Exercises.

1. Consider the CAT MAP
   (a) Depict the image of the fundamental domain $[0,1)^2$.
   (b) Show that f is a diffeo.
   (c) Show that f is Anosov.
   (d) Show that every $x \in \mathbb{Q}^2$ is periodic.
   (e) Show that $[\ell_1], [\ell_2]$ are dense in $\mathbb{T}^2$.

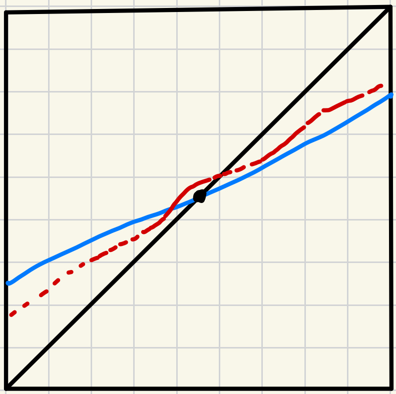
2. Show that $x \mapsto E_x^{s/u}$ is continuous in the definition of Anosov diffeo. Hence $\dim E^{s/u}$ is constant

3. (Adapted metric) Let f Anosov. For $K \in (\lambda, 1)$ and $N > 1$, define $\langle \cdot, \cdot \rangle_{new}$ by:

$$\begin{cases} \langle v_1, v_2 \rangle_{new} = \sum_{n=0}^{N} K^{2n} \cdot \langle df^n v_1, df^n v_2 \rangle, & v_1, v_2 \in E^s \\ \langle v_1, v_2 \rangle_{new} = \sum_{n=0}^{N} K^{2n} \langle df^{-n} v_1, df^{-n} v_2 \rangle, & v_1, v_2 \in E^u \\ \langle v_1, v_2 \rangle_{new} = 0, & v_1 \in E^s, v_2 \in E^u. \end{cases}$$

Show that if N is large, then $\langle \cdot, \cdot \rangle_{new}$ is adapted.

4. Let $f: \mathbb{R} \to \mathbb{R}$ with $f(0) = 0$, $|f'(0)| < 1$. Prove that if $g \overset{C^1}{\sim} f$ then g has a fixed point $x \approx 0$ with $|g'(x)| < 1$.



5. Show that the same happens if we perturb the CAT MAP.

6. $f: M \to M$ is transitive if $\forall A, B \subset M$ open there is $n > 0$ s.t. $f^n A \cap B \neq \emptyset$.

   (a) Show that the CAT MAP is transitive.

   (b) Show that f is transitive $\Leftrightarrow \exists x$ s.t. $\overline{O(x)} = M$.

7. Let f be Anosov. Prove that:

   (a) $v \in T_x M \setminus E^s_x \Rightarrow \|df^n v\| \to \infty$ exponentially fast

   (b) $E^s_x = \{ v \in T_x M : \|df^n v\| \text{ is bounded} \}$

8. We call f expansive if $\exists\, \varepsilon > 0$ s.t.

$$d(f^n x, f^n y) < \varepsilon,\ \forall\, n \in \mathbb{Z} \implies x = y.$$

Prove that every Anosov diffeo is expansive

9. Let f Anosov. Prove that $\exists\, \delta > 0$ s.t.

$$x, y \in M,\ d(x,y) < \delta \implies W^s_\varepsilon(x) \cap W^u_\varepsilon(y) \text{ is a single point.}$$