the complexity is represented by Hausdorff dimension of orbit closure. These results can be applied to some dynamical systems such as β-transformations, conformal expanding repeller, etc. We also determine the dimension of the Furstenberg level set, which is related to a problem of Furstenberg on the orbits under two multiplicatively independent maps.

P06

Classification of Ising Model Configurations in Two and Three Dimensions using Convolutional Neural Networks

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A Convolutional Neural Network (CNN) is a widely used deep learning tool for processing and classifying images [1]. In physics, a system of interest is the Ising model, which is widely used for studying phase transition phenomena in magnetic materials [2]. This model consists of a network of spins organized in space [2]. The spatial distribution of the spins leads to the formation of visual patterns characterized by regions where they exhibit a specific orientation. This distribution varies depending on whether the system is two-dimensional or three-dimensional. Distinguishing between the patterns formed by both distributions is not easily discernible to the naked eye. The aim of this project is to determine whether the system described by the Ising model is two-dimensional or three-dimensional based on these patterns, regardless of whether it is in a steady state or in a transient regime. A CNN is employed to classify images of these patterns, generated through Monte Carlo simulations of a two and three-dimensional Ising model. The network is capable of classifying these patterns with remarkable effectiveness for states sufficiently distant from a random state, suggesting that it is possible to determine the dimensionality of the system from these patterns.

[1] Rawat, W., Wang, Z., Neural computation **29**, 2352–2449 (2017)
[2] Barry A. Cipa, The American Mathematical Monthly **94**, 937-959 (1987).

P07

Invariant densities for intermittent maps with critical points

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For a class of piecewise convex maps T with an indifferent fixed point and critical points on the interval [0, 1], we show that T has a unique absolutely continuous invariant probability measure μ, and the invariant density has a upper bound and a lower bound. The Frobenius–Perron operator of T is asymptotically stable. We also obtain the polynomial decay rate of correlations with respect to μ by using the probabilistic method proposed by Liverani, Saussol and Vaienti.

[1] Hongfei Cui, J. Differ. Equations Appl. 27(3), 404-421(2021).

P08

From Decay of Correlations to Recurrence Times in Systems with Contracting Directions

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Classic results by L.-S. Young show that the decay of correlations for systems that admit inducing schemes can be obtained through the recurrence rates of the inducing scheme. Reciprocal results were obtained for non-invertible systems (without contracting directions). Here, we obtain reciprocal results also for invertible systems (with contracting directions).

[1] AIMINO, R., AND FREITAS, J. M. Large deviations for dynamical systems with stretched exponential decay of correlations. *Portugaliae Mathematica 76*, 2 , 143–152, (2020)

[2] ALVES, J. F. *Nonuniformly hyperbolic attractors. Geometric and probabilistic aspects*. Springer Monographs in Mathematics. Springer International Publishing, (2020).

[3] ALVES, J. F., FREITAS, J. M., LUZZATTO, S., AND VAIENTI, S. From rates of mixing to recurrence times via large deviations. *Adv. Math. 228*, 2 , 1203–1236, (2011)

[4] ALVES, J. F., MATIAS, J. S. From Decay of Correlations to Recurrence Times in Systems with Contracting Directions arXiv preprint arXiv:2401.06024., (2024)

[5] BUNIMOVICH, L. A., AND SU, Y. Maximal large deviations and slow recurrences in weakly chaotic systems. *Advances in Mathematics 432*, 109267, (2023)

[6] YOUNG, L.-S. Statistical properties of dynamical systems with some hyperbolicity. *Ann. of Math. (2) 147*, 3 , 585–650, (1998)

P09

Besov-ish spaces of distributions as particle systems

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Atomic decomposition has been a longstanding idea in mathematics. Literature abounds with numerous examples demonstrating how to express various types of function spaces using a simpler subclass of its elements. For example, we can mention the decomposition of Besov Spaces by Souza [6] or Frazier and Jawerth [2], as well as the groundbreaking result by Girardi and Sweldens [3], which offers a decomposition of L^p in terms of Haar wavelets. In a series of recent works, Smania et.al [1, 5, 4] employed Haar wavelets and the notion of a measure space with a good grid to define a family of spaces, denoted as $\mathcal{B}^s_{p,q}$, which generalize the classic Besov spaces for values of $p \in [1, \infty)$, $q \in [1, \infty]$ and $s \in (0, 1/p)$ and establish results about transfer operators on such spaces. In our current investigation, we aim to expand this definition to encompass the case where $p = q = \infty$ and to provide an atomic decomposition statement for some Besov-ish spaces of distributions in terms of objects that we call **Particle Systems**.

[1] Arbieto, A., Smania, D. (2019). Transfer operators and atomic decomposition. arXiv e-prints. doi:10.48550/arXiv.1903.0694

[2] Frazier, M., Jawerth, B. (1985). Decomposition of Besov Spaces. Indiana University Mathematics Journal, 34(4), 777–799. http://www.jstor.org/stable/24893969

[3] Girardi, M., Sweldens, W. A new class of unbalanced Haar wavelets that form an unconditional basis for L^p on general measure spaces. The Journal of Fourier Analysis and Applications 3, 457–474 (1997). https://doi.org/10.1007/BF02649107

[4] Smania, D. (2021). Classic and exotic Besov Spaces induced by good grids. Journal of Geometric Analysis, 31(3), 2481-2524. doi:10.1007/s12220-020-00361-x

[5] Smania, D. (2022). Besov-ish spaces through atomic decomposition. Analysis & PDE, 15(1). doi:10.2140/apde.2022.15.123

[6] Souza, Geraldo Soares de. The atomic decomposition of Besov-Bergman-Lipschitz Spaces. Proceedings of the American Mathematical Society 94, no. 4 (1985): 682–86. https://doi.org/10.2307/2044886.

P10

Dynamics of Heteroclinic Networks: Unraveling Complex Interactions and Stability

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Heteroclinic networks, intricate structures within dynamical systems, are pivotal in understanding complex interactions and stability phenomena. This research investigates the dynamics of heteroclinic networks, aiming to elucidate their formation, evolution, and stability characteristics. Through mathematical modeling, numerical simulations, and network analysis techniques, we delve into the intricate dynamics of interconnected trajectories approaching distinct equilibrium points. Our study explores the influence of network topology on the stability and transitions of heteroclinic networks, shedding light on their role in triggering system-level behaviors. By unraveling these complexities, we aim to contribute to fundamental understanding in dynamical systems theory and pave the way for applications across diverse fields, from neuroscience to engineering. This research endeavors to provide insights into the dynamics of complex interactions, offering valuable implications for the control and prediction of real-world systems.

[1] Kirk. V, Rucklidge A. M., The effect of symmetry breaking on the dynamics near a structurally stable heteroclinic cycle between equilibria and a periodic orbit, Dynamical Systems: An International Journal, 23(1):43-74, (2008).

[2] Kirk V. J., Lane E., Postlethwaite C. M., Rucklidge A. M., Silber M., Mechanism for switching near a heteroclinic network, Dynamical Systems- An International Journal, 25(3):323-349, (2010).

[3] H. Meyer-Ortmanns, Heteroclinic networks for brain dynamics, Frontier Network. Physiology. 3:1276401 (2023).

[4] C. Bick, S. V.D. Gracht, Heteroclinic dynamics in network dynamical systems with high-order interactions, Journal of Complex Networks (2024)2.

P11

A convergence rate for Birkhoff means of certain uniquely ergodic toral maps

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One of the fundamental results in ergodic theory – the Birkhoff theorem – refer to the almost everywhere convergence of additive ergodic processes. It is well known that given a uniquely ergodic system and a continuous observable, the corresponding Birkhoff averages converge everywhere and uniformly [1]. In this poster, we will present an estimate on the uniform convergence rate of the Birkhoff averages of a higher dimensional torus translation given by a frequency satisfying a generic arithmetic condition and a continuous observable. This convergence rate depends explicitly on the modulus of continuity of the observable and on the arithmetic properties of the frequency. Furthermore, we obtain similar results for affine skew product toral transformations and, in the case of one dimensional torus translation, these estimates are nearly optimal. This is a joint work with Xiao-Chuan Liu and Silvius Klein [2].

[1] G.D. Birkhoff, *Proof of the ergodic theorem*, Proc. Nat. Acad. Sci. USA, 17 (1931), pp. 656-660.

[2] S. Klein, X. Liu and A. Melo. *Uniform convergence rate for Birkhoff means of certain uniquely ergodic toral maps*, Ergodic Theory Dynam. Systems, 41.11, pp. 3363–3388, 2021.

P12

Constant Production in an Orchard: An Interaction-based Approach

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Abstract

Alternate bearing, the cyclic pattern of heavy and light fruit crops in fruit species, is a complex phenomenon influenced by both internal and external influences in an orchard. The impact of direct interactions practically realized through grafting and indirect interactions, which would be practically realized through pollination between two plants using the resource budget model was introduced in [1]. We have observed a fascinating phenomenon in our study, where the introduction of *mixed interaction* (direct and indirect) within a coupled map lattice not only fosters intricate dynamics but also gives rise to the intriguing concept of anti-synchronization. This remarkable phenomenon entails a synchronized pattern among paired plants, wherein one plant yields a crop in a given year while the other plant in the subsequent year. In an orchard comprising $2L^2$ trees, grafting, and pollination result in a distinct temporal pattern. Specifically, during a given period, approximately L^2 trees undergo an on-year, while the remaining trees experience an off-year. This cyclic alternation enables the total production of $2L^2$ trees to remain constant each year. We utilized two coupled tent map systems to derive the condition for the stability of the onset of the anti-synchronization state analytically. Our findings demonstrate that the strength of interaction plays a significant role in the occurrence of anti-synchronization thereby controlling alternate bearing. In general, our results provide insights into the interactions involved in the phenomenon of alternate bearing in an orchard and may have practical implications for sustainable and efficient crop production.

References

[1] A. Prasad, K. Sakai, Y. Hoshino, Direct coupling: a possible strategy to control fruit production in alternate bearing, Scientific reports 7 (1) (2017) 39890.

P13

Ergodicity and Accessibility of Partially Hyperbolic Diffeomorphisms in the Absence of Periodic Points

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Almost 30 years ago, Pugh and Shub proposed their Stable Ergodicity Conjecture that "stable ergodicity holds in an open and dense set of conservative partially hyperbolic diffeomorphisms". It is proved by F. Hertz, J. Hertz and R. Ures in the one-dimensional center case, and K. Burns and A. Wilkinson in the center-bunching setting. Both proofs implies that in three-dimensional manifolds, ergodicity is an abundant property for conservative partially hyperbolic diffeomophisms.

In the search for a more precise description of the abundance of ergodicity, F. Hertz, J. Hertz and R. Ures formulated their Ergodicity Conjecture (HHU Conjecture) that all partially hyperbolic diffeomorphisms are ergodic unless in three precise types of manifolds. This conjecture has motivated several works and has been verified in certain particular cases.

We provide an affirmative answer to HHU Conjecture for a new class of partially hyperbolic diffeomorphisms.

Any C^2 conservative partially hyperbolic diffeomorphism with no periodic points in a closed 3-manifold is ergodic.

Moreover, we also show the following result on accessibility:

Let $f : M^3 \to M^3$ be a C^1 partially hyperbolic diffeomorphism of a closed 3-manifold with no periodic points. If the fundamental group of the manifold is not virtually solvable, then f is accessible.

P14

Title: Modeling and analyzing the dynamics of a glucose-insulin regulation system using integer and fractional order systems.

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The study of the glycose-insulin interaction is very important to understand the mechanisms linked to glucose dynamics in the body. Its dysfunction is not without consequences and can lead to many diseases such as anxiety, coma, vision impairments, retina microvascular connection, neuronal connections and above all diabetes. The need to detect diabetic risk factors and treat diabetes-related disorders and complications has led to an increase in the number of glycoregulation models and simulation platforms designed primarily to analyze the various pathologies. In this work, we study the dynamics of a glucose-insulin regulatory system at both integer and fractional order. We highlight certain differences linked to their dynamics characteristics. The numerical simulation methods used for these various analyses are those of Runge Kutta of order 4 and Grünwald-Letniko**v.** The study of the dynamics is mainly carried out by plotting bifurcation diagrams and Lyapunov maximum exponents. The resulting analysis shows chaotic behavior (presence of a disease) and periodic behavior (absence of disease). The mathematical models and algorithms used in this study reveal the harmful consequences of excess glucose on health. These advances will lead to a better understanding of how glucose is regulated in the blood, improved methods for managing blood sugar levels and a significant improvement in quality of life.

Key words: Chaos, Insulin, Glucose, Fractional order,

P15

Computation of Microcanonical Entropy at Fixed Magnetization of the Long-Range Interacting System

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We developed the method to determine the microcanonical entropy at fixed magnetization starting from the canonical partition function. The presented method is based on the introduction of one (or more) auxiliary variables and on a min-max procedure, where the minimization is performed on the variable β, which can be both positive or negative. We emphasized that the method can be very useful where direct counting is not applicable or very difficult/convoluted. We applied our results to the case of systems having long- and short-range (possibly competing) interactions.

P16

On the modified complex balance harmonic method for seventh-order galloping oscillations

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This paper introduces a novel modified complex balance harmonic method (MCBHM) based on the displacement and velocity complex variables, which is compared with the multiple scales method (MSM) when applied to the Duffing oscillator under a seventh-order aeroelastic galloping force. Furthermore, we obtain the approximate solutions by using MCBHM and compare them to those obtained by the MSM. The Raphsen-Newton method is employed to solve the obtained polynomial equations for steady states, confirming the validity of the proposed method through numerical simulations. The stability chart is established by utilizing the Jacobian matrix of slow-flow equations.

[1] H. Nayfeh, D.T. Mook, Nonlinear Oscillations. (Wiley, New York, 1979).

[2] Weerakoon, S. Fernando, T.G.I., A variant of Newton's method with accelerated third-order convergence, Applied Mathematics Letters **13**,87-93 (2000).

P17

Shock waves in nonlinear transmission lines

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Abstract

We consider interaction between the small amplitude travelling waves (""sound"") and the shock waves in the transmission line containing both nonlinear capacitors and nonlinear inductors. We calculate for the ""sound"" wave the coefficient of reflection from (the coefficient of transmission through) the shock wave. These coefficients are expressed in terms of the wave speeds and the wave impedances. When only the capacitors or only the inductors are nonlinear, the coefficients are expressed in terms of the wave speeds only. We explicitly include into consideration of the shocks the dissipation, introducing ohmic resistors shunting the inductors and also in series with the capacitors. This allows us to describe the shocks as physical objects of finite width and study their profiles. In some particular cases the profiles were obtained in terms of elementary functions.

1 Circuit equations

The transmission line constructed from nonlinear inductors, nonlinear capacitors and ohmic resistors.

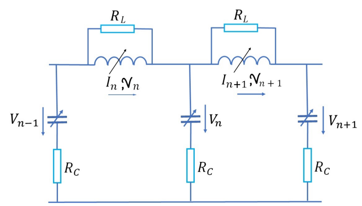

Figure 1: Lossless nonlinear transmission line.

The circuit equations are

$$\frac{dQ_n}{dt} = I_n - I_{n+1} + \frac{1}{R_L}\frac{d}{dt}(\Phi_n - \Phi_{n+1}),$$
$$\frac{d\Phi_{n+1}}{dt} = V_n - V_{n+1} + R_C\frac{d}{dt}(Q_n - Q_{n+1}),$$

where $\Phi_n \equiv \int \mathcal{V}_n dt$. In the continuum approximation the equations take the form

$$\frac{\partial Q}{\partial t} = -\frac{\partial I}{\partial x} - \frac{1}{R_L}\frac{\partial^2 \Phi}{\partial x \partial t},$$
$$\frac{\partial \Phi}{\partial t} = -\frac{\partial V}{\partial x} - R_C\frac{\partial^2 Q}{\partial x \partial t}.$$

$$Q = Q(V),$$
$$\Phi = \Phi(I).$$

2 The "sound" and the shocks

The ""sound"" waves have the velocity $u_0^2 = 1/(L_0C_0)$, and the current and the voltage in the waves are connected by the equations

$$v = \pm Z_0 i,$$

where

$$Z_0 = \sqrt{L_0/C_0}.$$

is the linear impedance of the transmission line.

For the travelling waves we obtain

$$\frac{1}{R_L}\frac{d\Phi}{d\tau} = UQ - I(\Phi) + \text{const},$$
$$R_C\frac{dQ}{d\tau} = U\Phi - V(Q) + \text{const},$$

To study the shock waves we should impose the boundary conditions

$$\lim_{\tau\to-\infty} Q(\tau) = Q_1; \quad \lim_{\tau\to-\infty} \Phi(\tau) = \Phi_1;$$
$$\lim_{\tau\to+\infty} Q(\tau) = Q_2; \quad \lim_{\tau\to+\infty} \Phi(\tau) = \Phi_2.$$

(Q_1, Φ_1) and (Q_2, Φ_2) are the fixed point, hence we obtain the jump conditions

$$U_{21}(Q_2 - Q_1) = I_2 - I_1,$$
$$U_{21}(\Phi_2 - \Phi_1) = V_2 - V_1,$$

where the indices 1 and 2 refer to the quantities before and after the shock respectively, and U_{21} is the shock wave speed. (Q_1, Φ_1) is the unstable fixed point, (Q_2, Φ_2) is the stable fixed point, hence we obtain

$$u_2^2 > U_{21}^2 > u_1^2.$$

The shock speed is higher than the "sound" speed before the shock but lower than the "sound" speed after the shock.

3 "Sound" reflection and transmission

We are interested in two problems. The first one: A ""sound"" wave is incident from the rear on a shock wave

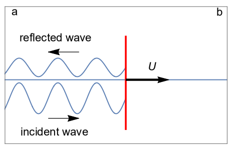

Figure 2: Reflection of a ""sound"" wave from a shock wave.

The second problem: A ""sound"" wave is incident from the front on a shock wave

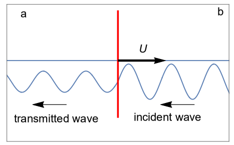

Figure 3: Transmission of a ""sound"" wave through a shock wave.

We calculate the reflection and the transmission coefficients

$$R \equiv \frac{i^{(r)}}{i^{(in)}} = \frac{u_{in}(Z_{ab} - Z_a)}{u_r(Z_{ab} + Z_a)},$$
$$T \equiv \frac{i^{(t)}}{i^{(in)}} = \frac{u_a u_{in}(Z_{ab} + Z_b)}{u_b u_t(Z_{ab} + Z_a)},$$

where $u_{in} = u_a - U$ is the speed of the incident ""sound"" wave relative to the shock wave, $u_r = u_a + U$ is the speed of the reflected ""sound"" wave relative to the shock wave, $u_t = u_a + U$ is the speed of the transmitted ""sound"" wave relative to the shock wave. and we introduced the impedance of the shock wave

$$Z_{ab} \equiv \frac{V_a - V_b}{I_a - I_b} = \frac{\Phi_a - \Phi_b}{Q_a - Q_b}.$$

4 Shock wave profile

For weak shocks we obtain the equation

$$\frac{LC_sR_C}{R_L}\frac{d^2V}{d\tau^2} + \tau_R\frac{dV}{d\tau} = \frac{(V - V_1)[Q(V) - Q_2] - (2 \leftrightarrows 1)}{Q_2 - Q_1},$$

where $\tau_R \equiv R_CC_s + L/R_L$.

The equation doesn't contain explicitly the independent variable τ. This prompts the idea to consider V as the new independent variable and $E \equiv dV/d\tau$ as the new dependent variable. In the new variables the equation is

$$\frac{LC_sR_C}{2R_L}\frac{dE^2}{dV} + \tau_R E = \frac{(V - V_1)[Q(V) - Q_2] - (2 \leftrightarrows 1)}{Q_2 - Q_1}.$$

The solution should satisfy the boundary conditions

$$\lim_{V\to V_1,V_2} E(V) = 0.$$

Since the differential equation is the first order, the problem is overdetermined. However, not only we are able to find the solutions satisfying the boundary conditions - the solutions turn out to be simple polynomial (fractional polynomial) functions.

We'll consider two models

$$Q = Q_0 + C_0(V + \beta V^2),$$
$$Q = Q_0 + C_0(V + \beta V^2 + \gamma V^3).$$

For definite relations between the parameters of the equation the solutions are

$$E(V) \sim (V - V_1)(V - V_2),$$
$$E(V) \sim (V - V_2)\left[\left(\frac{V - V_2}{V_1 - V_2}\right)^{1/2} - 1\right].$$

which leads to

$$V = V_2 + \frac{V_1 - V_2}{\left[\exp\left(\frac{1}{2}\chi\tau\right) + 1\right]^2},$$
$$V = \frac{V_1 + V_2}{2} - \frac{V_1 - V_2}{2}\tanh\left(\frac{1}{2}\psi\tau\right).$$

References

[1] E. Kogan, **The shocks in Josephson transmission line revisited**, Phys. Stat. Sol. (b) **261**, 2300336 (2024).

[2] E. Kogan, **Exact analytical solutions for the kinks, the solitons and the shocks in discrete nonlinear transmission line with nonlinear capacitance**, Phys. Stat. Sol. (b) **261**, 2400140 (2024).

[3] E. Kogan, **Shock waves in nonlinear transmission lines**, Phys. Stat. Sol. (b), submitted for publication.

P18

Abstract for the Workshop on Dynamic Systems from 5th-9th August 2024, Titled: Phase transitions and delay in swarmalator systems

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Swarmalator systems form a recent and interesting research profile in the family of complex systems where elements oscillate and swarm and their rich dynamics are part of most biological systems, chemistry agglomerations and enzymes formations, interacting electronics components and some few optics behaviours [1,2,3]. Though recent works focus on this research field, many are still to be studied and a more realistic system of these interacting particles will be that into which delay [4] is taken into account which is still greatly lacking.

Here, we analyse a system of swarmalators into which delay was introduced at the internal phase dynamics and observe its effects on the group of interacting elements. We lay emphasis on the effect of delay on the phase transitions from one observable state to another. We characterise observed states and name new ones and determine the influence of other deterministic parameters coupled to the introduced delay. The system considered here takes into consideration the attractive and repulsive forces of its creation [5] and the observed outcomes are determined using order parameters coupled with observation of time series of the group of elements. We find that increasing delay in the internal phase state forces the elements of the systems to synchronize in phase and also agglomerate in space leading to knew yet unobserved phenomena of great importance to animal tissues formation and to large numbers of interacting bodies in motion. Mainly we have first order transitions [6,7] and a first-time appearance of a two step first order transition new in the field of swarmalator systems showing rich dynamics.

Our work is a primer of its own as it considers the aspect of synchronization transitions induced by delay and sheds more light into swarmalator systems and their behaviours be it for life systems or interacting swarmalator bots.

[1] P. Japón, F. Jiménez-Morales and F. Casares Cells Dev. **169**, 203726 (2022)..
[2] Y. Origane and D. Kurabayashi, Symmetry **14,** 1578 (2022)
[3] S. Ceron , K. O'Keeffe , and K. Petersen , Nat. Commun. **14(1)**, 940 (2023).
[4] N. Blum , A. Li , K. O'Keeffe , and O. Kogan , Phys. Rev. E. **109(1)**, 014205 (2024).
[5]K. P. O'Keeffe, H. Hong and S. H. Strogatz, Nat. Commun. **8**,1--13 (2017)
[6] S. J. Kongni, V. Nguefoue., T. Njougouo , P. Louodop , F. F. Ferreira, R. Tchitnga,and H. A. Cerdeira A., Phys. Rev. E. **108(3)**, 034303 (2023).
[7] J. Gómez-Gardenes, S. Gómez , A. Arenas , Y. Moreno, Phys. Rev. lett. **106(12)**, 128701 (2011).

P19

Flexibility of generalized entropy for wandering dynamics

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A classical problem in dynamical systems is to measure the complexity of a map in terms of its orbits, and one of the main concepts used to achieve this goal is entropy. The notion of generalized entropy extends the classical notion of entropy and it is a useful tool to study dynamical systems with zero topological entropy. We show a flexibility result in the context of generalized entropy. The space of dynamical systems we work with is homeomorphisms on the sphere whose non-wandering set consist in only one fixed point.

[1] J. Correa and H. de Paula, *Flexibility of generalized entropy for wandering dynamics*, ArXiv./abs/2303.14780, (2023).

[2] J. Correa and H. de Paula, *Polynomial entropy of Morse-Smale diffeomorphisms on surfaces*, Bull. Sci. Math., Vol. **182**, (2023).

[3] J. Correa and E. R. Pujals, *Orders of growth and generalized entropy*, J. Inst. Math. Jussieu, Vol. **22(4)**, (2023).

[4] L. Hauseux and F. Le Roux, *Entropie polynomiale des homéomorphismes de Brouwer*, Annales Henri Lebesgue, Vol. **2**, (2019).

P20

Moduli of Continuity of Lyapunov Exponents of Random Matrix Products

Presenting: Yingjian Liu

IMPA

Given any compactly supported probability distribution on $\mathrm{GL}(d)$, the Lyapunov exponents are defined. It is proven in [1] that the Lyapunov exponents vary continuously. What about the moduli of continuity? For the case $d = 2$, it has been proved in [2] that they are pointwise Hölder continuous at points with simple Lyapunov spectrum, and pointwise log-Hölder continuous at the points with equal Lyapunov exponents, hence all points.

[1] A. Avila, A. Eskin, M. Viana, Continuity of the Lyapunov exponents of random matrix products. preprint arXiv:2305.06009, 2023.

[2] EH Tall, M. Viana, Moduli of continuity for the Lyapunov exponents of random GL(2)-cocycles. Trans. Am. Math. Soc. **373(2)**, 1343-1383 (2020).

P21

H usdorff-Box Dimension Discrep ncy: Origins nd Intuitions

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This t lk ex mines the rel tionship between H usdorff nd box dimensions. We begin by defining both dimensions, describing the box dimension through Minkowski-Boulig nd nd box-counting fr meworks. We then present theory of sp ces where these dimensions coincide, where every neighborhood of ny point h s uniform non-degener te box dimension. Next, we explore theory of sp ces where H usdorff nd box dimensions m y differ, ch r cterized by neighborhoods outside cert in filter m int ining the s me non-degener te box dimension. These two theories should provide n intuition on when one is to exepect the dimensions to m tch, nd when not.

P22

Equilibrium States for Open Zooming Systems

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The zooming systems were introduced by Pinheiro in [2] and the open zooming systems were introduced in [3] and a Markov structure adapted to special holes was constructed. In the context of non-uniformly expanding maps and the so-called hyperbolic potentials, possibly with the presence of a critical set, the authors establish the finiteness of equilibrium states in [1]. The zooming systems generalize the non-uniformly expanding maps by allowing contractions beyond the exponential context.

In the context of open zooming systems (see [3]), possibly with the presence of a critical/singular set, we prove the existence of finitely many ergodic zooming equilibrium states for zooming potentials that are also Hölder continuous. Among the examples of zooming potentials are the so-called *hyperbolic potentials* and also what we call *pseudo-geometric potentials* $\phi_t = -t \log J_\mu f$, where $J_\mu f$ is a Jacobian of the reference zooming measure. We prove uniqueness of equilibrium state when the system is also strongly topologically transitive and backward separated.

Then, our results generalize the Markov structure in [2] to the context of open zooming systems and the result in [1] for equilibrium states. As an application, we prove uniqueness of measure of maximal entropy for the important class well known as Viana maps, introduced in [4].

[1] J. F. Alves, K. Oliveira, E. Santana, *Equilibrium States for Hyperbolic Potentials via Inducing Schemes*, arXiv: 2003.11620

[2] V. Pinheiro, *Expanding Measures*, Annal. de l'Inst. Henri Poincaré, **28**, 889 (2011).

[3] E. Santana, *Equilibrium States for Open Zooming Systems*, arXiv: 2010.08143

[4] M. Viana, *Multidimensional NonHyperbolic Attractors*, Pub. Math. de l'Inst. des Haut. Étud. Scient., **85**, 1, 62 (1997).

P23

Maximal entropy measures for certain partially hyperbolic diffeomorphisms

Juan Carlos Mongez

UFRJ

We will present the results obtained in [2] where we considered partially hyperbolic diffeomorphisms f with a one-dimensional central direction such that the unstable entropy exceeds the stable entropy. Our main result proves that such maps have a finite number of ergodic measures of maximal entropy. Moreover, any C^{1+} diffeomorphism near f in the C^1 topology possesses at most the same number of ergodic measures of maximal entropy. These results extend the findings in [1] to arbitrary dimensions and provide an open class of non Axiom A systems of diffeomorphisms exhibiting a finite number of ergodic measures of maximal entropy.

[1] Buzzi, Jérôme; Crovisier, Sylvain; Sarig, Omri. **Measures of maximal entropy for surface diffeomorphisms.** Annals of Mathematics, v. 195, n. 2, p. 421-508, 2022.

[2] Mongez, Juan Carlos; Pacifico, Maria Jose. **Finite Measures of Maximal Entropy for an Open Set of Partially Hyperbolic Diffeomorphisms.** arXiv preprint arXiv:2401.02776, 2024.

P24

On the Phase Transition of the Ising Model with Competing Interactions on (m,k)-Ary Trees.

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Abstract

The Ising model was initially introduced by Lenz 1920 and further improved by Ising 1925 who solved the 1D version of the model. Subsequently, in 1944, Lars Osanger made a significant breakthrough by solving the 2D case on a square lattice with the absence of an external field. Although the Ising model is primarily a physical model, it has also made a major revolution in statistical mechanics and has significant contributions to the mathematics literature, its formulation involves tools from mathematical fields such as measure theory, graph theory, combinatorics, and convex analysis, among others. The interdisciplinary nature of the Ising model has made it a subject of interest for both mathematicians and physicists. Over time innovative techniques have been developed to address problems associated with the Ising model, leading to the emergence of independent fields within mathematical physics, such as integrable systems, graphical representations, and rigorous renormalization methods. In the first phase of our study, we restrict to the nearest-neighbour Isingng model on special graphs known by (m,k)-ary trees, which have significant applications in socio-physics and biology. We essentially focus on the critical points at which phase transition occurs. At the second stage, we address the question of whether a phase transition occurs when the Ising model with nearest neighbour and one level neighbours considered on the same type of trees.

[1] F.Mukhamedov, R.Utkir, On Some Remarks On The Ising Model With Competing Interactions On A Cayley Tree, arXiv preprint arXiv:0510022v1 (2005).
[2] U. Rozikov, Gibbs measures on Cayley trees,World Scientific, Hackensack, NJ, (2013).

P25

Abstract for poster presentation

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For a transcendental entire function with sufficiently small growth, Baker raised the question whether it has no unbounded Fatou components. We have shown that if the function is of order strictly less than half, minimal type, then it has no unbounded Fatou components. This, in particular gives a partial answer to Baker's question. In addition, we have addressed Wang's question on Fejér gaps. Certain results about functions with Fabry gaps and of infinite order have also been generalized.

[1] J. M. Anderson and A. Hinkkanen, Unbounded domains of normality, Proc. Amer. Math. Soc. **126** (1998), no. 11, 3243–3252.
[2] I. N. Baker, Zusammensetzungen ganzer Funktionen, Math. Z. **69** (1958), 121–163.
[3] I. N. Baker, The iteration of polynomials and transcendental entire functions, J. Austral. Math. Soc. (Series A) **30** (1981), 483-495.
[4] A. F. Beardon, Iteration of Rational Functions, vol. 132 of Graduate Texts in Mathematics, Springer, New York, NY, USA, 1991.
[5] W. Bergweiler, Iteration of meromorphic functions, Bull. Amer. Math. Soc. 29 (1993) 151–188.
[6] W. H. J. Fuchs, Proof of a conjecture of G. Pólya concerning gap series, Illinois J. Math. **7** (1963), 661–667.
[7] A. Hinkkanen, Entire functions with no unbounded Fatou components, in Complex Analysis and Dynamical Systems II, Contemp. Math. **382**, Amer. Math. Soc., Providence, RI, 2005, pp. 217–226.
[8] X. H. Hua, C.C. Yang, Dynamics of Transcendental Functions, Gordon and Breach Science Pub., 1998.
[9] J. K. Langley, *Postgraduate notes on complex analysis,* 2007.
[10] T. Murai, The deficiency of entire functions with Fejér gaps, Ann. Inst. Fourier (Grenoble) **33** (1983), no. 3, 39–58.
[11] P. J. Rippon and G. M. Stallard, Functions of small growth with no unbounded Fatou components, J. Anal. Math. **108** (2009), 61–86.
[12] A. P. Singh, Composite entire functions with no unbounded Fatou components, Journal of Mathematical Analysis and Applications, vol. 335, no. 2, pp. 907–914, 2007.
[13] G. M. Stallard, The iteration of entire functions of small growth, Math. Proc. Cambridge Philos. Soc. **114** (1993), no. 1, 43–55.
[14] Y. Wang, Bounded domains of the Fatou set of an entire function, Israel J. Math. 121 (2001) 55–60.
[15] Y. Wang, On the Fatou set of an entire function with gaps, Tohoku Math. J. 53 (2001) 163–170.
[16] Jian-Hua. Zheng, Unbounded domains of normality of entire functions of small growth, Math. Proc. Cambridge Philos. Soc. **128** (2000), no. 2, 355–361.

P26

Non-uniform hyperbolicity of maps on T^2

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In this work we prove that the homotopy class of non-homothety linear endomorphisms on T^2 with determinant greater than 2 contains a C^1 open set of non-uniformly hyperbolic endomorphisms [2]. Furthermore, we prove that the homotopy class of non-hyperbolic elements (having either 1 or -1 as an eigenvalue) whose degree is large enough contains non-uniformly hyperbolic endomorphisms that are also C^2 stably ergodic. These results provide partial answers to certain questions posed in [1].

[1] M. Andersson, P. Carrasco and R. Saghin, Non-uniformly hyperbolic endomorphisms, *arXiv:2206.08295v2*, 2022.
[2] S. Ramirez and K. Vivas, Non-uniform hyperbolicity of maps on T^2, *arXiv:2312.16742*, 2023.

P27

Abstract template for dynamial system conference

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We establish a new stable phenomenon for linear cocycles over chaotic systems in Hölder regularity. We show that every linear cocycle over a shift of finite type either admits a dominated splitting or is $\mathcal{C}^0$-approximated by a cocycle which $\mathcal{C}^\alpha$-stably exhibits bounded orbits, $\alpha > 0$.

P28

Historical behavior of skew products and arcsine laws

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We consider skew products over Bernoulli shifts, where the fibered dynamics are given by diffeomorphisms of the interval. We study the historical behavior, referred to as non-convergence, of the Birkhoff average. We establish a connection between historical behavior and the arcsine law, which allows us to construct large classes of dynamics that provide an affirmative solution to Takens' last problem. These classes include one-dimensional dynamics such as Thaler's interval maps, and skew products whose interval fiber maps have a zero Schwarzian derivative.

[1] A. Bonifant and J. Milnor. Schwarzian derivatives and cylinder maps. In Holomorphic dynamics and renormalization, volume 53 of Fields Inst. Commun., pages 1–21. Amer. Math. Soc., Providence, RI, 2008

[2] D. Coates and S. Luzzatto. Persistent non-statistical dynamics in one-dimensional maps, 2023.

[3] D. Coates, S. Luzzatto, and M. Muhammad. Doubly intermittent full branch maps with critical points and singularities. Comm. Math. Phys., 402(2):1845–1878, 2023.

[4] S. Crovisier, D. Yang, and J. Zhang. Empirical measures of partially hyperbolic attractors. Comm. Math. Phys., 375(1):725–764, 2020

[5] F. Takens. Orbits with historic behaviour, or non-existence of averages. Nonlinearity, 21(3):T33–T36, 2008.

[6] M. Thaler. A limit theorem for sojourns near indifferent fixed points of one-dimensional maps. Ergodic Theory Dynam. Systems, 22(4):1289–1312, 2002.

P29

Dynamical behavior of finance systems

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In this work we investigate the dynamically invariant structures and chaotic behavior in finance systems. We perform an analysis of the Huang–Li financial model [1] for which we compute the Poincaré map [2] and common chaos indicators, namely, the Lyapunov Exponents [3] and the Smaller Alignment Index (SALI) [4].

[1] D. Huang, H. Li, Sichuan University Press, Chengdu (1993).
[2] M. Henon, C. Heiles, The Astr. Journal **69**, 1 (1964).
[3] G. Benettin, L. Galgani, A. Giorgilli, J. Strelcyn, Meccanica **15**, 9–20 (1980).
[4] C. Skokos, G. A. Gottwald, J. Laskar, Springer-Verlag Berlin Heidelberg **15** (2016).

P30

A generalization of distality

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In this poster/talk we will present some properties of N-distal homeomorphims. To this end, we will show some results contained in [3] where in particular, we define N-equicontinuity and prove that every N-equicontinuous systems is N-distal. We introduce the notion of N-distal extensions and N-distal factors and show that M-distal extensions of N-distal homeomorphisms are MN-distal. We use the Ellis semigroups theory to obtain a criterion for existence of non-trivial distal factors for N-distal homeomorphisms and a restriction on the number of minimal subsystems for a transitive N-distal systems. Finally, we prove that topological entropy vanishes for N-distal systems on compact metric spaces. These results generalize previous ones for distal systems [1], [2].

[1] Fürstenberg, H. . *The structure of distal flows*, Amer. J. Math., **85**, (1963).
[2] Parry, W. . *Zero entropy of distal and related transformations*, Topological Dynamics (Symposium, Colorado State Univ., Ft. Collins, Colo.,1967), Benjamin, New York, (1968).
[3] Rego, E. and Salcedo, J.C. . *On N-distal homeomorphism*, Qual. Theory Dyn. Syst., **22**, 138 (2023).

P31

Example of Discontinuity for the Lyapunov Exponents for $\mathbf{SL}(2, \mathbb{R})$-cocycles

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In this work, we study an example of a discontinuity point for the Lyapunov exponents as function of the cocycle, relative to the α- Hölder topology, motivated by [2]. In [1], Backes-Brown- Butler proved that if a Hölder continuous $\mathrm{SL}(2, \mathbb{R})$-valued cocycle satisfies the fiber bunching condition, it is a continuity point for the Lyaunov exponents. Then, in our example, we consider a $\mathrm{SL}(2, \mathbb{R})$- valued cocycle very far from being fiber-bunched.

[1] L. Backes, A. Brown and C. Butler. Continuity of Lyapunov exponents for cocycles with invariant holonomies. J. Mod. Dyn., **12**, 223-260 (2018).

[2] C. Butler. Discontinuity of Lyapunov exponents near fiber bunched cocycles. Ergodic Theory Dynam. Systems, **38(2)**, 523-539 (2018).

P32

On the expansiveness of invariant measures under pseudogroups

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The expansiveness of a measure plays an important role in the study of dynamical systems, providing properties with respect to classical dynamic objects. Based on the work of Arbieto and Morales [1], we will define the concept of expansiveness of a measure from the point of view of pseudogroups and then we will discuss an implication in what we will define as the analogue of stable sets for pseudogroups.

[1] A.Arbieto, C.Morales Some properties of positive entropy maps. Ergodic Theory and Dynamical Systems. **34**, 765 (2013).

P33

Global dynamics of homogeneous polynomial Gradient system

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The classical Center-Focus Problem posed by H. Poincaré in 1880s is concerned on the existence of a center or focus at a singular point of a polynomial differential system of the form

$$\dot{x} = -y + P_n(x, y), \quad \dot{y} = x + Q_n(x, y), \tag{1}$$

with P_n and Q_n homogeneous polynomials of degree n. The center is a type of singular point where the trajectories of the system are periodic, while the focus is a type of singular point where the trajectories spiral around it .There are several approaches to solving the center-focus problem, including algebraic methods and geometric methods. The centers of the polynomial differential systems (1) have been studied for $n = 2$,3, 4 and 5.

While the phase portraits of systems with centers of degrees 2 and 3 have been classified in the Poincaré disc [1] and [2], this is not the case for the centers of degrees 4 and 5. Following the same approach as the study done for the centers of systems (1), this paper study the classification of the phase portrait of the homogeneous Gradient systems of degree 1, 2, 3 and 4 in the Poincaré disc. These systems are represented by the equations

$$\dot{x} = \frac{\partial G_n(x,y)}{\partial x}, \qquad \dot{y} = \frac{\partial G_n(x,y)}{\partial y}, \tag{2}$$

where $G_n(x, y)$ is a homogeneous polynomial of degree n with $n \in \{1, 2, 3, 4\}$.

[1] D. SCHLOMIUK, Algebraic particular integrals, integrability and the problem of the center, Trans. Amer. Math. Soc. **338**, 799-841 (1993).

[2] C.A. BUZZI, J. LLIBRE AND J.C. MEDRADO, it Phase Portraits of reversible linear differential systems with cubic homogeneous polynomial nonlinearities having a non-degenerate center at the origin, Qual. Theory Dyn. Syst. **7**, 369-403 (2009).

P34

Advances and open problems on symbolic dynamics over infinite alphabets

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The aim of this work is to present some recent results on the theory of shift spaces over infinite alphabets.

The first part of this presentation begins by examining the core features of classical definitions, such as shifts of finite type (SFT), sofic shifts, and sliding block codes, as they are given within the context of finite-alphabet shift spaces. From our observations, we propose general definitions that fit in both the contexts of finite and infinite alphabets [1]. The proposed definitions are used to recover several results in a more general setting [3, 4]. In particular, we present results that enable the characterization of shifts of finite type and sofic shifts in terms of (infinite) labeled graphs. Moreover, we define two new classes of shift spaces that can only exist in the context of infinite alphabets: *weakly sofic shifts* and *shifts of variable length (SVL)*. While the class of weakly sofic shifts extends the class of sofic shifts, the shifts of variable length can be viewed as the topological counterparts of variable length Markov chains. Together with shifts of finite type, shifts of variable length form a special class called *finitely defined shifts (FDS)*.

In the second part of this work, we introduce the *blur shift spaces* [2]. These are symbolic systems on the monoid $\mathbb{N}$, constructed from classical shift spaces by selecting sets of infinitely many symbols to be represented by a single new symbol, and defining a suitable topology. Blur shifts can act as a compactification scheme for classical shift spaces and serve to generalize the constructions proposed in [5] and [6], which were used to find out equivalence between the topological conjugacy of Markovian shifts and the isomorphism of their associated C^*-algebras.

We conclude by presenting a list of open problems related to the subjects.

[1] M. Sobottka, Bulletin of the Brazilian Mathematical Society, New Series, **53**, 981–1031 (2022).
[2] T. Z. Almeida, M. Sobottka, Bulletin des Sciences Mathématiques. **173**, 103069 (2021).
[3] U. B. Darji, D. Gonçalves, M. Sobottka, Advances in Mathematics. **385**, 107760 (2021).
[4] D. A. Lind, B. Marcus, An introduction to symbolic dynamics and coding. Cambridge, Cambridge University Press. Second Ed. (2021).
[5] W. Ott, M. Tomforde, P. N. Willis, One-sided shift spaces over infinite alphabets, New York Journal of Mathematics. NYJM Monographs 5. State University of New York, University at Albany, Albany, NY. 54 pp. (2014).
[6] D. Gonçalves, D. Royer, Int. Math. Res. Not., **2019**, 2177-2203 (2019).
[7] A. Author, B. Coauthor, J. Sci. Res. **13**, 1357 (2012).
[8] A. Author, B. Coauthor, J. Sci. Res. **17**, 7531 (2013).

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Scaled Thermodynamic Formalism for the Metric Mean Dimension

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Metric mean dimension is a geometric invariant of dynamical systems (X, d, T) with infinite topological entropy. It quantifies the rate at which the amount of ε-distinguishable orbits goes to infinity as $\varepsilon \to 0$. As in the topological pressure of finite entropy systems, one can add the dependence on a continuous potential $\varphi\colon X \to \mathbb{R}$.

Being a renormalization of the entropy, which now depends on the choice of equivalent metric to generate the topology, it is natural to search for a measure-theoretic notion of metric mean dimension $\mathcal{H}$ satisfying the classical variational principle, namely

$$\mathrm{mdim}_M(X, d, T, \varphi) = \sup_{\mu} \Big\{ \mathcal{H}(\mu) + \int \varphi \, d\mu \Big\}.$$

We will define such an object, state the variational principle and compute it explicitly for some classical examples of the theory. On these examples the obtained formula for the metric mean dimension with potential will be given in terms of ergodic optimization.

[1] M. Carvalho, G. Pessil and P. Varandas. A convex analysis approach to the metric mean dimension: limits of scaled pressures and variational principles. Adv. Math. 436 (2024) 109407

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The existence of a semi-dispersive billiard with infinite topological entropy

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A theorem of D.Burago, S.Ferleger and A. Kononenko [1], states that for a non-degenerate semi-dispersive billiard table, the time-one map of the billiard flow has finite topological entropy. In this work, we consider an example, originally due to Burago [2], of a degenerate semi-dispersive billiard table for which the time-one map of the billiard flow has infinite topological entropy.

[1] D. Burago, S. Ferleger, A. Kononenko, Topological entropy of semi-dispersing billiards. In: Ergodic Theory and Dynamical Systems, **18**, pp. 791-805 (1998).

[2] D. Burago, Semi-dispersing billiards of infinite topologycal entropy. In : Ergodic theory and Dynamical Systems **26**, pp.45-52 (1) (2006).

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Historical behavior for skew product diffeomorphisms

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In [1], the authors constructed a family of skew product diffeomorphisms of the cylinder which displays different dynamical features according to the Schwarzian derivative of their fiber maps, including intermingled basins and physical measures. In particular, they proved that when the maps have zero Schwarzian and the diffeomorphism is replaced by a step skew product, the system has historical behavior, that is, the empirical measures don't converge in a positive volume of points. The question of whether historical behavior is also present in the original cylinder map was left open.

In this work, we apply ideas from the article and [2] to investigate the presence of historical behavior in the original example and related maps, by employing probabilistic tools such as the almost sure invariance principle (approximation by Brownian motions) and arc-sine laws. This is in collaboration with Douglas Coates.

[1] A. Bonifant and J. Milnor, "Schwarzian derivatives and cylinder maps," *American Mathematical Society*, pp. 1–24, (2008).

[2] S. Crovisier, D. Yang, and J. Zhang, "Empirical Measures of Partially Hyperbolic Attractors," *Communications in Mathematical Physics*, vol. 375, no. 1, pp. 725–764, (2020).