

## WORKSHOP EXERCISES FOR TRIESTE LECTURE COURSE ON ERGODIC GEOMETRY, JULY 2024

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For the workshops, we will focus on BACKGROUND in dynamics of expanding and hyperbolic maps, and particularly the entropy theory. This will support the topic of the lectures, which will cover some of this background and focus on applications to the theory of geodesic flows. A strong grounding in the dynamics of systems such as hyperbolic toral automorphisms and expanding maps of the circle is an excellent idea to understand the context of the results we are talking about in the geodesic flow setting in the lectures. Excellent resources for self-study are available in these settings, which will allow for individuals with varying amounts of prior knowledge of the field to go at the speed most suitable for them.

We use the book ‘**Dynamical Systems by Example**’ by Barreira and Valls as a source of background exercises, which is available for free online at ITCP. The book is best used as a companion to another dynamical systems text. For those looking to study independently, excellent resources include the books ‘Foundations of Ergodic Theory’ by Viana and Oliveira; ‘Ergodic Theory and Dynamical Systems’ by Coudène; ‘Ergodic Dynamics’ by Hawkins.

For the ergodic theory of geodesic flows, ‘Equilibrium states in negative curvature’ by Paulin, Pollicott and Schapira is the closest reference to the topic of the course. Be aware that it has no exercises, and is a very advanced reference book. As an introduction to this area with exercises, I recommend the book ‘Hyperbolic flows’ by Fisher and Hasselblatt, in particular chapter 2 on Hyperbolic geodesic flow.

**Select problems based on your level and interest, and use them as a jumping off point for discussion and questions and learning the theory. You do not need to go through in order or solve every problem.** The problem selections in bold are particularly relevant for the material in the lectures, although those who do not have much familiarity with dynamics are better advised to start in chapter 1.

Recommended problems from ‘**Dynamical Systems by Example**’ by Barreira and Valls:

- (1) Chapter 1 - Selection of problems 1.1-1.22 about periodic orbits as needed.
- (2) Chapter 1 - Problems 1.23-1.26 about toral automorphisms
- (3) Chapter 1 - Problems 1.27-1.29 (optional)
- (4) Chapter 2 - Limit sets 2.1-2.3
- (5) Chapter 2 - Nonwandering sets 2.9-2.13 (and possibly 2.14-2.16)
- (6) Chapter 2 - **Topological entropy 2.30-2.36.**
- (7) Chapter 4 - Smale horseshoes and toral automorphisms 4.5-4.8, 4.22, 4.23
- (8) Chapter 5 - **Measure entropy, 6.32-6.40** (most of these statements have proof and context in Viana and Oliveira, and in Coudène)

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